

The Bond and Money Markets:
Strategy, Trading, Analysis

The Bond and Money Markets: *Strategy, Trading, Analysis*

Moorad Choudhry

BUTTERWORTH
HEINEMANN

OXFORD AUCKLAND BOSTON JOHANNESBURG MELBOURNE NEW DELHI

Butterworth-Heinemann
Linacre House, Jordan Hill, Oxford OX2 8DP
225 Wildwood Avenue, Woburn, MA 01801-2041
A division of Reed Educational and Professional Publishing Ltd

 A member of the Reed Elsevier plc group

First published 2001
© Moorad Choudhry 2001

All rights reserved. No part of this publication may be reproduced in any material form (including photocopying or storing in any medium by electronic means and whether or not transiently or incidentally to some other use of this publication) without the written permission of the copyright holder except in accordance with the provisions of the Copyright, Designs and Patents Act 1988 or under the terms of a licence issued by the Copyright Licensing Agency Ltd, 90 Tottenham Court Road, London, England W1P 0LP. Applications for the copyright holder's written permission to reproduce any part of this publication should be addressed to the publishers

The views, thoughts and opinions expressed in this book are those of the author in his individual capacity and should not in any way be attributed to JPMorgan Chase & Co, or to Moorad Choudhry as a representative, officer, or employee of JPMorgan Chase & Co

British Library Cataloguing in Publication Data

A catalogue record for this book is available from the British Library

Library of Congress Cataloguing in Publication Data

A catalogue record for this book is available from the Library of Congress

ISBN 0 7506 4677 2

Produced by ReadyText Publishing Services, Bath, Great Britain
Printed and bound in Great Britain by the Bath Press



FOR EVERY TITLE THAT WE PUBLISH, BUTTERWORTH-HEINEMANN
WILL PAY FOR BTCV TO PLANT AND CARE FOR A TREE.

For my Mum and Dad

Butterworth-Heinemann –The Securities Institute

A publishing partnership

About The Securities Institute

Formed in 1992 with the support of the Bank of England, the London Stock Exchange, the Financial Services Authority, LIFFE and other leading financial organizations, the Securities Institute is the professional body for practitioners working in securities, investment management, corporate finance, derivatives and related businesses. Their purpose is to set and maintain professional standards through membership, qualifications, training and continuing learning and publications. The Institute promotes excellence in matters of integrity, ethics and competence.

About the series

Butterworth-Heinemann is pleased to be the official **Publishing Partner** of the Securities Institute with the development of professional level books for: Brokers/Traders; Actuaries; Consultants; Asset Managers; Regulators; Central Bankers; Treasury Officials; Compliance Officers; Legal Departments; Corporate Treasurers; Operations Managers; Portfolio Managers; Investment Bankers; Hedge Fund Managers; Investment Managers; Analysts and Internal Auditors, in the areas of: Portfolio Management; Advanced Investment Management; Investment Management Models; Financial Analysis; Risk Analysis and Management; Capital Markets; Bonds; Gilts; Swaps; Repos; Futures; Options; Foreign Exchange; Treasury Operations.

Series titles

■ Professional Reference Series

The Bond and Money Markets: *Strategy, Trading, Analysis*

■ Global Capital Markets Series

The REPO Handbook

The Gilt-Edged Market

Foreign Exchange and Money Markets: *theory, practice and risk management*

For more information

For more information on **The Securities Institute** please visit their Web site:

www.securities-institute.org.uk

and for details of all **Butterworth-Heinemann Finance** titles please visit Butterworth-Heinemann:

www.bh.com

Contents

	Foreword	xix
	Preface	xxi
	About the author	xxviii
Part I	Introduction to the Bond Markets	1
Chapter 1	The Debt Capital Markets	3
	1.1 Description	4
	1.2 Bond issuers	5
	1.3 Capital market participants	6
	1.4 World bond markets	8
	1.5 Overview of the main bond markets	10
	1.6 Financial engineering in the bond markets	12
Chapter 2	Financial Markets Arithmetic	17
	2.1 Simple and compound interest	17
	2.2 The time value of money	20
	2.3 Multiple cash flows	24
	2.4 Corporate finance project appraisal	29
	2.5 Interpolation and extrapolation	31
	2.6 Measuring the rate of return	32
	2.7 Indices	34
Chapter 3	Traditional Bond Pricing	41
	3.1 Pricing a conventional bond	41
	3.2 Pricing zero-coupon bonds	45
	3.3 Clean and dirty bond prices	46
	3.4 Bond price and yield relationship	48
Chapter 4	Bond Yield Measurement	53
	4.1 Current yield	54
	4.2 Simple yield to maturity	55
	4.3 Yield to maturity	56
	4.4 Yield on a zero-coupon bond	60
	4.5 Modifying bond yields	61
	4.6 Converting bond yields	62
	4.7 Assumptions of the redemption yield calculation	64
	4.8 Holding-period yield	65
	4.9 Bonds with embedded options	65
	4.10 Index-linked bonds	71
	4.11 Yields on floating-rate bonds	75
	4.12 Measuring yield for a bond portfolio	76
	4.13 The price/yield relationship	80
	4.14 Summary	80
Chapter 5	Review of Bond Market Instruments	86
	5.1 Floating Rate Notes	86
	5.2 Inverse/Reverse floating-rate bonds	89

	5.3	Asset-backed bonds	90
	5.4	PIBS	93
	5.5	Callable bonds	94
	5.6	Index-linked bonds	98
Chapter 6		The Yield Curve	102
	6.1	Using the yield curve	102
	6.2	Yield-to-maturity yield curve	103
	6.3	The coupon yield curve	104
	6.4	The par yield curve	104
	6.5	The zero-coupon (or spot) yield curve	105
	6.6	The forward yield curve	110
	6.7	The annuity yield curve	114
	6.8	Analysing and interpreting the yield curve	114
	6.9	Interpreting the yield curve	124
	6.10	Fitting the yield curve	126
	6.11	Spot and forward rates in the market	130
	6.12	Examples, exercises and calculations	137
	6.13	Case Study: Deriving a discount function	140
	6.14	Case Study exercise: Deriving the theoretical zero-coupon (spot) rate curve	144
Chapter 7		Price, Yield and Interest Rate Risk I	155
	7.1	Revisiting the bond price/yield relationship	155
	7.2	Duration	158
	7.3	A summary of the duration measure	171
	7.4	Duration for other securities	172
Chapter 8		Price, Yield and Interest Rate Risk II	175
	8.1	Basis point value	175
	8.2	Yield value of price change	177
	8.3	Hedging using basis point value	178
	8.4	Volatility weighting for hedging	179
Chapter 9		Price, Yield and Interest Rate Risk III	182
	9.1	Convexity	182
	9.2	Summarising the properties of convexity	188
	9.3	Dispersion	189
Chapter 10		Price, Yield and Interest Rate Risk IV	194
	10.1	Yield curve changes	194
	10.2	Portfolio duration and changes in the yield curve	195
	10.3	Hedging strategy and duration	197
Part II		Government Bond Markets	201
Chapter 11		The United Kingdom Gilt Market	203
	11.1	Introduction and history	203
	11.2	Market instruments	206
	11.3	Taxation	215
	11.4	Market structure	216
	11.5	Market makers and brokers	216
	11.6	Issuing gilts	218

11.7	The DMO and secondary market trading	220
11.8	Settlement	221
11.9	Index-linked gilts analytics	223
11.10	Gilt strips	228
11.11	Zero-coupon bond trading and strategy	233
11.12	Strips market anomalies	235
11.13	Trading strategy	235
11.14	Illustration: Yield and cash flow analysis	240
11.15	Future developments in strips	242
11.16	HM Treasury and the remit of the Debt Management Office	243
11.17	Gilt derivatives and repo markets	244
11.18	The Minimum Funding Requirement	246
11.19	Developments in electronic trading	247
Chapter 12	The US Treasury Bond Market	257
12.1	The US Treasury	257
12.2	The Federal Reserve	258
12.3	Market convention	258
12.4	The Primary Market	261
12.5	The Secondary Market	262
12.6	Treasury strips	264
12.7	Inflation-protected Treasury bonds	265
12.8	Treasury repo market	266
12.9	Federal Agency bonds	267
12.10	Derivatives markets	269
12.11	Historical long-bond yields	271
Chapter 13	International Bond Markets	277
13.1	Overview of government markets	279
13.2	Germany	282
13.3	Italy	284
13.4	France	287
13.5	Japan	292
13.6	Australia	294
13.7	New Zealand	297
13.8	Canada	298
13.9	Hungary	301
13.10	South Africa	301
13.11	Egypt	303
Part III	Corporate Debt Markets	317
Chapter 14	Corporate Debt Markets	319
14.1	Introduction	320
14.2	Determinants of the development of a corporate market	322
14.3	The primary market	322
14.4	The secondary market	323
14.5	Fundamentals of corporate bonds	324
14.6	Bond security	326
14.7	Redemption provisions	328
14.8	Corporate bond risks	329
14.9	High-yield corporate bonds	332
14.10	Corporate bond offering circular	332

Chapter 15	Analysis of Bonds With Embedded Options	339
15.1	Understanding embedded option elements in a bond	339
15.2	The Binomial tree of short-term interest rates	341
15.3	Pricing callable bonds	345
15.4	Price and yield sensitivity	347
15.5	Price volatility of bonds with embedded options	349
15.6	Sinking funds	350
Chapter 16	Convertible Bonds I	357
16.1	Basic description	357
16.2	Advantages of issuing and holding convertibles	362
Chapter 17	Convertible Bonds II	364
17.1	Traditional valuation methodology	364
17.2	Fair value of a convertible bond	365
17.3	Further issues in valuing convertible bonds	371
17.4	Convertible bond default risk	372
Chapter 18	The Eurobond Market I	378
18.1	Eurobonds	378
18.2	Foreign bonds	379
18.3	Eurobond instruments	380
18.4	The issuing process: market participants	382
18.5	Fees, expenses and pricing	385
18.6	Issuing the bond	386
18.7	Covenants	388
18.8	Trust services	388
18.9	Form of the bond	390
18.10	Clearing systems	391
18.11	Market associations	392
18.12	Secondary market	393
18.13	Settlement	393
Chapter 19	Eurobonds II	394
19.1	Legal and tax issues	394
19.2	The secondary market	394
19.3	Eurobonds and swap transactions	395
Chapter 20	Warrants	396
20.1	Introduction	396
20.2	Analysis	396
20.3	Bond warrants	397
20.4	Comparison of warrants and convertibles	398
Chapter 21	Medium-term Notes	400
21.1	Introduction	401
21.2	The primary market	401
21.3	MTNs and corporate bonds	403
21.4	Issue mechanism	405
21.5	The secondary market	406
21.6	The Euro-MTN market	408

21.7	Structured MTNs	408
Chapter 22	Commercial Paper	414
22.1	Commercial Paper programmes	415
22.2	Commercial paper yields	416
Chapter 23	Preference Shares and Preferred Stock	418
23.1	The size of the market	418
23.2	Description and definition of preference shares	419
23.3	Cost of preference share capital	422
23.4	The preference share market	423
23.5	Auction market preferred stock (Amps)	423
Chapter 24	The US Municipal Bond Market	427
24.1	Description of municipal bonds	427
24.2	The municipal bond market	429
24.3	Municipal bonds credit ratings	430
24.4	Bond insurance	431
24.5	Taxation issues	431
24.6	Exotic municipal bonds	431
24.7	Municipal money market instruments	432
Chapter 25	Asset-Backed Bonds I: Mortgage-backed Securities	434
25.1	Mortgage-backed securities	434
25.2	Cash flow patterns	440
25.3	Evaluation and analysis of mortgage-backed bonds	444
Chapter 26	Mortgage-backed Bonds II	452
26.1	Basic concepts	452
26.2	Pricing and modelling techniques	452
26.3	Interest rate risk	455
26.4	Portfolio performance	457
Chapter 27	Asset-backed Securities III	459
27.1	Collateralised mortgage securities	459
27.2	Non-agency CMO bonds	465
27.3	Commercial mortgage-backed securities	467
27.4	Motor-car-backed securities	468
27.5	Credit card asset-backed securities	470
27.6	Static spread analysis of asset-backed bonds	473
27.7	Conclusion	475
Chapter 28	Collateralised Debt Obligations	478
28.1	An overview of CDOs	478
28.2	Relative value analysis	484
28.3	Credit derivatives	485
Chapter 29	High-yield Bonds	489
29.1	Growth of the market	489
29.2	High-yield securities	490
29.3	High-yield bond performance	492

Chapter 30	Corporate Bonds and Credit Analysis	496
30.1	Credit ratings	496
30.2	Credit analysis	498
30.3	Industry-specific analysis	502
30.4	The art of credit analysis	503
Part IV	The Money Markets	505
Chapter 31	The Money Markets	507
31.1	Introduction	507
31.2	Securities quoted on a yield basis	508
31.3	Securities quoted on a discount basis	512
31.4	Foreign exchange	517
Chapter 32	Banking Regulatory Capital Requirements	526
32.1	Regulatory issues	526
32.2	Capital adequacy requirements	527
32.3	Proposed changes to Basle rules	529
Chapter 33	Asset and Liability Management	532
33.1	Introduction	532
33.2	The ALM desk	534
33.3	Liquidity and interest-rate risk	536
33.4	Critique of the traditional approach	543
33.5	Securitisation	544
Chapter 34	The Repo Markets	550
34.1	Development of the repo market	550
34.2	Introduction to repo	551
34.3	Uses and economic functions of repo	552
34.4	Repo mechanics	554
34.5	Other repo structures	556
34.6	Pricing and margin	558
34.7	Risks in dealing repo	560
34.8	Legal issues	562
34.9	Accounting, Tax and capital issues	563
34.10	Market participants	565
34.11	The United Kingdom gilt repo market	565
34.12	Market structure	567
34.13	Trading patterns	568
34.14	Open market operations	570
34.15	Gilts settlement and the CREST service	571
34.16	Gilt repo Code of Best Practice	572
34.17	Trading approach	572
34.18	Electronic repo trading	579
34.19	Repo netting	580
34.20	The implied repo rate and basis trading	582
34.21	Repo market structures	590
34.22	Central bank repo and overseas markets	593

Chapter 35	Money Markets Derivatives	599
35.1	Forward rate agreements	599
35.2	FRA mechanics	600
35.3	Long-dated FRAs	606
35.4	Forward contracts	607
35.5	Short-term interest rate futures	607
Part V	Risk Management	619
Chapter 36	Risk Management	621
36.1	Introduction	621
36.2	Risk management	622
36.3	Non-VaR measure of risk	624
Chapter 37	Bank Risk Exposure and Value-at-Risk	625
37.1	Value-at-Risk	625
37.2	Explaining Value-at-Risk	627
37.3	Variance-covariance Value-at-Risk	628
37.4	Historical VaR methodology	635
37.5	Simulation methodology	635
37.6	Value-at-risk for fixed interest instruments	636
37.7	Derivative products and Value-at-Risk	640
37.8	Stress testing	643
37.9	Value-at-Risk methodology for credit risk	645
Chapter 38	Interest-rate Risk and a Critique of Value-at-Risk	661
38.1	Interest-rate risk	661
38.2	Comparison with traditional duration-based risk measurement	663
38.3	A critique of Value-at-Risk	663
Part VI	Derivative Instruments	667
Chapter 39	Swaps I	669
39.1	Introduction	669
39.2	Interest rate swaps	672
39.3	Relationship between interest-rate swaps and FRAs	680
39.4	Generic swap valuation	680
39.5	Zero-coupon swap pricing	681
39.6	Non-vanilla interest-rate swaps	689
39.7	Cancelling a swap	692
39.8	Swaptions	692
39.9	Cross-currency swaps	694
39.10	Credit risk	696
Chapter 40	Swaps II	705
40.1	Using swaps	705
40.2	Hedging an interest-rate swap	707
40.3	The convexity bias	711
40.4	Swaps netting	716

Chapter 41	Bond Futures	720
41.1	Introduction	720
41.2	Futures pricing	723
41.3	Hedging using futures	726
41.4	The margin process	730
Chapter 42	Options I	734
42.1	Introduction	734
42.2	Option instruments	738
42.3	Options and payoff profiles	739
42.4	Option pricing parameters	740
Chapter 43	The Dynamics of Asset Prices	745
43.1	The behaviour of asset prices	745
43.2	Stochastic calculus models: Brownian motion and Itô calculus	752
43.3	Perfect capital markets	755
Chapter 44	Options II: Pricing and Valuation	762
44.1	Option pricing	762
44.2	Pricing derivative securities	763
44.3	Simulation methods	770
44.4	Valuation of bond options	771
44.5	Interest-rate options and the Black model	772
44.6	Critique of the Black-Scholes model	773
44.7	The Barone-Adesi and Whaley model	774
44.8	Valuation of American options	775
44.9	Describing stochastic volatilities	778
44.10	A final word on (and summary of) the models	780
Chapter 45	Options III: The Binomial Pricing Model	788
45.1	The binomial option pricing model	788
45.2	The binomial approach for interest-rate options	790
45.3	Comparison with B-S model	791
Chapter 46	Options IV: Pricing Models for Bond Options	794
46.1	Introduction	794
46.2	Pricing bond options	795
46.3	Using option models to price corporate bonds	797
Chapter 47	Options V –Managing an Option Book	801
47.1	Behaviour of option prices	801
47.2	Measuring option risk: The Greeks	802
47.3	The option smile	809
Chapter 48	Options VI: Strategies and Uses	812
48.1	Introduction	812
48.2	Spreads	812
48.3	Volatility trades	816
48.4	Collars, caps and floors	820
48.5	Using options in bond markets	822

Chapter 49	Options VII: Exotic Options	832
49.1	Options with modified contract terms	832
49.2	Path-dependent options	833
49.3	Multi-asset options	837
49.4	Pricing and hedging exotic options	838
49.5	Using exotic options: case studies	840
Part VII	Approaches to Trading and Hedging	845
Chapter 50	Approaches to Trading and Hedging	847
50.1	Futures trading	848
50.2	Yield curves and relative value	851
50.3	Yield spread trades	855
50.4	Hedging bond positions	856
50.5	Introduction to bond analysis using spot rates and forward rates in continuous time	858
Part VIII	Advanced Fixed Income Analytics	865
Chapter 51	Interest-rate Models I	873
51.1	Introduction	873
51.2	Interest-rate processes	874
51.3	One-factor models	876
51.4	Arbitrage-free models	880
51.5	Fitting the model	884
51.6	Summary	886
Chapter 52	Interest-rate Models II	888
52.1	Introduction	888
52.2	The Heath–Jarrow–Morton model	888
52.3	Multi-factor term structure models	892
52.4	Assessing one-factor and multi-factor models	895
Chapter 53	Estimating and Fitting the Term Structure	901
53.1	Introduction	901
53.2	Bond market information	903
53.3	Curve-fitting techniques: parametric	904
53.4	The cubic spline method for estimating and fitting the yield curve	907
53.5	The Anderson–Sleath evaluation	910
Chapter 54	Advanced Analytics for Index-Linked Bonds	918
54.1	Index-linked bonds and real yields	918
54.2	Duration and index-linked bonds	919
54.3	Estimating the real term structure of interest rates	921
Chapter 55	Analysing the Long Bond Yield	926
55.1	Theories of long-dated bond yields	926
55.2	Pricing a long bond	929
55.3	Further views on the long-dated bond yield	931
55.4	Analysing the convexity bias in long-bond yields	932

Chapter 56	The Default Risk of Corporate Bonds	934
56.1	Corporate bond default spread risk	934
56.2	Default risk and default spreads	935
Part IX	Portfolio Management	939
Chapter 57	Portfolio Management I	941
57.1	Generic portfolio management	941
57.2	Active bond portfolio management	943
Chapter 58	Portfolio Management II	948
58.1	Overview	948
58.2	Structured portfolio strategies	952
58.3	Immunisation	955
58.4	Extending traditional immunisation theory	959
58.5	Multiple liabilities immunisation	960
Chapter 59	Portfolio Management III	966
59.1	Introduction	966
59.2	Performance evaluation	966
Chapter 60	Portfolio Yield Measurement	969
60.1	Portfolio yield	969
60.2	Value-weighted portfolio yield	970
Chapter 61	Bond Indices	972
61.1	Overview	972
61.2	Maturity of an index	973
61.3	Responding to events	974
61.4	Composition of the index	974
61.5	Calculation of index value	975
Chapter 62	International Investment	978
62.1	Arguments for investing in international bonds	978
62.2	International portfolio management	979
Part X	Technical Analysis	981
Chapter 63	Technical Analysis	983
63.1	Introduction	983
63.2	Trading market profile	985
63.3	Dow theory	986
63.4	Chart construction	987
63.5	Trend analysis	989
63.6	Reversal patterns	995
63.7	Continuation patterns	1000
63.8	Point and figure charting	1003
63.9	Mathematical approaches	1005
63.10	Contrary opinion theory	1009
63.11	Volume and open interest	1009

63.12	Candlestick charting	1010
63.13	Elliott wave theory	1017
63.14	Stop losses	1019
63.15	Concluding remarks	1019
Part XI	Introduction to Credit Derivatives	1027
Chapter 64	Introduction to Credit Derivatives	1029
64.1	Overview	1029
64.2	Pricing	1032
64.3	Regulatory issues	1036
Chapter 65	Credit Derivatives II	1045
65.1	Theoretical pricing models	1045
65.2	Credit spread options	1047
65.3	Default options pricing	1048
Chapter 66	Credit Derivatives III: Instruments and Applications	1052
66.1	Credit default swaps	1052
66.2	Total return swap	1053
66.3	Credit options	1054
66.4	Credit-linked notes	1055
66.5	Applications	1055
Part XII	Emerging Bond Markets	1059
Chapter 67	Emerging Bond Markets and Brady Bonds	1061
67.1	Overview	1061
67.2	Key features	1063
67.3	Trading in the emerging bond markets	1064
67.4	Brady bonds	1067
Chapter 68	Emerging Bond Markets II	1073
68.1	Analysing of relative value	1073
68.2	Selected emerging bond markets	1075
Concluding Remarks		1081
Glossary		1085
Index		1107

Foreword

The world's bond markets have a value of more than \$30 trillion. They form a vital source of finance both for governments and companies. For investors they provide an invaluable home for capital, offering a range of risks and rewards, and yet they are little understood outside the arcane spheres in which bankers and brokers move. Further, the bond markets have grown enormously both in size and complexity in the last quarter of the twentieth century. Long gone are the days when Galsworthy's fictional Forsyte family were content to lodge their fortunes in 'Consols', earning interest at 3 per cent a year, secure in the belief that both their capital and income was safe.

The development of the international bond markets, the advent of swaps and the arrival of sophisticated computer bond trading programs, are only a few of the changes over the last 25 years which have made the bond markets more intricate. More volatile interest rates and the abandonment of fixed exchange rates have introduced greater turbulence into the markets, requiring ever more agility from participants. The relative decline of bond issuance by governments –for the most part, at least among developed nations, regarded as highly unlikely to default on their debts –and the expansion of borrowing by corporate entities, has added a greater element of credit risk to the equation.

Indeed, a recent substantial and authoritative report by the investment bank Merrill Lynch concludes that the traditional risk-free asset presents challenges to investors and issuers alike. The report¹ is a most comprehensive analysis, which shows that the world bond market has experienced dramatic growth in its size and major shifts in its structure. The report highlights the extent of change in the current bond market world-wide, the US Fixed Income market, the divergent trends in government and corporate bonds and emerging market local currency debt securities.

Some of the most significant facts and developments in recent years have included the following:

- the global bond market maintains a robust rate of growth, while undergoing a dramatic shift in its composition;
- the world bond market grew 8.1% in 1999, due to a sharp increase in the non-government debt arena;
- the size of the world bond market at the end of 1999 was estimated to be US\$ 31.1 trillion;
- government bonds continue to see their share of the world market fall, to a new low of 54% at the end of 1999, while corporate bonds and Eurobonds volumes rose to 42% of the world bond market capitalisation at the same time;
- bonds in the world's three largest currencies (US\$, euro and Japanese yen) account for 88% of the total size of the world bond market;
- emerging market local currency debt securities are estimated at \$1.2 trillion as of year-end 1999.

It is thus a particularly apposite time for the publication of this book. *The Bond and Money Markets* provides a wide-ranging and detailed examination of the global markets for debt capital. Starting from first principles, and proceeding to explain the technicalities of bond valuation and trading strategies, this book will be of particular benefit to newcomers to the subject as well as to more advanced students, and experienced practitioners.

Moorad Choudhry has the experience of a number of years working in the bond markets, which has given him the knowledge to write this book. I have known Moorad since 1992, when we worked together at Hoare Govett Securities Limited. Although initially a sterling bond house, it later became an UK gilt-edged market maker, thus becoming one of the few houses to cover all sterling bonds. Moorad traded the short-dated gilt-edged market as well as the Treasury book, a possibly almost unique experience and exposure to money markets, repo, off-balance sheet and bond trading all at the same time. From there he went on to proprietary trading at Hambros Bank Limited. This background is evident in the style and accessibility of his book, which has been written from a practitioner viewpoint, with an emphasis on clarity of approach. Readers will find much of interest and value within the following pages.

Sean Baguley
Director, Merrill Lynch

¹ *Size and structure of the World Bond Market: The Decline of the Risk-Free Asset*, Merrill Lynch, April 2000.

Preface

This book is about the bond and money markets. This means that it is about the instruments used in the world's debt capital markets, because bonds are debt capital market products. One could stock an entire library with books about the fixed income markets and about how bonds are traded and analysed. In such a library one would also expect to find related books dealing with derivative instruments, bond portfolio management, technical analysis, financial market mathematics, yield curve modelling and so on. The subject matter is indeed a large one. In bringing together all the different strands into one volume one is forced to sacrifice some of the depth afforded by dedicated texts. However the purpose of this book is to present a fairly comprehensive and in-depth look at all aspects of the debt capital markets. Therefore while we start right from the beginning we hope, by the book's conclusion, to have presented the reader with most of the information required to be fully conversant with bonds and related derivatives, whether this is the terminology, analytical techniques, financial mathematics, or trading strategy.

The bond markets, also known as the fixed interest or *fixed income* markets, are an important part of the global financial markets, and a vital conduit through which capital is raised and invested.¹ Over the last two decades the growth in trading volumes has been accompanied by the introduction of ever more sophisticated financial engineering techniques, such that the bond markets today are made up of a large variety of structures. Banks can tailor packages to suit the most esoteric of requirements for their customers, so that bond cash flows and the hedging instruments available for holders of bonds can be far removed from the conventional fixed interest instruments that originally made up the market. Instruments are now available that will suit the needs of virtually all users of the financial markets, whether they are investors or borrowers.

The purpose of this book is to provide an introductory description and analysis of the bond markets as a whole. However we seek to leave the reader with sufficient information and worked examples to enable him or her to be at ease with all the different aspects of the markets. Hence we begin by considering conventional bonds and elementary bond mathematics, before looking at the array of different instruments available. This includes an overview of off-balance sheet instruments and their uses. We also consider the analytical techniques used by the markets, valuation of securities and basic trading and hedging strategy. We then develop the concepts further and look at constructing and managing portfolios, speculation and arbitrage strategies and hedging strategies. The basic principles apply in all bond markets around the world, but there are details differences across countries and currencies and so we also provide brief descriptions of some of the major bond markets. The exception to this is the United Kingdom government bond market, which is called the Gilt market, and which we look at in some detail.

One of the objectives behind writing this book was to produce something that had a high level of application to real-world situations, but maintained analytical rigour. We hope this objective has been achieved. There is no shortage of books in the market that are highly academic, perhaps almost exclusively so. Certain texts are essentially a collection of advanced mathematics. We have attempted to move seamlessly between academic principles and actual applications. Hence this book seeks to place every issue in context, and apply the contents to real-world matter. Where possible this is backed up by worked examples and case studies. Therefore the aim of this book is to be regarded as both an academic text as well as a practical handbook.

The capital markets

Capital markets is the term used to describe the market for raising and investing finance. The economies of developed countries and a large number of developing countries are based on financial systems that contain investors and borrowers, *markets* and trading arrangements. A market can be one in the traditional sense such as an exchange where *financial instruments* are bought and sold on a trading floor, or it may refer to one where participants deal with each other over the telephone or via electronic screens. The basic principles are the same in

¹ The term "fixed income" is something of a misnomer. Originally bonds were referred to as fixed income instruments because they paid a fixed rate of interest on the nominal value or "face amount" of the bond. In the sterling markets bonds were called "fixed interest" instruments. For some time now many instruments in the market have not paid a fixed coupon, for example floating-rate notes and bonds where the pay-off is linked to another reference rate or index. Although the term is still used, the fixed income markets are in reality the entire debt capital markets, and not just bonds that pay a fixed rate of interest.

any type of market. There are two primary users of the capital markets, lenders and borrowers. The source of lenders' funds is, to a large extent, the personal sector made up of household savings and those acting as their investment managers such as life assurance companies and pension funds. The borrowers are made up of the government, local governments and companies (called corporates). There is a basic conflict in the financial objectives of borrowers and lenders, in that those who are investing funds wish to remain *liquid*, which means they have easy access to their investments. They also wish to maximise the return on their investment. A corporate on the other hand, will wish to generate maximum net profit on its activities, which will require continuous investment in plant, equipment, human resources and so on. Such investment will therefore need to be as long-term as possible. Government borrowing as well is often related to long-term projects such as the construction of schools, hospitals and roads. So while investors wish to have ready access to their cash and invest short, borrowers desire as long-term funding as possible. An economist referred to this conflict as the "constitutional weakness" of financial markets (Hicks 1946), especially when there is no conduit through which to reconcile the needs of lenders and borrowers. To facilitate the efficient operation of financial markets and the price mechanism, intermediaries exist to bring together the needs of lenders and borrowers. A bank is the best example of this. Banks accept deposits from investors, which make up the *liability* side of their balance sheet, and lend funds to borrowers, which form the *assets* on their balance sheet. If a bank builds up a sufficiently large asset and liability base, it will be able to meet the needs of both investors and borrowers, as it can maintain liquidity to meet investors' requirements as well as create long-term assets to meet the needs of borrowers. The bank is exposed to two primary risks in carrying out its operations, one that a large number of investors decide to withdraw their funds at the same time (a "run" on the bank), or that a large number of borrowers go bankrupt and default on their loans. The bank in acting as a financial intermediary reduces the *risk* it is exposed to by spreading and pooling risk across a wide asset and liability base.

Corporate borrowers wishing to finance long-term investment can raise capital in various ways. The main methods are:

- continued re-investment of the profits generated by a company's current operations;
- selling shares in the company, known as equity capital, equity securities or *equity*, which confirm on buyers a share in ownership of the company. The shareholders as owners have the right to vote at general meetings of the company, as well as the right to share in the company's profits by receiving dividends;
- borrowing money from a bank, via a bank loan. This can be a short-term loan such as an overdraft, or a longer term loan over two, three, five, years or even longer. Bank loans can be at either a fixed or more usually, variable rate of interest;
- borrowing money by issuing debt securities, in the form of *bonds* that subsequently trade in the debt capital market.

The first method may not generate sufficient funds, especially if a company is seeking to expand by growth or acquisition of other companies. In any case a proportion of annual after-tax profits will need to be paid out as dividends to shareholders. Selling further shares is not always popular amongst existing shareholders as it dilutes the extent of their ownership; there are also a host of other factors to consider including if there is any appetite in the market for that company's shares. A bank loan is often inflexible, and the interest rate charged by the bank may be comparatively high for all but the highest quality companies. We say comparatively, because there is often a cheaper way for corporates to borrow money: by tapping the bond markets. An issue of bonds will fix the rate of interest payable by the company for a long-term period, and the chief characteristic of bonds –that they are *tradeable* – makes investors more willing to lend a company funds.

Bond markets play a vital and essential role in raising finance for both governments and corporations. In 1998 the market in dollar-denominated bonds alone was worth over \$11 trillion, which gives some idea of its importance. The basic bond instrument, which is a loan of funds by the buyer to the issuer of the bond, in return for regular interest payments up to the termination date of the loan, is still the most commonly issued instrument in the debt markets. Nowadays there is a large variety of bond instruments, issued by a variety of institutions. An almost exclusively corporate instrument, the international bond or Eurobond, is a large and diverse market. In 1998 the size of the Eurobond market was over \$1 trillion.

In every capital market the first financing instrument that was ever developed was the bond; today in certain developing economies the government bond market is often the only liquid market in existence. Over time as

financial systems develop and corporate debt and equity markets take shape, bond market retain their importance due to their flexibility and the ease with which (in theory!) transactions can be undertaken. In the advanced financial markets in place in developed countries today, the introduction of *financial engineering* techniques has greatly expanded the range of instruments that can be traded. These products include instruments used for *hedging* positions held in bonds and other *cash* products, as well as meeting the investment and *risk management* needs of a whole host of market participants. The debt capital markets have been and continue to be tremendously important to the economic development of all countries, as they have been the form of *intermediation* that allowed governments and corporates to finance their activities. In fact it is difficult to imagine long-term capital intensive projects such as those undertaken by say, petroleum, construction or aerospace companies, as well as sovereign governments, taking place without the existence of a debt capital market to allow the raising of vital finance.

Efficient markets

We often come across the term ‘free market’, and economists refer to ‘the price mechanism’. The role of the market in an economy is to allocate resources between competing interests in the most efficient way, and in a way that results in the resources being used in the most productive way. Where this takes place the market is said to be *allocatively efficient*. The term *operationally efficient* is used to describe a market where the *transaction costs* involved in trading are set competitively. Intermediaries in the capital markets do indeed determine their prices in relation to the competition, and because they depend on profits to survive there is always a cost associated with transacting business in the market. A market is described as *informationally efficient* if the price of any asset at any time fully reflects all available information that is available on the asset. A market that is allocatively, operationally and informationally efficient at the same time is *perfectly efficient*.

The concept of the efficient market was first described by Fama (1970). The *efficient markets hypothesis* is used to describe a market where asset prices fully reflect all available information. There are three types of the efficient markets hypothesis, which are:

- the *weak form*, which describes a situation where market prices reflect only historical data on the asset or security in questions;
- the *semi-strong form*, where prices reflect all publicly available information;
- the *strong form*, where prices reflect all known information, whether this information is publicly known or not.

The *weak-form efficient markets hypothesis* states that current market prices for assets fully reflects all information contained in the past history of asset prices. This implies therefore that historical prices provide no information on future prices of value to an investor seeking to make excess returns over the returns being earned by the market. Empirical evidence from market trading suggests that markets are indeed weak-form efficient, and that security prices incorporate virtually instantaneously all information reflected in past prices to enable investors to acquire any advantage.

The *semi-strong efficient markets hypothesis* states that current asset prices fully reflect all publicly available information about markets. If this is correct it means that any new information entering the public domain is incorporated almost instantaneously into the current price of the relevant security.² Once the security price has reacted to the new information there will be no more price changes as a result of that information. If markets are semi-strong efficient then this means that an investor who waits for the release of data before deciding which way to trade will be too late to make any gains. The evidence suggests that markets are also semi-strong efficient, and in fact security prices often change *before* the official release of information: this is because the markets have anticipated the content of the new data, through reading, say, media or brokers’ reports, and have ‘priced in’ the information accordingly. An example of this is where a central bank is expected to move interest rates a certain way; if the markets anticipate interest rates to be lowered and this view subsequently proves to be correct, the change in asset prices is much less than if the markets had guessed wrongly or were not expecting any change at all. So markets are not only weak-form but also semi-strong form efficient, and operating an investment policy of reacting to publicly available information will not generate returns that exceed those of the market itself.

² If it is macro-economic information the relevant securities could be all the stocks on the exchange, including the government bonds. If it is company specific information then it will probably be only that company’s shares, or companies in the same sector.

The *strong-form efficient markets hypothesis* states that securities prices reflect all known information about the securities and the market, including information that is available only privately. If this is true it implies that market prices respond so quickly that an investor with private (that is, *inside*) information would not be able to trade and generate excess returns above the market rate. This will not usually be the case, since someone armed with inside information can usually generate excess profits. However insider trading is illegal in most countries, so this suggests that strong-form inefficiency exists only through illegal activity. In recent years evidence has indicated that markets may be strong-form efficient as well, in the activities of fund managers. Where a portfolio of assets is actively managed by a fund manager with the objective of outperforming the market but does not, it indicates that even the possession of privately held information is insufficient to generate excess returns. We say “private” because fund managers undertake a large amount of research on companies and markets, the results of which are not available publicly. Over the years certain “active” fund managers have been much criticised for failing to outperform or even underperforming, the market. This has resulted in the popularity of “passive” fund managers, who simply structure their portfolios to replicate the constituents of the market, hoping to match overall market performance.³ So the growth of passive fund managers would seem to indicate that more and more investors believe markets to be strong-form efficient, and that it is impossible, over the long-term, to outperform the market.

We hope that this initial discussion on capital markets and market efficiency has set the scene for the discussions that follow. It is always worth keeping in mind the context within which the bond markets operate, and that debt capital trading exists in order to facilitate the efficient allocation of resources.

Intended audience

This book is aimed at a wide readership, from those with little or no previous understanding of or exposure to the bond markets to experienced practitioners. The subject matter is wide ranging and this makes the book useful for undergraduate and postgraduate students on finance or business courses. The second half of the book will be valuable for advanced level students and first-year researchers. Undergraduate students are recommended to tackle the book after initially studying the principles of finance, however the basic concepts required (such as present and future value) are covered and serve to make a complete volume. While most of the mathematics assumes a knowledge of basic algebra, some of the contents, particularly in the chapters dealing with derivatives and fixed income analytics, will require slightly higher level mathematical ability. It is not necessary to have degree-level or even A-level maths in order to understand the basic principles; however those with only elementary maths may find some of the chapters, particularly those on yield curve modelling, somewhat difficult. Complete beginners may wish to review first an elementary text on financial market mathematics. Nevertheless this book is intended to serve as a complete text, and takes readers from the first principles to advanced analysis. Note that by this we mean analysis of the bond and related derivatives markets; the budding rocket scientists among you may wish to consider books specifically concerned with say, option pricing, stochastic calculus or programme trading. For students wishing to enter a career in the financial services industry this book has been written to provide sufficient knowledge and understanding to be useful in their first job and beyond, thus enabling anyone to hit the ground running. It is also hoped that the book remains useful as a reference handbook.

The book is primarily aimed at people who work in the markets, including front office, middle office and back office banking and fund management staff who are involved to some extent in fixed interest markets. This includes traders, salespersons, money markets dealers, fund managers, stockbrokers and research analysts. Others including corporate and local authority treasurers, risk management personnel and operations staff will also find the contents useful, as will professionals who work in structured finance and other market sectors, such as accountants, lawyers and corporate financiers. As a source of reference the book should be valuable reading for management consultants and financial sector professionals, such as tax, legal and corporate finance advisors, and other professionals such as financial sector auditors and journalists.

The book is also aimed at postgraduate students and students sitting professional exams, including MBA students and those specialising in financial markets and financial market economics. Undergraduate students of business, finance or the securities markets will hopefully find the book to be a useful source of reference, while practitioners sitting the exams of various professional bodies may observe that there is much useful practical information that will help them to apply their studies to their daily work.

³ The rise of so-called “tracker” funds.

Comments on the text are welcome and should be sent to the author care of Butterworth-Heinemann. We apologise for any errors that are lurking in the text, and would appreciate being made aware of any that the reader might find. The author also welcomes constructive suggestions for improvement which we hope to incorporate in a second edition.

Organisation of the book

This book is organised into 12 parts. Each part introduces and then develops a particular aspect of the debt capital markets. Part I is the introduction to bonds as a debt market instrument. The ten chapters in Part I cover the basics of bonds, bond pricing and yield measurement, and interest-rate risk. There is also an initial look at the different types of bond instruments that trade in the market. For beginners, there is also a chapter on financial markets mathematics. Part II looks in detail at two government bond markets, the United Kingdom gilt market and the United States Treasury market. There is also a chapter looking briefly at selected government bond markets around the world. The type of subjects covered include market structure, the way bonds are issued and specific detail on the structure of the different markets.

The seventeen chapters in Part III look in some detail at the corporate debt markets. This sector of the bond markets is extremely diverse, and it is often in corporate markets that the latest and most exciting innovations are found. Some of the instruments used in the corporate markets demand their own particular type of analysis; to this end we review the pricing and analytics of callable bonds, asset-backed bonds and convertibles, among others. There is also a chapter on credit analysis.

In Part IV we review the money markets. The money markets are part of the debt markets and there is a relationship between the two, although as we shall see the money market yield curve sometimes trades independently of the bond yield curve. We cover in detail some related issues, which would probably be at home more in a book about banking than bond trading; these include bank capital requirements, asset and liability management (ALM) and the bond repurchase or *repo* market. Derivative instruments such as futures contracts play an important part in the money markets, and it was decided to include the chapter on money market derivatives in Part IV rather than in the section on derivatives, as it was felt that this would make Part IV complete in its own right.

In the capital markets and banking generally, risk management is a keenly-debated topic. A book dealing with capital markets trading would not be complete without a look at this topic, which is considered in Part V. We also cover one of the main risk management tools used today, the measurement methodology known as *value-at-risk*.

Part VI is a comprehensive review of derivative instruments. There are separate chapters on futures, swaps and options. Readers who have had only an introduction to this subject may find some of the chapters a little trying, particularly those dealing with stochastic processes and option pricing. We recommend perseverance, as the subject has been reviewed and summarised in a way that should be accessible to most, if not all. The mathematics has been kept to a minimum, and in most cases proofs and derivations are taken as given and omitted. The interested reader is directed to relevant texts that supply this detail in a bibliography at the end of each chapter.

Part VII is composed of a single chapter only, which deals with elementary trading and hedging strategy.

Part VIII on advanced fixed income analytics is the author's favourite and deals with a particularly exciting subject, interest-rate models and yield curve modelling. The main models that have emerged from leading academic writers are introduced, explained and summarised. We also cover fitting the yield curve, and there are additional chapters dealing with advanced analytics regarding index-linked bonds and the pricing of long-dated bonds.

The remainder of the book deals with related topics. In Part IX we consider portfolio management, essentially only the main strategies and techniques. There is also a chapter on constructing bond indices. Part X is another one that contains just one chapter; it sits on its own as it deals with technical analysis or "charting". In Part XI we introduce credit derivatives, which are relatively new instruments but are rapidly becoming an important part of the bond markets. The content is introductory however, and in fact there are a number of excellent texts on credit derivatives appearing in the market. The final part of the book looks at emerging markets and Brady bonds, and the additional considerations involved in investing across international markets. We conclude with a look at some likely future developments.

Study materials

Where possible the main concepts and techniques have been illustrated with worked examples and case studies. The case studies are examples of actual real-world happenings at a number of investment banks.

Questions and exercises are provided at the end of each chapter, which are designed to test readers' understanding of the material. Most of the questions can be answered using the content of the preceding chapters. Answers to the questions are available from the publishers for those involved in a teaching or lecturing capacity. Where appropriate a bibliography and list of selected references is provided so that the interested reader has a starting point for further research.

Some of the content in this book has been used to form part of bond market courses taught at a number of professional bodies and teaching institutions. This includes the material on the introduction to bonds, the gilt market, the repo market, value-at-risk, and advanced fixed income analytics. For these topics a number of Microsoft PowerPoint slides are available for use as teaching aids, and may be downloaded from the author's Web site at www.mchoudhry.co.uk. This Web site also contains details of training courses that are available on advanced bond market topics, run by the author and his associates.

Acknowledgements

As you might expect with a work such as this, various individuals have helped out both before and during the endeavour and I'd like to take this opportunity to thank them. First up there's Sherif who provided all that IT consulting advice, especially with regard to the kit this book was written on. Thanks, I really appreciated it. I've no benchmark to compare you with this time though, no Maurice Deebank, but I'm sure the advice was tops. And by the way, your work on "The Day After Tomorrow" was your best ever, it brings a smile to my face every time I hear it. I am equally indebted to Anuk Teasdale for her help with the graphics and illustrations used in the text, and also for her example of an hypothetical bond offering circular. John Paul George Bank indeed! Why not Ringo? Without her help many of the chapters would not have been possible.

My very best, as always, to Olga the Scottish wildcat, both elusive and exclusive...

I would like to thank Martin Barber for his continued help and support, and for inspiring me to do better. Thanks for everything Barberman, and also for agreeing with me that Mark Ramprakash should be England captain. For reviewing the book's manuscript, making constructive comments for improvement and being general all-round good eggs I'd like to thank Rod Pienaar at Deutsche Bank, and Richard Pereira at Dresdner Kleinwort Wasserstein, two top lads, and John Lenton, a true gent, excellent teacher and top Treasury man, latterly with King & Shaxson discount house and Woolwich plc. In particular Richard helped out with some of the more complex mathematics and structures, particularly the chapters on option pricing, yield curve modelling and stochastic calculus, and this was much appreciated. Any remaining errors are of course my own.

A very special thank you and best wishes to Sean Baguley, a truly splendid chap, for all his help in the past (including recruiting me into the market!) and for writing the excellent foreword.

For all kinds of assistance over the years I'd like to thank Derek Taylor at King & Shaxson Bond Brokers Limited –who knows everything there is to know about the sterling market, thanks very much Del-boy for all your help, (by the way, say hello to David Franklin for me, I did enjoy the morning joust with him and his team), and Ray Bieber at Garban ICAP, another sterling markets expert.

I'd like to thank Martin Davies for his help during the time he was at GNI, and later when he was at Yamaichi Securities, when together we were "the dream team," especially as I've since lost touch with him. If you're reading this Martin, please give me a call. (I'm still hoping for that invite to go and watch the Palace!) Thanks also to the head of GNI's gilts booth on the LIFFE floor in those days, Big John Howell, the chap in their short sterling booth, Ed Hardman, and Mr Tony Beard (now retired, previously at King & Shaxson discount house). Thanks to Ian Lincoln, now at Tullet & Tokyo I believe, for starting the trend in the sterling market of calling me "the Doctor", years before I ever started research towards my PhD! He was taking the mick, I'm sure...

For academic help in the past and currently I'd like to thank Matthew McQueen at the Department of Economics, University of Reading, Kerstin Barkman, Vicki Randall and Robin Theobald at the University of Westminster and Dr Ian Turner at Henley Management College. Mr McQueen especially first got me interested in writing so he's ultimately responsible! Thanks to Dr Christine Oughton at Birkbeck College, University of London for helping me to introduce some academic rigour into my research work.

A special thanks to Mike Cash and Nicki Kear, two very pleasant and charming individuals, and everyone at Butterworth-Heinemann. A big thank you to Graham Douglas, or ‘Captain Quirk’, who did the editorial project management, a mammoth task. He’s a perfectionist in the George Martin mould. In addition, Graham gets my thanks for deriving the Choudhry panic function, which describes emotional state leading up to publication.⁴

My appreciation to the following people for general assistance and providing prompt permission to reproduce various items in the book: Nimish Thakker at LIFFE, Ian Collett and Alison James at Garban InterCapital, Laura MacIntyre at Bridge Telerate, Peter Thomas at Reuters, Kim Pompilii at the Bank of England, Joyce Ruby at Bridge-Telerate, Andy Albert at Asset-Backed Alert, Bill Curtin at the US Bureau of the Public Debt, Christina Papavasiliou at the Copyright Clearance Centre in the USA, Linda Nicol at Cambridge University Press, and Mark Deacon and Steve Whiting at the United Kingdom Debt Management Office. Also at the DMO, Gurminder Bhachu helped out with detail on the gilts market, for which many thanks. The people at Bloomberg were also very prompt, especially Tibor Szigeti. But a collective thanks to all who work at that impressive organisation.

Parts of this book were originally written for the bond markets course run by the Securities Institute in London. My thanks to Zena Deane at the Institute for giving me the opportunity to teach this course, and for pointing out the possibility of getting some of my material published. I must mention Stafford Bent, because I said I would!

Thanks to Phil ‘the Mod’ Broadhurst, who can write better than anyone I know. I’d like to thank big time Alan Fulling for expert football coaching, he knows how to motivate a team, and my man Tony Holloway, always inspired to aim higher. Respect. Additional thanks to two long-standing friends, Clax, and Nik Slater.

A very big thank you, best wishes and warm regards to Dr Didier Joannas, the finest rocket scientist in the markets today (its ‘Goldfinger’, remember Dids!), Peter Matthews, from whom I learnt a great deal, Tony Alderman (ditto), Peter Capel, ‘Harry’ Cross, for composing that desk song on icing stock (and who knows the long end of the curve as well as anyone I’ve seen), Kevin Gaffney, a top chap to be a junior trader to, Graham Leach and Jim Harrison (your anecdote about the jobber on the old stock exchange floor is surely the best), all of whom were formerly with ABN AMRO Hoare Govett Sterling Bonds Limited or Hoare Govett Securities Limited, for all the times that they pointed out intricacies of bond trading that one can’t learn by reading a book! As for Nigel Marsh, well, just stay away from those relative value trades! My very best wishes gentlemen.

For inspiration I’d like to thank Derek Wanless, Stephan Harris, Luke Ding, Mr Frank Fabozzi, Kal Patel, Jonathan Ingersoll, Sheldon Ross, Bruce Foxton, Martin Hewes, Andy Rourke, Mike Joyce, and Simon Barnes’ writing in *The Times*. Out in a world of excellence and emotion of their own, thanks for inspiration to Lloyd Cole, Neil Clarke, Blair Cowan, Lawrence Donegan and Stephen Irvine.

Finally thanks to you for taking the time to read this book. The bond markets are an incredibly fascinating and exciting subject, as well as being extremely dynamic, and I have had tremendous fun writing about them and all the market instruments. It is certainly a subject that one could spend endless fascinating days discussing and debating about. I hope my enthusiasm has carried over onto the pages and that, having digested the contents, the reader will carry on his or her research and knowledge gathering to greater heights. I sincerely hope that this book has contributed to a greater understanding of and familiarity with the debt capital and derivatives markets for all of you, for both the sterling markets and the global debt markets. If readers spot any errors (and there will be a fair few I’m sure!) or have any other comments do please write to me care of the publishers and let me know –I very much look forward to hearing from you.

Moorad Choudhry
Surrey, England
May 2000

⁴ $P(t, \text{Publication date}) = \lambda_{\text{Moorad}} e^{1/(\text{Publication date} - t)}$, where λ_{Moorad} is a constant.

About the author

Moorad Choudhry is a vice-president with JPMorgan in London. He began his career in 1989 at the London Stock Exchange, before joining the sterling Eurobond desk at Hoare Govett Fixed Interest. He was later employed as a gilt-edged market maker and treasury trader at ABN Amro Hoare Govett Sterling Bonds Limited, where he ran the short-dated gilt book, the gilt repo book and the sterling money markets book, and was also responsible for stock lending and interbank funding. From there he moved on to Hambros Bank Limited, where he set up and ran the Treasury division's sterling proprietary trading desk. He then worked as a strategy and risk management consultant to some of the world's leading investment banks, before joining JPMorgan in March 2000.

Moorad has an MA in Econometrics from the University of Reading and an MBA from Henley Management College. He has taught courses on bond and money markets subjects for organisations both in the City of London and abroad, including the International Faculty of Finance and FinTuition Limited, and has lectured at City University Business School. He is a Fellow of the Securities Institute and a member of the Global Association of Risk Professionals, and previously sat on the supervisory committee of the Co-operative Society's 'rainbow' credit union. He currently sits on the Securities Institute Diploma examination panel.

Moorad is currently engaged in research towards a PhD degree in financial market economics, specialising in advanced yield curve analytics, at Birkbeck College, University of London. His previous published works include *An Introduction to Repo Markets* and *The Gilt Strips Market*, both published by SI (Services) Publishing.

Moorad was born in Bangladesh but moved at an early age to Surrey in the United Kingdom, where he lives today.

References

Fama, E., *Foundations of Finance*, Basic Books, 1970.
Hicks, J., *Value and Capital*, OUP, 1946.

Part I Introduction to the Bond Markets

We begin by describing the main instruments that go to make up the bond markets. So in Part I we explain the structure of bonds, and the variety of instruments available. This includes bond pricing and yield, and an initial look at the yield curve. Chapter 6 on the yield curve is a fairly long one and looks not only at the different types of yield curve that may be encountered, but also the issue of spot and forward interest rates, and how to interpret the shape of the yield curve. The remaining four chapters consider interest-rate risk, namely duration, modified duration and convexity, and how these measures are used to analyse and manage bond market risk.

“What’s the secret, Sean?”

“Buy cheap, sell dear...!”

1 The Debt Capital Markets

Readers will be familiar with the cursory slot on evening television news, where the newscaster informs viewers where the main stock market index closed that day and where key foreign exchange rates closed at. In the United States most bulletins go one better and also tell us at what yield the Treasury long bond closed at. This is because bond prices are affected directly by economic and political events, and yield levels on certain government bonds are fundamental indicators of the economy. The yield level on the US Treasury long bond reflects the market's view on US interest rates, inflation, public sector debt and economic growth. Reporting the bond yield level reflects the importance of the bond market to a country's economy, as important as the level of the equity stock market.

Bond and shares form part of the *capital markets*. Shares are *equity capital* while bonds are *debt capital*. So bonds are a form of debt, much like how a bank loan is a form of debt. Unlike bank loans however bonds can be *traded* in a market. A bond is a debt capital market instrument issued by a borrower, who is then required to repay to the lender/investor the amount borrowed plus interest, over a specified period of time. Bonds are also known as *fixed income* instruments, or *fixed interest* instruments in the sterling markets. Usually bonds are considered to be those debt securities with terms to maturity of over one year. Debt issued with a maturity of less than one year is considered to be *money market* debt. There are many different types of bonds that can be issued. The most common bond is the *conventional* (or *plain vanilla* or *bullet*) *bond*. This is a bond paying regular (annual or semi-annual) interest at a fixed rate over a fixed period to maturity or redemption, with the return of *principal* (the par or nominal value of the bond) on the maturity date. All other bonds will be variations on this.

A bond is therefore a financial contract, in effect an IOU from the person or body that has issued the bond. Unlike shares or equity capital, bonds carry no ownership privileges. An investor who has purchased a bond and thereby lent money to an institution will have no voice in the affairs of that institution and no vote at the annual general meeting. The bond remains an interest-bearing obligation of the issuer until it is repaid, which is usually the maturity date of the bond. The issuer can be anyone from a private individual to a sovereign government.¹

There is a wide range of participants involved in the bond markets. We can group them broadly into borrowers and investors, plus the institutions and individuals who are part of the business of bond trading. Borrowers access the bond markets as part of their financing requirements; hence borrowers can include sovereign governments, local authorities, public sector organisations and corporates. Virtually all businesses operate with a financing structure that is a mixture of debt and equity finance. The debt finance almost invariably contains a form of bond finance, so it is easy to see what an important part of the global economy the bond markets are. As we shall see in the following chapters, there is a range of types of debt that can be raised to meet the needs of individual borrowers, from short-term paper issued as part of a company's cash flow requirements, to very long-dated bonds that form part of the financing of key projects. An example of the latter was the recent issue of 40-year bonds by London and Continental Railways to finance the Channel Tunnel rail link, and guaranteed by the United Kingdom government. The other main category of market participant are investors, those who lend money to borrowers by buying their bonds. Investors range from private individuals to fund managers such as those who manage pensions funds. Often an institution will be active in the markets as both a borrower and an investor. The banks and securities houses that facilitate trading in bonds in both the *primary* and *secondary* markets are also often observed to be both borrowers and investors in bonds. The bond markets in developed countries are large and *liquid*, a term used to describe the ease with which it is possible to buy and sell bonds. In emerging markets a debt market usually develops ahead of an equity market, led by trading in government *bills* and bonds. This reflects the fact that, as in developed economies, government debt is usually the largest volume debt in the domestic market and the highest quality paper available.

The different types of bonds in the market reflect the different types of issuers and their respective requirements. Some bonds are safer investments than others. The advantage of bonds to an investor is that they represent a

¹ The musician David Bowie has issued bonds backed with royalties payable from purchases of his back catalogue. Governments have issued bonds to cover expenditure from early times, such as the issue by King William in 1694 to pay for the war against France, in effect the first United Kingdom gilt issue. That was also the year the Bank of England was founded.

fixed source of current income, with an assurance of repayment of the loan on maturity. Bonds issued by developed country governments are deemed to be guaranteed investments in that the final repayment is virtually certain. In the event of default of the issuing entity, bondholders rank above shareholders for compensation payments. There is lower risk associated with bonds compared to shares as an investment, and therefore almost invariably a lower return over the longer term.

We can now look in more detail at some important features of bonds.

1.1 Description

We have said that a bond is a debt instrument, usually paying a fixed rate of interest over a fixed period of time. Therefore a bond is a collection of cash flows and this is illustrated at Figure 1.1. In our hypothetical example the bond is a six-year issue that pays fixed interest payments of $C\%$ of the *nominal* value on an annual basis. In the sixth year there is a final interest payment and the loan proceeds represented by the bond are also paid back, known as the maturity proceeds. The amount raised by the bond issuer is a function of the price of the bond at issue, which we have labelled here as the issue proceeds.

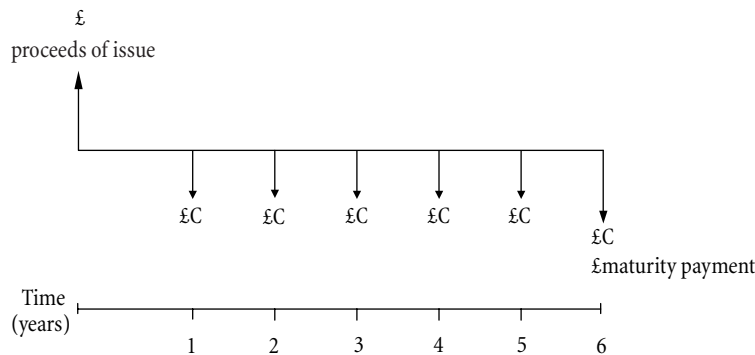


Figure 1.1: Cash flows associated with a six-year annual coupon bond.

The upward facing arrow represents the cash flow paid and the downward facing arrows are the cash flows received by the bond investor. The cash flow diagram for a six-year bond that had a 5% fixed interest rate, known as a 5% *coupon*, would show interest payments of £5 per every £100 of bonds, with a final payment of £105 in the sixth year, representing the last coupon payment and the redemption payment. Again, the amount of funds raised per £100 of bonds depends on the price of the bond on the day it is first issued, and we will look into this later. If our example bond paid its coupon on a semi-annual basis, the cash flows would be £2.50 every six months until the final redemption payment of £102.50.

Let us examine some of the key features of bonds.

- **Type of issuer** A primary distinguishing feature of a bond is its issuer. The nature of the issuer will affect the way the bond is viewed in the market. There are four issuers of bonds: sovereign governments and their agencies, local government authorities, supranational bodies such as the World Bank and corporations. Within the corporate bond market there is a wide range of issuers, each with differing abilities to satisfy their contractual obligations to lenders. The largest bond markets are those of sovereign borrowers, the government bond markets. The United Kingdom government issues bonds known as *gilts*. In the United States government bonds are known as *Treasury Notes* and *Treasury Bonds*, or simply *Treasuries*.
- **Term to maturity** The *term to maturity* of a bond is the number of years after which the issuer will repay the obligation. During the term the issuer will also make periodic interest payments on the debt. The *maturity* of a bond refers to the date that the debt will cease to exist, at which time the issuer will redeem the bond by paying the principal. The practice in the market is often to refer simply to a bond's "term" or "maturity". The provisions under which a bond is issued may allow either the issuer or investor to alter a bond's term to maturity after a set notice period, and such bonds need to be analysed in a different way. The term to maturity is an important consideration in the make-up of a bond. It indicates the time period over which the bondholder can expect to receive the coupon payments and the number of years before the principal will be paid in full. The

bond's *yield* is also depends on the term to maturity. Finally, the price of a bond will fluctuate over its life as yields in the market change and as it approaches maturity. As we will discover later, the *volatility* of a bond's price is dependent on its maturity; assuming other factors constant, the longer a bond's maturity the greater the price volatility resulting from a change in market yields.

- **Principal and coupon rate** The *principal* of a bond is the amount that the issuer agrees to repay the bondholder on the maturity date. This amount is also referred to as the *redemption value*, *maturity value*, *par value*, *nominal value* or *face amount*, or simply *par*. The *coupon rate* or *nominal rate* is the interest rate that the issuer agrees to pay each year. The annual amount of the interest payment made is called the *coupon*. The coupon rate multiplied by the principal of the bond provides the cash amount of the coupon. For example a bond with a 7% coupon rate and a principal of £1,000,000 will pay annual interest of £70,000. In the United Kingdom, United States and Japan the usual practice is for the issuer to pay the coupon in two semi-annual instalments. For bonds issued in European markets and the Eurobond market coupon payments are made annually. On rare occasions one will encounter bonds that pay interest on a quarterly basis. Certain bonds pay monthly interest. All bonds make periodic interest payments except for *zero-coupon bonds*. These bonds allow a holder to realise interest by being sold substantially below their principal value. The bonds are redeemed at par, with the interest amount then being the difference between the principal value and the price at which the bond was sold. We will explore zero-coupon bonds in greater detail later.
- **Currency** Bonds can be issued in virtually any currency. The largest volume of bonds in the global markets are denominated in US dollars; other major bond markets are denominated in euros, Japanese yen and sterling, and liquid markets also exist in Australian, New Zealand and Canadian dollars, Swiss francs and other major currencies. The currency of issue may impact on a bond's attractiveness and liquidity which is why borrowers in developing countries often elect to issue in a currency other than their home currency, for example dollars, as this will make it easier to place the bond with investors. If a bond is aimed solely at a country's domestic investors it is more likely that the borrower will issue in the home currency.

1.2 Bond issuers

In most countries government expenditure exceeds the level of government income received through taxation. This shortfall is made up by government borrowing and bonds are issued to finance the government's debt. The core of any domestic capital market is usually the government bond market, which also forms the benchmark for all other borrowing. Figure 1.2 illustrates UK gilt price and yield quotes as listed in the *Financial Times* for 23 July 1999.

Gilts are identified by their coupon rate and year of maturity; they are also given names such as *Treasury* or *Exchequer*. There is no significance attached to any particular name, all gilts are equivalent irrespective of their name. From Figure 1.2 we see that the 5¾% 2009 stock closing price from the day before was 104.79, which means £104.79 of par value. (Remember that par is the lump sum paid at maturity.) This price represents a *gross redemption yield* of 5.15%. If we pay £104.79 per £100 of stock today, we will receive £100 per £100 of stock on maturity. At first sight this appears to imply we will lose money, however we also receive coupon payments every six months, which for this bond is £2.875 per £100 nominal of stock. Also from Figure 1.2 we see the change in price from the day before for each gilt; in the case of the 5¾% 2009 the price was up 0.18 from the previous day's closing price.

Government agencies also issue bonds. Such bonds are virtually as secure as government bonds. In the United States agencies include the Federal National Mortgage Association. Local authorities issue bonds as part of financing for roads, schools, hospitals and other capital projects.

Corporate borrowers issue bonds both to raise finance for major projects and also to cover ongoing and operational expenses. Corporate finance is a mixture of debt and equity and a specific capital project will often be financed as a mixture of both.

UK GILTS PRICES																	
Yield						Yield						Yield					
Notes						Notes						Notes					
Int						Int						Int					
Red						Red						Red					
Price £ + or -						Price £ + or -						Price £ + or -					
52 week						52 week						52 week					
High						High						High					
Low						Low						Low					
Shorts* (Lives up to Five Years)																	
Treas 6½pc 1999	6.00	4.91	100.05	-01	100.50	98.70	Treas 12½pc 2003-5	9.90	5.58	126.26	+04	135.55	126.22	Index-Linked	(b)		
Treas 2½pc 1999	1.26	-	198.90	-02	199.60	191.47	Treas 8½pc 2005	7.32	5.48	116.06	+10	125.31	113.85	2½pc '01	(78.3) 2.78 3.33 204.26 -06 206.40 198.10		
Conv 10¼pc 1999	10.07	4.79	101.76	-02	105.37	101.76	Conv 9¼pc 2006	7.76	5.46	125.57	+16	136.21	123.98	2½pc '03	(78.8) 2.38 2.68 205.29 -13 207.83 195.70		
							Treas 7¼pc 2006	6.85	5.49	113.19	+12	122.40	110.47	4½pc '04	(135.6) 1.88 2.12 133.34 -08 134.77 126.55		
							Treas 8pc 2002-6	7.49	5.66	106.75	-10	111.73	105.05	2pc '06	(69.5) 1.62 1.79 237.40md -23 239.80 212.21		
							Treas 7½pc 2006	6.68	5.46	112.27	+13	121.62	109.33	2½pc '09	(78.8) 1.75 1.88 218.00 -28 221.45 192.71		
							Treas 11¼pc 2003-7	9.85	5.61	119.25md	-08	128.29	119.25	2½pc '11	(74.6) 1.88 1.99 230.05 -37 235.62 202.19		
							Treas 8½pc 2007	7.12	5.46	119.44	+16	129.86	116.89	2½pc '13	(89.2) 1.88 1.97 194.19 -31 199.07 168.63		
							Treas 7¼pc 2007	6.46	5.41	112.27	+17	122.55	109.27	2½pc '16	(81.6) 1.86 1.94 215.25md -31 221.10 183.37		
Treas 8½pc 2000	8.35	4.81	101.85md	-03	103.54	101.81	Treas 13½pc 2004-8	10.19	5.52	132.47	+08	142.61	132.39	2½pc '20	(83.0) 1.84 1.91 215.71 -25 221.68 178.82		
Conv 9pc 2000	8.78	4.77	102.52	-02	104.38	102.52	Treas 9pc 2008	7.12	5.34	126.37	+12	138.52	123.05	2½pc '24	(97.7) 1.81 1.87 187.39 -26 193.74 151.30		
Treas 13pc 2000	12.08	4.91	107.61	-05	113.28	107.61	Treas 8pc 2009	6.58	5.25	121.43	+19	132.91	117.03	4½pc '30	(135.1) 1.76 1.81 185.99md -28 193.23 149.11		
Treas 8pc 2000	7.70	5.25	103.86	-06	105.84	102.24											
Treas 5pc Rate 2001	0.32	100.35	-09	100.82	100.30												
Treas 10pc 2001	9.34	5.28	107.12	-05	113.37	107.12											
Conv 9½pc 2001	8.81	5.26	107.84	-05	110.89	106.73											
Conv 9¾pc 2001	8.98	5.25	106.63	-05	111.80	107.58											
Treas 7pc 2001	6.78	5.46	103.25	-06	106.51	100.83											
Conv 10pc 2002	9.03	5.66	110.79	-08	116.41	110.79											
Treas 7pc 2002	6.74	5.52	103.89	-06	108.13	101.52											
Conv 9½pc 2002	8.63	5.65	110.14	-08	116.64	109.81											
Treas 9½pc 2002	8.74	5.64	111.53	-08	117.29	111.19											
Exch 8½pc 2002	8.18	5.63	110.07	-08	115.62	108.87											
Conv 9½pc 2003	8.56	6.11	113.94	-08	120.19	112.78											
Treas 8pc 2003	7.37	5.53	108.51	+05	114.74	106.70											
Treas 10pc 2003	8.62	5.60	116.02	+04	123.52	115.58											
Treas 13¼pc 2000-3	12.66	5.49	106.58md	-06	115.00	108.58											
Treas 6½pc 2003	6.26	5.50	103.84	-05	110.21	101.10											
Treas 11pc 2004-4	10.49	5.34	109.64	+04	113.90	109.64											
Treas 10½pc 2001	8.44	5.59	118.62	+04	126.55	117.36											
Over Fifteen Years																	
Treas 8pc 2015	6.04	5.05	132.54	-29	144.12	123.43											
Treas 8½pc 2017	6.06	5.00	144.38	-41	156.08	133.09											
Exch 12pc 2013-17	7.17	5.26	167.45	-01	183.80	161.32											
Treas 8pc 2021	5.63	4.85	142.11	-67	153.21	127.51											
Treas 6pc 2028	4.94	4.65	121.48	-63	131.17	104.17											
Undated																	
Consols 4pc	5.13	-	77.98	-54	87.19	64.75											
War Loan 3½pc	4.95	-	70.73	-55	79.83	59.16											
Conv 3½pc '61 Alt.	4.23	-	82.73	-55	95.68	76.16											
Treas 3pc '66 Alt.	5.39	-	55.70	-42	61.93	46.98											
Consols 2½pc	4.93	-	50.70	-42	56.93	41.98											
Treas 2½pc	5.00	-	50.00	-18	55.17	41.20											
Other Fixed Interest																	
Notes						Notes						Notes					
Int						Int						Int					
Red						Red						Red					
Price £ + or -						Price £ + or -						Price £ + or -					
52 week						52 week						52 week					
High						High						High					
Low						Low						Low					
Asian Dev 10¼pc 2009	7.89	6.11	129½	-	140½	128½											
B'ham 11½pc 2012	7.93	6.40	145	-	158½	145											
Leeds 13½pc 2006	9.78	-	138	-	152	138											
Liverpool 3½pc Irred.	5.83	-	60	-	70½	55											
LCC 3pc '20 Alt.	5.88	-	51	-	59	47											
Manchester 11½pc 2007	8.58	6.28	134	-	151½	132											
Met. Wtr. 3pc '8	3.41	6.45	88	-	95	85											
N'wide Anglia 5½pc IL 2021	-	3.17	188½	-	192½	172½											
4½pc IL 2024	-	3.13	182	-	187½	164½											

● 'Tap' stock. All UK Gilts are tax-free to non-residents on application. E Auction basis. Ex dividend. Closing mid-prices are shown in pounds per £100 nominal of stock. Prospective real index-linked redemption yields are calculated by HSBC Greenwell from Gamma discount prices. * Indicative price.

Figure 1.2: UK gilts prices page. Reprinted from *The Financial Times*, 23 July 1999.© *Financial Times*. Used with permission.

1.3 Capital market participants

The debt capital markets exist because of the financing requirements of governments and corporates. The source of capital is varied, but the total supply of funds in a market is made up of personal or household savings, business savings and increases in the overall money supply. Growth in the money supply is a function of the overall state of the economy, and interested readers may wish to consult the bibliography at the end of this chapter which includes several standard economic texts. Individuals save out of their current income for future consumption, while business savings represent retained earnings. The entire savings stock represents the capital available in a market. As we saw in the preface however the requirements of savers and borrowers differs significantly, in that savers have a short-term investment horizon while borrowers prefer to take a longer term view. The “constitutional weakness” of what would otherwise be *unintermediated* financial markets led, from an early stage, to the development of financial intermediaries.

1.3.1 Financial Intermediaries

In its simplest form a financial intermediary is a *broker* or *agent*. Today we would classify the broker as someone who acts on behalf of the borrower or lender, buying or selling a bond as instructed. However intermediaries originally acted between borrowers and lenders in placing funds as required. A broker would not simply on-lend funds that have been placed with it, but would accept deposits and make loans as required by its customers. This resulted in the first banks. A *retail bank* deals mainly with the personal financial sector and small businesses, and in addition to loans and deposits also provides cash transmission services. A retail bank is required to maintain a minimum cash reserve, to meet potential withdrawals, but the remainder of its deposit base can be used to make loans. This does not mean that the total size of its loan book is restricted to what it has taken in deposits: loans can also be funded in the wholesale market. An *investment bank* will deal with governments, corporates and institutional investors. Investment banks perform an agency role for their customers, and are the primary vehicle through which a corporate will borrow funds in the bond markets. This is part of the bank’s corporate finance function; it will also act as wholesaler in the bond markets, a function known as *market making*. The bond issuing function of an

investment bank, by which the bank will issue bonds on behalf of a customer and pass the funds raised to this customer, is known as *origination*. Investment banks will also carry out a range of other functions for institutional customers, including export finance, corporate advisory and fund management.

Other financial intermediaries will trade not on behalf of clients but for their own *book*. These include *arbitrageurs* and speculators. Usually such market participants form part of investment banks.

1.3.2 Investors

There is a large variety of players in the bond markets, each trading some or all of the different instruments available to suit their own purposes. We can group the main types of investors according to the time horizon of their investment activity.

- **Short-term institutional investors.** These include banks and building societies, money market fund managers, central banks and the treasury desks of some types of corporates. Such bodies are driven by short-term investment views, often subject to close guidelines, and will be driven by the total return available on their investments. Banks will have an additional requirement to maintain *liquidity*, often in fulfilment of regulatory authority rules, by holding a proportion of their assets in the form of easily tradeable short-term instruments.
- **Long-term institutional investors.** Typically these types of investors include pension funds and life assurance companies. Their investment horizon is long-term, reflecting the nature of their liabilities; often they will seek to match these liabilities by holding long-dated bonds.
- **Mixed horizon institutional investors.** This is possibly the largest category of investors and will include general insurance companies and most corporate bodies. Like banks and financial sector companies, they are also very active in the primary market, issuing bonds to finance their operations.
- **Market professionals.** This category includes the banks and specialist financial intermediaries mentioned above, firms that one would not automatically classify as “investors” although they will also have an investment objective. Their time horizon will range from one day to the very long term. They include the proprietary trading desks of investment banks, as well as bond market makers in securities houses and banks who are providing a service to their customers. Proprietary traders will actively position themselves in the market in order to gain trading profit, for example in response to their view on where they think interest rate levels are headed. These participants will trade direct with other market professionals and investors, or via brokers. Market makers or *traders* (also called *dealers* in the United States) are wholesalers in the bond markets; they make two-way prices in selected bonds. Firms will not necessarily be active market makers in all types of bonds, smaller firms often specialise in certain sectors. In a two-way quote the *bid price* is the price at which the market maker will buy stock, so it is the price the investor will receive when selling stock. The *offer price* or *ask price* is the price at which investors can buy stock from the market maker. As one might expect the bid price is always lower than the offer price, and it is this *spread* that represents the theoretical profit to the market maker. The bid–offer spread set by the market maker is determined by several factors, including supply and demand and liquidity considerations for that particular stock, the trader’s view on market direction, *volatility* of the stock itself and the presence of any market intelligence. A large bid–offer spread reflects low liquidity in the stock, as well as low demand.

As mentioned above *brokers* are firms that act as intermediaries between buyers and sellers and between market makers and buyers/sellers. Floor-based stock exchanges such as the New York Stock Exchange (NYSE) also feature a *specialist*, members of the exchange who are responsible for maintaining an orderly market in one or more securities. These are known as *locals* on the London International Financial Futures and Options Exchange (LIFFE)². Locals trade securities for their own account to counteract a temporary imbalance in supply and demand in a particular security; they are an important source of *liquidity* in the market. Locals earn income from brokerage fees and also from pure trading, when they sell securities at a higher price than the original purchase price.

1.3.3 Markets

Markets are that part of the financial system where capital market transactions, including the buying and selling of securities, takes place. A market can describe a traditional stock exchange, a physical trading floor where securities

² Since this was written, trading on LIFFE has moved off the exchange floor and is conducted electronically via screens.

trading occurs. Many financial instruments are traded over the telephone or electronically over computer links; these markets are known as *over-the-counter* (OTC) markets. A distinction is made between financial instruments of up to one year's maturity and instruments of over one year's maturity. Short-term instruments make up the *money market* while all other instruments are deemed to be part of the *capital market*. There is also a distinction made between the *primary market* and the *secondary market*. A new issue of bonds made by an investment bank on behalf of its client is made in the primary market. Such an issue can be a *public* offer, in which anyone can apply to buy the bonds, or a *private* offer where the customers of the investment bank are offered the stock. The secondary market is the market in which existing bonds and shares are subsequently traded.

A list of selected world stock exchanges is given in Appendix 1.1.

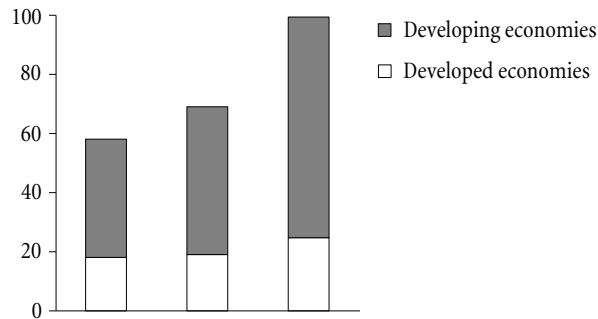


Figure 1.3: Number of stock exchanges around the world. Source: World Bank, OECD.

1.4 World bond markets

The origin of the spectacular increase in the size of global financial markets was the rise in oil prices in the early 1970s. Higher oil prices stimulated the development of a sophisticated international banking system, as they resulted in large capital inflows to developed country banks from the oil-producing countries. A significant proportion of these capital flows were placed in *Eurodollar* deposits in major banks. The growing trade deficit and level of public borrowing in the United States also contributed. The 1980s and 1990s saw tremendous growth in capital markets volumes and trading. As capital controls were eased and exchange rates moved from fixed to floating, domestic capital markets became internationalised. Growth was assisted by the rapid advance in information technology and the widespread use of financial engineering techniques. Today we would think nothing of dealing in virtually any liquid currency bond in financial centres around the world, often at the touch of a button. Global bond issues, underwritten by the subsidiaries of the same banks, are commonplace. The ease with which transactions can be undertaken has also contributed to a very competitive market in liquid currency assets.

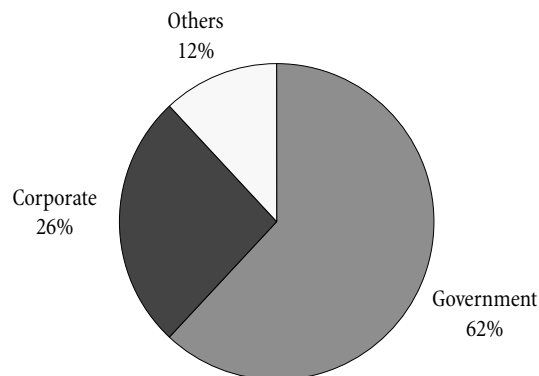


Figure 1.4: Global bond market issuers, December 1998. Source: IFC, 1998.

The world bond market has increased in size more than fifteen times since the 1970s. As at the end of 1998 outstanding volume stood at over \$26 trillion. The majority of this debt is issued by governments, as shown in Figure 1.4.

The market in US Treasury securities is the largest bond market in the world. Like the government bond markets in the UK, Germany, France and other developed economies it also very liquid and transparent. Table 1.1 lists the major government bond markets in the world; the US market makes up nearly half of the total. The Japanese market is second in size, followed by the German market. A large part of the government bond market is concentrated therefore in just a few countries. Government bonds are traded on major exchanges as well as *over-the-counter* (OTC). Generally OTC refers to trades that are not carried out on an exchange but directly between the counterparties. Bonds are also listed on exchanges, for example the NYSE had over 600 government issues listed on it at the end of 1996, with a total par value of \$2.6 billion.

Country	Nominal value (\$ billion)	Percentage (rounded)
United States	5,490	48.5
Japan	2,980	26.3
Germany	1,236	10.9
France	513	4.5
Canada	335	3.0
United Kingdom	331	2.9
Netherlands	253	2.2
Australia	82	0.7
Denmark	72	0.6
Switzerland	37	0.3
Total	11,329	100

Table 1.1: Major government bond markets, December 1998. Source: IFC 1998.

The corporate bond market varies in liquidity, depending on the currency and type of issuer of any particular bond. Outstanding volume as at the end of 1998 was over \$5.5 trillion. The global distribution of corporate bonds is shown at Figure 1.5, broken down by currency. The introduction of the euro across eleven member countries of the European Union in January 1999 now means that corporate bonds denominated in that currency form the second highest group.

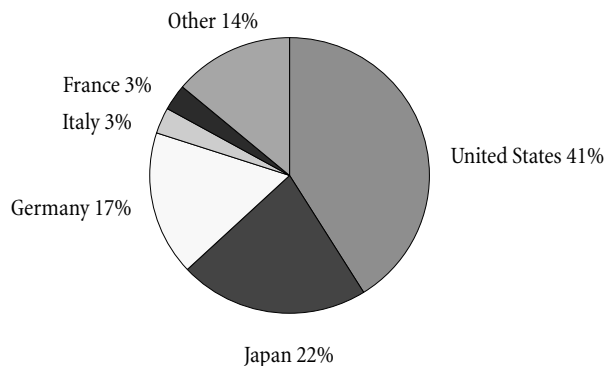


Figure 1.5: Global distribution of corporate bonds by currency, December 1998. Source: OECD.

Companies finance their operations in a number of ways, from equity to short term debt such as bank overdrafts. It is often advantageous for companies to fix longer term finance, which is why bonds are so popular. Bonds are also attractive as a means of raising finance because the interest payable on them to investors is tax deductible for the company. Dividends on equity are not tax deductible. A corporate needs to get a reasonable mix of

debt versus equity in its funding however, as a high level of interest payments will be difficult to service in times of recession or general market downturn. For this reason the market views unfavourably companies that have a high level of debt. Corporate bonds are also traded on exchanges and OTC. One of the most liquid corporate bond types is the *Eurobond*, which is an international bond issued and traded across national boundaries. Sovereign governments have also issued Eurobonds.

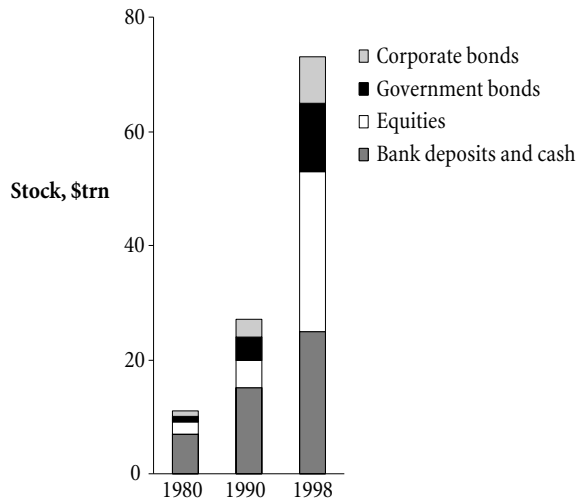


Figure 1.6 Global capital markets. Source: Strata Consulting.

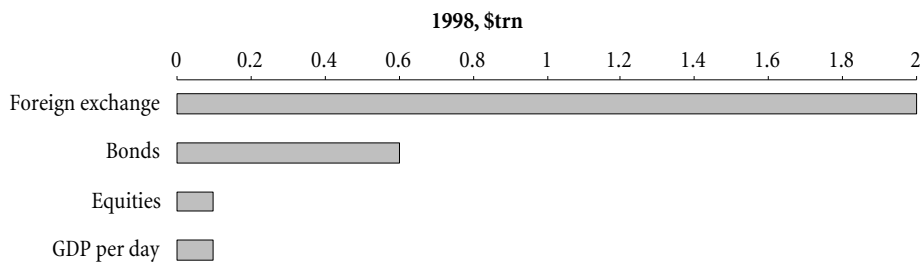


Figure 1.7: Global capital markets turnover. Source: Strata Consulting.

1.5 Overview of the main bond markets

So far we have established that bonds are debt capital market instruments, which means that they represent loans taken out by governments and corporations. The duration of any particular loan will vary from two years to thirty years or longer. In this chapter we introduce just a small proportion of the different bond instruments that trade in the market, together with a few words on different country markets. This will set the scene for later chapters, where we look at instruments and markets in greater detail.

1.5.1 Domestic and international bonds

In any market there is a primary distinction between *domestic* bonds and other bonds. Domestic bonds are issued by borrowers domiciled in the country of issue, and in the currency of the country of issue. Generally they trade only in their original market. A *Eurobond* is issued across national boundaries and can be in any currency, which is why they are also called *international* bonds. It is now more common for Eurobonds to be referred to as international bonds, to avoid confusion with “euro bonds”, which are bonds denominated in *euros*, the currency of twelve countries of the European Union (EU). As an issue of Eurobonds is not restricted in terms of currency or country,

the borrower is not restricted as to its nationality either. There are also *foreign* bonds, which are domestic bonds issued by foreign borrowers. An example of a foreign bond is a *Bulldog*, which is a sterling bond issued for trading in the United Kingdom (UK) market by a foreign borrower. The equivalent foreign bonds in other countries include *Yankee* bonds (United States), *Samurai* bonds (Japan), *Alpine* bonds (Switzerland) and *Matador* bonds (Spain).

There are detail differences between these bonds, for example in the frequency of interest payments that each one makes and the way the interest payment is calculated. Some bonds such as domestic bonds pay their interest *net*, which means net of a withholding tax such as income tax. Other bonds including Eurobonds make *gross* interest payments.

1.5.2 Government bonds

As their name suggests government bonds are issued by a government or *sovereign*. Government bonds in any country form the foundation for the entire domestic debt market. This is because the government market will be the largest in relation to the market as a whole. Government bonds also represent the best *credit risk* in any market as people do not expect the government to go bankrupt. As we see in a later chapter, professional institutions that analyse borrowers in terms of their credit risk always rate the government in any market as the highest credit available. While this may sometimes not be the case, it is usually a good rule of thumb.³ The government bond market is usually also the most *liquid* in the domestic market due to its size and will form the benchmark against which other borrowers are rated. Generally, but not always, the yield offered on government debt will be the lowest in the market.

- **United States.** Government bonds in the US are known as *Treasuries*. Bonds issued with an original maturity of between two and ten years are known as *notes* (as in “Treasury note”) while those issued with an original maturity of over ten years are known as *bonds*. In practice there is no real difference between notes and bonds and they trade the same way in the market. Treasuries pay semi-annual coupons. The US Treasury market is the largest single bond market anywhere and trades on a 24-hour basis all around the world. A large proportion of Treasuries are held by foreign governments and corporations. It is a very liquid and *transparent* market.
- **United Kingdom.** The UK government issues bonds known as *gilt-edged securities* or *gilts*.⁴ The gilt market is another very liquid and transparent market, with prices being very competitive. Many of the more esoteric features of gilts such as “tick” pricing (where prices are quoted in 32nds and not decimals) and *special ex-dividend* trading have recently been removed in order to harmonise the market with euro government bonds. Gilts still pay coupon on a semi-annual basis though, unlike euro paper. The UK government also issues bonds known as *index-linked* gilts whose interest and redemption payments are linked to the rate of inflation. There are also older gilts with peculiar features such as no redemption date and quarterly-paid coupons.
- **Germany.** Government bonds in Germany are known as *bunds*, BOBLs or *Schatze*. These terms refer to the original maturity of the paper and has little effect on trading patterns. Bunds pay coupon on an annual basis and are of course, now denominated in euros.

We will look at these markets and other government markets in greater detail in Chapter 13.

1.5.3 Non-conventional bonds

The definition of bonds given earlier in this chapter referred to conventional or *plain vanilla* bonds. There are many variations on vanilla bonds and we introduce a few of them here.

- **Floating Rate Notes.** The bond marked is often referred to as the *fixed income* market, or the *fixed interest* market in the UK. Floating rate notes (FRNs) do not have a fixed coupon at all but instead link their interest payments to an external reference, such as the three-month bank lending rate. Bank interest rates will fluctuate constantly during the life of the bond and so an FRNs cash flows are not known with certainty. Usually FRNs pay

³ Occasionally one may come across a corporate entity, such as Gazprom in Russia, that one may view as better rated in terms of credit risk compared to the government of the country in which the company is domiciled (in this case the Russian government).

⁴ This is because early gilt issues are said to have been represented by certificates that were edged with gold leaf, hence the term gilt-edged. In fact the story is almost certainly apocryphal and it is unlikely that gilt certificates were ever edged with gold!

a fixed margin or *spread* over the specified reference rate; occasionally the spread is not fixed and such a bond is known as a *variable rate note*. Because FRNs pay coupons based on the three-month or six-month bank rate they are essentially money market instruments and are treated by bank dealing desks as such.

- **Index-linked bonds.** An index-linked bond as its coupon and redemption payment, or possibly just either one of these, linked to a specified index. When governments issue Index-linked bonds the cash flows are linked to a price index such as consumer or commodity prices. Corporates have issued index-linked bonds that are connected to inflation or a stock market index.
- **Zero-coupon bonds.** Certain bonds do not make any coupon payments at all and these are known as *zero-coupon bonds*. A zero-coupon bond or *strip* has only cash flow, the redemption payment on maturity. If we assume that the maturity payment is say, £100 per cent or *par* the issue price will be at a discount to par. Such bonds are also known therefore as *discount* bonds. The difference between the price paid on issue and the redemption payment is the interest realised by the bondholder. As we will discover when we look at strips this has certain advantages for investors, the main one being that there are no coupon payments to be invested during the bond's life. Both governments and corporates issue zero-coupon bonds. Conventional coupon-bearing bonds can be *stripped* into a series of individual cash flows, which would then trade as separate zero-coupon bonds. This is a common practice in government bond markets such as Treasuries or gilts where the borrowing authority does not actually issue strips, and they have to be created via the stripping process.
- **Amortised bonds.** A conventional bond will repay on maturity the entire nominal sum initially borrowed on issue. This is known as a *bullet* repayment (which is why vanilla bonds are sometimes known as bullet bonds). A bond that repays portions of the borrowing in stages during its life is known as an *amortised* bond.
- **Bonds with embedded options.** Some bonds include a provision in their offer particulars that gives either the bondholder and/or the issuer an option to enforce early redemption of the bond. The most common type of option embedded in a bond is a *call feature*. A call provision grants the issuer the right to redeem all or part of the debt before the specified maturity date. An issuing company may wish to include such a feature as it allows it to replace an old bond issue with a lower coupon rate issue if interest rates in the market have declined. As a call feature allows the issuer to change the maturity date of a bond it is considered harmful to the bondholder's interests; therefore the market price of the bond at any time will reflect this. A call option is included in all asset-backed securities based on mortgages, for obvious reasons (asset-backed bonds are considered in a later chapter). A bond issue may also include a provision that allows the investor to change the maturity of the bond. This is known as a *put feature* and gives the bondholder the right to sell the bond back to the issuer at par on specified dates. The advantage to the bondholder is that if interest rates rise after the issue date, thus depressing the bond's value, the investor can realise par value by *putting* the bond back to the issuer. A *convertible* bond is an issue giving the bondholder the right to exchange the bond for a specified amount of shares (equity) in the issuing company. This feature allows the investor to take advantage of favourable movements in the price of the issuer's shares. The presence of embedded options in a bond makes valuation more complex compared to plain vanilla bonds, and will be considered separately.
- **Bond warrants.** A bond may be issued with a *warrant* attached to it, which entitles the bond holder to buy more of the bond (or a different bond issued by the same borrower) under specified terms and conditions at a later date. An issuer may include a warrant in order to make the bond more attractive to investors. Warrants are often detached from their host bond and traded separately.

Finally there is a large class of bonds known as *asset-backed securities*. These are bonds formed from pooling together a set of loans such as mortgages or car loans and issuing bonds against them. The interest payments on the original loans serve to back the interest payable on the asset-backed bond. We will look at these instruments in a later chapter.

1.6 Financial engineering in the bond markets

A quick glance through this book will show that we do not confine ourselves to the cash bond markets alone. The last twenty years has seen tremendous growth in the use of different and complex financial instruments, a result of *financial engineering*. These instruments have been introduced by banks to cater for customer demand, which includes the following:

- the demand for greater yield and diversification: investors in the capital markets are continually seeking opportunities to enhance yield and diversify across different markets, leading to products being introduced in new markets;
- the demand for lower borrowing costs: borrowers can issue bonds in virtually any currency where there is a demand, but still raise finance in the currency of their choice by means of a *swap* transaction;
- the demand to reduce risk exposure: market participants increasingly wish to transfer the risk that their operations expose them to, often by means of tailor-made *option* contracts.

Banks develop new instruments in response to customer demand, which will vary according to the current circumstances. This can include reaction to high levels of inflation, highly volatile interest or foreign exchange rates and other macroeconomic factors. In this book we will explore and analyse all these developments, including a detailed look at *derivatives* and risk management as well as bond analysis, trading and portfolio strategies.

Appendices

APPENDIX 1.1 List of World Stock Exchanges

Exchange	Country	Year established
Melbourne	SE Australia	1865
Dhaka SE	Bangladesh	1988
Brussels SE	Belgium	1801
Sao Paulo SE	Brazil	1850
Montreal SE	Canada	1874
Toronto SE	Canada	1852
Vancouver SE	Canada	1907
Copenhagen SE	Denmark	1690
Paris SE	France	1871
Frankfurt SE	Germany	1802
Hong Kong SE	Hong Kong	1891
Budapest SE	Hungary	1989
Milan SE	Italy	1808
Tokyo SE	Japan	1878
Amsterdam SE	Netherlands	1611
SE of Singapore	Singapore	1973
Madrid SE	Spain	1831
Zurich SE	Switzerland	1877
Taiwan SE	Taiwan	1961
London SE	United Kingdom	1805
New York SE	United States	1792

“SE” is a common abbreviation for Stock Exchange.

APPENDIX 1.2 Market-determined interest rates

This chapter has introduced bonds as a package of cash flows. As the cash flows in a plain vanilla bond are known with certainty we can use the principles of *discounting* and *present value* to calculate the price of a bond, and this is considered in Chapter 2. In this appendix we will look at how the discount rates used in present value calculations are determined by the market.

Market interest rates

An investor purchasing a bond security is sacrificing the consumption power of current funds in return for a future return. Whether the security is very short-dated or long-dated, the investor is locking away funds for a period of time, with the additional risk that the investment may not be returned because say, the borrower defaults on the loan. To compensate the investor on both these grounds the bond must pay an adequate return, sufficient to satisfy

the investor that the expected return for locking funds away is worth the risk of so doing. The return from any investment will be in the form of *income*, by way of dividends or periodic interest payments, and *capital gain*, which is when the value of the original investment rises. It is reasonable to generalise that the return on equity investments is expected to be more in the form of capital gain while the return on a bond investment is expected to be more in the form of interest income. There are of course exceptions to this.

The *real interest rate* applicable to any investment is the interest rate minus the rate of *inflation*. In a zero inflation environment the real interest rate is the quoted interest rate for the instrument. The interest rate is determined by the time preference of individual investors, which is a function of investors' willingness to forgo current consumption in return for an increased consumption in the future. The investor will therefore in the first instance set the interest rate. If an investor will accept a rate of return of 5% over a period of one year, this means that she is indifferent to consuming £1 today or £1.05 in one year's time. In a world of just one borrower and one lender, the borrower will have to offer a rate of interest of at least 5% (in our example) in order to be able to sell his bond. In a world of many borrowers and lenders, if the interest rate was below the rate demanded, there would be an excess of borrowers over lenders, while if the rate was too high there would be an excess of lenders. The *market determined* rate of interest is therefore that rate which balances the supply of lent funds with the demand for borrowed funds. This is also known as the *equilibrium* rate of interest.

Real interest rates and inflation

The market determined interest rate is the rate that would apply in a market without inflation and no concern for *liquidity* (see below). Investors placing their funds with borrowers for any length of time over say, three months will require compensation for any inflation in the economy, otherwise the borrower would gain from paying back funds that had depreciated in real value. In an environment of high inflation investors will require a rate of return that is at least equal to the rate of inflation in order to be certain they are receiving back funds with the same level of purchasing power as originally lent. They will also add to this the rate of interest they require on the loan.

Consider an economy with an inflation rate of 10%. A lender will require a minimum of £1.10 on a loan of £1 at the end of one year simply to compensate for the effects of inflation. If the equilibrium rate of interest is 5%, the amount returned on an investment of £1 at the end of one year will be £1.1550. This is shown below.

$$\begin{aligned}
 \text{Return on loan} &= £1 \times (1 + \text{real interest rate}) \times \frac{\text{inflation effect at end of year}}{\text{amount of loan}} \\
 &= £1 \times (1.05) \times \frac{£1.10}{£1} = £(1.05)(1.10) \\
 &= £1.1550.
 \end{aligned} \tag{1.1}$$

Therefore the *nominal* interest rate required is 15½%.

The “headline” quoted interest rate is therefore always taken to be the nominal interest rate that takes into account both the equilibrium interest rate and the rate of inflation. It is determined from equation (1.2) and is also known as the *Fisher equation* (1930):

$$\begin{aligned}
 1 + r &= (1 + \rho)(1 + i) \\
 &= 1 + \rho + i + (\rho)(i)
 \end{aligned} \tag{1.2}$$

where

- r is the nominal interest rate
- ρ is the real interest rate
- i is the expected inflation rate.

In (1.2) the product of $(\rho)(i)$ is often negligible and it is customary to ignore it, which simplifies the Fisher equation to (1.3) below.

$$r = \rho + i. \tag{1.3}$$

Note that the inflation rate is defined as the *expected* rate, the one expected to apply at the end of the investment period. The rate quoted in the media is always an historical one, for example the monthly retail price index quoted in the UK press is always the rate that applied in the previous month.

Liquidity premium

We saw earlier in the preface notes that it is common for investors to prefer to lend for short periods, as there is less risk involved and it is easier to convert investments back into cash if needed. Borrowers on the other hand prefer to fix their financing for as long as possible, as their cost of capital is known with certainty into the future. Short-term borrowing would also force them continually to refinance their activities, which exposes them to the risk that interest rates will rise and force up their financing costs. Because of this natural conflict between lenders and borrowers, the interest rate on longer term borrowing has to be higher than short-term borrowing, to compensate investors for the increased risk. In a conventional market environment therefore the interest rate for money increases steadily for investments as the maturity increases. The relationship between the maturity of the loan and the interest rate applicable to it is known as the *yield curve* or the *term structure of interest rates* (although strictly speaking this term should be reserved for the zero-coupon yield curve only) and this is examined in depth in later chapters. The yield curve usually slopes upwards as shown in Figure 1.8 and illustrates the *liquidity premium* associated with longer maturity loans.

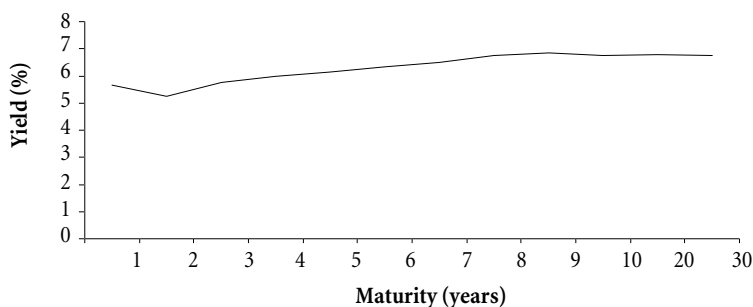


Figure 1.8: The yield curve

The liquidity premium will apply to all classes of investments, although it is greater for long-term instruments that are not as marketable as others. All buyers of long-term debt will require a liquidity premium to compensate them for locking their funds away. The liquidity premium will impact the required nominal interest rate, so we can adjust the Fisher equation as shown in (1.4) below.

$$r = \rho + i + l \quad (1.4)$$

where l is the liquidity premium.

Selected bibliography and references

- Cohen, K., Maier, S., Schwartz, R., Whitcomb, D., *The Microstructure of the Securities Markets*, Prentice Hall, 1986.
Hicks, *Value and Capital*, OUP, 1939.
Fama, E., Efficient Capital Markets, *Journal of Finance*, pp. 383–417 (May 1970).
Fisher, I., *The theory of interest*, NY: Macmillan, 1930.
Friedman, M., *A theoretical framework for monetary analysis*, paper 112, NY: NBER, 1971.
Madura, J., *International Financial Management*, 3rd edition, West Publishing Co., 1992.
Mishkin, F., *The economics of money, banking and financial markets*, 3rd edition, NY: Harper Collins, 1992.
Solnik, B., *International Investments*, 2nd edition, Addison-Wesley, 1991.
Van Horne, J., *Financial Markets Rates and Flows*, 3rd edition, Prentice-Hall, 1990.

Questions and exercises

1. Identify and classify the main participants in the capital markets.
2. What are gilts? What are the main ways that gilts differ from Eurobonds?
3. What are the main ways that corporations raise finance?
4. Consider your local street market. Would you classify it as an efficient market?
5. Two acquaintances wish to borrow funds from you and each insists that you charge interest. One promises to repay the funds within a month, whereas the other is asking for a repayment window of between six and twelve months. Do you charge them the same interest rate? If not, why not?

2 Financial Markets Arithmetic

Since the early 1970s the mathematics required for the analysis of capital market instruments has become steadily more complex. Bonds have been no exception to this. In later chapters we will introduce and develop some of the analytical techniques important to an understanding of bonds, but to begin with in this chapter we introduce the basic building blocks of corporate finance. These include the principles of compounded interest, the time value of money, and future and present values. The concepts in this chapter are important in all aspects of finance and are a vital part of capital market mathematics. It is essential to have a firm understanding of the main principles before moving on to other areas. When reviewing the concept of the time value of money, assume that the interest rates used the market determined rates of interest which we discussed in Appendix 1.2 of Chapter 1.

2.1 Simple and compound interest

The principles of financial arithmetic have long been used to illustrate that £1 received today is not the same as £1 received at a point in the future. Faced with a choice between receiving £1 today or £1 in one year's time we would not be indifferent, given a rate of interest of say, 10% and provided that this rate is equal to our required nominal rate. Our choice would be between £1 today or £1 plus 10p – the interest on £1 for one year at 10% per annum. The notion that money has a time value is a basic concept in the analysis of financial instruments. Money has time value because of the opportunity to invest it at a rate of interest.

2.1.1 Simple interest

A loan that has one interest payment on maturity is accruing *simple interest*. On short-term instruments there is usually only the one interest payment on maturity, hence simple interest is received when the instrument expires. The terminal value of an investment with simple interest is given by (2.1):

$$FV = PV(1 + r) \quad (2.1)$$

where

FV is the terminal value or *future value*
 PV is the initial investment or *present value*
 r is the interest rate.

So for example if PV is £100, r is 5% and the investment is one year then

$$FV = 100(1 + r) = 105.$$

The market convention is to quote interest rates as *annualised* interest rates, which is the interest that is earned if the investment term is one year. Consider a three-month deposit of £100 in a bank, placed at a rate of interest of 6%. In such an example the bank deposit will earn 6% interest for a period of 90 days. As the annual interest gain would be £6, the investor will expect to receive a proportion of this, which is calculated below:

$$£6.00 \times \frac{90}{365}.$$

So the investor will receive £1.479 interest at the end of the term. The total proceeds after the three months is therefore £100 plus £1.479. If we wish to calculate the terminal value of a short-term investment that is accruing simple interest we use the following expression:

$$FV = PV \left(1 + r \times \frac{\text{days}}{\text{year}} \right). \quad (2.2)$$

The fraction $\frac{\text{days}}{\text{year}}$ refers to the numerator, which is the number of days the investment runs, divided by the denominator which is the number of days in the year. In the sterling markets the number of days in the year is taken to be 365, however certain other markets (including the euro currency markets) have a 360-day year convention. For this reason we simply quote the expression as “days” divided by “year” to allow for either convention.

2.1.2 Compound interest

Let us now consider an investment of £100 made for three years, again at a rate of 6%, but this time fixed for three years. At the end of the first year the investor will be credited with interest of £6. Therefore for the second year the interest rate of 6% will be accruing on a principal sum of £106, which means that at the end of year 2 the interest credited will be £6.36. This illustrates how *compounding* works, which is the principle of earning interest on interest. What will the terminal value of our £100 three-year investment be?

In compounding we are seeking to find a *future value* given a *present value*, a *time period* and an *interest rate*. If £100 is invested today (at time t_0) at 6%, then one year later (t_1) the investor will have $£100 \times (1 + 0.06) = £106$. In our example the capital is left in for another two years, so at the end of year 2 (t_2) we will have:

$$\begin{aligned} £100 \times (1 + 0.06) \times (1 + 0.06) &= £100 \times (1 + 0.06)^2 \\ &= £100 \times (1.06)^2 \\ &= £112.36. \end{aligned}$$

The outcome of the process of compounding is the *future value* of the initial amount. We don't have to calculate the terminal value long-hand as we can use the expression in (2.3).

$$FV = PV(1 + r)^n \quad (2.3)$$

where

- r is the periodic rate of interest (expressed as a decimal)
- n is the number of periods for which the sum is invested.

In our example the initial £100 investment becomes $£100 \times (1 + 0.06)^3$ which is equal to £119.10.

When we compound interest we have to assume that the reinvestment of interest payments during the investment term is at the same rate as the first year's interest. That is why we stated that the 6% rate in our example was *fixed* for three years. We can see however that compounding increases our returns compared to investments that accrue only on a simple interest basis. If we had invested £100 for three years fixed at a rate of 6% but paying on a simple interest basis our terminal value would be £118, which is £1.10 less than our terminal value using a compound interest basis.

2.1.3 Compounding more than once a year

Now let us consider a deposit of £100 for one year, again at our rate of 6% but with quarterly interest payments. Such a deposit would accrue interest of £6 in the normal way but £1.50 would be credited to the account every quarter, and this would then benefit from compounding. Again assuming that we can reinvest at the same rate of 6%, the total return at the end of the year will be:

$$100 \times ((1 + 0.015) \times (1 + 0.015) \times (1 + 0.015) \times (1 + 0.015)) = 100 \times (1 + 0.015)^4$$

which gives us 100×1.06136 , a terminal value of £106.136. This is some 13 pence more than the terminal value using annual compounded interest.

In general if compounding takes place m times per year, then at the end of n years mn interest payments will have been made and the future value of the principal is given by (2.4) below.

$$FV = PV \left(1 + \frac{r}{m}\right)^{mn}. \quad (2.4)$$

As we showed in our example the effect of more frequent compounding is to increase the value of the total return when compared to annual compounding. The effect of more frequent compounding is shown below, where we consider the annualised interest rate factors, for an annualised rate of 5%.

<i>Compounding frequency</i>	<i>Interest rate factor</i>
Annual	$(1 + r) = 1.050000$
Semi-annual	$\left(1 + \frac{r}{2}\right)^2 = 1.050625$
Quarterly	$\left(1 + \frac{r}{4}\right)^4 = 1.050945$
Monthly	$\left(1 + \frac{r}{12}\right)^{12} = 1.051162$
Daily	$\left(1 + \frac{r}{365}\right)^{365} = 1.051267.$

This shows us that the more frequent the compounding the higher the interest rate factor. The last case also illustrates how a limit occurs when interest is compounded continuously. Equation (2.4) can be rewritten as follows:

$$\begin{aligned}
 FV &= PV \left(\left(1 + \frac{r}{m}\right)^{m/r} \right)^m = PV \left(\left(1 + \frac{1}{m/r}\right)^{m/r} \right)^m \\
 &= PV \left(\left(1 + \frac{1}{n}\right)^n \right)^m
 \end{aligned} \tag{2.5}$$

where $n = m/r$. As compounding becomes continuous and m and hence n approach infinity, the expression in large brackets in (2.5) above approaches a value known as e , which is shown below.

$$e = \lim_{n \rightarrow \infty} \left(1 + \frac{1}{n}\right)^n = 2.718281 \dots$$

If we substitute this into (2.5) this gives us:

$$FV = PV e^m \tag{2.6}$$

where we have continuous compounding. In (2.6) e^m is known as the *exponential function* of m and it tells us the continuously compounded interest rate factor. If $r = 5\%$ and $n = 1$ year then:

$$e^r = (2.718281)^{0.05} = 1.051271.$$

This is the limit reached with continuous compounding. From our initial example, to illustrate continuous compounding the future value of £100 at the end of three years when the interest rate is 6% is given by:

$$FV = 100e^{(0.06) \times 3} = £119.72.$$

2.1.4 Effective interest rates

The interest rate quoted on a deposit or loan is usually the *flat* rate. However we are often required to compare two interest rates which apply for a similar investment period but have different interest payment frequencies, for example a two-year interest rate with interest paid quarterly compared to a two-year rate with semi-annual interest payments. This is normally done by comparing equivalent *annualised* rates. The annualised rate is the interest rate with annual compounding that results in the same return at the end of the period as the rate we are comparing.

The concept of the effective interest rate allows us to state that:

$$PV \times \left(1 + \frac{r}{n}\right)^n = PV \times (1 + aer) \tag{2.7}$$

where *aer* is the equivalent annual rate. Therefore if r is the interest rate quoted which pays n interest payments per year, the *aer* is given by (2.8):

$$aer = \left[\left(1 + \frac{r}{n} \right)^n - 1 \right]. \quad (2.8)$$

The equivalent annual interest rate *aer* is known as the *effective* interest rate. We have already referred to the quoted interest rate as the “nominal” interest rate. We can rearrange equation (2.8) above to give us (2.9) which allows us to calculate nominal rates.

$$r = \left((1 + aer)^{\frac{1}{n}} - 1 \right) \times n. \quad (2.9)$$

We can see then that the effective rate will be greater than the flat rate if compounding takes place more than once a year. The effective rate is sometimes referred to as the *annualised percentage rate* or APR.

EXAMPLE 2.1

- Farhana has deposited funds in a building society 1-year fixed rate account with interest quoted at 5%, payable in semi-annual instalments. What is the effective rate that she earns at the end of the period?

$$\left[\left(1 + \frac{0.05}{2} \right)^2 - 1 \right] = 5.0625\%.$$

- Rupert is quoted a nominal interest rate of 6.40% for a 1-year time deposit where the interest is credited at maturity. What is the equivalent rate for the same building society's 1-year account that pays interest on a monthly basis?

$$\left((1 + 0.064)^{\frac{1}{12}} - 1 \right) \times 12 = 6.2196\%.$$

2.1.5 Interest rate conventions

The convention in both wholesale or personal (retail) markets is to quote an annual interest rate. A lender who wishes to earn the interest at the rate quoted has to place their funds on deposit for one year. Annual rates are quoted irrespective of the maturity of a deposit, from overnight to ten years or longer. For example if one opens a bank account that pays interest at a rate of 3.5% but then closes it after six months, the actual interest earned will be equal to 1.75% of the sum deposited. The actual return on a three-year building society bond (fixed deposit) that pays 6.75% fixed for three years is 21.65% after three years. The quoted rate is the annual one-year equivalent. An overnight deposit in the wholesale or *interbank* market is still quoted as an annual rate, even though interest is earned for only one day.

The convention of quoting annualised rates is to allow deposits and loans of different maturities and different instruments to be compared on the basis of the interest rate applicable. We must also be careful when comparing interest rates for products that have different payment frequencies. As we have seen from the foregoing paragraphs the actual interest earned will be greater for a deposit earning 6% on a semi-annual basis compared to 6% on an annual basis. The convention in the money markets is to quote the equivalent interest rate applicable when taking into account an instrument's payment frequency.

2.2 The time value of money

2.2.1 Present values with single payments

Earlier in this chapter we saw how a *future value* could be calculated given a known *present value* and rate of interest. For example £100 invested today for one year at an interest rate of 6% will generate $100 \times (1 + 0.06) = £106$ at the end of the year. The future value of £100 in this case is £106. We can also say that £100 is the *present value* of £106 in our example.

In equation (2.3) we established the following future value relationship:

$$FV = PV(1 + r)^n.$$

By reversing this expression we arrive at the present value formula (2.10):

$$PV = \frac{FV}{(1 + r)^n} \quad (2.10)$$

where terms are as before. Equation (2.10) applies in the case of annual interest payments and enables us to calculate the present value of a known future sum.

EXAMPLE 2.2

■ Naseem is saving for a trip around the world after university and needs to have £1000 in three years' time. He can invest in a building society bond at 7% guaranteed fixed for three years. How much does he need to invest now?

To solve this we require the PV of £1000 received in three years' time.

$$PV = \frac{1000}{(1 + .07)^3} = \frac{1000}{1.225043} = 816.29787.$$

Naseem therefore needs to invest £816.30 today.

To calculate the present value for a short-term investment of less than one year we will need to adjust what would have been the interest earned for a whole year by the proportion of days of the investment period. Rearranging the basic equation, we can say that the present value of a known future value is

$$PV = \frac{FV}{\left(1 + r \times \frac{\text{days}}{\text{year}}\right)}. \quad (2.11)$$

Given a present value and a future value at the end of an investment period, what then is the interest rate earned? We can rearrange the basic equation again to solve for the *yield*.

$$\text{yield} = \left(\frac{FV}{PV} - 1\right) \times \frac{\text{year}}{\text{days}}. \quad (2.12)$$

Using equation (2.12) will give us the interest rate for the actual period. We can then convert this to an effective interest rate using (2.13).

$$r = \left(1 + \text{yield} \times \frac{\text{days}}{\text{year}}\right)^{\frac{365}{\text{days}}} - 1. \quad (2.13)$$

When interest is compounded more than once a year, the formula for calculating present value is modified, as shown by (2.14):

$$PV = \frac{FV}{(1 + (r/m))^m} \quad (2.14)$$

where as before *FV* is the cash flow at the end of year *n*, *m* is the number of times a year interest is compounded, and *r* is the rate of interest or discount rate. Illustrating this therefore, the present value of £100 that is received at the end of five years at a rate of interest rate of 5%, with quarterly compounding is:

$$PV = \frac{100}{(1 + (0.05/4))^{(4)(5)}} = £78.00.$$

2.2.2 Discount factors

The calculation of present values from future values is also known as *discounting*. The principles of present and future values demonstrate the concept of the *time value* of money which is that in an environment of positive interest rates a sum of money has greater value today than it does at some point in the future because we are able to invest the sum today and earn interest. We will only consider a sum in the future compared to a sum today if we are compensated by being paid interest at a sufficient rate. Discounting future values allows us to compare the value of a future sum with a present sum.

Another way to write the expression in example (2.14) is to say that we multiply £1000 by $1/(1.05)^5$, which is the *reciprocal* of $(1.05)^5$ and is denoted in this case as $(1 + 0.05)^{-5}$. The rate of interest r that we use in Example 2.2 is known as the *discount rate* and is the rate we use to *discount* a known future value in order to calculate a present value. We can rearrange equation (2.14) to give:

$$PV = FV(1 + r)^{-n}$$

and the term $(1 + r)^{-n}$ is known as the n -year discount factor.

$$df_n = (1 + r)^{-n} \quad (2.15)$$

where df_n is the n -year discount factor.

The three-year discount factor when the discount rate is 9% is:

$$df_3 = (1 + 0.09)^{-3} = 0.77218.$$

We can calculate discount factor for all possible interest rates and time periods to give us a *discount function*. Fortunately we don't need to calculate discount factors ourselves as this has been done for us and discount tables for a range of rates are provided in Appendix 2.2.

EXAMPLE 2.3 Formula summary

Discount factor with simple interest: $df = \frac{1}{\left(1 + r \frac{\text{days}}{\text{year}}\right)}$.

Discount factor with compound interest: $df_n = \left(\frac{1}{1 + r}\right)^n$.

Earlier we established the continuously compounded interest rate factor as e^m . Using a continuously compounded interest rate therefore we can establish the discount factor to be:

$$df = \frac{1}{1 + \left(e^{r \times \frac{\text{days}}{\text{year}}} - 1\right)} = e^{-r \times \frac{\text{days}}{\text{year}}} \quad (2.16)$$

$$\therefore df_n = e^{-m}.$$

The continuously compounded discount factor is part of the formula used in option pricing models, which are discussed in the chapter on options.

It is possible to calculate discount factors from the prices of government bonds. The traditional approach described in most textbooks requires that we first use the price of a bond that has only one remaining coupon, its last one, and calculate a discount factor from this bond's price. We then use this discount factor to calculate the discount factors of bonds with ever-increasing maturities, until we obtain the complete discount function. This method, which is illustrated in the box below, suffers from certain drawbacks and in practice more sophisticated techniques are used, which we will introduce later in the book.

EXAMPLE 2.4 Discount factors

- The following hypothetical government bonds pay coupon on a semi-annual basis. Consider the bond prices indicated, and assume that the first bond has precisely six months to maturity, so that it has only one more cash flow to pay, the redemption value and final coupon. Assume further that the remaining bonds mature at precise six-month intervals.

Bond	Price
8% June 2000	101.09
7% December 2000	101.03
7% June 2001	101.44
6.5% December 2001	101.21

The first bond has a redemption payment of 104.00, comprised of the redemption payment and the final coupon payment (remember that this is a semi-annual coupon bond). The present value of this bond is 101.09. This allows us to determine the discount factor of the bond as follows:

$$\begin{aligned} 101.09 &= 104.00 \times df_{6\text{-month}} \\ 0.97202 &= df_{6\text{-month}}. \end{aligned}$$

This shows that the six-month discount factor is 0.97202. We use the second bond in the table, which has cash flows of 3.50 and 103.50, to calculate the next period discount factor, using the following expression:

$$101.03 = 3.50 \times df_{6\text{-month}} + 103.5 \times df_{1\text{-year}}.$$

We have already calculated the six-month discount factor, and use this to calculate the one-year discount factor from the above expression, which solves to give 0.94327. We then carry on this procedure for the next bond, leaving us the following discount factors:

Bond	Price	Discount factor
8% June 2000	101.09	0.97202
7% December 2000	101.03	0.94327
7% June 2001	101.44	0.91533
6.5% December 2001	101.21	0.89114

Note how the discount factors progressively reduce in value over an increasing maturity period. Using one of a number of techniques (which we consider later in the book) we can graph the set of discount factors above to obtain the two-year discount function. In the same way, if we have government bond prices for all maturities from six months to 30 years, we can obtain the complete discount function for that currency.

2.2.3 Present values with multiple discounting

Present values for short-term investments of under one year maturity often involve a single interest payment. If there is more than one interest payment then any discounting needs to take this into account. If discounting takes place m times per then we can use equation (2.4) to derive the present value formula as follows.

$$PV = FV \left(1 + \frac{r}{m} \right)^{-mn}. \quad (2.17)$$

For example, what is the present value of the sum of £1000 that is to be received in five years where the discount rate is 5% and there is semi-annual discounting?

Using (2.17) above we see that

$$PV = 1000 \left(1 + \frac{0.05}{2} \right)^{-2 \times 5} = £781.20.$$

The effect of more frequent discounting is to lower the present value. As with continuous compounding, the limiting factor is reached with continuous discounting and we can use equation (2.6) to derive the present value formula for continuous discounting

$$PV = FVe^{-m}. \quad (2.18)$$

Using this expression, if we consider the same example as before but now with continuous discounting we calculate the present value of £1000 to be received in five years' time as:

$$PV = 1000e^{-(0.05) \times 5} = £778.80.$$

EXAMPLE 2.5 Calculation summaries

- Grant invests £250 in a bank account for five years at a rate of 6.75%. What is the future value of this sum assuming annual compounding?

$$250 \times (1.0675)^5 = £346.56.$$

- After 180 days Grant decides to close the account and withdraw the cash. What is the terminal value?

$$250 \times (1 + 1.0675 \times 180/365) = £258.32.$$

- To pay off a personal loan Phil requires £500 in 30 days' time. What must he invest now if he can obtain 12% interest from a bank?

$$500/(1 + 0.12 \times 30/365) = £495.12.$$

- If Phil deposits £1000 today and receives a total of £1021 after 90 days, what yield has he earned on the investment?

$$((1021/1000) - 1) \times 365/90 = 8.52\%.$$

- What is the 180-day discount factor earned during this period is 6.15%? The ten-year discount factor?

$$1/(1 + 0.0615 \times 180/365) = 0.97056$$

$$1/(1 + 0.0615)^{10} = 0.55055.$$

- What is the present value of £100 in ten years' time at this discount rate?

$$100 \times 0.55055 = £55.06.$$

2.3 Multiple cash flows

2.3.1 Future values

In Chapter 1 we introduced bonds by describing them as a packages of cash flows. Up to now we have considered future values of a single cash flow. Of course the same principles of the time value of money can be applied to a bundle of cash flows. A series of cash flows can be regular or at irregular intervals. If we wish to calculate the total future value of a set of irregular payments made in the future we need to calculate each payment separately and then sum all the cash flows. The formula is represented with the equation given at (2.19):

$$FV = \sum_{n=1}^N C_n (1 + r)^{N-n} \quad (2.19)$$

where C_n is the payment in year n and the symbol Σ means “the sum of”. We assume that payment is made and interest credited at the end of each year.

It is much more common to come across a regular stream of future payments. Such a cash flow is known as an *annuity*. In an annuity the payments are identical and so C_n as given in (2.19) simply becomes C . We can then rearrange (2.19) as shown below.

$$FV = C \sum_{n=1}^N (1 + r)^{N-n}. \quad (2.20)$$

This equation can be simplified to give us the expression at (2.21):¹

$$FV = C \left(\frac{(1 + r)^N - 1}{r} \right). \quad (2.21)$$

¹ If we multiply both sides of (2.20) by $1 + r$ and then subtract the result from (2.20) we obtain:

$$\begin{aligned} FV - (1 + r)FV &= C \left(\sum_{n=1}^N (1 + r)^{N-n} - \sum_{n=1}^N (1 + r)^{N-n+1} \right) \\ &= -C((1 + r)^N - 1). \end{aligned}$$

This formula can be used to calculate the future value of an annuity. For example, if we consider an annuity that pays £500 each year for ten years at a rate of 6%, its future value is given by:

$$FV = 500 \left(\frac{(1.06)^{10} - 1}{0.06} \right) = £6,590.40.$$

EXAMPLE 2.6 Calculating pension contributions

- We can use the future value equation (2.21) to calculate the size of contributions required to establish a pension fund on retirement. If we rearrange (2.21) to obtain the size of the annuity C we obtain:

$$C = FV \left(\frac{r}{(1+r)^N - 1} \right).$$

- Lita wishes to have a savings pool of £250,000 to fund her pension when she retires in 30 years' time. What annual pension contribution is required if the rate of interest is assumed to be a constant 7.9%?

$$C = 250,000 \left(\frac{0.079}{(1.079)^{30} - 1} \right) = £2247.65.$$

The common definition of an annuity is a continuous stream of cash flows. In practice the pension represented by an annuity is usually paid in monthly instalments, similar to an employed person's annual salary. Certain regular payments compound interest on a more frequent basis than annually, so our formula at (2.20) needs to be adjusted slightly. If compounding occurs m times each year, then (2.20) needs to be altered to (2.22) to allow for this.

$$FV = C \sum_{n=1}^N \left(1 + \frac{r}{m} \right)^{m(N-n)}. \quad (2.22)$$

To make calculations simpler we can multiply both sides of (2.22) by $(1 + (r/m))$ and subtract the result from (2.22).² Simplifying this will then result in (2.23) below.

$$FV = C \left(\frac{(1 + (r/m))^{mN} - 1}{(1 + (r/m))^m - 1} \right). \quad (2.23)$$

For example a ten year annuity that has annual payments of £5000 each year, but compounded on a quarterly basis at a rate of 5% will have a future value of £63,073 as shown below.

$$FV = 5000 \left(\frac{(1.025)^{20} - 1}{(1.025)^2 - 1} \right) = £63,073.$$

Where there is continuous compounding, as before the limiting factor will result in (2.23) becoming (2.24):

$$FV = C \left(\frac{e^{rN} - 1}{e^r - 1} \right). \quad (2.24)$$

Equations (2.23) and (2.24) can be adjusted yet again to allow for frequent payments together with frequent compounding, but such a stream of cash flows is rarely encountered in practice. For reference, in the case of continuous compounding of continuous payments, the limiting factor expression is as shown at (2.25):

$$FV = C \left(\frac{e^{rN} - 1}{r} \right). \quad (2.25)$$

² The process is:

$$\begin{aligned} FV - [1 + (r/m)]^m FV &= C \left(\sum_{n=1}^N [1 + (r/m)]^{M(N-n)} - \sum_{n=1}^N [1 + (r/m)]^{M(N-n)+m} \right) \\ &= -C([1 + (r/m)]^{mN} - 1). \end{aligned}$$

2.3.2 Present values

Using similar principles as we have employed for calculating future values, we can calculate present values for a stream of multiple of cash flows. The method employed is slightly different according to whether the cash flows are regular or irregular.

For irregular payments we calculate present value by applying the conventional present value formula to each separate cash flow and then summing the present values. This is represented by (2.26):

$$PV = \sum_{n=1}^N C_n (1 + r)^{-n} \quad (2.26)$$

where C_n is the cash flow made in year n .

Consider a series of annual cash payments made up of £100 in the first year and then increasing by £100 each year until the fifth year. The present value of this cash flow stream is:

$$\begin{aligned} PV &= 100(1.05)^{-1} + 200(1.05)^{-2} + 300(1.05)^{-3} + 400(1.05)^{-4} + 500(1.05)^{-5} \\ &= 95.24 + 181.41 + 259.15 + 329.08 + 391.76 \\ &= £1256.64. \end{aligned}$$

The more frequently encountered type of cash flow stream is an *annuity*, regular annual payments with annual discounting. To calculate the present value of an annuity we can use a variation of (2.21) as shown at (2.27):

$$\begin{aligned} PV &= \frac{FV}{(1 + r)^N} \\ &= C \left(\frac{(1 + r)^N - 1}{r} \right) \left(\frac{1}{(1 + r)^N} \right) \\ &= C \left(\frac{1 - (1 + r)^{-N}}{r} \right). \end{aligned} \quad (2.27)$$

Consider now an annuity paying £5000 each year for twenty years at an interest rate 4.5%. The present value of this annuity is:

$$\begin{aligned} PV &= 5000 \left(\frac{1 - (1.045)^{-20}}{0.045} \right) \\ &= 65,039.68. \end{aligned}$$

We illustrated this principle using a 20-year annuity that employed annual discounting. If a cash flow stream employs more frequent discounting we need to adjust the formula again. If an annuity discounts its cash flows m times each year then the present value of its cash flow stream is found using the present value adjusted equation from (2.23). This becomes (2.28).

$$PV = \frac{FV}{\left(1 + \frac{r}{m}\right)^{mN}} = C \left(\frac{1 - \left(1 + \frac{r}{m}\right)^{-mN}}{\left(1 + \frac{r}{m}\right)^m - 1} \right). \quad (2.28)$$

If continuous discounting is employed then this results again in the limiting factor for continuous discounting, so we adjust (2.28) and the new expression is given at (2.29):

$$PV = C \left(\frac{1 - e^{-rN}}{e^r - 1} \right). \quad (2.29)$$

The last case to consider is that of the payments stream that has more frequent cash flows in addition to more frequent discounting. Such a payments stream will have m cash flows each year which are also discounted m times per year. To calculate the present value of the cash flows we use (2.30):

$$PV = \frac{FV}{(1 + (r/m))^{mN}} = C \frac{1 - (1 + (r/m))^{-mN}}{r/m} \quad (2.30)$$

The limiting factor for continuous discounting of continuous payments is given by (2.31):

$$PV = C \left(\frac{1 - e^{-rN}}{r} \right) \quad (2.31)$$

Payment streams that have cash flow frequencies greater than annually or semi-annually occur quite often in the markets. To illustrate how we might use (2.30), consider a mortgage-type loan taken out at the beginning of a period. If the borrower is able to fix the interest rate being charged to the whole life of the mortgage, she can calculate the size of the monthly payments that are required to pay off the loan at the end of the period.

For example consider a repayment mortgage of £76,000 taken out for 25 years at a fixed rate of interest of 6.99%. The monthly repayments that would be charged can be calculated using (2.30) as shown in (2.32):

$$C_i = \frac{C}{12} = \frac{PV}{12} \left(\frac{r}{1 - (1 + (r/m))^{-12 \times N}} \right) \quad (2.32)$$

where C_i is the size of the monthly payment. Substituting the terms of the mortgage payments in to the equation we obtain:

$$C_i = \frac{76,000}{12} \left(\frac{0.0699}{1 - (1 + (0.0699/12))^{-12 \times 25}} \right) = £536.67.$$

The monthly repayment is therefore £536.67 and includes the interest chargeable in addition to a repayment of some of the principal (hence the term *repayment* mortgage, as opposed to *endowment* mortgages which only pay off the monthly interest charge). A repayment mortgage is also known as an *amortised* mortgage. An amortised loan is one for which a proportion of the original loan capital is paid off each year. Loans that require the borrower to service the interest charge only each year are known as *straight* or *bullet* loans. It is for this reason that plain vanilla bonds are sometimes known as bullet bonds, since the capital element of a loan raised through a vanilla bond issue is repaid only on maturity.

2.3.3 Perpetual cash flows

The type of annuity that we as individuals are most familiar with is the *annuity pension*, purchased from a life assurance company using the proceeds of a pension fund at the time of retirement. Such an annuity pays a fixed annual cash amount for an undetermined period, usually up until the death of the beneficiary. An annuity with no set finish date is known as a *perpetuity*. As the end date of a perpetuity is unknown we are not able to calculate its present value with exact certainty, however a characteristic of the term $(1 + r)^{-N}$ is that it approaches zero as N tends to infinity. This fact reduces our present value expression to:

$$PV = \frac{C}{r} \quad (2.33)$$

and we can use this formula to approximate the present value of a perpetuity.

The UK gilt market includes four gilts that have no redemption date, so-called *undated* bonds. The largest issue amongst the undated gilts is the 3½% War Loan, a stock originally issued at the time of the 1914–18 war. This bond pays a coupon of £3½ per £100 nominal of stock. Since the cash flow structure of this bond matches a perpetual, its present value using (2.33) when long dated market interest rates are at say, 5% would be:

$$PV = \frac{3.5}{0.05} = £70.$$

The present value of the cash flow stream represented by War Loan when market rates are 5% would therefore be £70 per £100 nominal of stock. In fact because this bond pays coupon on a semi-annual basis we should adjust the calculation to account for the more frequent payment of coupons and discounting, so the present value (price) of the bond is more accurately described as:

$$PV = \frac{C/2}{r/2} = \frac{1.75}{0.025}$$

although as we would expect this still gives us a price of £70 per cent!

2.3.4 Varying discount rates

During our discussion we have assumed that a single discount factor is used whenever we wish to obtain the present value of a cash flow stream. In fact it is more realistic to expect that cash flows over a period of time are discounted at different rates. This is logical when one considers that over a certain period, different interest rates will apply to different time periods. If we wish to calculate the present value of a series of cash flows it is usually more realistic to do this by discounting each payment at its own appropriate rate and then taking the sum of all the individual present values.

Consider a financial instrument that pays £5 each year for four years, with a final payment of £105 in the fifth year (this sounds very similar to a 5% coupon five-year bond!). If the market required interest rate for five-year money is 5%, then all the cash flows will be discounted at 5%, as shown in Table 2.1.

Period	Cash flow	Present value
1	5	4.7619
2	5	4.5351
3	5	4.3192
4	5	4.1135
5	105	82.2702
Total PV =		100.0000

Table 2.1: Present values using one discount rate.

Using this discount rate, the total present value of the cash flow stream is exactly £100. If however the cash flows are treated as individual payments in their own right we may opt to discount each cash flow at a discount rate that is more appropriate to it. On this basis it may be that the first payment is discounted at a lower rate because it occurs earlier, and that the payments are all discounted at lower than 5%, except for the final payment. If we assign discount rates that fit the following market requirements, we find that the total present value of the cash flow stream is now higher, at £100.2129. This is shown in Table 2.2.

Period	Cash flow	Required interest rate	Present value
1	5	4%	4.8077
2	5	4.25%	4.6006
3	5	4.50%	4.3815
4	5	4.75%	4.1529
5	105	5%	82.2702
Total PV =			100.2129

Table 2.2: Present values using unique discount rates.

2.3.5 The discount function

Discount factors can be calculated for any discount rate that apply to any term to maturity, using the standard formulae. The complete range of discount factors for any particular rate is known as the *discount function*. Figure 2.1 illustrates the discount function when the discount rate selected is 5%. This is obtained by plotting continuous rather than discrete discount factors for a given rate. A discount factor table for selected rates and investment terms is given at Appendix 2.2.

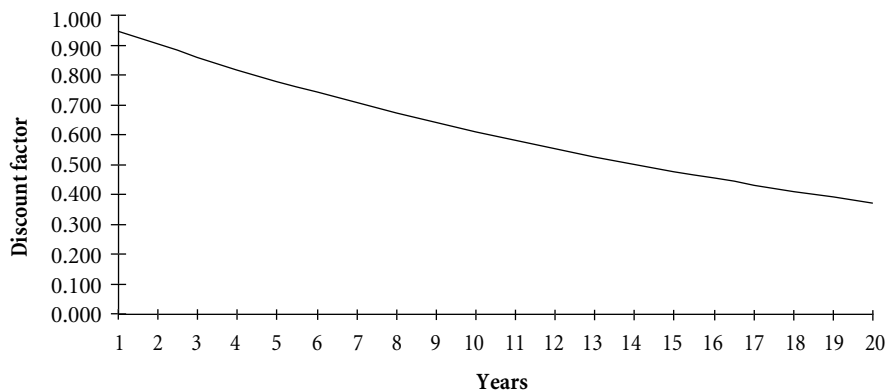


Figure 2.1: Discount function with the rate at 5%.

2.4 Corporate finance project appraisal

Two common techniques used by corporates and governments to evaluate whether a project is worth undertaking are *net present value* and *internal rate of return*. Both techniques evaluate the anticipated cash flows associated with a project, using the discounting and present value methods described so far in this chapter. Generally speaking it is a company's *cost of capital* that is used as the discount rate in project appraisal, and most companies attempt to ascertain the true cost of their capital as accurately as possible. As most corporate financing is usually a complex mixture of debt and equity this is sometimes problematic. A discussion of cost of capital is outside the scope of this book and we recommend the text in the reference section for readers wishing to know more about this subject.

2.4.1 Net present value

In the case of an investment of funds made as part of a project, we would have a series of cash flows of which some would be positive and others negative. Typically in the early stages of a project we would forecast negative cash flows as a result of investment outflows, followed by positive cash flows as the project began to show a return. Each cash flow can be present valued in the usual way. In project appraisal we would seek to find the present value of the entire stream of cash flows, and the sum of all positive and negative present values added together is the *net present value* (NPV). As the appraisal process takes place before the project is undertaken, the future cash flows that we are concerned with will be estimated forecasts and may not actually be received once the project is underway.

The present value equation is used to show that:

$$NPV = \sum_{n=1}^N \frac{C_n}{(1+r)^n} \quad (2.34)$$

where C_n is the cash flow in the project in period N . The rate r used to discount the cash flows can be the company's cost of capital or the rate of return required by the company to make the project viable.

Companies will apply NPV analysis to expected projected returns because funds invested in any undertaking has a time-related cost, the opportunity cost that is the corporate cost of capital. In effect NPV measures the present value of the gain achieved from investing in the project (provided that it is successful!). The general rule of thumb applied is that any project with a positive NPV is worthwhile, whereas those with a negative NPV, discounted at the required rate of return or the cost of capital, should be avoided.

EXAMPLE 2.7

■ What is the NPV of the following set of expected cash flows, discounted at a rate of 15%?

Year 0: −£23,000

Year 1: +£8,000

Year 2: +£8,000

Year 3: +£8,000

Year 4: +£11,000

$$NPV = 23,000 - \frac{8,000}{(1.15)} + \frac{8,000}{(1.15)^2} + \frac{8,000}{(1.15)^3} + \frac{11,000}{(1.15)^4} = £1,554.$$

2.4.2 The internal rate of return

The internal rate of return (IRR) for an investment is the discount rate that equates the present value of the expected cash flows (the NPV) to zero. Using the present value expression we can represent it by that rate r such that:

$$\sum_{n=0}^N \frac{C_n}{(1+r)^n} = 0 \quad (2.35)$$

where C_n is the cash flow for the period N , n is the last period in which a cash flow is expected, and Σ denotes the sum of discounted cash flows at the end of periods 0 through n . If the initial cash flow occurs at time 0, equation (2.35) can be expressed as follows:

$$C_0 = \frac{C_1}{(1+r)} + \frac{C_2}{(1+r)^2} + \dots + \frac{C_N}{(1+r)^N}. \quad (2.36)$$

In corporate finance project appraisal, C_0 is a cash outflow and C_1 to C_N are cash inflows. Thus r is the rate that discounts the stream of future cash flows (C_1 through C_N) to equal the initial outlay at time 0 — C_0 . We must therefore assume that the cash flows received subsequently are reinvested to realise the same rate of return as r . Solving for the internal rate of return r cannot be found analytically and has to be found through numerical iteration, or using a computer or programmable calculator. For the complete mathematical background to IRR consult Appendix 2.1.

To illustrate IRR consider the earlier project cash flows given in Example 2.7. If we wish to find the IRR long-hand then we would have to obtain the NPV using different discount rates until we found the rate that gave the NPV equal to zero. The quickest way to do this manually is to select two discount rates, one of which gives a negative NPV and the other a positive NPV, and then *interpolate* between these two rates. This method of solving for IRR is known as an *iterative* process, and involves converging on a solution through trial and error. This is in fact the only way to calculate the IRR for a set of cash flows and it is exactly an iterative process that a computer uses (the computer is just a touch quicker!). If we have two discount rates, say x and y that give positive and negative NPVs respectively for a set of cash flows, the IRR can be estimated using the equation at (2.37):

$$\text{IRR estimate} = x\% + (y\% - x\%) \times (+ve \text{ NPV value} / (+ve \text{ NPV value} - (-NPV \text{ value}))). \quad (2.37)$$

EXAMPLE 2.8

■ In Example 2.7 using a discount rate of 15% produced a positive NPV. Discounting the cash flows at 19% produces an NPV of $-\text{£}395$. Therefore the estimate for IRR is:

$$15\% + 4\% \times 1554 / (1554 - (-395)) = 18.19\%.$$

The IRR is approximately 18.19%. This can be checked using a programmable calculator or spreadsheet programme, or may be checked manually by calculating the NPV of the original cash flows using a discount rate of 18.19%; it should come to $-\text{£}23,000$. Using an HP calculator we obtain an IRR of 18.14%.

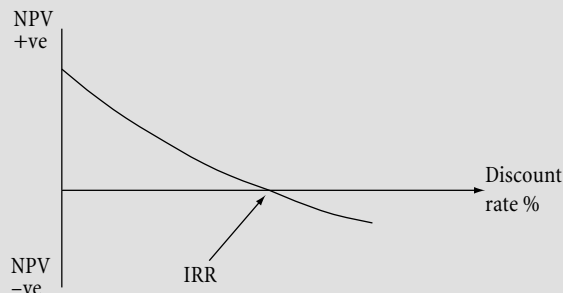


Figure 2.2: Relationship between NPV and IRR

The relationship between the IRR and the NPV of an investment is that while the NPV is the value of the projected returns from the investment using an appropriate discount rate (usually the company's cost of capital),

the IRR is the discount rate which results in the NPV being zero. For this reason it is common to hear the IRR referred to as a project's *breakeven* rate. A conventional investment is considered attractive if the IRR exceeds a company's cost of capital, as well as if the NPV is positive. In the context of the bond markets, if we assume that the discount rate applicable does indeed remain constant for the reinvestment of all cash flows arising from a financial instrument, the IRR can then be assumed to be the *yield to maturity* for that instrument. The yield to maturity is the main measure of the rate of return achieved from holding a bond, and is explored in depth in a later chapter.

2.5 Interpolation and extrapolation

Interest rates in the money markets are always quoted for standard maturities, for example overnight, "tom next" (the overnight interest rate starting tomorrow, or "tomorrow to the next"), spot next (the overnight rate starting two days forward), 1 week, 1 month, 2 months and so on up to 1 year. Figure 2.3 shows a typical brokers screen as seen on news services such as Reuters and Telerate.

Dow Jones Markets : martin_g Telerate 4734 Wed Dec 08 09:00:44 1999

08/12	8:54 GMT	[GARBAN INTERCAPITAL-EUROPE]				12/08 02:12	4734
	FRA	GBP CDS	DEPO	GBP INTERBANK	DEP	GBP REPO(GC)	
1X4	6.020-990	O/N	-	5 1/16-4	15/16	-	O/N
2X5	6.110-080	T/N	-	5 3/8 -5	1/4	-	T/N
3X6	6.230-200	1WK	-	5 1/4 -5	1/8	-	1WK
4X7	6.330-300	1MO	5 25/32-5 23/32	5 7/8 -5	13/16	-	2WK
5X8	6.420-390	2MO	5 15/16-5 7/8	5 31/32-5	29/32	-	3WK
6X9	6.510-480	3MO	6-5 15/16	6-5	15/16	-	1MO
9X12	6.760-730	4MO	6 1/32-5 31/32	6 1/32-5	31/32	-	2MO
		5MO	6 1/16-6	6 1/16-	6	-	3MO
1X7	6.240-210	6MO	6 1/8 -6 1/16	6 5/32-6	3/32	-	4MO
2X8	6.330-300	7MO	6 5/32-6 3/32	6 7/32-6	5/32	-	5MO
3X9	6.420-390	8MO	6 7/32-6 5/32	6 9/32-6	7/32	-	6MO
4X10	6.520-490	9MO	6 9/32-6 7/32	6 5/16-6	1/4	-	9MO
5X11	6.610-580	10M	6 11/32-6 9/32	6 3/8 -6	5/16	-	1YR
6X12	6.700-670	11M	6 13/32-6 11/32	6 7/16-6	3/8	-	
		12M	6 15/32-6 13/32	6 1/2 -6	7/16	-	

FRA 695-2040, EUROSTG 695-2030, GBP REPOS 695-2255

Figure 2.3: Broker's rates screen. ©Garban ICAP ©Dow-Jones Telerate. Reproduced with permission

If a bank or corporate customer wishes to deal for non-standard periods, an interbank desk will calculate the rate chargeable for such an "odd date" by *interpolating* between two standard period interest rates. If we assume that the rate for all dates in between two periods increases at the same steady state, we can calculate the required rate using the formula for *straight line* interpolation, shown at (2.38).

$$r = r_1 + (r_2 - r_1) \times \frac{n - n_1}{n_2 - n_1} \quad (2.38)$$

where

- r is the required odd-date rate for n days
- r_1 is the quoted rate for n_1 days
- r_2 is the quoted rate for n_2 days.

Let us imagine that the 1-month (30-day) offered interest rate is 5.25% and that the 2-month (60-date) offered rate is 5.75%. If a customer wishes to borrow money for a 40-day period, what rate should the bank charge? We can calculate the required 40-day rate using the straight line interpolation process. The increase in interest rates from 30 to 40 days is assumed to be 10/30 of the total increase in rates from 30 to 60 days. The 40-day offered rate would therefore be:

$$5.25 + (5.75 - 5.25) \times 10/30 = 5.4167\%.$$

EXAMPLE 2.9

- On an interbank desk Hussein is quoting the 7-day offered rate (the rate at which a bank will *offer* or lend money) at $5\frac{11}{16}\%$, while the 14-day rate is $5\frac{13}{16}\%$. What rate should he quote for the 10-day offered rate?

$$5.6875 + (5.8125 - 5.6875) \times 3/7 = 5.7411\%.$$

What about the case of an interest rate for a period that lies just before or just after two known rates and not roughly in between them? When this happens we *extrapolate* between the two known rates, again assuming a straight line relationship between the two rates and for a period after (or before) the two rates.

EXAMPLE 2.10

■ The 1-month offered rate is 5.25% while the 2-month rate is 5.75% as before. What is the 64-day rate?

$$5.25 + (5.75 - 5.25) \times 34/30 = 5.8167\%.$$

2.6 Measuring the rate of return

Rates of return are calculated in the market and by investors in order to measure the gain that has been achieved, as well as to compare the gains made by different investments. This is the market convention, presenting the profit made by an investment in percentage return figures rather than actual cash increments. In addition to comparing the performance of different investments, there are three other uses for rates of return.

- **Measuring historical performance:** a frequently used measure of investment performance is the historical rate of return, or the *realised rate of return* on an investment. This is the return that has already been realised as opposed to return anticipated in the future. In the US market this is also known as the *ex-post* rate of return.
- **Determining future investment:** investors often use historical rates of return to estimate future returns, and to gauge the level of risk associated with a particular security. Over a period of time, because of the higher associated risk, investors will expect a higher return from a higher risk stock compared to a less volatile one.
- **Estimating the cost of capital:** rates of return are also used to estimate a firm's cost of capital. Corporate decision makers use their firm's cost of capital when making capital allocation and budget decisions, and one method for estimating the appropriate discount rate to apply in NPV calculations uses the company's historical rate of return on equity.

The rate of return on an investment can be calculated in several ways and we will look at some of the methods in the rest of this section.

2.6.1 Simple rate of return

The simple rate of return measures the increase or decrease in the value of a given investment over a specified period of time. It is given by:

$$R = \frac{P_2 - P_1 + I}{P_1} \quad (2.39)$$

where

- P_1 is the initial value of the investment
- P_2 is the investment value at the end of the period
- I is the income earned during the investment.

For example a bond is purchased at a price of £100 and held for a year, during which a coupon of £8 is paid. At the end of the period the bond price is £108. The rate of return is:

$$\frac{108 - 100 + 8}{100} = 0.16 \text{ or } 16\%.$$

The simple rate of return is effective when measuring investment performance that has only one dividend payment at the end of the period. If dividends are paid more frequently or during the period the measurement loses accuracy.

2.6.2 The time-weighted rate of return

The simple rate of return can be adjusted to account for the timing of dividend or coupon payments and this is known as the *time weighted rate of return* or the *geometric mean rate of return*. If we take P as the initial value of an investment, FV as the final value, C_n as the payment received by the investment in year n and MV_n as the value of the investment when a dividend is received, the time-weighted rate of return is given by (2.40):

$$(1 + r)^N = \left(\frac{MV_1}{P} \right) \left(\frac{MV_2}{MV_1 + C_1} \right) \left(\frac{MV_3}{MV_2 + C_2} \right) \cdots \left(\frac{FV}{MV_{N-1} + C_{N-1}} \right). \quad (2.40)$$

Given that $MV_1/P = (1 + r_1)$ or one plus the return on the investment in the first period, and that $MV_2/(MV_1 + C_1) = (1 + r_2)$ and so on, the expression can be rewritten as shown at (2.41):

$$(1 + r)^N = (1 + r_1)(1 + r_2)(1 + r_3) \cdots (1 + r_N). \quad (2.41)$$

We can rearrange (2.41) to solve for r and this gives us (2.42) as shown.

$$r = ((1 + r_1)(1 + r_2)(1 + r_3) \cdots (1 + r_N))^{1/T} - 1. \quad (2.42)$$

Expression (2.42) illustrates how the time-weighted return is in fact the *geometric* return of each individual period return.

EXAMPLE 2.11

■ An initial investment of \$1000 is made which subsequently earns \$64 at the end of the first year, when the investment was valued at \$1118. At the end of the second year another \$64 is earned, when the investment value is \$1250. At the end of the third year the investment value is \$1339. What is the time-weighted rate of return earned by the investment?

$$\begin{aligned} r &= \left(\left(\frac{1118}{1000} \right) \left(\frac{1250}{1118 + 64} \right) \left(\frac{1339}{1250 + 64} \right) \right)^{1/3} - 1 \\ &= ((1.118)(1.0575)(1.019))^{1/3} - 1 = (1.2047)^{1/3} - 1 \\ &= 0.064 \text{ or } 6.40\%. \end{aligned}$$

2.6.3 Inflation-adjusted rate of return

Up to now we have been discussing rates of return calculated as the gain in the nominal cash value of an investment. In certain cases it is desirable to adjust the rate of return calculated on an investment to allow for the effects of inflation. In an inflationary environment the purchasing power of the domestic currency is eroded, so measurement of return can be modified to reflect this. In the UK inflation is measured using the *retail prices index* or RPI. The RPI measures the change in price of a specified basket of consumer goods. While RPI is the inflation index itself, there are three key percentage indicators that are used; these are the *headline* rate of inflation or RPI, the rate of inflation but excluding mortgage interest payments or RPIX and inflation excluding any rises in value-added tax rates or RPIY. In the US the consumer price index or CPI measures essentially the same thing.

The rate of inflation in any period can be measured by comparing the levels for two index numbers and is given by (2.43):

$$i = \frac{RPI_1 - RPI_0}{RPI_0} \quad (2.43)$$

where

i is the rate of inflation
 RPI_0 is the inflation index at the start of the period
 RPI_1 is the inflation index at the end of the period.

For example in June 1998 the UK's RPI was 163.4 and this had risen to 165.6 in June 1999. Therefore the inflation rate for the period June 1998-June 1999 is calculated as:

$$i = \frac{165.6 - 163.4}{163.4} = 0.01346 \text{ or } 1.355\%.$$

The *real rate of return* is the nominal rate of return adjusted for inflation. It can be calculated using (2.44).

$$R_{real} = \frac{1 + R_{nom}}{1 + i} - 1 \quad (2.44)$$

where

R_{real} is the real rate of return and R_{nom} is the nominal rate of return.

Equation (2.44) can be approximated by $R_{real} = R_{nom} - i$, and is derived from the Fisher relationship (Fisher 1930).

Note that if there is zero or very low inflation, the real rate of return will be equal to the nominal rate of return. If investments are made in an inflationary environment and the nominal rate of return is equal to the rate of inflation such that $R_{nom} = i$ then in real terms the rate of return is zero. This often leads to negative returns where rates are very low, for example with some types of bank accounts. In the UK current accounts and some deposit and instant access accounts are offered with interest rates below 1%. As the inflation rate has been consistently higher than 1% in the UK for some time now, account values will be declining in real terms.

2.6.4 Average rates of return

Where an investment is made up of a portfolio of assets the gain on the portfolio is calculated as an average return. An average is also used when measuring the return on a single asset over a period of years. The two main methods used are *arithmetic average* and *geometric average*.

The expression for calculating the average rate of return is given at (2.45):

$$R_A = \frac{\sum_{t=1}^m R_t}{m} \quad (2.45)$$

where

R_A is the average arithmetic rate of return
 t is the length of the period
 m is the number of observations.

The geometric method is an averaging method that compounds rates of return. That is, if £1 is invested in the first period, its future value will be $(1 + R_1)$ at the end of the period. We then assume that $£(1 + R_1)$ is invested in the second period, and at the end of period 2 the investment value will have risen to the value at the beginning of period 2 multiplied by the value of £1 invested in period 2. We illustrate this by saying that the investment value at the end of the second period is $(1 + R_1)(1 + R_2)$. More formally we can express the geometric return as:

$$R_G = \left(\prod_{t=1}^m (1 + R_t) \right)^{1/m} - 1 \quad (2.46)$$

where R_G is the geometric rate of return and the symbol $\prod$ means “take the product of”.³

The geometric average can be considered to be the actual growth rate of the assets. The arithmetic average should be used, on the other hand, when estimating the average performance across different securities for one period of time. The arithmetic average is also an unbiased estimate of future expected rates of return, and will exceed the geometric average whenever the rates of return are not constant.

2.7 Indices

An index is used to measure the rate of return for a basket of securities. In the UK the most familiar index is the FTSE-100, whose level is faithfully reported daily in the media. The FTSE-100 is made up of the largest 100 stocks, measured by market capitalisation, traded on the London Stock Exchange. The change in its index value can be taken to reflect broadly the overall performance of the stock market, although this is a very large approximation since there are over 4000 stocks listed on the London market. That said, an index level is often a useful indicator, and it is rare to find an index level rising if the general health of the economy is declining, or vice versa. There is a wide range of indices used across markets internationally, and they are all used to measure historical returns for a

³ For example $y = \prod_{i=1}^5 x_i$ means $y = (x_1)(x_2)(x_3)(x_4)(x_5)$.

group of securities, in the same way that the RPI measures the rise in retail prices. Indices are also used as a benchmark against which to measure a fund manager's performance. They are differentiated in the following three ways:

- by the type of securities included in them, such as equities or bonds, and the sector they are part of (for example, utilities stocks, or emerging markets stocks), as well as the number of securities included in each index;
- by the way the index is adjusted for any changes to its constituent stocks (such as a merger or takeover);
- by the method used to calculate the index level.

There are three main types of index, price-weighted, value-weighted and equally-weighted indices. Let us look at the way each of these index levels is calculated.

Index	Valuation method	Stocks represented
AMEX Composite	Value-weighted	The American Stock Exchange
AIM	Value-weighted	The Alternative Investment Market, London Stock Exchange smaller stock market
Australian Options Market	Value-weighted	Australian Stock Exchange
CAC 40	Value-weighted	MATIF exchange in France, largest 40 stocks
Dow Jones	Price-weighted	The 30 US companies of the Dow Jones industrial average
FTSE-100	Value-weighted	The Financial Times-Stock Exchange index, 100 largest London Stock Exchange companies by market capitalisation
Nikkei 225	Value-weighted	The 225 largest Japanese stocks
NYSE Composite	Value-weighted	All the stocks on the New York Stock Exchange
S&P 500	Value-weighted	The 500 largest US companies
SMI	Value-weighted	The 25 largest companies in Switzerland
TOPIX	Value-weighted	The Tokyo Stock Price Index, the 1100 stocks on the Tokyo Stock Exchange
Toronto 35	Value-weighted	The 35 largest Canadian stocks
Value Line	Geometric	Small company US shares

Table 2.3: Table of selected global stock indices.

2.7.1 The price-weighted index

In a price-weighted index the value is found by adding all the security prices and dividing by a *divisor*. The value of a price-weighted index at time t is given by (2.47).

$$I = \frac{1}{divisor} \sum_{i=1}^n P_{it} \quad (2.47)$$

where

- I is the index level
- P_{it} is the price of asset i in period t
- n is the number of stocks in the index.

The *divisor* is a number that is adjusted periodically for stock dividends and any other corporate actions.

The price-weighted index return is a relatively simple concept. A fund manager who wished to track such an index would simply purchase the same number of shares of each stock in the index. The Dow Jones Industrial Average index in the US is an example of a price-weighted index. However this method tends to result in higher priced stocks having greater influence in the level of the index, so it is not very common. The value-weighted index has been designed to remove this bias.

2.7.2 The value-weighted index

A value-weighted index is based on the total market capitalisation of the company whose security is represented in the index. The index level therefore takes into account share value rather than the absolute price level of individual stocks.

The level of a value-weighted index is given by (2.48):

$$I = \left(\frac{100}{\sum_{i=1}^N N_{i1} P_{i1}} \right) \sum_{i=1}^N N_{it} P_{it} \quad (2.48)$$

where

N_i is the number of shares of company i at time t
 P_i is the price of company i shares at time t .

The numerator 100 is taken to be the starting value of the index. Therefore if calculating the level of the FTSE-100 one would use 1000 in the numerator, as this index was re-based to 1000 in 1984.

The level of a value-weighted index is not affected by a corporate action such as a dividend payment or a rights issue, and this is considered to be an advantage of the method over the price-weighted index valuation.

2.7.3 The equal weight index

This index is calculated by assigning the same weight to each constituent security regardless of the security's price or the company's market capitalisation. If a fund manager wished to replicate the performance of an equally-weighted index, she would purchase an equal cash amount of each security. There are two ways to calculate the value of an equally-weighted index, the arithmetic method and the geometric method. In both cases the rate of return measured for each security over a specific period (usually one day) is measured. The arithmetic method value is given by (2.49):

$$I = I_{t-1} \left(1 + \frac{1}{n} \sum_{i=1}^N R_{it} \right) \quad (2.49)$$

where $\frac{1}{n} \sum R$ is the arithmetic average of the rates of return of all the index securities.

The geometric method, as its name suggests, takes the geometric average of the return for each security in the index over a specified time period. The value is given by

$$I = I_{t-1} \left(\prod_{i=1}^N (1 + R_{it}) \right)^{1/n} \quad (2.50)$$

In general the arithmetic method produces higher values over time compared to the geometric method.

2.7.4 Bond Indices

In the same way as the more familiar equity market indices, bond indices measure the return generated by a basket of fixed income stocks. Unlike an equity index such as the Dow or the FTSE-100 however, a bond index presents some complications that may make index valuation problematic. First, since bonds are always approaching in maturity, and because some are redeemed early, the set of bonds in a basket changes more frequently than the shares in an equity index. If we consider an hypothetical international "ten-year benchmark index", as a bond falls to less than say eight years maturity, it may be replaced by the current ten-year benchmark bond. This will have different risk characteristics to the bond it replaced and will trade differently in the market as a result. As the constituents of a bond index have to change more frequently, we may not always be comparing like-for-like when we consider historical index values. There is also the issue of bond coupon payments, which make up a significant proportion of a bond's overall return, and which must therefore be incorporated in the index valuation. Nevertheless bond indices are important for the same reason that equity indices are, and form the benchmark against which fund managers' performance is measured. Figure 2.4 summaries the attributes of three commonly-used US market bond indices.

	<i>Number of Issues</i>	<i>Maturity</i>	<i>Weighting</i>	<i>Reinvestment of coupons</i>
Lehman Brothers	6500	Over 1 year	Value-weighted	No
Merrill Lynch	5000	Over 1 year	Value-weighted	Yes
Salomon Brothers	5000	Over 1 year	Value-weighted	Yes, at 1-month T-Bill rate

Figure 2.4: Bond Market Indices. Source: Strata Consulting.

Appendices

APPENDIX 2.1 The Internal Rate of Return

A common method of measuring the return on an investment, the internal rate of return (IRR) is also frequently used in corporate finance analysis and project analysis. It is also known as the *money-weighted rate of return*. If there is only a single cash flow to consider the calculation involves the application of basic time value of money principles. Since $FV = PV(1 + r)^n$, the annual IRR on an investment that pays PV in n years' time can be obtained by solving the expression:

$$(1 + r)^n = FV/PV$$

which we can rearrange to calculate the IRR, which would be:

$$r = (FV/PV)^{1/n} - 1. \quad (2.51)$$

Consider an investment of £100 today which returns £131 in five years' time. The annual IRR is given by:

$$r = (131/100)^{1/5} - 1, \text{ which is } 0.05549 \text{ or } 5.55\%.$$

The above example assumes annual compounding. If compounding occurs more than once a year, the expression for the annual IRR is modified as shown at (2.52):

$$\left(1 + \frac{r}{m}\right)^{mN} = FV/PV \quad (2.52)$$

which is then rearranged to solve for r :

$$r = m\left((FV/PV)^{1/mN} - 1\right). \quad (2.53)$$

Let us consider our £100 investment example again but this time with semi-annual compounding. The annual IRR in such a case would then be:

$$r = 2\left((131/100)^{1/(2 \times 5)} - 1\right), \text{ which gives us } 5.47\%.$$

The limiting factor resulting from continuous compounding is given by (2.54).

$$e^{rN} = FV/PV. \quad (2.54)$$

A general principle is that if $x^b = y$, then b is known as the *logarithm* of y to base x . That is, we can say that $b = \log_x(y)$. If we apply this to (2.54) this results in:

$$rN = \log_e(FV/PV). \quad (2.55)$$

which means that rN is the logarithm of FV/PV to base e . Logarithms to base e are called natural logarithms and are shown as $b = \ln(y)$. This results in (2.55) being changed to:

$$r = \frac{1}{N} \ln(FV/PV). \quad (2.56)$$

If we apply (2.56) to our earlier example of the £100 investment over five years, but now assuming continuous compounding, the annual IRR is given by:

$$r = \frac{1}{5} \ln(131/100) = 0.0540054$$

or 5.4%. As in earlier illustrations the continuous compounding effect has reduced the IRR.

If there is more than one payment in a future cash flow, for example multiple payments arising out of a continuous project or bond instrument, it will not be possible to solve for the IRR analytically. Where this happens the solution for r can be found by using either (2.57) and (2.58) which can be solved using a numerical process of trial and error.

$$FV = \sum_{n=1}^N C_n (1+r)^{N-n}. \quad (2.57)$$

$$PV = \sum_n C_n (1+r)^{-n}. \quad (2.58)$$

The process of solving for multiple payments is known as numerical iteration and involves using the formula for linear interpolation. In the case of an irredeemable bond with perpetual coupon payments the IRR is more straightforward to calculate and is given by (2.59):

$$r = C/PV. \quad (2.59)$$

APPENDIX 2.2 Discount Factor Table

Years	Discount rate (%)												
	1	2	3	4	5	6	7	8	9	10	12	15	20
1	0.990099	0.980392	0.970874	0.961538	0.952381	0.943396	0.934579	0.925926	0.917431	0.909091	0.892857	0.869565	0.833333
2	0.980296	0.961169	0.942596	0.924556	0.907029	0.889996	0.873439	0.857339	0.841680	0.826446	0.797194	0.756144	0.694444
3	0.970590	0.942322	0.915142	0.888996	0.863838	0.839619	0.816298	0.793832	0.772183	0.751315	0.711780	0.657516	0.578704
4	0.960980	0.923845	0.888487	0.854804	0.822702	0.792094	0.762895	0.735030	0.708425	0.683013	0.635518	0.571753	0.482253
5	0.951466	0.905731	0.862609	0.821927	0.783526	0.747258	0.712986	0.680583	0.649931	0.620921	0.567427	0.497177	0.401878
6	0.942045	0.887971	0.837484	0.790315	0.746215	0.704961	0.666342	0.630170	0.596267	0.564474	0.506631	0.432328	0.334898
7	0.932718	0.870560	0.813092	0.759918	0.710681	0.665057	0.622750	0.583490	0.547034	0.513158	0.452349	0.375937	0.279082
8	0.923483	0.853490	0.789409	0.730690	0.676839	0.627412	0.582009	0.540269	0.501866	0.466507	0.403883	0.326902	0.232568
9	0.914340	0.836755	0.766417	0.702587	0.644609	0.591898	0.543934	0.500249	0.460428	0.424098	0.360610	0.284262	0.193807
10	0.905287	0.820348	0.744094	0.675564	0.613913	0.558395	0.508349	0.463193	0.422411	0.385543	0.321973	0.247185	0.161506
11	0.896324	0.804263	0.722421	0.649581	0.584679	0.526788	0.475093	0.428883	0.387533	0.350494	0.287476	0.214943	0.134588
12	0.887449	0.788493	0.701380	0.624597	0.556837	0.496969	0.444012	0.397114	0.355535	0.318631	0.256675	0.186907	0.112157
13	0.878663	0.773033	0.680951	0.600574	0.530321	0.468839	0.414964	0.367698	0.326179	0.289664	0.229174	0.162528	0.093464
14	0.869963	0.757875	0.661118	0.577475	0.505068	0.442301	0.387817	0.340461	0.299246	0.263331	0.204620	0.141329	0.077887
15	0.861349	0.743015	0.641862	0.555265	0.481017	0.417265	0.362446	0.315242	0.274538	0.239392	0.182696	0.122894	0.064905
16	0.852821	0.728446	0.623167	0.533908	0.458112	0.393646	0.338735	0.291890	0.251870	0.217629	0.163122	0.106865	0.054088
17	0.844377	0.714163	0.605016	0.513373	0.436297	0.371364	0.316574	0.270269	0.231073	0.197845	0.145644	0.092926	0.045073
18	0.836017	0.700159	0.587395	0.493628	0.415521	0.350344	0.295864	0.250249	0.211994	0.179859	0.130040	0.080805	0.037561
19	0.827740	0.686431	0.570286	0.474642	0.395734	0.330513	0.276508	0.231712	0.194490	0.163508	0.116107	0.070265	0.031301
20	0.819544	0.672971	0.553676	0.456387	0.376889	0.311805	0.258419	0.214548	0.178431	0.148644	0.103667	0.061100	0.026084
21	0.811430	0.659776	0.537549	0.438834	0.358942	0.294155	0.241513	0.198656	0.163698	0.135131	0.092560	0.053131	0.021737
22	0.803396	0.646839	0.521893	0.421955	0.341850	0.277505	0.225713	0.183941	0.150182	0.122846	0.082643	0.046201	0.018114
23	0.795442	0.634156	0.506692	0.405726	0.325571	0.261797	0.210947	0.170315	0.137781	0.111678	0.073788	0.040174	0.015095
24	0.787566	0.621721	0.491934	0.390121	0.310068	0.246979	0.197147	0.157699	0.126405	0.101526	0.065882	0.034934	0.012579
25	0.779768	0.609531	0.477606	0.375117	0.295303	0.232999	0.184249	0.146018	0.115968	0.092296	0.058823	0.030378	0.010483
26	0.772048	0.597579	0.463695	0.360689	0.281241	0.219810	0.172195	0.135202	0.106393	0.083905	0.052521	0.026415	0.008735
27	0.764404	0.585862	0.450189	0.346817	0.267848	0.207368	0.160930	0.125187	0.097608	0.076278	0.046894	0.022970	0.007280
28	0.756836	0.574375	0.437077	0.333477	0.255094	0.195630	0.150402	0.115914	0.089548	0.069343	0.041869	0.019974	0.006066
29	0.749342	0.563112	0.424346	0.320651	0.242946	0.184557	0.140563	0.107328	0.082155	0.063039	0.037383	0.017369	0.005055
30	0.741923	0.552071	0.411987	0.308319	0.231377	0.174110	0.131367	0.099377	0.075371	0.057309	0.033378	0.015103	0.004213

References

Higson, C., *Business Finance*, Butterworth, 1995.

Fabozzi, F., *Fixed Income Mathematics*, FF Associates, 1996.

Tuckman, B., *Fixed Income Securities*, John Wiley and Sons, 1996.

Weston, J.F., Copeland, T.E., *Managerial Finance*, Dryden, 1986.

Van Horne, J., *Financial Management and Policy*, 10th edition, Prentice Hall International, 1995, Chapters 16–20.

Questions and exercises

1. What is the future value of £250 in five years' time, assuming an interest rate of 5% per annum?
2. What is the present value of £2561 received in three years at a fixed interest rate of 8%, assuming:
 - (a) annual discounting and
 - (b) semi-annual discounting?
3. Aftab withdraws all the funds from his savings account, which amount to £849.16 after a period of 291 days at a rate of 6.49%. How much did he open the account with?
4. The rate earned on a 180-day deposit is 4.25%. What is the 180-day discount factor?
5. Nik is offered a choice between receiving £990 today or £1000 in three months' time. If interest rates are 7.75% which option should he choose?
6. The simple interest on a loan is 1.96% compounded monthly. What is the effective rate of interest?
7. An investment fund is used to buy an annuity that pays £2500 each year for ten years. If the annuity has a present value of £29,685, what is its money-weighted rate of return?
8. The rate quoted on a bank savings account is 3.1% paid semi-annually. What is the effective annual equivalent rate?
9. Fenton opens a building society step-up bond, which is a fixed-term investment paying 5.25% in the first year, 6% in the second year, 7% in the third year and 8.5% in the final year. How much will he receive at the end of the term? What is the equivalent annual compounded interest rate?
10. If an investment pays back £137 at the end of five years from an initial deposit of £100, what annual yield has been earned? Assume no interest payments during the term.
11. A deposit of £5000 for five years pays interest of 4.5% in the first two years and 5% in the final three years. What is the total value of the investment at the end of the term assuming quarterly compounding of interest?
12. On retirement Anita purchases a perpetual annuity with a yield of 4.5% per annum. How much must she invest in the annuity now in order to receive £4000 each year?
13. A corporate customer asks where you offer 22-day money at the moment. If the 2-week rate is 5.5625% and the 1-month rate is 5.6875%, what do you quote?
14. If the semi-annual yield to maturity of a bond is 5.38%, what is
 - (a) the equivalent annual yield to maturity?
 - (b) the equivalent quarterly yield to maturity?
15. The yield on a UK government bond is 6.5% while the yield on a sterling eurobond is 6.75%. Which paper is offering the better return?
16. A friend of yours is an independent financial advisor and one of her clients is about to retire with savings of £180,000. The client is keen to increase the level of his consumption by a fixed amount each year for the next 20 years and run the savings fund down to a zero balance at the end of the 20-year period. Assuming a rate of return on the fund of 6.8% fixed each year (and no withdrawals until one year has elapsed), how much should your IFA friend tell her client is the sum that can be spent each year?

17. Shailesh buys a house with a 25-year repayment mortgage of £120,000, and has managed to fix the interest rate chargeable at 8.45% per annum but with interest charged on a monthly basis. As it is a repayment mortgage, equal payments covering both principal and interest are made every month for the life of the mortgage. What monthly payments does Shailesh make?
18. Breeda is considering a business proposal, which sets out the following cash flow pattern. Using a discount rate of 15%, which she calculates to be roughly her cost of capital, what is the NPV of the cash flows? What is the IRR? Should Breeda invest in the project?

Today	—£25,000
1 year	£2,000
2 years	£5,000
3 years	£9,000
4 years	£15,000
5 years	£22,000

19. You are given the following government bonds and prices as at 19 April 2000. Calculate the relevant term discount factors.

<i>Coupon</i>	<i>Maturity</i>	<i>Price</i>
6.50%	7 December 2003	101.21
5.00%	7 June 2004	96.46
8.50%	7 December 2005	112.49
7.50%	7 December 2006	109.33
7.25%	7 December 2007	109.62
5.75%	7 December 2009	103.38

In constructing a discount function using the calculated discount factors, a junior analyst simply plots a graph between the points. Can you think of any flaws with this technique? (Hint: note that the data points are not available for the continuous range of maturities from the calculation date to the final 2009 date.)

3 Traditional Bond Pricing

In Chapter 2 we reviewed the concept of the time value of money. It is important to understand the principles of present and future value, compound interest and discounting, because they are all connected with bond pricing. The principles of pricing in the bond market are exactly the same as those in other financial markets, which states that the price of any financial instrument is equal to the net present value today of all the future cash flows from the instrument. For a bond the price is expressed as per 100 nominal of the bond, or “per cent”. So for example if the all-in price of a US dollar denominated bond is quoted as “98.00”, this means that for every \$100 nominal of the bond a buyer would pay \$98. The interest rate or discount rate used as part of the present value (price) calculation is key to everything, as it reflects where the bond is trading in the market and how it is perceived by the market. All the determining factors that identify the bond – those discussed in Chapter 1 and including the nature of the issuer, the maturity, the coupon and the currency – influence the interest rate at which a bond’s cash flows are discounted, which will be comparable to the rate used for comparable bonds. In this chapter we consider bond pricing for a plain vanilla instrument, and we make certain assumptions in order to keep the analysis simple. This is the “traditional” approach to bond pricing. In practice, more sophisticated techniques, some using forward rates, are used to calculate bond prices. We consider these, as well techniques for dealing with embedded options, later in the book.

3.1 Pricing a conventional bond

Since the price of a bond is equal to the present value of its cash flows, first we need to know the bond’s cash flows before determining the appropriate interest rate at which to discount the cash flows. We can then compute the price of the bond.

3.1.1 Bond cash flows

As we illustrated in Chapter 1 a vanilla bond’s cash flows are the interest payments or coupons that are paid during the life of the bond, together with the final redemption payment. It is possible to determine the cash flows with certainty only for conventional bonds of a fixed maturity. So for example, we do not know with certainty what the cash flow are for bonds that have embedded options and can be redeemed early. The coupon payments for conventional bonds are made annually, semi-annually or quarterly. Some bonds pay monthly interest.

Therefore a conventional bond of fixed redemption date is made up of an annuity (its coupon payments) and the maturity payment. If the coupon is paid semi-annually, this means exactly half the coupon is paid as interest every six months. Both gilts and US Treasuries pay semi-annual coupons. For example, the 5% gilt 2004 has the following cash flows:

$$\text{Semi-annual coupon} = £100 \times 0.025 = £2.50$$

$$\text{Redemption payment} = £100.$$

The bond was issued on 23 June 1999, is redeemed in June 2004, and pays coupon on 7 June and 7 December each year, so the bond is made up of 10 cash flows of £2.50 and one of £100. The time between coupon payments for any bond is counted as 1 period, so there are 10 periods between the first and last cash flows for the gilt in our example. The maturity payment is received 10 periods from issue.

3.1.2 The discount rate

The interest rate that is used to discount a bond’s cash flows (therefore called the *discount* rate) is the rate required by the bondholder. It is therefore known as the bond’s *yield*. The yield on the bond will be determined by the market and is the price demanded by investors for buying it, which is why it is sometimes called the bond’s *return*. The required yield for any bond will depend on a number of political and economic factors, including what yield is being earned by other bonds of the same class. Yield is always quoted as an annualised interest rate, so that for a semi-annually paying bond exactly half of the annual rate is used to discount the cash flows. The effective interest rate of half of an annual rate is higher than the annual rate and this was discussed in Chapter 2.

3.1.3 Bond pricing

The *fair price* of a bond is the present value of all its cash flows. Therefore when pricing a bond we need to calculate the present value of all the coupon interest payments and the present value of the redemption payment, and sum these. The price of a conventional bond that pays annual coupons can therefore be given by (3.1):

$$\begin{aligned}
 P &= \frac{C}{(1+r)} + \frac{C}{(1+r)^2} + \frac{C}{(1+r)^3} + \cdots + \frac{C}{(1+r)^N} + \frac{M}{(1+r)^N} \\
 &= \sum_{n=1}^N \frac{C}{(1+r)^n} + \frac{M}{(1+r)^N}
 \end{aligned} \tag{3.1}$$

where

- P is the price
- C is the annual coupon payment
- r is the discount rate (therefore, the required yield)
- N is the number of years to maturity (therefore, the number of interest periods in an annually-paying bond; for a semi-annual bond the number of interest periods is $N \times 2$)
- M is the maturity payment or par value (usually 100 per cent of currency).

For long-hand calculation purposes the first half of (3.1) is usually simplified and in fact can be expressed in two ways as shown by (3.2):

$$C \left(\frac{1 - \left(\frac{1}{(1+r)^N} \right)}{r} \right) \text{ or } \frac{C}{r} \left(1 - \frac{1}{(1+r)^N} \right). \tag{3.2}$$

The price of a bond that pays semi-annual coupons is given by the expression at (3.3), which is our earlier expression modified to allow for the twice-yearly discounting:

$$\begin{aligned}
 P &= \frac{C/2}{(1+\frac{1}{2}r)} + \frac{C/2}{(1+\frac{1}{2}r)^2} + \frac{C/2}{(1+\frac{1}{2}r)^3} + \cdots + \frac{C/2}{(1+\frac{1}{2}r)^{2N}} + \frac{M}{(1+\frac{1}{2}r)^{2N}} \\
 &= \sum_{t=1}^{2N} \frac{C/2}{(1+\frac{1}{2}r)^t} + \frac{M}{(1+\frac{1}{2}r)^{2N}} \\
 &= \frac{C}{r} \left(1 - \frac{1}{(1+\frac{1}{2}r)^{2N}} \right) + \frac{M}{(1+\frac{1}{2}r)^{2N}}.
 \end{aligned} \tag{3.3}$$

Note how we set $2N$ as the power to which to raise the discount factor, as there are two interest payments every year for a bond that pays semi-annually. Therefore a more convenient function to use might be the number of interest periods in the life of the bond, as opposed to the number of years to maturity, which we could set as n , allowing us to alter the equation for a semi-annually paying bond as:

$$P = \frac{C}{r} \left(1 - \frac{1}{(1+\frac{1}{2}r)^n} \right) + \frac{M}{(1+\frac{1}{2}r)^n}. \tag{3.4}$$

The formula at (3.4) calculates the fair price on a coupon payment date, so that there is no *accrued interest* incorporated into the price. It also assumes that there is an even number of coupon payments dates remaining before maturity.

The date used as the point for calculation is the *settlement date* for the bond, the date on which a bond will change hands after it is traded. For a new issue of bonds the settlement date is the day when the stock is delivered to investors and payment is received by the bond issuer. The settlement date for a bond traded in the *secondary market* is the day that the buyer transfers payment to the seller of the bond and when the seller transfers the bond to the buyer. Different markets will have different settlement conventions, for example UK gilts normally settle one business day after the trade date (the notation used in bond markets is “T + 1”) whereas Eurobonds settle on T + 3.

The term *value date* is sometimes used in place of settlement date, however the two terms are not strictly synonymous. A settlement date can only fall on a business date, so that a gilt traded on a Friday will settle on a Monday. However a value date can sometimes fall on a non-business day, for example when *accrued interest* is being calculated.

If there is an odd number of coupon payment dates before maturity the formula at (3.4) is modified as shown in (3.5):

$$P = \frac{C}{r} \left(1 - \frac{1}{\left(1 + \frac{1}{2}r\right)^{2N+1}} \right) + \frac{M}{\left(1 + \frac{1}{2}r\right)^{2N+1}}. \quad (3.5)$$

The standard formula also assumes that the bond is traded for a settlement on a day that is precisely one interest period before the next coupon payment. The price formula is adjusted if dealing takes place in between coupon dates. If we take the *value date* (almost always the settlement date, although unlike the settlement date the value date can fall on a non-working day) for any transaction, we need to calculate the number of calendar days from this day to the next coupon date. We then use the following ratio i when adjusting the exponent for the discount factor:

$$i = \frac{\text{Days from value date to next coupon date}}{\text{Days in the interest payment}}.$$

The number of days in the interest period is the number of calendar days between the last coupon date and the next one, and it will depend on the day count basis used for that specific bond; this is covered in the section on day counts. The price formula is then modified as shown at (3.6).

$$P = \frac{C}{(1+r)^i} + \frac{C}{(1+r)^{1+i}} + \frac{C}{(1+r)^{2+i}} + \cdots + \frac{C}{(1+r)^{n-1+i}} + \frac{M}{(1+r)^{n-1+i}} \quad (3.6)$$

where the variables C , M , n and r are as before. Note that (3.6) assumes r for an annually-paying bond and is adjusted to $r/2$ for a semi-annually paying bond.

EXAMPLE 3.1

In these examples we illustrate the long-hand price calculation, using both expressions for the calculation of the present value of the annuity stream of a bond's cash flows.

3.1(i)

- Calculate the fair pricing of a UK Gilt, the 9% Treasury 2008, which pays semi-annual coupons, with the following terms:

$C = \text{£}9.00$ per £100 nominal

$M = \text{£}100$

$N = 10$ years (that is, the calculation is for value the 13 October 1998)

$r = 4.98\%$

$$\begin{aligned} P &= \frac{\text{£}9.00}{0.0498} \left(1 - \frac{1}{\left(1 + \frac{1}{2}(0.0498)\right)^{20}} \right) + \frac{\text{£}100}{\left(1 + \frac{1}{2}(0.0498)\right)^{20}} = \text{£}70.2175 + \text{£}61.1463 \\ &= \text{£}131.3638. \end{aligned}$$

The fair price of the gilt is £131.3638, which is composed of the present value of the stream of coupon payments (£70.2175) and the present value of the return of the principal (£61.1463).

3.1(ii)

- What is the price of a 5% coupon sterling bond with precisely 5 years to maturity, with semi-annual coupon payments, if the yield required is 5.40%?

As the cash flows for this bond are 10 semi-annual coupons of £2.50 and a redemption payment of £100 in 10

six-month periods from now, the price of the bond can be obtained by solving the following expression, where we substitute $C = 2.5$, $n = 10$ and $r = 0.027$ into the price equation (the values for C and r reflect the adjustments necessary for a semi-annual paying bond):

$$P = 2.5 \left(\frac{1 - \left(\frac{1}{(1.027)^{10}} \right)}{0.027} \right) + \frac{100}{(1.027)^{10}} = 21.65574 + 76.61178 \\ = \text{£}98.26752.$$

The price of the bond is £98.2675 per £100 nominal.

3.1(iii)

- What is the price of a 5% coupon euro bond with five years to maturity paying annual coupons, again with a required yield of 5.4%?

In this case there are five periods of interest, so we may set $C = 5$, $n = 5$, with $r = 0.05$.

$$P = 5 \left(\frac{1 - \left(\frac{1}{(1.054)^5} \right)}{0.054} \right) + \frac{100}{(1.054)^5} = 21.410121 + 76.877092 \\ = \text{£}98.287213.$$

Note how the annual-paying bond has a slightly higher price for the same required annualised yield. This is because the semi-annual paying sterling bond has a higher *effective* yield than the euro bond, resulting in a lower price.

3.1(iv)

- Consider our 5% sterling bond again, but this time the required has risen and is now 6%. This makes $C = 2.5$, $n = 10$ and $r = 0.03$.

$$P = 2.5 \left(\frac{1 - (1/(1.03)^{10})}{0.03} \right) + \frac{100}{(1.03)^{10}} = 21.325507 + 74.409391 = \text{£}95.7349.$$

As the required yield has risen, the discount rate used in the price calculation is now higher, and the result of the higher discount is a lower present value (price).

3.1(v)

- Calculate the price of our sterling bond, still with five years to maturity but offering a yield of 5.1%.

$$P = 2.5 \left(\frac{1 - (1/(1.0255)^{10})}{0.0255} \right) + \frac{100}{(1.0255)} = 21.823737 + 77.739788 = \text{£}99.563525.$$

To satisfy the lower required yield of 5.1% the price of the bond has fallen to £99.56 per £100.

3.1(vi)

- Calculate the price of the 5% sterling bond one year later, with precisely four years left to maturity and with the required yield still at the original 5.40%. This sets the terms in 3.1(i) unchanged, except now $n = 8$.

$$P = 2.5 \left(\frac{1 - (1/(1.027)^8)}{0.027} \right) + \frac{100}{(1.027)^8} = 17.773458 + 80.804668 = \text{£}98.578126.$$

The price of the bond is £98.58. Compared to 3.1(ii) this illustrates how, other things being equal, the price of a bond will approach par (£100 per cent) as it approaches maturity.

3.1.4 Pricing undated bonds

There also exist *perpetual* or *irredeemable* bonds which have no redemption date, so that interest on them is paid indefinitely. They are also known as *undated* bonds or *consols*. An example of an undated bond is the 3½% War

Loan, a gilt originally issued in 1916. Most undated bonds date from a long time in the past and it is unusual to see them issued today. In structure the cash flow from an undated bond can be viewed as a continuous annuity. The fair price of such a bond is given from (3.4) by setting $T = \infty$, such that

$$P = C/r$$

where the inputs C and r are as before.

3.1.5 Bond price quotations

The convention in most bond markets is to quote prices as a percentage of par. The value of par is assumed to be 100 units of currency unless otherwise stated. A sterling bond quoted at an *offer* price of £98.45 means that £100 nominal of the bond will cost a buyer £98.45. A bond selling at below par is considered to be trading at a *discount*, while a price above par means the bond is trading at a *premium* to par. Do not confuse the term trading at a discount with a discount instrument however, which generally refers to a zero-coupon bond.

In most markets bond prices are quoted in decimals, in minimum increments of 1/100ths. This is the case with Eurobonds, euro denominated bonds and gilts. Certain markets including the US Treasury market and certain Commonwealth markets, such as Indian and South African government bonds, quote prices in *ticks*, where the minimum increment is 1/32nd. One tick is therefore equal to 0.03125. A US Treasury might be priced at “98-05” which means “98 and 5 ticks”. This is equal to 98 and 5/32nds which is 98.15625.

EXAMPLE 3.2

- What is the total consideration for £5 million nominal of a gilt, where the price is 114.50?

The price of the gilt is £114.50 per £100, so the consideration is $1.145 \times 5,000,000 = £5,725,000$.

- What consideration is payable for \$5 million nominal of a US Treasury, quoted at an all-in price of 99-16?

The US Treasury price is 99-16, which is equal to 99 and 16/32, or 99.50 per \$100. The consideration is therefore $0.9950 \times 5,000,000 = \$4,975,000$.

If the price of a bond is below par the total consideration is below the nominal amount, whereas if it is priced above par the consideration will be above the nominal amount.

3.2 Pricing zero-coupon bonds

The previous section dealt with pricing for conventional coupon-bearing bonds. Bonds that do not pay a coupon during their life are known as zero-coupon bonds, and the price for these bonds is determined by modifying (3.1) to allow for the fact that $C = 0$. We know that the only cash flow is the maturity payment, so we may set the price as:

$$P = \frac{M}{(1 + r)^N} \quad (3.7)$$

where M and r are as before and N is the number of years to maturity. The important factor is to allow for the same number of interest periods as coupon bonds of the same currency. That is, even though there are no actual coupons, we calculate prices and yields on the basis of a *quasi-coupon* period. For a US dollar or a sterling zero-coupon bond, a five-year zero-coupon bond would be assumed to cover ten quasi-coupon periods, which would set the price equation as:

$$P = \frac{M}{(1 + \frac{1}{2}r)^n} \quad (3.8)$$

We have to note carefully the quasi-coupon periods in order to maintain consistency with conventional bond pricing.

EXAMPLE 3.3(i)

- Calculate the price of a gilt *strip* with a maturity of precisely 5 years, where the required yield is 5.40%.

These terms allow us to set $N = 5$ so that $n = 10$, $r = 0.054$ (so that $r/2 = 0.027$), with $M = 100$ as usual.

$$P = 100/(1.027)^{10} = £76.611782.$$

3.3(ii)

- Calculate the price of a French government bond zero-coupon bond with precisely five years to maturity, with the same required yield of 5.40%.

$$P = 100 / (1.054)^5 = 76.877092.$$

3.3 Clean and dirty bond prices**3.3.1 Accrued interest**

Our discussion of bond pricing up to now has ignored coupon interest. All bonds (except zero-coupon bonds) accrue interest on a daily basis, and this is then paid out on the coupon date. The calculation of bond prices using present value analysis does not account for coupon interest or *accrued interest*. In all major bond markets the convention is to quote price as a *clean price*. This is the price of the bond as given by the net present value of its cash flows, but excluding coupon interest that has accrued on the bond since the last dividend payment. As all bonds accrue interest on a daily basis, even if a bond is held for only one day, interest will have been earned by the bondholder.¹ However we have referred already to a bond's *all-in price*, which is the price that is actually paid for the bond in the market. This is also known as the *dirty price* (or *gross price*), which is the clean price of a bond plus accrued interest. In other words the accrued interest must be added to the quoted price to get the total consideration for the bond.

Accruing interest compensates the seller of the bond for giving up all of the next coupon payment even though they will have held the bond for part of the period since the last coupon payment. The clean price for a bond will move with changes in market interest rates; assuming that this is constant in a coupon period, the clean price will be constant for this period. However the dirty price for the same bond will increase steadily from one interest payment date until the next one. On the coupon date the clean and dirty prices are the same and the accrued interest is zero. Between the coupon payment date and the next *ex dividend* date the bond is traded *cum dividend*, so that the buyer gets the next coupon payment. The seller is compensated for not receiving the next coupon payment by receiving accrued interest instead. This is positive and increases up to the next *ex dividend* date, at which point the dirty price falls by the present value of the amount of the coupon payment. The dirty price at this point is below the clean price, reflecting the fact that accrued interest is now negative. This is because after the *ex dividend* date the bond is traded “*ex dividend*”; the seller not the buyer receives the next coupon and the buyer has to be compensated for not receiving the next coupon by means of a lower price for holding the bond.

The net interest accrued since the last *ex dividend* date is determined as follows:

$$AI = C \times \left(\frac{N_{xt} - N_{xc}}{\text{Day Base}} \right) \quad (3.9)$$

where

AI	is the next accrued interest
C	is the bond coupon
N_{xc}	is the number of days between the <i>ex dividend</i> date and the coupon payment date (7 business days for UK gilts)
N_{xt}	is the number of days between the <i>ex dividend</i> date and the date for the calculation
Day Base	is the day count base (usually 365 or 360).

Interest accrues on a bond from and including the last coupon date up to and excluding what is called the *value date*. The value date is almost always the *settlement* date for the bond, or the date when a bond is passed to the buyer and the seller receives payment. Interest does not accrue on bonds whose issuer has subsequently gone into default. Bonds that trade without accrued interest are said to be trading *flat* or *clean*. By definition therefore,

¹ This is an accounting convention, the idea is that interest paid on one day in the year is in fact “accrued” on a pro-rata basis for each day that the bond is held. Certain authors have suggested that accrued interest treatment has no real economic meaning (see, for example, Van Deventer and Imai (1997), Chapter 1).

$$\text{Clean price of a bond} = \text{Dirty price} - AI.$$

For bonds that are trading ex-dividend, the accrued coupon is negative and would be subtracted from the clean price. The calculation is given by (3.10):

$$AI = -C \times \frac{\text{days to next coupon}}{\text{Day Base}}. \quad (3.10)$$

Certain classes of bonds, for example US Treasuries and Eurobonds, do not have an *ex dividend* period and therefore trade *cum dividend* right up to the coupon date.

3.3.2 Accrual day-count conventions

The accrued interest calculation for a bond is dependent on the day-count basis specified for the bond in question. We have already seen that when bonds are traded in the market the actual consideration that changes hands is made up of the clean price of the bond together with the accrued that has accumulated on the bond since the last coupon payment; these two components make up the dirty price of the bond. When calculating the accrued interest, the market will use the appropriate day-count convention for that bond. A particular market will apply one of five different methods to calculate accrued interest; these are:

actual/365	Accrued = Coupon $\times$ days/365
actual/360	Accrued = Coupon $\times$ days/360
actual/actual	Accrued = Coupon $\times$ days/actual number of days in the interest period
30/360	See below
30E/360	See below.

When determining the number of days in between two dates, include the first date but not the second; thus, under the actual/365 convention, there are 37 days between 4 August and 10 September. The last two conventions assume 30 days in each month, so for example there are “30 days” between 10 February and 10 March. Under the 30/360 convention, if the first date falls on the 31st, it is changed to the 30th of the month, and if the second date falls on the 31st *and* the first date is on the 30th or 31st, the second date is changed to the 30th. The difference under the 30E/360 method is that if the second date falls on the 31st of the month it is automatically changed to the 30th.

The day count basis, together with the coupon frequency, of selected major government bond markets around the world is given in Table 3.1.

Market	Coupon frequency	Day-count basis	Ex-dividend period
Australia	Semi-annual	actual/actual	Yes
Austria	Annual	actual/actual	No
Belgium	Annual	actual/actual	No
Canada	Semi-annual	actual/actual	No
Denmark	Annual	30E/360	Yes
Eurobonds	Annual	30/360	No
France	Annual	actual/actual	No
Germany	Annual	actual/actual	No
Eire	Annual	actual/actual	No
Italy	Annual	actual/actual	No
New Zealand	Semi-annual	actual/actual	Yes
Norway	Annual	actual/365	Yes
Spain	Annual	actual/actual	No
Sweden	Annual	30E/360	Yes
Switzerland	Annual	30E/360	No
United Kingdom	Semi-annual	actual/actual	Yes
United States	Semi-annual	actual/actual	No

Table 3.1: Government bond market conventions.

With the introduction of the Euro in eleven countries of the European Union in 1999, member countries harmonised certain aspects across their respective bond markets, including day-count basis; the euro day basis is act/act for bonds issued in euros.

EXAMPLE 3.4 Accrual calculation for 7% Treasury 2002

- This gilt has coupon dates of 7 June and 7 December each year. £100 nominal of the bond is traded for value 27 August 1998. What is accrued interest on the value date?

On the value date 81 days has passed since the last coupon date. Under the old system for gilts, act/365, the calculation was:

$$7 \times \frac{81}{365} = 1.55342.$$

Under the current system of act/act, which came into effect for gilts in November 1998, the accrued calculation uses the actual number of days between the two coupon dates, giving us:

$$7 \times \frac{81}{183} \times 0.5 = 1.54918.$$

EXAMPLE 3.5

- Mansur buys £25,000 nominal of the 7% 2002 gilt for value on 27 August 1998, at a price of 102.4375. How much does he actually pay for the bond?

The clean price of the bond is 102.4375. The dirty price of the bond is $102.4375 + 1.55342 = 103.99092$. The total consideration is therefore $1.0399092 \times 25,000 = £25,997.73$.

EXAMPLE 3.6

- A Norwegian government bond with a coupon of 8% is purchased for settlement on 30 July 1999 at a price of 99.50. Assume that this is 7 days before the coupon date and therefore the bond trades ex-dividend. What is the all-in price?

The accrued interest = $-8 \times 7/365 = -0.153424$. The all-in price is therefore $99.50 - 0.1534 = 99.3466$.

EXAMPLE 3.7

- A bond has coupon payments on 1 June and 1 December each year. What is the day-base count if the bond is traded for value date on 30 October, 31 October and 1 November 1999 respectively? There are 183 days in the interest period.

	30 October	31 October	1 November
Act/365	151	152	153
Act/360	151	152	153
Act/Act	151	152	153
30/360	149	150	151
30E/360	149	150	150

3.4 Bond price and yield relationship

An examination of the bond price formula tells us that the yield and price for a bond are closely related. A key aspect of this relationship is that the price changes in the opposite direction to the yield. This is because the price of the bond is the net present value of its cash flows; if the discount rate used in the present value calculation increases, the present values of the cash flows will decrease. This occurs whenever the yield level required by bondholders increases. In the same way if the required yield decreases, the price of the bond will rise. This property was observed in Example 3.1. As the required yield decreased the price of the bond increased, and we observed the same relationship when the required yield was raised.

Table 3.2 shows the prices for a hypothetical 7% coupon, quoted for settlement on 10 August 1999 and maturing on 10 August 2004. The bond pays annual coupons on a 30/360 basis. The prices are calculated by inserting the required yield values into the standard formulae for a set of cash flows; we can calculate the present value of the annuity

stream represented by the bond and the present value of the final maturity payment. Note that when the required yield is at the same level as the bond's fixed coupon (in this case 7%) the price of the bond is 100 per cent, or *par*.

Yield (%)	Price	Yield (%)	Price
4.0	113.3555	7.5	97.9770
4.5	110.9750	8.0	96.0073
5.0	108.6590	8.5	94.0890
5.5	106.4054	9.0	92.2207
6.0	104.2124	9.5	90.4007
6.5	102.0778	10.0	88.6276
7.0	100.0000		

Table 3.2: Prices and yields for a 7% five-year bond.

The relationship between any bond's price and yield at any required yield level is illustrated in exaggerated fashion in Figure 3.1 which is obtained if we plot the yield against the corresponding price; this shows a *convex* curve.

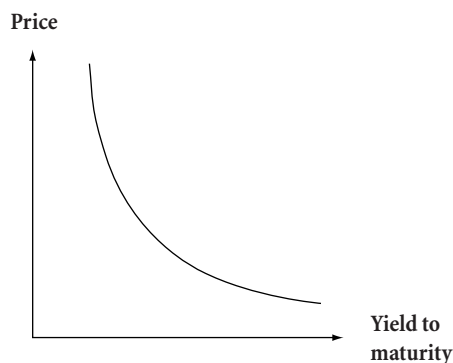


Figure 3.1: The price/yield relationship.

3.4.1 Coupon, yield and price relationship

The bond markets are also known as the “fixed income” or “fixed interest” markets. This reflects the fact that the coupon for conventional bonds is fixed, and in most cases the maturity date is also fixed. Therefore when required yield levels in the market change, the price is the only factor that can change to reflect the new market yield levels. We saw in Table 3.2 how the price of our hypothetical 7% five-year bond changed as the required yield changed. This is an important result. Let us consider the situation; if the required yield in the market for our 7% bond is fixed, investors will be happy to hold the bond. What if required yields subsequently rise above the 7% level? Bondholders will be unhappy as they are now being paid 7% when elsewhere in the market higher yields are available. The market price of the bond will therefore change so that the yield on the bond changes, to compensate bondholders. If the required yield for the bond changes to 8%, we see from Table 3.2 that the price has fallen from par to 96.00. If this did not happen bondholders would sell the 7% issue and buy a bond that was yielding 8%; the market price mechanism ensures that this does not happen. If the situation is reversed the price mechanism will again operate to equalise the yield on the bond with those prevalent in the marketplace. If required yields drop to 6% the bond price rises to 104.21, because bondholders would now be receiving 7% when yields available in the market are now only 6%. We can see then that when required yield in the market is equal to a bond's coupon, the bond price will be par; the price will move respectively above or below par if required yields are below or above the coupon rate.

When merchant banks issue bonds on behalf of borrowers, they will set a bond's coupon at the level that would make the bond price equal to par. This means that the bond coupon will be equal to the yield required by the market at the time of issue. The bond's price would then fluctuate as market yields changed, as we showed in Table 3.2. Investors generally prefer to pay par or just under par when they buy a new issue of bonds, which is why a merchant

bank will set the coupon that equates the bond price to par or in a range between 99.00 and par. The reason behind this preference to pay no more than par is often purely emotional, since an issue price above par would simply indicate a coupon higher than the market rate. However as many fund managers and investors buy a bond and hold it until maturity, one often finds this prejudice against paying over par for a new issue.

What will happen if market yields remain unchanged during the life of a bond from when it was issued? In this unlikely scenario the price of a conventional bond will remain unchanged at par. The price of any bond will ultimately equal its redemption value, which is par. Therefore a bond that is priced at a premium or a discount will gradually converge to par as it approaches maturity. This is illustrated in Tables 3.3 and 3.4 where we show the price of our hypothetical 7% bond at each coupon date if the required yield is at 6% and 8% throughout the life of the bond.

As a bond approaches maturity there are fewer and fewer coupon payments, so that progressively more of the bond's price is made up of the present value of the final redemption payment. The present value of this payment will steadily increase as the maturity date is approached, since it is being discounted over a shorter period of time. In Table 3.3 our hypothetical bond is priced at a premium, reflecting the 6% required yield level. Assuming the required yield remains at 6% (a very unrealistic assumption), the bond price progressively decreases as it converges towards the maturity price of par. The present value of the annuity cash flows of the bond steadily declines as we get fewer of them, and this is not offset by the increase in value of the maturity payment. Hence the price of the bond steadily declines. The opposite happens when the bond price starts off at a discount, where the increase in the value of the maturity payment outweighs the decrease in the price of the coupons, so that the bond price steadily converges to par.

Year	Price
10-Aug-99	104.2124
10-Aug-00	103.4651
10-Aug-01	102.6730
09-Aug-02	101.8334
11-Aug-03	100.9398
Maturity date	100.0000

Table 3.3: Price of 7% five-year bond approaching maturity, yield of 6%

Year	Price
10-Aug-99	96.0073
10-Aug-00	96.6879
10-Aug-01	97.4229
09-Aug-02	98.2137
11-Aug-03	99.0750
Maturity date	100.0000

Table 3.4: Price of 7% five-year bond approaching maturity, yield of 8%

EXAMPLE 3.8 Illustration of price/yield changes for two selected gilts

- Prices of selected gilts on 14 July 1999 for next-day settlement.

UK Treasury stock 5% 2004		UK Treasury stock 5¾% 2009	
Price	Yield%	Price	Yield%
98.61750	5.324	104.73750	5.155
98.64875	5.317	104.76875	5.151
98.68000	5.309*	104.80000	5.147 *
98.71100	5.302	104.83125	5.143
98.74200	5.295	104.86250	5.140

*actual quoted price at the time of asking

SUMMARY: Of the price/yield relationship

- At issue if a bond is priced at par, its coupon will equal the yield that the market requires from the bond
- If the required yield rises above the coupon rate, the bond price will decrease
- If the required yield goes below the coupon rate, the bond price will increase.

3.4.2 Determinants of the required market yield

In this chapter we have referred to the yield that is “required” by the market at any one time. Just as there are many different types of bond and many different types of borrower, so there are different types of yield. The price of any bond will change in line with changes in required yield, so that straight away we can see that it is not price changes that we are really interested in, but yields. It is a change in required yield that will drive a change in price.² So in the bond market we are concerned with examining the determinants of market yields.

The principal quoted yield in any market is the government bond yield. This is the yield on a domestic market’s government bonds. The required yield on these bonds is mainly a function of the central bank’s *base rate* or *minimum lending rate*, set by the government or central bank. Other factors will also impact the yield, including the relative size of the public sector budget deficit and national debt as a percentage of the national product (usually measured as Gross Domestic Product or Gross National Product), the economic policies that are adopted, and of course supply and demand for government bonds themselves. A change in any of these factors can and do affect government bond prices. While it is common to view government bonds as the safest credit for investors, this really only applies to the largest developed country markets. Certain countries within the Organisation for Economic Co-operation and Development (OECD) for example, such as Italy, Greece, South Korea and Mexico do not have their government debt given the highest possible rating by credit analysts.

Bonds issued by non-sovereign borrowers will be priced off government bonds, which means that the yields required on them will be at some level above their respective government bond yields, if they are domestic currency bonds. Bond yields are often quoted as a *yield spread* over the equivalent government bond. This is known as the *credit spread* on a bond. A change in the required credit spread for any bond will affect the bond’s price. Credit spreads will fluctuate for a variety of reasons, including when there is a change in the way the borrower is perceived in the market (such as a poor set of financial results by a corporate), which will affect the rate at which the borrower can raise funds. Credit spreads can sometimes change because comparable bonds yields change, as well as due to supply and demand factors and liquidity factors. As one might imagine it is important for investors and traders to be aware of changes in market yields and requirements, and staying ahead of market intelligence is a key part of bond trading. All these issues will be looked at in depth in later chapters.

Selected bibliography and references

Steiner, R., *Mastering Financial Calculations*, FT Pitman, 1998.

Fabozzi, F., *Bond Markets, Analysis and Strategies*, Prentice Hall, 1989, Chapter 2.

Van Deventer, D., Imai, K., *Financial Risk Analytics*, Irwin, 1997, Chapter 1.

Questions and exercises

1. A bond is traded for settlement on 1 August 1999. The bond has a coupon of 7%, pays annual coupon on 1 August each year and matures on 1 August 2009. What is the price of the bond if the required yield for the bond is 6%?
2. What is the price of the bond in question 1 if it paid semi-annual coupon?
3. A Eurobond is trading at 103.47 when the yield required on it by the market rises by 10 basis points. What will happen to the bond price?
4. A UK government bond (gilt) with a coupon of 7½% will be redeemed at par on 15 May 2007. What is the price of the bond on 16 May 1998 if the yield to maturity is 9%?
5. Abeda is expecting life assurance contracts in her name to expire in three years and four years. Both contracts will pay her £50,000. If she present values both sums at the current high-street yield rate of 4¾%, which contract has the higher value?

² We have observed how the price of a bond will gradually converge towards par as it approaches maturity, irrespective of the price it trades at during its life. However this is an automatic process and a mechanical price change that is part of the properties of a bond. As such it need not concern us unduly.

6. A gilt with coupon of $7\frac{1}{4}\%$ pays interest on 7 June and 7 December each year. What is the accrued interest if it is traded for value on 1 September 1999?
7. A fund manager buys €10million nominal of a 9% five-year Eurobond on its issue date, priced at par. What will the holding be worth on maturity if the coupons are re-invested at a rate of $7\frac{3}{4}\%$?
8. A 6% gilt is trading at par (£100) and had precisely five years to maturity. Calculate:
 - (a) the price of the gilt when the yield is 5.43%
 - (b) the change in the price if the yield changes to 5.3%
 - (c) the change in price if the yield subsequently changes to 5.5%
9. the accrued interest on the bond if it trades for value on 19 October 1999, with coupon dates on 7 June and 7 December.
10. Ingrid purchases an 8% coupon gilt at a price of 104.5675, which also has 83 days interest accrued. What total consideration does she pay if he buys £50,000 nominal?
11. What is the total consideration for \$10 million of a 5% bond trading at a price of 98.7687 with:
 - (a) 90 days accrued interest
 - (b) when it is ex-dividend and 5 days before the coupon date. Assume the bond has an actual/365 day-count basis.
12. Kirsten invests in a ten-year 8% government bond at a price of 100.45. One year later the price has gone down to 98.00. What factors *might* have contributed to the decline in price of her bond holding? What has happened to market yields over the year?
13. Calculate the prices of the following bonds, assuming:
 - (a) annual coupon payments
 - (b) semi-annual coupons payments and
 - (c) quarterly coupon payments.

Bond	Coupon(%)	Term to maturity	Yield (%)
1	5	5	5.05
2	7	7	5.15
3	7	8	5.25
4	8	10	5.45
5	6	20	5.65

4 Bond Yield Measurement

The discussion in Chapter 3 concentrated on how to calculate the price of a bond using an appropriate discount rate known as the bond's *yield*. Our brief discussion of corporate finance project appraisal tells us that this is the same as calculating a *net present value* at the selected discount rate. We can use reverse this procedure to find the yield of a bond where the price is known, which would be equivalent to calculating the bond's *internal rate of return* (IRR). As we saw from the previous chapter there is no equation for this calculation and a solution is obtained using numerical iteration. The IRR calculation is taken to be a bond's *yield to maturity* or *redemption yield* and is one of various yield measures used in the markets to estimate the return generated from holding a bond. In this chapter we will consider these various measures as they apply to plain vanilla bonds.

In most markets bonds are generally traded on the basis of their prices but because of the complicated patterns of cash flows that different bonds can have, they are generally compared in terms of their yields. This means that a market-maker will usually quote a two-way price at which she will buy or sell a particular bond, but it is the *yield* at which the bond is trading that is important to the market-maker's customer. This is because a bond's price does not actually tell us anything useful about what we are getting. Remember that in any market there will be a number of bonds with different issuers, coupons and terms to maturity. Even in a homogeneous market such as the gilt market, different gilts will trade according to their own specific characteristics. To compare bonds in the market therefore we need the yield on any bond and it is yields that we compare, not prices. A fund manager quoted a price at which she can buy a bond will be instantly aware of what yield that price represents, and whether this yield represents *fair value*.

So it is the yield represented by the price that is the important figure for bond traders. We can illustrate this by showing the gilts prices from two newspapers, reproduced below. Figure 4.1 (reproduced from Figure 1.2) shows both prices and yields and allows us to compare returns from different bonds. Figure 4.2 lists only prices for selected stocks on the same day. Figure 4.2 is useful only to say, private investors, who have purchased stock at a certain price and now wish to see where it is trading; it does not allow us to compare returns from the different bonds listed.

UK GILTS PRICES																					
Notes			Yield		Price		Yield		Price		Notes			Yield		Price		Yield		Price	
			Int		+ or -		52 week							Int		+ or -		52 week			
			Red				High		Low					Red				High		Low	
Shorts (Lives up to Five Years)																					
Treas 12½pc 2000			8.35	4.81	101.85d	-03	103.54	101.81			Treas 12½pc 2000-5			9.90	5.58	126.26	+04	135.55	126.22		
Treas 9½pc 2000			8.78	4.77	102.52	-01	100.50	96.70			Treas 9½pc 2005			7.32	4.58	116.07	+10	125.31	112.85		
Conv 9½pc 2000			8.95	4.77	102.52	-01	100.50	96.70			Conv 9½pc 2006			7.36	5.46	125.56	+16	136.21	129.98		
Treas 10pc 2000			12.08	4.91	107.61	-05	113.28	107.61			Treas 7½pc 2006			6.85	5.49	113.19	-12	122.40	110.47		
Treas 10½pc 2000			12.08	4.91	107.61	-05	113.28	107.61			Treas 8pc 2002-6			7.49	6.66	106.75	-10	111.73	105.05		
Treas 10½pc 2001			0.32	103.05		-09	100.82	100.30			Treas 7½pc 2006			6.56	5.46	112.27	-13	121.62	109.33		
Treas 10pc 2001			9.34	5.28	107.12	-05	110.37	107.11			Treas 11½pc 2003-7			9.85	5.81	119.25d	-08	126.29	119.25		
Conv 9½pc 2001			8.81	5.26	107.84	-05	110.37	106.73			Treas 8½pc 2007			7.12	5.46	119.4d	-08	126.29	116.89		
Treas 10½pc 2002			8.85	5.65	103.84	-05	108.14	103.84			Treas 13½pc 2004-8			10.19	5.52	132.47	+08	148.61	132.39		
Treas 10½pc 2002			8.85	5.65	103.84	-05	108.14	103.84			Treas 8pc 2008			7.12	5.34	126.37	-12	138.52	123.05		
Treas 7pc 2001			6.78	5.46	103.25	-06	106.51	100.79			Conv 9½pc 2008			6.95	5.25	124.13	-19	139.21	117.03		
Conv 10pc 2002			9.03	5.66	110.78	-08	116.41	110.79			Treas 9½pc 2009			6.49	5.15	132.49	-18	141.67	99.36		
Treas 7pc 2002			6.74	5.52	103.89	-06	108.13	101.52			Treas 9½pc 2010			5.74	5.21	132.83	-02	146.78	103.24		
Treas 7pc 2002			6.83	5.65	110.14	-08	115.64	104.69			Conv 9½pc Le 2011			6.77	5.25	104.99	-01	145.37	127.66		
Conv 9½pc 2002			8.74	5.64	111.53	-08	117.29	111.53			Treas 8pc 2012			6.66	5.24	135.19	-05	144.74	129.18		
Treas 7pc 2002			6.74	5.52	103.89	-06	108.13	101.52			Treas 5½pc 2008-12			5.38	5.18	129.25	-02	112.72	99.60		
Conv 9½pc 2002			8.74	5.64	111.53	-08	117.29	111.53			Treas 7pc 2012-15			6.26	5.18	128.08	-10	139.64	120.56		
Treas 7pc 2002			6.74	5.52	103.89	-06	108.13	101.52			Treas 10pc 2006			6.35	5.32	121.85d	-04	133.56	116.74		
Conv 9½pc 2002			8.74	5.64	111.53	-08	117.29	111.53													
Treas 7pc 2002			6.74	5.52	103.89	-06	108.13	101.52													
Conv 9½pc 2002			8.74	5.64	111.53	-08	117.29	111.53													
Treas 7pc 2002			6.74	5.52	103.89	-06	108.13	101.52													
Conv 9½pc 2002			8.74	5.64	111.53	-08	117.29	111.53													
Treas 7pc 2002			6.74	5.52	103.89	-06	108.13	101.52													
Conv 9½pc 2002			8.74	5.64	111.53	-08	117.29	111.53													
Treas 7pc 2002			6.74	5.52	103.89	-06	108.13	101.52													
Conv 9½pc 2002			8.74	5.64	111.53	-08	117.29	111.53													
Treas 7pc 2002			6.74	5.52	103.89	-06	108.13	101.52													
Conv 9½pc 2002			8.74	5.64	111.53	-08	117.29	111.53													
Treas 7pc 2002			6.74	5.52	103.89	-06	108.13	101.52													
Conv 9½pc 2002			8.74	5.64	111.53	-08	117.29	111.53													
Treas 7pc 2002			6.74	5.52	103.89	-06	108.13	101.52													
Conv 9½pc 2002			8.74	5.64	111.53	-08	117.29	111.53													
Treas 7pc 2002			6.74	5.52	103.89	-06	108.13	101.52													
Conv 9½pc 2002			8.74	5.64	111.53	-08	117.29	111.53													
Treas 7pc 2002			6.74	5.52	103.89	-06	108.13	101.52													
Conv 9½pc 2002			8.74	5.64	111.53	-08	117.29	111.53													
Treas 7pc 2002			6.74	5.52	103.89	-06	108.13	101.52													
Conv 9½pc 2002			8.74	5.64	111.53	-08	117.29	111.53													
Treas 7pc 2002			6.74	5.52	103.89	-06	108.13	101.52													
Conv 9½pc 2002			8.74	5.64	111.53	-08	117.29	111.53													
Treas 7pc 2002			6.74	5.52	103.89	-06	108.13	101.52													
Conv 9½pc 2002			8.74	5.64	111.53	-08	117.29	111.53													
Treas 7pc 2002			6.74	5.52	103.89	-06	108.13	101.52													
Conv 9½pc 2002			8.74	5.64	111.53	-08	117.29	111.53													
Treas 7pc 2002			6.74	5.52	103.89	-06	108.13	101.52													
Conv 9½pc 2002			8.74	5.64	111.53	-08	117.29	111.53													
Treas 7pc 2002			6.74	5.52	103.89	-06	108.13	101.52													
Conv 9½pc 2002			8.74	5.64	111.53	-08	117.29	111.53													
Treas 7pc 2002			6.74	5.52	103.89	-06	108.13	101.52													
Conv 9½pc 2002			8.74	5.64	111.53	-08	117.29	111.53													
Treas 7pc 2002			6.74	5.52	103.89	-06	108.13	101.52													
Conv 9½pc 2002			8.74	5.64	111.53	-08	117.29	111.53													
Treas 7pc 2002			6.74	5.52	103.89	-06	108.13	101.52													
Conv 9½pc 2002			8.74	5.64	111.53	-08	117.29	111.53													
Treas 7pc 2002			6.74	5.52	103.89	-06	108.13	101.52													
Conv 9½pc 2002			8.74	5.64	111.53	-08	117.29	111.53													
Treas 7pc 2002			6.74	5.52	103.89	-06	108.13	101.52													
Conv 9½pc 2002			8.74	5.64	111.53	-08	117.29	111.53													
Treas 7pc 2002			6.74	5.52	103.89	-06	108.13	101.52													
Conv 9½pc 2002			8.74	5.64	111.53	-08	117.29	111.53													
Treas 7pc 2002			6.74	5.52	103.89	-06	108.13	101.52													
Conv 9½pc 2002			8.74	5.64	111.53	-08	117.29	111.53													
Treas 7pc 2002			6.74	5.52	103.89	-06	108.13	101.52													
Conv 9½pc 2002			8.74	5.64	111.53	-08	117.29	111.53													
Treas 7pc 2002			6.74	5.52	103.89	-06	108.13	101.52													
Conv 9½pc 2002			8.74	5.64	111.53	-08	117.29	111.53													
Treas 7pc 2002			6.74	5.52	103.89	-06	108.13	101.52													
Conv 9½pc 2002			8.74	5.64	111.53	-08	117.29	111.53													
Treas 7pc 2002			6.74	5.52	103.89	-06	108.13	101.52													
Conv 9½pc 2002			8.74	5.64	111.53	-08	117.29	111.53													
Treas 7pc 2002			6.74	5.52	103.89	-06	108.13	101.52													
Conv 9½pc 2002			8.74	5.64	111.53	-08	117.29	111.53													
Treas 7pc 2002			6.74	5.52	103.89	-06	108.13	101.52													
Conv 9½pc 2002			8.74	5.64	111.53	-08	117.29	111.53													
Treas 7pc 2002			6.74	5.52	103.89	-06	108.13	101.52													
Conv 9½pc 2002			8.74	5.64	111.53	-08	117.29	111.53													
Treas 7pc 2002			6.74	5.52	103.89	-06	108.13	101.52													
Conv 9½pc 2002			8.74	5.64	111.53	-08	117.29	111.53													
Treas 7pc 2002			6.74	5.52	103.89	-06	108.13	101.52													
Conv 9½pc 2002			8.74	5.64	111.53	-08	117.29	111.53													
Treas 7pc 2002			6.74	5.52	103.89	-06	108.13	101.52													
Conv 9½pc 2002			8.74	5.64	111.53	-08	117.29	111.53													
Treas 7pc 2002			6.74	5.52	103.89	-06	108.13	101.52													
Conv 9½pc 2002			8.74	5.64	111.53	-08	117.29	111.53													
Treas 7pc 2002			6.74	5.52	103.89	-06	108.13	101.52													
Conv 9½pc 2002			8.74	5.64	111.53	-08	117.29	111.53													
Treas 7pc 2002			6.74	5.52	103.89	-06	108.13	101.52													
Conv 9½pc 2002			8.74	5.64																	

GILT EDGED SHORT			GILT EDGED LONG		
Cv 10y 99	£101.25	+0.01	104.32	101.25	
Cv 10y 00	£102.12	+0.01	104.40	102.13	
Tr 08 00	£103.29	+0.07	105.85	103.05	
Tr 10 01	£106.32	+0.04	110.40	105.56	
Tr 13 00	£106.73	+0.03	111.65	106.70	
Tr 15 99	£100.03		100.50	100.03	
Tr 7 01	£102.67	+0.08	112.77	102.02	
GILT EDGED MEDIUM			GILT EDGED INDEX LINKED		
Cv 9 11	£134.32	+0.97	145.30	130.66	
Cv 9y 04	£117.32		126.40	115.56	
Cv 9y 05	£118.46	+0.29	125.25	116.30	
Tr 3y 99 04	£93.54	+0.20	98.00	91.22	
Tr 07y 06	£112.72	+0.49	122.42	110.07	
Tr 08 03	£107.72	+0.17	114.74	106.34	
Tr 10 03	£115.06	+0.20	123.51	113.58	
Tr 11y 01 04	£108.72	+0.08	113.64	108.51	
Tr 11y 03 07	£117.98		126.25	117.18	
Tr 12y 03 05	£124.63		135.43	123.56	
Tr 6y 10	£110.01	+0.81	118.77	106.84	
Tr 6y 04	£105.37	+0.22	113.27	103.40	
Tr 7y 06	£111.80	+0.51	121.64	109.08	
Tr 8 02 06	£105.50	+0.15	111.78	105.10	
Tr 8 09	£121.81	+0.63	132.91	103.31	
Tr 8y 05	£115.30	+0.31	125.38	112.87	
Tr 8y 07	£119.04	+0.57	129.86	116.05	
Tr 9 08	£126.32	+0.65	138.50	122.60	
Tr 9y 02	£110.70	+0.15	117.28	109.81	
Ex 12 13 17	£167.93		183.75	117.50	
Tr 5y 08 12	£102.20	+0.57	112.25	99.90	
Tr 7y 12 15	£123.14	+0.88	133.45	119.91	
Tr 8 13	£129.71	+1.21	139.68	126.44	
Tr 8 15	£134.05	+1.53	144.18	130.72	
Tr 9 21	£143.59	+2.09	153.27	138.80	
Tr 9y 17	£146.32	+1.75	156.12	142.00	
Tr 9 12	£136.67	+1.00	147.74	133.15	
Tr 2y 10 06	£231.48	+0.09	239.62	231.19	
Tr 2y 10 01	£202.51	+0.08	207.70	202.01	
Tr 2y 10 03	£201.71	+0.08	207.83	201.46	
Tr 2y 10 09	£210.94	+0.07	221.38	210.65	
Tr 2y 10 11	£221.78	+0.06	235.76	221.48	
Tr 2y 10 13	£186.24	+0.05	199.14	186.02	
Tr 2y 10 16	£206.49	+0.06	221.31	206.18	
Tr 2y 10 20	£207.04	+0.06	221.81	206.69	
Tr 2y 10 24	£180.22	+0.05	183.87	179.92	
Tr 4y 10 30	£175.38	+0.06	183.28	178.08	
Tr 4y 10 04	£130.24	+0.08	134.84	130.07	
GILT EDGED UNDATED					
Cv 2y	£50.78	+1.18	57.00	48.12	
Cv 4	£79.64	+1.83	88.15	75.15	
Cv 2y	£77.00	-1.00	80.00	75.02	
Tr 2y	£50.07	+1.15	56.00	47.25	
Tr 3	£57.00	-1.50	58.50	52.50	
Wf 3y	£72.13	+1.76	79.65	68.02	

Source: Standard & Poor's Comstock / Hesse

~ Company name change
 † Dealings suspended
 * Ex-dividend
 † Ex-rights issue
 a Ex-all
 c Ex-capitalisation issue

Figure 4.2: Evening Standard shares page (gilts only) from 25 August, 1999.

© Associated News Media. Reproduced with permission.

The yield on any investment is the interest rate that will make the present value of the cash flows from the investment equal to the initial cost (price) of the investment. Mathematically the yield on any investment, represented by r , is the interest rate that satisfies equation (4.1):

$$P = \sum_{n=1}^N \frac{C_n}{(1+r)^n} \quad (4.1)$$

where

C_n is the cash flow in year n
 P is the price of the investment
 n is the number of years.

The yield calculated from this relationship is the *internal rate of return*.

But as we have noted there are other types of yield measure used in the market for different purposes. The most important of these are bond redemption yields, *spot* rates and *forward* rates. We will now discuss each type of yield measure and show how they are computed, followed by a discussion of the relative usefulness of each measure.

4.1 Current yield

The simplest measure of the yield on a bond is the *current yield*, also known as the *flat yield*, *interest yield* or *running yield*. The running yield is given by (4.2):

$$rc = \frac{C}{P} \times 100 \quad (4.2)$$

where

C is the bond coupon
 rc is the current yield
 P is the clean price of the bond.

In (4.2) C is not expressed as a decimal. Current yield ignores any capital gain or loss that might arise from holding and trading a bond and does not consider the time value of money. It essentially calculates the bond coupon income as a proportion of the price paid for the bond, and to be accurate would have to assume that the bond was more like an annuity rather than a fixed-term instrument. It is not really an “interest rate” though.

The current yield is useful as a “rough-and-ready” interest rate calculation; it is often used to estimate the cost of or profit from a short-term holding of a bond. For example if other short-term interest rates such as the one-week or three-month rates are higher than the current yield, holding the bond is said to involve a *running cost*. This is also

known as *negative carry* or *negative funding*. The term is used by bond traders and market makers and *leveraged* investors. The *carry* on a bond is a useful measure for all market practitioners as it illustrates the cost of holding or *funding* a bond. The funding rate is the bondholder's short-term cost of funds. A private investor could also apply this to a short-term holding of bonds.

EXAMPLE 4.1 Running yield

- A bond with a coupon of 6% is trading at a clean price of 97.89. What is the current yield of the bond?

$$rc = \frac{6.00}{97.89} \times 100 = 6.129\%.$$

EXAMPLE 4.2

- What is the current yield of a bond with 7% coupon and a clean price of 103.49?

$$rc = \frac{7}{103.49} \times 100 = 6.76\%.$$

Note from Examples 4.1 and 4.2 that the current yield of a bond will lie above the coupon rate if the price of the bond is below par, and vice versa if the price is above par.

EXAMPLE 4.3

- Badur buys a bond with coupon of 10% at a price of 110.79, with funds borrowed at 8.75% via a special-rate credit card offer and holds the bond for three months. Has he made money during the course of the investment (ignore transaction costs)?

The running yield on the bond is 9.026% while Badur has paid interest on his borrowed funds at 8.75%. Therefore he has earned approximately 0.276% net carry or funding return on his investment, ignoring any capital gain or loss he may have suffered when he sold the bond.

4.2 Simple yield to maturity

The *simple yield to maturity* makes up for some of the shortcomings of the current yield measure by taking into account capital gains or losses. The assumption made is that the capital gain or loss occurs evenly over the remaining life of the bond. The resulting formula is:

$$rs = \frac{C}{P} + \frac{100 - P}{nP} \quad (4.3)$$

where

- P is the clean price
- rs is the simple yield to maturity
- n is the number of years to maturity.

For the bond discussed in Example 4.1 and assuming $n = 5$ years,

$$\begin{aligned} rs &= \frac{6.00}{97.89} + \frac{100 - 97.89}{5 \times 97.89} = 0.06129 + 0.00431 \\ &= 6.560\%. \end{aligned}$$

The simple yield measure is useful for rough-and-ready calculations. However its main drawback is that it does not take into account compound interest or the time value of money. Any capital gain or loss resulting is amortised equally over the remaining years to maturity. In reality as bond coupons are paid they can be reinvested, and hence interest can be earned. This increases the overall return from holding the bond. As such the simple yield measure is not overly useful and it is not commonly encountered in say, the gilt market. However it is often the main measure used in the Japanese government bond market.

4.3 Yield to maturity

4.3.1 Calculating bond yield to maturity

The *yield to maturity* (YTM) or *gross redemption yield* is the most frequently used measure of return from holding a bond.¹ Yield to maturity takes into account the pattern of coupon payments, the bond's term to maturity and the capital gain (or loss) arising over the remaining life of the bond. We saw from our bond price formula in Chapter 3 that these elements were all related and were important components determining a bond's price. If we set the IRR for a set of cash flows to be the rate that applies from a start-date to an end-date we can assume the IRR to be the YTM for those cash flows. The YTM therefore is equivalent to the *internal rate of return* on the bond, the rate that equates the value of the discounted cash flows on the bond to its current price. The calculation assumes that the bond is held until maturity and therefore it is the cash flows to maturity that are discounted in the calculation. It also employs the concept of the time value of money.

As we would expect the formula for YTM is essentially that for calculating the price of a bond. For a bond paying annual coupons the YTM is calculated by solving equation (4.4), and we assume that the first coupon will be paid exactly one interest period now (which, for an annual coupon bond is exactly one year from now).

$$P_d = \frac{C}{(1+rm)^1} + \frac{C}{(1+rm)^2} + \frac{C}{(1+rm)^3} + \cdots + \frac{C}{(1+rm)^n} + \frac{M}{(1+rm)^n} \quad (4.4)$$

where

P_d	is the bond dirty price
M	is the par or redemption payment (100)
n	the number of interest periods
C	is the coupon rate
rm	is the annual yield to maturity (the YTM).

Note that the number of interest periods in an annual-coupon bond are equal to the number of years to maturity, and so for these bonds n is equal to the number of years to maturity.

We can simplify (4.4) using Σ :

$$P_d = \sum_{n=1}^N \frac{C}{(1+rm)^n} + \frac{M}{(1+rm)^n}. \quad (4.5)$$

Note that the expression at (4.5) has two variable parameters, the price P_d and yield rm . It cannot be rearranged to solve for yield rm explicitly and must be solved using numerical iteration. The process involves estimating a value for rm and calculating the price associated with the estimated yield. If the calculated price is higher than the price of the bond at the time, the yield estimate is lower than the actual yield, and so it must be adjusted until it converges to the level that corresponds with the bond price.²

For YTM for a semi-annual coupon bond we have to adjust the formula to allow for the semi-annual payments. Equation (4.5) is modified as shown by (4.6) again assuming there are precisely six months to the next coupon payment.

$$P_d = \sum_{n=1}^N \frac{C/2}{(1+\frac{1}{2}rm)^n} + \frac{M}{(1+\frac{1}{2}rm)^n} \quad (4.6)$$

where n is now the number of interest periods in the life of the bond and therefore equal to the number of years to maturity multiplied by 2.

For yield calculations carried out by hand ("long-hand") we can simplify (4.5) and (4.6) to reduce the amount of arithmetic. For a semi-annual coupon bond with an actual/365 day-base count (4.6) can be written out long-hand and rearranged to give us (4.7):

¹ In this book the terms *yield to maturity* and *gross redemption yield* are used synonymously. The latter term is common in sterling markets.

² Bloomberg® also uses the term *yield-to-workout* where "workout" refers to the maturity date for the bond.

$$P_d = \left(\frac{1}{(1 + \frac{1}{2}rm)^{N_{tc}/182.5}} \right) \times \left(\frac{C}{rm} \left(1 + \frac{1}{2}rm \right) - \frac{1}{(1 + \frac{1}{2}rm)^{n-1}} \right) + \frac{M}{(1 + \frac{1}{2}rm)^{n-1}} \quad (4.7)$$

where

- P_d is the dirty price of the bond
 rm is the yield to maturity
 N_{tc} is the number of days between the current date and the next coupon date
 n is the number of coupon payments before redemption; if T is the number of complete years before redemption, then $n = 2T$ if there is an even number of coupon payments before redemption, and $n = 2T + 1$ if there is an odd number of coupon payments before redemption.

All the YTM equations above use rm to discount a bond's cash flows back to the next coupon payment and then discount the value at that date back to the date of the calculation. In other words rm is the *internal rate of return* (IRR) that equates the value of the discounted cash flows on the bond to the current dirty price of the bond (at the current date). The internal rate of return is the discount rate which, if applied to all of the cash flows will solve for a number that is equal to the dirty price of the bond (its present value). By assuming that this rate will be unchanged for the reinvestment of all the coupon cash flows, and that the instrument will be held to maturity, the IRR can then be seen as the yield to maturity. In effect both measures are identical; the assumption of uniform reinvestment rate allows us to calculate the IRR as equivalent to the redemption yield. It is common for the IRR measure to be used by corporate financiers for project appraisal, while the redemption yield measure is used in bond markets. The solution to the equation for rm cannot be found analytically and has to be solved through numerical iteration, that is, by estimating the yield from two trial values for rm , then solving by using the formula for linear interpolation. It is more common nowadays to use a spreadsheet programme or programmable calculator such as an Hewlett-Packard calculator.

For the equation at (4.7) we have altered the exponent used to raise the power of the discount rate in the first part of the formula to $N/182.5$. This is a special case and is only applicable to bonds with an actual/365 day-count base. The YTM in this case is sometimes referred to as the *consortium yield*, which is a redemption yield that assumes exactly 182.5 days between each semi-annual coupon date. As most developed-country bond markets now use actual/actual day bases it is not common to encounter the consortium yield equation.

EXAMPLE 4.4 Yield to maturity for semi-annual coupon bond

- A semi-annual paying bond has a dirty price of £98.50, an annual coupon of 6% and there is exactly one year before maturity. The bond therefore has three remaining cash flows, comprising two coupon payments of £3 each and a redemption payment of £100. Equation (4.7) can be used with the following inputs:

$$98.50 = \frac{3.00}{(1 + \frac{1}{2}rm)} + \frac{103.00}{(1 + \frac{1}{2}rm)^2}.$$

Note that we use half of the YTM value rm because this is a semi-annual paying bond. The expression above is a quadratic equation, which is solved using the standard solution for quadratic equations, which is noted below.

$$ax^2 + bx + c = 0 \Rightarrow x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

In our expression if we let $x = (1 + rm/2)$ we can rearrange the expression as follows:

$$98.50x^2 - 3.0x - 103.00 = 0.$$

We then solve for a standard quadratic equation, and as such there will be two solutions, only one of which gives a positive redemption yield. The positive solution is $rm/2 = 0.037929$ so that $rm = 7.5859\%$.

As an example of the iterative solution method, suppose that we start with a trial value for rm of $r_1 = 7\%$ and insert this into the right-hand side of equation (4.7). This gives a value for the right-hand side of $RHS_1 = 99.050$ which is higher than the left-hand side ($LHS = 98.50$); the trial value for rm was therefore too low. Suppose then

that we try next $r_2 = 8\%$ and use this as the right-hand side of the equation. This gives $RHS_2 = 98.114$ which is lower than the LHS. Because RHS_1 and RHS_2 lie on either side of the LHS value we know that the correct value for rm lies between 7% and 8%. Using the formula for linear interpolation,

$$rm = r_1 + (r_2 - r_1) \frac{RHS_1 - LHS}{RHS_1 - RHS_2}$$

our linear approximation for the redemption yield is $rm = 7.587\%$, which is near the exact solution.

EXAMPLE 4.5

- We wish to calculate the gross redemption yield for the bond in Example 4.1. If we assume that the analysis is performed with precisely five years to maturity, with a settlement date of 3 August 1999 and that it is a semi-annual coupon bond, the bond will comprise cash flows of ten coupon payments of £3 every six months and a redemption payment of £100 five years from now. In order to calculate the redemption yield rm long-hand we need to try different trial levels for the discount rate rm until we obtain the cash flows present value total of 97.89. We know that the YTM must be greater than the coupon rate of 6% because the bond is trading at a price below par. In the table below we use different trial values for rm until we reach the semi-annual discount rate of 3.25% which is equal to a YTM of 6.50%. In practice we would have obtained two rates that gave present value total above and below the price of 97.89 and then used the formula for numerical iteration to solve for rm .

Annual yield (%)	s/a yield (%)	PV of all 10 coupon payments	PV of redemption payment	Total PV
5.0	2.50	26.2563	78.1120	104.3638
5.5	2.75	25.9202	76.2398	102.1600
6.0	3.00	25.5906	74.4094	100.0000
6.5	3.25	25.2672	72.6272	97.8944
7.0	3.50	24.9498	70.8919	95.8417

When calculating yields long-hand we can use the following formulas to calculate cash flow present values, where n is the number of interest periods during the life of the bond.

■ Present value of coupon payments:

For annual coupon bonds

$$C \left(\frac{1 - (1/(1 + rm)^n)}{rm} \right)$$

For semi-annual coupon bonds

$$C \left(\frac{1 - (1/(1 + rm/2)^n)}{rm/2} \right)$$

■ Present value of redemption payment:

For annual coupon bonds

$$M \left(\frac{1}{(1 + rm)^n} \right)$$

For semi-annual coupon bonds.

$$M \left(\frac{1}{(1 + (rm/2))^n} \right)$$

Note that the redemption yield as calculated as discussed in this section is the *gross redemption yield*, the yield that results from payment of coupons without deduction of any withholding tax. The *net redemption yield* is obtained by multiplying the coupon rate C by $(1 - \text{marginal tax rate})$. The net yield is what will be received if the bond is traded in a market where bonds pay coupon *net*, which means net of a withholding tax. The net redemption yield is always lower than the gross redemption yield.

4.3.2 Using the redemption yield calculation

We have already alluded to the key assumption behind the YTM calculation, namely that the rate rm remains stable for the entire period of the life of the bond. By assuming the same yield we can say that all coupons are reinvested at

the same yield rm . For the bond in Example 4.5 this means that if all the cash flows are discounted at 6.5% they will have a total present value or NPV of 97.89. At the same time if all the cash flows received during the life of the bond are reinvested at 6.5% until the maturity of the bond, the final redemption yield will be 6.5%. This is patently unrealistic since we can predict with virtual certainty that interest rates for instruments of similar maturity to the bond at each coupon date will not remain at 6.5% for five years. In practice however investors require a rate of return that is equivalent to the price that they are paying for a bond and the redemption yield is, to put it simply, as good a measurement as any. A more accurate measurement might be to calculate present values of future cash flows using the discount rate that is equal to the markets view on where interest rates will be at that point, known as the *forward* interest rate. However forward rates are *implied* interest rates, and a YTM measurement calculated using forward rates can be as speculative as one calculated using the conventional formula. This is because the *actual* market interest rate at any time is invariably different from the rate implied earlier in the forward markets. So a YTM calculation made using forward rates would not be realised in practice either.³ We shall see later in this chapter how the *zero-coupon* interest rate is the true interest rate for any term to maturity, however the YTM is, despite the limitations presented by its assumptions, still the main measure of return used in the markets.

EXAMPLE 4.6 Comparing the different yield measures

- The examples in this section illustrated a five-year bond with a coupon of 6% trading at a price of 97.89. Using the three common measures of return we have:

Running yield = 6.129%

Simple yield = 6.560%

Redemption yield = 6.50%

The different yield measures are illustrated graphically in Figure 4.3 below.

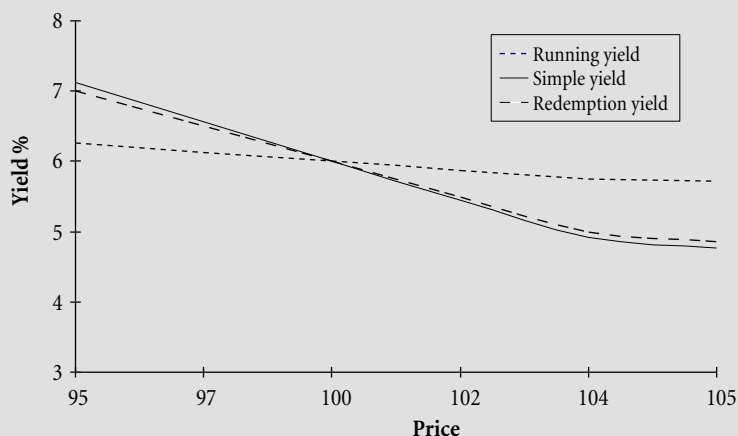


Figure 4.3: Comparing yield measures for a 6% bond with five years to maturity.

4.3.3 Calculating redemption yield between coupon payments

The yield formula in paragraph 4.3.1 can be used whenever the settlement date for the bond falls on a coupon date, so that there is precisely one interest period to the next coupon date. If the settlement date falls in between coupon dates, the same price/yield relationship holds and the YTM is the interest rate that equates the NPV of the bond's cash flows with its dirty price. However the formula is adjusted to allow for the uneven interest period, and this is given by (4.8) for an annual coupon bond.

$$P_d = \frac{C}{(1 + rm)^w} + \frac{C}{(1 + rm)^{1+w}} + \frac{C}{(1 + rm)^{2+w}} + \dots + \frac{C}{(1 + rm)^{n-1+w}} + \frac{M}{(1 + rm)^{n-1+w}} \quad (4.8)$$

³ Such an approach is used to price interest-rate swaps, however.

where w is $\frac{\text{number of days between the settlement date and the next coupon date}}{\text{number of days in the interest period}}$

and n is the number of coupon payments remaining in the life of bond. The other parameters are as before. As before the formula can be shortened as given by (4.9):

$$P_d = \sum_{n=1}^N \frac{C}{(1 + rm)^{n-1+w}} + \frac{M}{(1 + rm)^{n-1+w}}. \quad (4.9)$$

4.3.4 Yield represented by par bond price

In Chapter 3 we illustrated a characteristic of bonds in that when the required yield is the same as a bond's coupon rate, the price will be par (100 per cent). We expect this because the cash flows represented by a bond result from a fixed coupon payment, and discounting these cash flows at the coupon rate will result in a net present value (NPV) of 100 again. As the yield required for the bond drops below the coupon rate the NPV will rise, and vice versa if the required yield moves above the coupon rate. At any time we can *approximate* a bond's price to par at *any* time that the yield is the same as the coupon. However the price of a bond is only precisely equal to par in this case when the calculation is made on a coupon date. On any other date the price will not be exactly par when the yield equals the coupon rate. This is because accrued interest on the bond is calculated on a simple interest basis, whereas bond prices are calculated on a compound interest basis. The rule is that on a non-coupon date where a bond is priced at par, the redemption yield is just below the coupon rate. This effect is amplified the further away the bond settlement date lies from the coupon date and will impact more on short-dated bonds. In most cases however this feature of bonds does not have any practical impact.

In the context of yield represented by a par bond price, one might occasionally encounter *yield to par* or *par yield*, which is the yield for a bond trading at or near its par value (100 per cent). In practice this will refer to a bond price between 99 and 101 per cent, and the par yield is essentially the coupon rate for a bond trading at or near par.

4.4 Yield on a zero-coupon bond

Zero-coupon bonds, sometimes known as *strips*, have only one cash flow, the redemption payment on maturity. Hence the name: strips pay no coupon during their life. In virtually all cases zero-coupon bonds make one payment on redemption, and this payment will be par (100). Therefore a zero-coupon bond is sold at a discount to par and trades at a discount during its life. For a bond with only one cash flow it is obviously not necessary to use (4.4) and we can use (4.10) instead.

$$P = \frac{C}{(1 + rm)^n} \quad (4.10)$$

where C is the final redemption payment, usually par (100). This is the “traditional” approach. Note that P , the price of a zero-coupon bond, has only one meaning because there is no “dirty” price, since no accrued interest arises.

Equation (4.10) still uses n for the number of interest periods in the bond's life. Because no interest is actually paid by a zero-coupon bond, the interest periods are known as *quasi-interest periods*. A quasi-interest period is an assumed interest period, where the assumption is that the bond pays interest. It is important to remember this because zero-coupon bonds in markets that use a semi-annual convention will have n equal to double the number of years to maturity. For annual coupon bond markets n will be equal to the number of years to redemption. We can rearrange (4.10) for the yield, rm :

$$rm = \left(\frac{C}{P} \right)^{\frac{1}{n}} - 1. \quad (4.11)$$

EXAMPLE 4.7(i)

- A zero-coupon bond with five years to maturity is trading at €77.795. What is its yield to maturity?

$$rm = \left(\frac{100}{77.795} \right)^{\frac{1}{5}} - 1 = 0.0515009. \text{ The yield of the bond is 5.15\%.}$$

4.7(ii)

- A zero-coupon bond with five years to maturity is trading at £77.795. What is its yield to maturity?

Because this is a sterling bond it is assumed to have two quasi-coupon periods each year. Therefore n is equal to 5×2 or 10 interest periods.

$$rm = \left(\frac{100}{77.795} \right)^{\frac{1}{10}} - 1 = 0.02543.$$

The calculation is 2.543%. To obtain the bond-equivalent yield to maturity we double this figure, giving us a yield of 5.0854%.

4.7(iii)

- When zero-coupon bonds are analysed in between quasi-coupon dates the calculation is adjusted to allow for days, exactly as with conventional bonds. Consider the following zero-coupon bond stripped from a bond that pays semi-annual coupons:

Maturity date:	1 December 2009
Settlement date:	1 August 1999
Yield:	5.465%
Day basis:	Actual/Actual

What is the price of the bond?

The quasi-coupon dates are 1 June and 1 December each year. On the act/act basis the “accrued interest” day-count is 122/183. The price is therefore:

$$\frac{100}{\left(1 + \frac{0.05465}{2} \right)^{\frac{122}{183} + 20}} = 57.2846.$$

4.7(iv)

- If the zero-coupon bond in Example 4.7 (iii) was from a euro currency strips market the calculation needs to be adjusted to allow for the different conventions. The price is then given by:

$$57.2846 = \frac{100}{(1 + rm)^{\frac{243}{365} + 10}}.$$

$$\text{We can rearrange this to solve for the yield } rm = \sqrt[10.665753]{\frac{100}{57.2846}} - 1 = 5.362\%.$$

The examples of zero-coupon bond yields illustrated in Example 4.7 shows how the yields for such bonds are different according to the conventions that apply in each market. We expect the yields to differ because two of them were annual yields while the other two were semi-annual yields. In the markets the convention is to convert one to the other to make them equivalent, in order to enable us to compare yields. What happens if we convert the yield in 4.7 (iii) above to an annualised yield? Our first thought must be that the yields should be equal because both bonds are trading at the same price. If we check this, using $(1 + 0.05465/2)^2 - 1 = 5.540\%$, we see that there is still a discrepancy and the annualised yield for the “semi-annual” zero-coupon bond is higher. This is due to the way the day-count calculation for actual/actual will always produce unequal interest periods over a year for bonds with semi-annual coupon payments.

4.5 Modifying bond yields

4.5.1 Very short-dated bonds

A bond with one coupon remaining often trades as a money market instrument because it has only cash flow, its redemption payment and final coupon on maturity. In fact such a bond that was also trading below par would have

the characteristics of a short-dated zero-coupon bond. The usual convention in the markets is to adjust bond yields using money market convention by discounting the cash flow at a simple rate of interest instead of a compound rate.

The bond yield formula in the case of a short-date final coupon bond is then given by (4.12):

$$rm = \left(\frac{\text{final cash flow}}{P_d} - 1 \right) \times \frac{B}{\text{days to maturity}} \quad (4.12)$$

where B is the day-base count for the bond (360, 365 or 366).

4.5.2 Money market yields⁴

Price and yield conventions in domestic money markets are often different to those in use in the same country's bond markets. For example the day-count convention for the US money market is actual/360 while the Treasury bond markets uses actual/actual. A list of the conventions in use in selected countries is given in Appendix 4.1.

The different conventions in use in money markets compared to bond markets results in some slight difficulty when comparing yields across markets. It is important to adjust yields when comparisons are made, and the usual practice is to calculate a money market-equivalent yield for bond instruments. We can illustrate this by comparing the different approaches used in the Certificate of Deposit (CD) market compared to bond markets. Money market instruments make their interest payments as actual and not rounded amounts. A long-dated CD price calculation uses exact day counts, as opposed to the regular time intervals assumed for bonds, and the final discount from the nearest coupon date to the settlement date is done using simple rather than continuous compounding. In order to compare bond yields to money market paper therefore we calculate a money market yield for the bond. In the US market the adjustment of the bond yield is given by (4.13), which shows the adjustment required to the act/360 day-count convention.

$$P_d = \frac{M}{(1 + ((r_{me}/n) \times t))} \cdot \left(\frac{C}{n} \times \frac{1 - 1/\left(1 + (r_{me}/n) \times \frac{365}{360}\right)^N}{1 - 1/\left(1 + (r_{me}/n) \times \frac{365}{360}\right)} + \frac{1}{\left(1 + (r_{me}/n) \times \frac{365}{360}\right)^{N-1}} \right) \quad (4.13)$$

where

r_{me} is the bond money-market yield
 t is the fraction of the bond coupon period, on a money market basis.

4.5.3 Moosmuller yield

Certain German government bonds and domestic bond use what is known as the Moosmuller yield method. This method is similar to a money market yield calculation, because it discounts the next coupon from the settlement date using simple rather than compounding interest. The day-count basis remains the one used in the bond markets. The yield calculation is given by (4.14) below.

$$P_d = \frac{M}{1 + ((r_M/n) \times t)} \cdot \left(\frac{C}{n} \times \frac{1 - 1/(1 + (r_M/n))^N}{1 - 1/(1 + (r_M/n))} + \frac{1}{(1 + (r_M/n))^{N-1}} \right) \quad (4.14)$$

where r_M is the Moosmuller yield.

4.6 Converting bond yields

4.6.1 Discounting and coupon frequency

In our discussion on yield we noted the difference between calculating redemption yield on the basis of both annual and semi-annual coupon bonds. Analysis of bonds that pay semi-annual coupons incorporates semi-annual discounting of semi-annual coupon payments. This is appropriate for most UK and US bonds. However government bonds in most of continental Europe, and most Eurobonds for example pay annual coupon payments, and the appropriate method of calculating the redemption yield is to use annual discounting. The two yields measures are not therefore directly comparable. We could make a eurobond directly comparable with a UK gilt by using semi-annual discount-

⁴ Money markets are covered in detail in a separate chapter.

ing of the eurobond's annual coupon payments. Alternatively we could make the gilt comparable with the eurobond by using annual discounting of its semi-annual coupon payments.

The price/yield formulae for the different discounting possibilities we encounter in the markets are listed below (as usual we assume that the calculation takes place on a coupon payment date so that accrued interest is zero).

- Semi-annual discounting of semi-annual payments:

$$P_d = \frac{C/2}{(1 + \frac{1}{2}rm)} + \frac{C/2}{(1 + \frac{1}{2}rm)^2} + \frac{C/2}{(1 + \frac{1}{2}rm)^3} + \dots + \frac{C/2}{(1 + \frac{1}{2}rm)^N} + \frac{M}{(1 + \frac{1}{2}rm)^{2N}}. \quad (4.15)$$

- Annual discounting of annual payments:

$$P_d = \frac{C}{(1 + rm)} + \frac{C}{(1 + rm)^2} + \frac{C}{(1 + rm)^3} + \dots + \frac{C}{(1 + rm)^N} + \frac{M}{(1 + rm)^N}. \quad (4.16)$$

- Semi-annual discounting of annual payments:

$$P_d = \frac{C}{(1 + \frac{1}{2}rm)^2} + \frac{C}{(1 + \frac{1}{2}rm)^4} + \frac{C}{(1 + \frac{1}{2}rm)^6} + \dots + \frac{C}{(1 + \frac{1}{2}rm)^{2N}} + \frac{M}{(1 + \frac{1}{2}rm)^{2N}}. \quad (4.17)$$

- Annual discounting of semi-annual payments:

$$P_d = \frac{C/2}{(1 + rm)^{\frac{1}{2}}} + \frac{C/2}{(1 + rm)} + \frac{C/2}{(1 + rm)^{\frac{3}{2}}} + \dots + \frac{C/2}{(1 + rm)^N} + \frac{M}{(1 + rm)^N}. \quad (4.18)$$

Earlier we considered a bond with a dirty price of 97.89, a coupon of 6% and a five years to maturity. This bond would have the following gross redemption yields under the different yield calculation conventions:

Discounting	Payments	Yield to Maturity (%)
Semi-annual	Semi-annual	6.500
Annual	Annual	6.508
Semi-annual	Annual	6.428
Annual	Semi-annual	6.605

This proves what we have already observed, namely that the coupon and discounting frequency will impact the redemption yield calculation for a bond. We can see that increasing the frequency of discounting will the lower the yield, while increasing the frequency of payments will raise the yield. When comparing yields for bonds that trade in markets with different conventions it is important to convert all the yields to the same calculation basis.

4.6.2 Converting yields

Intuitively we might think that doubling a semi-annual yield figure will give us the annualised equivalent; in fact this will result in an inaccurate figure due to the multiplicative effects of discounting and one that is an underestimate of the true annualised yield. The correct procedure for producing an annualised yields from semi-annual and quarterly yields is given by the expressions below.

The general conversion expression is given by (4.19):

$$rm_a = (1 + \text{interest rate})^m - 1 \quad (4.19)$$

where m is the number of coupon payments per year. Specifically, we can convert between annual and semi-annual compounded yields using the expressions given at (4.20):

$$\begin{aligned} rm_a &= \left((1 + \frac{1}{2}rm_s)^2 - 1 \right) \\ rm_s &= \left((1 + rm_a)^{1/2} - 1 \right) \times 2 \end{aligned} \quad (4.20)$$

or between annual and quarterly compounded yields using (4.21):

$$\begin{aligned} rm_a &= \left(\left(1 + \frac{1}{4} rm_q \right)^4 - 1 \right) \\ rm_q &= \left((1 + rm_a)^{1/4} - 1 \right) \times 4 \end{aligned} \quad (4.21)$$

where rm_q , rm_s and rm_a are respectively the quarterly, semi-annually and annually compounded yields to maturity.

EXAMPLE 4.8

- A UK gilt paying semi-annual coupons and a maturity of 10 years has a quoted yield of 4.89%. A European government bond of similar maturity is quoted at a yield of 4.96%. Which bond has the higher effective yield?

The effective annual yield of the gilt is: $rm = \left(1 + \frac{1}{2} \cdot 0.0489 \right)^2 - 1 = 4.9498\%$.

Therefore the gilt does indeed have the lower yield.

The market convention is sometimes simply to double the semi-annual yield to obtain the annualised yields, despite the fact that this produces an inaccurate result. It is only acceptable to do this for rough calculations. An annualised yield obtained by multiplying the semi-annual yield by two is known as a *bond equivalent yield*.

4.7 Assumptions of the redemption yield calculation

While YTM is the most commonly used measure of yield, it has one major disadvantage. Implicit in the calculation of the YTM is the assumption that each coupon payment as it becomes due is re-invested at the rate rm . This is clearly unlikely, due to the fluctuations in interest rates over time and as the bond approaches maturity. In practice the measure itself will not equal the actual return from holding the bond, even if it is held to maturity. That said, the market standard is to quote bond returns as yields to maturity, bearing the key assumptions behind the calculation in mind. We can demonstrate the inaccuracy of the assumptions by multiplying both sides of (4.15), the price/yield formula for a semi-annual coupon bond, by $(1 + \frac{1}{2} rm)^{2N}$, which gives us (4.22):

$$P_d (1 + \frac{1}{2} rm)^{2N} = (C/2)(1 + \frac{1}{2} rm)^{2N-1} + (C/2)(1 + \frac{1}{2} rm)^{2N-2} + \dots + (C/2) + M. \quad (4.22)$$

The left-hand side of (4.22) gives the value of the investment in the bond on the maturity date, with compounding at the redemption yield. The right-hand side gives the terminal value of the returns from holding the bond. The first coupon payment is reinvested for $(2N - 1)$ half-years at the yield to maturity, the second coupon payment is reinvested for $(2N - 2)$ half-years at the yield to maturity, and so on. This is valid only if the rate of interest is constant for all future time periods, that is, if we had the same interest rate for all loans or deposits, irrespective of the loan maturity. This would only apply under a flat *yield curve* environment. However a flat yield curve implies that the yields to maturity of all bonds should be identical, and is very rarely encountered in practice. So we can discount the existence of flat yield curves in most cases.

Another disadvantage of the yield to maturity measure of return is where investors do not hold bonds to maturity. The redemption yield will not be of great where the bond is not being held to redemption. Investors might then be interested in other measures of return, which we can look at later.

To reiterate then, the redemption yield measure assumes that:

- the bond is held to maturity;
- all coupons during the bond's life are reinvested at the same (redemption yield) rate.

Therefore the YTM can be viewed as an *expected* or *anticipated* yield and is closest to reality perhaps where an investor buys a bond when it is first issued and holds it to maturity. Even then the actual realised yield on maturity would be different from the YTM figure because of the inapplicability of the second condition above.

In addition, as coupons are discounted at the yield specific for each bond, it actually becomes inaccurate to compare bonds using this yield measure. For instance the coupon cash flows that occur in two years' time from both a two-year and five-year bond will be discounted at different rates (assuming we do not have a flat yield curve). This would occur because the YTM for a five-year bond is invariably different to the YTM for a two-year bond. However it would clearly not be correct to discount the two-year cash flows at different rates, because we can see that the present value calculated today of a cash flow in two years' time should be the same whether it is sourced from a

short- or long-dated bond. Even if the first condition noted above for the YTM calculation is satisfied, it is clearly unlikely for any but the shortest maturity bond that all coupons will be reinvested at the same rate. Market interest rates are in a state of constant flux and hence this would affect money reinvestment rates. Therefore although yield to maturity is the main market measure of bond levels, it is not a true interest rate. This is an important result and we shall explore the concept of a true interest rate later in the book.

4.8 Holding-period yield

Investors often hold a bond for a period of time in between the issue and maturity dates. The return generated by such a bond holding would then be a function of the purchase price and the sale price on disposal, in addition to the coupons received. In such cases investors may then calculate a *holding-period yield*. The holding-period yield (also known as the *reinvestment yield*) is the average yield realised during the holding period, taking into account changes in the *rollover rate* (the interest rate at which coupon payments are reinvested). Since the rollover interest rate will fluctuate in between coupon dates there is a chance that it will be below the bond's redemption yield. The risk that the rollover rate is less than the yield to maturity is known as *reinvestment risk*.

The holding-period yield rh is calculated using the expression given by (4.23). Again this assumes that the bond is bought on a coupon payment date (so that accrued interest is zero) and sold an even number of coupon dates later (so that N is an integer). Note that equation (4.23) is for a semi-annual coupon bond.

$$P_d \left(1 + \frac{1}{2}rh\right)^{2N} = (C/2)\left(1 + \frac{1}{2}r_1\right)^{2N-1} + (C/2)\left(1 + \frac{1}{2}r_2\right)^{2N-2} + \dots + (C/2) + P_1. \quad (4.23)$$

The expression at (4.23) can be rearranged and simplified as shown at (4.24).

$$rh = \left\{ \left(\frac{(C/2)\left(1 + \frac{1}{2}r_1\right)^{2N-1} + \dots + (C/2) + P_1}{P_d} \right)^{\frac{1}{2N}} - 1 \right\} \times 2 \quad (4.24)$$

where r_i is the rollover rate of interest earned by the i th coupon payment and P_1 is the price at which the bond was sold. The original purchase price is the denominator P_d .

EXAMPLE 4.9

- Our (by now familiar!) 6% five-year bond is purchased at 97.89 (representing yield of 6.50%) for settlement on a coupon date and sold precisely two years later at 98.95. If we imagine that the coupon reinvestment rates were 5.5%, 5.1% and 6.8% for the three coupons earned during the holding period, what was the return on the bond during the two years?

Using (4.24) we calculate the holding period yield as:

$$\begin{aligned} rh &= \left\{ \left(\frac{3\left(1 + \frac{1}{2}(0.055)\right)^3 + 3\left(1 + \frac{1}{2}(0.051)\right)^2 + 3\left(1 + \frac{1}{2}(0.068)\right) + (3 + 98.95)}{97.89} \right)^{\frac{1}{4}} - 1 \right\} \times 2 \\ &= 0.0797451. \end{aligned}$$

The holding-period return is 7.97%, compared to the YTM measure of 6.50%.

The approach we have described to now is very much the traditional one. Research and academic texts published during the 1980s and 1990s expressed bond prices and yields in different terms, which we consider in Part VIII. The terminology is that used in leading texts such as Jarrow (1996), Neftci (1996), Baxter and Rennie (1996), Hull (1997), Ross (1999) and James and Webber (2000), among others.

4.9 Bonds with embedded options

Up to now we have discussed essentially plain vanilla coupon bonds. Bonds with certain special characteristics require slightly different yield analysis and we will introduce some the key considerations here. A common type of non-vanilla bond is one described as featuring an *embedded option*. This refers to a bond that can be redeemed ahead of its specified maturity date. A bond that carries uncertainty as to its redemption date introduces an extra

element to its yield analysis. This is because aspect of its cash flows, such as the timing or present value of individual payments, are not known with certainty. Contrast this with conventional bonds that have clearly defined cash flow patterns. It is the certainty of the cash flows associated with vanilla bonds that enables us to carry out the yield to maturity analysis we have discussed so far; when cash flows are not known with certainty we need to adjust our yield analysis. In this section we discuss the yield calculations for bonds with embedded options built into their features. The main types of such bonds are *callable bonds*, *putable bonds* and bonds with a *sinking fund* attached to them.

A callable bond is a bond containing a provision that allows the borrower to redeem all or part of the issue before the stated maturity date, which is referred to as *calling* the bond. The issuer therefore has a *call option* on the bond. The price at which the issuer will call the bond is known as the *call price*, and this might be a fixed price from a specific point in time or a series of prices over time, known as a *call schedule*. The bond might be callable over a continuous period of time or at certain specified dates, which is referred to as being *continuously* or *discretely* callable. In many cases an issue will not be callable until a set number of years has passed, in which case the bond is said to have a *deferred call*.

A bond that allows the bondholder to sell back the bond, or *put* the bond, to the issuer at par on specified dates during its life is known as a puttable bond. Therefore a bond with a put provision allows investors to change the term to maturity of the bond. The issue terms of a puttable bond will specify periods during which or dates on which the bond can be put back to the issuer. The put price is usually par but sometimes there is a schedule of prices at which the bond may be put.

Some bonds are issued with a *sinking fund* attached to them. A sinking fund is a deposit of funds kept in a separate custody account that is used to redeem the nominal outstanding on a bond in accordance with a pre-determined schedule, regardless of changes in price of the bond in the secondary market. It is common for sinking fund requirements to be satisfied through the redemption of a specified amount of the issue at certain points during the bond's life. In other cases the sinking fund requirement is met through purchases of amounts of the issue in the open market. Whatever the specific terms for the sinking fund provision is, all or part of the bond will be redeemed ahead of the maturity date. This makes yield analysis more complex.

The usual way to look at callable and puttable bond yields is by using binomial models and option-adjusted spread analysis, and this is discussed in a later chapter. The key issues associated with all bonds with embedded options are the same, and in this section we introduce the basic concepts associated with callable and puttable bonds.

4.9.1 Yield analysis for callable bonds

An issuer of a callable bond retains the right to buy, or call, the issue from bondholders before the specified redemption date. The details of the call structure are set out in the bond's issue *prospectus*. The issuer will generally call a bond when falling interest rates or an improvement in their borrowing status makes it worthwhile to cancel existing loans and replace them at a lower rate of interest. Since a callable bond can be redeemed early at a point when investors would probably prefer to hold onto it, the call provision acts as a "cap" on the value of the bond, so that the price does not rise above a certain point (more relevantly, its yield does not fall below a certain point at which the bond would be called). A callable bond will have more than one possible maturity date, comprised of the call dates (or any date during a continuous call period) and the final maturity date. Therefore the cash flows associated with the issue are not explicit. The uncertainty of the cash flows arises from this fact, and yield calculations for such bonds are based on assumed maturity dates.

As a callable bond can be called at the option of the issuer it is likely to be called when the market rate of interest is lower than the coupon on the bond. When this occurs the bond will be trading above the call price (usually par). However consider a bond that is issued with 15 years to maturity on 3 August 1999 and is callable after 5 years, so that on issue the bond had a 5-year deferred call. The bond has the following call schedule:

<i>Call date</i>	<i>Call price</i>
3 August 2004	102.00
3 August 2005	101.50
3 August 2006	101.00
3 August 2007	100.50
3 August 2008 onwards	100.00

The first par call date for the bond is 3 August 2008 and it callable at par from that date until maturity in August 2014. The *yield to first call* is the yield to maturity assuming that the bond is redeemed on the first call date. It can be calculated using equation (4.4) and setting $M = 102.00$. Similarly the *yield to next call* is the yield to maturity assuming the bond is called on the next available call date. The *operative life* of a callable bond is the bond's expected life. This will depend on both the current price of the bond and the call schedule. If the bond is currently trading below par its operative life is likely to be the number of years to maturity, as this will mean that the market interest rate is above the coupon rate and there is therefore no advantage to the issuer in calling the bond. If the current price is above par, the operative life of the bond will depend on the date on which the call price falls below the current price. For example if the current price for the bond is 101.65, the bond is not likely to be called until a year before the par call date. Similarly the *operative yield* is either the yield to maturity or the yield to relevant call depending on whether the bond is trading above or below par.

In general then, for a callable bond a bondholder will calculate YTM based on an assumed call date. This *yield-to-call* is the discount rate that equates the NPV of the bonds cash flows (up to the assumed call date) to the current dirty price of the bond. For an annual coupon bond the general expression for yield-to-call, assuming settlement on a coupon date, is given by the expression at (4.25):

$$P_d = \frac{C}{(1 + r_{ca})} + \frac{C}{(1 + r_{ca})^2} + \frac{C}{(1 + r_{ca})^3} + \cdots + \frac{C}{(1 + r_{ca})^{N_c}} + \frac{P_c}{(1 + r_{ca})^{N_c}} \quad (4.25)$$

where

- r_{ca} is the yield-to-call
- N_c is the number of years to the assumed call date
- P_c is the call price at the assumed call date (par, or as given in call schedule).

For a semi-annual coupon bond (4.25) is adjusted to allow for semi-annual discounting and semi-annual coupon payments, in the normal manner for vanilla bonds.

Expression (4.25) is usually written as (4.26):

$$P_d = \sum_{n=1}^{N_c} \frac{C}{(1 + r_{ca})^n} + \frac{P_c}{(1 + r_{ca})^{N_c}}. \quad (4.26)$$

EXAMPLE 4.10

- Consider again the bond from Example 4.1 with a gross redemption yield of 6.50%. If we assume that this bond is callable after two years at a call price of £102 per cent for settlement date on 3 August 1999, we can calculate the yield-to-call to the first call date, so that the bond will comprise cash flows of four coupon payments of £3 every six months and a redemption payment of £102 two years or four interest periods from now. The yield-to-call r_{ca} will be the discount rate which equates the NPV of the bond's (assumed) cash flows to the bond price of 97.89. We again use trial values for r_{ca} that will result in the correct price.

Annual yield %	s/a yield %	PV of all 4 coupon payments	PV of call payment	Total PV (102)
8.0	4.00	10.8897	87.1900	98.0797
8.1	4.05	10.8769	87.0226	97.8994
8.2	4.10	10.8641	86.8555	97.7196
8.4	4.20	10.8386	86.5225	97.3911
9.0	4.50	10.7625	85.5333	96.2956

The present value of the coupon payments is calculated using:

$$3 \left(1 - \left(\frac{1}{(1 + r_{ca})^4} \right) \right) \frac{1}{r_{ca}}$$

The present value of the call payment is calculated using $102/(1 + r_{ca})^4$.

The yield-to-call to the first call date is an annualised bond equivalent of 8.10% which corresponds to a price roughly near to the bond price of 97.89. Compare this to the yield-to-maturity for the bond of 6.50% for the 5-year conventional bond.

4.9.2 Yield analysis for puttable bonds

A puttable bond grants the bondholder the right to sell the bond back to the issuer, usually in accordance with specified terms and conditions. For a puttable bond we calculate a *yield-to-put*, assuming a selected put date for the bond. The yield to put is calculated in a similar manner to the yield to call. The difference is that a bond with a put feature is redeemable at the option of the investor, and the investor will exercise the put only if this maximises the value of the bond. Therefore a bond with a put option will always trade on the basis of the yield to maturity or the yield to put, whichever is greater.

On the whole the price/yield formula we use to calculate yield-to-put is identical to the formula used for yield-to-call, exchanging assumed call date and call price for assumed put date and put price. The general expression for a bond paying annual coupons and valued for settlement on a coupon date is given at (4.27):

$$P_d = \sum_{n=1}^N \frac{C}{(1 + r_p)^n} + \frac{P_p}{(1 + r_p)^N} \quad (4.27)$$

where

- r_p is the yield-to-put
- N is the number of years to the assumed put date
- P_p is the put price on the assumed put date.

4.9.3 Bonds with call and put features

The US domestic markets includes bonds that are both callable and puttable under certain conditions. For these bonds it is possible to calculate both a yield-to-call and a yield-to-put for selected future dates, under the usual assumptions. In these circumstances the practice is to calculate the yield to maturity for the bond to the redemption date that gives the lowest possible yield for the bond. Bloomberg™ analytics uses the expression *yield to worst* in relation to calculating such a yield. The term is commonly encountered for bonds with embedded options. It is used to describe the yield to the first callable date for a bond trading above par and callable at par; where the bond is trading below par the yield to worst will be the yield to maturity. A bond that is callable at par and trading above par will indeed be called by the issuer, as the price indicates that the issuer can borrow money at a cheaper rate. Therefore the appropriate date to use when calculating the redemption yield is the first call date, hence the term yield to worst. For a bond that was both callable and puttable, yield-to-worst is the yield to the redemption date that gives the lowest yield, irrespective of whether this is a call or put date (or indeed if it both callable and puttable on that date).

4.9.4 Yield to average life

As we noted at the start of this section, some corporate bonds have a sinking fund or *purchase fund* attached to them. Where bonds are issued under these provisions, a certain amount of the issue is redeemed before maturity, ranging from a set percentage to the entire issue. The partial or full redemption process is carried out randomly on the basis of selecting bond serial numbers, or sometimes through direct purchase of some of the issue in the secondary market. Bonds are issued with a redemption schedule that specifies the dates, the proportions and (in the case where the process takes place randomly) the values of the redemption payments.

Investors holding and trading bonds that are issued with these provisions sometimes use a different yield measure to the conventional YTM one. A common measure of return is the *yield to average life*. The *average life* of a bond is defined as the weighted average time to redemption using relative redemption cash flows as the weights. The expression for calculating average life is given by equation (4.28):

$$\text{Average life} = \frac{\sum_{t=1}^N A_t \times M_t \times t}{\sum_{t=1}^N A_t \times M_t} \quad (4.28)$$

where

A_t is the proportion of bonds outstanding redeemed in year t
 M_t is the redemption price of bonds redeemed in year t
 N is the number of years to maturity.

The yield to average life calculation makes use of the bond's *average redemption price*, which can be determined from the redemption schedule. The average redemption price is calculated using (4.29):

$$P_{\text{ave}} = \sum_{t=1}^N A_t \times M_t \quad (4.29)$$

where P_{ave} is the average redemption price.

Example 4.11 illustrates the calculation of the yield to average life for a hypothetical 6% bond with exactly five years to maturity. The size of the issue is £200 million, and the bond has a fixed redemption schedule as shown in the table below.

EXAMPLE 4.11 Yield-to-average-life

- A bond issue of £200 million is raised with a sinking fund incorporated into its terms and conditions. The bond has a coupon of 6% payable annually, and had precisely five years to maturity. The following redemption schedule is in place:

6% 5-year sterling bond, £200m issue

Year (t)	Amount redeemed (£m)	Proportion redeemed (A_t)	Redemption price (M_t)	$A_t \times M_t \times t$	$A_t \times M_t$
1	10	0.05	102.00	5.10	5.100
2	10	0.05	101.50	10.15	5.075
3	10	0.05	101.00	15.15	5.050
4	10	0.05	100.50	20.10	5.025
5	160	0.80	100.00	400.00	80.000

Table 4.1: Bond redemption schedule.

If we assume our analysis is performed with exactly five years to maturity, we can use the expression at (4.28) to determine the bond's *average life* at that point. This is shown to be:

$$\text{Average life} = \frac{5.1 + 10.15 + 15.15 + 20.10 + 400}{5.1 + 5.075 + 5.05 + 5.025 + 80}$$

giving an average life of 4.494 years.

If we now consider the situation after say, three years, £30 million of the issue has been redeemed. This leaves a further £10 million of the bonds to be redeemed in the following year, which is 5.88% of the remaining balance of £170. This would then leave 94.12% of the bonds, or £160 million of the remaining £170 million, to be paid back in the final year. Therefore the average remaining life of the bond after the first three years has elapsed is given by:

$$\text{Average life} = \frac{(0.0588 \times 100.50 \times 1) + (0.9412 \times 100.00 \times 2)}{(0.0588 \times 100.50) + (0.9412 \times 100)}$$

or 1.9409 years.

The *average redemption price* of the bond at any point is calculated using (4.29). Therefore after three years the average redemption price for our bond is $(0.0588 \times 100.5) + (0.9412 \times 100)$ or 100.0294.

We can then calculate the yield to average life at this point. The yield to average life is the bond's redemption yield or yield to maturity assuming that the bond matures on the day given by its average life, and at the *average redemption price* instead of the par price. If we say that the bond is trading at an actual price of £99.875 per cent at this time, the yield to average life *ral* is the solution to the expression below.

$$99.875 = \frac{6}{(1 + ral)} + \frac{6}{(1 + ral)^{1.9409}} + \frac{100.0294}{(1 + ral)^{1.9409}}.$$

This expression can be solved in the same fashion as for the conventional redemption yield calculation. One way to obtain the solution is by using a spreadsheet such as Microsoft Excel® and this is detailed in Appendix 4.2.

The bond price/yield formula above gives us a yield to average life of 6.267%. At the same time we can calculate the conventional yield to maturity at the price of 99.875 and this is 6.068%. This is below the yield to average life, which is explained by the fact that the bond is trading below par in our example and the redemption schedule reduces the time over which the bondholder receives the capital gain, so that the average life yield is increased.

4.9.5 Yield to equivalent life

In some markets bonds with sinking funds are analysed using a measure known as *yield to equivalent life*. The *equivalent life* of a bond is defined as the weighted average redemption date using the *present values* of relative redemption cash flows as weights. The equivalent life therefore takes into account the fact that the redemption payments are received at different times.

To determine a bond's equivalent life we use the expression at (4.30):

$$\text{Equivalent life} = \frac{\sum_{t=1}^N A_t \times PVM_t \times t}{\sum_{t=1}^N A_t \times PVM_t} \quad (4.30)$$

where PVM_t is the present value of the redemption price of the bonds redeemed in year t and A_t and N are as defined in (4.29).

In the case of the yield to equivalent life the discount factor used to calculate the present values of the redemption prices of the bonds is the yield itself, so it is necessary to calculate the yield to equivalent life before we can calculate the bond's equivalent life. The yield to equivalent life *rel* is found by solving equation (4.31) for a bond with annual coupon payments, and again assuming the analysis is carried out on a coupon payment date so that accrued interest is zero.

$$P_d = \frac{C + (A_1 \times M_1)}{(1 + rel)} + \frac{C + (A_2 \times M_2)}{(1 + rel)^2} + \dots + \frac{C + (A_N \times M_N)}{(1 + rel)^N}. \quad (4.31)$$

To illustrate using this equation consider again our bond in Example 4.11, the five-year paper that is purchased after three years. Equation (4.31) would then become:

$$99.875 = \frac{6 + (0.0588 \times 100.50)}{(1 + rel)} + \frac{6 + (0.9412 \times 100)}{(1 + rel)^2}.$$

This is solved to give a yield to equivalent maturity *rel* of 6.262%. The spreadsheet calculation for this is shown in Appendix 4.2. Compare this to the yield to average life level of 6.267%. We can now determine the corresponding equivalent life of the bond by using equation (4.30) and this is shown below.

$$\text{Equivalent life} = \frac{(0.0588 \times (\frac{100.50}{1.06262}) \times 1) + (0.9412 \times (\frac{100.00}{1.06262^2}) \times 2)}{(0.0588 \times (\frac{100.50}{1.06262})) + (0.9412 \times (\frac{100.00}{1.06262^2}))}$$

which gives us an equivalent life of 1.937 years. This is shorter than the average life of the bond at the same point, which was 1.9409 years.

The equivalent life and the yield to equivalent life are always lower than the corresponding average life and yield to average life. This is expected because using the present values of redemption cash flows, rather than actual values, gives more weight to earlier cash flows so that the resulting yield is lower.

As with callable bonds the operative life and operative yield of a bond with a sinking fund depends on both the current price of the bond and the redemption schedule. If the bond is trading below par the issuer is likely to repurchase the bond in the market rather than pay the stated redemption price if this is higher. So if the bond is trading below par the bond's yield is its yield to maturity. If however the bond is trading above the redemption price the issuer is likely to redeem bonds at the stated redemption price. So when the bond is trading above par the usual bond yield measure to use is the yield to equivalent life.

4.10 Index-linked bonds

4.10.1 Calculating index-linked yields

In certain markets including the United Kingdom market, the government and certain corporates issue bonds that have either or both of their coupon and principal linked to a price index, such as the retail price index (RPI), a commodity price index (for example, wheat) or a stock market index.⁵ In the UK we refer to the RPI whereas in other markets the price index is the consumer price index (CPI). If we wish to calculate the yield on such *index-linked bonds* it is necessary to make forecasts of the relevant index, which are then used in the yield calculation. As an example we can use the case of index-linked government bonds, which were first introduced in the UK in March 1981. Both the principal and coupons on UK index-linked government bonds are linked to the RPI and are therefore designed to give a constant *real* yield. When index-linked gilts were first introduced the Bank of England allowed only pension funds to buy them, under the reasoning that these funds had index-linked pensions to deliver to their clients. However a wide range of investors are potential investors in bonds whose cash flows are linked to prices, so that shortly after their introduction all investors were permitted to hold index-linked gilts. Most of the index-linked stocks that have been issued by the UK government have coupons of 2 or 2½ per cent. This is because the return from an index-linked bond represents in theory *real* return, as the bond's cash flows rise in line with inflation. Historically, the real rate of return on UK market debt stock over the long term has been roughly 2½ per cent.

Indexed bonds differ in their make-up across markets. In some markets only the principal payment is linked, whereas other indexed bonds link only their coupon payments and not the redemption payment. Indexed gilts⁶ in the UK market link both their coupons and principal payment to the RPI. Therefore each coupon and the final principal on index-linked gilts are scaled up by the ratio of two values of the RPI. The main RPI measure is the one reported for eight months before the issue of the gilt, and is known as the *base RPI*. The base RPI is the denominator of the index measure. The numerator is the RPI measure for eight months prior to the month coupon payment, or eight months before the bond maturity date.

The coupon payment of an index-linked gilt is given by (4.32) below.

$$\text{Coupon payment} = (C/2) \times \frac{RPI_{C-8}}{RPI_0}. \quad (4.32)$$

Expression (4.32) shows the coupon divided by two before being scaled up, because index-linked gilts pay semi-annual coupons. The formula for calculating the size of the coupon payment for an annual-paying indexed bond is modified accordingly. The principal repayment is given by (4.33):

$$\text{Principal repayment} = 100 \times \frac{RPI_{M-8}}{RPI_0} \quad (4.33)$$

where

C is the annual coupon payment
 RPI_0 is the RPI value eight months prior to the issue of the bond (the *base RPI*)

⁵ "Principal" is the usual term for the maturity or redemption payment of a bond.

⁶ Called "linkers" in the gilt market. Like conventional gilts, these pay semi-annual coupon.

RPI_{C-8} is the RPI value eight months prior to the month in which the coupon is paid
 RPI_{M-8} is the RPI value eight months prior to the bond redemption.

Note how in the UK gilt market the indexation calculation given by (4.32) and (4.33) uses the RPI values for eight months prior to the date of each cash flow. This is for technical reasons, because the payment authority (in this case the Bank of England) needs to know each coupon payment six months before it is paid in order to determine the interest accruing between coupon payments. This would call for a time lag of at least six months. The two additional months are due to the one-month delay in publishing the RPI (for example, January's RPI is not published until February) plus additional time to make the payment calculations. The time lag is therefore eight months. Price indices are occasionally "re-based" which means that the index is set to a base level again. In the UK the RPI has been re-based twice, the last occasion being in January 1987, when it was set to 100 from the January 1974 value of 394.5.

EXAMPLE 4.12

- An index-linked gilt with coupon of 4.625% was issued in April 1988 and matured in April 1998. The base measure required for this bond is the RPI for August 1987, which was 102.1. The RPI for August 1997 was 158.5. We can use these values to calculate the actual cash amount of the final coupon payment and principal repayment in April 1998, as shown below.

$$\text{Coupon payment} = (4.625/2) \times \frac{158.5}{102.1} = \text{£}3.58992$$

$$\text{Principal repayment} = 100 \times \frac{158.5}{102.1} = \text{£}155.23996.$$

We can determine the accrued interest calculation for the last six-month coupon period (October 1997 to April 1998) by using the final coupon payment, given below.

$$3.58992 \times \frac{\text{No. of days accrued}}{\text{actual days in period}}.$$

The markets use two main yield measures for index-linked bonds, both of which are a form of yield-to-maturity. These are the *money* (or *nominal*) *yield*, and the *real yield*.

In order to calculate a money yield for an indexed bond we require forecasts of all future cash flows from the bond. Since future cash flows from an index-linked bond are not known with certainty, we therefore require a forecast of all the relevant future RPIs, which we then apply to all the cash flows. In fact the market convention is to take the latest available RPI and assume a constant inflation rate thereafter, usually 2½% or 5%. By assuming a constant inflation rate we can set future RPI levels, which in turn allow us to calculate future cash flow values.

We obtain the forecast for the first relevant future RPI using (4.34).

$$RPI_1 = RPI_0 \times (1+\tau)^{m/12} \quad (4.34)$$

where

RPI_1 is the forecast RPI level
 RPI_0 is the latest available RPI
 τ is the assumed future annual inflation rate
 m is the number of months between RPI_0 and RPI_1 .

Consider an indexed bond that pays coupons every June and December. For analysis we require the RPI forecast value for eight months prior to June and December, which will be for October and April. If we are now in February, we require a forecast for the RPI for the next April. This sets $m = 2$ in our equation at (4.34). We can then use (4.35) to forecast each subsequent relevant RPI required to set the bond's cash flows.

$$RPI_{j+1} = RPI_1 \times (1 + \tau)^{j/2} \quad (4.35)$$

where j is the number of semi-annual forecasts after RPI_1 (which was our forecast RPI for April). For example if the February RPI was 163.7 and we assume an annual inflation rate of 2.5%, then we calculate the forecast for the RPI for the following April to be $RPI_1 = 163.7 \times (1.025)^{2/12} = 164.4$, and for the following October it would be $RPI_3 = 164.4 \times (1.025) = 168.5$. Once we have determined the forecast RPIs we can calculate the yield. Adopting our familiar assumption that the analysis is carried out on a coupon date, so that accrued interest on the bond is zero, we can calculate the money yield (ri) by solving equation (4.36):

$$P_d = \frac{(C/2)(RPI_1/RPI_0)}{(1 + \frac{1}{2}ri)} + \frac{(C/2)(RPI_2/RPI_0)}{(1 + \frac{1}{2}ri)^2} + \dots + \frac{((C/2) + M)(RPI_N/RPI_0)}{(1 + \frac{1}{2}ri)^N} \quad (4.36)$$

where

ri is the semi-annualised money yield-to-maturity
 N is the number of coupon payments (interest periods) up to maturity.

Equation (4.36) is for semi-annual paying indexed bonds such as index-linked gilts. The equation for annual coupon indexed bonds is given at (4.37):

$$P_d = \frac{C(RPI_1/RPI_0)}{(1 + ri)} + \frac{C(RPI_2/RPI_0)}{(1 + ri)^2} + \dots + \frac{(C + M)(RPI_N/RPI_0)}{(1 + ri)^N}. \quad (4.37)$$

The real yield (ry) is related to the money yield through equation (4.38), as it applies to semi-annual coupon bonds, which was first described by Fisher in *Theory of Interest* (1930).

$$(1 + \frac{1}{2}ry) = (1 + \frac{1}{2}ri) / (1 + \tau)^{\frac{1}{2}} \quad (4.38)$$

To illustrate this, if the money yield is 5.5% and the forecast inflation rate is 2.5%, then the real yield is calculated using (4.38) as shown below.

$$ry = \left(\frac{(1 + \frac{1}{2}(0.055))}{(1 + (0.025))^{\frac{1}{2}}} - 1 \right) \times 2 = 0.0297 \text{ or } 2.97\%.$$

We can rearrange equation (4.36) and use (4.38) to solve for the real yield, shown at (4.39) and applicable to semi-annual coupon bonds. Again, we use (4.39) where the calculation is performed on a coupon date:

$$\begin{aligned} P_d &= \frac{RPI_a}{RPI_0} \left(\frac{(C/2)(1 + \tau)^{\frac{1}{2}}}{(1 + \frac{1}{2}ri)} + \frac{(C/2)(1 + \tau)}{(1 + \frac{1}{2}ri)^2} + \dots + \frac{((C/2) + M)(1 + \tau)^{\frac{N}{2}}}{(1 + \frac{1}{2}ri)^N} \right) \\ &= \frac{RPI_a}{RPI_0} \left(\frac{(C/2)}{(1 + \frac{1}{2}ry)} + \dots + \frac{(C/2) + M}{(1 + \frac{1}{2}ry)^N} \right) \end{aligned} \quad (4.39)$$

where $RPI_a = RPI_1 / (1 + \tau)^{1/2}$ and RPI_0 is the base index level as initially described. RPI_a / RPI_0 is the rate of inflation between the bond's issue date and the date the yield calculation is carried out. It is best to think of the equations for money yield and real yield by thinking of which discount rate to employ when calculating a redemption yield for an indexed bond. Equation (4.36) can be viewed as showing that the money yield is the appropriate discount rate for discounting money or nominal cash flows. We then rearrange this equation as given in (4.39) to show that the real yield is the appropriate discount rate to use when discounting real cash flows.

4.10.2 Assessing yield for index-linked bonds

Index-linked bonds do not offer *complete* protection against a fall in real value of an investment. That is, the return from index-linked bonds including index-linked gilts is not in reality a guaranteed real return, in spite of the cash flows being linked to a price index such as the RPI. The reason for this is the lag in indexation, which for index-linked gilts is eight months. The time lag means that an indexed bond is not protected against inflation for the last interest period of its life, which for gilts is the last six months. Any inflation occurring during this final interest period will not be reflected in the bond's cash flows and will reduce the real value of the redemption payment and

hence the real yield. This can be a worry for investors in high inflation environments. The only way effectively to eliminate inflation risk for bondholders is to reduce the time lag in indexation of payments to something like one or two months.

Bond analysts frequently compare the yields on index-linked bonds with those on conventional bonds, as this implies the market's expectation of inflation rates. To compare returns between index-linked bonds and conventional bonds analysts calculate the *break-even inflation rate*. This is the inflation rate that makes the money yield on an index-linked bond equal to the redemption yield on a conventional bond of the same maturity. Roughly speaking, the difference between the yield on an indexed bond and a conventional bond of the same maturity is what the market expects inflation during the life of the bond to be; part of the higher yield available on the conventional bond is therefore the inflation *premium*. In August 1999 the redemption yield on the 5¾% Treasury 2009, the 10-year benchmark gilt, was 5.17%. The real yield on the 2½% index-linked 2009 gilt, assuming a constant inflation rate of 3%, was 2.23%. Using (4.36) this gives us an implied break-even inflation rate of:

$$\tau = \left(\frac{\left(1 + \frac{1}{2}(0.0517)\right)}{\left(1 + \frac{1}{2}(0.0223)\right)} \right)^2 - 1 = 0.029287 \text{ or } 2.9\%.$$

If we accept that an advanced, highly developed and liquid market such as the gilt market is of at least semi-strong form, if not strong form, then the inflation expectation in the market is built into these gilt yields. However if this implied inflation rate understated what was expected by certain market participants, investors will start holding more of the index-linked bond rather than the conventional bond. This activity will then force the indexed yield down (or the conventional yield up). If investors had the opposite view and thought that the implied inflation rate overstated inflation expectations, they would hold the conventional bond. In our illustration above, the market is expecting long-term inflation to be at around 2.9% or less, and the higher yield of the 5¾% 2009 bond reflects this inflation expectation. A fund manager will take into account her view of inflation, amongst other factors, in deciding how much of the index-linked gilt to hold compared to the conventional gilt. It is often the case that investment managers hold indexed bonds in a portfolio against specific index-linked liabilities, such as pension contracts that increase their payouts in line with inflation each year.

The premium on the yield of the conventional bond over that of the index-linked bond is therefore compensation against inflation to investors holding it. Bondholders will choose to hold index-linked bonds instead of conventional bonds if they are worried by unexpected inflation. An individual's view on expected inflation will depend on several factors, including the current macro-economic environment and the credibility of the monetary authorities, be they the central bank or the government. In certain countries such as the UK and New Zealand, the central bank has explicit inflation targets and investors may feel that over the long-term these targets will be met. If the track record of the monetary authorities is proven, investors may feel further that inflation is no longer a significant issue. In these situations the case for holding index-linked bonds is weakened.

The real-yield level on indexed bonds in other markets is also a factor. As capital markets around the world have become closely integrated in the last twenty years, global capital mobility means that high inflation markets are shunned by investors. Therefore over time expected returns, certainly in developed and liquid markets, should be roughly equal, so that real yields are at similar levels around the world. If we accept this premise, we would then expect the real yield on index-linked bonds to be at approximately similar levels, whatever market they are traded in. For example we would expect indexed bonds in the UK to be at level near to that in say, the US market. In fact in May 1999 long-dated index-linked gilts traded at just over 2% real yield, while long-dated indexed bonds in the US were at the higher real yield level of 3.8%. This was viewed by analysts as reflecting that international capital was not as mobile as had previously been thought, and that productivity gains and technological progress in the US economy had boosted demand for capital there to such an extent that real yield had had to rise. However there is no doubt that there is considerable information content in index-linked bonds and analysts are always interested in the yield levels of these bonds compared to conventional bonds.

4.11 Yields on floating-rate bonds

4.11.1 Floating Rate Notes

In a later chapter we will discuss in greater detail securities known as *floating rate notes* (FRNs) which are bonds that do not pay a fixed coupon but instead pay coupon that changes in line with another specified reference interest rate. The FRN market in countries such as the US and UK is large and well-developed; floating-rate bonds are particularly popular with short-term investors and financial institutions such as banks. With the exception of its coupon arrangement, an FRN is similar to a conventional bond. Maturity lengths for FRNs range from two years to over 30 years. The coupon on a floating-rate bond “floats” in line with market interest rates. According to the payment frequency, which is usually quarterly or semi-annually, the coupon is re-fixed in line with a money market index such as the London Inter-bank Offer Rate or LIBOR. Often an FRN will pay a *spread* over LIBOR, and this spread is fixed through the life of the bond. For example a sterling FRN issued by the Nationwide Building Society in the UK maturing in August 2001 pays semi-annual coupons at a rate of LIBOR plus 5.7 basis points.⁷ This means that every six months the coupon is set in line with the 6-month LIBOR rate, plus the fixed spread.

The rate with which the FRN coupon is set is known as the *reference rate*. This will be 3-month or 6-month LIBOR or another interest rate index. In the US market FRNs frequently set their coupons in line with the Treasury bill rate. The spread over the reference note is called the *index spread*. The index spread is the number of basis points over the reference rate; in a few cases the index spread is negative, so it is subtracted from the reference rate.

4.11.2 Yield measurement

As the coupon on an FRN is re-set every time it is paid, the bond’s cash flows cannot be determined with certainty in advance. It is not possible therefore to calculate a conventional yield to maturity measure for an FRN. Instead the markets use a measure called the *discounted margin* to estimate the return received from holding a floating-rate bond. The discounted margin measures the average spread (margin) over the reference rate that is received during the life of the bond, assuming that the reference rate stays at a constant level throughout. The assumption of a stable reference rate is key to enabling the calculation to be made, and although it is slightly unrealistic it does enable comparisons to be made between yields on different bonds. In addition the discount margin method also suffers from the same shortcoming as the conventional redemption yield calculation, namely the assumption of a stable discount rate.

To calculate discounted margin, select a reference rate and assume that this remains unchanged up to the bond’s maturity date. The common practice is to set the rate at its current level. The bond’s margin or index spread is then added to this reference rate (or subtracted if the spread is negative). With a “fixed” rate in place, it is possible to determine the FRN’s cash flows, which are then discounted at the fixed rate selected. The correct discount rate will be the one that equates the present values of the discounted cash flows to the bond’s price. Since the reference rate is fixed, we need to alter the margin element in order to obtain the correct result. When we have equated the NPV to the price at a selected discount rate, we know what the discounted margin for the bond is.

Due to the way that each coupon is re-set every quarter or every six months, FRNs trade very close to par (100 per cent) and on the coupon re-set date the price is always par. When a floating-rate bond is priced at par the discounted margin is identical to the fixed spread over the reference rate. Note that some FRNs feature a spread over the reference rate that is itself floating; such bonds are known as *variable rate notes*.

EXAMPLE 4.13

- Consider a floating-rate note trading at a price of £99.95 per cent. The bond pays semi-annual coupons at 6-month LIBOR plus 10 basis points and has precisely three years to maturity (so that the discount margin calculation is carried out for value on a coupon date). Table 4.2 shows the present value calculations, from which we can see that at the price of 99.95 this represents a discounted margin of 12 basis points on an annualised basis.

3-year FRN, Price: 99.95, 6-mo Libor + 10 basis points, Libor 5.5%.

⁷ A basis point is one-hundredth of 1 per cent, that is $1\text{bp} = 0.01\%$.

PV of cash flows: at selected spread over reference rate (basis points)									
Interest period (n)	LIBOR _{6mo}	s/a	Cash flow	10	11	12	13	14	15
1	5.50%	2.75%	2.80	2.72374	2.72360	2.72347	2.72334	2.72321	2.72307
2	5.50%	2.75%	2.80	2.64955	2.64929	2.64903	2.64878	2.64852	2.64826
3	5.50%	2.75%	2.80	2.57738	2.57701	2.57663	2.57625	2.57589	2.57550
4	5.50%	2.75%	2.80	2.50718	2.50669	2.50621	2.50572	2.50523	2.50474
5	5.50%	2.75%	2.80	2.43889	2.43830	2.43771	2.43711	2.43652	2.43593
6	5.50%	2.75%	102.80	87.10326	87.07785	87.05244	87.02704	87.00166	86.97628
Net Present Value:				100.00000	99.97274	99.94549	99.91824	99.89103	99.86378
PV calculated using				1.028	1.02805	1.0281	1.02815	1.0282	1.02825
$C/(1 + \text{ref rate} + \text{margin})^n$									

Table 4.2: Discounted margin calculation for an FRN.
Calculation carried out on coupon date.

In our calculation the 6-month LIBOR rate is 5.50%, so we assume that this rate stays constant for the life of the bond. As this is an annual rate, the semi-annual equivalent for our purposes is 2.75%. There are two coupon payments per year, so that there are six interest periods and six remaining cash flows until maturity. As we have assumed a constant reference rate, we can set the bond coupons and redemption payment, shown as “cash flow” in the table. Each coupon payment is half of the annual reference rate plus half the annual spread. This works out as 2.75% plus five basis points, which is a semi-annual coupon of £2.80. The last cash flow is the final coupon of £2.80 plus the redemption payment of £100.00. We then discount all the cash flows at the selected margin levels until we find a level that results in a net present value to equal the current bond price. From the table we see that the price equates to a discounted margin of six basis points on a semi-annual basis, or 12 basis points annually.

4.12 Measuring yield for a bond portfolio

An investor holding a portfolio of two or more bonds will often be interested in measuring the return from the portfolio as a whole, rather than simply the return from individual bonds. This is often the case when for example a portfolio is tasked with tracking or beating a particular bond index, or has a target return to aim for. It then becomes important to measure portfolio return and not bond return. The markets adopt two main methods of measuring portfolio return, the weighted average yield and the IRR or total rate of return. The internal rate of return method has more basis in logic, however it is common to encounter the weighted average method, used as a “quick-and-dirty” yield measure.

4.12.1 Weighted average portfolio yield

When determining the yield for a portfolio of bonds, investors commonly calculate the weighted average portfolio, using the redemption yields of individual constituent bonds in the portfolio. This is not an accurate measure however and it should be avoided for all portfolios that do not hold roughly equal amounts of each bond, each of which is roughly similar maturity.

To calculate the weighted average yield, set the following, where:

- MV_p is the *market* value of bond p as a proportion of total portfolio market value
- r_p is the yield-to-maturity for bond p
- n is the number of securities in the portfolio.

The weighted average yield for the portfolio is then given by (4.40):

$$r_{\text{port}} = MV_1 r_1 + MV_2 r_2 + MV_3 r_3 + \cdots + MV_n r_n \quad (4.40)$$

where r_{port} is the weighted average portfolio yield.

EXAMPLE 4.14

- Consider a gilt portfolio made up of the following five bonds on 25 August 1999. The portfolio in Table 4.3 holds gilts ranging in maturity from two years to nine years, with a spread between the highest and lowest yielding bonds of only 36 basis points. Therefore an investor may wish to calculate portfolio yield using the weighted average method.

Bond	Nominal Amount (£m)	Price	Yield to Maturity (%)	Market Value (£m)
7% 2001	10	102.59	5.72	10.259
7% 2002	20	103.15	5.76	20.630
8% 2003	30	107.54	5.76	32.262
5% 2004	20	97.64	5.57	19.528
9% 2008	15	125.65	5.40	18.8475
MV_{port}				101.5265

Table 4.3: Portfolio weighted average yield calculation.

The market value of the portfolio is £101,526,500. There are five bonds in the portfolio, so we can set the calculation parameters as follows:

$$MV_1 = 10.259/101.5265 = 0.10105 \quad r_1 = 5.72\%$$

$$MV_2 = 20.630/101.5265 = 0.20320 \quad r_2 = 5.76\%$$

$$MV_3 = 32.262/101.5265 = 0.31777 \quad r_3 = 5.76\%$$

$$MV_4 = 19.528/101.5265 = 0.19234 \quad r_4 = 5.57\%$$

$$MV_5 = 18.8475/101.5265 = 0.18564 \quad r_5 = 5.40\%.$$

Using (4.40) we calculate the weighted average yield of the portfolio as follows:

$$\begin{aligned} &0.10104(0.0572) + 0.20320(0.0576) + 0.31777(0.0576) + 0.19234(0.057) + 0.18564(0.0540) \\ &= 0.0567753 \text{ or } 5.678\%. \end{aligned}$$

The weighted average yield of the portfolio is 5.678%.

In Example 4.14 we are comfortable with the calculation result because of the nature of the portfolio itself. A portfolio with highly unequal holdings of say, very short-dated and very long-dated bonds cannot have its yield measured accurately using the weighed-average method. Therefore it is important to be careful when using this yield measurement.

4.12.2 Portfolio internal rate of return

In the previous section we discussed the weighted-average rate of return for a bond portfolio, and while this may appear intuitively to be the best way to measure yield for a portfolio, the arithmetically more accurate method is to apply the NPV procedure used for individual bonds to the entire portfolio. This is measuring essentially the yield to maturity for the portfolio as a whole, and is also known as the portfolio internal rate of return.

We encountered internal rate of return (IRR) in the earlier chapter on financial arithmetic. The IRR of a bond, if we assume a constant reinvestment rate up to the maturity of the bond, is the bond's yield to maturity. We can calculate IRR for a portfolio in the same manner, by applying the procedure to the portfolio's cash flows. The IRR or yield is calculated by ascertaining the cash flows resulting from the portfolio as a whole and then determining the internal rate of return for these cash flows in the normal manner. For a portfolio holding say, five bonds, whose coupons all fall on the same day, the IRR will be the rate that equates the present value of the total cash flows with the current market value of the portfolio. The portfolio market value is simply the sum of the market values of all five bonds. Example 4.15 illustrates an IRR calculation for an hypothetical portfolio of five bonds, each paying annual coupons. We also assume the analysis is carried out on a coupon date, and that all five bonds have identical coupon

dates.⁸ Since the bonds in this example (4.15) pay annual coupons, the number of interest periods N is equal to the number of years to maturity.

EXAMPLE 4.15

- Our hypothetical portfolio is comprised of the following €-denominated bonds as at 3 August 1999, shown at Table 4.4.

Bond	Nominal	Price	Yield
8% 2001	2m	102.50	6.6245%
7% 2002	3m	103.20	5.8071%
6% 2003	5m	103.05	5.1371%
5% 2004	3m	97.60	5.5630%
5% 2005	1m	98.25	5.3486%

Table 4.4: Bond portfolio

These bonds generate annual cash flows as shown in Table 4.5, comprised of the annual coupon and final redemption payment. The total cash payments amount to €17,200,000.

Sett date 3 August 1999 HP yields									
Nominal		2m	3m	5m	3m	1m			
Price		102.5	103.2	103.05	97.6	98.25			
Yield		6.6245%	5.8071%	5.1371%	5.5630%	5.3486%	MV = 14,209,000		
Bond		8% 2-year	7% 3-year	6% 4-year	5% 5-year	5% 6-year	Totals	PV >	
Year:	1	160,000	210,000	300,000	150,000	50,000	870,000	870,000	828,571.43
	2	2,160,000	210,000	300,000	150,000	50,000	2,870,000	2,870,000	2,603,174.60
	3		3,210,000	300,000	150,000	50,000	3,710,000	3,710,000	2,738,365.19
	4		5,300,000		150,000	50,000	5,500,000	5,500,000	4,524,863.61
	5				3,150,000	50,000	3,200,000	3,200,000	2,507,283.73
	6					1,050,000	1,050,000	1,050,000	783,526.17
							17,200,000	17,200,000	13,985,784.73
									14,949,611.32

Portfolio IRR = 4.76841%

Table 4.5: Portfolio IRR calculation.

From the bond prices we determine the market value of the portfolio to be €14,209,000. To calculate the portfolio IRR or redemption yield we need to obtain the discount rate that gives us a net present value of the portfolio cash flows equal to €14,209,000. In normal circumstances we can obtain the required rate using a Hewlett-Packard calculator or spreadsheet programme. On this occasion we have calculated the IRR by using two trial discount rates and then interpolating between them.

A trial discount rate of 5% produces an NPV of €13,985,784.73, while a rate of 4% produces an NPV of €14,949,611.32. Using the formula for numerical iteration, the portfolio yield of 4.768% as shown below.

The portfolio “price” is its market value of €14,209,000. The yield is therefore:

⁸ This last is not such an unrealistic assumption. In the UK gilt market for example, “benchmark” gilts have identical coupon dates (7 June and 7 December each year) and are also “strippable”.

$$\begin{aligned}
 IRR_{\text{port}} &= 4\% + (5\% - 4\%) \cdot \frac{14,949,611 - 14,209,000}{14,949,611 - 13,985,784} \\
 &= 4.76841\%.
 \end{aligned}$$

How does the portfolio IRR compare to a weighted-average yield measurement? Table 4.6 shows the market value of each bond and its weight in the portfolio, which is used to calculate the weighted-average yield as in Example 4.14. The result is a weighted-average yield of 5.614%:

MV_1	0.144275	(2050000/14209000)
MV_2	0.21789	(3096000/14209000)
MV_3	0.362622	(5152500/14209000)
MV_4	0.206067	(2928000/14209000)
MV_5	0.069146	(982500/14209000)
Weighted average yield = 0.056138759 or 5.614%.		

Table 4.6: Portfolio weighted average yield.

4.12.3 Total rate of return

We have seen that in calculating a bond's redemption yield one is required to make certain assumptions; where these assumptions are not met the yield measure will be inaccurate. The assumption that coupon payments are reinvested at the same rate as the redemption yield is not realistic. Interest earned on coupon interest may be responsible for a significant proportion of a bond's total return; reinvestment at a lower rate than the yield will result in a total return being lower than the redemption yield. If an investor wishes to calculate a total rate of return from holding a bond, it is still necessary to make an assumption, an explicit one about the reinvestment rate that will be realised during the life of the bond. For instance in a steep yield curve or high-yield environment, the investor may wish to assume a higher rate of reinvestment and apply this to the yield calculation. The reinvestment rate assumed will need to be based on the investor's personal view of future market conditions. The *total rate of return* is the yield measure that incorporates this assumption of a different reinvestment rate.

When computing total return, a portfolio manager will first calculate the total cash value that will accrue from investing in a bond assuming an explicit reinvestment rate. The total return is then calculated as the interest rate that will make the initial investment in the bond increase to the calculated total cash value. The procedure for calculating the total return for a bond held over a specified investment horizon, and paying semi-annual coupons is summarised below.

First we calculate the value of all coupon payments plus the interest-on-interest, based on the assumed interest rate. If we denote this as I then this can be done using equation (4.41):

$$I = C \left(\frac{(1 + r)^N - 1}{r} \right) \quad (4.41)$$

where r is the semi-annual reinvestment rate and N is the number of interest periods to maturity (for a semi-annual coupon bond it will be twice the number of years to maturity). The investor then needs to determine the projected sale price at the end of the planned investment horizon. The projected sale price will be a function of the anticipated required yield at the end of the investment horizon, and will equal the present value of the remaining cash flows of the bond, discounted at the anticipated required yield. The sum of these two values, the total interest income and projected sale price, is the total cash value that will be received from investing in the bond, using our assumed reinvestment rate and the projected required yield at the end of the investment horizon. To calculate the semi-annual total return we use (4.42):

$$\left(\frac{\text{Total cash value}}{\text{Purchase price of bond}} \right)^{\frac{1}{n}} - 1 \quad (4.42)$$

where n is the number of six-month periods in the investment horizon. This formula is derived from (4.11), which was our formula for obtaining the yield for a zero-coupon bond. This semi-annual total return is doubled to obtain the total return. The equations are modified in the usual fashion for bonds that pay annual coupons.

4.13 The price/yield relationship

The last two chapters have illustrated a fundamental property of bonds, namely that an upward change in the price results in a downward move in the yield, and vice-versa. This is of course immediately apparent since the price is the present value of the cash flows; as the required yield for a bond say, decreases the present value and hence the price of the cash flow for the bond will increase. It also reflects the fact that for plain vanilla bonds the coupon is fixed, therefore it is the price of the bond that will need to fluctuate to reflect changes in market yields. It is useful sometimes to plot the relationship between yield and price for a bond. A typical price/yield profile is represented graphically in Figure 4.4, which shows a *convex* curve. To reiterate, for a plain vanilla bond with a fixed coupon, the price is the only variable that can change to reflect changes in the market environment. When the coupon rate of a bond is equal to the market rate, the bond price will be par (100). If the required interest rate in the market moves above a bond's coupon rate at any point in time, the price of the bond will adjust downward in order for the bondholder to realise the additional return required. Similarly if the required yield moves below the coupon rate, the price will move up to equate the yield on the bond to the market rate. As a bond will redeem at par, the capital appreciation realised on maturity acts as compensation when the coupon rate is lower than the market yield.

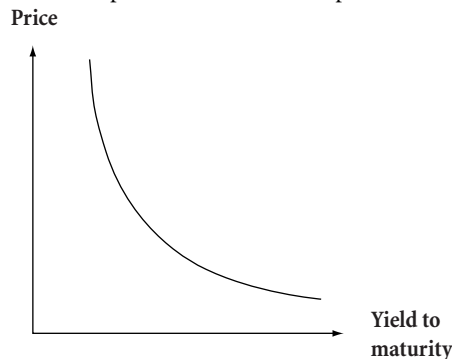


Figure 4.4: The bond price/yield relationship.

The price of a bond will move for a variety of reasons, including the market-related ones noted here:

- when there is a change in the yield required by the market, either because of changes in the base rate or a perceived change in credit quality of the bond issuer (credit considerations do not affect developed country government bonds);
- when there is a change because as the bond is approaching maturity, its price moves gradually towards par;
- when there is a change in the market-required yield due to a change in the yield on comparable bonds.

Bond prices also move for liquidity reasons and normal supply-and-demand reasons, for example if there is a large amount of a particular bond in issue it is easier to trade the bond; also if there is demand due to a large customer base for the bond. Liquidity is a general term used here to mean the ease with which a market participant can trade in or out of a position. If there is always a ready buyer or seller for a particular bond, it will be easier to trade in the market.

4.14 Summary

In this and the previous chapter we have discussed the market conventions for calculating bond prices and yields. The key issues are the price/yield relationship that all bonds exhibit and the assumptions behind the various yield calculations. The different conventions used across various markets, although not impacting the economic fundamentals of any particular instrument, make it important to convert yields to the same calculation basis, to ensure that one is comparing like-for-like. In some markets such as the UK gilt strips market, bonds trade on a yield rather than price basis. We suggested earlier in this chapter it is the yield that is the primary information required by

investors. When analysing bonds across markets, the pricing and yield conventions in themselves are not important as such, but serve to remind us of what is important – that we conduct analysis under the same terms for all financial instruments.

The other main topics we have considered are:

- **bond day-count convention** – the basis used to calculate accrued interest on a bond, such as actual/365 or actual/actual;
- **adjustments for non-business days** – bond accrued interest runs for every calendar day, whereas actual settlement can only take place on a working day. Most markets ignore non-business days for bond calculations, although the UK gilt market for example discounts cash flows to the dirty price using the actual payment date for each cash flow;
- **simple interest and compound interest** – certain markets including the US Treasury and the German government bond market calculate the yield for a bond with only one remaining coupon using simple rather than compound interest;
- **compounding method** – the most common market convention discounts cash flows using compounding in whole years. This results in a discount factor as given by (4.43):

$$\left(\frac{1}{1+r} \right)^{\frac{\text{days to next coupon}}{\text{days in interest period}} + N} . \quad (4.43)$$

It is also common to encounter the more accurate measurement for compounding as given by (4.44):

$$\left(\frac{1}{1+r} \right)^{\frac{\text{days to next coupon}}{\text{year}} \times N} ; \quad (4.44)$$

- **annualised yields** – the yield quoted for a bond generally follows the coupon payment frequency for that bond, so that annual coupon bonds quote an annualised yield, semi-annual bonds a semi-annual yield and so on. When comparing bonds of different coupon conventions it is important to convert one so that we are comparing like-for-like.

The redemption yield or yield to maturity for a bond is the bond's IRR, assuming that re-investment rates for subsequent cash flows are identical to the yield itself. This is an important feature of the redemption yield measurement. We have also discussed a variation on the basic calculation, whereby a selected fixed reinvestment rate is assumed and then used to determine future cash flows from a bond. In later chapters we will discuss zero-coupon or *spot* yields, and the concept of a true interest rate.

Appendices

APPENDIX 4.1 Day-count conventions in selected bond markets

	Market	Day-count basis	Coupon
<i>Australia</i>	Money market	act/365	
	Bond market	act/act	semi-annual
<i>Austria</i>	Money market	act/360	
	Bond market	act/act	annual
<i>Belgium</i>	Money market	act/360	
	OLO	act/act	annual
<i>Canada</i>	Money market	act/365	
	Bond market	act/365	semi-annual
<i>Denmark</i>	Money market	act/360	
	Bond market	30(E)/360	annual
<i>Euromarket</i>	Money market	act/360	
	Eurobonds	30(E)/360	annual
<i>Finland</i>	Money market	act/360	
	Bond market	act/act	annual
<i>France</i>	Money market	act/360	
	OAT, BTAN	act/act	annual
<i>Germany</i>	Money market	act/360	
	Bund, OBL	act/act	annual
<i>Ireland</i>	Money market	act/360	
	Bond market	act/act	annual and semi-annual
<i>Italy</i>	Money market	act/360	
	BTP	act/act	annual
<i>Japan</i>	Money market	act/365	
	JGB	act/365	semi-annual
<i>Netherlands</i>	Money market	act/360	
	DSL	act/act	annual
<i>Norway</i>	Money market	act/360	
	T-bill	act/365	
<i>Spain</i>	Bond market	act/365	annual
	Money market	act/360	
<i>Sweden</i>	Bono	act/act	annual
	Money market	act/360	
<i>South Africa</i>	T-bill	30(E)/360	
	Bond market	30(E)/360	annual
<i>Switzerland</i>	Money market	act/365	
	Bond market	act/365	semi-annual
<i>United Kingdom</i>	Money market	act/360	
	Bond market	30(E)/360	annual
<i>United States</i>	Money market	act/365	
	Gilts	act/act	semi-annual (rarely, quarterly)
<i>United States</i>	Money market	act/360	
	Treasuries	act/act	semi-annual
	Federal Agency	30(A)/360	semi-annual, quarterly, monthly
	Corporate bonds	30(A)/360	semi-annual

Table 4.7: Day-count conventions in selected bond markets.

APPENDIX 4.2 Yield-to-average life

In determining the yield to average life of the bond in Example 4.11 we have to solve for *ral* as given in the expression below:

$$99.875 = \frac{6}{(1 + ral)} + \frac{106.02946}{(1 + ral)^{1.9409}}.$$

We can set up the present value calculation in our Microsoft Excel spreadsheet as shown in Figure 4.5.

Cell	D	E	F	
7		discount rate =	0.06267275	
8				
9				
10	Year	Cash flow	PV	
11	0.0000	−99.875	−99.875	"=E11/(1+F7)^D11
12	1.0000	6	5.64614084	"=E12/(1+F7)^D12
13	1.9409	106.0294	94.2296181	"=E13/(1+F7)^D13
14				
15		NPV =	0.00075892	"=SUM(F11:F13)
16				

Figure 4.5: Using Microsoft Excel® to calculate an internal rate of return.

To obtain *ral* we solve the expression as we would a conventional IRR or NPV calculation. Thus the opening cash flow is the purchase price of the bond, in this case 99.875 and this is entered as a negative value. The cash flows of the bond are the two coupon payments and the redemption payment. We then enter the formula for the present values of each cash flow, using a discount factor that we select ourselves. The sum of the present values is the NPV value as shown. We then select “Tools” and then “Goal Seek...”, setting the NPV cell to zero and selecting that the value of the discount factor be changed to that which gives us this zero NPV. The spreadsheet then works out the discount factor required, which is 6.262%. To solve for the yield to maturity the table looks like Figure 4.6.

		discount rate =	0.060682542	
Year	Cash flow	PV		
0	−99.875	−99.875	"=D9/(1+\$E\$5)^C9	
1	6	5.656734943	"=D10/(1+\$E\$5)^C10	
2	106.0000	94.21824784	"=D11/(1+\$E\$5)^C11	
	NPV =	−1.72149E−05	"=SUM(E9:E11)	

Figure 4.6: Calculating the yield to maturity.

Finally, the spreadsheet calculation for the yield to equivalent life measure discussed above is shown at Figure 4.7.

	discount rate =	0.062621
Year	Cash flow	PV
0	−99.8750	−99.875
1	11.9094	11.20757
2	100.1200	88.66741
	NPV =	−1.8E−05

Figure 4.7: Calculating the yield to equivalent life.

Selected bibliography and references

Fabozzi, F., *Bond Markets, Analysis and Strategies*, Chapter 2, Prentice Hall, 1989.

Frost, A.J., Hager D.P., *Debt Securities*, Heinemann, 1990.

Jarrow, R., Turnbull, S., *Derivative Securities*, 2nd edition, South-Western, 2000.

Jones, P., *Introduction to Financial Market Mathematics*, Astra Books, 1999.

The following texts were referred to in the text but are more relevant to the contents of Parts VI and VIII in this book:

Baxter, M., Rennie, A., *Financial Calculus*, Cambridge University Press, 1996.

Hull, J., *Options, Futures and Other Derivatives*, 3rd edition, Prentice Hall, 1997.

Neftci, S., *Mathematics of Financial Derivatives*, Academic Press, 1996.

James, J., Webber, N., *Interest Rate Modelling*, John Wiley and Sons, 2000.

Jarrow, R., *Modelling Fixed Income Securities and Interest Rate Options*, McGraw-Hill, 1996.

Questions and exercises

1. A UK government bond with a coupon of 6½% will be redeemed at par on 7 June 2003. What is the price of the bond on 7 June 1999 if the yield to maturity is 5.70%?
2. If the semi-annual yield to maturity of a bond is 6.38%, what is
 - (a) the equivalent annual yield to maturity?
 - (b) the equivalent quarterly yield to maturity?
3. The yield on a UK government bond is 6.5% while the yield on a sterling eurobond is 6.75%. Which paper is offering the better return?
4. Alamgir buys a €-denominated eurobond via his execution-only stockbroker at a price of €102.45. The bond has a coupon of 8% and has precisely five years to maturity. What is the bond's running yield and its simple yield to maturity?
5. Calculate the first three coupon payments for an index-linked gilt issued on 1 September 1997, given the following UK retail price indices:

	1997	1998		1997	1998
January	154.4	159.9	July	157.5	163
February	155.0	160.3	August	158.5	163.7
March	155.4	160.8	September	159.3	164.4
April	156.3	162.6	October	159.5	164.5
May	156.9	163.5	November	159.6	164.4
June	157.5	163.4	December	160.0	164.4

6. In August 1999 the five-year benchmark gilt was trading at a yield to maturity of 5.57%, while the five-year index-linked gilt had a real yield (assuming 3% inflation) of 2.55%. What is the break-even inflation rate? Austin is considering which of the two gilts to invest in, and his personal view is that inflation will be running at over 3.5% over the next five years. What bond should Austin hold?
7. Calculate the redemption yield of the following bond, which trades on an actual/actual basis and pays annual coupons:
- Coupon: 6%
Maturity date: 1 December 2009
Settlement date: 1 September 1999
Price: 101.15
8. Nasima buys the 6% 2009 bond on 1 September 1999 but then sells it for value on 1 March 2000 at a price of 102.10. What is her simple rate of return during this period, on an actual/365 basis, and her effective rate of return?
9. You hold the following portfolio of three €-denominated bonds on 25 August 1999:

	<i>Nominal (€m)</i>	<i>Price</i>	<i>Yield(%)</i>
5% 2004	5m	101.45	4.668
6% 2001	20m	102.14	4.852
5½% 2003	10m	102.67	4.75

What is the weighted-average yield of your portfolio? If all of the bonds pay coupon on the same date, calculate the portfolio internal rate of return.

5 Review of Bond Market Instruments

The development of financial engineering techniques in banks around the world has resulted in a great variety of financial instruments being traded. The bond markets are no exception to this and there are a range of instruments in the debt market with special features. Such bonds require a variation of the basic tools we have considered up to now for their analysis. So far in this book we have been content to limit our discussion and analysis to conventional or plain vanilla bonds, that is, instruments with a fixed coupon and term to maturity. In the previous chapter we introduced some variation on this when we looked at yield measurement for floating-rate and index-linked bonds. We introduced the concept of the zero-coupon bond yield, and this subject will be developed in the next chapter which looks at the term structure of interest rates. Before that we will use this chapter to introduce in overview form some variations on the basic plain vanilla bond. This is to give readers some flavour of the other types of bonds that exist in the market, and which we will look at in greater detail in Part III.

5.1 Floating Rate Notes

Floating rate notes (FRNs) are bonds that have variable rates of interest; the coupon rate is linked to a specified index and changes periodically as the index changes. An FRN is usually issued with a coupon that pays a fixed spread over a reference index; for example the coupon may be 50 basis points over the six-month interbank rate. An FRN whose spread over the reference rate is not fixed is known as a *variable rate note*. Since the value for the reference benchmark index is not known, it is not possible to calculate the redemption yield for an FRN. Additional features have been added to FRNs, including *floors* (the coupon cannot fall below a specified minimum rate), *caps* (the coupon cannot rise above a maximum rate) and *callability*. There also exist perpetual FRNs. As in other markets borrowers frequently issue paper with specific or even esoteric terms in order to meet particular requirements or meet customer demand, for example a US bank recently issued US dollar-denominated FRNs with interest payments indexed to the €uribor rate, and another FRN with its day count basis linked to a specified Libor range.

Generally the reference interest rate for FRNs is the London interbank rate; the *offered* rate, that is the rate at which a bank will lend funds to another bank is LIBOR. An FRN will pay interest at LIBOR plus a quoted margin (or spread). The interest rate is fixed for a three-month or six-month period and is reset in line with the LIBOR *fixing* at the end of the interest period. Hence at the coupon re-set date for a sterling FRN paying six-month Libor + 0.50%, if the Libor fix is 7.6875%, then the FRN will pay a coupon of 8.1875%. Interest therefore will accrue at a daily rate of £0.0224315.

On the coupon reset date an FRN will be priced precisely at par. Between reset dates it will trade very close to par because of the way in which the coupon is reset. If market rates rise between reset dates an FRN will trade slightly below par, similarly if rates fall the paper will trade slightly above. This makes FRNs very similar in behaviour to money market instruments traded on a yield basis, although of course FRNs have much longer maturities. Investors can opt to view FRNs as essentially money market instruments or as alternatives to conventional bonds. For this reason one can use two approaches in analysing FRNs. The first approach is known as the *margin method*. This calculates the difference between the return on an FRN and that on an equivalent money market security. There are two variations on this, simple margin and discounted margin.

The simple margin method is sometimes preferred because it does not require the forecasting of future interest rates and coupon values. *Simple margin* is defined as the average return on an FRN throughout its life compared with the reference interest rate. It has two components: a *quoted margin* either above or below the reference rate, and a capital gain or loss element which is calculated under the assumption that the difference between the current price of the FRN and the maturity value is spread evenly over the remaining life of the bond. Simple margin uses the expression at (5.1):

$$\text{Simple margin} = \frac{(M - P_d)}{(100 \times T)} + M_q \quad (5.1)$$

where

- P_d is $P + AI$, the dirty price
- M is the par value
- T is the number of years from settlement date to maturity
- M_q is the quoted margin.

A quoted margin that is positive reflects yield for an FRN that is offering a higher yield than the comparable money market security.

EXAMPLE 5.1 Simple margin

- An FRN with a par value of £100, a quoted margin of 10 basis points over six-month Libor is currently trading at a clean price of 98.50. The previous LIBOR fixing was 5.375%. There are 90 days of accrued interest, 92 days until the next coupon payment and five years from the next coupon payment before maturity. Therefore we have:

$$P_d = 98.50 + \frac{90}{365} \times 5.375 = 99.825.$$

We obtain T as shown:

$$T = 10 + \frac{92}{365} = 10.252.$$

Inserting these results into (5.1) we have the following simple margin:

$$\text{Simple margin} = \frac{100 - 99.825}{100 \times 10.252} + 0.0010 = 0.00117$$

or 11.7 basis points.

At certain time the simple margin formula is adjusted to take into account of any change in the reference rate since the last coupon reset date. This is done by defining an adjusted price, which is either:

$$AP_d = P_d + (re + QM) \times \frac{N_{sc}}{365} \times 100 - \frac{C}{2} \times 100$$

or

$$(5.2)$$

$$AP_d = P_d + (re + QM) \times \frac{N_{sc}}{365} \times P_d - \frac{C}{2} \times 100$$

where

- AP_d is the adjusted dirty price
- re is the current value of the reference interest rate (such as Libor)
- $C/2$ is the next coupon payment (that is, C is the reference interest rate on the last coupon reset date plus M_q)
- N_{sc} is the number of days between settlement and the next coupon date.

The upper equation in (5.2) above ignores the current yield effect: all payments are assumed to be received on the basis of par, and this understates the value of the coupon for FRNs trading below par and overstates the value when they are trading above par. The lower equation in (5.2) takes account of the current yield effect.

The adjusted price AP_d replaces the current price P_d in (5.1) to give an *adjusted simple margin*. The simple margin method has the disadvantage of amortising the discount or premium on the FRN in a straight line over the remaining life of the bond rather than at a constantly compounded rate. The discounted margin method uses the latter approach. The distinction between simple margin and discounted margin is exactly the same as that between simple yield to maturity and yield to maturity. The discounted margin method does have a disadvantage in that it requires a forecast of the reference interest rate over the remaining life of the bond.

The discounted margin is the solution to equation (5.3) shown below, given for an FRN that pays semi-annual coupons.

$$P_d = \left(\frac{1}{\left(1 + \frac{1}{2}(re + DM)\right)^{\text{days / year}}} \right) \times \left(\frac{C}{2} + \sum_{t=1}^{N-1} \frac{(re^* + QM) \times 100 / 2}{\left(1 + \frac{1}{2}(re^* + DM)\right)^t} + \frac{M}{\left(1 + \frac{1}{2}(re^* + DM)\right)^{N-1}} \right) \quad (5.3)$$

where

- DM is the discounted margin
- re is the current value of the reference interest rate
- re^* is the assumed (or forecast) value of the reference rate over the remaining life of the bond
- M_q is the quoted margin
- N is the number of coupon payments before redemption.

Equation (5.3) may be stated in terms of discount factors instead of the reference rate. This version is given in Appendix 5.1. The *yield to maturity spread* method of evaluating FRNs is designed to allow direct comparison between FRNs and fixed-rate bonds. The yield to maturity on the FRN (rmf) is calculated using (5.3) with both $(re + DM)$ and $(re^* + DM)$ replaced with rmf . The yield to maturity on a reference bond (rmb) is calculated using 4.4. The *yield to maturity spread* is defined as:

$$\text{Yield to maturity spread} = rmf - rmb.$$

If this is positive the FRN offers a higher yield than the reference bond.

Table 5.1 shows an extract from the screen of a market maker in sterling FRNs as at May 1997. The screen shows bonds at the price they are offered to investors, as well as the discount margin in basis points and the credit rating for the bond. The “amount” is the amount of paper that the bank had to offer at that time.

Name	Maturity	Price	d/m	Amount (£m)	Rating	Notes
Bradford & Bingley	Oct-99	100.00	6.5	4	A1	
Bradford & Bingley	Mar-01	100.14	7.5	3		
Bradford & Bingley	Nov-01	99.89	8.1	4.9	A2	
Britannia BS	Jan-00	100.12	6	3		
Britannia BS	Feb-01	99.89	8.7	1.5		
Bankers Trust	Feb-02	99.78	11.8	5		
Irish Permanent	Oct-98	100.37	4	10	A2	
Nationwide	Aug-01	99.78	5.7	5	A1	Callable Aug-00 at 99.93
Midland	Mar-02	99.97	0.5	5		
Midland	May-01	99.65	20	8		Sub-ordinated issue
Woolwich	Mar-99	100.14	6.1	0.930	A1	
Woolwich	Mar-01	99.87	6.8	0.600		
Royal Bank of Scotland	Jan-04	99.93	3.5	10	AA3/AA-	
Royal Bank of Scotland	Jun-05	100.05	36.5	4.5		Sub-ordinated issue; callable after Jun-00

Table 5.1: Market maker’s FRN Offer Page.

In addition to plain vanilla FRNs, some of the other types of floating-rate bonds that have traded in the market are:

- **Collared FRNs:** these offer caps and floors on an instrument, thus establishing a maximum and minimum coupon on the deal. Effectively these securities contain two embedded options, the issuer buying a cap and selling a floor to the investor.
- **Step-up recovery FRNs:** where coupons are fixed against comparable longer maturity bonds, thus providing investors with the opportunity to maintain exposure to short-term assets while capitalising on a positive sloping yield curve.

- **Corridor FRNs:** these were introduced to capitalise on expectations of comparative interest rate inactivity. A high-risk/high-reward instrument, it offers investors a very substantial uplift over a chosen reference rate. But rates have to remain within a relatively narrow corridor if the interest payment is not to be forfeited entirely.

5.2 Inverse/Reverse floating-rate bonds

5.2.1 Introduction

The coupon rate for some FRNs is set so that it does not change in the same direction as the reference interest rate, but rather in the opposite direction to the change in the reference rate. This means that if the reference rate increases from its level at the previous coupon reset date, the coupon rate on the bond will decline. These bonds are known as *inverse floating-rate notes*. Generally the coupon is calculated as a fixed rate, less a floating reference rate, implying that the product has an interest rate cap equal to the fixed rate element. Inverse FRNs originated in the US municipal and mortgage-backed bond markets. One of the first examples was a combined vanilla and inverse *collateralised mortgage obligation* FRN issued by Lehman Brothers in 1986, created by converting an existing fixed-rate bond issue into the combined FRN structure. They are common in the US corporate, municipal and collateralised mortgage obligation markets. The combined structure typically has an FRN with a coupon reset at a specified margin to an index such as Libor or (in the US market) the Federal Home Loan Cost of Funds Index or COFI. The coupon of the associated inverse FRN moves inversely with the specified index. Vanilla and inverse FRNs combination structures usually have caps and floors that set the maximum and minimum coupon payable on both bonds. The cap and floor may be explicit, in the form a specified maximum rate payable, or implicit, for example setting a floor equal to the margin if the index rate fell below say, 1%. Caps and floors may be fixed throughout the life of the bond or may be altered during its life. The inverse floater is sometimes issued with a set level from which the reference rate is subtracted, for example it may pay a rate of $10\% - \text{Libor}$. In some cases a *multiplier* is used, for example an inverse FRN paying a coupon of $20\% - (2 \times \text{Libor})$ has a multiplier of two.

£200m 10% bond, five-year maturity

Used to create:

100m FRN coupon set as $\text{Libor} + 25$ basis points, cap 9%

100m Inverse FRN coupon set as $9.75 - \text{Libor}$, floor 1%

Libor	FRN coupon	Inverse FRN coupon	Total coupon
0.00	0.25	9.75	10
1.00	1.25	8.75	10
2.00	2.25	7.75	10
3.00	3.25	6.75	10
4.00	4.25	5.75	10
5.00	5.25	4.75	10
6.00	6.25	3.75	10
7.00	7.25	2.75	10
8.00	8.25	1.75	10
9.00	9.00	1.00	10
10.00	9.00	1.00	10
12.00	9.00	1.00	10
15.00	9.00	1.00	10

Table 5.2: Creation of FRN and inverse FRN from conventional coupon bond

Where a combined structure has been created from an existing fixed coupon bond, the nominal amounts of the two new FRNs will add to the total size of the original issue. The sum of interest paid on the vanilla and inverse FRNs will also equal the coupon paid on the source bond. If a multiplier is used for the inverse FRN, its nominal size will need to be less than that of the vanilla FRN, in order to keep the combined interest payable at the same level as the

original bond's coupon. The original conventional bond from which the floating-rate and inverse floater bonds are created is called the *collateral*.

Table 5.2 illustrates an hypothetical example for a combined vanilla and inverse FRN from a conventional 10% coupon bond. The two FRNs have their coupon set in line with Libor; the vanilla FRN coupon is fixed at Libor plus 25 basis points, with a cap at 9%, while the inverse FRN coupon is set as $9.75 - \text{Libor}$. It has a floor of 1%. There is no multiplier so therefore the two bonds have equal nominal values.

5.2.2 Modified duration¹

As we noted above an inverse floater is a security whose coupon rate changes inversely with the change in the specified reference rate. In many cases inverse floating-rate bonds are created from conventional bonds. The duration of such an inverse floater is a multiple of the duration of the collateral bond from which it was created. To illustrate this, consider a 25-year conventional bond with a market value of £100 million that is split into a floating-rate bond and an inverse floater security, which have market values of £80 million and £20 million respectively. Assume that the conventional bond has a modified duration of 9.50; for a 1% change in yield, the value of this bond will change by approximately 9.5% or £9.5 million. Therefore, the two securities created by splitting the collateral bond must also change in value by a total of £9.5 million for the same change in rates. As we see in Chapter 7, both the duration and the modified duration of an FRN are relatively small, due to the nature of the coupon re-set making the bond more of a money market security (that is, a semi-annually paying FRN has an interest-rate sensitivity similar to a six-month instrument). This means that the change in value of the new portfolio must come virtually entirely from the inverse floater bond. In our example, the modified duration of this bond must be 47.50. A modified duration of 47.50 means that there will be a 47.5% change in the value of the inverse floater for a 1% change in interest rates, a change in value of £9.5 million. Inverse floater securities have the highest modified duration measures of any instrument in the bond markets.

From the example given we see that the modified duration of the inverse floater bond is higher than the number of years to maturity of the collateral bond. This is an unexpected result for those who are more familiar with the duration measure referring to years, as originally defined by Macaulay, as the inverse floater bond has a duration higher than the bond from which it was created.

The general expression for the modified duration MD of an inverse floating rate security is given by (5.4), which assumes that the modified duration for the FRN is close to zero.

$$MD_{\text{inverse}} = (1 + l)(MD_{\text{collateral}}) \times \frac{P_{\text{collateral}}}{P_{\text{inverse}}} \quad (5.4)$$

where l is the leverage level of the inverse floating-rate security.

5.3 Asset-backed bonds

5.3.1 Introduction

There is a large group of bond instruments that trade under the overall heading of *asset-backed bonds*. These are bundled securities, so called because they are marketable instruments that result from the bundling or packaging together of a set of non-marketable assets. This process is known as *securitisation*, when an institution's assets or cash flow receivables are removed from its balance sheet and packaged together as one large loan, and then "sold" on to an investor, or series of investors, who then receive the interest payments due on the assets until they are redeemed. The purchasers of the securitised assets often have no recourse to the original borrowers, in fact the original borrowers are not usually involved in the transaction or any of its processes. In this section we provide a brief overview of asset-backed bonds.

Securitisation was introduced in the US market and this market remains the largest for asset-backed bonds. The earliest examples of such bonds were in the US mortgage market, where residential mortgage loans made by a *thrift* (building society) were packaged together and sold on to investors who received the interest and principal payments made by the borrowers of the original loans. The process benefited the original lender in a number of ways. One key

¹ Duration and modified duration are reviewed in Chapter 7.

benefit was that removing assets from the balance sheet reduced risk exposure for the bank and enhanced its liquidity position.

The effect of these benefits are increased with the maturity of the original loans. For example in the case of mortgage loans, the term to maturity can be up to 25 years, perhaps longer. The bulk of these loans are financed out of deposits that can be withdrawn on demand, or at relatively short notice. In addition it is often the case that as a result of securitisation, the packaged loans are funded at a lower rate than that charged by the original lending institution. This implies that the bundled loans can be sold off at a higher value than the level at which the lending institution valued them. Put another way, securitising loans adds value to the loan book and it is the original lender that receives this value. Another benefit is that as a result of securitisation, the total funding available to the lending institution may well increase due to its access to capital markets; in other words, the firm becomes less dependent on its traditional deposit base. And finally by reducing the level of debt on the lending institution's balance sheet, it will improve the firm's gearing ratio.²

The main advantage to the investor of securitisation is that it offers a marketable asset-backed instrument to invest in. Often the instrument offers two levels of protection, the original assets and credit enhancement. The original assets will provide good security if they are well-diversified and equivalent in terms of quality, terms and conditions (for example the repayment structure and maturity of assets). A diversified asset base reduces the risk of a single drastic failure, while homogeneous assets make it more straightforward to analyse the loan base. If there is little or no liquidity in the original loans (no secondary market) then investors will often require *credit enhancement* in the form of an insurance contract, letters of credit, subordination of a second tranche which absorbs losses first³, over-collateralisation or a reserve fund, for the instrument to be sold at a price acceptable to the original lender. Ironically, by implementing one or more of the protection features described, securitisation provides a better credit risk for the investor than the loans represented to the original lender.

Securitisation began in the US housing market in 1970 after the Government National Mortgage Association (GNMA or "Ginnie Mae") began issuing *mortgage pass-through certificates*. A pass-through is a security representing ownership in a pool of mortgages. The mortgages themselves were sold through a grantor trust and the certificates sold in the capital markets. As with standard mortgages the interest and amortised principal were paid monthly. Later on *mortgage-backed bonds* were issued with semi-annual payments and maturities of up to 15 years, which were terms familiar to domestic bondholders. In 1983 *collateralised mortgage obligations* were issued, the collateral provided by mortgages issued by the Federal Home Loans Mortgage Corporation. Being government agencies, the bonds that they issue are guaranteed and as such carry little additional risk compared to US Treasury securities. They can therefore be priced on the same basis as Treasuries. However they present an additional type of risk, that of *prepayment risk*. This is the risk that mortgages will be paid off early, ahead of their term, a risk that increases when mortgages have been taken out at high fixed interest rates and rates have subsequently fallen. The existence of this risk therefore dictates that these bonds pay a higher return than corresponding Treasury bonds. The term *average life* is used to describe the years to maturity for asset-backed bonds that have an element of prepayment risk about them, and is obviously an estimate used by bond analysts.

Securitisation was introduced in the UK market in 1985. A number of institutions were established for the purpose of securitising mortgages and other assets such as car loans and credit card debt. These included National Home Loans Corporation, Mortgage Funding Corporation and First Mortgage Securities.

5.3.2 Credit rating

In the sterling market all public mortgage-backed and asset-backed securities (MBS/ABS) are explicitly rated by one or both of two of the largest credit-rating agencies, Moody's and Standard & Poor's. In structured financings it is normal for the rating of the paper to be investment grade, with most issues at launch being rated Aaa and/or AAA. We can briefly touch on the issues involved in rating such paper. The rating of the issue is derived from a combination of factors. As it cannot generally be expected that investors will be sufficiently protected by the performance of the collateral alone, the rating agencies look to minimise the risk of principal default and ensure timely payment of interest coupons by requiring additional credit enhancement. The percentage of additional enhancement is determined by analysing the "riskiness" of the collateral under a range of stress-tested environments which seek to

² Gearing is a firm's debt-to-equity ratio.

³ A second tranche of bonds that is junior to the first, so it would absorb any losses first.

quantify the effect of various interest rate, foreclosure and loss scenarios, which are largely based on the expected performance of the collateral base in a recession. Much of the analysis is based on performance in the US markets, and the rating agencies try to establish criteria for each market and collateral type that is rated. The amount of enhancement required depends on the rating required at launch, for instance less is required for a lower rated issue. In many cases issues will be backed by a larger nominal value of collateral, for example an issue size of £100 million is formed out of assets composed of say, £110 million or a higher amount.

Enhancement levels are also determined by the agencies reviewing the legal risks in the transaction. The legal analysis examines the competing rights and interests in the assets, including those of the bondholders and various third parties. Mortgage-backed securities (MBS) and asset-backed securities (ABS) are typically issued out of low capitalised “special purpose vehicle” companies (SPV), established solely for the purpose of issuing the securities. The rating agencies need to be assured that there is no risk to the bondholders in the event of the originator, that is the seller of the assets to the SPV, becoming insolvent, and to be certain that a receiver or administrator cannot seize the assets or obtain rights to the SPV’s cash flows. In addition the agencies need to be satisfied that the SPV will be able to meet its obligations to its investors in circumstances where the service body (the entity responsible for administering the collateral, usually the originator) becomes insolvent. Consequently significant emphasis is placed on ensuring that all primary and supporting documentation preserves the rights of investors in the security. An independent Trustee is appointed to represent the interests of investors. Providing Trustee services results in valuable fee-based income for banks.

A change in rating for an ABS or MBS issue may occur due to deterioration in performance of the collateral, heavy utilisation of credit enhancement, or downgrade of a supporting rating, for example an insurance company that was underwriting insurance on the pool of the assets.

5.3.3 Credit enhancement

Credit support enhancement for ABS and MBS issues is usually by either of the following methods:

- **Pool insurance:** an insurance policy provided by a composite insurance company to cover the risk of principal loss in the collateral pool. The claims paying rating of the insurance company is important in determining the overall rating of the issue. In many cases in the past the rating of the insurance company at launch proved to be insufficient to achieve the desired rating and a reinsurance policy was entered into with a higher rated company in order to achieve the desired rating.
- **Senior/Junior note classes:** credit enhancement is provided by subordinating a class of notes (“class B” notes) to the senior class notes (“class A” notes). The class B note’s right to its proportional share of cash flows is subordinated to the rights of the senior noteholders. Class B notes do not receive payments of principal until certain rating agency requirements have been met, specifically satisfactory performance of the collateral pool over a pre-determined period, or in many cases until all of the senior note classes have been redeemed in full.
- **Margin step-up:** a number of ABS issues incorporate a step-up feature in the coupon structure, which typically coincides with a call date. Although the issuer is usually under no obligation to redeem the notes at this point, the step-up feature is an added incentive for investors, to convince them from the outset that the economic cost of paying a higher coupon would be unacceptable and that the issuer will seek to refinance by exercising its call option.
- **Substitution:** this feature enables the issuer to utilise principal cash flows from redemptions to purchase new collateral from the originator. This has the effect of lengthening the effective life of the transaction as the principal would otherwise have been used to redeem the notes. The issuer is usually under no obligation to substitute and it is an option granted by the investor.

5.3.4 Redemption mechanism

ABS and MBS issue terms usually incorporate one of two main methods through which redeeming principal can be passed back to investors.

- **Drawing by lot:** the available principal from the relevant interest period is repaid to investors by the international clearing agencies Euroclear and Clearstream drawing notes, at random, for cancellation. Notes will therefore trade at their nominal value.

- **Pro rata:** The available principal for the interest period is distributed among all investors, dependent upon their holding in the security. A *pool factor* is calculated, which is the remaining principal balance of the Note expressed as a factor of one. For instance if the pool factor is 0.62557, this means that for each Note of £10,000 nominal, £3,744.30 of principal has been repaid to date. A pool factor value is useful to investors since early repayment of say, mortgages reduces the level of asset backing available for an issue, the outstanding value of such an issue is reduced on a pro-rata basis, like early redemption, by a set percentage so that the remaining amount outstanding is adequately securitised.

5.3.5 Additional features

Some ABS structures will incorporate a *call option* feature. In some cases the terms of the issue prevent a call being exercised until a certain percentage of the issue remains outstanding, usually 10 per cent, and a certain date has been passed.

It is common for ABS issues to have an *average life* quoted for them. This says that based on the most recent principal balance for the security, it is assumed that a redemption rate is applied such that the resultant average life equals the number of months left from the last interest payment date until 50 per cent of the principal balance remains. Some issuers will announce the expected average life of their paper, and yield calculations are based on this average life.

5.3.6 Analysis

It is necessary to allow for the particular characteristics of ABS and MBS bonds. Reinvestment risk, the risk that the coupon payments are reinvested at a lower rate than the redemption yields, is more acute for ABS bonds because often payments are received as frequently as every month. The yield calculation, or *cash flow yield*, is dependent on realisation of the projected cash flow according to a set prepayment rate. If actual prepayments differ from that set by the prepayment rate, the cash flow yield will not be realised.

At the time that an investor purchases an MBS, it is not possible to calculate an exact yield; this will depend on the actual prepayments of mortgages in the pool. The convention in the market is to quote the yield as a spread over a comparable government bond. The repayment of principal over time makes it inaccurate to compare the yield of an MBS to a gilt or Treasury of a stated maturity. Market participants instead use two measures: Macaulay duration and *average life*. The average life is the average time to receipt of principal payments (projected scheduled principal payments and projected principal repayments), weighted by the amount of principal expected, and divided by the total principal to be repaid. We can represent this using equation (5.5):

$$\text{Average life} = \frac{1}{12} \frac{\sum_{t=1}^n t (\text{Principal expected at time } t)}{\text{Total principal}} \quad (5.5)$$

where n is the number of months remaining.

5.4 PIBS

PIBS are a type of bond peculiar to the sterling market in London. The term comes from *Permanent Interest Bearing Shares*, and they are issued exclusively by UK building societies. PIBS are very similar to preference shares issued by banks and other corporates, that is they are irredeemable (like preference shares and ordinary shares) and they are loss absorbing, again like preference and ordinary shares, in that a building society can elect not to pay the coupon (dividend) due on PIBS if by so doing it would leave the society insolvent. The principal difference between PIBS and bank preference shares is that PIBS coupon payments are tax deductible for a building society whereas a preference share dividend is not tax deductible for a bank. PIBS are thus very attractive to building societies and can therefore be issued at higher margins over gilts than bank preference shares and still appear relatively cheap to the issuer.

The first issue of PIBS was in 1991 when Hoare Govett Securities Limited raised £75 million for Leeds Permanent Building Society (subsequently merged with and now part of Halifax plc). Further issues followed and a list of current PIBS is shown at Table 5.3.

Further developments included the introduction of floating rate PIBS. Issuing such paper resulted in the building societies involved being rated for the first time. The highest rated societies such as Halifax and Cheltenham & Gloucester subsequently either converted into banks, with shares listed on the London Stock Exchange, or were taken over by banks. In theory PIBS offer no certainty as to capital value because the issuing building society is

under no obligation to repay the principal. However no building society has ever gone out of business in the history of the movement (the oldest building society currently still in existence dates from 1845); where individual societies have found themselves in difficulties in the past, they have been taken over by another society. PIBS have delivered significant increases in capital gain for their holders, as sterling interest rates have fallen greatly from the levels that existed at the time most of the bonds were issued. They continue to trade in a liquid market.

	Coupon	Current price	Yield %	Issue price	Minimum nominal amount (£)
Birmingham Midshires BS	9.375%	146.00	6.42	100.17	1,000
Bradford & Bingley BS	11.625%	187.00	6.22	100.13	10,000
Britannia BS	13.000%	197.00	6.60	100.42	1,000
Coventry BS	12.125%	195.00	6.22	100.75	1,000
Leeds & Holbeck BS	13.375%	200.00	6.69	100.23	1,000
Newcastle BS	10.750%	175.00	6.14	100.32	1,000
Skipton BS	12.875%	214.00	6.02	100.48	1,000
Bristol & West	13.375%	193.00	6.93	100.34	1,000
Cheltenham & Gloucester	11.750%	183.00	6.40	100.98	50,000
Halifax plc	12.000%	174.00	6.90	100.28	50,000
Northern Rock plc	12.625%	195.00	6.47	100.14	1,000

Table 5.3: PIBS and perpetual subordinated bonds of building societies and former building societies as at March 1999. Source: Barclays Capital.

The building society movement has contracted somewhat in recent years, as the largest societies converted to banks, with shares publicly listed on the Stock Exchange, or were taken over by banking groups.⁴ This resulted in the PIBS sector experiencing a decline in recent years as no new issues were placed in the market. In October 1999 Manchester Building Society issued £5 million nominal of paper, this being the first new issue of PIBS since 1993. The bonds were placed by Barclays Capital, and the small size of the issue indicates that the paper was aimed largely at retail customers rather than institutions. The bonds had a coupon of 8% and were priced over the long-dated UK gilt, the 6% 2028.

5.5 Callable bonds

A callable bond is one that allows the issuer to *call* the issue before its stated maturity date, that is, redeem the bond early. This provision is sometimes referred to as a *call feature* and is widely used by corporate and local authority issuers of debt. A bond with a call provision carries two disadvantages for the bondholder. First an issuer will call a bond when the market interest rates are lower than the issue's coupon rate; for example if the coupon on a callable bond is 10 per cent and market interest rates are 8 per cent, it will be advantageous for the issuer to call the 10 per cent issue and refinance it with an 8 per cent issue. The investor is faced with reinvesting the monies received as a result of the call at a lower interest rate. Thus callable bonds present investors with a higher level of reinvestment risk. Secondly the potential for the price of a callable bond to appreciate in a falling interest rate environment is less than that of a conventional bond. This is because the price of a callable bond will remain at or near its call price (the price above which it is economical for the issuer to redeem the bond) rather than rise to the higher price that would result for an otherwise comparable non-callable bond. Because callable bonds carry such disadvantages for the investor, the market also requires them to offer sufficient compensation for bondholders, in the form of higher yield compared to conventional bonds. The higher yield on callable bonds is the price they pay in return for asking investors to bear the associated call risk.

⁴ In 1980 there were nearly 200 building societies in the UK. At October 1999 this number had been reduced to 69, with another society (Bradford & Bingley) in the process of converting to a publicly listed bank. Many of the largest institutions, such as the Halifax, Woolwich and Alliance & Leicester building societies, had earlier converted to banks. The oldest surviving building society, the Chesham Building Society, dates from 1845.

The *call price* for a callable bond is the price paid to bondholders when the bond is called. It is usually par, but it may be above par or set at different levels for different times. The prices and times at which a bond are set out in the issue terms and form part of the *call schedule*. The most common reason why a bond is called is because the issuer can re-finance borrowing requirements at a lower interest rate, for example because it has achieved an improved credit rating. When lower borrowing costs are available an issuer will call an existing issue and replace it with a bond carrying a lower coupon. Another reason why a bond may be called is when the issuer can re-fund existing debt at a lower rate; in this case, following a drop in interest rates the current issue is called and replaced, again with a bond paying a lower coupon.

Investors in a callable bond are aware of the call feature attached to the bond, and the potential downside of holding the bond. For this reason, investors require a higher yield from holding callable bonds compared to non-callable bonds of similar maturity issued by the same or similar issuers. There is often a period of *call protection* built into the bond's terms, during which the bond may not be called. If we assume an efficient market mechanism, bondholders will have the same information about the market as the bond issuer, and the premium required by them when compared to the return from holding a non-callable bond is known at the time of issue of the callable bond. In an efficient market therefore there is no advantage to a borrower in issuing a callable bond compared to a non-callable one, since the advantages of the call feature are balanced by the higher funding cost associated with incorporating a call feature.

In certain cases callable bonds are issued with a period of *call protection* included in the terms. This is a period during which the bond cannot be called, for example a callable bond with a 10-year term to maturity that is not callable for the first five years of its life. Call protection is often provided by the issuer as an added incentive to investors to buy the bond.

5.5.1 Analysis

For callable bonds the market calculates a yield to maturity in the normal fashion but also calculates a *yield to call* for the bond. The yield to call is calculated on the assumption that the bond will be called at the first call date. The procedure is the same as for the normal yield calculation, that is, to determine the market interest rate of return that will equate the present value of all the cash flows to the current price. For yield to call the cash flows are those to the first call date. The normal practice for a bond trading above par is to calculate the yield to maturity and the yield to call and take the lower value as the bond's yield. In fact bond investors and analysts will usually calculate the yield to the first call date and also the yield to all possible call dates, and then use the lowest of these values as the bond's yield. Bloomberg analysis refers to this value as the *yield to worst*. This yield calculation suffers from the same problems as yield to maturity, in that figure computed assumes reinvestment of coupon at the same rate, and that bondholders will hold the bond to maturity. Yield to call also assumes that the issuer will call the bond on the assumed call date.

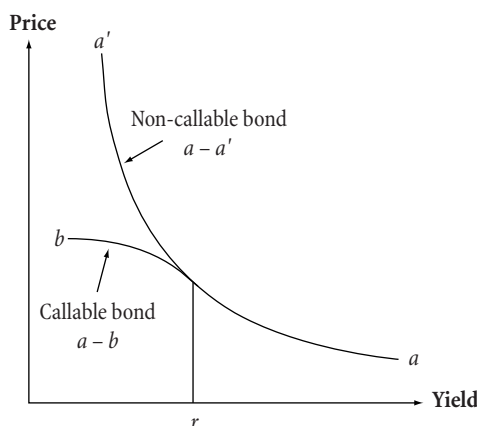


Figure 5.1: Price/yield relationship for callable bond.

The price/yield relationship for a conventional bond is convex. For a bond with an option feature, such as a callable bond, the relationship differs at a certain yield point, such that the bond is said to then display *negative convexity*. This is discussed in greater detail in the chapter on duration and convexity.

5.5.2 Constituents of a callable bond

The best way to analyse a callable bond is to consider it as one for which the bondholder has sold the issuer an option that allows the issuer to redeem the bond, at his discretion, from the first call date until maturity. The bondholder has effectively entered into two separate transactions, first the purchase of a non-callable bond from the issuer at a price, and secondly the sale to the issuer of a *call option*, for which they receive a price (it is a call option as the issuer has the right, but not the obligation, to buy back the bond from the bondholder at a call date(s), which is in effect the expiry date(s) of the option). The position of the bondholder is therefore:

$$\text{Long position in callable bond} = \text{Long position in non-callable bond} + \text{short position in call option.}$$

The price of the callable bond is therefore equal to the net position of the two component parts.

$$\text{Price of callable bond} = \text{Price of non-callable bond} - \text{price of call option.}$$

If you think about it, the bondholder has sold the issuer a call option, therefore the “proceeds” of this sale are subtracted from the price of the hypothetical non-callable bond. This is shown in Figure 5.1; at point r the price of the non-callable bond and the callable bond is the price of the call option on the bond.

As most callable bonds have a set of dates on which they can be called, or even a time period during which the bond can be called at any time after a requisite notice period, the bondholder can be said to have sold not one call option on the bond, but a strip of options. The underlying asset for these call options is the set of cash flows that are not paid if the issuer exercises the call.

5.5.3 Constituents of a puttable bond

The analysis we have applied to callable bonds can be repeated for bonds that have an embedded *put* feature. With a puttable bond the bondholder has the right to sell the bond back to the issuer at a designated price and time. Again a puttable bond can be viewed as representing two separate transactions; first the bondholder buys a non-puttable bond, secondly the investor buys an option from the issuer that allows him to sell the bond to the issuer. This type of option is called a *put option*. The price of the puttable bond is therefore:

$$\text{Price of puttable bond} = \text{Price of non-puttable bond} + \text{price of put option.}$$

5.5.4 Refunding

The term refunding is used to describe the process when a borrower calls an existing bond issue and replaces it with a new lower coupon bond issue. This is at the discretion of the issuer. A company that has borrowed funds via a callable bond will want to aware of the best time at which to call the issue. The primary issues are if there is any net gain to be had from an immediate refinancing, and what the appropriate replacement bond should be.

On issue the yield of the callable yield will lie above a non-callable bond of the same maturity; this reflects the call premium benefit to the bondholder. At any point during the bond’s life market interest rates may fall far enough below the coupon level such that it becomes attractive to the issuer to call in the bonds (before the stated maturity date) and replace the borrowing. The first issue for the borrower is what type of bond should be used to replace the called debt. If the replacement bond is non-callable and has the same maturity date as the original issue, it would have to offer the same yield as the existing issue in order to be attractive to investors. However the lower general level of interest rates may result in the issuer needing to offer a spread over, say, the government yield level that is lower than the spread offered on the original issue.

Borrowers also need to consider the timing of the refinancing and whether it is advantageous to delay calling the issue until a later date. However this is a judgement call based on the corporate treasurer’s view of market interest rates. The decision to refinance after a material change in the general level of interest rates is an important one. If interest rates have fallen sufficiently an issuing company has an opportunity to refinance at a new (lower) borrowing rate, and this will be at the expense of the existing bondholders. Thus the existence of a call option provision as part of the bond’s terms is an advantage to the borrower, and this option needs to be exercised in order for the borrower to gain from any fall in the level of interest rates.

5.5.5 Advance refunding

It is common for corporate borrowers to issue callable bonds that include a period of call protection. If there is a significant fall in interest rates during the period of this call protection, the borrower may consider an *advance refunding*. This is where the borrower offers to buy call-protected bonds at a price above the call price; when this happens bondholders are not obliged to respond to the call but may be tempted to do so because the terms of the call are attractive, while still enabling the borrower to achieve a funding gain.

EXAMPLE 5.2

- A callable bond with a coupon of 8% has two years remaining on its call protection provision. The call price is £110 per cent once the bond becomes callable. Market interest rates are now at 5%. The firm may wish to refund immediately, rather than wait for two years and refund at an as yet unknown rate, providing it can offer a call price attractive enough to the bondholders whilst retaining an element of funding benefit for itself.

The fair value of the bond is the present value of the £8 coupon for the next two years together with the present value of the call price two years from now. This is given at (5.6):

$$P = \frac{8}{(1+r)} + \frac{8}{(1+r)^2} + \frac{110}{(1+r)^2}. \quad (5.6)$$

At a discount rate of 5% the price of the bond is 114.6485. If it wishes to advance refund, the issuer will have to set a call price above this level in order to induce to bondholders to sell their bonds. However this allows it to gain a refunding advantage immediately instead of having to wait two years. Any funding gain must take into account the transaction costs associated with calling back the bonds and issuing new debt.

5.5.6 Sinking funds

In certain domestic markets including the UK and US markets, corporate bonds are issued with a *sinking fund* attached. This requires the borrower to redeem a part of the bond issue at regular intervals during the bond's life. An example of a sinking fund provision is illustrated at Figure 5.2.

	Year												
Time periods	0	...	10	11	12	13	14	15	16	17	18	19	20
Sinking fund payments as 5% of outstanding nominal (£m)			10	9.5	9.025	8.57375	8.14506	7.73781	7.35092	6.98337	6.63420	6.30249	
Final principal payment (£m)													119.7473867

Figure 5.2: £200 million 20-year bond issue: bond sinking fund payments.

In the example in Figure 5.2 a bond with a twenty-year maturity and a total issue size of £200 million has 5% of its outstanding issue size redeemed each year after ten years have elapsed, leaving £119.75 million to be paid back on maturity. At each anniversary of the sinking fund dates the borrowing firm can either purchase the required number of bonds in the open market or call the same number of bonds. The exact procedure will be set out in the bond issue terms, and the firm will elect the cheapest option.

The main reason why an issuer will include a sinking fund provision is to provide comfort to potential investors. A sinking fund will indicate a lower level of risk of default, since the availability of funds to repay a set proportion of the debt indicates a healthy cash flow state. However if the sinking fund is sourced from external funds this may not actually be the case. Nevertheless in theory a sinking fund signals favourable future prospects for the borrowing company, so as a result the yield required by investors from holding the bond consequently is lower.

5.5.7 Analysing call provisions

Callable bonds are very popular in corporate debt markets, significantly so in the United States. A study by Kish (1992) found that approximately 80% of corporate debt in the US market in the second half of the 1980s was issued with call provisions attached. It is often observed that only issuers whose paper is highly sought-after by investors raise funds without the existence of any call provision, and consequently at a lower yield.

In an economist's perfect market the increased yield available on a callable bond together with any period of call protection should take into account the probability of the call option being exercised. In theory therefore there should be no advantage either way to issuing a bond with a call provision attached. In practice however corporate issuers continue to issue a large volume of debt incorporating call features. Previous studies including the one by Kish have attempted to explain this phenomenon. These are summarised below.

- **Corporate management superior knowledge on direction of interest rates.** While an above-average forecasting ability on the direction of interest rates would indeed be a valuable skill, and would be advantageous to the issuers of callable bonds, it is highly unlikely that corporate managers possess such a skill. In any case the investors who purchase callable debt carry out their own market analysis and one would not expect their forecasting ability to be any poorer than that of corporate borrowers.
- **The risk averse nature of borrowers and lenders.** This premise suggests that borrowers do not favour locking in to high interest rates for anything more than short periods of time, while lenders wish to avoid large downward fluctuations in bond prices, and do not attach great value to the call feature. Again this is unrealistic with regard to the lenders position, since the conventional preference for lenders is to lock in a fixed return over complete term of a bond's life.
- **Corporate management knowledge of the firm's prospects.** In this analysis the suggestion is that managers possess superior knowledge of their firms' prospects over that of investors, who view the firm in less favourable light and therefore demand a higher yield on the debt. The managers expect the firm's circumstances to improve and refinance the borrowing at a better rate when this occurs. However this analysis does not explain why even well-rated firms issue callable bonds, or why firms issue both callable and non-callable debt.

Whatever the primary rationale behind the issue of callable debt, investors are frequently interested in increasing the yields on their investments and callable bonds provide one of the ways of achieving this. Therefore it is advantageous to both investors and borrowers for corporates to issue some callable debt as part of their overall funding requirement.

5.6 Index-linked bonds

In certain countries there is a market in bonds whose return, both coupon and final redemption payment, is linked to the consumer prices index. The exact design of such *index-linked bonds* varies across different markets. This of course makes the comparison of measures such as yield difficult and has in the past acted as a hindrance to arbitrageurs seeking to exploit real yield differentials. In an earlier chapter we looked at the calculation of yield for index-linked bonds; in this section we present an overview of indexed bonds and how they differ from the conventional market. Not all index-linked bonds link both coupon and maturity payments to a specified index; in some markets only the coupon payment is index-linked. Generally the most liquid market available will be the government bond market in index-linked instruments.

The structure of index-linked bond markets differs across the world, including in those areas noted below.

- **Choice of index.** In principle bonds can be indexed to any number of variables, including various price indices, earnings, output, specific commodities or foreign currencies. Although ideally the chosen index would reflect the hedging requirements of both parties, these may not coincide. In practice most bonds have been linked to an index of consumer prices such as the UK Retail Price Index, since this is usually widely circulated and well understood and issued on a regular basis.
- **Indexation lags.** In order to construct precise protection against inflation, interest payments for a given period would need to be corrected for actual inflation over the same period. However unavoidable lags between the movements in the price index and the adjustment to the bond cash flows distort the inflation-proofing properties of indexed bonds. The lags arise in two ways. First, inflation statistics can only be calculated and published with a delay. Secondly in some markets the size of the next coupon payment must be known before the start of the

coupon period in order to calculate the accrued interest; this leads to a delay equal to the length of time between coupon payments.

- **Coupon frequency.** Index-linked bonds often pay interest on a semi-annual basis.
- **Indexing the cash flows.** There are four basic methods of linking the cash flows from a bond to an inflation index. These are:
 - ▶ **Interest-indexed bonds:** these pay a fixed real coupon and an indexation of the fixed principal every period; the principal repayment at maturity is not adjusted. In this case all the inflation adjustment is fully paid out as it occurs and does not accrue on the principal. These type of bonds have been issued in Australia, although the most recent issue was in 1987.
 - ▶ **Capital-indexed bonds:** the coupon rate is specified in real terms. Interest payments equal the coupon rate multiplied by the inflation-adjusted principal amount. At maturity the principal repayment is the product of the nominal value of the bond multiplied by the cumulative change in the index. Compared with interest-indexed bonds of similar maturity, these bonds have higher duration and lower reinvestment risk. These type of bonds have been issued in Australia, Canada, New Zealand, the UK and the USA.
 - ▶ **Zero-coupon indexed bonds:** as their name implies these pay no coupons but the principal repayment is scaled for inflation. These have the highest duration of all indexed bonds and have no reinvestment risk. These bonds have been issued in Sweden.
 - ▶ **Indexed-annuity bonds:** the payments consist of a fixed annuity payment and a varying element to compensate for inflation. These bonds have the lowest duration and highest reinvestment risk of all index-linked bonds. They have been issued in Australia, although not by the central government.
- **Coupon stripping feature.** Allowing market practitioners to strip indexed bonds enables them to create new inflation-linked products that are more specific to investors needs, such as indexed annuities or deferred payment indexed bonds. In markets which allow stripping of indexed government bonds, a strip is simply an individual uplifted cash flow. An exception to this is in New Zealand, where the cash flows are separated into three components: the principal, the principal inflation adjustment and the set of inflation-linked coupons (that is, an indexed annuity).

Country	First issue date	Index used	Inflation in year before first issue
Australia	1985	consumer prices	4.5%
Brazil	1991	wholesale prices	1477%
Canada	1991	consumer prices	4.8%
Chile	1967	consumer prices	17%
Colombia	1995	consumer prices	22.8%
Hungary	1995	consumer prices	22.1%
Iceland	1955	credit terms index	102.7%*
Israel	1955	consumer prices	12.3%
Mexico	1989	consumer prices	114.8%
New Zealand	1995	consumer prices	2.8%
Poland	1992	service price indices	60.4%
Sweden	1994	consumer prices	4.4%
Turkey	1997	consumer prices	84.9%
United Kingdom	1981	retail prices	14%
United States	1997	commodity prices	2.9%

* between 1949 and 1954.

Table 5.4: Current issuers of inflation-indexed government bonds.
Source: Bank of England.

Appendices

APPENDIX 5.1 Floating rate note discount margin equation using discount factor

The discounted margin for FRNs redeemed on a normal coupon date is given by (5.7):

$$P \cdot \left(1 + \frac{re + DM}{100} B\right) = C + \sum_{n=1}^{n-1} \frac{(re_2 + QM)}{m} v^n + Mv^{n-1} \quad (5.7)$$

where

- DM is the required discounted margin %
- P is the dirty price
- re is the current market reference rate from the value date to the first coupon date
- re_2 is the assumed market reference rate for subsequent coupon payments
- B is the number of days from the value date to the next coupon date, as a fraction of the day-count base (360, 365 or 365/366 days)
- C is the next coupon payment %
- n is the number of future coupon payments
- QM is the quoted margin %
- m is the number of coupon payments per year adjusted for the assumed number of days in the year
- M is the redemption value
- v is the discount factor, that is $v = 1/(1 + (re_2 + DM)/100h)$.

The left-hand side of (5.7) represents the cost of the bond adjusted using current reference rates to the next coupon date.

For perpetual or undated FRNs (5.7) is simplified as shown at (5.8):

$$P \cdot \left(1 + \frac{re + DM}{100} \cdot B\right) = C + 100 \left(\frac{re_2 + QM}{re_2 + DM} \right). \quad (5.8)$$

A gross redemption yield for an FRN can only be calculated if one assumes unchanged market reference rates during the remaining life of the bond, so that future coupon rates may be predicted. Under this restrictive assumption we may set an FRNs redemption yield as (5.9) below.

$$P = \sum_{n=1}^N C_i \cdot v^{B+ti} + M \cdot v^{B+tn} \quad (5.9)$$

where

- n is the number of coupon payments to redemption
- t_i is the time in periods from the next to the i th coupon payment
- v is the discount factor, that is, $v = 1/(1 + y/h)$
- y is the required redemption yield compounded m times per year.

Selected bibliography and references

- Brown, P.J., *Bond Markets: Structures and yield calculations*, Glenlake Publishing Company, 1998.
- Kish, R.J., Livingston, M., "The determinants of the call feature on corporate bonds", *Journal of Banking and Finance*, 16, 1992, pp. 687–703.
- Levy, H., *Introduction to Investments*, South-Western, 1999.
- Livingston, M., *Money and Capital Markets*, 2nd edition, New York Institute of Finance, 1993.
- Narayan, M.P., Lim, S.P., "The call provision on corporate zero-coupon bonds", *Journal of Financial and Quantitative Analysis*, 24, March 1989, pp. 91–103.

Questions and exercises

1. What advantages are there to a borrow of funds by issuing debt that includes a call feature?
2. Adel plc issues a bond with a 10-year maturity that is callable after the first five years. In the sixth year after issue Adel's treasurer decides not to call the issue immediately but to wait and refund the borrowing at a later date. For what reasons might the treasurer wish to do this? Are there any disadvantages to such action?
3. A sterling FRN is redeemable at par on 30 June 2005. Its coupon dates are 30 June and 30 December each year, with the coupon set at 20 basis points above the six-month sterling Libor rate. On 30 December 1998 the coupon for 30 June 1999 was set relative to a Libor fix of 6.375%, while on 28 February 1999 the Libor fix was 5.625%. If the FRN was priced at 99.75, what is the simple margin?
4. If we assume a six-month Libor at a constant rate of 5.875% during the life of the FRN described in the previous question, what is the discounted margin (DM)?
5. A euro-denominated FRN pays quarterly interest on the 15th of January, April, July and October of each year, and matures on 15 October 2008. It pays interest at 10 basis points over three-month Euribor. The current Euribor rate is 3.15%, while the next interest payment on 15 January is set at €0.7875, based on a Euribor rate of 3.05%. The price of the bond for value on 15 December 1999 was 99.75, with accrued interest of 0.2712. Calculate the redemption yield.
6. Explain the advantages and disadvantages to a borrower of incorporating a sinking fund with a specific issue of bonds.

6 The Yield Curve

So far we have considered the main measure of return associated with holding bonds, which is the *yield to maturity* or *gross redemption yield*. In developed markets, as well as a fair number of developing ones, there is usually a large number of bonds trading at one time, at different yields and with varying terms to maturity. Investors and traders frequently examine the relationship between the yields on bonds that are in the same class; plotting yields of bonds that differ only in their term to maturity produces what is known as a *yield curve*. The yield curve is an important indicator and knowledge source of the state of a debt capital market.¹ It is sometimes referred to as the *term structure of interest rates*, but strictly speaking this is not correct, as this term should be reserved for the zero-coupon yield curve only. We shall examine this in detail later.

Much of the analysis and pricing activity that takes place in the bond markets revolves around the yield curve. The yield curve describes the relationship between a particular redemption yield and a bond's maturity. Plotting the yields of bonds along the maturity term structure will give us our yield curve. It is very important that only bonds from the same class of issuer or with the same degree of liquidity are used when plotting the yield curve; for example a curve may be constructed for UK gilts or for AA-rated sterling Eurobonds, but not a mixture of both, because gilts and Eurobonds are bonds from different class issuers. The primary yield curve in any domestic capital market is the government bond yield curve, so for example in the US market it is the US Treasury yield curve. With the advent of the euro currency in 11 countries of the European Union, in theory any euro-currency government bond can be used to plot a default-free euro yield curve. In practice only bonds from the same government are used, as for various reasons different country bonds within euroland trade at different yields. Outside the government bond markets yield curves are plotted for Eurobonds, money market instruments, off-balance sheet instruments, in fact virtually all debt market instruments. So it is always important to remember to compare like-for-like when analysing yield curves across markets.

In this chapter we will consider the yield to maturity yield curve as well as other types of yield curve that may be constructed. Later in this chapter we will consider how to derive spot and forward yields from a current redemption yield curve. In Part VIII of the book we examine more advanced techniques for fitting, analysing and interpreting the yield curve.

6.1 Using the yield curve

Let us first consider the main uses of the yield curve. All participants in the debt capital markets have an interest in the current shape and level of the yield curve, as well as what this information implies for the future. The main uses are summarised below.

- **Setting the yield for all debt market instruments.** The yield curve essentially fixes the cost of money over the maturity term structure. The yields of government bonds from the shortest-maturity instrument to the longest set the benchmark for yields for all other debt instruments in the market, around which all debt instruments are analysed. Issuers of debt (and their underwriting banks) therefore use the yield curve to price bonds and all other debt instruments. Generally the zero-coupon yield curve is used to price new issue securities, rather than the redemption yield curve.
- **Acting as an indicator of future yield levels.** As we discuss later in this chapter, the yield curve assumes certain shapes in response to market expectations of the future interest rates. Bond market participants analyse the present shape of the yield curve in an effort to determine the implications regarding the future direction of market interest rates. This is perhaps one of the most important functions of the yield curve, and it is as much

¹ The author was excited to read this description from someone obviously as excited about the yield curve as he is; Ryan (1997) writes:

“The future ... of the global economy ... may well rest on the success of how we finance [the yield] curve. God bless the Treasury Yield Curve!”

an art as a science. The yield curve is scrutinised for its information content not just by bond traders and fund managers but also by corporate financiers as part of project appraisal. Central banks and government treasury departments also analyse the yield curve for its information content, not just regarding forward interest rates but also with regard to expected inflation levels.

- **Measuring and comparing returns across the maturity spectrum.** Portfolio managers use the yield curve to assess the relative value of investments across the maturity spectrum. The yield curve indicates the returns that are available at different maturity points and is therefore very important to fixed-income fund managers, who can use it to assess which point of the curve offers the best return relative to other points.
- **Indicating relative value between different bonds of similar maturity.** The yield curve can be analysed to indicate which bonds are cheap or dear to the curve. Placing bonds relative to the zero-coupon yield curve helps to highlight which bonds should be bought or sold either outright or as part of a bond spread trade.
- **Pricing interest-rate derivative securities.** The price of derivative securities revolves around the yield curve. At the short-end, products such as Forward Rate Agreements are priced off the futures curve, but futures rates reflect the market's view on forward three-month cash deposit rates. At the longer end, interest-rate swaps are priced off the yield curve, while hybrid instruments that incorporate an option feature such as convertibles and callable bonds also reflect current yield curve levels. The “risk-free” interest rate, which is one of parameters used in option pricing, is the T-bill rate or short-term government repo rate, both constituents of the money market yield curve.

6.2 Yield-to-maturity yield curve

The most commonly occurring yield curve is the yield to maturity yield curve. The equation used to calculate the yield to maturity was shown in Chapter 4. The curve itself is constructed by plotting the yield to maturity against the term to maturity for a group of bonds of the same class. Three different examples are shown at Figure 6.1. Bonds used in constructing the curve will only rarely have an exact number of whole years to redemption; however it is often common to see yields plotted against whole years on the x-axis. This is because once a bond is designated the *benchmark* for that term, its yield is taken to be the representative yield. For example, the then ten-year benchmark bond in the UK gilt market, the 5¾% Treasury 2009, maintained its benchmark status throughout 1999 and into 2000, even as its term to maturity fell below ten years. The yield to maturity yield curve is the most commonly observed curve simply because yield to maturity is the most frequent measure of return used. The business sections of daily newspapers, where they quote bond yields at all, usually quote bond yields to maturity.

As we might expect, given the source data from which it is constructed, the yield to maturity yield curve contains some inaccuracies. We have already come across the main weakness of the yield to maturity measure, which is the assumption of a constant rate for coupon reinvestment during the bond's life at the redemption yield level. Since market rates will fluctuate over time, it will not be possible to achieve this (a feature known as *reinvestment risk*). Only zero-coupon bondholders avoid reinvestment risk as no coupon is paid during the life of a zero-coupon bond.

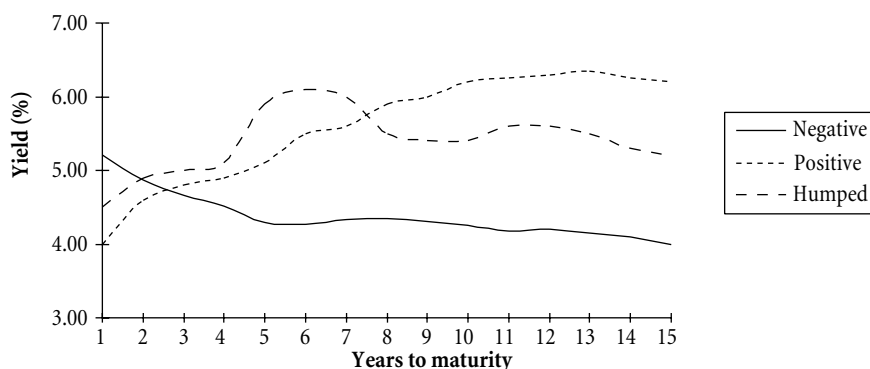


Figure 6.1: Yield to maturity yield curves.

The yield to maturity yield curve does not distinguish between different payment patterns that may result from bonds with different coupons, that is, the fact that low-coupon bonds pay a higher portion of their cash flows at a later date than high-coupon bonds of the same maturity. The curve also assumes an even cash flow pattern for all bonds. Therefore in this case cash flows are not discounted at the appropriate rate for the bonds in the group being used to construct the curve. To get around this bond analysts may sometimes construct a *coupon yield curve*, which plots yield to maturity against term to maturity for a group of bonds with the same coupon. This may be useful when a group of bonds contains some with very high coupons; high coupon bonds often trade “cheap to the curve”, that is they have higher yields, than corresponding bonds of same maturity but lower coupon. This is usually because of reinvestment risk and, in some markets (including the UK), for tax reasons.

For the reasons we have discussed the market often uses other types of yield curve for analysis when the yield to maturity yield curve is deemed unsuitable.

6.3 The coupon yield curve

The *coupon yield curve* is a plot of the yield to maturity against term to maturity for a group of bonds with the same coupon. If we were to construct such a curve we would see that in general high-coupon bonds trade at a discount (have higher yields) relative to low-coupon bonds, because of reinvestment risk and for tax reasons (in the UK for example, on gilts the coupon is taxed as income tax, while any capital gain is exempt from capital gains tax; even in jurisdictions where capital gain on bonds is taxable, this can often be deferred whereas income tax cannot). It is frequently the case that yields vary considerably with coupon for the same term to maturity, and with term to maturity for different coupons. Put another way, usually we observe different coupon curves not only at different levels but also with different shapes. Distortions arise in the yield to maturity curve if no allowance is made for coupon differences. For this reason bond analysts frequently draw a line of “best fit” through a plot of redemption yields, because the coupon effect in a group of bonds will produce a curve with humps and troughs. Figure 6.2 shows a hypothetical set of coupon yield curves, however since in any group of bonds it is unusual to observe bonds with the same coupon along the entire term structure this type of curve is relatively rare.

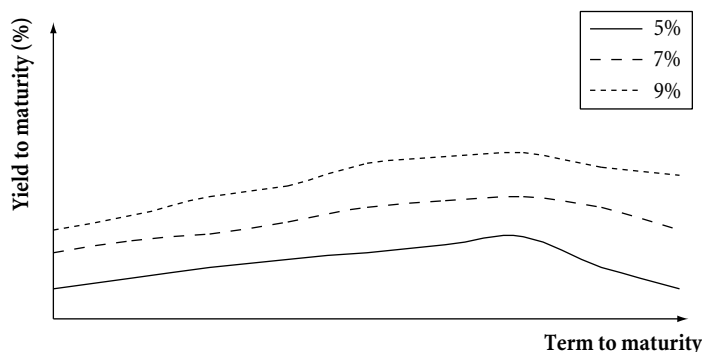


Figure 6.2: Coupon yield curves.

6.4 The par yield curve

The *par yield curve* is not usually encountered in secondary market trading, however it is often constructed for use by corporate financiers and others in the new issues or *primary* market. The par yield curve plots yield to maturity against term to maturity for current bonds trading at par.² The par yield is therefore equal to the coupon rate for bonds priced at par or near to par, as the yield to maturity for bonds priced exactly at par is equal to the coupon rate. Those involved in the primary market will use a par yield curve to determine the required coupon for a new bond that is to be issued at par. This is because investors prefer not to pay over par for a new-issue bond, so the bond requires a coupon that will result in a price at or slightly below par.

² Par price for a bond is almost invariably 100 per cent. Certain bonds have par defined as 1000 per 1000 nominal of paper.

The par yield curve can be derived directly from bond yields when bonds are trading at or near par. If bonds in the market are trading substantially away from par then the resulting curve will be distorted. It is then necessary to derive it by iteration from the spot yield curve. As we would observe at almost any time, it is rare to encounter bonds trading at par for any particular maturity. The market therefore uses actual non-par vanilla bond yield curves to derive *zero-coupon yield curves* and then constructs hypothetical par yields that would be observed were there any par bonds being traded.

6.5 The zero-coupon (or spot) yield curve

The *zero-coupon* (or *spot*) *yield curve* plots zero-coupon yields (or spot yields) against term to maturity. A zero-coupon yield is the yield prevailing on a bond that has no coupons. In the first instance if there is a liquid zero-coupon bond market we can plot the yields from these bonds if we wish to construct this curve. However it is not necessary to have a set of zero-coupon bonds in order to construct the curve, as we can derive it from a coupon or par yield curve; in fact in many markets where no zero-coupon bonds are traded, a spot yield curve is derived from the conventional yield to maturity yield curve. This is of course a *theoretical* zero-coupon (spot) yield curve, as opposed to the *market* or *observed* spot curve that can be constructed using the yields of actual zero-coupon bonds trading in the market.³

6.5.1 Basic concepts

Spot yields must comply with equation (6.1). This equation assumes annual coupon payments and that the calculation is carried out on a coupon date so that accrued interest is zero.

$$\begin{aligned}
 P_d &= \sum_{n=1}^N \frac{C}{(1 + rs_n)^n} + \frac{M}{(1 + rs_N)^N} \\
 &= \sum_{n=1}^N C \times df_n + M \times df_N
 \end{aligned}
 \tag{6.1}$$

where

rs_n is the spot or zero-coupon yield on a bond with n years to maturity
 $df_n \equiv 1/(1 + rs_n)^n$ = the corresponding *discount factor*.

In (6.1) rs_1 is the current one-year spot yield, rs_2 the current two-year spot yield, and so on. Theoretically the spot yield for a particular term to maturity is the same as the yield on a zero-coupon bond of the same maturity, which is why spot yields are also known as zero-coupon yields.

This last is an important result, as spot yields can be derived from redemption yields that have been observed in the market.

As with the yield to redemption yield curve, the spot yield curve is commonly used in the market. It is viewed as the true term structure of interest rates because there is no reinvestment risk involved; the stated yield is equal to the actual annual return. That is, the yield on a zero-coupon bond of n years maturity is regarded as the true n -year interest rate. Because the observed government bond redemption yield curve is not considered to be the true interest rate, analysts often construct a theoretical spot yield curve. Essentially this is done by breaking down each coupon bond being observed into its constituent cash flows, which become a series of individual zero-coupon bonds. For example, £100 nominal of a 5% two-year bond (paying annual coupons) is considered equivalent to £5 nominal of a one-year zero-coupon bond and £105 nominal of a two-year zero-coupon bond.

Let us assume that in the market there are 30 bonds all paying annual coupons. The first bond has a maturity of one year, the second bond of two years, and so on out to thirty years. We know the price of each of these bonds, and we wish to determine what the prices imply about the market's estimate of future interest rates. We naturally expect

³ It is common to see the terms spot rate and zero-coupon rate used synonymously. However the spot rate is a theoretical construct and cannot be observed in the market. The definition of the spot rate, which is the rate of return on a single cash flow that has been dealt today and is received at some point in the future, comes very close to that of the yield on a zero-coupon bond, which can be observed directly in the market. Zero-coupon rates can therefore be taken to be spot rates in practice, which is why the terms are frequently used interchangeably.

interest rates to vary over time, but that all payments being made on the same date are valued using the same rate. For the one-year bond we know its current price and the amount of the payment (comprised of one coupon payment and the redemption proceeds) we will receive at the end of the year; therefore we can calculate the interest rate for the first year: assume the one-year bond has a coupon of 5%. If the bond is priced at par and we invest £100 today we will receive £105 in one year's time, hence the rate of interest is apparent and is 5%. For the two-year bond we use this interest rate to calculate the future value of its current price in one year's time: *this is how much we would receive if we had invested the same amount in the one-year bond*. However the two-year bond pays a coupon at the end of the first year; if we subtract this amount from the future value of the current price, the net amount is what we should be giving up in one year in return for the one remaining payment. From these numbers we can calculate the interest rate in year two.

Assume that the two-year bond pays a coupon of 6% and is priced at 99.00. If the 99.00 was invested at the rate we calculated for the one-year bond (5%), it would accumulate £103.95 in one year, made up of the £99 investment and interest of £4.95. On the payment date in one year's time, the one-year bond matures and the two-year bond pays a coupon of 6%. If everyone expected that at this time the two-year bond would be priced at more than 97.95 (which is 103.95 minus 6.00), then no investor would buy the one-year bond, since it would be more advantageous to buy the two-year bond and sell it after one year for a greater return. Similarly if the price was less than 97.95 no investor would buy the two-year bond, as it would be cheaper to buy the shorter bond and then buy the longer-dated bond with the proceeds received when the one-year bond matures. Therefore the two-year bond must be priced at exactly 97.95 in 12 months' time. For this £97.95 to grow to £106.00 (the maturity proceeds from the two-year bond, comprising the redemption payment and coupon interest), the interest rate in year two must be 8.20%. We can check this using the present value formula covered earlier. At these two interest rates, the two bonds are said to be in equilibrium.

This is an important result and shows that (in theory) there can be no arbitrage opportunity along the yield curve; using interest rates available today the return from buying the two-year bond must equal the return from buying the one-year bond and rolling over the proceeds (or *reinvesting*) for another year. This is the known as the *breakeven principle*, a law of no-arbitrage.

Using the price and coupon of the three-year bond we can calculate the interest rate in year three in precisely the same way. Using each of the bonds in turn, we can link together the *implied one-year rates* for each year up to the maturity of the longest-dated bond. The process is known as *bootstrapping*. The “average” of the rates over a given period is the spot yield for that term: in the example given above, the rate in year one is 5%, and in year two is 8.20%. An investment of £100 at these rates would grow to £113.61. This gives a total percentage increase of 13.61% over two years, or 6.588% per annum (the average rate is not obtained by simply dividing 13.61 by 2, but – using our present value relationship again – by calculating the square root of “1 plus the interest rate” and then subtracting 1 from this number). Thus the one-year yield is 5% and the two-year yield is 8.20%.

In real-world markets it is not necessarily as straightforward as this; for instance on some dates there may be several bonds maturing, with different coupons, and on some dates there may be no bonds maturing. It is most unlikely that there will be a regular spacing of bond redemptions exactly one year apart. For this reason it is common for analysts to use a software model to calculate the set of implied spot rates which best fits the market prices of the bonds that do exist in the market. For instance if there are several one-year bonds, each of their prices may imply a slightly different rate of interest. We choose the rate which gives the smallest average price error. In practice all bonds are used to find the rate in year one, all bonds with a term longer than one year are used to calculate the rate in year two, and so on. The zero-coupon curve can also be calculated directly from the coupon yield curve using a method similar to that described above; in this case the bonds would be priced at par and their coupons set to the par yield values.

The zero-coupon yield curve is ideal to use when deriving implied forward rates, which we consider next, and defining the term structure of interest rates. It is also the best curve to use when determining the *relative value*, whether cheap or dear, of bonds trading in the market, and when pricing new issues, irrespective of their coupons. However it is not an absolutely accurate indicator of average market yields because most bonds are not zero-coupon bonds.

6.5.2 Zero-coupon discount factors

Having introduced the concept of the zero-coupon curve in the previous paragraph, we can illustrate more formally the mathematics involved. When deriving spot yields from redemption yields, we view conventional bonds as being made up of an *annuity*, which is the stream of fixed coupon payments, and a zero-coupon bond, which is the redemption payment on maturity. To derive the rates we can use (6.1), setting $P_d = M = 100$ and $C = rm_N$, as shown in (6.2) below. This has the coupon bonds trading at par, so that the coupon is equal to the yield.

$$\begin{aligned} 100 &= rm_N \times \sum_{n=1}^N df_n + 100 \times df_N \\ &= rm_N \times A_N + 100 \times df_N \end{aligned} \quad (6.2)$$

where rm_N is the par yield for a term to maturity of N years, where the discount factor df_N is the fair price of a zero-coupon bond with a par value of £1 and a term to maturity of N years, and where

$$A_N = \sum_{n=1}^N df_n = A_{N-1} + df_N \quad (6.3)$$

is the fair price of an annuity of £1 per year for N years (with $A_0 = 0$ by convention). Substituting (6.3) into (6.2) and rearranging will give us the expression below for the N -year discount factor, shown at (6.4):

$$df_N = \frac{1 - rm_N \times A_{N-1}}{1 + rm_N}. \quad (6.4)$$

If we assume one-year, two-year and three-year redemption yields for bonds priced at par to be 5%, 5.25% and 5.75% respectively, we will obtain the following solutions for the discount factors:

$$\begin{aligned} df_1 &= \frac{1}{1 + 0.05} = 0.95238 \\ df_2 &= \frac{1 - (0.0525)(0.95238)}{1 + 0.0525} = 0.90261 \\ df_3 &= \frac{1 - (0.0575)(0.95238 + 0.90261)}{1 + 0.0575} = 0.84476. \end{aligned}$$

We can confirm that these are the correct discount factors by substituting them back into equation (6.2); this gives us the following results for the one-year, two-year and three-year par value bonds (with coupons of 5%, 5.25% and 5.75% respectively):

$$\begin{aligned} 100 &= 105 \times 0.95238 \\ 100 &= 5.25 \times 0.95238 + 105.25 \times 0.90261 \\ 100 &= 5.75 \times 0.95238 + 5.75 \times 0.90261 + 105.75 \times 0.84476. \end{aligned}$$

Now that we have found the correct discount factors it is relatively straightforward to calculate the spot yields using equation (6.1), and this is shown below:

$$\begin{aligned} df_1 &= \frac{1}{(1 + rs_1)} = 0.95238 \text{ which gives } rs_1 = 5.0\% \\ df_2 &= \frac{1}{(1 + rs_2)^2} = 0.90261 \text{ which gives } rs_2 = 5.269\% \\ df_3 &= \frac{1}{(1 + rs_3)^3} = 0.84476 \text{ which gives } rs_3 = 5.778\%. \end{aligned}$$

Equation (6.1) discounts the n -year cash flow (comprising the coupon payment and/or principal repayment) by the corresponding n -year spot yield. In other words rs_n is the *time-weighted rate of return* on a n -year bond. Thus as we said in the previous section the spot yield curve is the correct method for pricing or valuing any cash flow, including an irregular cash flow, because it uses the appropriate discount factors. That is, it matches each cash flow

to the discount rate that applies to the time period in which the cash flow is paid. Compare this to the approach for the yield-to-maturity procedure discussed earlier, which discounts all cash flows by the same yield to maturity. This illustrates neatly why the N -period zero-coupon interest rate is the true interest rate for an N -year bond.

The expressions above are solved algebraically in the conventional manner, although those wishing to use a spreadsheet application such as Microsoft Excel® can input the constituents of each equation into individual cells and solve using the “Tools” and “Goal Seek” functions.

EXAMPLE 6.1 Zero-coupon yields

■ Consider the following zero-coupon market rates:

One-year (1y)	5.000%
2y	5.271%
3y	5.598%
4y	6.675%
5y	7.213%

- Calculate the zero-coupon discount factors and the prices and yields of:
 - (a) a 6% two-year bond, and
 - (b) a 7% five-year bond.

Assume both are annual coupon bonds.

The zero-coupon discount factors are:

$$\begin{aligned}
 1y: \quad 1/1.05 &= 0.95238095 \\
 2y: \quad 1/(1.05271)^2 &= 0.90236554 \\
 3y: \quad 1/(1.05598)^3 &= 0.84924485 \\
 4y: \quad 1/(1.06675)^4 &= 0.77223484 \\
 5y: \quad 1/(1.07213)^5 &= 0.70593182.
 \end{aligned}$$

The price of the 6% two-year bond is then calculated in the normal fashion using present values of the cash flows:

$$(6 \times 0.95238095) + (106 \times 0.90236554) = 101.365.$$

The yield to maturity is 5.263%, obtained using the iterative method, with a spreadsheet function such as Microsoft Excel® “Goal Seek” or a Hewlett Packard (HP) calculator.

The price of the 7% five-year bond is:

$$\begin{aligned}
 &(7 \times 0.95238095) + (7 \times 0.90236554) + (7 \times 0.84924485) + (7 \times 0.77223484) + (107 \times 0.70593182) \\
 &= 99.869.
 \end{aligned}$$

The yield to maturity is 7.032%.

FORMULA SUMMARY

Example 6.1 illustrates that if the zero-coupon discount factor for n years is df_n and the par yield for N years is rp , then the expression at (6.5) is always true.

$$\begin{aligned}
 (rp \times df_1) + (rp \times df_2) + \cdots + (rp \times df_N) + (1 \times df_N) &= 1 \\
 \Rightarrow rp \times (df_1 + df_2 + \cdots + df_N) &= 1 - df_N \\
 \Rightarrow rp &= \frac{1 - df_N}{\sum_{n=1}^N df_n}.
 \end{aligned} \tag{6.5}$$

6.5.3 Using spot rates in bond analysis

The convention in the markets is to quote the yield on a non-government bond as a certain *spread* over the yield on the equivalent maturity government bond, usually using gross redemption yields. Traders and investment managers will assess the relative merits of holding the non-government bond based on the risk associated with the bond's issuer and the magnitude of its yield spread. For example, in the UK at the beginning of 1999 companies such as National Grid, Severn Trent Water, Abbey National plc and Tesco plc issued sterling denominated bonds, all of which paid a certain spread over the equivalent gilt bond.⁴ Figure 6.3 shows the average yield spreads of corporate bonds over gilts in the UK market through 1998/99.

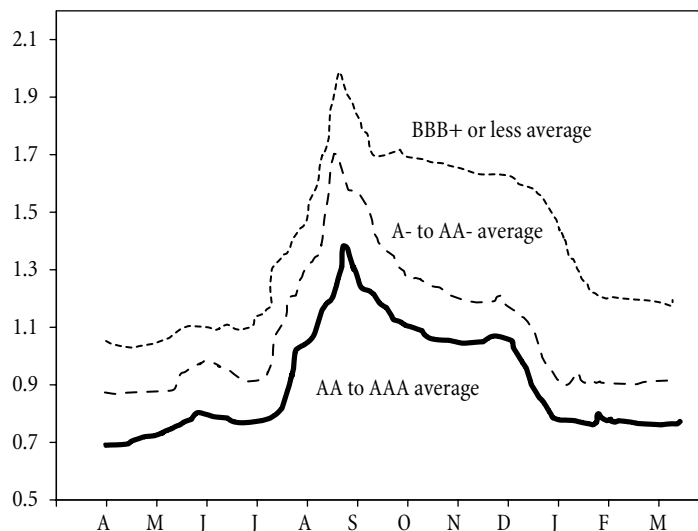


Figure 6.3: Average yield spreads of UK corporate bonds versus gilts, 1998/1999. Source: Halifax plc.

Traditionally investors will compare the redemption yield of the bond they are analysing with the redemption yield of the equivalent government bond. Just as with the redemption yield measure however there is a flaw with this measure, in that the spread quoted is not really comparing like-for-like, as the yields do not reflect the true term structure given by the spot rate curve. There is an additional flaw if the cash flow stream of the two bonds do not match, which in practice they will do only rarely.

Therefore the correct method for assessing the yield spread of a corporate bond is to replicate its cash flows with those of a government bond, which can be done in theory by matching the cash flows of the corporate bond with a package of government zero-coupon bonds of the same nominal value. If no zero-coupon bond market exists, the cash flows can be matched synthetically by valuing a coupon bond's cash flows on a zero-coupon basis. The corporate bond's price is of course the sum of the present value of all its cash flows, which should be valued at the spot rates in place for each cash flow's maturity. It is the yield spread of each individual cash flow over the equivalent maturity government spot rate that is then taken to be the true yield spread.

This measure is known in US markets as the *zero-volatility spread* or *static spread*, and it is a measure of the spread that would be realised over the government spot rate yield curve if the corporate bond were to be held to maturity. It is therefore a different measure to the traditional spread, as it is not taken over one point on the (redemption yield) curve but over the whole term to maturity. The zero-volatility spread is that spread which equates the present value of the corporate bond's cash flows to its price, where the discount rates are each relevant government spot rate. The spread is found through an iterative process, and it is a more realistic yield spread measure than the traditional one.

⁴ The spread is, of course, not fixed and fluctuates with market conditions and supply and demand.

6.6 The forward yield curve

6.6.1 Forward yields

Most transactions in the market are for immediate delivery, which is known as the *cash* market, although some markets also use the expression *spot* market, which is more common in foreign exchange. Cash market transactions are settled straight away, with the purchaser of a bond being entitled to interest from the settlement date onwards.⁵ There is a large market in *forward* transactions, which are trades carried out today for a forward settlement date. For financial transactions that are forward transactions, the parties to the trade agree today to exchange a security for cash at a future date, but at a price agreed today. So the *forward rate* applicable to a bond is the spot bond yield as at the forward date. That is, it is the yield of a zero-coupon bond that is purchased for settlement at the forward date. It is derived today, using data from a present-day yield curve, so it is not correct to consider forward rates to be a prediction of the spot rates as at the forward date.

Forward rates can be derived from spot interest rates. Such rates are then known as *implied* forward rates, since they are implied by the current range of spot interest rates. The *forward* (or *forward-forward*) *yield curve* is a plot of forward rates against term to maturity. Forward rates satisfy expression (6.6):

$$\begin{aligned} P_d &= \frac{C}{(1 + {}_0rf_1)} + \frac{C}{(1 + {}_0rf_1)(1 + {}_1rf_2)} + \cdots + \frac{M}{(1 + {}_0rf_1)\cdots(1 + {}_{N-1}rf_N)} \\ &= \sum_{n=1}^N \frac{C}{\prod_{i=1}^n (1 + {}_{i-1}rf_i)} + \frac{M}{\prod_{i=1}^N (1 + {}_{i-1}rf_i)} \end{aligned} \quad (6.6)$$

where ${}_{n-1}rf_n$ is the implicit forward rate (or forward-forward rate) on a one-year bond maturing in year N .

As a forward or forward-forward yield is implied from spot rates, the forward rate is a forward zero-coupon rate. Comparing (6.1) and (6.6) we see that the spot yield is the *geometric mean* of the forward rates, as shown below:

$$(1 + rs_n)^n = (1 + {}_0rf_1)(1 + {}_1rf_2)\cdots(1 + {}_{n-1}rf_n). \quad (6.7)$$

This implies the following relationship between spot and forward rates:

$$\begin{aligned} (1 + {}_{n-1}rf_n) &= \frac{(1 + rs_n)^n}{(1 + rs_{n-1})^{n-1}} \\ &= \frac{df_{n-1}}{df_n}. \end{aligned} \quad (6.8)$$

Using the spot yields we calculated in the earlier paragraph we can derive the implied forward rates from (6.8). For example, the two-year and three-year forward rates are given by:

$$(1 + {}_1rf_2) = \frac{(1 + 0.05269)^2}{(1 + 0.05)} = 5.539\% \quad (1 + {}_2rf_3) = \frac{(1 + 0.05778)^3}{(1 + 0.05269)^2} = 6.803\%.$$

Using our expression gives us ${}_0rf_1$ equal to 5%, ${}_1rf_2$ equal to 5.539% and ${}_2rf_3$ as 6.803%. This means for example that given current spot yields, which we calculated from the one-year, two-year and three-year bond redemption yields (which were priced at par), the market is expecting the yield on a bond with one year to mature in three years' time to be 6.803% (that is, the three year one-period forward-forward rate is 6.803%).

The relationship between the par yields, spot yields and forward rates is shown in Table 6.1.

⁵ We refer to "immediate" settlement, although of course there is a delay between trade date and settlement date, which can be anything from one day to seven days, or even longer in some markets. The most common settlement period is known as "spot" and is two business days.

Year	Coupon yield (%)	Zero-coupon yield (%)	Forward rate (%)
1	5.000	5.000	5.000
2	5.250	5.269	5.539
3	5.750	5.778	6.803

Table 6.1: Coupon, spot and forward yields.

Figure 6.4 highlights our results for all three yield curves graphically. This illustrates another important property of the relationship between the three curves, in that as the original coupon yield curve was positively sloping, so the spot and forward yield curves lie above it. The reasons behind this will be considered later in the chapter.

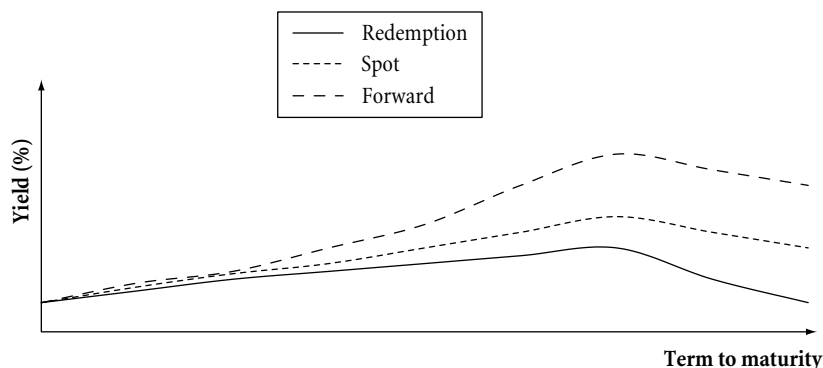


Figure 6.4: Redemption, spot and forward yield curves: traditional analysis.

Let us now consider the following example. Suppose that a two-year bond with cash flows of £5.25 at the end of year 1 and £105.25 at the end of year 2 is trading at par, hence it has a redemption yield (indeed a par yield) of 5.25% (this is the bond in our table above). As we showed in the section on zero-coupon yields and the idea of the break-even principle, in order to be regarded as equivalent to this a pure zero-coupon bond or discount bond making a lump sum payment at the end of year 2 only (so with no cash flow at the end of year 1) would require a rate of return of 5.269%, which is the spot yield. That is, for the same investment of £100 the maturity value would have to be £110.82 (this figure is obtained by multiplying 100 by $(1 + 0.05269)^2$).

This illustrates why the zero-coupon curve is important to corporate financiers involved in new bond issues. If we know the spot yields then we can calculate the coupon required on a new three-year bond that is going to be issued at par in this interest-rate environment by making the following calculation:

$$100 = \frac{C}{(1.05)} + \frac{C}{(1.05269)^2} + \frac{C + 100}{(1.05778)^3}.$$

This is solved in the conventional algebraic manner to give C equal to 5.75%.

The relationship between spot yields and forward rates was shown at (6.8). We can illustrate it as follows. If the spot yield is the *average return*, then the forward rate can be interpreted as the *marginal return*. If the marginal return between years 2 and 3 increases from 5.539% to 6.803%, then the average return increases from 5.269% up to the three-year spot yield of 5.778% as shown below:

$$\left((1.05269)^2 (1.06803)\right)^{1/3} - 1 = 0.05777868$$

or 5.778%, as shown in Table 6.1.

FORMULA SUMMARY

- The forward zero-coupon rate from interest period a to period b is given by (6.9).

$${}_a r f_b = \left(\frac{(1 + rs_b)^b}{(1 + rs_a)^a} \right)^{1/(b-a)} - 1 \quad (6.9)$$

where rs_a and rs_b are the a and b period spot rates respectively.

- The forward rate from interest period a to period $(a + 1)$ is given by (6.10):

$${}_a r f_{a+1} = \frac{(1 + rs_{a+1})^{a+1}}{(1 + rs_a)^a} - 1. \quad (6.10)$$

6.6.2 Calculating spot rates from forward rates

The previous section showed the relationship between spot and forward rates. Just as we have derived forward rates from spot rates based on this mathematical relationship, it is possible to reverse this and calculate spot rates from forward rates. If we are presented with a forward yield curve, plotted from a set of one-period forward rates, we can use this to construct a spot yield curve. Equation (6.7) states the relationship between spot and forward rates, rearranged as (6.11) to solve for the spot rate:

$$rs_n = ((1 + {}_1 r f_1) \times (1 + {}_2 r f_1) \times (1 + {}_3 r f_1) \times \cdots \times (1 + {}_n r f_1))^{1/n} - 1 \quad (6.11)$$

where ${}_1 r f_1$, ${}_2 r f_1$, ${}_3 r f_1$ are the one-period versus two-period, two-period versus three-period forward rates up to the $(n - 1)$ period versus n -period forward rates.

Remember to adjust (6.11) as necessary if dealing with forward rates relating to a deposit of a different interest period. If we are dealing with the current six-month spot rate and implied six-month forward rates, the relationship between these and the n -period spot rate is given by (6.11) in the same way as if we were dealing with the current one-year spot rate and implied one-year forward rates.

EXAMPLE 6.2(i)

- The one-year cash market yield is 5.00%. Market expectations have priced one-year rates in one year's time at 5.95% and in two years' time at 7.25%. What is the current three-year spot rate that would produce these forward rate views?

To calculate this we assume an investment strategy dealing today at forward rates, and calculate the return generated from this strategy. The return after a three-year period is given by the future value relationship, which in this case is $1.05 \times 1.0595 \times 1.0725 = 1.1931$.

The three-year spot rate is then obtained by:

$$\left(\frac{1.1931}{1} \right)^{\frac{1}{3}} - 1 = 6.062\%.$$

6.2(ii)

- Consider the following six-month implied forward rates, when the six-month spot rate is 4.0000%:

${}_1 r f_1$	4.0000%
${}_2 r f_1$	4.4516%
${}_3 r f_1$	5.1532%
${}_4 r f_1$	5.6586%
${}_5 r f_1$	6.0947%
${}_6 r f_1$	7.1129%

An investor is debating between purchasing a three-year zero-coupon bond at a price of £72.79481 per £100 nominal or buying a six-month zero-coupon bond and then rolling over her investment every six months for the three year term. If the investor was able to re-invest her proceeds every six months at the actual forward rates in

place today, what would her proceeds be at the end of the three year term?

An investment of £72.79481 at the spot rate of 4% and then re-invested at the forward rates in our table over the next three years would yield a terminal value of:

$$72.79481 \times (1.04)(1.044516)(1.051532)(1.056586)(1.060947)(1.071129) = 100.$$

This merely reflects our spot and forward rates relationship, in that if all the forward rates are indeed realised, our investor's £72.79 will produce a terminal value that matches the investment in a three-year zero-coupon bond priced at the three-year spot rate. This illustrates the relationship between the three-year spot rate, the six-month spot rate and the implied six-month forward rates. So what is the three-year zero-coupon bond trading at? Using (6.11) the solution to this is given by:

$$rs_6 = ((1.04)(1.044516)(1.051532)(1.056586)(1.060947)(1.071129))^{\frac{1}{6}} - 1 = 5.4346\%$$

which solves our three-year spot rate rs_6 as 5.4346%. Of course we could have also solved for rs_6 using the conventional price/yield formula for zero-coupon bonds, however the calculation above illustrates the relationship between spot and forward rates.

6.6.3 An important note on spot and forward rates

Forward rates that exist at any one time reflect everything that is known in the market *up to that point*. Certain market participants may believe that the forward rate curve is a forecast of the future spot rate curve. This is implied by the *unbiased expectations hypothesis* that we consider below. In fact there is no direct relationship between the forward rate curve and the spot rate curve; for an excellent analysis of this see Jarrow (1996). It is possible for example for the forward rate curve to be upward sloping at the same time that short-dated spot rates are expected to decline.

To view the forward rate curve as a predictor of rates is a misuse of it. The derivation of forward rates reflects all currently known market information. Assuming that all developed country markets are at least semi-strong form⁶, to preserve market equilibrium there can only be one set of forward rates from a given spot rate curve. However this does not mean that such rates are a prediction because the instant after they have been calculated, new market knowledge may become available that alters the markets view of future interest rates. This will cause the forward rate curve to change.

Forward rates are important because they are required to make prices today for dealing at a future date. For example a bank's corporate customer may wish to fix today the interest rate payable on a loan that begins in one year from now; what rate does the bank quote? The forward rate is used by market makers to quote prices for dealing today, and is the best *expectation* of future interest rates given everything that is known in the market up to now, but it is not a prediction of future spot rates. What would happen if a bank was privy to insider information, for example it knew that central bank base rates would be changed very shortly? A bank in possession of such information (if we ignore the ethical implications) would not quote forward rates based on the spot rate curve, but would quote rates that reflected its insider knowledge.

6.6.4 Bond valuation using forward rates

That there is a relationship between spot rates and implied forward rates, although it is not necessarily a straight-forward one, should tell us that, in theory there is no difference in valuing a conventional bond with either spot rates or forward rates. The present value of a cash flow C received in period n using forward rates is given by (6.12):

$$PV_C = \frac{C}{(1 + rs_1)(1 + {}_1rf_1)(1 + {}_2rf_1) \dots (1 + {}_nrf_1)}. \quad (6.12)$$

Therefore we use (6.12) to assemble the expression for valuing an N -period term bond using implied forward rates, with coupon C , given at (6.13). Note that we use the six-month or one-year spot rates and the six-month or one-year implied forward rates for the forward dates according to whether the bond pays annual or semi-annual coupons. Equation (6.13) assumes an exact number of interest periods to maturity.

⁶ See the Preface.

$$\begin{aligned}
P_d = & \frac{C}{(1 + rs_1)} + \frac{C}{(1 + rs_1)(1 + {}_0rf_1)} + \frac{C}{(1 + rs_1)(1 + {}_1rf_1)(1 + {}_2rf_1)} + \dots \\
& \dots + \frac{C + M}{(1 + rs_1)(1 + {}_1rf_1)(1 + {}_2rf_1)\dots(1 + {}_Nrf_1)}.
\end{aligned} \tag{6.13}$$

Although bond analysts may use either spot or implied forward rates to present value cash flow streams, the final valuation will be the same regardless of whichever rates are used.

6.7 The annuity yield curve

Life assurance companies and other providers of personal pensions are users of the *annuity yield curve*, which is a plot of annuity yields against term to maturity. The *annuity yield* is the implied yield on an annuity where the annuity is valued using spot yields. In (6.2) above we decomposed a bond into an annuity and a zero-coupon discounted bond. We used the spot yield to price the discount bond component. Now we are concerned with the annuity or pure coupon component.

The value of the annuity component of a bond is given by (6.14):

$$\begin{aligned}
A'_N &= \sum_{n=1}^N \frac{C}{(1 + rs_n)^n} = \sum_{n=1}^N C \times df_n \\
&= C \times A_N
\end{aligned} \tag{6.14}$$

where rs_n and df_n are the n -period spot rate and discount factor, as defined earlier, and A_N is the fair price of an N -year annuity of £1. However, A_N is also given by the standard formula (6.15):

$$A_N = \frac{1}{ra_N} \left(1 - \frac{1}{(1 + ra_N)^N} \right) \tag{6.15}$$

where ra_N is the annuity yield on an N -year annuity.

Again using the same rates as before, consider a three-year bond with a coupon of 5.75%. To obtain the value of the annuity portion of the bond we would use (6.14) to give us:

$$\begin{aligned}
A'_3 &= \frac{5.75}{1.05} + \frac{5.75}{(1.05269)^2} + \frac{5.75}{(1.05778)^3} \\
&= 15.52.
\end{aligned}$$

This indicates a value for A_3 of £2.70, obtained by dividing 15.52 by 5.75. We can then use this value to solve for the annuity yield ra using (6.15) or using annuity tables; in this case the three-year annuity yield is 5.46%.

The relationship between the spot and annuity yield curves will depend on the level of market rates. With a positive-sloping spot yield curve, the annuity yield is below the end-of-period spot yield; with a negative sloping spot yield curve, the annuity yield curve lies above it. With a low-coupon bond the present value will be dominated by the terminal payment and the annuity curve will lie close to the spot curve; with a high-coupon bond the two curves will be further apart.

6.8 Analysing and interpreting the yield curve

From observing yield curves in different markets at any time, we notice that a yield curve can adopt one of four basic shapes, which are:

- *normal* or *conventional*: in which yields are at “average” levels and the curve slopes gently upwards as maturity increases;
- *upward sloping* or *positive* or *rising*: in which yields are at historically low levels, with long rates substantially greater than short rates;
- *downward sloping* or *inverted* or *negative*: in which yield levels are very high by historical standards, but long-term yields are significantly lower than short rates;
- *humped*: where yields are high with the curve rising to a peak in the medium-term maturity area, and then sloping downwards at longer maturities.

Sometimes yield curves will incorporate a mixture of the above features.

A great deal of effort is expended by bond analysts and economists analysing and interpreting yield curves. There is often a considerable information content associated with any curve at any time. For example Figure 6.5 shows the UK gilt redemption yield curve at three different times in the ten years from June 1989 to June 1999. What does the shape of each curve tell us about the UK debt market, and the UK economy at each particular time?

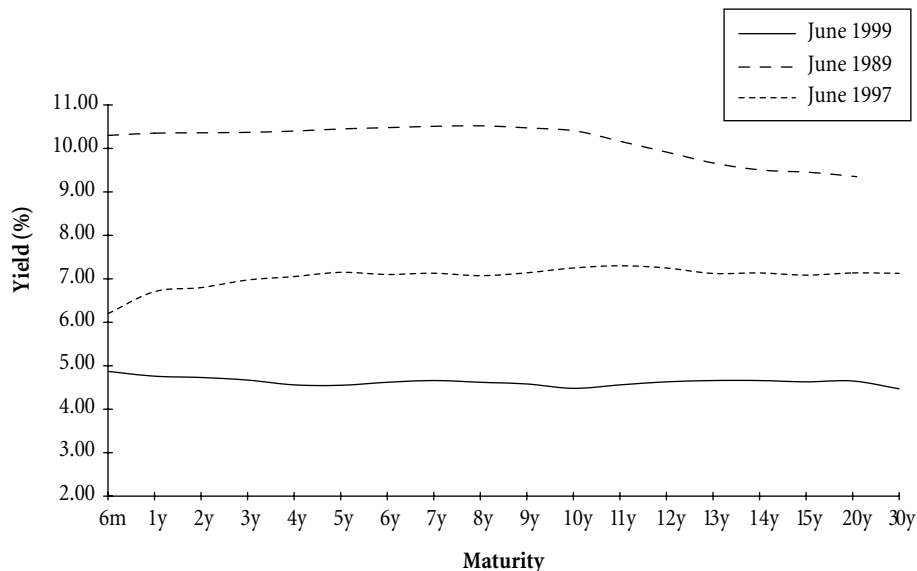


Figure 6.5: UK gilt redemption yield curves. Source: Bloomberg.

In this section we will consider the various explanations that have been put forward to explain the shape of the yield curve at any one time. None of the theories can adequately explain everything about yield curves and the shapes they assume at any time, so generally observers seek to explain specific curves using a combination of the accepted theories. This subject is a large one, indeed we could devote several books to it, so at this stage we will introduce the main ideas, reserving a more detailed investigation for Part VIII.

The existence of a yield curve itself indicates that there is a cost associated with funds of different maturities, otherwise we would observe a flat yield curve. The fact that we very rarely observe anything approaching a flat yield suggests that investors require different rates of return depending on the maturity of the instrument they are holding. In this section we review the main theories that have put forward to explain the shape of the yield curve, which all have fairly long-dated antecedents.

An excellent account of the term structure is given in *Theory of Financial Decision Making* by Jonathan Ingersoll (1987), Chapter 18. In fact it is worth purchasing this book just for Chapter 18 alone. Another quality account of the term structure is by Shiller (1990). In the following section we provide an introductory review of the research on this subject to date.

6.8.1 The expectations hypothesis

The expectations hypothesis suggests that bondholders' expectations determine the course of future interest rates. There are two main competing versions of this hypothesis, the *local expectations hypothesis* and the *unbiased expectations hypothesis*. The *return-to-maturity expectations hypothesis* and *yield-to-maturity expectations hypothesis* are also quoted (see Ingersoll 1987). The local expectations hypothesis states that all bonds of the same class but differing in term to maturity will have the same expected holding period rate of return. This suggests that a six-month bond and a twenty-year bond will produce the same rate of return, on average, over the stated holding period. So if we intend to hold a bond for six months, we will receive the same return no matter what specific bond we buy. The author feels that this theory is not always the case nor relevant, despite being mathematically neat; however it is worth spending a few moments discussing it and related points. Generally holding period returns from longer-dated

bonds are on average higher than those from short-dated bonds. Intuitively we would expect this, with longer-dated bonds offering higher returns to compensate for their higher price volatility (risk). The local expectations hypothesis would not agree with the conventional belief that investors, being risk averse, require higher returns as a reward for taking on higher risk; in addition it does not provide any insight about the shape of the yield curve. An article by Cox, Ingersoll and Ross (1981) showed that the local expectations hypothesis best reflected equilibrium between spot and forward yields. This was demonstrated using a feature known as Jensen's inequality, which is described in Appendix 6.2. Robert Jarrow (1996) states

“... in an economic equilibrium, the returns on ... similar maturity zero-coupon bonds cannot be too different. If they were too different, no investor would hold the bond with the smaller return. This difference could not persist in an economic equilibrium”.

(Jarrow 1996, p. 50)

This reflects economic logic, but in practice other factors can impact on holding period returns between bonds that do not have similar maturities. For instance investors will have restrictions as to which bonds they can hold, for example banks and building societies are required to hold short-dated bonds for liquidity purposes. In an environment of economic disequilibrium, these investors would still have to hold shorter-dated bonds, even if the holding period return was lower.

So although it is economically neat to expect that the return on a long-dated bond is equivalent to rolling over a series of shorter-dated bonds, it is often observed that longer-term (default-free) returns exceed annualised short-term default-free returns. So an investor that continually rolled over a series of short-dated zero-coupon bonds would most likely receive a lower return than if she had invested in a long-dated zero-coupon bond. Rubinstein (1999) gives an excellent, accessible explanation of why this should be so. The reason is that compared to the theoretical model, in reality future spot rates are not known with certainty. This means that short-dated zero-coupon bonds are more attractive to investors for two reasons; first, they are more appropriate instruments to use for hedging purposes, and secondly they are more liquid instruments, in that they may be more readily converted back into cash than long-dated instruments. With regard to hedging, consider an exposure to rising interest rates. If the yield curve shifts upwards at some point in the future, the price of long-dated bonds will fall by a greater amount. This is a negative result for holders of such bonds, whereas the investor in short-dated bonds will benefit from rolling over his funds at the (new) higher rates. With regard to the second issue, Rubinstein (1999) states

“...it can be shown that in an economy with risk-averse individuals, uncertainty concerning the timing of aggregate consumption, the partial irreversibility of real investments (longer-term physical investments cannot be converted into investments with earlier payouts without sacrifice), [and] ... real assets with shorter-term payouts will tend to have a ‘liquidity’ advantage.”

(Rubinstein 1999, p. 84–85)

Therefore the demand for short-term instruments is frequently higher, and hence short-term returns are often lower than long-term returns.

The *pure or unbiased expectations hypothesis* is more commonly encountered and states that current implied forward rates are unbiased estimators of future spot interest rates.⁷ It assumes that investors act in a way that eliminates any advantage of holding instruments of a particular maturity. Therefore if we have a positive-sloping yield curve, the unbiased expectations hypothesis states that the market expects spot interest rates to rise. Equally, an inverted yield curve is an indication that spot rates are expected to fall. If short-term interest rates are expected to rise, then longer yields should be higher than shorter ones to reflect this. If this were not the case, investors would only buy the shorter-dated bonds and roll over the investment when they matured. Likewise if rates are expected to fall then longer yields should be lower than short yields. The unbiased expectations hypothesis states that the long-term interest rate is a geometric average of expected future short-term rates. This was in fact the theory that was used to derive the forward yield curve using (6.5) and (6.7) previously. This gives us:

⁷ For original discussion, see Lutz (1940) and Fisher (1986, although he formulated his ideas earlier).

$$(1 + rs_N)^N = (1 + rs_1)(1 + {}_1rf_2) \dots (1 + {}_{N-1}rf_N) \quad (6.16)$$

or

$$(1 + rs_N)^N = (1 + rs_{N-1})^{N-1} (1 + {}_{N-1}rf_N) \quad (6.17)$$

where rs_N is the spot yield on a N -year bond and ${}_{n-1}rf_n$ is the implied one-year rate n years ahead. For example if the current one-year spot rate is $rs_1 = 5.0\%$ and the market is expecting the one-year rate in a year's time to be ${}_1rf_2 = 5.539\%$, then the market is expecting a £100 investment in two one-year bonds to yield:

$$£100(1.05)(1.05539) = £110.82$$

after two years. To be equivalent to this an investment in a two-year bond has to yield the same amount, implying that the current two-year rate is $rs_2 = 5.7\%$, as shown below:

$$£100(1 + rs_2)^2 = £110.82$$

which gives us rs_2 equal to 5.27%, and gives us the correct future value as shown below:

$$£100(1.0527)^2 = £110.82.$$

This result must be so, to ensure no arbitrage opportunities exist in the market and in fact we showed as much earlier in the chapter when we considered forward rates. According to the unbiased expectations hypothesis therefore the forward rate ${}_0rf_2$ is an unbiased predictor of the spot rate ${}_1rs_1$ observed one period later; on average the forward rate should equal the subsequent spot rate. The hypothesis can be used to explain any shape in the yield curve.

A rising yield curve is therefore explained by investors expecting short-term interest rates to rise, that is ${}_1rf_2 > rs_2$. A falling yield curve is explained by investors expecting short-term rates to be lower in the future. A humped yield curve is explained by investors expecting short-term interest rates to rise and long-term rates to fall. Expectations, or views on the future direction of the market, are a function mainly of the expected rate of inflation. If the market expects inflationary pressures in the future, the yield curve will be positively shaped, while if inflation expectations are inclined towards disinflation, then the yield curve will be negative. However, several empirical studies including one by Fama (1976) have shown that forward rates are essentially biased predictors of future spot interest rates, and often over-estimate future levels of spot rates. The unbiased hypothesis has also been criticised for suggesting that investors can forecast (or have a view on) very long-dated spot interest rates, which might be considered slightly unrealistic. As yield curves in most developed country markets exist to a maturity of up to thirty years or longer, such criticisms may have some substance. Are investors able to forecast interest rates 10, 20 or 30 years into the future? Perhaps not, nevertheless this is indeed the information content of say, a thirty-year bond; since the yield on the bond is set by the market, it is valid to suggest that the market has a view on inflation and future interest rates for up to thirty years forward.

The expectations hypothesis is stated in more than one way; we have already encountered the local expectations hypothesis. Other versions include the *return-to-maturity* expectations hypothesis, which states that the total return from a holding a zero-coupon bond to maturity will be equal to the total return that is generated by holding a short-term instrument and continuously rolling it over the same maturity period. A related version, the *yield-to-maturity* hypothesis, states that the periodic return from holding a zero-coupon bond will be equal to the return from rolling over a series of coupon bonds, but refers to the annualised return earned each year rather than the total return earned over the life of the bond. This assumption enables a zero-coupon yield curve to be derived from the redemption yields of coupon bonds. The unbiased expectations hypothesis of course states that forward rates are equal to the spot rates expected by the market in the future. The Cox–Ingersoll–Ross article suggests that only the local expectations hypothesis describes a model that is purely arbitrage-free, as under the other scenarios it would be possible to employ certain investment strategies that would produce returns in excess of what was implied by today's yields. Although it has been suggested⁸ that the differences between the local and the unbiased hypotheses

⁸ For example, see Campbell (1986) and Livingstone (1990).

are not material, a model that describes such a scenario would not reflect investors' beliefs, which is why further research is ongoing in this area.

The unbiased expectations hypothesis does not by itself explain all the shapes of the yield curve or the information content contained within it, so it is often tied in with other explanations, including the liquidity preference theory.

6.8.2 Liquidity preference theory

Intuitively we might feel that longer maturity investments are more risky than shorter ones. An investor lending money for a five-year term will usually demand a higher rate of interest than if they were to lend the same customer money for a five-week term. This is because the borrower may not be able to repay the loan over the longer time period as they may for instance, have gone bankrupt in that period. For this reason longer-dated yields should be higher than short-dated yields, to recompense the lender for the higher risk exposure during the term of the loan.⁹

We can consider this theory in terms of inflation expectations as well. Where inflation is expected to remain roughly stable over time, the market would anticipate a positive yield curve. However the expectations hypothesis cannot by itself explain this phenomenon, as under stable inflationary conditions one would expect a flat yield curve. The risk inherent in longer-dated investments, or the *liquidity preference theory*, seeks to explain a positive shaped curve. Generally borrowers prefer to borrow over as long a term as possible, while lenders will wish to lend over as short a term as possible. Therefore, as we first stated, lenders have to be compensated for lending over the longer term; this compensation is considered a premium for a loss in *liquidity* for the lender. The premium is increased the further the investor lends across the term structure, so that the longest-dated investments will, all else being equal, have the highest yield. So the liquidity preference theory states that the yield curve should almost always be upward sloping, reflecting bondholders preference for the liquidity and lower risk of shorter-dated bonds. An inverted yield curve could still be explained by the liquidity preference theory when it is combined with the unbiased expectations hypothesis. A *humped* yield curve might be viewed as a combination of an inverted yield curve together with a positive-sloping liquidity preference curve.

The difference between a yield curve explained by unbiased expectations and an actual observed yield curve is sometimes referred to as the *liquidity premium*. This refers to the fact that in some cases short-dated bonds are easier to transact in the market than long-term bonds. It is difficult to quantify the effect of the liquidity premium, which in any cases is not static and fluctuates over time. The liquidity premium is so-called because, in order to induce investors to hold longer-dated securities, the yields on such securities must be higher than those available on short-dated securities, which are more liquid and may be converted into cash more easily. The liquidity premium is the compensation required for holding less liquid instruments. If longer-dated securities then provide higher yields, as is suggested by the existence of the liquidity premium, they should generate on average higher total returns over an investment period. This is not consistent with the local expectations hypothesis. More formally we can write:

$$0 = L_1 < L_2 < L_3 < \dots < L_n \text{ and } (L_2 - L_1) > (L_3 - L_2) > \dots (L_n - L_{n-1})$$

where L is the premium for a bond with term to maturity of n years, which states that the premium increases as the term to maturity rises and that an otherwise flat yield curve will have a positively sloping curve, with the degree of slope steadily decreasing as we extend along the yield curve. This is consistent with observation of yield curves under "normal" conditions.

The expectations hypothesis assumes that forward rates are equal to the expected future spot rates, that is as shown in (6.18):

$${}_{n-1}rf_n = E({}_{n-1}rs_n) \quad (6.18)$$

where $E(\cdot)$ is the expectations operator for the current period. This assumption implies that the forward rate is an unbiased predictor of the future spot rate, as we suggested in the previous paragraph. Liquidity preference theory on the other hand, recognises the possibility that the forward rate may contain an element of liquidity premium which declines over time as the period approaches, given by (6.19):

⁹ For original discussion, see Hicks (1946).

$${}_{n-1}rf_n > E({}_{n-1}rs_n). \quad (6.19)$$

If there was uncertainty in the market about the future direction of spot rates and hence where the forward rate should lie, (6.19) is adjusted to give the reverse inequality.

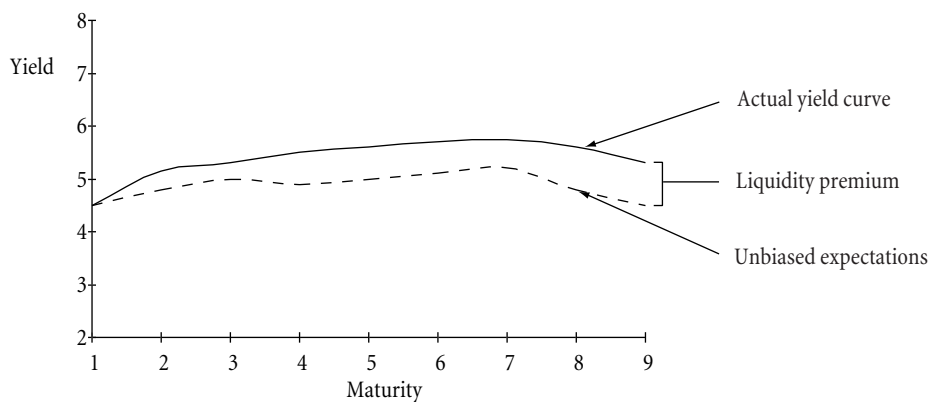


Figure 6.6: Yield curve explained by expectations hypothesis and liquidity preference.

6.8.3 Money substitute hypothesis

A particular explanation of short-dated bond yield curves has been attempted by Kessel (1965). In the *money substitute* theory short-dated bonds are regarded as substitutes for holding cash. Investors hold only short-dated market instruments because these are viewed as low or negligible risk. As a result the yields of short-dated bonds are depressed due to the increased demand and lie below longer-dated bonds. Borrowers on the other hand prefer to issue debt for longer maturities, and on as few occasions as possible to minimise costs. Therefore the yields of longer-dated paper are driven upwards due to a combination of increased supply and lower liquidity. In certain respects the money substitute theory is closely related to the liquidity preference theory, and by itself does not explain inverted or humped yield curves.

6.8.4 Segmentation hypothesis

The capital markets are made up of a wide variety of users, each with different requirements. Certain classes of investors will prefer dealing at the short-end of the yield curve, while others will concentrate on the longer end of the market. The *segmented markets* theory suggests that activity is concentrated in certain specific areas of the market, and that there are no inter-relationships between these parts of the market; the relative amounts of funds invested in each of the maturity spectrum causes differentials in supply and demand, which results in humps in the yield curve. That is, the shape of the yield curve is determined by supply and demand for certain specific maturity investments, each of which has no reference to any other part of the curve.

For example banks and building societies concentrate a large part of their activity at the short end of the curve, as part of daily cash management (known as *asset and liability management*) and for regulatory purposes (known as *liquidity requirements*). Fund managers such as pension funds and insurance companies are active at the long end of the market. Few institutional investors however have any preference for medium-dated bonds. This behaviour on the part of investors will lead to high prices (low yields) at both the short and long ends of the yield curve and lower prices (higher yields) in the middle of the term structure.

Since according to the segmented markets hypothesis a separate market exists for specific maturities along the term structure, interest rates for these maturities are set by supply and demand.¹⁰ Where there is no demand for a particular maturity, the yield will lie above other segments. Market participants do not hold bonds in any other area

¹⁰ See Culbertson (1957).

of the curve outside their area of interest¹¹ so that short-dated and long-dated bond yields exist independently of each other. The segmented markets theory is usually illustrated by reference to banks and life companies. Banks and building societies hold their funds in short-dated instruments, usually no longer than five years in maturity. This is because of the nature of retail banking operations, with a large volume of instant access funds being deposited at banks, and also for regulatory purposes. Holding short-term, liquid bonds enables banks to meet any sudden or unexpected demand for funds from customers. The classic theory suggests that as banks invest their funds in short-dated bonds, the yields on these bonds is driven down. When they then liquidate part of their holding, perhaps to meet higher demand for loans, the yields are driven up and prices of the bonds fall. This affects the short end of the yield curve but not the long end.

The segmented markets theory can be used to explain any particular shape of the yield curve, although it fits best perhaps with positive sloping curves. However it cannot be used to interpret the yield curve whatever shape it may be, and therefore offers no information content during analysis. By definition the theory suggests that for investors bonds with different maturities are not perfect substitutes for each other. This is because different bonds would have different holding period returns, making them imperfect substitutes of one another.¹² As a result of bonds being imperfect substitutes, markets are segmented according to maturity.

The segmentations hypothesis is a reasonable explanation of certain features of a conventional positively-sloping yield curve, but by itself is not sufficient. There is no doubt that banks and building societies have a requirement to hold securities at the short end of the yield curve, as much for regulatory purposes as for yield considerations, however other investors are probably more flexible and will place funds where value is deemed to exist. Nonetheless the higher demand for benchmark securities does drive down yields along certain segments of the curve.

A slightly modified version of the market segmentation hypothesis is known as the *preferred habitat theory*. This suggests that different market participants have an interest in specified areas of the yield curve, but can be induced to hold bonds from other parts of the maturity spectrum if there is sufficient incentive. Hence banks may at certain times hold longer-dated bonds once the price of these bonds falls to a certain level, making the return on the bonds worth the risk involved in holding them. Similar considerations may persuade long-term investors to hold short-dated debt. So higher yields will be required to make bond holders shift out of their usual area of interest. This theory essentially recognises the flexibility that investors have, outside regulatory or legal requirements (such as the terms of an institutional fund's objectives), to invest in whatever part of the yield curve they identify value.

6.8.5 Humped yield curves

When plotting a yield curve of all the bonds in a certain class, it is common to observe humped yield curves. These usually occur for a variety of reasons. In line with the unbiased expectations hypothesis, humped curves will be observed when interest rates are expected to rise over the next several periods and then decline. On other occasions humped curves can result from skewed expectations of future interest rates. This is when the market believes that fairly constant future interest rates are likely, but also believes that there is a small probability for lower rates in the medium term. The other common explanation for humped curves is the preferred habitat theory.

6.8.6 The combined theory

The explanation for the shape of the yield curve at any time is more likely to be described by a combination of the pure expectations hypothesis and the liquidity preference theory, and possibly one or two other theories. Market analysts often combine the unbiased expectations hypothesis with the liquidity preference theory into an "eclectic" theory. The result is fairly consistent with any shape of yield curve, and is also a predictor of rising interest rates. In the combined theory the forward interest rate is equal to the expected future spot rate, together with a quantified liquidity premium. This is shown at (6.20):

$${}_0rf_i = E({}_{i-1}rs_1) + L_i \quad (6.20)$$

¹¹ For example, retail and commercial banks hold bonds in the short dates, while life assurance companies hold long-dated bonds.

¹² *Ibid.*

where L_i is the liquidity premium for a term to maturity of i . The size of the liquidity premium is expected to increase with increasing maturity.¹³ An illustration is given at Example 6.3.

EXAMPLE 6.3 Positive yield curve with constant expected future interest rates

- Consider the interest rates structure in Table 6.2.

Period n	0	1	2	3	4	5
$E(rs)$		4.5%	4.5%	4.5%	4.5%	4.5%
Forward rate ${}_0rf_n$		5.00%	5.50%	6.00%	6.50%	7.50%
Spot rate rs_n	5%	5.30%	5.80%	6.20%	6.80%	7%

Table 6.2: Positive yield curve with constant expected future rates.

The current term structure is positive sloping since the spot rates increase with increasing maturity. However the market expects future spot rates to be constant at 4.5%. The forward and spot rates are also shown, however the forward rate is a function of the expected spot rate and the liquidity premium. This premium is equal to 0.50% for the first year, 1.0% in the second and so on.

The combined theory is consistent with an inverted yield curve. This will apply even when the liquidity premium is increasing with maturity, for example where the expected future spot interest rate is declining. Typically this would be where there was a current term structure of falling yields along the term structure. The spot rates might be declining where the fall in the expected future spot rate exceeds the corresponding increase in the liquidity premium.

6.8.7 The flat yield curve

The conventional theories do not seek to explain a flat yield curve. Although it is rare to observe flat curves in a market, certainly for any length of time, at times they do emerge in response to peculiar economic circumstances. In the conventional thinking, a flat curve is not tenable because investors should in theory have no incentive to hold long-dated bonds over shorter-dated bonds when there is no yield premium, so that as they sell off long-dated paper the yield at the long end should rise, producing an upward sloping curve. In previous circumstances of a flat curve, analysts have produced different explanations for their existence. In November 1988 the US Treasury yield curve was flat relative to the recent past; researchers contended that this was the result of the market's view that long-dated yields would fall as bond prices rallied upwards.¹⁴ One recommendation is to buy longer maturities when the yield curve is flat, in anticipation of lower long-term interest rates, which is the direct opposite to the view that a flat curve is a signal to sell long bonds. In the case of the US market in 1988, long bond yields did in fact fall by approximately 2% in the following 12 months. This would seem to indicate that one's view of future long-term rates should be behind the decision to buy or sell long bonds, rather than the shape of the yield curve itself. A flat curve may well be more heavily influenced by supply and demand factors than anything else, with the majority opinion eventually winning out and forcing a change in the curve to a more conventional shape.

6.8.8 Yield curves as a function of the stochastic behaviour of interest rates

As a result of research into the behaviour of asset prices more recent explanations for the shape of the yield curve have sought to describe it as reflecting the behaviour of interest rates and the process that interest rates follow. These explanations are termed *stochastic processes*. A stochastic process is one where random phenomena evolve over time, and these may be asset prices, interest rates, returns on an investment portfolio and so on. In Parts VI and VIII we discuss stochastic processes in greater detail. Under these explanations then, yield curves reflect the following:

- bond yields follow a stochastic process over time, and hence the yield curve reflects this;
- bond yields at any one time satisfy the no-arbitrage pricing rule for spot and forward rates.

The model of the term structure as being an arbitrage-free stochastic process evolved with option pricing theory and was described separately.¹⁵ Such models sought to describe the term structure in terms of the short-term

¹³ So that $L_i > L_{i-1}$.

¹⁴ See Levy (1999).

¹⁵ See Black and Scholes (1973) and Merton (1973).

interest rate only, more recent models describe the whole term structure as part of a stochastic process.¹⁶ This subject is key to yield curve modelling and so we will hold discussion about it until Part VIII.

6.8.9 Further views on the yield curve

In this and the previous chapters our discussion of present values, spot and forward interest rates assumed an economist's world of the *perfect market* (also sometimes called the *frictionless* financial market). Such a perfect capital market is characterised by:

- perfect information;
- no taxes;
- bullet maturity bonds;
- no transaction costs.

Of course in practice markets are not completely perfect. However assuming perfect markets makes the discussion of spot and forward rates and the term structure easier to handle. When we analyse yield curves for their information content, we have to remember that the markets that they represent are not perfect, and that frequently we observe anomalies that are not explained by the conventional theories.

At any one time it is probably more realistic to suggest that a range of factors contributes to the yield curve being one particular shape. For instance short-term interest rates are greatly influenced by the availability of funds in the money market. The slope of the yield curve (usually defined as the 10-year yield minus the three-month interest rate) is also a measure of the degree of tightness of government monetary policy. A low, upward sloping curve is often thought to be a sign that an environment of cheap money, due to a more loose monetary policy, is to be followed by a period of higher inflation and higher bond yields. Equally a high downward sloping curve is taken to mean that a situation of tight credit, due to more strict monetary policy, will result in falling inflation and lower bond yields. Inverted yield curves have often preceded recessions; for instance *The Economist* in an article from April 1998 remarked that in the United States every recession since 1955 bar one had been preceded by a negative yield curve. The analysis is the same: if investors expect a recession they also expect inflation to fall, so the yields on long-term bonds will fall relative to short-term bonds. So the conventional explanation for an inverted yield curve is that the markets and the investment community expect either a slow-down of the economy, or an outright recession.¹⁷ In this case one would expect the monetary authorities to ease the money supply by reducing the base interest rate in the near future: hence an inverted curve. At the same time, a reduction of short-term interest rates will affect short-dated bonds and these are sold off by investors, further raising their yield.

While the conventional explanation for negative yield curves is an expectation of economic slow-down, on occasion other factors will be involved. In the UK in the period July 1997–June 1999 the gilt yield curve was inverted.¹⁸ There was no general view that the economy was heading for recession however, in fact the new Labour government led by Tony Blair inherited an economy believed to be in good health. Instead the explanation behind the inverted shape of the gilt yield curve focused on two other factors: first, the handing of responsibility for setting interest rates to the Monetary Policy Committee (MPC) of the Bank of England, and secondly the expectation that the UK would over the medium term, abandon sterling and join the euro currency. The yield curve in this time suggested that the market expected the MPC to be successful and keep inflation at a level around 2.5% over the long term (its target is actually a 1% range either side of 2.5%), and also that sterling interest rates would need to come down over the medium term as part of *convergence* with interest rates in euroland. These are both medium-term expectations however, and in the author's view not logical at the short-end of the yield curve. In fact the term structure moved to a positive-sloped shape up to the 6–7 year area, before inverting out to the long-end of the curve, in June 1999. This is a more logical shape for the curve to assume, but it was short-lived and returned to being inverted after the 2-year term.

¹⁶ For example, see Heath, Jarrow and Morton (1992).

¹⁷ A recession is formally defined as two successive quarters of falling output in the domestic economy.

¹⁸ Although the curve briefly went positively sloped out to 7–8 years in July 1999, it very quickly reverted to being inverted throughout the term structure, and remained so at the time of writing.

There is therefore significant information content in the yield curve, and economists and bond analysts will consider the shape of the curve as part of their policy making and investment advice. The shape of parts of the curve, whether the short-end or long-end, as well that of the entire curve, can serve as useful predictors of future market conditions. As part of an analysis it is also worthwhile considering the yield curves across several different markets and currencies. For instance the interest-rate swap curve, and its position relative to that of the government bond yield curve, is also regularly analysed for its information content. In developed country economies the swap market is invariably as liquid as the government bond market, if not more liquid, and so it is common to see the swap curve analysed when making predictions about say, the future level of short-term interest rates. We will consider the swap curve again in Part VI.

Government policy will influence the shape and level of the yield curve, including policy on public sector borrowing, debt management and open-market operations. The markets perception of the size of public sector debt will influence bond yields; for instance an increase in the level of debt can lead to an increase in bond yields across the maturity range. Open-market operations, which refers to the daily operation by the Bank of England to control the level of the money supply (to which end the Bank purchases short-term bills and also engages in repo dealing), can have a number of effects. In the short-term it can tilt the yield curve both upwards and downwards; longer term, changes in the level of the base rate will affect yield levels. An anticipated rise in base rates can lead to a drop in prices for short-term bonds, whose yields will be expected to rise; this can lead to a temporary inverted curve. Finally debt management policy will influence the yield curve. (In the United Kingdom this is now the responsibility of the Debt Management Office.) Much government debt is rolled over as it matures, but the maturity of the replacement debt can have a significant influence on the yield curve in the form of humps in the market segment in which the debt is placed, if the debt is priced by the market at a relatively low price and hence high yield.

EXAMPLE 6.4 The information content of the UK gilt curve: a special case

In the first half of 1999 various factors combined to increase the demand for gilts, especially at the long-end of the yield curve, at a time of a reduction in the supply of gilts as the government's borrowing requirement was falling. This increased demand led to a lowering in market liquidity as prices rose and gilts became more expensive (that is, lower yielding) than government securities in most European countries. This is a relatively new phenomenon, witness 10-year UK government yields at 5.07% compared to US and Germany at 6.08% and 5.10% respectively at one point in August 1999.¹⁹ At the long end of the yield curve, UK rates were, for the first time in over 30 years, below both German and US yields, reflecting the market's positive long-term view of the UK economy. At the end of September 1999, the German 30-year bond (the 4¾% July 2028) was yielding 5.73% and the US 6.125% 2027 was at 6.29%, compared to the UK 6% 2028, which was trading at a yield of 4.81%.²⁰

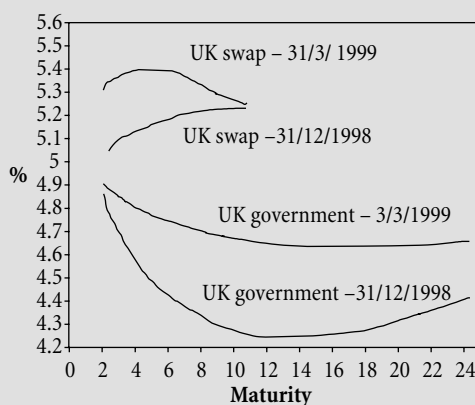


Figure 6.7: UK gilt and swap yield curves. Source: BoE.

¹⁹ Yields obtained from Bloomberg.

²⁰ *Ibid.*

The relative dearth of UK gilts was reflected in the yield spread of interest-rate swaps versus gilts (swaps are covered in Part VI). For example in March 1999 10-year swap spreads over government bonds were over 80 basis points in the UK compared to 40 basis points in Germany. This was historically large and was more than what might be required to account purely for the credit risk of swaps. It appears that this reflected the high demand for gilts, which had depressed the long end of the yield curve. At this point the market contended that the gilt yield curve no longer provided an accurate guide to expectations about future short-term interest rates. The sterling swap market, where liquidity is always as high as the government market and (as on this occasion) often higher, was viewed as being a more accurate prediction of future short-term interest rates. In hindsight this view turned out to be correct; swap rates fell in the UK in January and February 1999, and by the end of the following month the swap yield curve had become slightly upward-sloping, whereas the gilt yield curve was still inverted. This does indeed suggest that the market foresaw higher future short-term interest rates and that the swap curve predicted this, while the gilt curve did not. Figure 6.7 shows the change in the swap yield curve to a more positive slope from December 1998 to March 1999, while the gilt curve remained inverted. This is an occasion when the gilt yield curve's information content was less relevant than that in another market yield curve, due to the peculiar circumstances resulting from lack of supply to meet increased demand.

6.9 Interpreting the yield curve

We illustrate some of the points we have raised here using yield curves from the United Kingdom gilt market during the 1990s. It is of course easier to analyse them for their information content in hindsight than it might have been at the time, but these observations illustrate the general conclusions that might be made.

To begin consider the observed yield curves for January and June 1990 in Figures 6.8 and 6.9.²¹

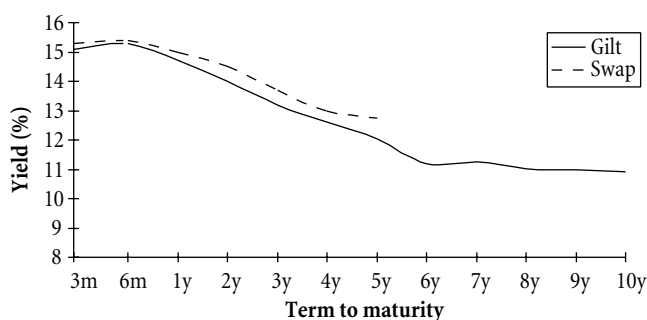


Figure 6.8: January 1990.

The yield curve for the first date suggests declining short-term rates and this is indicated by the yield curve for June 1990. The prediction is approximately accurate.

That the yield curves were inverted is not surprising given that the UK economy was shortly to enter an economic recession, however at that date – January 1990 – it had not yet begun. The evidence of GDP output data is that the recession took place during 1990–91. The markets were therefore expecting some loosening of monetary policy given the economic slow-down during 1989. There was an additional anomaly in that during 1988 the government had been paying off public sector debt, leading some commentators to suggest that gilt supply would be reduced in coming years. This may have contributed to the depressed yields at the long end in January 1990 but almost certainly had less influence in June 1990, when the recession was underway.

²¹ All government yield curves shown are fitted par yield curves, using the Bank of England's internal model (see Mastronikola (1991) and Anderson and Sleath (1999)), except where indicated. Source data is the Bank of England. The other curve is the interest-rate swap curve, also called the Libor curve, for sterling swaps. In practice interest-rate swaps are priced off the government yield curve, and reflect the market's view of interbank credit risk, as the swap rate is payable by a bank (or corporate) viewed as having an element of credit risk. All swap curves are drawn using interest data from Bloomberg.

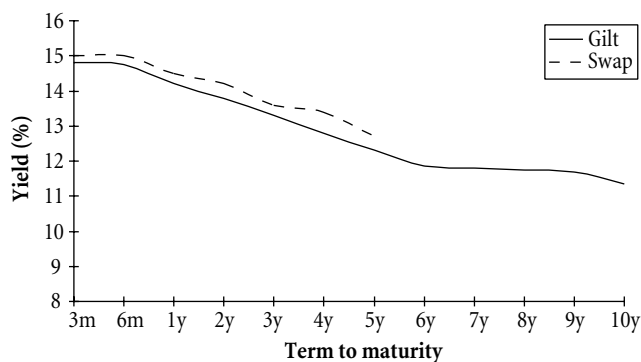


Figure 6.9: June 1990.

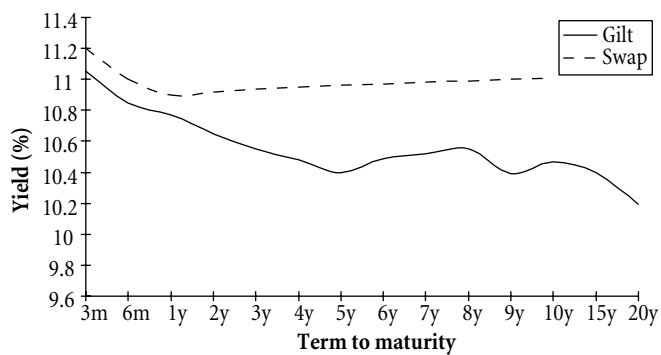


Figure 6.10: June 1991.

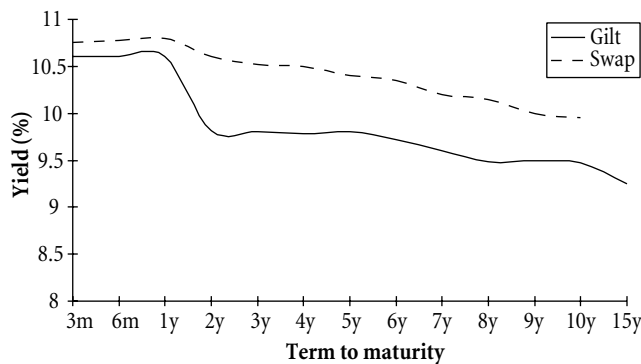


Figure 6.11: January 1992.

Consider the same curves for June 1991 and January 1992, shown as Figures 6.10 and 6.11 respectively. In this case the money market has priced swaps to give a different shape yield curve; does this mean that the money market has a different view of forward rates to government bond investors? Observing the yield curves for January 1992 suggests not: the divergence in the swap curve reflects credit considerations that price long maturity corporate rates at increasing yield spreads. This is common during recessions, and indeed the curves reflect recession conditions in the UK economy at the time, as predicted by the yield curves during 1990. Another indicator is the continuation of the wide swap spread as the term to maturity increases to ten years, rather than the conventional mirroring of the government curve. Note also that the short-term segment of the swap curve in June 1991 (Figure 6.9) matches the

government yield curve, indicating that the market agreed with the short-term forward rate prediction of the government curve.

During 1992 as the UK economy came out of recession the government yield curve changed from inverted to steadily positive, while the swap curve mirrored the shape of the government curve. The swap spread itself has declined, to no more than 10 basis points at the short end. This was most pronounced shortly after sterling fell out of the European Union's exchange rate mechanism, shown in Figure 6.12, the curves for November 1992.

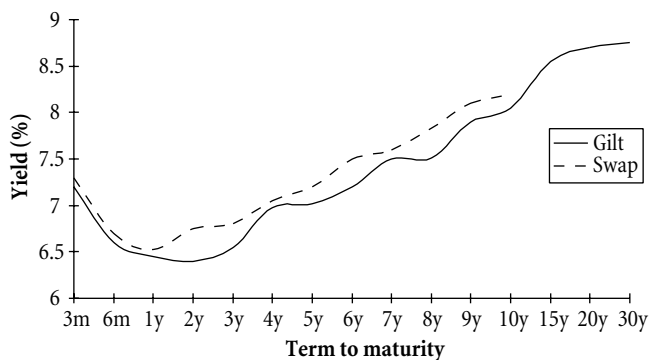


Figure 6.12: 5 November 1992.

Both yield curves had reverted to being purely positive sloping by January 1993.

The illustrations used are examples where government yield curves were shown to be accurate predictors of the short-term rates that followed, more so than the swap curve. This is to be expected: the government or *benchmark* yield curve is the cornerstone of the debt markets, and is used by the market to price all other debt instruments, including interest-rate swaps. This reflects both the liquidity of the government market and its risk-free status.

6.10 Fitting the yield curve

When graphing a yield curve, we plot a series of discrete points of yield against maturity. Similarly for the term structure of interest rates, we plot spot rates for a fixed time period against that time period. The yield curve itself however is a smooth curve drawn through these points. Therefore we require a method that allows us to fit the curve as accurately as possible, known as *yield curve modelling* or *estimating the term structure*. There are several ways to model a yield curve, which we introduce in this section. We will return to the subject in greater depth in Part VIII.

Ideally the fitted yield curve should be a continuous function, with no gaps in the curve, while passing through the observed yield vertices. The curve also needs to be “smooth”, as kinks in the curve will produce sudden sharp jumps in derived forward rates. We have stated how it is possible to calculate a set of discrete discount factors or a continuous discount function. It has been shown that the discount function, par yield curve, spot rate yield curve and forward rate curve are all related mathematically, such that if one knows any one of these, the other three can be derived. In practice in many markets it is not possible usually to observe the curves directly, hence they need to be derived from coupon bond prices and yields. In attempting to model a yield curve from bond yields we need to consider the two fundamental issues introduced above. First is the problem of gaps in the maturity spectrum, as in reality there will not be a bond maturing at regular intervals along the complete term structure. For example in the UK gilt market, currently there is no bond at all maturing between 2017 and 2021, or between 2021 and 2028. Secondly as we have seen the term structure is formally defined in terms of spot or zero-coupon interest rates, but in many markets there is no actual zero-coupon bond market. In such cases spot rates cannot be inferred directly but must be implied from coupon bonds. Where zero-coupon bonds are traded, for example in the US and UK government bond markets, we are able to observe zero-coupon yields directly in the market.

Further problems in fitting the curve arise from these two issues. How is the gap in maturities to be tackled? Analysts need to choose between “smoothness” and “responsiveness” of the curve estimate. Most models opt for a smooth fitting, however enough flexibility should be retained to allow for true movements in the term structure where indicated by the data. Should the yield curve be estimated from the discount function or say, the par yield curve? There are other practical factors to consider as well, such as the effect of withholding tax on coupons, and the

size of bond coupons themselves. We will consider the issues connected with estimating the yield curve in a later chapter; at this point we confine ourselves to introducing the main methods.

6.10.1 Interpolation

The simplest method that is employed to fit a curve is *linear interpolation*, which involves drawing a straight line joining each pair of yield vertices. To calculate the yield for one vertex we use (6.21):

$$rm_t = rm_i + \frac{t - n_i}{n_{i+1} - n_i} \times (rm_{i+1} - rm_i) \quad (6.21)$$

where rm_t is the yield being estimated and n is the number of years to maturity for yields that are observed. For example consider the following redemption yields:

1 month:	4.00%
2 years:	5.00%
4 years:	6.50%
10 years:	6.75%

If we wish to estimate the six-year yield we calculate it using (6.21), which is:

$$\begin{aligned} rm_{6y} &= 6.50\% + \frac{6 - 4}{10 - 4} \times 6.75\% - 6.50\% \\ &= 6.5833\%. \end{aligned}$$

The limitations of using linear interpolation are that first, the curve can have sharp angles at the vertices where two straight lines meet, resulting in unreasonable jumps in the derived forward rates. Second and more fundamentally being a straight-line method, it assumes the yield between two vertices should automatically be rising (in a positive yield curve environment) or falling. This assumption can lead to gross inaccuracies in the fitted curve.

Another approach is to use *logarithmic interpolation*, which involves applying linear interpolation to the natural logarithms of the corresponding discount factors. Therefore given any two discount factors we can calculate an intermediate discount factor using (6.22):

$$\ln(df_t) = \ln(df_{n_i}) + \frac{t - n_i}{n_{i+1} - n_i} \times (\ln(df_{n_{i+1}}) - \ln(df_{n_i})). \quad (6.22)$$

To calculate the six-year yield from the same yield structure above, we use the following procedure:

- calculate the discount factors for years 4 and 10 and then take the natural logarithms of these discount factors;
- perform a linear interpolation on these logarithms;
- take the anti-log of the result, to get the implied interpolated discount factor;
- calculate the implied yield in this discount factor.

Using (6.22) we obtain a six-year yield of 6.6388%. The logarithmic interpolation method reduces the sharpness of angles on the curve and so makes it smoother, but it retains the other drawbacks of the linear interpolation method.

6.10.2 Polynomial models²²

The most straightforward method for estimating the yield curve involves fitting a single polynomial in time. For example a model might use an F -order polynomial, illustrated with (6.23):

$$rm_i = \alpha + \beta_1 N_i + \beta_2 N_i^2 + \cdots + \beta_F N_i^F + u_i \quad (6.23)$$

where

rm_i is the yield to maturity of the i -th bond

²² These are standard econometric techniques. For an excellent account see Campbell *et al.* (1997).

N_i is the term to maturity of the i -th bond
 α, β_i are coefficients of the polynomial
 u_i is the residual error on the i -th bond

To determine the coefficients of the polynomial we minimise the sum of the squared residual errors, given by:

$$\sum_i^T u_i^2 \quad (6.24)$$

where T is the number of bonds used. This is represented graphically in Figure 6.13.

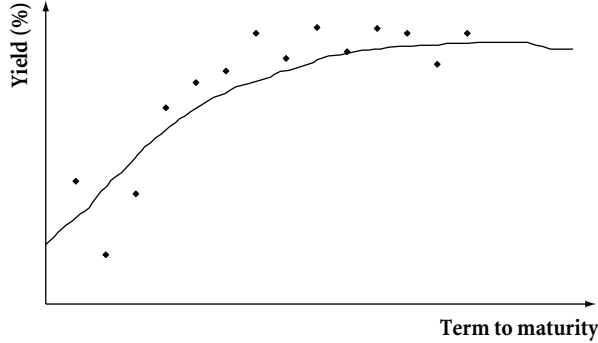


Figure 6.13: Polynomial curve fitting.

The type of curve that results is a function of the order of the polynomial F . If F is too large the curve will not be smooth but will be in effect too “responsive”, such that the curve runs through every point, known as being “over-fitted”. The extreme of this is given when $F = T - 1$. If F is too small the curve will be an over-estimation.

The method described above has been supplanted by a more complex method, which fits different polynomials over different but overlapping terms to maturity. The fitted curves are then spliced together to produce a single smooth curve over the entire term structure. This is known as a *spline* curve and is the one most commonly encountered in the markets. For an accessible introduction to spline techniques see James and Webber (2000) and Choudhry *et al.* (2001).

The limitation of the polynomial method is that a blip in the observed series of vertices, for instance a vertex which is out of line with others in the series, produces a “wobbled” shape, causing wild oscillations in the corresponding forward yields. This can result in the calculation of negative long-dated forward rates.

6.10.3 Cubic splines

The cubic spline method involves connecting each pair of yield vertices by fitting a unique cubic equation between them. This results in a yield curve where the whole curve is represented by a chain of cubic equations, instead of a single polynomial. This technique adds some “stiffness” to the yield curve, while at the same time preserving its smoothness.²³

Using the same example as before, we wish to fit the yield curve from 0 to 10 years.

There are four observed vertices, so we require three cubic equations, $rm_{(i,t)}$, each one connecting two adjacent vertices n_i and n_{i+1} as follows:

$$\begin{aligned}
 rm_{(0,t)} &= a_0 n^3 + b_0 n^2 + c_0 n + d_0, \text{ which connects vertex } n_0 \text{ with } n_1, \\
 rm_{(1,t)} &= a_1 n^3 + b_1 n^2 + c_1 n + d_1, \text{ which connects vertex } n_1 \text{ with } n_2, \text{ and} \\
 rm_{(2,t)} &= a_2 n^3 + b_2 n^2 + c_2 n + d_2, \text{ which connects vertex } n_2 \text{ with } n_3,
 \end{aligned}$$

where a, b, c , and d are unknowns. The equations each contain four unknowns (the coefficients a to d), and there are three equations so we require twelve conditions in all to solve the system. The cubic spline method imposes certain

²³ In case you’re wondering, a spline is a tool used by a carpenter to draw smooth curves.

conditions on the curves which makes it possible to solve the system. The solution for this set is summarised in Appendix 6.3.

The three cubic equations for the data in this example are:

$$\begin{aligned} rm_{(0,t)} &= 0.022 \times n^3 + 0.413 \times n + 4.000 \text{ for vertices } n_0 - n_1, \\ rm_{(1,t)} &= -0.047 \times n^3 + 0.411 \times n^2 - 0.410 \times n + 4.548 \text{ for vertices } n_1 - n_2, \text{ and} \\ rm_{(2,t)} &= 0.008 \times n^3 - 0.249 \times n^2 + 2.230 \times n + 1.029 \text{ for vertices } n_2 - n_3. \end{aligned}$$

Using a cubic spline produces a smoother curve for both the spot rates and the forward rates, while the derived forward curve will have fewer “kinks” in it.

To calculate the estimated yield for the 6-year maturity we apply the third cubic equation, which spans the 4–10 year vertices, which is $rm_{(2,t)} = 0.008 \times 6^3 - 0.249 \times 6^2 + 2.230 \times 6 + 1.029 = 7.173\%$.

From Appendix 6.3 it is clear that simply to fit a 4-vertex spline requires the inversion of a fairly large matrix. In practice more efficient mathematical techniques, known as basis splines or *B-splines* are typically used when there are a larger number of observed yield vertices. This produces results that are very close to what we would obtain by simple matrix inversion.

6.10.4 Regression models

A variation on polynomial fitting is regression analysis. In this method bond prices are used as the dependent variable, with the coupon and maturity cash flows of the bonds being the independent variables. This is given by (6.25):

$$P_{di} = \beta_1 C_{1i} + \beta_2 C_{2i} + \cdots + \beta_N (C_{Ni} + M) + u_i \quad (6.25)$$

where

P_{di}	is the dirty price of the i -th bond
C_{ni}	is the coupon of the i -th bond in period n
β_n	is the coefficient of the regression equation
u_i	is the residual error in the i -th bond

In fact, the coefficient in (6.25) is an estimate of the discount factor, as shown by (6.26) and can be used to generate the spot interest rate curve.

$$\beta_n = df_n = \frac{1}{(1 + rs_n)^n}. \quad (6.26)$$

In the form shown, (6.25) cannot be estimated directly. This is because individual coupon payment dates will differ across different bonds, and in a semi-annual coupon market there will be more coupons than bonds available. In practice therefore the term structure is divided into specific dates, known as *grid points*, along the entire maturity term; coupon payments are then allocated between two grid points. The allocation between two points is done in such a way so that the present value of the coupon is not altered. This is shown in Figure 6.14.

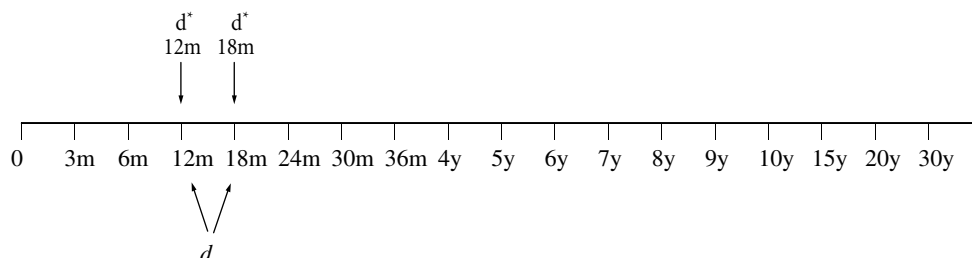


Figure 6.14: Grid point allocation in regression analysis.

Note how there are more grid points at the short end of the term structure, with progressively fewer points as we reach the longer end. This is because the preponderance of the data is invariably at the shorter end of the curve,

which makes yield curve fitting more difficult. At the long end however the shortage of data, due to the relative lack of issues, makes curve estimation more inaccurate.

The actual regression equation that is used in the analysis is given at (6.27) where d_{ni}^* represents the grid points.

$$P_{di} = \beta_1 d_{1i}^* + \beta_2 d_{2i}^* + \cdots + \beta_N (d_{Ni}^* + M) + u_i. \quad (6.27)$$

The two methodologies described above are the most commonly encountered in the market. Generally models used to estimate the term structure generally fall into two distinct categories, these being the ones that estimate the structure using the par yield curve and those that fit it using a discount function. We shall examine these in greater detail in a later chapter.

6.11 Spot and forward rates in the market

6.11.1 Using spot rates

The concepts discussed in this chapter are important and form a core part of debt markets analysis. It may appear that the content is largely theoretical, especially since many markets do not trade zero-coupon instruments and so spot rates are therefore not observable in practice; however the concept of the spot rate is an essential part of bond (and other instruments') pricing. In the first instance we are already aware that bond redemption yields do not reflect a true interest rate for that maturity, for which we use the spot rate. For relative value purposes, traders and portfolio managers frequently compare a bond's actual market price to its theoretical price, calculated using specific zero-coupon yields for each cash flow, and determine whether the bond is "cheap" or "dear". Even where there is some misalignment between the theoretical price of a bond and the actual price, the decision to buy or sell may be based on judgemental factors, since there is often no zero-coupon instrument against which to effect an arbitrage trade. In a market where no zero-coupon instruments are traded, the spot rates used in the analysis are theoretical and are not represented by actual market prices. Traders therefore often analyse bonds in terms of relative value against each other, and the redemption yield curve, rather than against their theoretical zero-coupon based price.

What considerations apply where a zero-coupon bond market exists alongside a conventional coupon-bond market? In such a case, in theory arbitrage trading is possible if a bond is priced above or below the price suggested by zero-coupon rates. For example, a bond priced above its theoretical price could be sold, and zero-coupon bonds that equated its cash flow stream could be purchased; the difference in price is the arbitrage profit. Or a bond trading below its theoretical price could be purchased and its coupons "stripped" and sold individually as zero-coupon bonds; the proceeds from the sale of the zero-coupon bonds would then exceed the purchase price of the coupon bond. In practice often the existence of both markets equalises prices between both markets so that arbitrage is no longer possible, although opportunities will still occasionally present themselves.

6.11.2 Using forward rates

Newcomers to the markets frequently experience confusion when first confronted with forward rates. Do they represent the market's expectation of where interest rates will actually be when the forward date arrives? If forward rates are a predictor of future interest rates, exactly how good are they at making this prediction? Empirical evidence²⁴ suggests that in fact forward rates are not accurate predictors of future interest rates, frequently overstating them by a considerable margin. If this is the case, should we attach any value or importance to forward rates?

The value of forward rates does not lie however in its track record as a market predictor, but moreover in its use as a hedging tool. As we illustrate in Example 6.5 the forward rate is calculated on the basis that if we are to price say, a cash deposit with a forward starting date, but we wish to deal today, the return from the deposit will be exactly the same as if we invested for a start date today and rolled over the investment at the forward date. The forward rate allows us to lock in a dealing rate now. Once we have dealt today, it is irrelevant what the actual rate pertaining on the forward date is – we have already dealt. Therefore forward rates are often called *hedge* rates, as they allow us to lock in a dealing rate for a future period, thus removing uncertainty.

The existence of forward prices in the market also allows us to make an investment decision, based on our view compared to the market view. The forward rate implied by say, government bond prices is in effect the market's

²⁴ Including Fama (1976).

view of future interest rates. If we happen not to agree with this view, we will deal accordingly. In effect we are comparing our view on future interest rates with that of the market, and making our investment decision based on this comparison.

EXAMPLE 6.5 Spot and forward rates calculation: bootstrapping from the par yield curve

■ Zero-coupon rates

As we have seen, zero-coupon (or spot), par and forward rates are linked. The term “zero-coupon” is a bond market term and describes a bond which has no coupons. The yield on a zero-coupon bond can be viewed as a true yield, if the paper is held to maturity as no reinvestment is involved and there are no interim cash flows vulnerable to a change in interest rates. Because zero-coupon rates can be derived from coupon rates, we can assume that:

- zero-coupon rates can be derived at any time where there is a liquid government bond market;
- these zero-coupon rates can then be used as the benchmark term structure.

Where zero-coupon bonds are traded the yield on a zero-coupon bond of a particular maturity is the zero-coupon rate for that maturity. However it is not necessary to have zero-coupon bonds in order to deduce zero-coupon rates. It is possible to calculate zero-coupon rates from a range of market rates and prices, including coupon bonds, interest-rate futures and currency deposits. The price of a zero-coupon bond of a particular maturity defines directly the value today of a cash flow due on the bond's redemption date, and indirectly the zero-coupon rate for that maturity. It is therefore that term to maturity's true interest rate.

■ Discount factors and the discount function

As we have seen already it is possible to determine a set of *discount factors* from market rates. A discount factor is the number in the range zero to one which can be used to obtain the present value of some future value.

Because

$$PV = \frac{FV}{(1 + r)^n} \quad (6.28)$$

where PV is the present value FV is the future value
 r is the required interest rate n is the number of interest rate periods

we know that

$$PV_n = df_n \times FV_n \quad (6.29)$$

where PV_n is the present value of the future cash flow occurring at time n
 FV_n is the future cash flow occurring at time n
 df_n is the discount factor for cash flows occurring at time n .

Discount factors can be calculated most easily from zero-coupon rates; equations (6.30) and (6.31) apply to zero-coupon rates for periods up to one year (which would be money market terms) and over one year (bond market maturities) respectively.

$$df_n = \frac{1}{(1 + rs_n T_n)} \quad (6.30)$$

$$df_n = \frac{1}{(1 + rs_n)^{T_n}} \quad (6.31)$$

where

df_n is the discount factor for cash flows occurring at time n
 rs_n is the zero-coupon or spot rate for the period to time n
 T_n is the time from the value date to time n , expressed in years and fractions of a year.

Individual zero-coupon rates allow discount factors to be calculated at specific points along the maturity

spectrum. As cash flows may occur at any time in the future, and not necessarily at convenient times like in three months or one year, discount factors often need to be calculated for every possible date in the future. The complete set of discount factors is called the *discount function*.

■ Implied spot and forward rates²⁵

In this section we describe how to obtain zero-coupon and forward interest rates from the yields available from coupon bonds, using a method known as *bootstrapping*. In a government bond market such as that for US Treasuries or UK gilts, the bonds are considered to be *default-free*. The rates from a government bond yield curve describe the risk-free rates of return available in the market *today*, however they also *imply* (risk-free) rates of return for *future time periods*. These implied future rates, known as *implied forward rates*, or simply *forward rates*, can be derived from a given spot yield curve using bootstrapping. This term reflects the fact that each calculated spot rate is used to determine the next period spot rate, in successive steps.

Table 6.3 shows an hypothetical benchmark gilt yield curve for value as at 7 December 2000. The observed yields of the benchmark bonds that compose the curve are displayed in the last column. All rates are annualised and assume semi-annual compounding. The bonds all pay on the same coupon dates of 7 June and 7 December, and as the value date is a coupon date, there is no accrued interest on any of the bonds.²⁶ The *clean* and *dirty* prices for each bond are identical.

Bond	Term to maturity (years)	Coupon	Maturity date	Price	Gross redemption yield
4% Treasury 2001	0.5	4%	07-Jun-01	100	4%
5% Treasury 2001	1	5%	07-Dec-01	100	5%
6% Treasury 2002	1.5	6%	07-Jun-02	100	6%
7% Treasury 2002	2	7%	07-Dec-02	100	7%
8% Treasury 2003	2.5	8%	07-Jun-03	100	8%
9% Treasury 2003	3	9%	07-Dec-03	100	9%

Table 6.3: Hypothetical UK government bond yields as at 7 December 2000.

The gross redemption yield or *yield-to-maturity* of a coupon bond describes the single rate that present-values the sum of all its future cash flows to its current price. It is essentially the *internal rate of return* of the set of cash flows that make up the bond. This yield measure suffers from a fundamental weakness in that each cash-flow is present-valued at the same rate, an unrealistic assumption in anything other than a flat yield curve environment. So the yield to maturity is an *anticipated* measure of the return that can be expected from holding the bond from purchase until maturity. In practice it will only be achieved under the following conditions:

- the bond is purchased on issue;
- all the coupons paid throughout the bond's life are re-invested at the same yield to maturity at which the bond was purchased;
- the bond is held until maturity.

In practice these conditions will not be fulfilled, and so the yield to maturity of a bond is not a true interest rate for that bond's maturity period.

The bonds in table 1 pay semi-annual coupons on 7 June and 7 December and have the same time period – six months – between 7 December 2000, their valuation date and 7 June 2001, their next coupon date. However since each issue carries a different yield, the next six-month coupon payment for each bond is present-valued at a different rate. In other words, the six-month bond present-values its six-month coupon payment at its 4%

²⁵ This section follows the approach described in Windas (1993), Chapter 5.

²⁶ Benchmark gilts pay coupon on a semi-annual basis on 7 June and 7 December each year, so we've used these dates for our exercise.

yield to maturity, the one-year at 5%, and so on. Because each of these issues uses a different rate to present-value a cash flow occurring at the same future point in time, it is unclear which of the rates should be regarded as the true interest rate or benchmark rate for the six-month period from 7 December 2000 to 7 June 2001. This problem is repeated for all other maturities.

For the purposes of valuation and analysis however, we require a set of true interest rates, and so these must be derived from the redemption yields that we can observe from the benchmark bonds trading in the market. These rates we designate as rs_i , where rs_i is the *implied spot rate* or *zero-coupon rate* for the term beginning on 7 December 2000 and ending at the end of period i .

We begin calculating implied spot rates by noting that the six-month bond contains only one future cash flow, the final coupon payment and the redemption payment on maturity. This means that it is in effect trading as a zero-coupon bond, as there is only one cash flow left for this bond, its final payment. Since this cash flow's present value, future value and maturity term are known, the unique interest rate that relates these quantities can be solved using the compound interest equation (6.32):

$$\begin{aligned} FV &= PV \times \left(1 + \frac{rs_i}{m}\right)^{(nm)} \\ rs_i &= m \times \left(\sqrt[nm]{\frac{FV}{PV}} - 1\right) \end{aligned} \quad (6.32)$$

where

FV	is the future value
PV	is the present value
rs_i	is the implied i -period spot rate
m	is the number of interest periods per year
n	is the number of years in the term.

The first rate to be solved is referred to as the implied six-month spot rate and is the true interest rate for the six-month term beginning on 2 January and ending on 2 July 2000.

Equation (6.32) relates a cash flow's present value and future value in terms of an associated interest rate, compounding convention and time period. Of course if we rearrange it, we may use it to solve for an implied spot rate. For the six-month bond the final cash flow on maturity is £102, comprised of the £2 coupon payment and the £100 (*par*) redemption amount. So we have for the first term, $i=1$, $FV = £102$, $PV = £100$, $n = 0.5$ years and $m = 2$. This allows us to calculate the spot rate as follows:

$$\begin{aligned} rs_i &= m \times \left(\sqrt[nm]{\frac{FV}{PV}} - 1\right) \\ rs_1 &= 2 \times \left(\sqrt[0.5 \times 2]{\frac{£102}{£100}} - 1\right) \\ rs_1 &= 0.04000 \\ rs_1 &= 4.000\%. \end{aligned}$$

Thus the implied six-month spot rate or zero-coupon rate is equal to 4 per cent.²⁷ We now need to determine the implied one-year spot rate for the term from 7 December 2000 to 7 June 2001. We note that the one-year issue has a 5% coupon and contains two future cash flows: a £2.50 six-month coupon payment on 7 June 2001 and a £102.50 one-year coupon and principal payment on 7 December 2001. Since the first cash flow occurs on 7 June – six months from now – it must be present-valued at the 4 per cent six-month spot rate established above. Once this present value is determined, it may be subtracted from the £100 total present value (its current price) of the one-year issue to obtain the present value of the one-year coupon and cash flow. Again we then have a single cash flow with a known present value, future value and term. The rate that equates these quantities is the implied one-year

²⁷ Of course intuitively we could have concluded that the six-month spot rate was 4%, without the need to apply the arithmetic, as we had already assumed that the six-month bond was a quasi-zero-coupon bond.

spot rate. From equation (4) the present value of the six-month £2.50 coupon payment of the one-year benchmark bond, discounted at the implied six-month spot rate, is:

$$\begin{aligned} PV_{6\text{-mo cash flow, 1-yr bond}} &= £2.50/(1 + 0.04/2)^{0.5 \times 2} \\ &= £2.45098. \end{aligned}$$

The present value of the one-year £102.50 coupon and principal payment is found by subtracting the present value of the six-month cash flow, determined above, from the total present value (current price) of the issue:

$$\begin{aligned} PV_{1\text{-yr cash flow, 1-yr bond}} &= £100 - £2.45098 \\ &= £97.54902. \end{aligned}$$

The implied one-year spot rate is then determined by using the £97.54902 present value of the one-year cash flow determined above:

$$\begin{aligned} rs_2 &= 2 \times \left(\sqrt[1 \times 2]{£102.50/£97.54902} - 1 \right) \\ &= 0.0501256 \\ &= 5.01256\%. \end{aligned}$$

The implied 1.5 year spot rate is solved in the same way:

$$\begin{aligned} PV_{6\text{-mo cash flow, 1.5-yr bond}} &= £3.00/(1 + 0.04/2)^{0.5 \times 2} \\ &= £2.94118 \\ PV_{1\text{-yr cash flow, 1.5-yr bond}} &= £3.00/(1 + 0.0501256/2)^{1 \times 2} \\ &= £2.85509 \\ PV_{1.5\text{-yr cash flow, 1.5-yr bond}} &= £100 - £2.94118 - £2.85509 \\ &= £94.20373 \\ rs_3 &= 2 \times \left(\sqrt[1.5 \times 2]{£103/£94.20373} - 1 \right) \\ &= 0.0604071 \\ &= 6.04071\%. \end{aligned}$$

Extending the same process for the two-year bond, we calculate the implied two-year spot rate rs_4 to be 7.0906 per cent. The implied 2.5-year and three-year spot rates rs_5 and rs_6 are 8.1709 per cent 9.2879 per cent respectively.

The interest rates rs_1 , rs_2 , rs_3 , rs_4 , rs_5 and rs_6 describe the true zero-coupon interest rates for the six-month, one-year, 1.5-year, two-year, 2.5-year and three-year terms that begin on 7 December 2000 and end on 7 June 2001, 7 December 2001, 7 June 2002, 7 December 2002, 7 June 2003 and 7 December 2003 respectively. They are also called *implied spot rates* because they have been calculated from redemption yields observed in the market from the benchmark government bonds that were listed in Table 6.3.

Note that the 1-, 1.5-, 2-year, 2.5-year and 3-year implied spot rates are progressively greater than the corresponding redemption yields for these terms. This is an important result, and occurs whenever the yield curve is positively sloped. The reason for this is that the present values of a bond's shorter-dated cash flows are discounted at rates that are lower than the bond's redemption yield; this generates higher present values that, when subtracted from the current price of the bond, produce a lower present value for the final cash flow. This lower present value implies a spot rate that is greater than the issue's yield. In an inverted yield curve environment we observe the opposite result, that is implied rates that lie below the corresponding redemption yields. If the redemption yield curve is flat, the implied spot rates will be equal to the corresponding redemption yields.

The methodology we have described to construct theoretical spot rates from observed coupon bond yields is known as bootstrapping. This method while useful, is in practice more complex. A number of problems may present themselves. For instance within the group of bonds one is using to derive the spot curve, there may be more than one issue with the same maturity but trading at different yields. As we noted elsewhere, this may be due to differing coupon rates and their tax effects, or the presence of embedded option features such as call

provisions. In addition there may not be a current bond issue for every required maturity, so that a continuous curve cannot be calculated.

■ Calculating implied forward rates

Once we have calculated the spot or zero-coupon rates for the six-month, one-year, 1.5-year, 2-year, 2.5-year and 3-year terms, we can determine the rate of return that is implied by the yield curve for the sequence of six-month periods beginning on 7 December 2000, 7 June 2001, 7 December 2001, 7 June 2002 and 7 December 2002. These period rates are referred to as *implied forward rates* or *forward-forward rates* and we denote these as rf_i , where rf_i is the implied six-month forward interest rate today for the i th period.

Since the implied six-month zero-coupon rate (spot rate) describes the return for a term that coincides precisely with the first of the series of six-month periods, this rate describes the risk-free rate of return for the first six-month period. It is therefore equal to the first period spot rate. Thus we have $rf_1 = rs_1 = 4.0$ per cent, where rf_1 is the risk-free forward rate for the first six-month period beginning at period 1. We show now how the risk-free rates for the second, third, fourth, fifth and sixth six-month periods, designated rf_2 , rf_3 , rf_4 , rf_5 and rf_6 respectively may be solved from the implied spot rates.

The benchmark rate for the second semi-annual period rf_2 is referred to as the one-period forward six-month rate, because it goes into effect one six-month period from now (“one-period forward”) and remains in effect for six months (“six-month rate”). It is therefore the six-month rate in six months’ time, and is also referred to as the six-month forward-forward rate. This rate, in conjunction with the rate from the first period rf_1 , must provide returns that match those generated by the implied one-year spot rate for the entire one-year term. In other words, one pound invested for six months from 7 December 2000 to 7 June 2001 at the first period’s benchmark rate of 4 per cent and then reinvested for another six months from 7 June 2001 to 7 December 2001 at the second period’s (as yet unknown) implied *forward* rate must enjoy the same returns as one pound invested for one year from 7 December 2000 to 7 December 2001 at the implied one-year *spot* rate of 5.0125 per cent. This reflects the law of no-arbitrage.

A moment’s thought will convince us that this must be so. If this were not the case, we might observe an interest rate environment in which the return received by an investor over any given term would depend on whether an investment is made at the start period for the entire maturity term or over a succession of periods within the whole term and re-invested. If there were any discrepancies between the returns received from each approach, there would exist an unrealistic arbitrage opportunity, in which investments for a given term carrying a lower return might be sold short against the simultaneous purchase of investments for the same period carrying a higher return, thereby locking in a risk-free, cost-free profit. Therefore forward interest rates must be calculated so that they are *arbitrage-free*. Forward rates are not therefore a prediction of what spot interest rates are likely to be in the future, rather a mathematically derived set of interest rates that reflect the current spot term structure and the rules of no-arbitrage. Excellent mathematical explanations of the no-arbitrage property of interest-rate markets are contained in Ingersoll (1987), Jarrow (1996), and Shiller (1990) among others.

It is important to remember that our discussion centres on forward rates calculated for dealing *today*. If one wishes to enter into a bargain for a forward date but write the ticket today, we assume the principle of no-arbitrage pricing. If an actual strategy of investment for a shorter term followed by reinvestment at the actual *spot* rate (in place at the time) for the remainder of the term was followed, the final return would of course be different. We have no way of knowing what the actual result would be of dealing always at spot rates (or, if we did, we would not be here now!). But if we wish to deal today for a period that begins at a future date, there will be no difference if we follow a strategy of investing for the whole term or if we decide on reinvesting at a later date.

The existence of a no-arbitrage market of course makes it straightforward to calculate forward rates; we know that the return from an investment made over a period must equal the return made from investing in a shorter period and successively reinvesting to a matching term. If we know the return over the shorter period, we are left with only one unknown, the full-period forward rate, which is then easily calculated.

Returning to our example then, having established the rate for the first six-month period, the rate for the second six-month period – the one-period forward six-month rate – is determined below.

The future value of £1 invested at rf_1 , the period 1 forward rate, at the end of the first six-month period is

calculated as follows:

$$\begin{aligned} FV_1 &= £1 \times \left(1 + \frac{rf_1}{2}\right)^{(0.5 \times 2)} \\ &= £1 \times \left(1 + \frac{0.04}{2}\right)^1 \\ &= £1.02000. \end{aligned}$$

The future value of £1 at the end of the one-year term, invested at the implied benchmark one-year spot rate is determined as follows:

$$\begin{aligned} FV_2 &= £1 \times \left(1 + \frac{rs_2}{2}\right)^{(1 \times 2)} \\ &= £1 \times \left(1 + \frac{0.0501256}{2}\right)^2 \\ &= £1.050754. \end{aligned}$$

The implied benchmark one-period forward rate rf_2 is the rate that equates the value of FV_1 (£1.02) on 7 June 2001 to FV_2 (£1.050754) on 7 December 2001. From equation (4) we have:

$$\begin{aligned} rf_2 &= 2 \times \left(\sqrt[0.5 \times 2]{\frac{FV_2}{FV_1}} - 1 \right) \\ &= 2 \times \left(\sqrt[0.5 \times 2]{\frac{£1.050754}{£1.02}} - 1 \right) \\ &= 0.060302 \\ &= 6.0302\%. \end{aligned}$$

In other words, £1 invested from 7 December to 7 June at 4.0 per cent (the implied forward rate for the first period) and then reinvested from 7 June to 7 December 2001 at 6.0302 per cent (the implied forward rate for the second period) would accumulate the same returns as £1 invested from 7 December 2000 to 7 December 2001 at 5.01256 per cent (the implied one-year spot rate).

The rate for the third six-month period – the two-period forward six-month interest rate – may be calculated in the same way:

$$\begin{aligned} FV_2 &= £1.050754 \\ FV_3 &= £1 \times \left(1 + \frac{rs_3}{2}\right)^{1.5 \times 2} \\ &= £1 \times \left(1 + \frac{0.0604071}{2}\right)^3 \\ &= £1.093375 \\ rf_3 &= 2 \times \left(\sqrt[0.5 \times 2]{\frac{FV_3}{FV_2}} - 1 \right) \\ &= 2 \times \left(\sqrt[0.5 \times 2]{\frac{£1.093375}{£1.050754}} - 1 \right) \\ &= 0.081125 \\ &= 8.1125\%. \end{aligned}$$

In the same way the three-period forward six-month rate rf_4 is calculated to be 10.27247 per cent. The rest of the results are shown in Table 6.4. We say *one-period* forward rate because it is the forward rate that applies to the six-month period. The results of the implied spot (zero-coupon) and forward rate calculations along with the given redemption yield curve are illustrated graphically in Figure 6.15.

The simple bootstrapping methodology can be applied using a spreadsheet for actual market redemption yields. However in practice we will not have a set of bonds with exact and/or equal periods to maturity and

coupons falling on the same date. Nor will they all be priced conveniently at par. In designing a spreadsheet spot rate calculator therefore, the coupon rate and maturity date is entered as standing data and usually interpolation is used when calculating the spot rates for bonds with uneven maturity dates. A spot curve model that uses this approach in conjunction with the boot-strapping method is available for downloading at

<http://www.mchoudhry.co.uk>

Market practitioners usually use discount factors to extract spot and forward rates from market prices. For an account of this method, see the chapter by Dr Didier Joannas in Choudhry *et al.* (2001).

Term to maturity	Yield to maturity	Implied spot rate	Implied one-period forward rate
0.5	4.0000%	4.00000%	4.00000%
1	5.0000%	5.01256%	6.03023%
1.5	6.0000%	6.04071%	8.11251%
2	7.0000%	7.09062%	10.27247%
2.5	8.0000%	8.17090%	12.24833%
3	9.0000%	9.28792%	14.55654%

Table 6.4: Implied spot and forward rates.

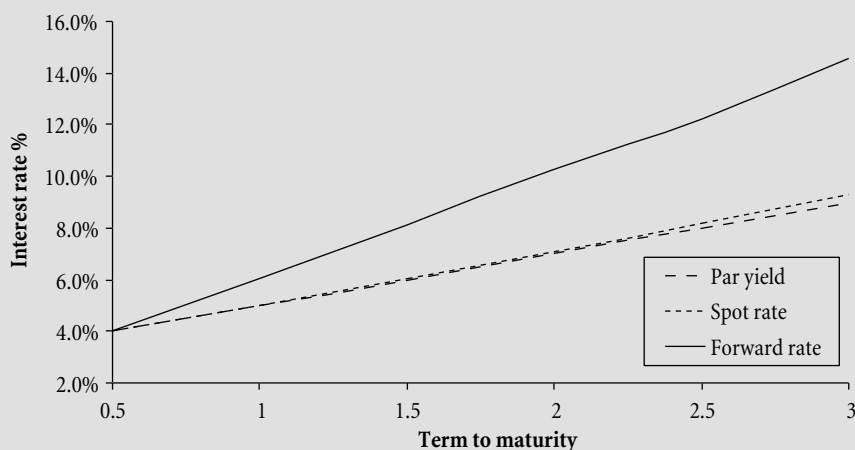


Figure 6.15: Par, spot and forward yield curves.

6.12 Examples, exercises and calculations

EXAMPLE 6.6 Forward rates: breakeven principle

■ Consider the following spot yields:

1-year 10%
2-year 12%

Assume that a bank's client wishes to lock in *today* the cost of borrowing 1-year funds in one year's time. The solution for the bank (and the mechanism to enable the bank to quote a price to the client) involves raising 1-year funds at 10% and investing the proceeds for two years at 12%. As we observed in Example 6.5 the no-arbitrage principle means that the same return must be generated from both fixed rate and reinvestment strategies.

In effect, we can look at the issue in terms of two alternative investment strategies, both of which must provide the same return.

Strategy 1: Invest funds for two years at 12%

Strategy 2: Invest funds for one year at 10%, and re-invest the proceeds for a further year at the forward rate calculated today.

The forward rate for strategy 2 is the rate that will be quoted to the client. Using the present value relationship we know that the proceeds from strategy 1 are:

$$FV = (1 + r_2)^2$$

while the proceeds from strategy 2 would be:

$$FV = (1 + r_1) \times (1 + R).$$

We know from the no-arbitrage principle that the proceeds from both strategies will be the same, therefore this enables us to set:

$$\begin{aligned} (1 + r_2)^2 &= (1 + r_1)(1 + R) \\ \therefore R &= \frac{(1 + r_2)^2}{(1 + r_1)} - 1. \end{aligned}$$

This enables us to calculate the forward rate that can be quoted to the client (together with any spread that the bank might add) as follows:

$$\begin{aligned} (1 + 0.12)^2 &= (1 + 0.10) \times (1 + R) \Rightarrow (1 + R) = (1 + 0.12)^2 / (1 + 0.10) \\ (1 + R) &= 1.14036 \\ R &= 14.04\%. \end{aligned}$$

This rate is the one-year forward-forward rate, or the implied forward rate.

EXAMPLE 6.7

■ If a 1-year AAA Eurobond trading at par yields 10% and a 2-year Eurobond of similar credit quality, also trading at par, yields 8.75%, what should be the price of a 2-year AAA zero-coupon bond? Note that Eurobonds pay coupon annually.

1. Cost of 2-year bond (per cent nominal):
= 100.
2. *less* amount receivable from sale of first coupon on this bond (that is, its present value):
= $8.75 / (1 + 0.10)$
= 7.95.
3. *equals* amount that must be received on sale of second coupon plus principal in order to break-even:
= 92.05.
4. calculate the yield implied in the cash flows below (that is, the 2-year zero-coupon yield):
 - receive 92.05;
 - pay out on maturity 108.75.

Therefore, solving $92.05 = 108.75 / (1 + R)^2$ gives R equal to 8.69%.

5. What is the price of a 2-year zero-coupon bond with nominal value 100, to yield 8.69%?
= $(92.05 / 108.75) \times 100$
= 84.84

EXAMPLE 6.8

■ A highly-rated customer asks you to fix a yield at which he could issue a 2-year zero-coupon USD Eurobond in three years' time. At this time the US Treasury zero-coupon rates were:

1 Yr	6.25%
2 Yr	6.75%
3 Yr	7.00%
4 Yr	7.125%
5 Yr	7.25%

1. Ignoring borrowing spreads over these benchmark yields, as a market maker you could cover the exposure created by borrowing funds for 5 years on a zero-coupon basis and placing these funds in the market for 3 years before lending them on to your client. Assume annual interest compounding (even if none is actually paid out during the life of the loans).

Borrowing rate for 5 years $R_5/100 = 0.0725$

Lending rate for 3 years $R_3/100 = 0.0700$

2. The key arbitrage relationship is:

Total cost of funding = Total Return on Investments

$$(1 + R_5)^5 = (1 + R_3)^3 \times (1 + R_{3 \times 5})^2.$$

Therefore the break-even forward yield is:

$$\begin{aligned} (1 + R_{3 \times 5})^2 &= \frac{(1 + 0.0725)^5}{(1 + 0.0700)^3} \Rightarrow (1 + R_{3 \times 5}) = \sqrt{\frac{(1 + 0.0725)^5}{(1 + 0.0700)^3}} \\ R_{3 \times 5} &= \sqrt{\frac{(1 + 0.0725)^5}{(1 + 0.0700)^3}} - 1 \\ &= 7.63\%. \end{aligned}$$

EXAMPLE 6.9 Forward rate calculation for money market term

Consider two positions:

- borrowing of £100 million from 2 January 2000 for 30 days at 6.500%;
- loan of £100 million from 2 January for 60 days at 6.625%.

The two positions can be said to be a 30-day forward 30-day interest rate exposure (a 30- versus 60-day forward rate). It is usually referred to as an interest-rate *gap* exposure. What forward rate must be used if the trader wished to hedge this exposure, assuming no bid-offer spreads?

The 30-day by 60-day forward rate can be calculated using the following formula:

$$rf_i = \left(\frac{1 + \left(rs_L \times \frac{n_L}{B} \right)}{1 + \left(rs_S \times \frac{n_S}{B} \right)} - 1 \right) \times \frac{B}{n_L - n_S} \quad (6.33)$$

where

rf_i	is the forward rate
rs_L	is the % long period rate
rs_S	is the % short period rate
n_L	is the long period days
n_S	is the short period days
B	is the day-count base, either 360 or 365.

Using this formula we obtain a 30v60 day forward rate of 6.713560%.

This interest rate exposure can be hedged using interest rate futures or Forward Rate Agreements (FRAs). Either method is an effective hedging mechanism, although the trader must be aware of:

- *basis* risk that exists between interbank deposit rates and the forward rates implied by futures and FRAs;
- date mismatches between expiry of futures contracts and the maturity dates of the deposit transactions.

■ Forward rates and compounding

Example 6.8 is for a forward rate calculation more than one year into the future, and therefore the formula used must take compounding of interest into consideration. Example 6.9 is for a forward rate within the next 12 months, with one-period bullet interest payments. A different formula is required to allow for this and is shown in the example, to adjust the interest rates quoted for the money market (below one-year) maturities involved

■ Understanding forward rates

Spot and forward rates that are calculated from current market rates follow mathematical principles to establish what the market believes the arbitrage-free rates for dealing *today* at rates that are effective at some point in the future. As such forward rates are a type of market view on where interest rates will be (or should be!) in the future. As we have already noted however, forward rates are not a prediction of future rates. It is important to be aware of this distinction. If we were to plot the forward rate curve for the term structure in three months' time, and then compare it in three months with the actual term structure prevailing at the time, the curves would almost certainly not match. However this has no bearing on our earlier statement, that forward rates are the market's *expectation* of future rates. The main point to bear in mind is that we are not comparing like-for-like when plotting forward rates against actual current rates at a future date. When we calculate forward rates, we use the current term structure. The current term structure incorporates all known information, both economic and political, and reflects the market's views. This is exactly the same as when we say that a company's share price reflects all that is known about the company and all that is expected to happen with regard to the company in the near future, including expected future earnings. The term structure of interest rates reflects everything the market knows about relevant domestic and international factors. It is this information then, that goes into the forward rates calculation. In three months' time though, there will be new developments that will alter the market's view and therefore alter the current term structure; these developments and events were (by definition, as we cannot know what lies in the future!) not known at the time we calculated and used the three-month forward rates. This is why rates actually turn out to be different from what the term structure predicted at an earlier date. However for dealing today we use today's forward rates, which reflect everything we know about the market today.

6.13 Case Study: Deriving a discount function²⁸

In this example we present a traditional bootstrapping technique for deriving a discount function for yield curve fitting purposes. This technique has been called "naïve" (for instance see James and Webber (2000), page 129) because it suffers from a number of drawbacks, for example it results in an unrealistic forward rate curve, which means that it is unlikely to be used in practice. We review the drawbacks at the end of the case study; more advanced techniques are considered in Part VIII.

- Today is 14 July 2000. The following rates are observed in the market. We assume that the day-count basis for the cash instruments and swaps is act/365. Construct the money market discount function.

<i>Money Market rates</i>	<i>Rate (%)</i>	<i>Expiry</i>	<i>Days</i>
One month (1m)	4 7/32	14/8/00	31
3m	4 1/4	16/10/00	94
6m	4 1/2	15/1/01	185

²⁸ In this illustration, the discount function is derived using interest rate data from two off-balance instruments, futures and swaps, as well as money market deposit rates. Derivative instruments are covered in later chapters.

Future prices

Sep-00	95.60	20/9/00	68
Dec-00	95.39	20/12/00	159
Mar-01	95.25	21/3/01	249
Jun-01	94.80	20/6/01	340

Swap rates

One year (1y)	4.95	16/7/01	367
2y	5.125	15/7/02	731
3y	5.28	14/7/03	1095
4y	5.55	14/7/04	1461
5y	6.00	14/7/05	1826

Creating the discount function

Using the cash money market rates we can create discount factors up to a maturity of six months, using the expression at (6.34):

$$df = \frac{1}{\left(1 + r \times \frac{\text{days}}{365}\right)}. \quad (6.34)$$

The resulting discount factors are shown below.

From	To	Days	r%	df
14/7/00	14/8/00	31	4 7/32	0.99642974
	16/10/00	94	4 1/4	0.98917329
	15/1/01	185	4 1/2	0.97770040

We can also calculate forward discount factors from the rates implied in the futures prices, which are shown below.

From	To	Days	r%	df
20/9/00	20/12/00	91	4.40	0.98914917
20/12/00	21/3/01	91	4.61	0.98863717
21/3/01	20/6/01	91	4.75	0.98829614
20/6/01	19/9/00	91	5.20	0.98720154

In order to convert these values into zero-coupon discount factors, we need to first derive a cash “stub” rate up to the expiry of the first futures contract. The most straightforward way to do this is by linear interpolation of the one-month and three-month rates, as shown in Figure 6.16 below.

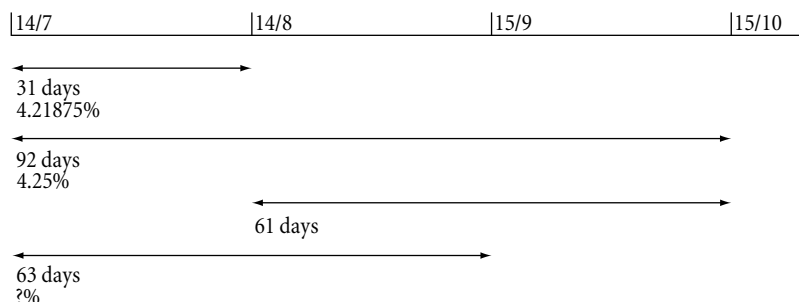


Figure 6.16: Linear interpolation of money market and futures rates.

For instance, the calculation for the term marked is

$$4.21875 + \left((4.25 - 4.21875) \times \frac{32}{61}\right) = 4.235143\%.$$

To convert this to a discount factor:

$$\frac{1}{1 + (0.04235143 \times \frac{63}{365})} = 0.99274308.$$

From the futures implied forward rates, the zero-coupon discount factors are calculated by successive multiplication of the individual discount factors. These are shown below.

<i>From</i>	<i>To</i>	<i>Days</i>	<i>df</i>
14/7/00	0/9/00	68	0.99172819
	20/12/00	159	0.98172542
	21/3/01	250	0.96992763
	20/6/01	341	0.960231459
	19/9/01	432	0.948925494

For the interest-rate swap rates, to calculate discount factors for the relevant dates we use the boot-strapping technique.

■ **1y swap:**

We assume a par swap, the present value is known to be 100, and as we know the future value as well, we are able to calculate the one-year zero-coupon rate as shown from the one-year swap rate,

$$\begin{aligned} df_1 &= \frac{1}{1 + r} = \frac{100}{104.95} \\ &= 0.95283468. \end{aligned}$$

■ **2y swap:**

The coupon payment occurring at the end of period one can be discounted back using the one-year discount factor above, leaving a zero-coupon structure as before.

$$df_2 = \frac{100 - C \times Df_1}{105.125}.$$

This gives df_2 equal to 0.91379405.

The same process can be employed for the three, four and five-year par swap rates to calculate the appropriate discount factors.

$$df_3 = \frac{100 - C \times (df_1 + df_2)}{105.28}.$$

This gives df_3 equal to 0.87875624. The discount factors for the four-year and five-year maturities, calculated in the same way, are 0.82899694 and 0.77835621 respectively.

The full discount function is given in Table 6.5 and illustrated graphically at Figure 6.17.

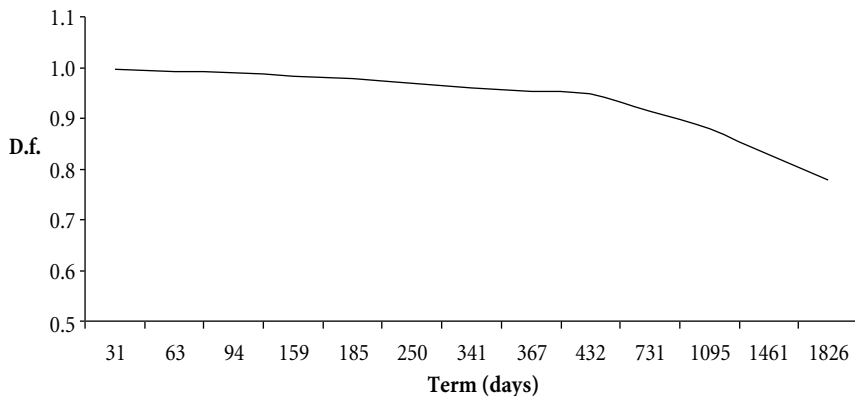


Figure 6.17: Discount function.

From	To	Days	Zero-coupon (%)	Discount factor	Source
14/07/2000	14/08/2000	31	4.21875	0.99642974	Money market
	20/09/2000	63	4.23500	0.99274308	Money market
	16/10/2000	94	4.25000	0.98917329	Money market
	20/12/2000	159	4.38000	0.98172542	Futures
	15/01/2001	185	4.50000	0.97777004	Money market
	21/03/2001	250	4.55000	0.96992763	Futures
	20/06/2001	341	4.73000	0.96023145	Futures
	16/07/2001	367	4.95000	0.95283468	Swap
	19/09/2001	432	5.01000	0.94892549	Futures
	15/07/2002	731	5.12500	0.91379405	Swap
	14/07/2003	1095	5.28000	0.87875624	Swap
	15/07/2004	1461	5.58000	0.82899694	Swap
	15/07/2005	1826	6.10000	0.77835621	Swap

Table 6.5: Discount factors.

6.13.1 Critique of the traditional technique

The method used to derive the discount function in the case study used three different price sources to produce an integrated function and hence yield curve. However there is no effective method by which the three separate curves, which are shown at Figure 6.18, can be integrated into one complete curve. The result is that a curve formed from the three separate curves will exhibit distinct kinks or steps at the points at which one data source is replaced by another data source.

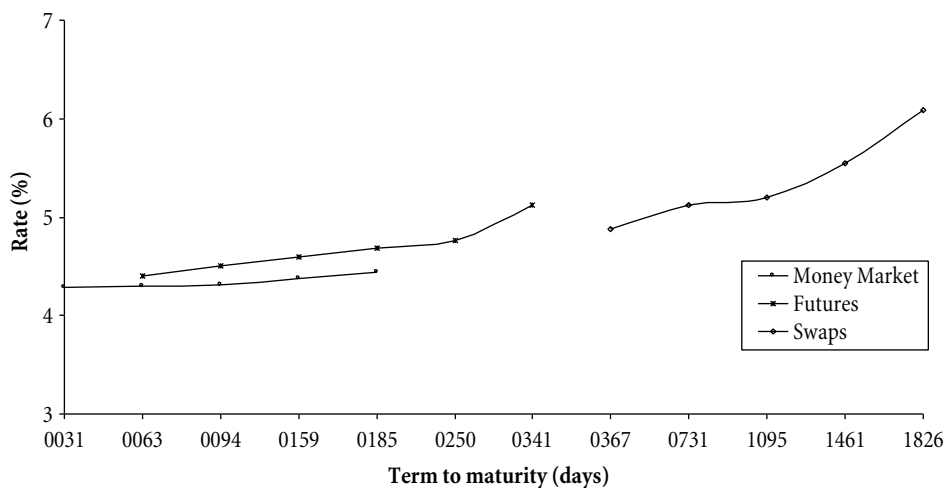


Figure 6.18: Comparison of money market curves.

The money market and swap rates incorporate a credit risk premium, reflecting the fact that interbank market counterparties carry an element of default risk. This means that money market rates lie above government repo rates. Futures rates do not reflect default risk, as they are exchange-traded contracts and the exchange clearing house takes on counterparty risk for each transaction. However futures rates are treated as one-point period rates, in effect making them equivalent to forward-rate agreement (FRA) rates. In practice, as the cash flow from FRAs is received as a discounted payoff at one point, whereas futures contract trades require a daily margin payment, a *convexity adjustment* is required to convert futures accurately to FRA rates. This adjustment is considered in another chapter.

Swap rates also incorporate an element of credit risk, although generally they are considered lower risk as they are off-balance sheet instruments and no principal is at risk. As liquid swap rates are only available for set maturity points, linear interpolation is used to plot points in between available rates. This results in an unstable forward rate curve calculated from the spot rate curve (see James and Webber 2000), due to the interpolation effect. Nevertheless market makers in certain markets price intermediate-dated swaps based on this linear interpolation method. Another drawback is that the bootstrapping method uses near-maturity rates to build up the curve to far-maturity rates. One of the features of a spot curve derived in this way is that even small changes in short-term rates causes excessive changes in long-dated spot rates, and oscillations in the forward curve. Finally, money market rates beyond the “stub” period are not considered once the discount factor to the stub date is calculated, so their impact is not felt.

For these reasons the traditional technique, while still encountered in textbooks and training courses (including this one), is not used very often in the markets.

6.14 Case Study exercise: Deriving the theoretical zero-coupon (spot) rate curve²⁹

Here we construct a theoretical spot interest rate curve using the yields observed on coupon bonds. To illustrate the methodology, we will use a hypothetical set of bonds that are trading in a positive yield curve environment. The maturity, price and yield for ten bonds are shown in Table 6.6. The prices shown are for settlement on 1 March 1999, and all the bonds have precisely 1, 1.5, 2 and so on years to maturity, that is they mature on 1 March or 1 September of their maturity year. That means that the first bond has no intermediate coupon before it is redeemed, and we can assume it trades as a zero-coupon bond. All bonds have no accrued interest because the settlement date is a coupon date. Bonds pay semi-annual coupon.

Maturity date	Years to maturity	Coupon (%)	Yield to maturity	Price
1-Sep-99	0.5	5.0	6.00	99.5146
1-Mar-00	1.0	10.0	6.30	103.5322
1-Sep-00	1.5	7.0	6.40	100.8453
1-Mar-01	2.0	6.5	6.70	99.6314
1-Sep-01	2.5	8.0	6.90	102.4868
1-Mar-02	3.0	10.5	7.30	108.4838
1-Sep-02	3.5	9.0	7.60	104.2327
1-Mar-03	4.0	7.3	7.80	98.1408
1-Sep-03	4.5	7.5	7.95	98.3251
1-Mar-04	5.0	8.0	8.00	100.0000

Table 6.6: Set of hypothetical bonds.

According to the principle of no-arbitrage pricing, the value of any bond should be equal to the value of the sum of all its cash flows, should these be stripped into a series of zero-coupon bonds whose last bond matures at the same time as the coupon bond. Consider the first bond in the table above. As it matures in precisely six months' time, it is effectively a zero-coupon bond; its yield of 6% is equal to the six-month spot rate. Given this spot rate we can derive the spot rate for a one-year zero-coupon gilt. The price of a one-year gilt strip must equal the present value of the two cash flows from the 10% one-year coupon gilt. If we use £100 as par, the cash flows from the one-year coupon bond are:

$$0.5 \text{ years} \quad 10\% \times £100 \times 0.5 = £5$$

$$1.0 \text{ years} \quad 10\% \times £100 \times 0.5 = £5 + £100 \text{ redemption payment.}$$

²⁹ This illustrates bootstrapping, as well as the principle behind stripping a coupon bond, and follows the approach described in Fabbozi (1993).

The present value of the total cash flow is $\frac{5}{(1 + \frac{1}{2}r_1)} + \frac{105}{(1 + \frac{1}{2}r_2)^2}$ where

r_1 is the six-month theoretical spot rate

r_2 is the one-year theoretical spot rate.

Therefore the present value of the one-year coupon gilt is $5/(1.03) + 105/(1 + \frac{1}{2}r_2)^2$. As the price of the one-year gilt is 103.5322, from the table above, the following relationship must be true:

$$103.5322 = \frac{5}{(1.03)} + \frac{105}{(1 + \frac{1}{2}r_2)^2}.$$

Using this relationship we are now in a position to calculate the one-year theoretical spot rate as shown below.

$$\begin{aligned} 103.5322 &= 4.85437 + \frac{105}{(1 + \frac{1}{2}r_2)^2} \\ 98.67783 &= 105/(1 + \frac{1}{2}r_2)^2 \\ (1 + \frac{1}{2}r_2)^2 &= 105/98.67783 = 1.064069 \\ (1 + \frac{1}{2}r_2) &= \sqrt{1.064069} \\ \frac{1}{2}r_2 &= 0.03154. \end{aligned}$$

Therefore r_2 is 0.06308, or 6.308%, which is the theoretical one-year spot rate. Now that we have obtained the theoretical one-year spot rate, we are in a position to calculate the theoretical 1.5-year spot rate. The cash flow for the 7% 1.5-year coupon gilt shown in the table is:

$$\begin{aligned} 0.5 \text{ years} & \quad 7\% \times £100 \times 0.5 = £3.5 \\ 1.0 \text{ years} & \quad 7\% \times £100 \times 0.5 = £3.5 \\ 1.5 \text{ years} & \quad 7\% \times £100 \times 0.5 = £3.5 + \text{redemption payment (£100)}. \end{aligned}$$

The present value of this stream of cash flows is:

$$\frac{3.5}{(1 + \frac{1}{2}r_1)} + \frac{3.5}{(1 + \frac{1}{2}r_2)^2} + \frac{103.5}{(1 + \frac{1}{2}r_3)^3}$$

where r_3 is the 1.5-year theoretical spot rate. We have established that the six-month and one-year spot rates are 6% and 6.308% respectively, so that r_1 is 0.06 and r_2 is 0.06308. Therefore the present value of the 7% 1.5-year coupon gilt is:

$$\frac{3.5}{(1.03)} + \frac{3.5}{(1.03154)^2} + \frac{103.5}{(1 + \frac{1}{2}r_3)^3}.$$

From the table the price of the 7% 1.5-year gilt is 100.8453; therefore the following relationship must be true:

$$100.8453 = \frac{3.5}{(1.03)} + \frac{3.5}{(1.03154)^2} + \frac{103.5}{(1 + \frac{1}{2}r_3)^3}.$$

This equation can then be solved to obtain r_3 :

$$\begin{aligned} 100.8453 &= 3.39806 + 3.28924 + 103.5/(1 + \frac{1}{2}r_3)^3 \\ 94.158 &= 103.5/(1 + \frac{1}{2}r_3)^3 \\ (1 + \frac{1}{2}r_3)^3 &= 1.099216 \\ \frac{1}{2}r_3 &= 0.032035. \end{aligned}$$

The theoretical 1.5-year spot rate bond equivalent yield is two times this result which is 6.407%.

6.14.1 For readers to complete...

Calculate the spot rates for the remaining terms in the table.

6.14.2 Mathematical relationship

The relationship used to derive a theoretical spot rate can be generalised, so that in order to calculate the theoretical spot rate for the n th six-month period, we use the following expression:

$$P_n = \frac{C/2}{(1 + \frac{1}{2}r)} + \frac{C/2}{(1 + \frac{1}{2}r_2)^2} + \frac{C/2}{(1 + \frac{1}{2}r_3)^3} + \dots + \frac{C/2 + 100}{(1 + \frac{1}{2}r_n)^n}$$

where

P_n is the dirty price of the coupon bond with n periods to maturity
 C is the annual coupon rate for the coupon bond
 r_n is the theoretical n -year spot rate.

We can re-write this expression as

$$P_n = \frac{C}{2} \sum_{t=1}^{n-1} \frac{1}{(1 + \frac{1}{2}r_t)^t} + \frac{C/2 + 100}{(1 + \frac{1}{2}r_n)^n}$$

where r_t for $t = 1, 2, \dots, n-1$ is the theoretical spot rates that are already known. This equation can be rearranged so that we may solve for r_n :

$$r_n = \left(\frac{C/2 + 100}{P_n - \frac{C}{2} \sum_{t=1}^{n-1} \frac{1}{(1 + \frac{1}{2}r_t)^t}} \right)^{\frac{1}{n}} - 1.$$

The methodology used here is the *bootstrapping* technique.

If we carry on the process for the bonds in the table we obtain the results shown in Table 6.7.

Maturity date	Years to maturity	Yield to maturity (%)	Theoretical spot rate (%)
1-Sep-99	0.5	6.00	6.000
1-Mar-00	1.0	6.30	6.308
1-Sep-00	1.5	6.40	6.407
1-Mar-01	2.0	6.70	6.720
1-Sep-01	2.5	6.90	6.936
1-Mar-02	3.0	7.30	7.394
1-Sep-02	3.5	7.60	7.712
1-Mar-03	4.0	7.80	7.908
1-Sep-03	4.5	7.95	8.069
1-Mar-04	5.0	8.00	8.147

Table 6.7: Theoretical spot rates.

6.14.3 Implied forward rates

We now use the theoretical spot rate curve to infer the market's expectations of future interest rates. Consider the following, where an investor with a one-year time horizon has the following two investment options:

- Option 1: Buy the one-year bond

- Option 2: Buy the six-month bond, and when it matures in six months buy another six-month bond

The investor will be indifferent between the two alternatives if they produce the same yield at the end of the one-year period. The investor knows the spot rate on the six-month bond and the one-year bond, but does not know what yield will be available on a six-month bond purchased six months from now. The yield on a six-month bond six months from now is the *forward rate*. Given the spot rate for the six-month bond and the one-year bond spot yield, the forward rate on a six-month bond that will make the investor indifferent to the two alternatives can be derived from the spot curve, shown below.

By investing in a one-year zero-coupon bond, the investor will receive the maturity value at the end of one year. The redemption proceeds of the one-year zero-coupon bond is £105. The price (cost) of this bond is:

$$105 / (1 + \frac{1}{2}r_2)^2$$

where r_2 is half the bond-equivalent yield of the theoretical one-year spot rate.

Suppose that the investor purchases a six-month gilt for P pounds. At the end of the six months the value of this investment would be $P(1 + \frac{1}{2}r_1)$ where r_1 is the bond-equivalent yield of the theoretical six-month spot rate.

Let f be the forward rate on a six-month bond available six months from now. The future value of this bond in one year from the $£P$ invested is given by $P(1 + \frac{1}{2}r_1)(1 + f)$.

How much would we need to invest to get £105 one year from now? This is found as follows,

$$P(1 + \frac{1}{2}r_1)(1 + f) = 105.$$

Solving this expression gives us:

$$P = \frac{105}{(1 + \frac{1}{2}r_1)(1 + f)}.$$

The investor is indifferent between the two methods if they receive £105 from both methods in one year's time. That is, the investor is indifferent if:

$$\frac{105}{(1 + \frac{1}{2}r_2)^2} = \frac{105}{(1 + \frac{1}{2}r_1)(1 + f)}$$

$$\text{Solving for } f \text{ gives us } f = \frac{(1 + \frac{1}{2}r_2)^2}{(1 + \frac{1}{2}r_1)} - 1.$$

Therefore f gives us the bond-equivalent rate for the six-month forward rate. We can illustrate this by using the spot rates from the bond table.

$$\text{Six-month bond spot rate} = 6\% \Rightarrow \frac{1}{2}r_1 = 0.03$$

$$\text{One-year bond spot rate} = 6.308\% \Rightarrow \frac{1}{2}r_2 = 0.03154.$$

$$\text{Substituting these values into the equation gives us } f = \frac{(1.03154)^2}{(1.03)} - 1 = 0.0330823.$$

We double this result to give the forward rate on a six-month bond as 6.6165%. As we use theoretical spot rates in its calculation, the resulting forward rate is called the *implied forward rate*.

We can use the same methodology to determine the implied forward rate six months from now for an investment period longer than six months. We can also look at forward rates that start more than six months from now.

We use the following notation for forward rates: ${}_nf_t$ = the forward rate n periods from now for t periods. We can use the following equation when calculating forward rates where the final maturity is one year or more from now,

$${}_nf_t = \left(\frac{(1 + \frac{1}{2}r_{n+t})^{n+t}}{(1 + \frac{1}{2}r_n)^n} \right)^{1/t} - 1$$

where r_n is the spot rate. The result ${}_nf_t$ gives us the implied forward rate on a bond-equivalent basis.

Complete the six-month forward rates in the table below.

Maturity date	Years to maturity	Yield to maturity (%)	Theoretical spot rate (%)	Forward rate (%)
1-Sep-99	0.5	6.00	6.000	–
1-Mar-00	1.0	6.30	6.308	–
1-Sep-00	1.5	6.40	6.407	–
1-Mar-01	2.0	6.70	6.720	7.133%
1-Sep-01	2.5	6.90	6.936	–
1-Mar-02	3.0	7.30	7.394	8.755%
1-Sep-02	3.5	7.60	7.712	–
1-Mar-03	4.0	7.80	7.908	9.465%
1-Sep-03	4.5	7.95	8.069	–
1-Mar-04	5.0	8.00	8.147	9.108%

Table 6.8: Forward rates.

Appendices

APPENDIX 6.1 Testing the unbiased expectations hypothesis

For empirical studies testing the unbiased expectations hypothesis see Kessel (1965) and Fama (1976). If we consider the expectations hypothesis to be true then the forward rate ${}_0rf_2$ should be an accurate predictor of the spot rate in period 2. Put another way, the mean of ${}_0rf_2$ should be equal to the mean of ${}_1rs_1$. In previous studies (*ibid.*) it has been shown that forward rates are in fact biased upwards in their estimates of future spot rates. That is, ${}_0rf_2$ is usually higher than the mean of ${}_1rs_1$. This bias tends to be magnified the further one moves along the term structure. We can test the unbiased expectations hypothesis by determining if the following condition holds:

$${}_1rs_1 = p + q({}_0rf_2). \quad (6.35)$$

In an environment where we uphold the expectations hypothesis, then p should be equal to zero and q equal to one. Outside of the very short end of the yield curve, there is no evidence that this is true. Another approach, adopted by Fama (1984), involved subtracting the current spot rate rs_1 from both sides of equation (6.36) and testing whether:

$${}_1rs_1 - {}_0rs_1 = p + q({}_0rf_2 - {}_0rs_1). \quad (6.36)$$

If the hypothesis were accurate, we would again have p equal to zero and q equal to one. This is because ${}_1rs_1 - {}_0rs_1$ is the change in the spot rate predicted by the hypothesis. The left-hand side of (6.36) is the actual change in the spot rate, which must equal the right-hand side of the equation if the hypothesis is true. Evidence from the earlier studies mentioned has suggested that q is a positive number less than one. This of course is not consistent with the unbiased expectations hypothesis. However the studies indicate that the prediction of changes in future spot rates is linked to actual changes that occur. This suggests then that forward rates are indeed based on the market's view of future spot rates, but not in a completely unbiased manner.

An earlier study was conducted by Meiselman (1962), referred to as his error-learning model. According to this, if the unbiased expectations hypothesis is true, forward rates are not then completely accurate forecasts of future spot rates. The study tested whether (6.37) was true.

$${}_1rf_n - {}_0rf_n = p + q({}_1rs_1 - {}_0rf_2). \quad (6.37)$$

If the hypothesis is true then p should be equal to zero and q should be positive. The error-learning model suggests a positive correlation between forward rates, but this would hold in an environment where the unbiased expectations hypothesis did not apply.

The empirical evidence suggests that the predictions of future spot rates reflected in forward rates is related to subsequent actual spot rates. So forward rates do include an element of market interest rate forecasts. However this would indicate more a biased expectations theory, rather than the pure unbiased expectations hypothesis.

APPENDIX 6.2 Jensen's inequality and the shape of the yield curve

In Cox, Ingersoll and Ross (1981) an analysis on the shape of the term structure used a feature known as *Jensen's inequality* to illustrate that the expectations hypothesis was consistent with forward rates being an indicator of future spot rates. Jensen's inequality states that the expected value of the reciprocal of a variable is not identical to the reciprocal of the expected value of that variable. Following this, the if the expected holding period returns on a set of bonds are all equal, the expected holding period returns on the bonds cannot then be equal over any other holding period. Applying this in practice, consider two zero-coupon bonds, a one-year bond with a yield of 11.11% and a two-year zero-coupon bond with a yield of 11.8034%. The prices of the bonds are as follows:

1 year: 90

2 year: 80

Assume that the price of the two-year bond in one year's time can be either 86.89 or 90.89, with identical probability. At the end of year 1, the total return generated by the two-year will be either $(86.89/80)8.6125\%$ or, $(90.89/80)13.6125\%$ while at this point the (now) one-year bond will offer a return of either $(100/86.89)15.089\%$ or $(100/90.89)10.023\%$. The two possible prices have been set deliberately so as to ensure that the expected return over one year for the two-year bond is equal to the return available today on the one-year bond, which is 11.11% as we noted at the start. The return expected on the two-year bond is indeed the same (provided either of the two prices is available), that is $((0.5) \times (86.89/80) + (0.5) \times (90.89/80))$ or 11.11%. Therefore it cannot also be true that the certain return over two years for the two-year bond is equal to the expected return for two years from rolling over the investment in the one-year bond. At the start of the period the two-year bond has a guaranteed return of $[100/80]25\%$ over its lifetime. However investing in the one-year bond and then re-investing at the one-year period after the first year will produce a return that is higher than this, as shown:

$$11.11\% \times ((0.5) \times (100/86.89) + (0.5) \times (100/90.89))$$

or 25.063%. Under this scenario then investors cannot expect equality of returns for all bonds over all investment horizons.

APPENDIX 6.3 Cubic spline interpolation

There are four observed vertices in the example quoted in the main text, which requires three cubic equations, $rm_{(i,t)}$, each one connecting two adjacent vertices n_i and n_{i+1} , as follows:

$$rm_{(0,t)} = a_0n^3 + b_0n^2 + c_0n + d_0, \text{ connecting vertex } n_0 \text{ with } n_1,$$

$$rm_{(1,t)} = a_1n^3 + b_1n^2 + c_1n + d_1, \text{ connecting vertex } n_1 \text{ with } n_2,$$

$$rm_{(2,t)} = a_2n^3 + b_2n^2 + c_2n + d_1, \text{ connecting vertex } n_2 \text{ with } n_3,$$

where a , b , c and d are unknowns. The three equations require 12 conditions in all. The cubic spline method imposes the following set of conditions on the curves. Each cubic equation must pass through its own pair of vertices. Thus, for the first equation:

$$a_0n_0^3 + b_0n_0^2 + c_0n_0 + d_0 = 4.00$$

$$a_0n_1^3 + b_0n_1^2 + c_0n_1 + d_0 = 5.00.$$

For the second and third equations:

$$a_1n_1^3 + b_1n_1^2 + c_1n_1 + d_1 = 5.00$$

$$a_1n_2^3 + b_1n_2^2 + c_1n_2 + d_1 = 6.50$$

$$a_2n_2^3 + b_2n_2^2 + c_2n_2 + d_2 = 6.50$$

$$a_2n_3^3 + b_2n_3^2 + c_2n_3 + d_2 = 6.75.$$

The resulting yield curve should be smooth at the point where one cubic equation joins with the next one. This is achieved by requiring the slope and the convexity of adjacent equations to be equal at the point where they meet,

ensuring a smooth rollover from one equation to the next. Mathematically, the first and second derivatives of all adjacent equations must be equal at the point where the equations meet:

Thus, at vertex n_1 :

$$\begin{aligned} 3a_0n_1^2 + 2b_0n_1 + 1 &= 3a_1n_1^2 + 2b_1n_1 + 1 && \text{(the first derivative)} \\ 6a_0n_1 + 2 &= 6a_1n_1 + 2 && \text{(the second derivative).} \end{aligned}$$

And at vertex n_2 :

$$\begin{aligned} 3a_1n_2^2 + 2b_1n_2 + 1 &= 3a_2n_2^2 + 2b_2n_2 + 1 && \text{(the first derivative)} \\ 6a_1n_2 + 2 &= 6a_2n_2 + 2 && \text{(the second derivative).} \end{aligned}$$

Finally, we may impose the condition that the splines tail off flat at the end vertices, or more formally we state mathematically that the second derivatives should be zero at the end points:

$$\begin{aligned} 6a_0n_0 + 2 &= 0 && \text{(first spline starts flat)} \\ 6a_2n_3 + 2 &= 0 && \text{(second spline ends flat).} \end{aligned}$$

These constraints together give us a system of 12 equations from which we can solve for the 12 unknown coefficients. The solution is usually using matrices, where the equations are expressed in matrix form. This is shown below.

$$\begin{bmatrix} n_0^3 & n_0^2 & n_0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ n_1^3 & n_1^2 & n_1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & n_1^3 & n_1^2 & n_1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & n_2^3 & n_2^2 & n_2 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & n_2^3 & n_2^2 & n_2 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & n_3^3 & n_3^2 & n_3 & 1 \\ 3n_1^2 & 2n_1 & 1 & 0 & -3n_1^2 & -2n_1 & -1 & 0 & 0 & 0 & 0 & 0 \\ 6n_1 & 2 & 0 & 0 & -6n_1 & -2 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 3n_2^2 & 2n_2 & 1 & 0 & -3n_2^2 & -2n_1 & -1 & 0 \\ 0 & 0 & 0 & 0 & 6n_2 & 2 & 0 & 0 & -6n_2 & -2 & 0 & 0 \\ 6n_0 & 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 6n_3 & 2 & 0 & 0 \end{bmatrix} \times \begin{bmatrix} a_0 \\ b_0 \\ c_0 \\ d_0 \\ a_1 \\ b_1 \\ c_1 \\ d_1 \\ a_2 \\ b_2 \\ c_2 \\ d_2 \end{bmatrix} = \begin{bmatrix} 4 \\ 5 \\ 6.5 \\ 6.75 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

Figure 6.19: Cubic spline interpolation matrix.

In matrix notation we have $[n] \times [\text{Coefficients}] = [rm]$ therefore the solution is $[\text{Coefficients}] = [n]^{-1} \times [rm]$. Inverting the matrix n and then pre-multiplying rm with the resulting inverse, we obtain the array of required coefficients:

$$\text{Coefficients} = [0.022, 0.000, 0.413, 4.000, -0.047, 0.411, -0.410, 4.548, 0.008, -0.249, 2.230, 1.029].$$

So the three cubic equations are specified as:

$$\begin{aligned} rm_{(0,t)} &= 0.022 \times n^3 + 0.413 \times n + 4.000 \text{ for vertices } n_0 - n_1, \\ rm_{(1,t)} &= -0.047 \times n^3 + 0.411 \times n^2 - 0.410 \times n + 4.548 \text{ for vertices } n_1 - n_2, \\ rm_{(2,t)} &= 0.008 \times n^3 - 0.249 \times n^2 + 2.230 \times n + 1.029 \text{ for vertices } n_2 - n_3. \end{aligned}$$

Selected bibliography and references

- Anderson, N., Sleath, J., *Bank of England Quarterly Bulletin*, November 1999.
- Campbell, J., “A Defence of Traditional Hypotheses about the Term Structure of Interest Rates”, *Journal of Finance*, March 1986, pp. 183–193.
- Campbell, J., Lo, A., MacKinlay, A., *The Econometrics of Financial Markets*. Princeton UP, 1997, Chapters 10–11.
- Choudhry, M., “The information content of the United Kingdom gilt yield curve”, unpublished MBA assignment, Henley Management College, 1998.
- Choudhry, M., Joannas, D., Pereira, R., Pienaar, R., *Capital Markets Instruments*, FT Prentice Hall, 2001.
- Cox, J., Ingersoll, J.E., Ross, S.A., “A re-examination of traditional hypothesis about the term structure of interest rates”, *Journal of Finance* 36, Sep 1981, pp. 769–99.
- Culbertson, J.M., “The term structure of interest rates,” *Quarterly Journal of Economics* 71, November 1957, pp. 485–517.
- The Economist*, “Admiring those shapely curves,” 4 April 1998, p. 117.
- Fabozzi, F., *Bond Markets, Analysis and Strategies*, 2nd edition, Prentice Hall, 1993.
- Fama, E.F., “Forward rates as predictors of future spot interest rates”, *Journal of Financial Economics*, Vol. 3, No.4, Oct 1976, pp. 361–377.
- Fama, E.F., “The information in the term structure,” *Journal of Financial Economics* 13, Dec 1984, pp. 509–528.
- Fisher, I., “Appreciation of Interest”, *Publications of the American Economic Association*, August 1986, pp. 23–39.
- Heath, D., Jarrow, R., Morton, A., “Bond pricing and the term structure of interest rates,” *Journal of Financial and Quantitative Analysis* 25, 1990, pp. 419–440.
- Hicks, J., *Value and Capital*, OUP, 1946.
- Ingersoll, J., *Theory of Financial Decision Making*, Rowman & Littlefield 1987, Chapter 18.
- James, J., Webber, N., *Interest Rate Modelling*, John Wiley and Sons, 2000.
- Jarrow, R., “Liquidity premiums and the expectations hypothesis”, *Journal of Banking and Finance* 5(4), 1981, pp. 539–546.
- Jarrow, R., *Modelling Fixed Income Securities and Interest Rate Options*, McGraw-Hill, 1996.
- Kessel, R.A., “The cyclical behaviour of the term structure of interest rates”, *Essays in Applied Price Theory*, University of Chicago, 1965.
- Levy, H., *Introduction to Investments*, 2nd edition, South-Western College Publishing, 1999.
- Livingstone, M., *Money and Capital Markets*, Prentice-Hall, 1990, pp. 254–256.
- Lutz, F., “The Structure of Interest Rates”, *Quarterly Journal of Economics*, November 1940, pp. 36–63.
- Mastronikola, K., “Yield curves for gilt-edged stocks: a new model”, *Bank of England Discussion Paper (Technical Series)*, No. 49, 1991.
- McCulloch, J.H., “An estimate of the liquidity premium,” *Journal of Political Economy* 83, Jan–Feb 1975, pp. 95–119.
- Meiselman, D., *The term structure of interest rates*, Prentice Hall, 1962.
- Ryan, R. (ed.), *Yield Curve Dynamics*, Glenlake Publishing Company, 1997.
- Rubinstein, M., *Rubinstein on Derivatives*, RISK 1999, pp. 84–85.
- Shiller, R., “The Term Structure of Interest Rates”, in Friedman, B., Hahn, F. (eds.), *Handbook of Monetary Economics*, North-Holland 1990, Chapter 13.
- Windas, T., *An Introduction to Option-Adjusted Spread Analysis*, Bloomberg Publishing, 1993.

Questions and exercises

- From the relevant daily newspaper, plot a graph of the gross redemption yields for the government bonds of a market of your choice.
- Consider the following zero-coupon bonds and prices:

Bond maturity (years)	Price (€)
1	95.2381
2	90.1868
3	84.7991
4	80.1125
5	75.4348

Calculate the spot interest rates, the forward rates (for a 1-year term starting from each maturity point in years) and then the relevant forward prices for each bond. Plot a graph of the spot and forward rates.

3. Suppose that the five-year spot interest rate is 8%, and the six-year spot rate is 7.9%. What is the forward rate for the sixth period? (Hint: solve this using our forward rates expression given below.)

$${}_{n-1}rf_n = \frac{(1 + rs_n)^n}{(1 + rs_{n-1})^{n-1}} - 1$$

What do we notice about the forward rate in relation to the five-year spot rate?

4. If \$100 nominal of zero-coupon bonds of two and three years' maturity are priced at \$90 and \$80 respectively, is there any arbitrage opportunity available to investors?
5. Khaleda sits on the money markets desk of a bank. Given that Df_1 is equal to 0.9523 and Df_2 is equal to 0.9434, she decides to short a one-year zero-coupon bond and buy a two-year zero-coupon bond. Has she created a synthetic forward position?
6. Note the following table of forward interest rates. What is the minimum value of the five-period spot interest rate, and what is the forward rate? (Hint: remember the relationship between spot and forward rates, which states that the n -period spot rate rs_n is the geometric mean of rs_1 and the forward rates. This is given by the expression below.)

$$rs_n = (rs_1 + rf_2 + rf_3 + \cdots + rf_n)/n$$

<i>Period</i>	<i>Forward rate (%)</i>
1	3.2%
2	3.5%
3	4.2%
4	5.5%

Plot a yield curve for a government bond market of your choice. How would you seek to explain its shape? Is there an information that can be gleaned from it?

7. Which of the conventional theories of the yield curve are consistent with a negative sloping (inverted) yield curve?
8. Brett is a junior research analyst with a securities house. He has been asked to write a few words explaining the following yield curve. Which theories (if any) should he use to explain the yield curve?

<i>Maturity (years)</i>	<i>Spot rate (%)</i>
1	5.10%
2	5.00%
3	5.05%
4	5.40%
5	5.85%
6	6.01%
7	5.98%
8	5.87%
9	5.35%
10	4.98%
15	4.85%
20	4.78%
30	4.56%

9. If we assume that the combined theory is a satisfactory explanation of the yield curve, and we have a one-period spot rate of 6.0%, an expected spot rate in the next period of 6.5% and a corresponding liquidity premium for that period of 0.8%, what is the two-period spot rate?
10. A five-year bond with 6.00% coupon is trading at €97.65. A five-year bond with a coupon of 3.00% is trading at €93.70. Calculate the five-year spot interest rate. (Hint: assume a long position in two 3.00% bonds and a short position in one of the 6.00% bonds.)
11. Consider the following yield curve for risk-free bonds paying semi-annual coupons.

<i>Maturity (years)</i>	<i>Redemption yield (%)</i>	<i>Spot rate (%)</i>
0.5	5.00	5.00
1.0	5.15	5.16
1.5	5.23	–
2.0	5.28	–
2.5	5.34	–
3.0	5.41	–
3.5	5.46	–
4.0	5.55	–
4.5	5.63	5.89
5.0	5.70	5.99

The 0.5-year bond is a zero coupon bond. Assume that all the other bonds are selling at par. Calculate the spot rates that are not shown. What should a four-year bond be priced at? What is the six-month forward rate in year 2.5?

12. “Cash is king in a negative yield curve environment.” Discuss.³⁰
13. How would a market maker price zero-coupon bonds in the gilt market? If the observed yield of a zero-coupon bond is above the theoretical yield implied for it by the par yield curve, what trade could the market maker put on in order to profit from the situation?
14. In a positive yield curve environment, place the following yields in increasing order of magnitude:
 - (a) the five-year spot rate
 - (b) the five-year bond yield to maturity
 - (c) the six-month forward rate starting in five years’ time
 Then assume a negative yield curve environment and place the rates accordingly.
15. Consider the following spot yield curve:

<i>Maturity (years)</i>	<i>Rate %</i>
1	3.50
2	3.54
3	3.65
4	3.83
5	3.99

What are the forward rates? Assume annual compounding.

16. The term structure is currently showing one-year and two-year spot interest rates both at 6.00%. What is the price of a bond with precisely three years to maturity and a coupon of 7.25% and a redemption yield of 6.85%? What is the three-year spot interest rate? Assume annual compounding.
17. Consider the following observed coupon bond yields:

³⁰ With apologies to Graham “Harry” Cross!

1 month:	5.00%
1 year:	5.65%
2 years:	5.75%
5 years:	6.00%
10 years:	6.75%

18. Calculate the theoretical yield for a four-year bond using the following techniques:
- linear interpolation
 - logarithmic interpolation
- What are the cubic equations required to estimate the four-year rate using cubic splines?
19. The coupon yield curve is upward sloping from two years to ten years. Where would the following lie in relation to each other:
- the redemption yield on a six-year plain vanilla bond
 - the six-year zero-coupon yield
 - the six-year interest-rate swap rate
 - the six-month forward-forward rate commencing in six years' time.
20. An analyst calculates an estimate for the three-year yield, given two-year and five-year yields of 5.00% and 6.50% respectively. Arrange the results he estimates for the three-year yield using the following methods, in order of magnitude only (there is no need to calculate the actual estimate itself):
- logarithmic interpolation
 - polynomial curve fitting
 - cubic splines
 - linear interpolation
21. The three-month and nine-month zero-coupon rates are currently both 5.50% annualised. The yield on a one-year 6% bond (annual coupon, only the last coupon remaining) is 6.25%. What is the price of the bond? What is the one-year zero-coupon rate?
22. Consider the following set of hypothetical bonds, which pay semi-annual coupon, and assume all bonds have just paid a coupon so there is no accrued interest:

Bond	Maturity	Coupon	Price	Yield
5% 2000	6 months	5.00%	99.8782	5.25%
8% 2001	1 year	8.00%	102.4005	5.50%
6.75% 2001	1.5 years	6.75%	101.4177	5.75%
7% 2002	2 years	7.00%	101.5769	6.15%
9.25% 2002	2.5 years	9.25%	106.2524	6.50%
6% 2003	3 years	6.00%	97.3357	7.00%

Calculate the zero-coupon rates from six months to three years, and the six-month forward rates for the same maturity structure. What would be the price and yield of a three-year bond with a coupon of 10%? How would you calculate the zero-coupon rates if you had just been given the bond prices and not the yields?

23. Review the various explanations behind the shape of the yield curve. Can the liquidity preference theory be used adequately to explain an inverted yield curve?
24. The UK gilt yield curve inverted in July 1997 and remained inverted for just under two years thereafter, following which it changed to positive sloping out to seven years and then inverted. Which of our yield curve hypotheses would cover this scenario?
25. Calculate the forward rates derived from a money market yield curve with the following spot rates: three-month 5%, six-month 5.14%, 12-month 5.21%, 15-month 5.28%, 18-month 5.35%. What is the price of a three-month T-Bill today? Purchased at auction in six months' time?

7 Price, Yield and Interest Rate Risk I

Although it is one of the key identifying features of bond instruments, the term to maturity of a bond does not tell us very much about the timing of its cash flows or its price behaviour in the market compared to other bonds. Bonds pay a part of their total return during their lifetime, in the form of coupon interest, so that the term to maturity does not reflect the true period over which the bond's return is earned. Additionally if we wish to gain an idea of the trading characteristics of a bond, and compare this to other bonds of say, similar maturity, term to maturity is insufficient and so we need a more accurate measure.

We begin this chapter by considering a more useful measure of the time period over which bond return is generated. In addition in this and the next three chapters we will look at the main methods used to measure the *risk* exposure inherent in holding bonds. The risk under consideration here is *interest rate risk*; there are of course other risks in holding a bond. However the main risk in holding developed country government bonds such as UK gilts will be interest rate risk, since gilts do not expose investors to any type of *credit risk*. Holders of say, corporate bonds are of course exposed to an element of credit risk, and this will vary with the credit quality of the issuer of the bond. Another type of risk is inflation risk. Our analysis of yield curve shapes referred to the market's view of inflation. Inflation risk is common to all bonds, and the required return on bonds will reflect an inflationary expectation element. This chapter is concerned with interest rate risk. Interest rate risk is also common to all bonds, and has an important effect on bond values. It is the risk that bond prices will fall if market interest rates rise, and is the main form of *market risk* for bonds and strips. The traditional measures of interest rate risk are *duration*, *modified duration* and *convexity*.

7.1 Revisiting the bond price/yield relationship

In Chapter 3 we showed how for a plain vanilla bond, because the coupon is fixed, the price is the only parameter of the bond's make-up that can change in response to a change in market interest rates. Hence a bond's price will move in the opposite direction to a move in market interest rates, where the latter has triggered a change in the required yield for that bond. Table 7.1 shows how the price of a set of hypothetical bonds changes with changes in yield. Assume the bonds pay annual coupon on an actual/actual basis. The bonds represent maturities of three, ten and 20 years and coupons of 0% (zero-coupon bond), 5% and 8%.

In an earlier chapter a generic graph of the bond price/yield relationship was shown to be convex in shape. This illustrates how the change in price for given changes in yield is not linear, but follows this convex relationship. In Figure 7.1 we draw the graph of the price/yield relationship for three more hypothetical bonds. This illustrates how the nature of the convex relationship between price and yield is different for each bond. The *convexity* of a bond is a key element in analysing its price behaviour and performance.

In examining the price changes in Table 7.1, we note that the change in price for a given change is not uniform for an upward or downward change in yield. That is, if we consider that 5% 2009 bond, an upward change in yield of 10 basis points, from 5.00% to 5.10%, results in a fall in price of £0.76844 per cent. However a downward change in yield of 10 basis points, to 4.90%, results in a rise in price of £0.77593 per cent. This is an important result and one which we will return to later when we discuss convexity in greater detail.

The immediate observation from Table 7.1 is that the change in price for a given change in yields differs for each of the bonds. Understanding this price volatility is important when analysing the properties of bonds that make up a portfolio. Table 7.1 illustrates the basic characteristics of the price/yield relationship for plain vanilla bonds, which are that (i) the price volatility of bonds differs amongst individual bonds and that (ii) price volatility is not symmetrical for all but the smallest changes in yields: as we demonstrated above, the change in price for a given change in yield differs for upward and downward moves in interest rates, a function of the convex price/yield relationship. Generally for a change in yield of over ten basis points, the increase in price for a downward move is more than the decrease in price resulting from the same size upward yield change. This can be gauged from Figure 7.1, which shows the relationship for three of our hypothetical bonds.

Bond	Maturity (years)	Yield										
		4.00%	4.90%	5.00%	5.10%	5.50%	6.00%	6.50%	7.00%	7.50%	8.00%	9.00%
0% 2002	3	88.89964	86.63104	86.38376	86.13742	85.16137	83.96193	82.78491	81.62979	80.49606	79.38322	77.21835
0% 2009	10	67.55642	61.97908	61.39133	60.80970	58.54306	55.83948	53.27260	50.83493	48.51939	46.31935	42.24108
0% 2019	20	45.63869	38.41406	37.68895	36.97819	34.27290	31.18047	28.37970	25.84190	23.54131	21.45482	17.84309
5% 2002	3	102.77509	100.27284	100.00000	99.72818	98.65103	97.32699	96.02728	94.75137	93.49869	92.26871	89.87482
5% 2009	10	108.11089	100.77593	100.00000	99.23156	96.23119	92.63991	89.21675	85.95283	82.83980	79.86976	74.32937
5% 2019	20	113.59033	101.25686	100.00000	98.76428	94.02481	88.53008	83.47224	78.81197	74.51377	70.54556	63.48582
8% 2002	3	111.10036	108.45791	108.16974	107.88264	106.74483	105.34602	103.97271	102.62431	101.30026	100.00000	97.46871
8% 2009	10	132.44358	124.05405	123.16520	122.28468	118.84406	114.72017	110.78325	107.02358	103.43204	100.00000	93.58234
8% 2019	20	154.36130	138.96253	137.38663	135.83593	129.87596	122.93984	116.52776	110.59401	105.09725	100.00000	90.87145

All bonds have a maturity date of 30 September in the final year (e.g., 8% 2009 issued 30/9/1999, matures 30/9/2009).

Table 7.1: Price/yield relationship for a set of bonds.

If we measure the change along the price axis for a fixed change in the yield axis, we can see most obviously for the 11% 2019 bond that the price increase is greater than the price decrease for a given change in yield. This is observed for all bonds where the price/yield relationship is convex. We can also see from Figure 7.1 that the 5% 2002 bond is less convex than the 11% 2019 bond; so, while the property of upward price changes outstripping downward price changes still holds, it is less obvious than for the longer-dated bond. This characteristic is important for traders and portfolio managers, because it tells us that the convexity relationship is positive for a bondholder. If one is holding a bond, there will be a capital gain arising from an appreciation in price that outstrips the loss that would arise for an equal (downward) change in yield. However a market maker running a short position in the bond will suffer a capital loss that is greater than any potential gain that would result if yields went up by the same number of basis points.

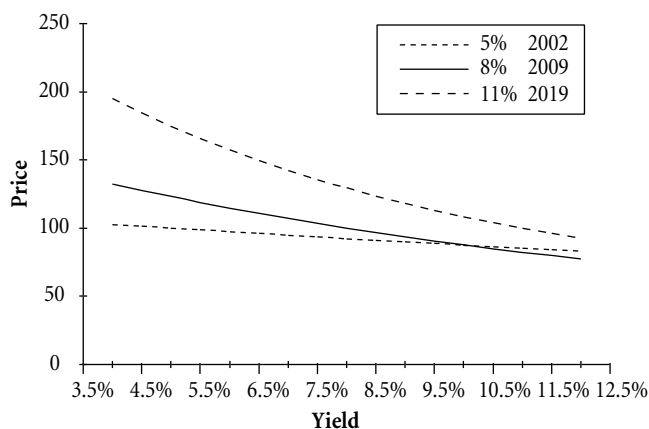


Figure 7.1: Price/yield relationship for three hypothetical bonds.

There are further price volatility characteristics that can be observed from Table 7.1. These include the common feature, for all but very long-dated bonds trading well below par, that price volatility is higher for bonds with lower coupon. Given a set term to maturity and from the same initial yield level, bonds with lower coupon will exhibit higher price volatility than bonds with higher coupon. In our table of hypothetical bonds, for any maturity the 5% bond will fall by a greater percentage than the same-maturity 8% bond. We also note that the zero-coupon bond for any maturity has the greatest price volatility of any of the bonds in the table.

Another feature that we can observe from our bonds is that, for a given coupon level and from the same initial yield, price volatility increases with term to maturity. Comparing any of the bonds with another with the same coupon, the change in price is greater for a given yield level when the term to maturity is longer. For a portfolio manager therefore a holding of long-dated bonds is advantageous if the market experiences a fall in interest rates, while the opposite is true if there is a rise in yields.

Initial yield	Price	Price at a 100bp lower yield	Price change	% change
6%	122.93984	137.38663	14.44679	11.75111
7%	110.59401	122.93984	12.34583	11.16320
8%	100.00000	110.59401	10.59401	10.59401
9%	90.87145	100.00000	9.12855	10.04556
10%	82.97287	90.87145	7.89858	9.51947

Table 7.2 Yield level and price volatility, 8% 2019 bond

Does the initial level for a bond have an impact on its price volatility? Table 7.2 shows us that it does. If we select one of our bonds from Table 7.2, the 8% 2019 we see that the percentage change in price for a 100 basis points drop in yield is greater if the initial yield level is lower. This feature can also be deduced from studying our convexity graph at Figure 7.1; when the initial yield level is high, a given change in interest rates produces a smaller change in

the price of the bond than when the initial yield level is lower. Market makers and portfolio managers are therefore aware of the fact that, when market yield levels are relatively high, the price volatility of bonds is comparatively low.

7.2 Duration

7.2.1 Basic concepts

A plain vanilla coupon bond pays out a proportion of its return during the course of its life, in the form of coupon interest. If we were to analyse the properties of a bond, we should conclude quite quickly that its maturity gives us little indication of how much of its return is paid out during its life, nor any idea of the timing or size of its cash flows, and hence its sensitivity to moves in market interest rates. For example, if comparing two bonds with the same maturity date but different coupons, the higher coupon bond provides a larger proportion of its return in the form of coupon income than does the lower coupon bond. The higher coupon bond provides its return at a faster rate; its value is theoretically therefore less subject to subsequent fluctuations in interest rates.

We may wish to calculate an average of the time to receipt of a bond's cash flows, and use this measure as a more realistic indication of maturity. However cash flows during the life of a bond are not all equal in value, so a more accurate measure would be to take the average time to receipt of a bond's cash flows, but weighted by the cash flows' present value. This is, in effect, *duration*. We can measure the speed of payment of a bond, and hence its price risk relative to other bonds of the same maturity by measuring the average maturity of the bond's cash flow stream. Bond analysts use duration to measure this property (it is sometimes known as *Macaulay's duration*, after its inventor, who first introduced it in 1938).¹ Duration is the weighted average time until the receipt of cash flows from a bond, where the weights are the present values of the cash flows, measured in years. At the time that he introduced the concept, Macaulay used the duration measure as an alternative for the length of time that a bond investment had remaining to maturity. We can illustrate Macaulay duration with a simple example, using an hypothetical 5-year bond with precisely five years to maturity and a coupon of 8 per cent. Assume that the bond is priced at par, giving a yield to maturity of 8 per cent. The bond's cash flows are shown in Table 7.3 along with a diagram of the timing of cash flows at Figure 7.2.

8% five-year bond			
Cash flow	Present value*	Timing (t)	$PV \times t$
8.00	7.41	1	7.41
8.00	6.86	2	13.72
8.00	6.35	3	19.05
8.00	5.88	4	23.52
108.00	73.50	5	367.51
	<u>100.00</u>		<u>431.21</u>

*Calculated as $C/(1 + r)^t$

Table 7.3 Example of duration calculation

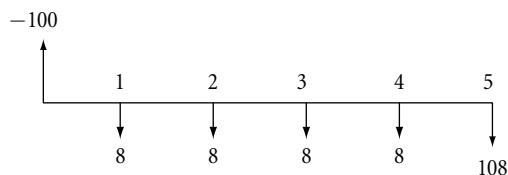


Figure 7.2: Receipt of cash flows for 8% 5-year bond.

¹ Macaulay, F., *Some theoretical problems suggested by the movements of interest rates, bond yields and stock prices in the United States since 1865*, National Bureau of Economic Research, NY, 1938. This is currently available from the *RISK Classics* library.

The present value of each cash flow is calculated in the normal way, hence for the period 2 cash flow of £8 is $8/(1.08)^2$, which gives us 6.859.

Duration is then calculated using (7.1):

$$D = \frac{\sum(\text{present value of cash flow} \times \text{time to cash flow})}{\sum(\text{present value of cash flow})}. \quad (7.1)$$

Mathematically this is written as (7.2) below:²

$$D = \frac{(1)PVC_1 + (2)PVC_2 + (3)PVC_3 + \cdots + (n)PVC_n}{\sum_{n=1}^n PVC_n} \quad (7.2)$$

where

- D is the Macaulay duration
- PVC_n is the present value of the n -period cash flow, discounted at the current yield to maturity
- n is the number of interest periods.

Note that the denominator, as the sum of all the present values of the cash flows, is in fact the price of the bond.

For a semi-annual coupon bond, the cash flows are discounted at half the current yield to maturity. Equation (7.2) solves for Macaulay duration in terms of the number of interest periods; we divided this by the number of interest periods per year (either 1 or 2 for annual or semi-annual coupon bonds) to obtain the Macaulay duration in years.

In our illustration of the 8% five-year bond, duration is calculated as $431.21/100$, which is equal to 4.31 years. This implies that the average time taken to receive the cash flows on this bond is 4.31 years. This is shown in Figure 7.3 as our “duration fulcrum”, with 4.31 years being the time to the pivot, marked from A to B. The coupons are shown as “C”, and these diminish progressively as their present value decreases.

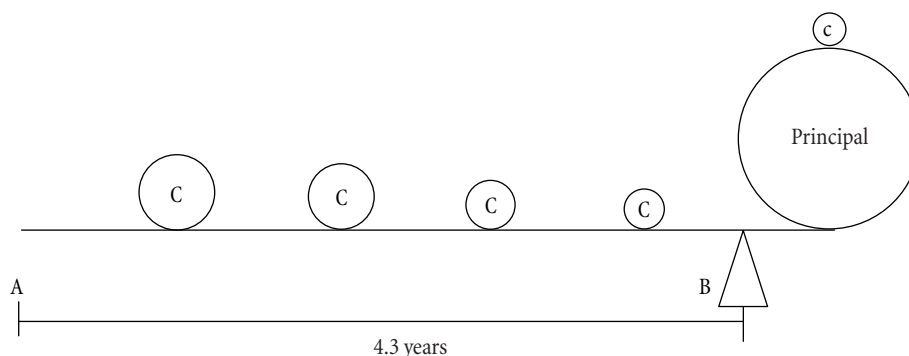


Figure 7.3: The Duration Fulcrum.

Example 7.1 calculates the Macaulay duration for one of our hypothetical bonds from Table 7.1, the 5% 2009 bond.

EXAMPLE 7.1 Calculating the Macaulay duration for the 8% 2009 annual coupon bond

Issued	30 September 1999
Maturity	30 September 2009
Price	102.497
Yield	7.634%

² This assumes the bond has a whole number of interest periods to maturity; if this is not the case we round down to the nearest whole number.

<i>Annual coupon</i>	8%	<i>Price</i>	102.497
<i>Issued</i>	30.09 1999	<i>Yield</i>	7.634%
<i>Maturity</i>	30.09 2009		
Period (<i>n</i>)	Cash flow	PV at current yield*	<i>n</i> × <i>PV</i>
1	8	7.43260	7.4326
2	8	6.90543	13.81086
3	8	6.41566	19.24698
4	8	5.96063	23.84252
5	8	5.53787	27.68935
6	8	5.14509	30.87054
7	8	4.78017	33.46119
8	8	4.44114	35.529096
9	8	4.12615	37.13535
10	108	51.75222	517.5222
	Total	<u>102.49696</u>	<u>746.54069</u>

*Calculated as $C/(1+r)^n$

Macaulay duration = $746.540686/102.497$
= 7.283539998 years

Modified duration = $7.28354/1.07634$
= 6.76695

Table 7.4: Duration and modified duration calculation for the 8% 2009 bond.

For a zero-coupon bond, where the present value of the coupon payments is zero because there are no coupon payments, the Macaulay duration can be shown to be equal to the number of interest periods remaining in the bond's life. If the bond is trading in a bond market with an annual coupon convention, the Macaulay duration is therefore equal to the number of years remaining to maturity. This can also be seen from the definition of a zero-coupon bond (all of its cash flow is received on maturity) and from (7.2) given that $C = 0$ and that $P = M/(1+r)^N$.

The concept of duration can be applied to any series of cash flows and therefore to a portfolio bondholding. Its importance lies in the fact that it is a measure of bond price sensitivity, and also in its application in portfolio immunisation, which we consider in Part IX.

7.2.2 Deriving the duration expression

Recall that the price/yield formula for a plain vanilla bond is as given at (7.3) below, assuming complete years to maturity paying annual coupons, and with no accrued interest at the calculation date. In this section we use r to denote the yield to maturity instead of the usual rm .

$$P = \frac{C}{(1+r)} + \frac{C}{(1+r)^2} + \frac{C}{(1+r)^3} + \cdots + \frac{C}{(1+r)^n} + \frac{M}{(1+r)^n}. \quad (7.3)$$

If we take the first derivative of this expression we obtain (7.4):

$$\frac{dP}{dr} = \frac{(-1)C}{(1+r)^2} + \frac{(-2)C}{(1+r)^3} + \cdots + \frac{(-n)C}{(1+r)^{n+1}} + \frac{(-n)M}{(1+r)^{n+1}}. \quad (7.4)$$

If we rearrange (7.4) we will obtain the expression at (7.5), which is our equation to calculate the approximate change in price for a small change in yield.

$$\frac{dP}{dr} = -\frac{1}{(1+r)} \left(\frac{1C}{(1+r)} + \frac{2C}{(1+r)^2} + \cdots + \frac{nC}{(1+r)^n} + \frac{nM}{(1+r)^n} \right). \quad (7.5)$$

Readers may feel a sense of familiarity regarding the expression in brackets in equation (7.5) as this is the weighted average time to maturity of the cash flows from a bond, where the weights are, as in our example above, the present values of each cash flow. The expression at (7.5) gives us the approximate measure of the change in price for a small change in yield. If we divide both sides of (7.5) by P we obtain the expression for the approximate percentage price change, given at (7.6):

$$\frac{dP}{dr} \frac{1}{P} = -\frac{1}{(1+r)} \left(\frac{1C}{(1+r)} + \frac{2C}{(1+r)^2} + \cdots + \frac{nC}{(1+r)^n} + \frac{nM}{(1+r)^n} \right) \frac{1}{P}. \quad (7.6)$$

If we divide the bracketed expression in (7.6) by the current price of the bond P we obtain the definition of Macaulay duration, given at (7.7):

$$D = \frac{\frac{1C}{(1+r)} + \frac{2C}{(1+r)^2} + \cdots + \frac{nC}{(1+r)^n} + \frac{nM}{(1+r)^n}}{P}. \quad (7.7)$$

Equation (7.7) is frequently re-written as (7.8):

$$D = \frac{\sum_{n=1}^N \frac{nC_n}{(1+r)^n}}{P} \quad (7.8)$$

where C represents the bond cash flow at time n .

Occasionally, equation (7.8) is observed in the following form:

$$D = \frac{C}{P} \sum_{n=1}^N \frac{n}{(1+r)^n} + \frac{M}{P} \frac{N}{(1+r)^N} \quad (7.9)$$

where

n is the time in years to the n th cash flow N is the time to maturity in years.

This is obviously measuring the same thing but the expression has been arranged in a slightly different way.

The markets commonly use a measure of bond price sensitivity to interest rates³ known as *modified duration*. If we substitute the expression for Macaulay duration (7.7) into equation (7.6) for the approximate percentage change in price we obtain (7.10):

$$\frac{dP}{dr} \frac{1}{P} = -\frac{1}{(1+r)} D. \quad (7.10)$$

This is the definition of modified duration, given as (7.11):

$$MD = \frac{D}{(1+r)}. \quad (7.11)$$

Modified duration is clearly related to duration then, in fact we can use it to indicate that, for small changes in yield, a given change in yield results in an inverse change in bond price. We can illustrate this by substituting (7.11) into (7.10), giving us (7.12):

$$\frac{dP}{dr} \frac{1}{P} = -MD. \quad (7.12)$$

We will examine modified duration in greater detail shortly.

³ Referred to as *interest rate sensitivity* or interest rate *risk*. Sometimes in the past referred to as volatility.

In a later section we note the use of Microsoft Excel® to calculate duration for a bond. If we are determining duration long-hand, there is another arrangement we can use to shorten the procedure. Instead of equation (7.3) we use (7.13) as the bond price formula, which calculates price based on a bond being comprised of an annuity stream and a redemption payment, and summing the present values of these two elements. Again we assume an annual coupon bond priced on a date that leaves a complete number of years to maturity and with no interest accrued.

$$P = C \left(\frac{1 - 1/(1+r)^n}{r} \right) + \frac{M}{(1+r)^n}. \quad (7.13)$$

We encountered this expression in Chapter 3 and it calculates the price of a bond as the present value of the stream of coupon payments and the present value of the redemption payment. If we take the first derivative of (7.13) and then divide this by the current price of the bond P , the result is another expression for the modified duration formula, given at (7.14):

$$MD = \frac{\frac{C}{r^2} (1 - 1/(1+r)^n) + \frac{n(M - (C/r))}{(1+r)^{n+1}}}{P}. \quad (7.14)$$

We have already shown that modified duration and duration are related; to obtain the expression for Macaulay duration from (7.14) we multiply it by $(1+r)$. This short-hand formula is demonstrated in Example 7.2 for the hypothetical bond used in the earlier example, the gilt 8% 2009.

EXAMPLE 7.2 8% 2009 bond: using equation (7.14) for the modified duration calculation

Coupon	8%, annual basis
Yield	7.634%
n	10
Price	102.497

Substituting the above terms into the equation we obtain:

$$MD = \frac{\frac{8}{(0.07634^2)} \left(1 - \frac{1}{(1.07634)^{10}} \right) + \frac{10(100 - \frac{8}{0.07634})}{(1.07634)^{11}}}{102.497}$$

$$= 6.76695.$$

To obtain the Macaulay duration we multiply the modified duration by $(1+r)$, in this case 1.07634, which gives us a value of 7.28354 years.

For an irredeemable bond duration is given by:

$$D = \frac{1}{rc} \quad (7.15)$$

where $rc = (C/P_d)$ is the *running yield* (or *current yield*) of the bond. This follows from equation (7.9) as $N \rightarrow \infty$, recognising that for an irredeemable bond $r = rc$. Equation (7.15) provides the limiting value to duration. For bonds trading at or above par duration increases with maturity and approaches this limit from below. For bonds trading at a discount to par duration increases to a maximum at around 20 years and then declines towards the limit given by (7.15). So in general, duration increases with maturity, with an upper bound given by (7.15).

7.2.3 Limiting duration

The definition of duration is linked to a bond's maturity and the size of its coupon payments, so we might expect that increasing maturity (and decreasing coupon) would lead to higher duration measures. All else being equal this is true only up to a point, as equation (7.15) illustrates. Figure 7.4 shows the effect of maturity on duration of increasing the maturity of a bond with a coupon of 5%, maintaining a yield to maturity of 9.50%. In this case the limiting value for duration is approximately 9.1 years, after which it declines and reaches a level around 7.8 years for increasing maturity.

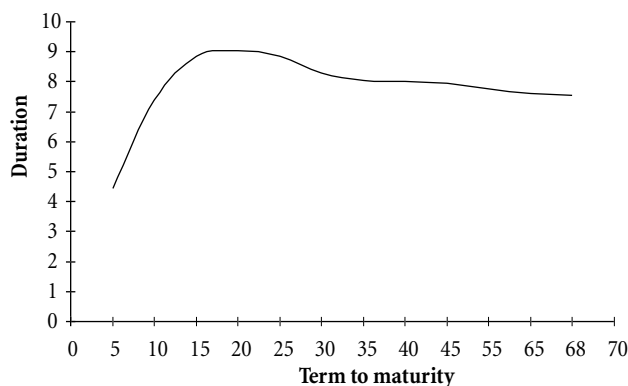


Figure 7.4: The limiting value to duration I.

For an hypothetical 25-year bond at a yield of 7.25%, altering the coupon size affects the duration value up to a lower limit of around 7.1 years. This is shown at Figure 7.5.

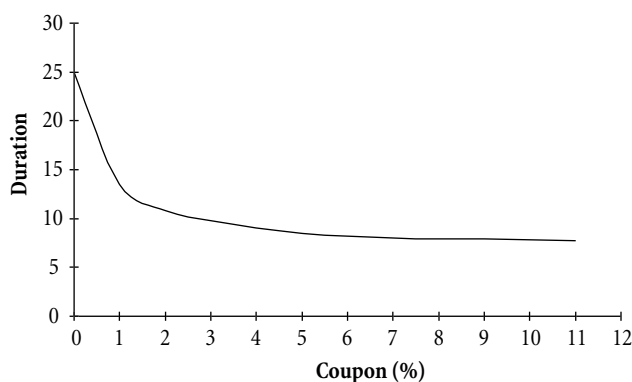


Figure 7.5: The limiting value to duration II.

7.2.4 Properties of Macaulay duration

Let us examine some of the properties of duration. A bond's duration is always less than its maturity. This is because some weight is given to the cash flows in the early years of the bond's life, which brings forward the average time at which cash flows are received. In the case of a zero-coupon bond, there is no present value weighting of the cash flows, for the simple reason that there are no cash flows, and so duration for a zero-coupon bond is equal to its term to maturity. Duration varies with coupon, yield and maturity. The following three factors imply higher duration for a bond:

- the lower the coupon;
- the lower the yield;
- broadly, the longer the maturity.

Duration increases as coupon and yield decrease. As the coupon falls, more of the relative weight of the cash flows is transferred to the maturity date and this causes duration to rise. Because the coupon on index-linked bonds is generally much lower than on vanilla bonds, this means that the duration of index-linked bonds will be much higher than for vanilla bonds of the same maturity. As yield increases, the present values of all future cash flows fall, but the present values of the more distant cash flows fall relatively more than those of the nearer cash flows. This has the effect of increasing the relative weight given to nearer cash flows and hence of reducing duration.

We can demonstrate that duration is a measure of interest rate risk. Remember that for vanilla bonds with an exact number of years to maturity the present value equation for an annual coupon bond is given by:

$$P = \sum_{n=1}^N \frac{C}{(1+r)^n} + \frac{M}{(1+r)^N}. \quad (7.16)$$

Differentiating this equation with respect to $(1+r)$ results in:

$$\frac{\Delta P}{\Delta(1+r)} = -C \sum_{n=1}^N \frac{n}{(1+r)^{n+1}} - M \frac{N}{(1+r)^{N+1}}. \quad (7.17)$$

Multiplying both sides of 7.17 by $(1+r)/P$ gives us:

$$\begin{aligned} \frac{\Delta P/P}{\Delta(1+r)/(1+r)} &= -\frac{C}{P} \sum_{n=1}^N \frac{n}{(1+r)^n} - \frac{M}{P} \frac{N}{(1+r)^N} \\ &= -D. \end{aligned} \quad (7.18)$$

The left-hand side of (7.18) is the elasticity of the bond price with respect to (one plus) the yield to maturity; the right-hand side is the negative of duration. So duration measures the interest rate elasticity of the bond price, and is therefore a measure of interest rate risk. The lower the duration, the less responsive is the bond's value to interest rate fluctuations.⁴

Let us look at the effect of other factors on duration during the life of a bond.

- **Duration between coupon dates.** If we think again of the “duration fulcrum” diagram at Figure 7.3 consider what happens as one day elapses. Each cash flow and the original fulcrum are now one “day” closer to the investor. If the position of the fulcrum does not change relative to the cash flows, then the duration will have decreased by one day. This is in effect the case. As one day elapses (with no change in yield) all of the cash flows will increase in present value, because the discounting period is being shortened. Duration therefore shortens by the same time elapsed.
- **The effect of the coupon frequency.** Certain bonds such as most Eurobonds pay coupon annually compared to say, gilts which pay semi-annual coupons. Again thinking of our duration fulcrum, if we imagine that every coupon is divided into two parts, with one part paid a half-period earlier than the other, this will represent a shift in weight to the left, as part of the coupon is paid earlier. Thus increasing the coupon frequency shortens duration, and of course decreasing coupon frequency has the effect of lengthening duration.
- **Duration as maturity approaches.** Using our definition of duration we can see that initially it will decline slowly, and then at a more rapid pace as a bond approaches maturity.
- **Duration of a portfolio.** Portfolio duration is a weighted average of the duration of the individual bonds. The weights are the present values of the bonds divided by the full price of the entire portfolio, and the resulting duration calculation is often referred to as a “market-weighted” duration. This approach is in effect the duration calculation for a single bond. Portfolio duration has the same application as duration for a individual bond, and can be used to structure an *immunised* portfolio.

Figure 3.1 illustrated the present value profile for a conventional bond (the price/yield relationship). There is a negative-sloping, convex relationship between the price of the bond and the yield to maturity. The slope of the present value profile at the current bond price and yield to maturity is equal to the (negative of the) duration of the bond. The flatter the present value profile, the lower the duration and the lower the interest rate risk. In Figure 7.1 we observed the difference in profile for three different bonds.

7.2.5 Bond duration with uneven cash flow stream

The standard duration calculation formula given at (7.8) assumes that cash flows are evenly spaced. For vanilla bonds this will usually apply except for the first coupon date whenever the value date does not fall on a coupon date, which will be the majority of transactions. Calculating the yield to maturity for a bond with an uneven number of spaces between cash flows was considered in Chapter 4; here we show the equivalent calculation for the duration measure.

⁴ Burghardt says in his book *The Treasury Bond Basis* (1994) that duration, measured in years, “is of no value to anyone”. We think this is a little harsh, but substantially accurate!

Consider our hypothetical 8% 2009 bond after the issue date and traded in between coupon dates; as this is an annual coupon bond this will be in most cases! If we say the bond has N payments of $C_t, C_{t+1}, C_{t+2}, \dots, C_{t+N-1}$, where $0 < t < 1$. This represents the case when the first cash flows payment is received at less than one interest period from today, and here the redemption payment M is also denoted by C . To recap from the earlier chapter, the price of this bond is given by (7.19):

$$P = \sum_{n=1}^N \frac{C_{t+n-1}}{(1+r)^{t+n-1}} = (1+r)^{1-t} \sum_{n=1}^N \frac{C_{t+n-1}}{(1+r)^n}. \quad (7.19)$$

The duration of this bond is given by (7.20):

$$D = \frac{1}{P} \sum_{n=1}^N \frac{(t+n-1)C_{t+n-1}}{(1+r)^{t+n-1}}. \quad (7.20)$$

Rearranging (7.20) gives us (7.21):

$$\begin{aligned} D &= \frac{1}{P} (1+r)^{1-t} \left[\sum_{n=1}^N \frac{nC_{n+t}}{(1+r)^n} + \sum_{n=1}^N \frac{(t-1)C_{n+t}}{(1+r)^n} \right] \\ &= \frac{1}{(1+r)^{1-t} \sum_{n=1}^N \frac{C_{n+t}}{(1+r)^n}} (1+r)^{1-t} \left[\sum_{n=1}^N \frac{nC_{n+t}}{(1+r)^n} + (t-1) \sum_{n=1}^N \frac{C_{n+t}}{(1+r)^n} \right] \\ &= \frac{1}{\sum_{n=1}^N \frac{C_{n+t}}{(1+r)^n}} \left[\sum_{n=1}^N \frac{nC_{n+t}}{(1+r)^n} \right] + t - 1. \end{aligned} \quad (7.21)$$

Equation (7.21) states that a bond with N cash flow payments, the first occurring at time t from today has a duration that is the sum of the duration of a bond with N payments spaced at even intervals and $t - 1$.

7.2.6 Modified duration

Although it is common for newcomers to the markets to think intuitively of duration much as Macaulay originally did, as a proxy measure for the time to maturity of a bond, such an interpretation is to miss the main point of duration, which is a measure of price volatility or interest rate risk.

Using the first term of a Taylor's expansion of the bond price function⁵ we can show the following relationship between price volatility and the duration measure, which is expressed as (7.22) below.

$$\Delta P = - \left[\frac{1}{(1+r)} \right] \times \text{Macaulay duration} \times \text{Change in yield} \quad (7.22)$$

where r is the yield to maturity for an annual-paying bond (for a semi-annual coupon bond, we use one-half of r). If we combine the first two components of the right-hand side, we obtain the definition of modified duration. Equation (7.22) expresses the approximate percentage change in price as being equal to the modified duration multiplied by the change in yield. We saw in the previous section how the formula for Macaulay duration could be modified to obtain the *modified duration* for a bond. There is a clear relationship between the two measures. From the Macaulay duration of a bond can be derived its modified duration, which gives a measure of the sensitivity of a bond's price to small changes in yield. As we have seen, the relationship between modified duration and duration is given by (7.23):

$$MD = \frac{D}{1+r} \quad (7.23)$$

where MD is the modified duration in years. For a bond that pays semi-annual coupons, the equation becomes:

⁵ This is explained in an appendix in Chapter 9.

$$MD = \frac{D}{(1 + \frac{1}{2}r)}. \quad (7.24)$$

This means that the following relationship holds between modified duration and bond prices:

$$\Delta P = MD \times \Delta r \times P. \quad (7.25)$$

In the UK markets the term *volatility* is sometimes used to refer to modified duration, but this is becoming increasingly uncommon in order to avoid confusion with option markets' use of the same term, which often refer to *implied volatility* and which is something quite different.

EXAMPLE 7.3 Using modified duration

An 8% annual coupon bond is trading at par with a duration of 2.85 years. If yields rise from 8% to 8.50%, then the price of the bond will fall by:

$$\begin{aligned} \Delta P &= -D \times \frac{\Delta(r)}{1+r} \times P = -(2.85) \times \left(\frac{0.005}{1.080}\right) \times 100 \\ &= -£1.3194. \end{aligned}$$

That is, the price of the bond will now be £98.6806. The modified duration of a bond with a duration of 2.85 years and yield of 8% is obviously:

$$MD = \frac{2.85}{1.08}$$

which gives us *MD* equal to 2.639 years.

In the earlier example of the five-year bond with a duration of 4.31 years, the modified duration can be calculated to be 3.99. This tells us that for a 1 per cent move in the yield to maturity, the price of the bond will move (in the opposite direction) by 3.99%.

We can use modified duration to approximate bond prices for a given yield change. This is illustrated with the following expression: $\Delta P = -MD \times (\Delta r) \times P$. For a bond with a modified duration of 3.99, priced at par, an increase in yield of 1 basis point (100 basis = 1 per cent) leads to a fall in the bond's price of:

$$\begin{aligned} \Delta P &= (-3.99/100) \times (+0.01) \times 100.00 \\ \Delta P &= £0.0399, \text{ or } 3.99 \text{ pence.} \end{aligned}$$

In this case 3.99 pence is the *basis point value* of the bond, which is the change in the bond price given a one basis point change in the bond's yield. The basis point value of a bond can be calculated using (7.26) and is examined in greater detail in the next chapter.

$$BPV = \frac{MD}{100} \cdot \frac{P}{100}. \quad (7.26)$$

7.2.7 Duration for a bond with an embedded option

The price/yield relationship for a conventional bond is convex. For a bond with an option feature, such as a callable bond, the relationship differs at a certain yield point, as shown in Figure 7.6.

The relationship changes because of the properties of a callable bond. When market interest rates are higher than the coupon rate on a callable bond (so that it is trading below par) the issuer is unlikely to call the bond. For example if market rates are at 8% and the coupon rate on the bond is 5% the issuer will have no need to call the bond and refinance, as there is no financial gain involved. At all times that this situation prevails, the price/yield relationship for a callable bond will mirror that of a conventional bond. There will be a slight variation if the market rate drops to just above the coupon rate, and investors will demand a premium over a comparable non-callable bond, because there is the possibility that market rates may drop further, thereby raising the prospect of the bond being called. As market rates drop further the possibility then exists that the bond will be called; this yield is indicated as *r* in Figure 7.6. At this point the price/yield relationship will move away from the conventional convex curve, towards a scenario where there is only very limited price appreciation as market interest rates fall. Here a

callable bond is displaying *negative convexity*. This means that a rise in market yields by any number of basis points will result in a greater price depreciation compared to the price appreciation that results if yields decline by the same number of basis points.

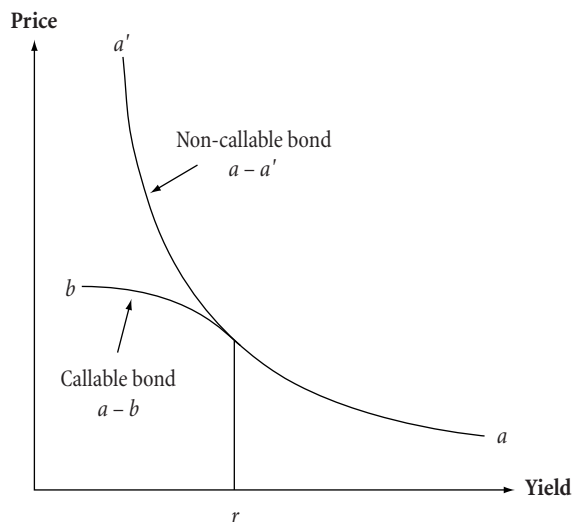


Figure 7.6: Present value relationship for non-callable and callable bond.

Modified duration is an ineffective measure of the price volatility of callable or putable bonds because it does not make any allowance for the change in future cash flows that may result from changes in the yield. As we said earlier, for a callable bond a fall in market interest rates may result in a change in the expectation of future cash flows, as the market may perceive that the bond will be called shortly. For certain asset-backed bonds such as mortgage-backed bonds, a change in market interest rates may well result in a change in prepayment amounts made by borrowers. Therefore for these types of bonds where the cash flow is sensitive to changes in yields, the sensitivity of the price to changes in yield must allow for the impact of yield changes on the expected cash flows of a bond, as well as its price. The measurement of duration that considers changes in cash flows resulting from yield changes is known as *effective duration*. Although we shall consider effective duration in more detail in the chapter on asset-backed bonds, it suffices to say here that the formula for calculating effective duration is similar to that used to calculate *approximate duration*, given at (7.27):

$$\text{Approximate duration} = \frac{P_- - P_+}{2P_0 (\Delta r)} \quad (7.27)$$

where

- P_0 is the initial price of the bond
- P_- is the estimated price of the security if the yield is decreased by Δr
- P_+ is the estimated value of the bond if the yield is increased by Δr .

7.2.8 Using modified duration

The modified duration measure for a bond is often used to determine the approximate percentage change in price of a bond for a given change in yield. As we shall see in Chapter 9, the measure becomes significantly inaccurate for anything greater than small changes in yield. In the same way however, we can determine the approximate change in the cash price of a bond if we know the bond's modified duration, the initial price and the yield change in basis points. This measure is known as the bond's *price duration* or, in the US markets, as the *dollar duration*. Essentially the term dollar duration is used synonymously with *basis point value*, which is the more common term in non-US markets. The dollar duration for a one basis point change in yield is given by (7.28):

$$\text{Dollar duration} = \frac{MD \times P}{10,000}. \quad (7.28)$$

To illustrate using one of our hypothetical bonds again, the 8% 2009 (which was trading at £102.497 to give a yield to maturity of 7.634%), we take the modified duration of 6.76695 to calculate the dollar duration, as shown below.

$$\text{Dollar duration} = \frac{6.76695 \times 102.497}{10,000} = 0.069359.$$

The most commonly observed use of the expression *dollar duration*, as we noted above, is for the price value of one basis point, or basis point value. This is the value we calculated above for the 8% 2009 bond, and it refers to a change in £0.06936 per cent of par value for every one basis point change in yield. This is of course an approximate value. In some markets the convention is different and participants refer to “dollar duration” as the value for a change in price resulting from a 100 basis point change in yield. In our example this value for dollar duration would then be £6.936.

The modified duration measurement is an approximation and only reasonably accurate for small changes in yield. This results from the convex nature of the price/yield relationship. Therefore the value for dollar duration is also an approximation, and will not be accurate for large changes in yield. This is illustrated in Example 7.4

EXAMPLE 7.4 The nature of the modified duration approximation

- Table 7.5 shows the change in price for one of our hypothetical bonds, the 8% 2009, for a selection of yields. We see that for a one basis point change in yield, the change in price given by the dollar duration figure, while not completely accurate, is a reasonable estimation of the actual change in price. For a large move however, say 200 basis points, the approximation is significantly in error and analysts would not use it. Notice also for our hypothetical bond how the dollar duration value, calculated from the modified duration measurement, underestimates the change in price resulting from a fall in yields but overestimates the price change for a rise in yields. This is a reflection of the price/yield relationship for this bond. Some bonds will have a more pronounced convex relationship between price and yield and the modified duration calculation will underestimate the price change resulting from both a fall or a rise in yields.

Bond	Maturity (years)	Modified duration	Price duration of basis point	Yield									
				6.00%	6.50%	7.00%	7.50%	7.99%	8.00%	8.01%	8.50%	9.00%	10.00%
8% 2009	10	6.76695	0.06936	114.72017	110.78325	107.02358	103.43204	100.067131	100.00000	99.932929	96.71933	93.58234	87.71087
				Yield change		Price change		Estimate using price duration					
				down 1 bp		0.06713		0.06936					
				up 1 bp		0.06707		0.06936					
				down 200 bp		14.72017		13.872					
				up 200 bp		12.28913		13.872					

Table 7.5: Nature of the modified duration approximation.

We examine the nature of the modified duration approximation, and the measure for estimating the approximation itself, in a later chapter.

EXAMPLE 7.5 Using Microsoft Excel®

- The Excel® spreadsheet package has two duration functions, *Duration* and *Mduration*, which can be used for the two main measures. Not all installations of the software may include these functions, which have to be installed using the “Analysis ToolPak” add-in macro. The syntax required for using these functions is the

same in both cases, and for Macaulay duration is:

Duration(settlement, maturity, coupon, yield, frequency, basis).

The dates that are used in the syntax for Excel® version 5 are serial dates, so the user may need to use the separate function available to convert conventional date formats to the serial date for Excel®. The latest versions of Excel recognise conventional date formats. The other parameters are:

settlement	the settlement date
maturity	the maturity date
coupon	the coupon level
yield	the yield to maturity
frequency	annual or semi-annual
basis	the day-count basis.

Once the parameters have been set up, Excel® will calculate the duration and modified duration for you.

7.2.9 Modified duration and volatility of yields

The modified duration measure refers to the percentage price change resulting from a 100 basis point change in bond yield. However bond analysts need to consider the yield volatility of a bond, more so than its price volatility. This becomes clear when one analyses non-government bonds. Given that modified duration is a function of redemption yield, a corporate bond of similar maturity to a government bond will trade at a higher yield; in the case of lower-rated bonds (*high-yield* or “junk” bonds) there will be a significant yield spread over the government bond. This would imply then that the government bond will have a higher modified duration than the corporate bond (other things being equal). However although this might suggest that the government bond then has a higher interest rate risk than the corporate, observation tells us that this would be incorrect. Corporate bonds invariably trade at higher levels of price volatility, indeed their yields will fluctuate relative to government bond yields even when the latter are stable. Across government markets the same is true, for example Japanese government bonds trade at considerable lower yields than say UK or US government bonds, and therefore have a higher duration. However their price volatility is generally lower. For this reason it is important to consider volatility of bond yields as well as price volatility as measured by modified duration. We consider this in a later chapter.

7.2.10 Portfolio duration

Duration and modified duration as discussed in this chapter can be applied in a portfolio context, since the main concept refers to a cash flow stream and a bond portfolio is essentially a series of cash flows. To calculate the duration of a portfolio the same concept is used as for a single bond, using the cash flows that arise from the instruments comprising the portfolio. However an approximation can be obtained by taking the weighted average of the duration of each constituent instrument. Hence the duration approximation for a portfolio is given at (7.29).

$$D_{port} = \frac{\sum(\text{duration of each instrument} \times \text{value of each instrument})}{\text{market value of portfolio}}. \quad (7.29)$$

Similarly the modified duration of a portfolio is obtained by calculating the weighted average of the modified duration of the bonds in the portfolio. This is given at (7.30):

$$MD_{port} = \frac{\sum(\text{modified duration of each instrument} \times \text{value of each instrument})}{\text{market value of portfolio}}. \quad (7.30)$$

Another way to write the portfolio modified duration formula is as shown at (7.31), the format preferred by Fabozzi (1997):

$$MD_{port} = w_1 MD_1 + w_2 MD_2 + w_3 MD_3 + \cdots + w_k MD_k \quad (7.31)$$

where

- w_1 is the market value of bond 1 / market value of the portfolio
- MD_1 is the modified duration of bond 1
- k is the number of bonds in the portfolio.

We can illustrate the calculation for portfolio modified duration, consider the following hypothetical portfolio, made up of some of the bonds from Table 7.1

Bond	Nominal value	Price	Yield	Market value	Modified duration
5% 2002	£5m	98.65103	5.50%	£4,932,551.5	2.65000
0% 2009	£10m	58.54310	5.50%	£5,854,310.0	9.47867
8% 2019	£3m	129.87596	5.50%	£3,896,278.8	13.14692
8% 2002	£2m	106.74483	5.50%	£2,134,896.6	2.32227
				<u>£16,818,036.9</u>	

Table 7.6: Hypothetical portfolio.

Bearing in mind the usual assumption, we can calculate the portfolio modified duration once we have modified duration values for the individual bonds. The total market value is known and there are four bonds in the portfolio. We therefore set k equal to 4 and calculate the modified duration as follows:

$$\begin{aligned}
 w_1 &= 4,932,551.5 / 16,818,036.9 = 0.29329 & MD_1 &= 2.65000 \\
 w_2 &= 5,854,310.0 / 16,818,036.9 = 0.34810 & MD_2 &= 9.47867 \\
 w_3 &= 3,896,278.8 / 16,818,036.9 = 0.23167 & MD_3 &= 13.14692 \\
 w_4 &= 2,134,896.6 / 16,818,036.9 = 0.12694 & MD_4 &= 2.32227.
 \end{aligned}$$

The portfolio modified duration is then:

$$0.29329(2.65) + 0.3481(9.47867) + 0.23167(13.14692) + 0.12694(2.32227) = 7.417.$$

We use the resulting portfolio modified duration value in exactly the same way, that is for a 100 basis point change in the yield for all four bonds, the market value of the entire portfolio will change by approximately 7.4173%. For this number to have any real value however, there has to be a parallel shift in the yield curve such that the required yield for all the bonds in the portfolio changes by the same amount. Notice also how the initial yield for all four bonds was at the same starting level – another heroic assumption.

Bond analysts frequently use the portfolio modified duration measure but it has limited application and its place as a measure of interest rate risk has been overtaken by the *value-at-risk* tool for market risk. We consider VaR in the chapter on risk management. Another issue to consider is the make-up of the portfolio. In our foregoing discussion we have assumed bonds of the same currency, and the example above automatically assumed that all four bonds were denominated in the same currency. How do we treat a portfolio made up of bonds denominated in different currencies? Although a weighted duration measure can be calculated as shown by equation (7.29), essentially there is no accurate modified duration measure that we can use. This is because modified duration measures bond price volatility with respect to a change in yield, but in a portfolio of different currency bonds we would not be using a single change in yield. In effect we would not be comparing like-for-like as yield movements would most likely be caused by unique factors not applicable to all the bonds in the (cross-currency) portfolio. Therefore there is no justification for calculating or using a modified duration measure for a cross-currency portfolio.

EXAMPLE 7.6 Portfolio duration and hedging

- A portfolio is made up of the following bonds:

Bond	Nominal value	Price	Modified duration
5% 2002	£5m	98.65103	2.65000
0% 2009	£10m	58.54310	9.47867
8% 2002	£2m	106.74483	2.32227

Table 7.7: Portfolio hedging

The portfolio manager wishes to hedge the portfolio with another bond, with the following characteristics:

Bond	Price	Modified duration
8% 2019	129.87596	13.14692

What nominal value of this bond should the portfolio manager short sell in order to protect the portfolio from small increases in market yields, effectively hedging the positions, assuming parallel shifts in the yield curve?

The hedge position must be of a sufficient size to offset the change in value of the portfolio resulting from a move in market rates. The nominal value to short sell is therefore:

$$\frac{(5\text{m} \times 0.9865 \times 2.65) + (10\text{m} \times 0.5854 \times 9.47) + (2\text{m} \times 1.0674 \times 2.32)}{1.2987 \times 13.14}.$$

The nominal value to sell of the 8% 2019 is therefore £4,304,811.

7.3 A summary of the duration measure

In this chapter we have considered Macaulay duration (often called simply “duration”) and modified duration measures of bond price volatility. Let us summarise here the main interpretations of the duration measures.

In its original formulation by Macaulay duration was defined as the weighted-average time until receipt of a financial instrument’s cash flows. The original formula for duration given as (7.32) can be re-written as (7.33).

$$D = \frac{1}{P} \sum_{n=1}^N \frac{nC_n}{(1+r)^n}. \quad (7.32)$$

$$D = \sum_{n=1}^N \left(\frac{C_n/P}{(1+r)^n} \right) \times n. \quad (7.33)$$

So expression (7.33) defines duration as the time-weighted average of a bond instrument’s discounted cash flows as a proportion of the bond’s price.

The duration concept was later developed as a measure of a bond’s price elasticity with respect to changes in its yield to maturity. This allowed us to view duration as a measure of a bond’s price volatility. To derive this measure, we needed to obtain the first derivative of the bond’s price with respect to its yield shown at (7.34)

$$\frac{dP}{dr} = \sum_{n=1}^N \frac{-nC_n}{(1+r)^{n+1}} \quad (7.34)$$

which can be rearranged using algebra⁶ to give (7.35):

$$\frac{dP}{dr} = -\frac{DP}{1+r}. \quad (7.35)$$

This enables us to interpret duration now as the bond yield elasticity of the bond price, as shown in (7.36).

$$\frac{dP/P}{dr/(1+r)} = \frac{\text{Percentage change in bond price}}{\text{Percentage change in bond yield}} = -D. \quad (7.36)$$

This further allows us to use the *modified* duration measure as the price volatility of a bond, if we rearrange (7.36) to give us (7.37):

$$\frac{dP}{P} = -D \frac{dr}{1+r}. \quad (7.37)$$

One further interpretation of the duration measure was given by Babcock (1985). This views definition as a weighted average of two factors, shown as (7.38):

$$D = N \left(1 - \frac{rc}{r} \right) + \frac{rc}{r} PV_{Am}(r, N) \times (1+r) \quad (7.38)$$

where

rc is the bond’s running yield

⁶ The process is illustrated at Appendix 7.1.

$$PV_{Ann} \text{ is the present value of an annuity, given as } PV_{Ann}(r, N) = \sum_{n=1}^N \frac{1}{(1+r)^n},$$

where the present value measure is for an N -period annuity.

Equation (7.38) gives us a further insight into the duration measure, which is that it can be considered a weighted average of the maturity of the bond and of $(1+r)$ multiplied by the present value of the annuity stream element of the bond. Note however that quite often the running yield of a bond is near in value to its redemption yield, so in these cases the duration value is close in value to the $(1+r)PV_{Ann}$ measurement.

Babcock's formula does not hold for a bond with varying coupon payments, that is where C_n is not uniform over time. This does not apply to the conventional interpretations of duration given earlier.

7.4 Duration for other securities

Our analysis of duration and modified duration has concentrated on conventional plain vanilla bonds. We can look briefly at the duration properties of other instruments in the market.

- **Money market instruments.** Most money market instruments such as commercial paper, bankers' acceptances and Treasury bills can be treated as short-term, zero-coupon bonds. As such their durations are equal to their maturities.
- **Bonds with embedded options.** The presence of option features in a bond will affect its price behaviour. The price of callable bonds tend to peak once they rise slightly above par, especially if a call date is near. Puttable bonds tend to trade near par (or slightly above) as put dates approach, because the bond is redeemable. Duration calculations are made difficult by the uncertainties of the cash flows associated with the bonds, in addition to the problem of determining the appropriate yield to use (usually we use *yield to worst*). Bonds with embedded options must be analysed with a model of price behaviour to properly determine duration. For puttable and callable bonds the model may simply combine the value of the underlying bond with the positive (put) or negative (call) value of the option position. The duration of the combined security is usually determined implicitly by estimating its price response to a change in yield level and then determining the duration that would lead to the same price change.
- **Futures contracts.** The duration of a futures contract cannot be determined using the standard calculation. There are no defined cash flows associated with a futures contract. We can view a contract as pure volatility, and because there is no cash outflow (price [we ignore margin payments]) paid to enter the contract, its percentage volatility (and thus its modified duration) is infinite. A long position adds volatility, while a short position reduces volatility in a portfolio (this assumes a portfolio that is net long; with a net short portfolio, a long position in futures will of course reduce volatility). We can determine the cash volatility of a futures contract relative to yield changes in the underlying bond or portfolio. This volatility can be added to portfolio volatility for a long position (or subtracted for a short position) and the result will be the net portfolio volatility. From this value we can calculate an implied duration figure. Hence it is not necessary to have an actual duration value for the futures contract, as we can still determine its effect on portfolio duration.
- **Floating rate notes.** Calculating duration for floating rate notes is problematic because the level of future cash flows is unknown. However if we wish to know the sensitivity of the bond to interest rate change, we can infer a duration value. If a particular FRN is reset every quarter to the then-prevailing three-month rate based on a specified index, the primary volatility of the bond will be the same as that of a three-month instrument. As time elapses and the coupon payment approaches, the implied primary duration will approach zero and will reset to three months on the coupon date. This measure is independent of the maturity of the FRN.

Another aspect of the price sensitivity of an FRN is indeed a function of its maturity. If market spreads change for FRNs, then the price change will vary according to the maturity. For example consider an FRN resetting at Libor flat, and assume that the market for new FRNs from issuers of similar quality is also Libor flat. The FRN would be priced at or near par. If the market then began demanding new issues (and re-priced old issues) at Libor + 20 basis points, then the price of this old FRN would reflect the number of remaining quarters in which the bondholder would receive the lower "historic" rate rather than the new rate of Libor + 20. In effect the investor has given up an annuity of 20 basis points, and the price should decline by the present value of this annuity.

In essence two volatility measures are needed for FRNs: the simple duration, which is the time until the next coupon payment and reset, and a “spread duration” that is a function of the maturity and the starting yield level.

- **Interest rate swaps.** An interest rate swap can be analysed as essentially an exchange of two securities, usually involving a fixed-rate and a floating-rate component. In a manner similar to futures contracts, an interest rate swap contract adds (or subtracts) volatility without involving a purchase “price”, as it is an off-balance sheet instrument. As a result it is impossible to determine duration as a percentage volatility measure, but it is possible to estimate the volatility characteristics of the swap and how entering into the swap affects the duration of an existing portfolio. As a combination of long and short positions in a fixed-rate instrument and an FRN, the volatility of an interest rate swap can be determined by netting the volatilities of each component. This volatility can then be aggregated with the volatility of the portfolio to back into the duration of the portfolio including the swap.

In practice many hedge transactions and relative value trades involving swaps are “duration weighted” using only the fixed-rate payment side of the swap. This is because the floating side is already offset by floating-rate liabilities in the portfolio. Thus the primary volatility at issue is the value of the fixed-rate side of the swap versus the value of fixed-rate bonds, for instance. If the market values of these components are equal and have the same duration, then the position is considered to be properly weighted. In the chapter on swaps we will look in greater detail at the duration of an interest rate swap, as well at using swaps to hedge bond positions and how to hedge a swap book with bond instruments.

Appendices

APPENDIX 7.1 Formal derivation of modified duration measure

Given that duration is defined as:

$$D = \frac{1}{P} \sum_{n=1}^N \frac{nC_n}{(1+r)^n}, \quad (7.39)$$

if we differentiate P with respect to r we obtain:

$$\frac{dP}{dr} = -\sum_{n=1}^N nC_n (1+r)^{-n-1}. \quad (7.40)$$

Multiplying (7.40) by $(1+r)$ we obtain:

$$(1+r) \frac{dP}{dr} = -\sum_{n=1}^N nC_n (1+r)^{-n}. \quad (7.41)$$

We then divide the expression by P giving us:

$$\frac{dP}{dr} \frac{1+r}{P} = -\sum_{n=1}^N \frac{nC_n}{(1+r)^n P} = -D. \quad (7.42)$$

If we then define modified duration as $D/(1+r)$ then:

$$-\frac{dP}{dr} \frac{1}{P} = MD. \quad (7.43)$$

Thus modified duration measures the proportionate impact on the price of a bond resulting from a change in its yield. The sign in (7.43) is negative because of the inverse relationship between bond prices and yields.⁷ So if a bond has a modified duration of 8.541, then a rise in yield of 1% means that the price of the bond will fall by 8.541%. As we discuss in the main text however, this is an approximation only and is progressively more inaccurate for greater changes in yield.

⁷ That is, rising yields result in falling prices.

Selected bibliography

- Babcock, G., "Duration as a weighted average of two factors", *Financial Analysts Journal*, Mar–Apr 1985, pp. 75–76.
- Bierwag, G.O., "Immunization, duration and the term structure of interest rates," *Journal of Financial and Quantitative Analysis*, December 1977 pp. 725–741.
- Bierwag, G.O., "Measures of duration," *Economic Inquiry* 16, October 1978, pp. 497–507.
- Fabozzi, F., *Bond Markets, Analysis and Strategies*, 2nd edition, Prentice Hall, 1993.
- Fabozzi, F., *Fixed Income Mathematics*, 3rd edition, McGraw-Hill, 1997, pp. 190–192.
- Gultekin, B., Rogalski, R.J., "Alternative duration specifications and the measurement of basis risk: empirical tests," *Journal of Business*, April 1984, pp. 241–264.
- Macaulay, F.R., *The movement of interest rates, bonds, yields and stock prices in the United States since 1865*, NY: Columbia University, 1983.

Questions and exercises

1. Explain why the change in a bond's price for a given change in yields is not uniform for both upward and downward movement in the bond's yield.

2. Consider these two bonds:

	(1)	(2)
Coupon	5%	6%
Yield	5.3%	5.3%
Maturity (years)	3	5
Price	99.188	103.006

Calculate the Macaulay duration and modified duration for bonds 1 and 2. What is the price of bond 1 if there is a 100 basis point upward change in yield? The price of bond 2 if there is a 29 basis point downward change in yield?

3. If the coupon level on a bond is raised, what effect does this have on its duration measure? Assume that there is no change in the yield to maturity.
4. Consider the following bonds:

Bond	Coupon	Maturity (years)
3	5%	10 years
4	9%	12 years
5	10%	13 years

If we assume that all three bonds are trading at the same yield to maturity, is it possible to tell which bond carries the greatest interest rate risk without making any calculations?

5. Explain why the modified duration measure is only effective for small changes in yield.
6. What is the effect on a bond's duration of increasing its term to maturity? What is the effect as the term to maturity approaches infinity, such that the bond becomes in effect an annuity stream?
7. The duration of an annuity stream is given by $(1 + r)/r$. Explain the relevance of this to the limiting value for the duration measure.
8. A zero-coupon bond with maturity of N has a single cash flow payment, the redemption payment on maturity. What is the duration of this bond?
9. In a negative yield curve environment, is there any use for the modified duration measure?
10. In the markets duration is often viewed as a proxy for the "riskiness" of a bond instrument. If all else is assumed to be equal, does a bond of lower duration present lower risk to a bondholder than one of higher duration? What is the logic behind this thinking?

8 Price, Yield and Interest Rate Risk II

We saw in the previous chapter how duration and modified duration for a bond or a portfolio of bonds were calculated. These are the main measures of interest rate risk for bonds. Portfolio managers and market makers use another measure, calculated from modified duration, known as *basis point value*. In this chapter we discuss basis point value and also a related measure, the yield value of a price change.

8.1 Basis point value

8.1.1 Basic concepts

The basis point value (BPV) is the change in price of a bond for a one basis point change in the bond's yield. It is also referred to as the *price value of a basis point* (PVBP) or the *dollar value of a basis point* (DV01).¹ BPV refers to the actual cash change in a bond's price, so it is a sterling, dollar, euro or other currency amount. It does not therefore refer to a percentage change in bond price for a change in yield. For very small changes in yield the change in price will be approximately similar irrespective of the direction of the yield change. However as we illustrated in the previous chapter the convex nature of the price/yield relationship means that, beyond a small yield change, the BPV will differ for upward and downward changes in yield. Basis point value is a key risk measure, used by bond trading desks to measure the extent of their interest rate risk. Trading books are often set trading exposure limits in terms of an overall basis point value. For management reporting purposes, a desk will aggregate its risk exposure as one number, so that senior executives will be aware of how much the value of the book will change for a 1 basis point change in interest rates. To cater for large market moves banks use *jump risk*, which we examine in the chapter on risk management.

Table 8.1 shows the BPV calculated for the set of hypothetical bonds introduced in the previous chapter. The table shows the change in price for a 1 basis point upward move in yield and a 1 basis point downward move in yield.

It is worth noting one or two points that stand out from Table 8.1. The short-dated bonds have BPVs that are roughly similar for both upward and downward changes in yield. For longer-dated bonds the BPVs begin to differ significantly. For longer maturity bonds any use of the BPV needs to note the yield change direction that it applies to. Given that BPV is derived from modified duration, its value is not static and changes with changes in the initial yield level. Another observation worth noting is that the BPV for a bond is higher when the yield level is lower. For example, at a yield of 5% the 5% 2019 bond has a BPV for a upward yield shift of 0.12452, whereas the same bond, when trading at a yield of 8% has a BPV of 0.07620. Figure 8.1 illustrates this graphically and is intended to show that the BPV of a bond changes at different yields. The higher the initial yield, the lower the BPV. Therefore in a market where bonds are trading at historically low yield levels, the price volatility is higher. If we were to plot a graph of the price/yield relationship for the bonds in Table 8.1 we would see that higher *convexity* in a bond will lead to a higher BPV (other things being equal). A graphical illustration would show us that by comparing curves, the BPV will be higher for more convex bonds.

¹ PVBP is also the acronym for *present value of a basis point*, which is of course the same thing as price value of a basis point. It is more usual for the term "present value of a basis point" to be encountered in the money markets, while the "price value" expression is used in the bond markets.

Bond	Maturity (years)	Yield		BPV	Yield	
		4.99%	5%		5.01%	BPV
0% 2002	3	86.40845	86.38376	0.02469	86.35908	0.02468
0% 2009	10	61.44982	61.39133	0.05849	61.33289	0.05844
0% 2019	20	37.76081	37.68895	0.07186	37.61723	0.07172
5% 2002	3	100.02724	100.0	0.02724	99.97277	0.02723
5% 2009	10	100.07725	100.0	0.07725	99.92282	0.07718
5% 2019	20	100.12473	100.0	0.12473	99.87548	0.12452
8% 2002	3	108.19851	108.16974	0.02877	108.14099	0.02875
8% 2009	10	123.25371	123.16520	0.08851	123.07678	0.08842
8% 2019	20	137.54308	137.38663	0.15645	137.23043	0.15620
0% 2002	3	79.40528	79.38322	0.02206	79.36118	0.02204
0% 2009	10	46.36226	46.31935	0.04291	46.27648	0.04287
0% 2019	20	21.49459	21.45482	0.03977	21.41513	0.03969
5% 2002	3	92.29309	92.26871	0.02438	92.24434	0.02437
5% 2009	10	79.92780	79.86976	0.05804	79.81176	0.05800
5% 2019	20	70.62188	70.54556	0.07632	70.46936	0.07620
8% 2002	3	100.02578	100.0	0.02578	99.97423	0.02577
8% 2009	10	100.06713	100.0	0.06713	99.93293	0.06707
8% 2019	20	100.09825	100.0	0.09825	99.90189	0.09811

All bonds have a maturity date of 30 September in the final year (eg., 8% 2009 issued 30/9/1999, matures 30/9/2009)

Table 8.1: Comparing basis point value for a set of bonds.

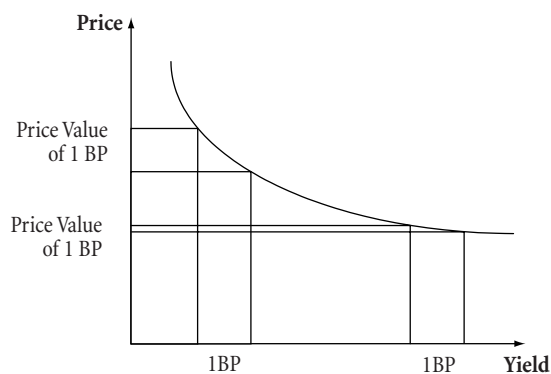


Figure 8.1: Illustrating bond basis point value.

Although BPV is a measure of change in the cash price of a bond, we can use it to calculate the percentage change in price resulting from a change in yield. This is given by (8.1):

$$\% \text{ price change} = \frac{\text{BPV}}{\text{Initial bond price}}. \quad (8.1)$$

Remember that the BPV for a bond is a snapshot measure; it is not static, and it is an approximation. It also differs according to whether the change in yield is an upward or downward move, except for short-dated bonds. Because BPV is an approximation it is not recommended as a risk measure to estimate exposure for large changes in yields. Nevertheless it is common for practitioners to use the BPV to estimate the price exposure for a large move, say 10 basis points. In this case the BPV is simply multiplied by the yield change to obtain the price value of a multiple basis point change. However such a measure has little real value as it will be substantially inaccurate for all but very short-dated bonds.

8.1.2 Portfolio BPV

Just as we calculated modified duration for a portfolio of bonds in the previous chapter, we can also calculate a portfolio BPV. Consider again our hypothetical portfolio introduced earlier, shown in Table 8.2.

Bond	Nominal value	Yield	Price	Market value	Yield	Price	Market value
5% 2002	£5m	5.50%	98.65103	£4,932,551.5	5.51%	98.62431	£4,931,215.50
0% 2009	£10m	5.50%	58.54310	£5,854,310.0	5.51%	58.48760	£5,848,760
8% 2019	£3m	5.50%	129.87596	£3,896,278.8	5.51%	129.73178	£3,891,953.40
8% 2002	£2m	5.50%	106.74483	£2,134,896.6	5.51%	106.71660	£2,134,332
				<u>£16,818,036.9</u>			
							<u>£16,806,260.90</u>

Table 8.2: Portfolio BPV.

If we know the individual BPVs for each bond we can work out the change in price for the basis point move; or, as we show here, we work out the new market values for the individual bonds resulting from the change in yield. In table we show the change after the yield has moved up 1 basis point to 5.51%. This shows a change in the market value of £11,776 and this is the portfolio basis point value.

8.2 Yield value of price change

In some markets portfolio managers use another measure of bond risk, known as yield value of a price change. This measures the change in the yield of a bond for a given change in price. To do this we calculate the new yield of a bond if the price is changed by a specified amount. There is no real market convention for this, in fact it is common to hear the expression “yield value of a basis point”, referring to the change in yield resulting from a 0.01 currency amount change in price. This is not recommended however because generally the market convention is to use the expression “basis point” as a unit of measurement of yield, not price.

We do not need a new figure to illustrate yield value of a price change, as the same graph applies. In Figure 8.1 it is equally valid to take a plot of the price on the y-axis and observe the new yield that results. The user selects the amount of price change to calculate. One point to note is that a smaller yield value of a price change indicates a higher bond price volatility. This is the natural result of the price/yield relationship, and means that a larger cash price move is required to change the yield by a given number of basis points.

EXAMPLE 8.1 Yield value of a basis point for 8% 2009 bond

■ At a price of £99.500 the bond trades at a yield of 8.07477%. What is the yield if the price falls to 99.490?

At a price of 99.49 the new yield is 8.07627. The “yield value of a basis point” is therefore 0.0015 (or 0.15 of a basis point).

The US Treasury market quotes bond prices in 32nds of a percentage point. A 32nd unit is known as a “tick”. For example a Treasury bond may be quoted at a price of “98-18”, which means that the price is \$98-18/32, or \$98.5625.² The value of one tick is therefore \$0.03125. For this reason investors in the US market often use a measure known as *yield value of a 32nd* which is the change in yield for a bond resulting from a one tick change in price. The

² This was also the convention in the UK gilt market until November 1998, when quotes changed to decimal pricing. Other markets that have tick pricing include the South African and Indian government bond markets (where the bonds are also known as gilts!).

yield value of a 32nd is lower for longer duration (and modified duration) bonds compared to shorter duration bonds and reflects the higher price volatility of longer duration bonds.

8.3 Hedging using basis point value

We have noted that modified duration and basis point value are measures of interest rate risk for bonds. How then do traders and fund managers use these measures as part of their risk management? The price/yield relationship for fixed interest instruments illustrates that a position in a bond carries with it interest rate risk exposure. If market yields rise, the required yield on the bond will rise, thus moving the bond price down. The bondholder then suffers a capital loss on her position. For this reason in a trading environment traders and fund managers may wish to *hedge* their positions to protect against capital loss.³ Note that a hedge is protection against loss; it does not improve the profit outlook of a position, and it does not allow for any upside gain. A perfect hedge should, from the day it is put on, result in a zero profit-and-loss position each day.

To hedge a bond position requires an opposite position to be taken in the hedging instrument. So if we are long of a 10-year bond, we may wish to sell short a similar 10-year bond as a hedge against it. Similarly a short position in a bond will be hedged through a purchase of an equivalent amount of the hedging instrument. There are a variety of hedging instruments available, both on- and off-balance sheet and we will discuss them in a later chapter. Once the hedge is put on, any loss in the primary position should in theory be offset by a gain in the hedge position, and vice-versa. How much of the hedging instrument should be bought or sold? This is where BPVs are used. The objective of a hedge is to ensure that the price change in the primary instrument is equal to the price change in the hedging instrument. If we are hedging a position with another bond, we use the BPVs of each bond to calculate the amount of the hedging instrument required. This is important because each bond will have different BPVs, so that to hedge a long position in say £1 million nominal of a 30-year bond does not mean we simply sell £1 million of another 30-year bond. This is because the BPVs of the two bonds will almost certainly be different. Also there may not be another 30-year bond in that particular bond. What if we have to hedge with a 10-year bond? How much nominal of this bond would be required?

We need to know the ratio given at (8.2) to calculate the nominal hedge position.

$$\frac{BPV_p}{BPV_h} \quad (8.2)$$

where

BPV_p is the basis point value of the primary bond (the position to be hedged)

BPV_h is the basis point value of the hedging instrument.

The *hedge ratio* is used to calculate the size of the hedge position and is given at (8.3):

$$\frac{BPV_p}{BPV_h} \times \frac{\text{Change in yield for primary bond position}}{\text{Change in yield for hedge instrument}} \quad (8.3)$$

The second ratio in (8.3) is known as the *yield beta* in the US market. Example 8.2 illustrates the use of the hedge ratio.

EXAMPLE 8.2 Calculating hedge size using basis point value

- A trader holds a long position in £1 million of the 8% 2019 bond. The modified duration of the bond is 11.14692 and its price is 129.87596. The basis point value of this bond is therefore 0.14477. The trader decides, to protect against a rise in interest rates, to hedge the position using the 0% 2009 bond, which has a BPV of 0.05549. If we assume that the yield beta is 1, what nominal value of the zero-coupon bond must be sold in order to hedge the position?

The hedge ratio is:

³ In an environment where bonds were purchased and held to maturity, there is some justification for not *marking-to-market* the bond price each day, that is, not recording profit or loss when bond prices moved. However the market convention for both traders and fund managers is to mark the prices of positions on a daily basis.

$$\frac{0.14477}{0.05549} \times 1 = 2.60894.$$

Therefore, to hedge £1 million of the 20-year bond the trader shorts £2,608,940 of the zero-coupon bond. If we use the respective BPVs to see the net effect of a 1 basis point rise in yield, the loss on the long position is approximately equal to the gain in the hedge position.

A hedge ratio can also be calculated using the yield value of a price change. The ratio given at (8.3) is adjusted and yield values for each bond used instead of basis point values.

8.4 Volatility weighting for hedging

In some markets modified duration is referred to as *volatility*. This is less frequent these days as the term can be confused with option traders use of both this term and the term *implied volatility*. As we have just seen traders will use modified duration properties of instruments when setting up hedges for positions they have on their books. When entering into a hedge transaction, we seek to minimise a risk arising from the primary position, presumably because we do not want to simply remove the position and eliminate the risk. For various reasons, including a desire not to crystallise a loss or because there are no counterparties available, this will frequently be the case. In its simplest form the hedge we have described attempts to offset price changes in one bond (that result from changes in market yield levels) with equal changes in another bond. Because most securities in the debt markets are positively correlated with one another in terms of price movement, a short position normally counteracts the price movement in a long position. We have already seen that different bonds experience different price changes as a result of the same change in yield, which is why we cannot simply hedge using an identical amount of bonds. Therefore we use the hedge ratio described in the previous section. Weighting by basis point value is not the same as “duration weighted”, although the terms are often used synonymously. The ratio of the durations is not in fact the correct hedge ratio, rather the ratio we should use is that of the basis point values.

The objective of weighting a hedge position is to equalise the total changes in value of the two offsetting positions. The hedge ratio is therefore expressed mathematically as (8.4):

$$HR = \frac{\Delta P_a}{\Delta r_a} \times \frac{\Delta r_a}{\Delta r_b} \times \frac{\Delta r_b}{\Delta P_b} \quad (8.4)$$

where

$\frac{\Delta P_a}{\Delta r_a}$ is the change in price of bond *a* for a given change in yield

$\frac{\Delta r_a}{\Delta r_b}$ is the change in yield on bond *a* relative to a change in yield for bond *b*

$\frac{\Delta r_b}{\Delta P_b}$ is the reciprocal of change in price of bond *b* for a given change in yield.

It is the coupon and term to maturity of conventional bonds that determine their price volatility. For a given term to maturity and initial yield, the price volatility of a bond is greater the lower the coupon rate. For a given coupon rate and initial yield, the longer the term to maturity the greater will be the price volatility. This tends to imply that bondholders who wish to increase a portfolio's price volatility because they expect interest rates to fall (and hence bond prices to rise) should hold long-dated bonds in the portfolio. If interest rates are expected to rise, investors should hold shorter-dated bonds.

EXAMPLE 8.3 Hedging using basis point value

- Consider a market maker's book position of £40 million in five-year bonds. The trader expects a rise shortly of 10 basis points in five-year bond yields. She decides to hedge the position using another bond. The basis point values for various bonds are shown below.

Bond	BPV
3-year	0.022898
5-year	0.041583
10-year	0.059404

The trader expects a higher rise in shorter-dated bond yields and decides to sell short the three-year bond. Calculate the nominal value of the new holding.

$$\frac{\text{BPV current position}}{\text{BPV proposed position}} = \frac{0.041583}{0.022898} = 1.816.$$

Therefore the hedge position will be short of $\text{£}40\text{m} \times 1.816 = \text{£}72.64$ million of the three-year bond.

■ Non-parallel shift

Assume that the yield curve changes shape, with a 10bp rise in five-year yields and a 8bp rise in three-year yields. Calculate the gain made by the fund manager in the three-year position, and compare this to the loss on the five-year position.

The profit/loss on a position is calculated using:

$$\frac{\text{BPV}}{100} \times \text{basis points} \times \text{£nominal}.$$

The gain in the three-year bond position is $\frac{0.022898}{100} \times 8 \times \text{£}72.64\text{m} = +\text{£}133,065$.

The loss on the five-year bond position is $\frac{0.041583}{100} \times 10 \times \text{£}40\text{m} = -\text{£}166,332$.

■ Parallel shift

Assume that the entire yield curve shifts upwards by 10 basis points. Using the BPVs, calculate the net p/l of the book. What can we see about the trader's risk position with respect to parallel shifts in the yield curve?

The loss on the long five-year position is $\text{£}166,332$ as before.

The gain on the new three-year hedge position is $\frac{0.022898}{100} \times 10 \times \text{£}72.64\text{m} = \text{£}166,331$.

We can see that the trader's long five-year position is hedged with respect to parallel shifts in the yield curve. The BPV of a bond is only a snap-shot in time and will fluctuate with changes in price, yield and duration.

■ Modified duration and BPV

■ Consider a 10% sterling Eurobond maturing on 10 February 2009. For settlement on 10 February 1999 it was trading at 118.00 with a Macaulay duration of 7.46 years and a yield of 7.535%. Calculate:

(a) the modified duration?

$$MD = \frac{\text{Macaulay duration}}{(1 + \text{yield})} = \frac{7.46}{1.07535} = 6.937.$$

(b) the basis point value?

$$BPV = \frac{MD}{100} \times \frac{PV}{100} = \frac{6.937}{100} \times 1.18 = 0.0819.$$

(c) What would the bond price be if its yield fell by 1 basis point? As the yield has fallen by 1 basis point, the price has risen and the new price will be $118.00 + 0.0819$, or 118.08.

(d) If the bond price rises to 120.00, resulting in a yield of 7.294% and a Macaulay duration of 7.49 years, what is the bond's new basis point value? The new modified duration is $7.49/1.07294$, which is 6.981. Using the same formula again we calculate the new basis point value to be 0.0838. The BPV has risen with a drop in yield, indicating (all else being equal) a higher inter-rate risk.

Selected bibliography and references

Tuckman, B., *Fixed Income Securities*, Wiley, 1996.

Questions and exercises

1. Consider these two bonds:

	(1)	(2)
Coupon	5%	6%
Yield	5.3%	5.3%
Maturity (years)	3	5
Price	99.188	103.006

In the previous you were asked to calculate the modified duration of these bonds. What is the BPV for the each of the bonds? If I hold €12.2 million of bond 1, how much of bond 2 would I need to short to hedge the position?

2. Examine the following statements and assess whether they are true or false. Explain your answers.
 - (a) The duration of a zero-coupon bond is equal in years to its term to maturity. Therefore its interest rate sensitivity does not change with changes in its yield.
 - (b) At low relative yield levels, using basis point values to estimate interest rate risk for a 50 basis point yield change will give us an accurate estimate of our exposure.
 - (c) If the two bonds have the same duration, yield and price, their BPVs will be identical.
 - (d) For a small change in yield level, the BPV value for a bond will give us a similar risk estimate as the yield value of a price change value.
3. Calculate the BPVs for each of the bonds in Table 8.1 from an initial yield level of 5%.
4. The hedge ratio is used to calculate the nominal value required of an hedging instrument. Do you think the trader can leave the hedge amount unchanged over time if the market is experiencing a large movement in yields? Why do you think the hedge will be less and less exact over time if there are large fluctuations in yield levels?

9 Price, Yield and Interest Rate Risk III

The previous chapters have discussed the bond price and yield relationship and have emphasised how the convex nature of this relationship gives rise to many of its properties. This chapter explores the relationship further, particularly how the convex shape of the price/yield curve affects bond trading.

9.1 Convexity

9.1.1 Approximate nature of the modified duration measure

An examination of Table 7.1 makes clear that the actual price change resulting from a 1% change in yield for a particular bond exceeds the estimate given by using say, that bond's basis point value. This is because the BPV is derived from the bond's modified duration, which itself derived from the duration measure. Figure 9.1 shows how the modified duration measure is actually an approximation of the change in price resulting from a change in yield, as it measures the tangent drawn at a particular yield. Since modified duration is therefore only accurate for very small changes in yield, it will produce errors if used to measure the price effect of a large change in yield. That is, duration and modified duration does not take into account the *convexity* of bond price with respect to its yield.

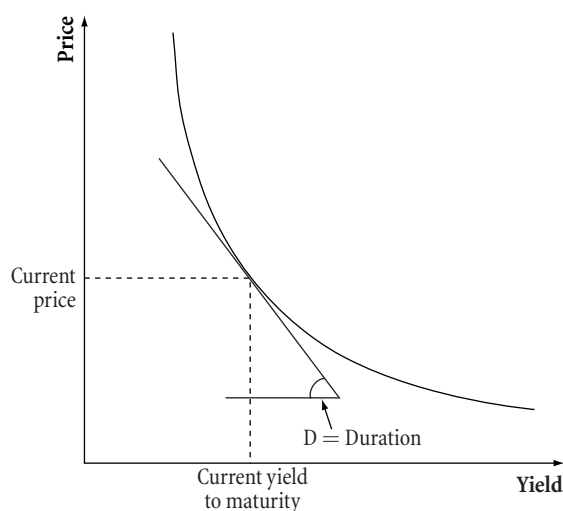


Figure 9.1: Drawing a tangent to the bond price/yield relationship curve.

Duration can be regarded as a first-order measure of interest rate risk: it measures the *slope* of the present value/yield profile. It is however only an approximation of the actual change in bond price given a small change in yield to maturity. Similarly for modified duration, which describes the price sensitivity of a bond to small changes in yield. As Figure 9.2 shows, this approximation breaks down under large changes in yield as it is a straight line approximation of one point on a non-linear curve. The tangent drawn through at the point of measurement has a slope proportional to the bond's modified duration, which is calculated as:

$$-MD \times P_d . \quad (9.1)$$

The tangent drawn against the price/yield curve can be used to measure the price change if the yield on the bond changes. However, as Figure 9.2 illustrates, the approximation is an underestimate of the actual price at the new yield. This is the weakness of the duration measure.

Convexity is a second-order measure of interest rate risk; it measures the *curvature* of the present value/yield profile. It can be regarded as an indication of the error we make when using duration and modified duration, as it measures the degree to which the curvature of a bond's price/yield relationship diverges from the straight-line

estimation. The convexity of a bond is positively related to the dispersion of its cash flows thus, other things being equal, if one bond's cash flows are more spread out in time than another's, then it will have a higher *dispersion* and hence a higher convexity. Convexity is also positively related to duration.

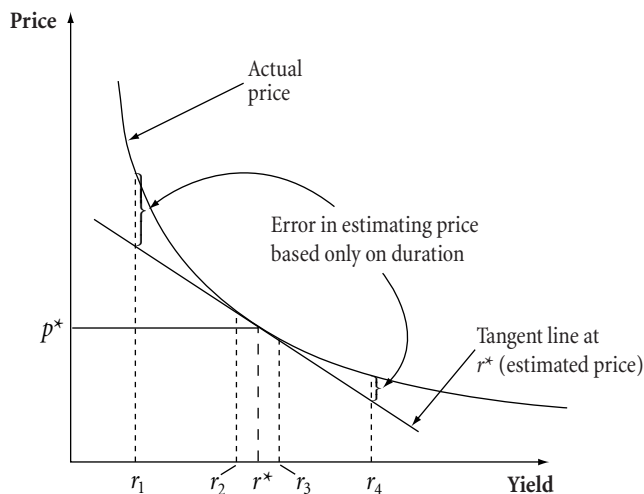


Figure 9.2: Approximation of the bond price change using modified duration.

9.1.2 The convexity measure

Figure 9.2 illustrates the weakness of the modified duration in estimating the price volatility of a bond under conditions of large yield changes. The markets therefore use another measure to assess the extent of the price change for a given change in yield, which is the *convexity* value of a bond. The second-order differential of the bond price equation with respect to the redemption yield r is:

$$\begin{aligned}\frac{\Delta P}{P} &= \frac{1}{P} \frac{\Delta P}{\Delta r} (\Delta r) + \frac{1}{2P} \frac{\Delta^2 P}{\Delta r^2} (\Delta r)^2 \\ &= -MD(\Delta r) + \frac{CV}{2} (\Delta r)^2\end{aligned}\tag{9.2}$$

where

- P is the bond price
- MD is the modified duration
- CV is the convexity.

From equation (9.2), convexity is the rate at which price variation to yield changes with respect to yield. That is, it describes a bond's modified duration changes with respect to changes in interest rates. It can be approximated by expression (9.3):

$$CV = 10^8 \left(\frac{\Delta P'}{P} + \frac{\Delta P''}{P} \right)\tag{9.3}$$

where

- $\Delta P'$ is the change in bond price if yield increases by 1 basis point (0.01)
- $\Delta P''$ is the change in bond price if yield decreases by 1 basis point (0.01).

Appendix 9.1 provides the mathematical derivation of the formula.

EXAMPLE 9.1

- A 5% annual coupon is trading at par with three years to maturity. If the yield increases from 5 to 5.01%, the price of the bond will fall (using the bond price equation) to:

$$P'_d = \frac{5}{(0.0501)} \left(1 - \frac{1}{(1.0501)^3} \right) + \frac{100}{(1.0501)^3} = 99.97277262$$

or by $\Delta P'_d = -0.02722738$. If the yield falls to 4.99%, the price of the bond will rise to:

$$P''_d = \frac{5}{(0.0499)} \left(1 - \frac{1}{(1.0499)^3} \right) + \frac{100}{(1.0499)^3} = 100.027237$$

or by $\Delta P''_d = 0.02723695$. Therefore

$$CV = 10^8 \left(\frac{-0.02722738}{100} + \frac{0.02723695}{100} \right) = 9.57$$

that is, a convexity value of approximately 9.57.

9.1.3 Calculating convexity

It is clear from Chapter 7 that each bond will have a unique price/yield relationship and therefore will also possess its own convexity measure. Figure 9.3 illustrates the price profile for two hypothetical bonds, one of which has a more convex price/yield relationship than the other. That means that this bond has a higher convexity. (In reality the actual profile for any bond will not be quite so convex, as generally for a vanilla bond the profile flattens out as the yield increases, but the graphs are exaggerated here for effect.)

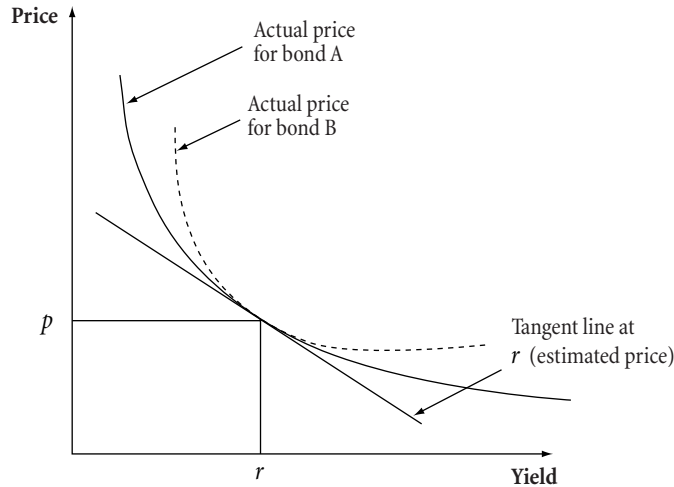


Figure 9.3: Convexity and the price/yield relationship for two hypothetical bonds.

The previous section demonstrated that convexity is a function of duration. Hence to calculate convexity long-hand we use the present value of the cash flows and yield for a bond, adjusted for the time period of each cash flow. With these convexity is calculated using the expression at (9.4), which gives us the convexity measurement as the number of interest rate periods to maturity, at a given yield level. The expression applies to annual coupon bond with a complete number of interest periods to maturity. For semi-annual coupon bonds we use $r/2$.

$$CV = \frac{1 \times 2PVC_1 + 2 \times 3PVC_2 + 3 \times 4PVC_3 + \dots + n \times (N + 1)PVC_n}{(1 + r)^2 P} \quad (9.4)$$

where PVC_n is the present value of the cash flow payable in interest period n .

The unit of measurement for convexity using (9.4) is the number of interest periods. For annual coupon bonds this is equal to the number of years; for bonds paying coupon on a different frequency we use (9.5) to convert the convexity measure to years.

$$CV_{years} = \frac{CV}{C^2} \quad (9.5)$$

where c is the number of coupon payments each year.

The convexity measure for a zero-coupon bond is given by (9.6):

$$CV = \frac{n(n+1)}{(1+r)^2}. \quad (9.6)$$

Note that another expression to calculate convexity long-hand is also encountered in some texts, given by (9.7):

$$CV = \frac{1^2 PVC_1 + 2^2 PVC_2 + 3^2 PVC_3 + \dots + n^2 PVC_n}{(1+r)^2 \times P}. \quad (9.7)$$

EXAMPLE 9.2 Convexity calculation for the 5% 2009 bond when trading at 5.50%

- This calculation is carried out when the bond has precisely 10 years to maturity.

5% 2009 bond
Price 96.23119
Yield 5.50%
Date 30/9/1999

Year	Interest period (n)	Cash flow	PVC _n	n(n+1)	PVC × n(n+1)
2000	1	5	4.73934	2	9.47868
2001	2	5	4.49226	6	26.95356
2002	3	5	4.25807	12	51.09684
2003	4	5	4.03608	20	80.7216
2004	5	5	3.82567	30	114.7701
2005	6	5	3.62623	42	152.30166
2006	7	5	3.43718	56	192.48208
2007	8	5	3.25799	72	234.57528
2008	9	5	3.08815	90	277.9335
2009	10	105	61.47021	110	6761.7231
					<u>7902.03640</u>

Table 9.1: Convexity calculation for 5% 2009 bond.

The convexity for the bond is therefore $\frac{7902.03640}{(1.055)^2 \times 96.23119} = 73.77653$.

Because our hypothetical bond is an annual coupon bond, this is also the convexity measure in years. If the bond paid on another interest basis, we would convert the value to years using (9.5).

If we consider Figure 9.3 then, convexity measures the relationship between the percentage price change and the percentage yield change. We label the percentage price change as dP/P and the percentage yield change as $dr/1+r$; the more convex the relationship between dP/P and $dr/1+r$, the bigger the error made from using modified duration to measure the interest rate risk. Appendix 9.3 shows how a Taylor expansion of the price/yield relationship is used to derive the convexity formula shown at the beginning of the chapter.

9.1.4 Approximation of convexity measure

Fabozzi (1997) describes an approximate convexity value for a vanilla bond using (9.8):

$$CV = \frac{P_+ + P_- - 2P_0}{P_0 (\Delta r)^2} \quad (9.8)$$

where

- P_0 is the initial price of the bond
- Δr is the change in yield of the bond
- P_+ is the price of the bond estimated for a fall in yield of Δr
- P_- is the price of the bond estimated for a rise in yield of Δr .

As the measure given by (9.8) is an estimate of what is, at best, an approximate value in the first place, it should only be used for rough-and-ready analysis and not as the basis for investment or trading decisions. Equation (9.8) is also used to calculate the convexity of a bond whose cash flows change with changes in yield. Such a convexity measure is also known as *effective convexity*.

9.1.5 Using convexity

Convexity is a second-order approximation of the change in price resulting from a change in yield. This is given by:

$$\Delta P = \frac{1}{2} \times CV \times (\Delta r)^2. \quad (9.9)$$

The reason we multiply the convexity by $\frac{1}{2}$ to obtain the convexity adjustment is because the second term in the Taylor expansion contains the coefficient $\frac{1}{2}$. The convexity approximation is obtained from a Taylor expansion of the bond price formula.

The formula is the same for a semi-annual coupon bond.

Note that the value for convexity given by the expression above will always be positive, as the approximate price change due to convexity is positive for both yield increases and decreases.

EXAMPLE 9.3 Second-order interest rate risk

- A 5% annual coupon bond is trading at par with a modified duration of 2.639 and convexity of 9.57. If we assume a significant market correction and yields rise from 5 to 7%, the price of the bond will fall by:

$$\begin{aligned} \Delta P_d &= -MD \times (\Delta r) \times P_d + \frac{CV}{2} \times (\Delta r)^2 \times P_d \\ &= -(2.639) \times (0.02) \times 100 + \frac{9.57}{2} \times (0.02)^2 \times 100 \\ &= -5.278 + 0.1914 = -£5.0866. \end{aligned}$$

to £94.9134. The first-order approximation, using the modified duration value of 2.639, is $-£5.278$, which is an overestimation of the fall in price by £0.1914.

EXAMPLE 9.4

- The 5% 2009 bond is trading at a price of £96.23119 (a yield of 5.50%) and has precisely ten years to maturity. If the yield rises to 7.50%, a change of 200 basis points, the percentage price change due to the convexity effect is given by:

$$(0.5) \times 96.23119 \times (0.02)^2 \times 100 = 1.92462\%.$$

If we use an HP calculator to find the price of the bond at the new yield of 7.50% we see that it is £82.83980 (see Table 7.1 in Chapter 7), a change in price of 13.92%. The convexity measure of 1.92462% is an approximation of the error we would make when using the modified duration value to estimate the price of the bond following the 200 basis point rise in yield.

If the yield of the bond were to fall by 200 basis points, the convexity effect would be the same, as given by (9.9).

In Example 9.3 we saw that the price change estimated using modified duration will be significantly inaccurate, and that the convexity measure is the approximation of the size of the inaccuracy. The magnitude of the price change as estimated by both duration and convexity is obtained by summing the two values. However it only makes any significant difference if the change in yield is very large. If we take our hypothetical bond again, the 5% 2009 bond, its modified duration is 7.64498. If the yield rises by 200 basis points, the approximation of the price change given by modified duration and convexity is:

$$\text{Modified duration} = 7.64498 \times 2 = -15.28996$$

$$\text{Convexity} = 1.92462.$$

Note that the modified duration is given as a negative value, because a rise in yields results in a fall in price. This gives us a net percentage price change of 13.36534. The actual percentage price change is 13.92%. So in fact using the convexity adjustment has given us a noticeably more accurate estimation. Let us examine the percentage price change resulting from a fall in yields of 1.50% from the same starting yield of 5.50%. This is a decrease in yield of 150 basis points, so our convexity measurement needs to be recalculated.

$$\text{The convexity value is } (0.5) \times 96.23119 \times (0.0150)^2 \times 100 = 1.0826\%.$$

So the price change is based on:

$$\text{Modified duration} = 7.64498 \times 1.5 = 11.46747$$

$$\text{Convexity} = 1.0826.$$

This gives us a percentage price change of 12.55007. The actual price change was 10.98843%, so here the modified duration estimate is actually closer! This illustrates that the convexity measure is effective for larger yield changes only; Example 9.5 shows us that for very large changes, a closer approximation for bond price volatility is given by combining the modified duration and convexity measures.

EXAMPLE 9.5

- The hypothetical bond is the 5% 2009, again trading at a yield of 5.50% and priced at 96.23119. If the yield rises to 8.50%, a change of 300 basis points, the percentage price change due to the convexity effect is given by:

$$(0.5) \times 96.23119 \times (0.03)^2 \times 100 = 4.3304\%.$$

Meanwhile as before the modified duration of the bond at the initial yield is 7.64498. At the new yield of 8.50% the price of the bond is 77.03528 (check using an HP calculator; the prices are also given in Table 7.1).

The price change can be approximated using:

$$\text{Modified duration} = 7.64498 \times 3.0 = -22.93494$$

$$\text{Convexity} = 4.3304.$$

This gives a percentage price change of 18.60454%. The actual percentage price change was 19.9477%, but our estimate is still closer than that obtained using only the modified duration measure. The continuing error reflects the fact that convexity is also a dynamic measure and changes with yield changes; the effect of a large yield movement compounds the inaccuracy given by modified duration.

9.1.6 Convexity cash value

Just as the modified duration measure for a bond is used to calculate the cash value resulting from a yield change, by multiplying the value by the initial price, so a cash convexity value is obtained by multiplying the convexity value by the initial price. In the US markets this is sometimes referred to as *dollar convexity*. To calculate the price change as a result of the convexity effect we use (9.10):

$$\Delta P_C = CV \times (\Delta r)^2 \quad (9.10)$$

where ΔP_C is the value of the price change due to the convexity effect. For a semi-annual coupon bond the right-hand side of (9.10) is multiplied by 0.5.

The convexity value of a bond is not static and moves with changes in yield. This is apparent from examining Figure 9.4 below, with which we are familiar from Chapter 7.

A particular convexity measure and cash convexity value will be measures calculated at the tangent drawn to one point on the price/yield curve. For a conventional vanilla bond, the properties of the price/yield curve are such that as the redemption yield rises above r (see Figure 9.4) the cash convexity value falls, and as the yield falls the convexity value increases. This convexity feature of a bond is of value to bondholders, because it illustrates that as the yield on a bond falls, the increase in price that results is greater for bonds with greater convexity. If there is a rise in yields, the resulting fall in price is minimised for bonds of the greatest convexity. In theory then investors will hold the bond with the highest convexity, all else being equal. However the market will have taken the convexity effect of a bond into account when setting its price, so that the expression “all else being equal” has little meaning in practice. If two bonds of similar duration are trading in the market, the bond with greater convexity will usually be priced to reflect a lower yield, since its higher convexity will make it more attractive to investors.

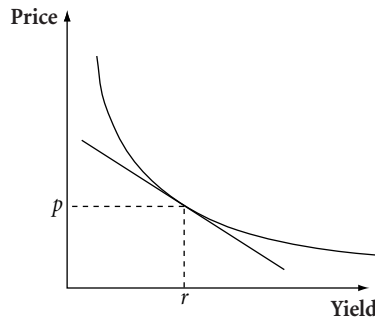


Figure 9.4

What level of premium will be attached to a bond's higher convexity? This is a function of the current yield levels in the market as well as market volatility. Remember that modified duration and convexity are functions of yield level, and that the effect of both is magnified at lower yield levels. As well as the relative level, investors will value convexity higher if the current market conditions are volatile. The cash effect of convexity is noticeable only for large moves in yield. If an investor expects market yields to move only by relatively small amounts, they will attach a lower value to convexity; and vice versa for large movements in yield. Therefore the yield premium attached to a bond with higher convexity will vary according to market expectations of the future size of interest rate changes.

9.2 Summarising the properties of convexity

The convexity measure increases with the square of maturity, and it decreases with both coupon and yield. As the measure is a function of modified duration, index-linked bonds have greater convexity than conventional bonds. We discussed how the price/yield profile will be more convex for a bond of higher convexity, and that such a bond will outperform a bond of lower convexity whatever happens to market interest rates. High convexity is therefore a desirable property for bonds to have. In principle a more convex bond should fall in price less than a less convex one when yields rise, and rise in price more when yields fall. That is, convexity can be equated with the potential to outperform. Thus other things being equal, the higher the convexity of a bond the more desirable it should in principle be to investors. In some cases investors may be prepared to accept a bond with a lower yield in order to gain convexity. We noted also that convexity is in principle of more value if uncertainty, and hence expected market volatility, is high, because the convexity effect of a bond is amplified for large changes in yield. The value of convexity is therefore greater in volatile market conditions.

For a conventional vanilla bond convexity is almost always positive. Negative convexity resulting from a bond with a concave price/yield profile would not be an attractive property for a bondholder; the most common occurrence of negative convexity in the cash markets is with callable bonds.

We illustrated that for most bonds, and certainly when the convexity measure is high, the modified duration measurement for interest rate risk becomes more inaccurate for large changes in yield. In such situations it becomes necessary to use the approximation given by our convexity equation, to measure the error we have made in estimating the price change based on modified duration only.

The following points highlight the main convexity properties for conventional vanilla bonds.

- **A fall in yields leads to an increase in convexity.** A decrease in bond yield leads to an increase in the bond's convexity; this is a property of positive convexity. Equally a rise in yields leads to a fall in convexity. This can be observed if from the price/yield profile shown at Figure 9.5. The profile is for one of the hypothetical bonds from Chapter 7, the 11% 2019, drawn when the bond has a term to maturity of precisely 20 years. The tangent lines for the three yields r_1 , r_2 , and r_3 show how the duration decreases as the bond yield increases. A property of positive convexity is that it is a function of modified duration, and that (as shown in Figure 9.5) it measures the rate of change of modified duration as the yield changes.

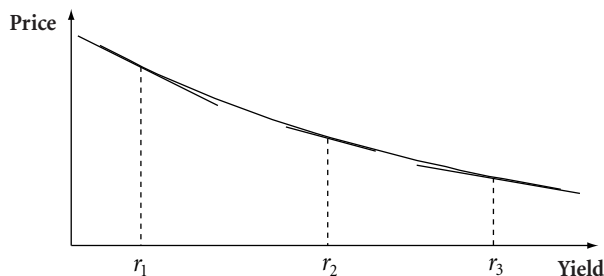


Figure 9.5: The 11% 2019 bond (term to maturity 20 years).

- **For a given term to maturity, higher coupon results in lower convexity.** For any given redemption yield and term to maturity, the higher a bond's coupon, the lower its convexity. Therefore among bonds of the same maturity, zero-coupon bonds have the highest convexity.
- **For a given modified duration, higher coupon results in higher convexity.** For any given redemption yield and modified duration, a higher coupon results in a higher convexity. Contrast this with the earlier property; in this case, for bonds of the same modified duration, zero-coupon bonds have the lowest convexity.
- **For increasing duration, bonds have increasing convexity.** Figure 9.6 illustrates how the convexity of a bond increases at a faster rate as the duration measure increases. The rate of increase is such that, at a given redemption yield level, a bond of twice the modified duration as another bond will have a convexity value greater than twice that of the other bond.

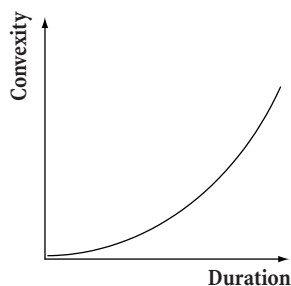


Figure 9.6: Duration and convexity relationship.

9.3 Dispersion

One other interest rate risk value is a little-used measure known as *dispersion*. It measures the variance in the timing of a bond's cash flows around its duration date, using the cash flows' present values as weights. The value is given by equation (9.11):

$$DS^2 = \frac{C}{P} \sum_{n=1}^N \frac{n^2}{(1+r)^n} + \frac{M}{P} \frac{N^2}{(1+r)^N} - D^2 \quad (9.11)$$

where DS^2 is dispersion.

When cash flows are widely dispersed, DS^2 will be a large number. Duration, convexity and dispersion are all related, as shown by equation (9.12).

$$CV = \frac{DS^2 + D^2 + D}{(1 + r)^2}. \quad (9.12)$$

Appendices

APPENDIX 9.1 Measuring convexity

In Chapter 8 we illustrated the first derivative of the bond price equation, with respect to the yield. From this we obtained the expressions for Macaulay duration and modified duration. The modified duration of a plain vanilla bond was given as:

$$MD = \frac{D}{(1 + r)}. \quad (9.13)$$

It was also shown that:

$$\frac{dP}{dr} \frac{1}{P} = -MD. \quad (9.14)$$

This shows that for a percentage change in the yield we have an inverse change in the price by the amount of the modified duration value.

If we multiply both sides of (9.14) by any particular change in the bond yield, given by dr , we obtain expression (9.15):

$$\frac{dP}{P} = -MD \times dr. \quad (9.15)$$

Using the first two terms of a Taylor expansion, we obtain an approximation of the bond price change, given by (9.16):

$$dP = \frac{dP}{dr} dr + \frac{1}{2} \frac{d^2P}{dr^2} (dr)^2 + \text{approximation error}. \quad (9.16)$$

If we divide both sides of (9.16) by P to obtain the percentage price change the result is (9.17):

$$\frac{dP}{P} = \frac{dP}{dr} \frac{1}{P} dr + \frac{1}{2} \frac{d^2P}{dr^2} \frac{1}{P} (dr)^2 + \frac{\text{approximation error}}{P}. \quad (9.17)$$

The first component of the right-hand side of (9.16) is the expression at (9.15), which is the cash price change given by the duration value. Therefore equation (9.16) is the approximation of the price change. Equation (9.17) is the approximation of the price change as given by the modified duration value. The second component in both expressions is the second derivative of the bond price equation. This second derivative captures the convexity value of the price/yield relationship and is the cash value given by convexity. As such it is referred to as *dollar convexity* in the US markets. The dollar convexity is stated as (9.18):

$$CV_{dollar} = \frac{d^2P}{dr^2}. \quad (9.18)$$

If we multiply the dollar convexity value by the square of a bond's yield change we obtain the approximate cash value change in price resulting from the convexity effect. This is shown by (9.19):

$$dP = (CV_{dollar})(dr)^2. \quad (9.19)$$

If we then divided the second derivative of the price equation by the bond price, we obtain a measure of the percentage change in bond price as a result of the convexity effect. This is the measure known as *convexity* and is the convention used in virtually all bond markets. This is given by (9.20):

$$CV = \frac{d^2P}{dr^2} \frac{1}{P}. \quad (9.20)$$

To measure the amount of the percentage change in bond price as a result of the convex nature of the price/yield relationship we can use (9.21):

$$\frac{dP}{P} = \frac{1}{2} CV (dr)^2. \quad (9.21)$$

For long-hand calculations note that the second derivative of the bond price equation is (9.22), which can be simplified to (9.24). The usual assumptions apply to the expressions, that the bond pays annual coupons and has a precise number of interest periods to maturity. If the bond is a semi-annual paying one the yield value r is replaced by $r/2$.

$$\frac{d^2P}{dr^2} = \sum_{n=1}^N \frac{n(n+1)C}{(1+r)^{n+2}} + \frac{n(n+1)M}{(1+r)^{n+2}}. \quad (9.22)$$

Alternatively we differentiate to the second order the bond price equation as given by (9.23), giving us the alternative expression (9.24).

$$P = \frac{C}{r} \left(1 - \frac{1}{(1+r)^n} \right) + \frac{100}{(1+r)^n}. \quad (9.23)$$

$$\frac{d^2P}{dr^2} = \frac{2C}{r^3} \left(1 - \frac{1}{(1+r)^n} \right) - \frac{2C}{r^2 (1+r)^{n+1}} + \frac{n(n+1)(100 - (C/r))}{(1+r)^{n+2}}. \quad (9.24)$$

APPENDIX 9.2 Duration and modified duration for the 5% 2009 bond (with precisely ten years to maturity), trading at a yield of 5.50%

Annual coupon:	5%
Issued:	30.09 1999
Maturity:	30.09 2009
Price:	96.23119
Yield:	5.500%

Period (n)	Cash flow	PV at current yield *	$n \times PV$
1	5	4.73934	4.73934
2	5	4.49226	8.98452
3	5	4.25807	12.77421
4	5	4.03608	16.14432
5	5	3.82567	19.12835
6	5	3.62623	21.75738
7	5	3.43718	24.06026
8	5	3.25799	26.06392
9	5	3.08815	27.79335
10	105	61.47021	614.7021
Total		<u>96.23118</u>	<u>776.14775</u>

* Calculated as $C/(1+r)^n$

Macauley duration = $776.14775 / 96.23119$

= 8.065449785 years.

Modified duration = $8.0654498 / 1.055 = 7.64498$.

Table 9.2: Duration and modified duration for 5% 2009 gilt (with precisely 10 years to maturity), trading at a yield of 5.50%.

APPENDIX 9.3 Taylor expansion of the price/yield function

The Taylor expansion dates from many years ago and was developed to perform calculations before the era of calculators. It is widely used in quantitative finance applications, for example the concepts of modified duration and convexity were developed from a Taylor expansion of the bond price/yield function. More recently the Taylor expansion was used to develop what is known as the “delta-gamma” adjustment in value-at-risk measurement.

Taylor’s expansion for a change in value G is

$$\Delta G = \frac{dG}{dx} \Delta x + \frac{1}{2} \frac{d^2 G}{dx^2} \Delta x^2 + \frac{1}{6} \frac{d^3 G}{dx^3} \Delta x^3 + \dots \quad (9.25)$$

In (9.25) the first term on the right-hand side is the first derivative and in terms of bond pricing would be the duration. The second term is the second derivative and is therefore equivalent to the convexity measure. The third term produces a negligible value and for this reason is often ignored in financial applications.

The formulas for duration and convexity were derived using Taylor’s expansion. We summarise the bond price formula as (9.26) where C represents the cash flows from the bond, including the redemption payment.

$$P = \sum_{n=1}^N \frac{C_n}{(1+r)^n}. \quad (9.26)$$

We therefore derive the following by differentiating first and then a second time:

$$\frac{dP}{dr} = - \sum_{n=1}^N \frac{n C_n}{(1+r)^{n+1}}, \quad (9.27)$$

$$\frac{d^2 P}{dr^2} = \sum_{n=1}^N \frac{n(n+1) C_n}{(1+r)^{n+2}}. \quad (9.28)$$

This then gives us:

$$\Delta P = \left[\frac{dP}{dr} \Delta r \right] + \left[\frac{1}{2!} \frac{d^2 P}{dr^2} (\Delta r)^2 \right] + \left[\frac{1}{3!} \frac{d^3 P}{dr^3} (\Delta r)^3 \right] + \dots \quad (9.29)$$

The first expression in (9.29) is the modified duration measure, while the second expression measures convexity. The more powerful the changes in yield, the more expansion is required to approximate the change to greater accuracy. Expression (9.29) therefore gives us the equations for modified duration and convexity, given by (9.30) and (9.31) respectively.

$$MD = - \frac{dP/dr}{P}. \quad (9.30)$$

$$CV = \frac{d^2 P/dr^2}{P}. \quad (9.31)$$

We can therefore state the following:

$$\frac{\Delta P}{P} = [-(MD) \Delta r] + \left[\frac{1}{2} (CV) (\Delta r)^2 \right] + \text{residual error}. \quad (9.32)$$

$$\Delta P = -[P(MD) \Delta r] + \left[\frac{P}{2} (CV) (\Delta r)^2 \right] + \text{residual error}. \quad (9.33)$$

EXAMPLE 9.6

- Consider a three-year bond with (annual) coupon of 5% and yield of 5%. At a price of par we have:

$$\frac{dP}{dr} = -\left(\frac{5}{(1.05)^2} + \frac{5(2)}{(1.05)^3} + \frac{105(3)}{(1.05)^4}\right) = 263.9048$$

$$D = \frac{dP}{dr} \left(\frac{1+r}{P}\right) = 263.9048 \left(\frac{1.05}{100}\right) = 2.771$$

$$MD = \frac{2.771}{1.05} = 2.639$$

$$\frac{d^2P}{dr^2} = \left(\frac{5(1)(2)}{(1.05)^3} + \frac{5(2)(3)}{(1.05)^4} + \frac{105(3)(4)}{(1.05)^5}\right) = 957.3179$$

$$CV = \frac{d^2P/dr^2}{P} = \frac{957.3179}{100} = 9.573$$

Selected bibliography

Allen, S.L., Kleinstein, A.D., *Valuing Fixed Income Investments and Derivative Securities*, New York Institute of Finance, 1991.

Bashir, A., *Bonds and interest rate risk*, OUP, 1996.

Bierwag, A., *Duration Analysis*, Ballinger, 1987.

Fabozzi, F., *Fixed Income Mathematics*, 3rd edition, McGraw-Hill, 1997.

Questions and exercises

1. Explain the limitations associated with using duration to estimate the extent of a bond's price sensitivity to changes in interest rates.
2. Consider these two bonds:

	(1)	(2)
Coupon	5%	6%
Yield	5.3%	5.3%
Maturity (years)	3	5
Price	99.188	103.006

In the previous chapter you were asked to calculate the modified duration of these bonds. Calculate the convexity of the two bonds, and estimate the change in price resulting from a 200 basis point rise in yields. By how much does this adjust the estimate made using only the modified duration measure?

3. In the previous question assess which bond would have greater price volatility for a 50 basis point change in yields.
4. Summarise the main properties of convexity with relation to yield, coupon, term to maturity and modified duration.

10 Price, Yield and Interest Rate Risk IV

The first part of this book has introduced bonds as financial instruments, and the concept of the yield curve. We have also introduced the concept of interest rate risk, as described by the duration, modified duration and convexity measures. So far we have described the primary analytical tools used in the market and we have also considered how these are not always accurate; for example the yield to maturity measurement holds only under certain restrictive assumptions, and the modified duration tool of risk measurement is a static approximation in a dynamic environment. The yield curve itself is not static and will change level and shape over relatively short periods of time. Changes in the shape of the yield curve will affect individual bonds in different ways, which is one reason why the modified duration measurement cannot be applied to a portfolio of bonds. In this chapter we will look at the different types of changes that affect yield curves and how risk measurement tools can be modified to suit.

10.1 Yield curve changes

The measurement tools we encountered in the previous three chapters are only effective in the case of a *parallel shift* in the yield curve. A parallel shift is where the yield curve changes by the same value at every point along the term structure, so that the overall shape of the curve is unchanged.¹ The duration and modified duration measures cannot be used effectively when there is a non-parallel shift in the yield curve. We could observe this for two hypothetical bond portfolios of the same duration; for a non-parallel shift the effect on the portfolios would be expected to be different. This would reflect the different convexity and yield of each bond in each portfolio. For portfolio analysis the traditional yield, modified duration and convexity tools are ineffective for measuring performance due to the changes in the yield curve that will occur over an investment time period.

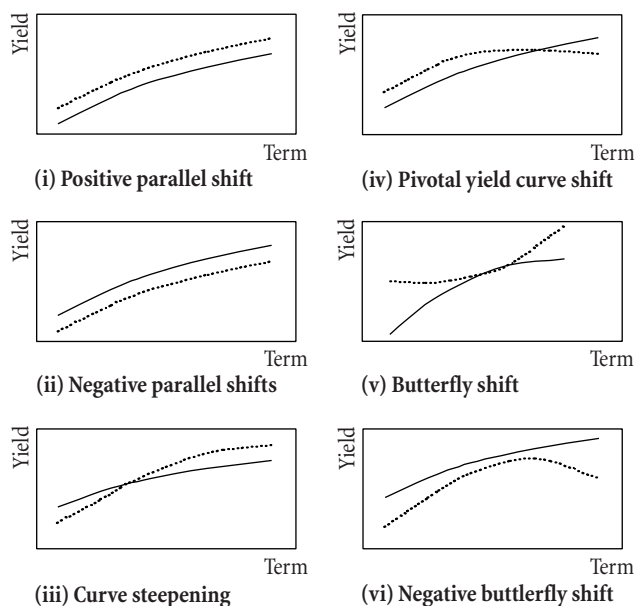


Figure 10.1: Typical yield curve changes.

¹ The term *parallel change* is also used to describe a parallel shift.

Let us review the main types of yield curve changes that are observed in the market. Figure 10.1 illustrates diagrammatically the types of curve change. The first two diagrams show a *parallel shift*. In a parallel shift the shape of the yield curve is unchanged after a yield change, as every bond that makes up the curve has changed in yield by exactly the same number of basis points. It is rare to observe a complete parallel shift in the market. A study by Jones (1991) found that over 90% of yield curve changes in the US Treasury market during 1979 to 1990 were either parallel shifts or *twists*, the US market term for yield curve steepening or flattening. However the great majority of these were twists in the curve. A twist occurs when the spread between various points in the yield curve widens or narrows. A widening in spread is called a *steepening* of the curve and is illustrated at Figure 10.1(iv). A narrowing of yield between the short-end and long-end of the curve is called yield curve *flattening*. The third type of change that occurs is when the shape changes into a humped curve, or the degree of “humpedness” of the curve is altered. The two basic types are illustrated in Figure 10.1 (v) and (vi).

The types of change are not mutually exclusive. In reality yield curve changes will reflect two, three or more types of those shown, so that for example we might observe a downward shift that is also steepening and humped at one end. Often the very short end of the yield curve will act independently of the very long end. A major change in market sentiment is usually reflected in a significant change in the yield curve; in the UK market the curve changed from positively sloping to negatively sloping (inverted) shortly after the change in government occasioned by the general election in May 1997. The shape of the yield curves for shortly before the election and at July 1997 are shown at Figure 10.2 (i) and(ii).

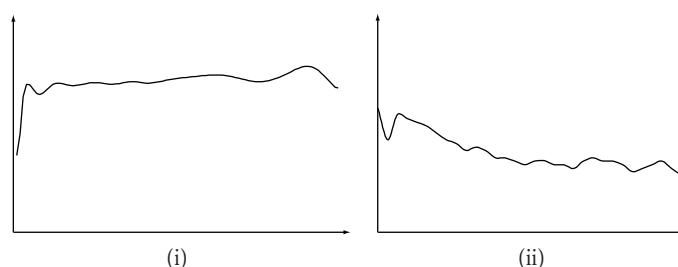


Figure 10.2: UK gilt yield curves May–July 1997. Source: Hambros Bank Ltd/author’s notes.

10.2 Portfolio duration and changes in the yield curve

The standard interest rate risk measurements do not apply in a portfolio environment because they are not effective in non-parallel shift scenarios. An approximate measure of risk can be calculated however, and one method adopted was that described by Klaffky, Ma and Nozari (1992). The measure they calculated was a portfolio interest rate risk measure known as *reshaping durations*. In this section we will use their hypothetical portfolios to illustrate the concept.

Bond	Yield	Modified duration	Composition	
			Portfolio A	Portfolio B
T 5.125% 94	5.18%	1.9	20%	39%
T 7.5% 02	7.33%	7.1	60%	20%
T 8% 21	7.84%	11.1	20%	41%

Table 10.1: Portfolio duration and the yield curve.

Table 10.1 shows the portfolios described by Klaffky, Ma and Nozari. They are both made up of actual US Treasury bonds of two, ten and 30-year maturities as at May 1992, trading at the yields shown. Portfolio A has a heavier weighting in the 10-year Treasury, while portfolio B is weighted percentage-wise more heavily at the two-year and 30-year end of the yield curve. These are examples of *bullet* and *barbell* portfolios. A bullet portfolio is made up of (or concentrated on) bonds with a very similar duration, while a barbell portfolio is comprised of bonds at either end of the maturity spectrum. The modified duration for the hypothetical portfolios A and B is exactly 7 in

both cases, although as we shall see each will respond differently to changes in the yield curve because they are constructed around different parts of the curve. Note that the measure used by Klaffky *et al.* in their analysis is actually *effective duration*, however for conventional vanilla bonds such as Treasuries there is very little difference between effective duration and modified duration so we will use the latter here. A difference between the two measures arises principally with instruments that contain an option element such as callable bonds, which doesn't apply here.

Using the modified duration measure for Klaffky *et al.*'s portfolio, we can see that a 50 basis point change in yield for all three bonds simultaneously would result in a portfolio yield change of 3.50%. What is the actual yield change? If there is a parallel shift in the yield curve the actual yield change for portfolio A is 3.52% and for portfolio B it is 3.56%. Therefore the change predicted by modified duration is a relatively close approximation for the bullet portfolio. Assume now however a yield change that results in a change in the shape of the curve, specifically a curve steepening resulting from a 50 basis point fall in the two-year bond yield. In this case the price of portfolio A will change by 0.19% but the portfolio B price changes by 0.36%. Portfolio B has therefore outperformed portfolio A. In a scenario where the yield curve flattens, for example the long-dated bond falls in yield, portfolio B will outperform portfolio A, but by a smaller amount.²

The conventional modified duration measure is not applicable in a portfolio situation where the market is experiencing non-parallel yield curve shifts. Klaffky *et al.* describe a new measure that considers a change in yield curve shape, one that approximates the change in yield spread between different points in the yield curve, specifically the two-year, ten-year and 30-year points also on the curve. The spread between the two-year and ten-year yields is known as the *short-end* spread while the other spread is the *long-end* spread. The measure used for interest rate risk at the short-end is referred to as short-end duration or *SEDUR*; the other measure is not unexpectedly long-end duration or *LEDUR*.

Using *SEDUR* and *LEDUR* gives us a more effective measure of sensitivity for a portfolio scenario. We calculate both measures using the percentage change in bond price for a given yield change in both a steepening and a flattening curve environment. The calculation is made using (10.1):

$$SEDUR = \frac{P_1 - P_2}{P_0} \times 100 \quad (10.1)$$

where

- P_1 is the bond price resulting from a yield curve steepening of x basis points
- P_2 is the bond price resulting from a yield curve flattening of x basis points
- P_0 is the current bond price.

For *SEDUR* it is the short-end of the yield curve that is being altered by the given number of basis points. The calculation for *LEDUR* is exactly the same except that we then use the long-end of the yield curve. The market convention is to use a 50 basis point steepening and flattening for both ends of the yield curve.

For the bonds used in the Klaffky *et al.* study, the two-year Treasury has a *SEDUR* of 1.9 and a *LEDUR* of zero. For the 30-year bond the values are zero and 11.1 respectively. In this case the scenarios are steepening and flattening of the yield curve while the ten-year Treasury remains at a constant yield. In such an environment both the *SEDUR* and *LEDUR* for the ten-year bond are zero.

We then use *SEDUR* and *LEDUR* values for the individual bonds of a portfolio to calculate a portfolio *SEDUR* and *LEDUR*. It is these measures that are then used as interest rate sensitivity measures for the portfolio. To calculate the values for a portfolio we use a weighted average of the individual *SEDUR*s and *LEDUR*s of the constituent bonds. In the example above that would give us a *SEDUR* for portfolios A and B as follows:

$$\begin{aligned} \text{Portfolio A: } & 0.2(1.9) + 0.4(0) + 0.2(0) = 3.8 \\ \text{Portfolio B: } & 0.39(1.9) + 0.2(0) + 0.41(0) = 7.4. \end{aligned}$$

These values then become are short-end yield curve sensitivity measures. For portfolio A a *SEDUR* of 3.8 is double the approximate change in portfolio value if the short-end of the yield curve moves by 50 basis points. We

² For a fuller account readers should refer to the original article by Klaffky *et al.*

can calculate sensitivities for set changes in yield by using any given basis point change in equation (10.1). Therefore in our example portfolio is more sensitive to change in the short-end of the yield curve as it has a *SEDUR* of 7.4, meaning (in this case) a change of portfolio value of approximately 3.7% for the same change in yield.

A portfolio *LEDUR* is calculated in the same way, using a weighted-average of individual bond *LEDUR*s.

The analysis developed by Klaffky *et al.* can be extended to any set of points along the yield curve, using as large a set of bonds as make up the portfolio. The convention is to calculate a spread in between the two-year and ten-year, and between the ten-year and the 30-year, as a proportion of the main spreads. So for example if the short-end spread is 100 basis points and the spread between the five-year and the ten-year is 50 basis points and there is a curve flattening of 10 basis points, we use this change as a proportion when measuring the five-year to ten-year spread. That is, a 10 basis point change is 10% of the short-end spread, therefore the five-year *SEDUR* is calculated using a 9 basis point change, as given by $10 - (0.10 \times 100)$. The same convention applies for calculating *LEDUR* for bonds lying between the ten-year and 30-year points on the yield curve.

10.3 Hedging strategy and duration

The concept of duration is an important one in the bond markets. It used widely as part of portfolio investment strategy as well as for hedging. Chapter 7 illustrated the duration measure and its calculation and we saw that the modified duration measure gave us the percentage change in bond price for a 1% change in the bond's yield. Although this is only accurate for small changes in yield, and for parallel shifts in the yield curve, it remains important for bond hedging analysis. In this section we consider hedging using a *bond futures* contract. Futures are discussed in detail in Chapter 41 but we can use them to illustrate duration-based hedging techniques.

In a parallel yield curve shift situation we know from previous chapters that (10.2) holds good.

$$\begin{aligned}\frac{\Delta P}{\Delta r} &= PD \\ \frac{\Delta P}{P} &= -D\Delta r\end{aligned}\tag{10.2}$$

which indicates that the percentage price change in bond price is equal to its duration times the size of the yield change.³

Let us now consider hedging a vanilla bond with a bond futures contract. From (10.2) we can construct the following expression, which is a close approximation in a parallel shift scenario:

$$\Delta P_{bond} = -P_{bond} D_{bond} \Delta r.\tag{10.3}$$

We can then use this to approximate the price change for our hedging instrument, given by (10.4):

$$\Delta P_{fut} = -P_{fut} D_{fut} \Delta r\tag{10.4}$$

where

P_{fut}	is the price of the futures contract
D_{fut}	is the duration of the notional bond underlying the futures contract
P_{bond}	is the price of the bond being hedged
D_{bond}	is the duration of the bond being hedged

Therefore the hedging calculation is derived using the expressions above; the number H of futures contracts we would use to hedge the bond position is given by (10.5):

$$H = \frac{P_{bond} D_{bond}}{P_{fut} D_{fut}}.\tag{10.5}$$

This expression is known as the *duration hedge ratio* or the *price sensitivity hedge ratio*. In a parallel shift scenario this gives us the number of futures contracts required to hedge a bond position such that the net duration of the combined bond and futures position would be zero, resulting in a theoretically perfect hedge. In practice there

³ We generally observe that markets prefer to use the modified duration measure instead, given by the expression:

$$MD = D / (1 + r).$$

will be complications in that the duration value for the futures contract uses the measure for the contract's *underlying* bond. This is known as the *cheapest-to-deliver* bond for the futures contract and will be investigated in detail in the chapter on futures.

Equation (10.5) assumes continuously compounded yields. As in practice bond yields are compounded annually or semi-annually the formula needs to be adjusted slightly, and this is shown in Appendix 10.1.

EXAMPLE 10.1 Yield and sensitivity analysis in the UK gilt market

- Table 10.2 lists yield and interest rate risk values for five selected benchmark gilts as at 20 October 1999 and illustrates the difference in interest rate sensitivity measures for bonds of increasing maturity. Figure 10.3 is a plot of the gross redemption yields for the selected bonds.

	8% Treasury 2000	7% Treasury 2002	5% Treasury 2004	5.75% Treasury 2009	6% Treasury 2028
Maturity	07-Dec-00	07-Jun-02	07-Jun-04	07-Dec-09	07-Dec-28
Price (£)	102.1700	101.5000	94.7400	99.8400	119.2500
Accrued	2.9726	2.6010	1.6393	2.1366	2.2295
Yield (%)	5.972	6.367	6.327	5.770	4.770
Duration (years)	1.072	2.388	4.104	7.652	15.031
Modified duration	1.041	2.315	3.979	7.437	14.681
BPV	0.01095	0.02410	0.03835	0.07584	0.17834
Convexity	0.016	0.068	0.191	0.699	3.225

Table 10.2: Yield and sensitivity analysis for five selected benchmark gilts.
Source: Bloomberg.

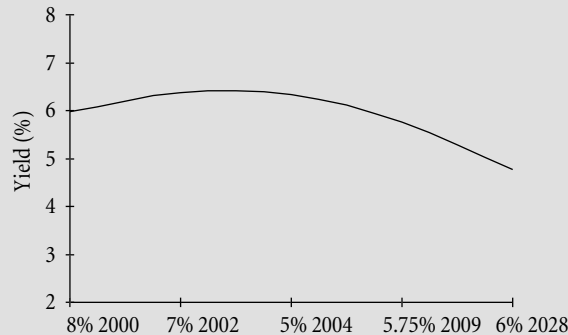


Figure 10.3: Plot of the yield curve for bonds in Table 10.2.

Our yield curve has been plotted using just five bonds. However the term to maturity ranges from one to 29 years; how accurate is a plot of a continuous curve through five discrete points? In Part VIII we discuss the concepts of modelling, estimating and fitting the yield curve. This is an important topic and will draw initially on the concepts we discussed in Part I of this book.

Appendices

APPENDIX 10.1

The futures contract duration hedge ratio in a parallel shift scenario is given by:

$$H = \frac{P_{bond} D_{bond}}{P_{fut} D_{fut}} \quad (10.6)$$

where H is the number of futures contracts required to hedge a bond position. However this expression assumes an environment of continuously compounded yields. Where yields are compounded on a different basis, (10.6) is modified to (10.7):

$$H = \frac{P_{bond} D_{bond} (1 + r_{bond}/m)}{P_{fut} D_{fut} (1 + r_{fut}/m)} \quad (10.7)$$

where

- m is the interest compounding frequency
- r_{bond} is the bond yield
- r_{fut} is the yield on the futures contract underlying bond.

Equation (10.7) will differ from (10.6) at all times except where $r_{bond} = r_{fut}$. This is because our assumption that $\Delta r_{bond} = \Delta r_{fut}$ when yields are continuously compounded is not equal to $\Delta r_{bond} = \Delta r_{fut}$ when yields are compounded on an annual or semi-annual basis.

Selected bibliography

Fabozzi, F., *Fixed Income Mathematics: Analytical and Statistical Techniques*, 3rd edition, McGraw-Hill, 1997, Chapters 11–15.

Jones, F.J., “Yield curve strategies,” *Journal of Fixed Income*, Sept. 1991, pp. 33–41.

Klaffky, T.E., Ma, Y.Y., Nozari, A., “Managing yield curve exposure: Introducing Reshaping Durations,” *Journal of Fixed Income*, Dec. 1992, pp. 49–66.

Kolb, R.W., *Interest Rate Futures*, R.F. Dame, 1982.

Kolb, R.W., Chiang, R., “Improving hedging performance using interest rate futures,” *Financial Management* 10, Autumn 1981, pp. 72–79.

Questions and exercises

1. Consider the following bond yields:

- Two-year: 4.95%
- Ten-year: 5.28%
- 30-year: 5.46%

What has happened to the yield curve if the following yield changes are observed:

- (a) a change in two-year yields to 5.08%
 - (b) a change in ten-year yields to 5.19%
 - (c) a change in two-year yields to 5.35%, ten-year yields to 5.13% and 30-year yields to 4.95%
 - (d) a change in ten-year yields to 5.65% and 30-year yields to 5.95%
 - (e) a change in two-year yields to 4.80% and ten-year yields to 5.35%? In cases (iv) and what would be the effect on market value of a portfolio made up in equal weights of two- and ten-year bonds? Do we have sufficient data to answer this question?
2. Camille is a junior trader on a bond desk that holds €10 million of German government bonds trading at a price of 92.755. The duration of this bond is 7.8 years. How many bund futures contracts must the desk sell to hedge this position if the future is trading at 101.40, if the cheapest-to-deliver bond has a duration of 8.9 years? What assumptions is Camille making in calculating this hedge?
3. A ten-year semi-annual paying 6% bond is trading at a yield of 5.65%. Calculate the following:
 - (a) the bond's price
 - (b) the bond's duration
 - (c) the bond's modified duration

What is the change in the bond's price if there is a 16 basis point rise in the bond's yield. Calculate the bond's price from scratch at the new yield and check how close the modified duration approximation is.

4. A fund manager is reviewing two portfolios. Portfolio 1 is made up of £10 million each of a two-year and 10-year bond, which have yields of 5.25% and 5.75% and durations of 1.8 and 8.7 respectively. Portfolio 2 is made up of the same 10-year bond and a 20-year bond with yield of 6.10% and duration of 13.2, in a 80:20 weighting. Calculate:
 - (a) each portfolio duration
 - (b) SEDUR and LEDUR for portfolio 1 and 2 respectively
 - (c) the percentage changes in the values of each portfolio after (i) a 20 basis point steepening shift at the short end and (ii) a 10 basis point flattening shift at the long end of the yield curve. Which portfolio has the greater interest rate sensitivity?
5. Rasheed securities is running a bond portfolio with a current market value of €28.2 million. The portfolio duration is 8.2 years. The equivalent exchange-traded bond futures contract is trading at a price of 104.26, and the underlying bond has a duration of 8.9 years. The securities house is expecting a rise in bond yields over the next few weeks; how can it protect against a fall in value of its portfolio? Calculate a possible interest rate hedge.
6. How can a hedge be put on for a portfolio with a market value of \$16 million and a duration of 3.2 years, where the Treasury exchange-traded futures contract is trading at a price of 102-15 and its underlying cheapest-to-deliver bond has a duration of 7.9 years? Given the duration of the portfolio, what might be a better instrument to use to hedge the position?

Part II Government Bond Markets

The government bond markets are the most important in terms of volume of trading, and as a benchmark against which all other debt capital market instruments are priced. It is doubtful whether the global capital markets could function efficiently without a large and liquid US Treasury government market and selected other government markets. With the emphasis in macroeconomic policy-making in recent years being directed at reducing public sector deficits and repaying national debt, this is a topical issue. What would be the impact of a greatly reduced supply of government bonds, such that demand could not be met, on the corporate bond and swap markets, for example? Another area that would be affected would be commercial banks, who hold short-dated government bonds as part of regulatory and capital requirements and for liquidity purposes. At the moment there is no obvious substitute for government bonds, and until there is we can but conclude that an efficient and liquid government market is essential to the well-being of the global capital markets as a whole.

In Part II of the book we look in detail at two government bond markets, the US Treasury market, and the UK gilt market. The Treasury market is by far the largest in the world and plays a pivotal role in the global economy as a source of risk-free investment and an interest-rate benchmark. The gilt market is one of the oldest, if not the oldest, dating from 1694. There is a detailed description of the functioning and structure of both markets; we also look briefly at other selected government markets.

11

The United Kingdom Gilt Market

Gilts are bonds issued by the United Kingdom government. The term “gilt” comes from *gilt-edged securities*, the official term for UK government bonds.¹ Gilts are the main method by which the UK government finances the shortfall between its expenditure and its tax revenues, and as they are direct obligations of the government they are the highest-rated securities in the sterling market. Their AAA-rating reflects in part the fact that the UK government has never defaulted on an interest payment or principal repayment in the history of the gilt market, as well as the strength and standing of the UK economy. The market is also at the centre of the sterling financial asset markets and forms the basis for pricing of all other sterling assets and financial instruments.

At the end of December 1999 there were 71 individual gilt issues in existence, representing nearly £270 billion nominal of debt outstanding. The majority of these are conventional fixed interest bonds, but there are also index-linked, double-dated, irredeemable and floating rate gilts. The UK government issues gilts to finance its income deficit, previously known as the Public Sector Borrowing Requirement but now known as the Public Sector Debt Requirement. The actual financing requirement each fiscal year is known as the Central Government Net Cash Requirement (CGNCR). The CGNCR, being the difference between central exchequer income and expenditure, always reflects a borrowing requirement except on rare occasions when the central government runs a surplus, as in 1988 and in 1998. New issues of debt are also made to cover repayment of maturing gilts.

The responsibility for issuing gilts now rests with the Debt Management Office (DMO), an executive agency of Her Majesty’s Treasury. The DMO is in charge of sterling debt management for the government. This transfer of duties traditionally performed by the Bank of England (BoE) was introduced after the Chancellor of the Exchequer handed over control of UK monetary policy to a committee of the Bank, the Monetary Policy Committee, in May 1997. The DMO was set up in 1998 and assumed its responsibilities on 1 April 1998. Prior to this the gilt market had been subject to a series of reforms in its structure and operation over the past few years, overseen by the BoE, designed to make gilts more competitive as investment instruments and to bring the market up-to-date. Recent innovations have included the introduction of an open market in gilt repo and a new market in zero-coupon bonds known as gilt strips, as well as improvements to the auction process. This included introduction of an auction calendar and a policy of building up large volume, liquid benchmark issues. The recent changes were designed to enhance the efficiency, liquidity and transparency of the market. The gilt market is an important world bond market and in this chapter we look at the structure and make-up of the market today, and in greater detail at recent reforms and structural changes. We also review related off-balance sheet products such as the exchange-traded gilt futures contract; readers may wish to read the chapters on derivatives and futures contracts, in Part VI before tackling this part of the chapter.

11.1 Introduction and history

United Kingdom national debt dates from 1694 when the government of King William raised £1.2 million in order to finance a war against France. The Bank of England was founded the following year. The currency itself however dates from much earlier than this, for instance the word *pondus* was a Latin word meaning “weight” but signified the weight of the coin of money; while the “£” sign originates from the designation for “Libra”, used to denote the pound. The term *sterling* originates from “esterling”, silver coins introduced during the reign of King Henry II in the twelfth century.² From this beginning the UK national debt has grown steadily, experiencing rapid growth during the wars in 1914–18 and 1939–45. The nominal value of the debt stood at £418 billion in March 1998.³ Appendix 11.8

¹ The legend is that the term “gilt-edged” was used to refer to British government bonds because certificates representing individual bond holdings were originally edged with gold leaf or *gilt*. This tale is almost certainly apocryphal. Another commonly heard explanation is that UK government bonds were deemed to be as “good as” (as safe as) gold, hence the term “gilts”.

² These and other fascinating facts are to be found in David Sinclair’s splendid book “The Pound”, published by Century (2000).

³ Source: DMO (1999).

lists the growth of the UK national debt from inception to date. Note that the size of total debt also includes public sector debt, including that representing nationalised corporations, and foreign currency debt, as well as gilts in issue. The pattern of steady growth in the size of the debt, and increased expansion in war years, is mirrored in other developed countries. In the post-war period government intervention in the economy and social welfare was also reflected in the size of the debt, as was the effect of inflation in the 1970s, which resulted in a greater nominal value of debt. Perhaps a better indicator of the growth of government debt is as its value as a proportion of domestic product. As a percentage this was highest at the end of the Second World War, and had been declining since. The 1998 national debt was a fraction under 50% of UK gross domestic product that year.

As the gilt market forms the cornerstone of the sterling asset markets, the gilt yield curve is the benchmark for banks and corporates when setting interest rates and borrowing funds. As is common in many countries, government bonds in the UK form the largest sector within the UK bond markets; in March 1998 the nominal value outstanding of £291 billion comprised just over 57% of the total nominal value of UK bonds. This figure was also larger than the total volume of sterling Eurobonds in issue at that time, approximately £200 billion. The remainder was made up of domestic bonds such as *debentures* and *Bulldogs*.

Gilts were one of the strongest performing government bond markets in 1997 and 1998 and yields fell across the curve through 1998, reaching their lowest levels since the mid-1950s. For instance yields on the 30-year benchmark gilt, the 6% Treasury 2028, fell by 120 basis points to 4.5 per cent, roughly the same level as the ten-year benchmark gilt and a lower yield than the ten-year US Treasury benchmark. Gilts also outperformed equities in 1998. In 1998/99 the government issued £8.2 billion of gilts and is expected to issue approximately £17.3 billion of gilts in 1999/00. A full list of gilts and nominal amounts outstanding as at December 1999 is given in Appendix 11.1.

UK GILTS PRICES

	... Yield ...	... 52 week ...		... Yield ...	... 52 week ...		... Yield ...	... 52 week ...			
	Notes	Int	Red	Price £ + or -	High	Low	Notes	(1) (2)	Price £ + or -	High	Low
Shorts* (Lives up to Five Years)											
Treas 6pc 1999.....	X	6.00	4.91	100.05	-01	100.50	98.70				
Treas 2½pc 1999.....	X	1.26	-	198.90	-02	190.60	191.47				
Conv 10¼pc 1999.....	X	10.07	4.79	101.76	-02	105.37	101.76				
Index-Linked (b)											
Treas 2½pc '01.....	(78.3)	2.78	3.33	204.26	-06	206.40	198.10				
2½pc '03.....	(78.8)	2.38	2.68	205.29	-13	207.83	195.70				
4½pc '04.....	(135.6)	1.88	1.12	133.34	-08	134.77	126.55				
2pc '06.....	(69.5)	1.62	1.79	237.40d	-23	239.80	212.21				
2½pc '09.....	(78.8)	1.75	1.88	218.00	-28	221.45	192.71				
2½pc '11.....	(74.4)	1.88	1.99	230.05	-37	235.62	202.19				
2½pc '13.....	(89.2)	1.88	1.97	194.19	-31	199.07	168.63				
2½pc '16.....	(81.6)	1.86	1.94	215.25d	-31	221.10	183.37				
2½pc '20.....	(83.0)	1.84	1.91	215.71	-25	221.68	178.82				
2½pc '24.....	(97.1)	1.81	1.87	187.39	-26	193.74	151.30				
4½pc '30.....	(135.1)	1.76	1.81	185.99d	-28	193.23	149.11				
Prospective real redemption rate on projected inflation of (1) 5% and (2) 3%.											
(b) Figures in parentheses show RPI base for indexing (ie 8 months prior to issue) and have been adjusted to reflect rebasing of RPI to 100 in February 1987. Conversion factor 3.945. RPI for November 1998: 164.4 and for June 1999: 165.6.											
Other Fixed Interest											
	... Yield ...	... 52 week ...		... Yield ...	... 52 week ...		... Yield ...	... 52 week ...			
	Notes	Int	Red	Price £ + or -	High	Low	Notes	(1) (2)	Price £ + or -	High	Low
Asian Dev 10¼pc 2009.....											
7.89 6.11 129%											
B'ham 11½pc 2012.....											
7.93 6.40 145											
Leeds 13½pc 2006.....											
9.78 - 138											
Liverpool 3½pc Ired.....											
5.83 - 60											
LCC 3pc '20 Alt.....											
5.88 - 51											
Manchester 11½pc 2007.....											
8.58 6.28 134											
Mt. W'pc 'B'.....											
3.41 6.45 85											
Nwide Angle 1½pc IL 2021.....											
- 3.17 188½											
4¼pc IL 2024.....											
- 3.13 182											
Five to Fifteen Years											
Treas 5pc 2004.....	X	5.10	5.45	98.10	+07	98.80	98.03				
Funding 3½pc 1999-4.....	X	3.77	17	92.76	+05	98.98	91.80				
Conv 9½pc 2004.....	X	6.08	5.53	117.90	+08	126.43	116.58				
Treas 6¾pc 2004.....	X	8.36	5.49	105.75	+09	113.28	103.09				
Conv 9½pc 2005.....	X	7.97	5.54	119.25	+13	128.16	117.84				
Exch 10½pc 2005.....	X	8.36	5.53	125.62	+13	135.64	124.50				
Undated											
Consols 4pc.....	X	5.13	-	77.98	-54	87.19	64.75				
War Loan 3½pc.....	X	4.95	-	70.73	-55	79.83	59.16				
Conv 3½pc '61 Alt.....	X	4.23	-	82.73	-55	96.88	76.16				
Consols 3pc '66 Alt.....	X	5.39	-	55.70	-42	61.93	46.98				
Consols 2½pc.....	X	4.93	-	50.70	-42	56.93	41.98				
Treas 2pc.....	X	5.00	-	50.00	-118	55.17	41.20				

● 'Tap' stock. All UK Gilts are tax-free to non-residents on application. E Auction basis. xd Ex dividend. Closing mid-prices are shown in pounds per £100 nominal of stock. Prospective real Index-Linked redemption yields are calculated by HSBC Greenwell from Gemma closing prices. * Indicative price.

Figure 11.1: UK gilts prices, 23 July 1999. ©Financial Times. Used with permission.

Figure 11.1 is an extract from the *Financial Times* for 23 July 1999, showing prices and yields for the main benchmark conventional and index-linked gilts in issue. Figure 11.2 shows the King & Shaxson Bond Brokers gilts page as it appears on Reuters; this was the page for 19 August 1999. King & Shaxson is part of the Gerrard & National

group and is a leading broker in the gilts market. The page shows indicative middle prices for the leading stocks. It also shows the gross redemption yield at the indicated price and the change on the previous day's closing price.

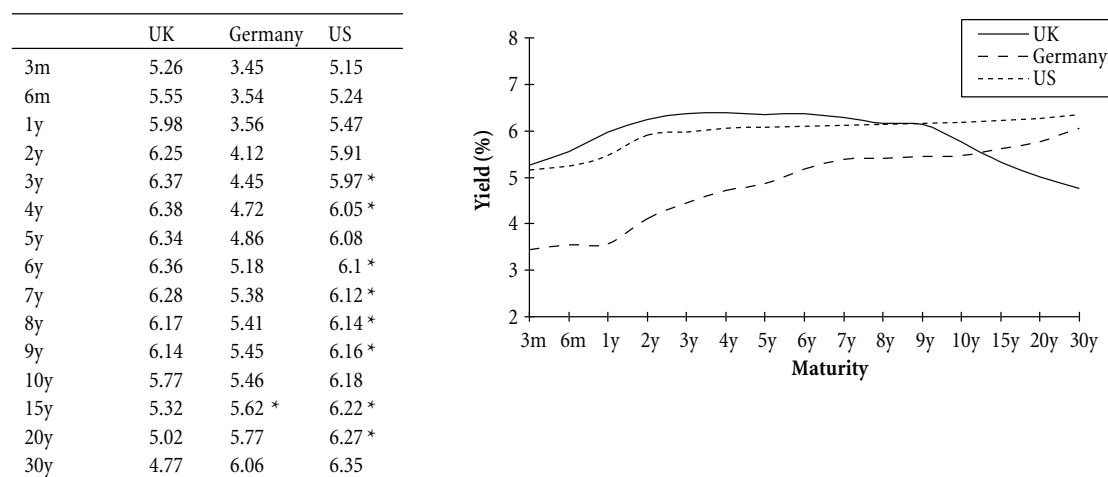
G.N.I LTD					Thursday, 19 August 1999 15:11:45				
14:08	19AUG99	KING & SHAXSON BOND BROKERS			626-7755	UK15110			KSBBL
STOCK	CLOSE	CURRENT	CHANGE	GRY	STOCK	CLOSE	CURRENT	CHANGE	GRY
10Q 99	101.36	101.37	+0.01	4.71	6Q 10	108.77	108.85	+0.08	5.18
9 2000	102.20	102.22	+0.02	4.78	9 11	132.80	132.90	+0.10	5.23
132000	106.87	106.90	+0.03	5.03	9 12	135.27	135.31	+0.04	5.20
8 2000	103.36	103.40	+0.04	5.23	08 12	101.10	101.29	+0.19	5.31
10 01	106.46	106.51	+0.05	5.44	8 13	128.22	128.37	+0.15	5.14
7 2001	102.69	102.76	+0.07	5.64	12 15	122.30	122.48	+0.18	5.26
7 2002	103.24	103.32	+0.08	5.69	8 15	132.43	132.53	+0.10	5.04
9T 02	110.70	110.79	+0.09	5.80	8T 17	144.69	144.72	+0.03	4.95
8 03	107.50	107.61	+0.11	5.74	8 21	141.82	141.71	-0.11	4.85
10 03	114.83	114.96	+0.13	5.80	6 28	120.60	120.49	-0.11	4.69
6H03	102.95	103.07	+0.12	5.67					
01 04	109.00	109.05	+0.05	5.44					
5 04	97.48	97.61	+0.13	5.57					
6T 04	104.95	105.11	+0.16	5.60					
9H 05	117.88	118.07	+0.19	5.71					
8H 05	114.66	114.84	+0.18	5.66					
7T 06	111.87	112.05	+0.18	5.65					
02 06	105.67	105.76	+0.09	5.93					
7H 06	110.92	111.10	+0.18	5.62					
8H 07	118.17	118.35	+0.18	5.59					
7Q 07	111.17	111.35	+0.18	5.52					
9 08	125.24	125.45	+0.21	5.43					
5T 09	104.30	104.38	+0.08	5.19					

All Prices in Decimals

Figure 11.2: King & Shaxson Bond Brokers gilts page, 19 August 1999.

©Reuters ©K&S Ltd. Used with permission.

Figure 11.3 compares the yield curves for benchmark government bonds in the UK, United States and Germany on 20 October 1999.



* Interpolated yield

(Source: Bloomberg)

Figure 11.3: Government bond yield curves.

The gilt yield curve in Figure 11.3 reflects historically high demand and low yields for gilts. This is explored later in this chapter. However the relative low yield at the long end of the market is potentially advantageous for the UK government and should assist the DMO to fulfil one of the objectives of debt management in the UK, which is that the cost of financing the governments expenditure be minimised over the long term. Overall debt management is the responsibility of the DMO and it seeks to meet its objectives through a regular programme of gilt issues and helping to maintain an orderly and efficient market.

	1997	1998				1999	
Quarterly average:		Q1	Q2	Q3	Q4	Q1	Q2
Gilts:							
■ Conventional	475	451	406	411	347	368	354
■ Index-linked	11	9	11	7	7	7	7
Total	486	460	417	418	354	375	360

£ billion nominal value

Table 11.1: Gilt market turnover. Source: BoE, DMO.

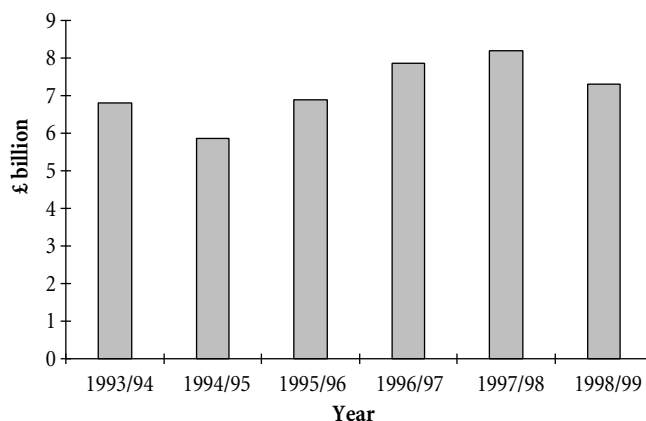


Figure 11.4: Average daily gilt market turnover. Source: DMO.

11.2 Market instruments

11.2.1 Conventional gilts

The gilt market is essentially plain vanilla in nature. The majority of gilt issues are conventional fixed interest bonds. Conventional gilts have a fixed coupon and maturity date. By volume they made up 76% of the market in September 1999. Coupon is paid on a semi-annual basis. The coupon rate is set in line with market interest rates at the time of issue, so the range of coupons in existence reflects the fluctuations in market interest rates. Unlike many government and corporate bond markets, gilts can be traded in the smallest unit of currency and sometimes nominal amounts change hands in amounts quoted down to one penny (£0.01) nominal size. Individual gilts are given names such as the 7% Treasury 2002 or the 9% Conversion 2011. There is no significance attached to the name given to a gilt, and they all trade in the same way; most issues in existence are now “Treasury” issues, although in the past it was sometimes possible to identify the purpose behind the loan by its name. For example a “Conversion” issue usually indicates a bond converted from a previous gilt. The 3½% War Loan on the other hand was issued to help finance the 1914–18 war. The 3% Gas 1995/98 was issued to finance the nationalisation of the gas industry and was redeemed in 1998.

The colour plate shows the certificate representing a retail holding of a conventional gilt, the 5% Treasury 2004, which matures on 7 June 2004.

Gilts are registered securities. All gilts pay coupon to the registered holder as at a specified record date; the record date is seven business date before the coupon payment date. The period between the record date and the coupon date is known as the *ex-dividend* or “ex-div” (“xd”) date; during the ex-dividend period the bond trades without accrued interest. This is illustrated in Figure 11.5.

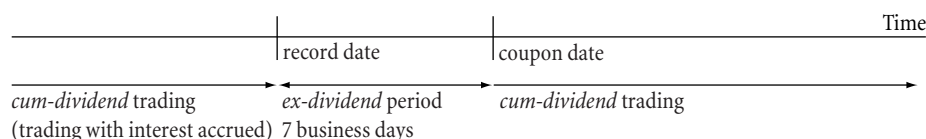


Figure 11.5: Gilt ex-dividend trading.

The ex-dividend period was reduced from three weeks to seven business days for all gilts in 1998; the facility to trade *special ex-div*, which was a two-week period prior to the start of the ex-dividend period during which transactions could be traded ex-div on agreement between buyer and seller, was also removed in 1998. The ex-dividend period for one issue, 3½% War Loan, was retained at three weeks (subsequently reduced to ten business days) due to the large number of retail holders of this bond. The floating-rate gilt 2001 has no ex-dividend period.

11.2.2 Index-linked gilts

The other major gilt instruments are *index-linked* (I-L) gilts, or “linkers”. The United Kingdom was one of the first countries to introduce index-linked government bonds, with an issue in 1981. I-L gilts are designed to provide investors with an inflation-protected, real return from their bondholding. In March 1999 approximately 21% of gilts in issue were linkers. I-L gilts link both their coupon and redemption payments to the UK retail prices index (RPI) and this adjustment should in theory preserve the real value of investors’ income and capital, independent of the rate of inflation. The RPI figure for eight months prior to the relevant cash flow is used to adjust the final value of each payment as it becomes due. To adjust a coupon payment therefore, the nominal value (this is the coupon rate) is adjusted using the RPI value recorded eight months prior to the bond’s issue date and the RPI value recorded eight months prior to the coupon payment date.

The use of an earlier RPI index level, known as an “indexation lag”, is due to the fact that the actual cash flow needs to be known six months before it is paid, so enabling the accrued interest on the bond to be calculated during the interest period. This accounts for six months of the lag; RPI figures themselves are always issued one month after the month to which they relate (for instance, September’s RPI is issued in October) and one extra month is allowed to enable the BoE to make the calculations. The final redemption is treated in the same way. The existence of this indexation lag means that in practice I-L gilt returns are not completely protected from inflation.

The coupon level for I-L gilts are typically 2% or 2½%, which is lower than conventional gilts issued since the 1950s. This reflects the fact that, as the coupon is protected from inflation the nominal coupon value is the real interest rate expected during the bond’s life. Historically the real return on gilt stock has been around 2–3%. The importance of indexed gilts in the market as a whole reflects the attraction that such debt has for investors. Index-linked stock allows both issuers and bondholders to reduce or eliminate the risk posed by unexpected changes in inflation to the real value of their obligations and liabilities. To the extent that either the UK government or investors are averse to this risk, this represents a market gain. From the government’s point of view indexed debt offers benefits in terms of potentially contributing to a reduction of costs and risks. For the investor the principal attractions are the hedge against inflation, which is valuable from the point of view of investors with their own indexed liabilities, plus the opportunity to trade the bonds as part of a view on real interest rates.

The market analyses the trading patterns and yield levels of index-linked gilts for their information content. The difference between the yield on I-L gilts and conventional gilts of the same maturity is an indication of the markets’ view on future inflation; where this difference is historically low it implies that the market considers that inflation prospects are benign. So the yield spread between index-linked gilts and the same maturity conventional gilt is roughly the market’s view of expected inflation levels over the long term. For example on 3 November 1999 the ten-year benchmark, the 5¾% Treasury 2009 had a gross redemption yield of 5.280%. The ten-year index-linked bond, the 2½% I-L Treasury 2009⁴ had a *money yield* of 5.046% and a real yield of 2.041%, the latter assuming an inflation rate of 3%. Roughly speaking this reflects a market view on inflation of approximately 3.24% in the ten years to maturity. Of course other factors drive both conventional and I-L bond yields, including supply and demand, and liquidity. Generally conventional bonds are more liquid than I-L bonds. An increased demand will depress yields for the conventional bond as well.⁵ However the inflation expectation will also be built into the conventional bond yield

⁴ This bond was issued in October 1982; as at October 1999 there was £2.625 billion nominal outstanding.

⁵ For example two days later, following a rally in the gilt market, the yield on the conventional gilt was 5.069%, against a real yield on the linker of 1.973%, implying that the inflation premium had been reduced to 3.09%. This is significantly lower

and it is reasonable to assume the spread to be an approximation of the market's view on inflation over the life of the bond. A higher inflation expectation will result in a greater spread between the two bonds, this reflecting a premium to holders of the conventional issue as a compensation against the effects of inflation. This spread has declined slightly from May 1997 onwards, the point at which the government gave up control over monetary policy and the Monetary Policy Committee (MPC) of BoE became responsible for setting interest rates.

Bonds	Yields (%)							
	Feb-97	Jun-97	Feb-98	Jun-98	Feb-99	Jun-99	Sep-99	Nov-99
2.5% I-L 2009 *	3.451	3.707	3.024	2.880	1.937	1.904	2.352	2.041
7.25% 2007	7.211	7.021	–	–	–	–	–	–
9% 2008	–	–	5.959	5.911	–	–	–	–
5.75% 2009	–	–	–	–	4.379	4.907	5.494	5.280
Spread	3.760	3.314	2.935	3.031	2.442	3.003	3.142	3.239

* Real yield, assuming 3% rate of inflation

Table 11.2: Real yield on the 2½% I-L 2009 versus the ten-year benchmark. Source: Bloomberg.

Table 11.2 above shows the real yield of the 2½% I-L 2009 bond at selected points since the beginning of 1997, alongside the gross redemption yield of the ten-year benchmark conventional bond at the time (we use the same I-L bond because there was no issue that matured in 2007 or 2008). Although the market's view on expected inflation rate over the ten-year period is on the whole stuck around the 3% level, there has been a downward trend in the period since the MPC became responsible for setting interest rates. As the MPC has an inflation target of 2.5%, the spread between the real yield on the ten-year linker and the ten-year conventional gilt implies that the market believes that the MPC will achieve its goal.⁶

The yield spread between I-L and conventional gilts fluctuates over time and is influenced by a number of factors and not solely by the market's view of future inflation (the implied forward inflation rate). As we discussed in the previous paragraph however the market uses this yield spread to gauge an idea of future inflation levels. The other term used to describe the yield spread is *breakeven inflation*, that is the level of inflation required that would equate nominal yields on I-L gilts with yields on conventional gilts. Figure 11.6 shows the implied forward inflation rate for the 15-year and 25-year terms to maturity as they fluctuated during 1998/1999. The data is from the Bank of England. For both maturity terms, the implied forward rate decreased significantly during the summer of 1998; analysts however ascribed this to the rally in the conventional gilts, brought on by the “flight to quality” after the emerging markets fallout beginning in July that year. This rally was not matched by I-L gilts performance. The 25-year implied forward inflation rate touched 1.66% in September 1999, which was considered excessively optimistic given that that BoE was working towards achieving a 2.5% rate of inflation over the long term! This suggested that conventional gilts were significantly overvalued.⁷ As we see in Figure 11.6 this implied forward rate for both maturity terms returned to more expected levels later during the year, slightly above 3%. This is viewed as more consistent with the MPC's target, and can be expected to fall to just over 2.5% over the long term.

than the premium just two days later, which may reflect the fact that the MPC had just raised interest rates by ¼% the day before, but the rally in gilts would impact the conventional bond more than the linker.

⁶ In fact the MPC target centres on the RPIX measure of inflation, the “underlying” rate. This is the RPI measure but with mortgage interest payments stripped out.

⁷ As we have noted, the yield spread between I-L and conventional gilts reflects other considerations in addition to the forward inflation rate. As well as specific supply and demand issues, considerations include inflation risk premium in the yield of conventional gilts, and distortions created when modelling the yield curve. There is also a liquidity premium priced into I-L gilt yields that would not apply to benchmark conventional gilts. The effect of these is to generally over-state the implied forward inflation rate. Note also that the implied forward inflation rate applies to RPI, whereas the BoE's MPC inflation objective targets the RPIX measure of inflation, which is the headline inflation rate minus the impact of mortgage interest payments.

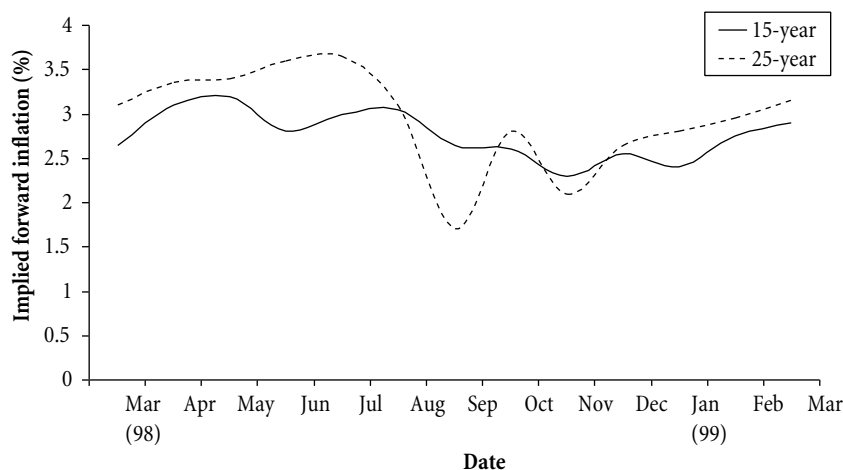


Figure 11.6: UK implied forward inflation rates during 1998/99. Source: BoE.

EXAMPLE 11.1 I-L gilt cash flow calculation

- As with conventional gilts, I-L gilts pay coupon on a semi-annual basis. Coupons are linked to the retail price index. To calculate any coupon payment and the redemption payment for an I-L gilt we require the RPI value for eight months prior to the issue of the bond and the value for eight months prior to the cash flow date. The BoE carries out and publishes the calculations for each I-L gilt.

Coupon and redemption payments are calculated using (11.1) and (11.2).

$$\frac{C}{2} \times \frac{RPI_C}{RPI_I} \quad (11.1)$$

$$100 \times \frac{RPI_M}{RPI_I} \quad (11.2)$$

where

- RPI_I is the RPI value recorded eight months prior to the bond issue date
- RPI_C is the RPI value recorded for the month eight months prior to the coupon date
- RPI_M is the RPI value recorded eight months prior to the maturity date.

RPI_I is also known as the *base RPI* value. A table of RPI values is given at Appendix 11.2.

The 2½% index-linked Treasury 2024 stock was issued in December 1986. Therefore the base RPI figure for this bond is 97.67, which is the index value for March 1986, published in April 1986. The bond has coupon dates each January and July, so to calculate the coupons we require the RPI values for the previous May and November respectively. The coupon payment in July 1999 was calculated as follows:

$$\frac{2.5}{2} \times \frac{164.4}{97.67} = 2.1040.$$

The RPI values used are those for April 1986 (the base RPI) and November 1998. Note that 164.4 is the RPI for October 1998, published one month later.

The bond's maturity date is 17 July 2024. The redemption payment for this bond will be calculated using the expression given below:

$$100 \times \frac{RPI_{Nov2023}}{RPI_{Apr1986}}.$$

From Figure 11.1 we can see that the usual measure of return used with gilts is the yield to maturity or gross redemption yield. This measure is described in Chapter 4. The yield calculation for I-L gilts assumes a constant level of inflation in order to fix the cash flows, which enables us to calculate the redemption yield. This is usually carried out using the current RPI level and then assuming a constant inflation rate from then on, scaling the RPI values up accordingly. The yield calculated using this method is often referred to as a *nominal yield*. The nominal yield reflects the assumed level of inflation, usually fixed at 3% or 5% by analysts in the sterling markets. In order to gauge an idea of the real return, the nominal yield is sometimes adjusted to give a real yield using the Fisher equation.⁸ The DMO⁹ uses the following form of the Fisher identity in its calculations:

$$\left(1 + \frac{r}{200}\right)^2 = \frac{(1 + (y/200))^2}{(1 + (i/100))} \quad (11.3)$$

where

- r is the real yield (%)
- y is the nominal yield (%)
- i is the assumed inflation rate (%).

The other yield calculation used in I-L gilt analysis is that for real redemption yield. The DMO in its *Gilt Review* 1998 uses equation (11.4) to calculate real yields, given that there is an indexation lag used in computing actual cash flows, based on the need to calculate accrued interest for a forthcoming interest period, and the delay inherent in publishing the RPI figure itself. As the DMO notes, this lag introduces a complication in real yield calculation, and also requires an assumed constant inflation rate to be used when computing real redemption yields.¹⁰

$$P = \frac{d_1}{\left(1 + \frac{\rho}{2}\right)^{\frac{r}{s}} (1 + i_0)^{\frac{r}{2s} - \frac{1}{6}} (1 + i_1)^{\frac{1}{6}}} + \frac{\frac{C}{2} \cdot \frac{RPI_{latest}}{RPI_{base}} (1 + i_0)^{\frac{k}{12}}}{\left(1 + \frac{\rho}{2}\right)^{\frac{r}{s} + 1} (1 + i_0)^{\frac{r}{2s} - \frac{1}{6}} (1 + i_1)^{\frac{1}{2}} (1 + i_2)^{\frac{1}{6}}} \quad (11.4)$$

$$+ \frac{\frac{C}{2} \cdot \frac{RPI_{latest}}{RPI_{base}} (1 + i_0)^{\frac{k}{12}} (1 + i_1)^{\frac{1}{2}}}{\left(1 + \frac{\rho}{2}\right)^{\frac{r}{s} + 2} (1 + i_0)^{\frac{r}{2s} - \frac{1}{6}} (1 + i_1)^{\frac{1}{2}} (1 + i_2)^{\frac{1}{2}} (1 + i_3)^{\frac{1}{6}}} + \dots$$

where

- P is the bond dirty price
- c is the real coupon
- ρ is the gross real redemption yield
- i_j is the future annual inflation rate for period j
- RPI_{latest} is the last published RPI
- RPI_{base} is the RPI relating to eight months prior to bond issue date
- d_1 is the next dividend payment
- r is the number of days from the settlement date to the next dividend date
- s is the number of days in the interest period in which the settlement date falls
- k is the number of months between latest RPI at time of settlement and the month of the RPI that defines the next but one dividend payment.

Equation (11.4) assumes that the bond does not settle during the ex-dividend period. It simplifies to (11.5):

⁸ From Chapter 2 remember that the Fisher identity states that nominal yield is comprised of real yield and expected inflation, given in its simplest form as $(1 + y) = (1 + r)(1 + i)$.

⁹ Review, 1998.

¹⁰ Note that equations (11.3) and (11.4) use the DMO's terminology.

$$P = \frac{d_1}{\left(1 + \frac{\rho}{2}\right)^{\frac{r}{s}} (1 + i_0)^{\frac{r}{2s} - \frac{1}{6}} (1 + i_1)^{\frac{1}{6}}} + \frac{\frac{C}{2} \cdot \frac{RPI_{latest}}{RPI_{base}}}{\left(1 + \frac{\rho}{2}\right)^{\frac{r}{s}+1} (1 + i_0)^{\frac{r}{2s} - \frac{1}{6} - \frac{k}{12}} (1 + i_1)^{\frac{1}{2}} (1 + i_2)^{\frac{1}{6}}} + \dots$$

$$\frac{\frac{C}{2} \cdot \frac{RPI_{latest}}{RPI_{base}}}{\left(1 + \frac{\rho}{2}\right)^{\frac{r}{s}+2} (1 + i_0)^{\frac{r}{2s} - \frac{1}{6} - \frac{k}{12}} (1 + i_2)^{\frac{1}{2}} (1 + i_3)^{\frac{1}{6}}} + \dots \quad (11.5)$$

From (11.5) we observe that the terms for the future rate of inflation in the numerator and denominator do not cancel out completely. This is due to the indexation lag. This is why it is necessary to assume a constant inflation rates for all forward coupon calculations, given by i_0 , i_1 , i_2 , and so on to i_n in order to calculate the real redemption yield. If we assume a constant inflation rate of π^e for all rates of inflation during the life of a bond, equation (11.5) becomes (11.6) below, using the DMO's terminology.

$$P = \frac{d_1}{\left(1 + \frac{\rho}{2}\right)^{\frac{r}{s}} (1 + \pi^e)^{\frac{r}{2s}}} + \frac{\frac{C}{2} \cdot \frac{RPI_{latest}}{RPI_{base}}}{\left(1 + \frac{\rho}{2}\right)^{\frac{r}{s}+1} (1 + \pi^e)^{\frac{r}{2s} - \frac{k}{12} + \frac{1}{2}}} + \frac{\frac{C}{2} \cdot \frac{RPI_{latest}}{RPI_{base}}}{\left(1 + \frac{\rho}{2}\right)^{\frac{r}{s}+2} (1 + \pi^e)^{\frac{r}{2s} - \frac{k}{12} + \frac{1}{2}}} + \dots \quad (11.6)$$

In addition to the process given by (11.4), another method used to calculate values given assumed inflation rates is to use the forward inflation rates implied by the difference between points on the nominal yield curve and the real yield curve. In the UK market the convention is to assume a constant inflation rate of 3%, although often analysts compute yields using both 3% and 5% assumed inflation. Using a higher inflation rate results in a lower real yield value.

In other index-linked bond markets, the calculation difficulty in computing real yields is removed by assuming that π^e is equal to zero, so that the inflation terms in (11.6) cancel out. Analysts in the US index-linked market for example, sometimes use this method to calculate real yields. This may have less impact since the indexation lag in the US market, at three months, is shorter than in the UK.

A table of RPI values from 1982 is given in Appendix 11.2.

EXAMPLE 11.2 Real yields versus conventional gross redemption yields

- On 10 October 1999 the 2% I-L Treasury 2006 gilt is trading at £231.90, a real yield of 2.209% (assuming an inflation rate of 3%). A double-dated gilt, the 3½% Funding 1999–2004, is trading at £92.72 which is equivalent to a gross redemption yield of 5.211%, assuming the stock is redeemed at the final maturity date. A private investor currently holding the I-L gilt is recommended by their stockbroker to switch into the double-dated issue. The investor is a higher-rate taxpayer, and there is no capital gains tax to pay on gilt investments. Do you agree with the stockbroker's recommendation? What reasons lie behind your decision?

In order to compare the two gilts we need to make an assumption about future inflation. The yield spread between the two bonds implies a forward inflation rate of just over 3%. Therefore if the actual inflation rate to 2004 averages more than 3%, the investor would have been better served by the I-L gilt. At a price of 92.72 the conventional gilt offers a capital gain on maturity of £7.28. The coupon on the investment is only 3.50% however. Compare this with an expected coupon of 4.50% for the I-L bond, if we assume that inflation will stay at or around the MPC's target rate of 2.5%. On maturity however the price of the I-L gilt will be £196.00, which we calculate using the current RPI level until maturity (this gives us 165.1×1.0257) and assuming 2.5% inflation to 2006. This will result in a capital loss for the investor. As there is income tax to pay on gilts, the conventional gilt probably then is the better option for the investor who will pay tax at the higher rate on the coupon income.

However, there are a range of factors for the investor to consider, and the most significant is the extent to which she wishes to protect herself against unexpected inflation. The final decision must also consider the investors portfolio as a whole; for example a high level of cash holdings would imply a greater exposure to inflation, which would swing the argument in favour of holding index-linked gilts.

Comparing the yield levels on indexed bonds across markets enables analysts to assess the inflation expectations in several countries at once. While this is carried out frequently, it is important to factor in the non-inflation expectation considerations that make up bond yields. A high level of demand for I-L bonds, or an overvalued conventional bond market can sometimes imply a lower forward inflation rate than is realistic. This is best illustrated using the implied forward inflation rates across four countries during 1998/1999, as shown in Figure 11.7. The inflation expectation for the UK was artificially depressed by the excessive institutional investor demand for I-L gilts, brought on in part by the Minimum Funding Requirement rules, as well as “flight to quality” during the 1998 bear market in bonds that lowered gilt yields versus Bund yields.

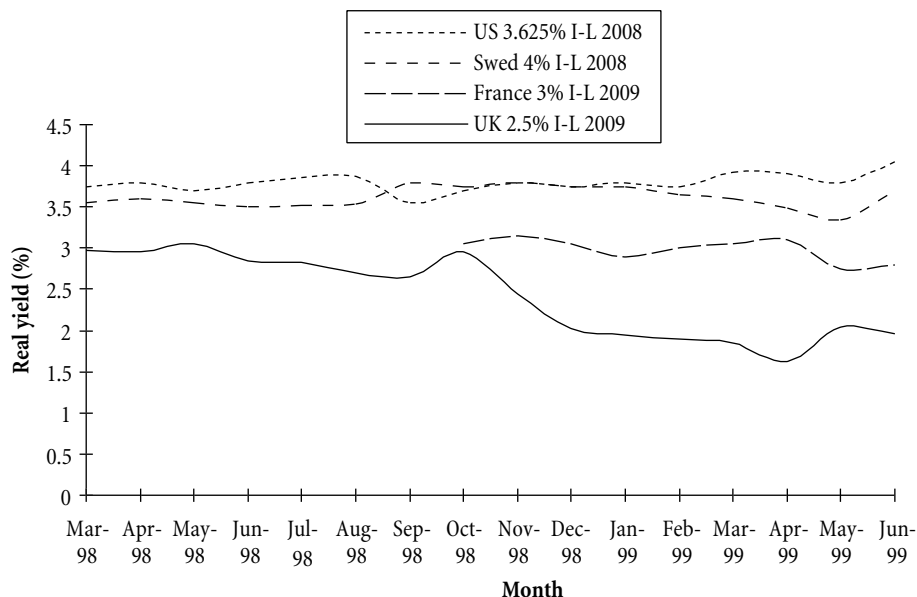


Figure 11.7: Real yields on index-linked bonds across four countries in 1998/1999. Source: BoE.

11.2.3 Double-dated gilts

Although they have not been issued for over ten years, there are currently ten double-dated gilts in existence, but they represent a small proportion of the market. Double-dated gilts have two maturity dates quoted, and under the terms of issue the government may redeem them on any day between the first and second maturity dates, providing at least three months' notice is given. As with callable bonds in the corporate market, the government will usually redeem a double-dated bond early if it is trading above par, since this indicates that the coupon on the bond is above the prevailing market interest rate. Where the price is below par the bond will be allowed to run to the final redemption date. An example of the latter is the 3½% Funding 1999–2004, which trades well below par and therefore can be expected to run to its final maturity date of 14 July 2004, although the government can redeem it at any point between now and July 2004 if it so wishes, providing it gives at least three months' notice.

Double-dated issue are usually less liquid than conventional or I-L gilts, mainly because there is a relatively small amount in issue and also because a larger proportion are held by personal investors. They also tend to have high coupons, reflecting the market rates in existence at the time they were issued.

11.2.4 Floating rate gilts

In recent years the government has issued conventional floating-rate gilts, one of which matured in March 1999, leaving only one in existence at the time of writing. Floating-rate gilts pay coupon on a quarterly basis at a rate of LIBID minus 12.5 basis points. The BoE calculates the coupon level based on the LIBID fixing for the day before the coupon payment is due. The liquidity of floating-rate gilts is comparable to conventional gilts.

11.2.5 Gilt strips

Gilt strips are zero-coupon bonds created from conventional coupon gilts. Only issues actually designated as strippable gilts may be stripped. They are considered separately later in this chapter.

11.2.6 Undated gilts

The most esoteric instruments in the gilt market are the undated gilts, also known as *irredeemable* gilts or *consols*. They are very old issues, indeed some date from the nineteenth century.¹¹ There are eight such issues, and they do not have a maturity date. Redemption is therefore at the discretion of the government. Some undated issues are very illiquid. The largest issue is the 3½% War Loan, with just over £1.9 billion in existence. In the past the BoE undertook conversions of the less liquid irredeemable bonds into War Loan, so that for all but this stock and the 2½% Treasury bond there are only *rump* amounts remaining. The government can choose to redeem an undated gilt provided a requisite notice period is given (this varies for individual issues but generally is three months), but in practice – given that the coupon on these bonds is very low – is unlikely to do so unless market interest rates drop below say, 3%. A peculiarity of three of the undated gilts is that they pay interest on a quarterly basis.¹²

11.2.7 Treasury bills

Strictly speaking Treasury bills are not part of the gilts market but form part of the sterling money markets. They are short-term government instruments, issued at a discount and redeemed at par. The most common bills are three-month (91-day) maturity instruments, although in theory any maturity between one-month and twelve-month may be selected. In the past the BoE has issued one-month and six-month bills in addition to the normal three-month maturity bills. Bills are issued via a weekly tender at which anyone may bid; generally clearing banks, building societies and *discount houses* take an active part in the bill market.

In debt capital markets the yield on a domestic government T-Bill is usually considered to represent the *risk-free* interest rate, since it is a short-term instrument guaranteed by the government. This makes the T-Bill rate, in theory at least, the most secure investment in the market. It is common to see the three-month T-Bill rate used in corporate finance analysis and option pricing analysis, which often refer to a risk-free money market rate.

The responsibility for bill issuance was transferred to the DMO from the BoE in 1999. The DMO set up a slightly changed framework¹³ in order to facilitate continued market liquidity. The main elements of the framework included a wider range of maturities and a larger minimum issue size at each weekly tender, plus a guaranteed minimum stock in issue of £5 billion. The DMO also pre-announces the maturities that will be available in the next quarter's tenders. The settlement of Treasury bills has been fixed at the next working day following the date of the tender.

11.2.8 Maturity breakdown of stock outstanding

Gilts are classified by the DMO and the Financial Times as “shorts” if maturing in 0–7 years, “mediums” if maturing in 7–15 years and “longs” if maturing in over 15 years' time. GEMMs usually apply a different distinction, with shorts being classified as 0–3 years, mediums as 4–10 years and longs as those bonds maturing in over 10 years. Of conventional gilts outstanding, the proportion of shorts:mediums:longs has remained fairly constant in recent years; the ratio was 46:35:19 in 1997, 45:35:20 in 1998, and 44:30:26 in 1999 (DMO 1999).

The distribution of holdings of gilts as at June 1999 is shown below.

Insurance companies and pension funds	65%
Overseas holders	18%
Households	9%
Banks and building societies	4%
Other institutions	3%
Local authorities and public corporations	1%

(Source: DMO)

¹¹ For instance the 2½% Annuities gilt was issued in 1853. You won't find too many market makers who are keen to trade in it though!

¹² These are 2½% Consolidated stock, 2½% Annuities and 2¾% Annuities.

¹³ This is set out in *The Future of UK Government Cash Management: The New Framework*, Debt Management Office, 4 December 1998.

Appendix 11.3 lists statistical data on the gilt portfolio and outstanding volume.

11.2.9 Market turnover

Figure 11.4 shows the annual turnover by value in gilts from the fiscal year 1993/94 to 1998/99. Average daily turnover in 1998/99 was £7.3 billion. The chart illustrates a general upward trend in market trading volumes, the trend being reversed during 1998. This reflects the contraction in global bond market trading in the aftermath of the financial crises of the second half of that year, typified by the technical default in a Russian long-dated bond and the Brazilian currency crisis.

Data on market turnover is published by the London Stock Exchange and the DMO on a regular basis.

11.2.10 Market trading conventions

Gilts are quoted on a clean price basis, for next day settlement. This is known as “cash” settlement or T+1. It is possible to trade for same day settlement (“cash-cash”) if dealing is carried out before mid-day and with the agreement of the market maker. It is also possible to trade for forward settlement. During 1998 there were major changes to gilt market trading conventions designed to bring market conventions into line with major European bond markets. These changes are detailed below.

- **Price quote.** From 1 November 1998 gilt prices changed from pricing in ticks (1/32 of a point; a tick was therefore equal to 0.03125. The tick price quote is employed in the US Treasury market) to pricing in decimals. Prices are now displayed as £ and pence per cent of stock. Auction bids are to two decimal places, as are GEMM reference prices. The bid-offer spread is very close in the gilt market, reflecting its liquidity and transparency. For bonds up to ten years in maturity it is possible to receive quotes as narrow as £0.01 between bid and offer; at the very short end institutional investors are often able to deal on “choice” prices (when the price for bid and offer is the same) if talking to two GEMMs simultaneously. For long-dated gilts the bid-offer spread can sometimes be as close as £0.06, roughly the equivalent of two ticks.
- **Daycount convention.** The daycount convention for the calculation of accrued interest was changed from actual/365 to actual/actual, from 1 November 1998. Appendix 11.6 illustrates the difference in calculations that has resulted from this change.

In addition after 31 July 1998 the special ex-dividend period arrangement for gilts was ended. This mechanism had allowed trading whereby up to 21 calendar days prior to the ex-dividend date, the parties to a gilt transaction could agree to trade on an *ex-dividend* basis (this refers to a trade in which the purchaser takes delivery of the gilts without the right to the next coupon payment). Although special ex-dividend arrangements have now been removed, gilts still automatically trade ex-dividend up to 7 business days before the calendar date (except for War Loan, for which the ex-dividend period was retained at 21 days, since reduced to 10 business days). There is no ex-dividend period for floating rate gilts. Gilt strips trade on a yield basis and of course there is no accrued interest element in the calculation of settlement proceeds in a transaction. The DMO has published a yield-to-price conversion formula, to ensure that a uniform calculation is used by all market participants.

- **Market screens.** There a number of news screens associated with the gilt market, which help to make it transparent. The DMO has screens on Reuters, Telerate and Bloomberg. Many GEMMs also post prices of gilts on their own news screens, although some firms only make their screens available to selected customers.¹⁴ Brokers’ screens are usually available to the market as a whole. Prices are indicative only and must be checked on the telephone with the market maker or broker before dealing. A typical broker screen as appearing on Bridge-Telerate, showing mid-prices for benchmark gilts, is shown at Figure 11.9. The screen also shows the most recent price change, the equivalent yield, and the yield spread to the interest rate swap curve (called the “Libor spread” on the screen shown).

¹⁴ The author’s screen on Reuters, AAHX, with short-dated gilt prices and sterling money market rates, was for some time still “alive” but is now sadly unavailable!

DESCRIPTION	TIME	MID PX	PX CHG	TRUE YIELD	LIBOR SPREAD
UKT 6	99	7:46	100.0900	—	4.6700 —0.403
UKT 8	00	9:02	104.0700	—	4.9413 —0.569
UKT 7	01	9:06	103.7400	−0.010	5.2590 −0.674
UKT 7	02	9:06	104.4700	−0.010	5.3116 −0.780
UKT 8	03	9:05	109.0000	−0.010	5.4096 −0.834
UKT 6.5	03	9:05	104.3600	—	5.3715 −0.858
UKT 6.75	04	9:06	106.3500	−0.010	5.3699 −0.868
UKT 8.5	05	9:06	116.4900	−0.020	5.4137 −0.825
UKT 7.5	06	9:06	112.4400	+0.010	5.4337 −0.731
UKT 7.25	07	9:06	112.4600	+0.070	5.3846 −0.725
UKT 9	08	9:06	126.4800	+0.040	5.3329 −0.807
UKT 9	12	9:06	135.6400	−0.020	5.2048 −0.857
UKT 8	15	9:06	133.0700	+0.060	5.0143 −0.987
UKT 8	21	9:06	142.1200	+0.010	4.8521 −1.152
UKT 6	28	9:05	120.9400	+0.110	4.6811 −1.096

Figure 11.8: Butlers Gilts screen on 14 July 1999. © Bridge-Telerate.

© GarbonICAP 1999. Used with permission.

11.3 Taxation

Coupon interest on registered gilts is payable gross, without deduction of withholding tax. Investors who are resident in the United Kingdom for tax purposes are liable to pay tax at their marginal rate on gilt coupons however. Therefore interest received, including the inflation uplift on I-L gilts, is declared on annual tax returns. Bondholders on the BoE register may request that their coupons be paid net of withholding tax.

The tax treatment is summarised below.

- **Domestic corporate investors.** The interest and capital gain earned on holdings of gilts are liable to tax at the corporate taxation rate. The corporate tax rate in 1999/2000 was 30%, and the tax is payable on a quarterly basis. The accounting treatment for gilt holdings is expected to be in line with the company's statutory accounts. This will be either a *mark-to-market* or *accruals* basis. In a mark-to-market approach, bonds are valued at their market dirty price on a daily basis; the valuation is the market value that applies at the end of the quarterly accounting period. The total return over the period is then deemed to be the increase in market value in that time. In an accruals approach, bonds are amortised on a clean price basis from the purchase date to maturity. The total return is deemed to be the change in the clean price plus income accrued over the accounting period.

The above applies equally to conventional gilts and gilt strips. For I-L gilts however UK corporates are not liable to tax on the inflation uplift earned on the principal payment received on maturity.

- **Resident private investors.** Individual private investors resident in the UK are liable for income tax on gilt coupon interest received. This includes accrued interest earned during a short-term holding. Capital gain made on a disposal of gilts is not liable to tax. Gains accruing on gilt strips are taxable however, on an annual basis. Strip earnings are taxed as income on an annual basis, irrespective of whether the strip has actually been sold. That is, the tax authorities deem a strip to have been sold and re-purchased at the end of tax year, with any gain taxed as income. A savings vehicle introduced in April 1999, known as an Individual Savings Account (ISA), allows individuals to hold gilts free of both income and capital taxes. There is a limit on the amount that may be held, which is a maximum of £5,000 per annum from tax year 2000/01 onwards. Gilt strips may also be held free of tax within an ISA, within the £5,000 yearly limit.
- **Overseas investors.** Investors who are resident overseas, both corporate and individual, are exempt from UK tax on gilt holdings. Prior to April 1998 only gilts designated as "Free of Tax to Residents Abroad" (FOTRA) paid gross coupons automatically to overseas residents; however from that date all gilts have been designated FOTRA stocks and thus overseas investors receive gross coupons.

Unlike UK equity market transactions, there is no stamp duty or stamp duty reserve tax payable on purchases or sales of gilts.

11.4 Market structure

The gilt market operates within the overall investment business environment in the United Kingdom. As such wholesale market participants are regulated by the Financial Services Authority (FSA), the central regulatory authority brought into being by the newly elected Labour government in 1997. The FSA regulates the conduct of firms undertaking business in the gilt market; it also supervises the exchanges on which trading in gilts and gilt derivatives takes place. The previous regulatory regime in the UK markets, as conducted under the Financial Services Act 1986, consisted of market participants being authorised by self-regulatory organisations such as the Securities and Futures Authority (SFA). The FSA was set up initially through merging all the different self-regulatory bodies. Hence gilt market makers (known as *gilt-edged market makers* or GEMMs) and brokers are authorised by the FSA on behalf of the SFA, while firms domiciled in European Union countries are authorised by their domestic regulatory authority. The full formal powers of the FSA will be assumed by that body once the relevant legislation has been passed, which is expected in 1999/2000.¹⁵

The gilt market is an “over-the-counter” market, meaning that transactions are conducted over the telephone between market participants. However all individual issues are listed on the London Stock Exchange, which as a Recognised Investment Exchange is also supervised by the FSA.

11.5 Market makers and brokers

Just as in the United States and France for example, there is a registered primary dealer system in operation in the UK government bond market. The present structure dates from “Big Bang” in 1986, the large-scale reform of the London market that resulted in the abolition of the old distinction between *jobbers* and *brokers* and allowed firms to deal in both capacities if they so wished; it also resulted in stock and share trading moving off the floor of the Stock Exchange and into the dealing rooms of banks and securities houses. Firms that wished to provide a two-way dealing service and act on their own account registered as gilt-edged market makers. In 1986 there were 29 companies so registered, most of whom were the gilts trading arms of the large banks. Firms registered as GEMMs with the Bank of England, and until 1998 there was a requirement for GEMMs to be separately capitalised if they were part of a larger integrated banking group. This requirement has since been removed. In September 1999 there were 16 firms registered as GEMMs, who must now be recognised as such by the DMO. The full list is provided at Appendix 11.4. GEMMs are also required to be members of the London Stock Exchange (LSE).

The key obligation of GEMMs is to make two-way prices on demand in all gilts, thereby providing liquidity to the market. Some firms observe this requirement more closely than others. Certain gilts that the DMO has designated as rump stocks do not form part of this requirement. GEMMs also must participate in gilt auctions, the main method through which the DMO issues gilts, making competitive bids in all auction programmes. The third primary requirement of a GEMM is to “provide information to the DMO on market conditions, [its] positions and turnover” (DMO 1999). In return for carrying out its market-making obligations, a GEMM receives certain privileges exclusive to it, which are:

- the right to make competitive telephone bids at gilt auctions and tap issues;
- a reserved amount of stock at each auction, available at non-competitive bid prices (currently this is ½% of the issue for each GEMM, or 10% if an I-L gilt);
- access to the DMO’s gilt dealing screens, through which the GEMM may trade or switch stocks;
- a trading relationship with the DMO whenever it wishes to buy or sell gilts for market management purposes;
- the facility to strip gilts;
- a quarterly consultation meeting with the DMO, which allows the GEMM to provide input on which type of stocks to auction in the next quarter, plus advice on other market issues;
- access to gilt inter-dealer broker (IDB) screens.

¹⁵ This will be the Financial Services and Markets Act.

In 1998 the DMO set up a separate category of GEMMs known as index-linked GEMMs (IG GEMMs). A firm could opt for registration for either or both category. The role of an IG GEMM is the same as that for conventional GEMM, as applied to index-linked gilts. An IG GEMM has the same obligations as a GEMM, with respect to I-L gilts, and an additional requirement that they must seek to maintain a minimum 3% market share of the I-L gilt market. Therefore an IG GEMM must participate actively in auctions for I-L stock. In addition to the privileges listed above, IG GEMMs also have the right to ask the DMO to bid for any I-L gilt. In September 1999 eight of the 16 GEMMs were also registered as IG GEMMs. No firm was registered solely as an IG GEMM.

11.5.1 Gilt market broker functions

There are at the time of writing two firms that provide an inter-dealer broker (IDB) service for GEMMs. This is a facility for GEMMs to advertise, via the IDB's screens, anonymous live dealing prices for any gilt, together with the nominal value of the stock that they are prepared to deal in. Only GEMMs have access to IDB screens, which assists in the provision of market liquidity. IDBs are required to be members of the London Stock Exchange and are recognised by the DMO. They are pure brokers and are prohibited from taking principal positions in any gilt or providing market intelligence outside their customer base.

A separate category of LSE broker is the broker/dealer. These are firms that act both for their own account or as agents for another party. Broker/dealers therefore trade with both market makers and client firms, and can deal with clients either as principal or agent. Broker/dealers may also act as wholesale broker dealers to GEMMs.

11.5.2 The role of the Bank of England

Although the responsibility for UK government debt management has been transferred to the DMO, the BoE continues to maintain a link with the gilt market. The Bank is also involved in monitoring other sterling markets such as gilt futures and options, swaps, strips, gilt repo and domestic bonds. The Bank's *Quarterly Bulletin* for February 1999 listed its operational role in the gilt market as:

- calculating and publishing the coupons for index-linked gilts, after the publication of each month's inflation data and inflation index;
- setting and announcing the dividend for floating-rate gilts, which is calculated as a spread under three-month Libid each quarter;
- operating the BoE brokerage service, a means by which private investors can buy and sell gilts by post instead of via a stockbroker. This service was previously operated by National Savings, the governments savings bank for private retail customers. Private investors sometimes wish to deal in gilts via the post as usually commission charges are lower and as it is a user-friendly service.

This is in addition to the normal daily money market operations, which keeps the Bank closely connected to the gilt repo market. The BoE's dealers also carry out orders on behalf of its customers, primarily other central banks.

The BoE has a duty to "protect the interests of index-linked gilt investors" (DMO 1999). This is a responsibility to determine whether any future changes in the composition of the RPI index would be materially harmful to I-L gilt holders, and to effect a redemption of any issue, via HM Treasury, if it feels any change had been harmful.

The BoE operates the Central Gilts Office (CGO) settlement mechanism on behalf of CrestCo Limited; the two bodies have now merged. It also acts as the central registrar for gilts. The settlement process for gilts is reviewed separately.

11.5.3 The role of the London Stock Exchange

The gilt market is an OTC market where trading is conducted over the telephone. However all gilts are listed instruments on the London Stock Exchange and GEMMs and inter-dealer brokers are members of the LSE. A new gilt issue and further issues of an existing stock are always listed on the LSE, usually on the day the auction or the further issue is announced.

Members of the LSE must follow its conduct of business rules, in addition there are specific rules that apply to GEMMs. This includes the requirement to report all gilt trades to the LSE, except gilt repo and stock loan trades.¹⁶ The LSE publishes market trading statistics that include the monthly turnover, by volume, of gilt transactions. It also publishes the Daily Official List, the list of closing prices for all securities listed on the London market.

¹⁶ Gilt repo is reviewed in Chapter 34.

11.6 Issuing gilts

Auctions are the primary means of issuance of all gilts, both conventional and index-linked. They are generally for £2-£3 billion nominal of stock on a competitive bid price basis. Auctions of index-linked gilts are for between £0.5 billion and £1.25 billion nominal. The programme of auctions is occasionally supplemented in between auctions by sales of stock “on tap”. This is an issue of a further tranche of stock of a current issue, usually in conditions of temporary excess demand in that stock or that part of the yield curve. That said, only one conventional stock has been tapped since 1996, a £400 million conventional tap in August 1999. The DMO has stated that tap issues of conventional gilts will only take place in exceptional circumstances.

After an auction the authorities generally refrain from issuing stocks of a similar type or maturity for a “reasonable” period. Such stock will only be issued if there is a clear demand. The 1996/97 remit for gilt issuance was accompanied by changes to the structure for gilt auctions. These changes were designed to encourage participation in auctions and to make the process more smooth. The average size of auctions was reduced and a monthly schedule put in place; periodic dual auctions were also introduced. Dual auctions allow the issue of two stocks of different maturity in the same month, which moderates the supply of any one maturity and also appeals to a wider range of investors. Market makers (GEMMs) were allowed to telephone bids in up to five minutes before the close of bidding for the auction, which allowed them to accommodate more client demand into their bids. Instituting a pre-announced auction schedule at the start of the fiscal year further assists market transparency and predictability in gilt auctions, which reduced market uncertainty. In theory a reduction in uncertainty should result in lower yields over the long term, which reduce government borrowing costs.

The DMO has a slightly different auction procedure for I-L gilts. Unlike conventional gilts, which are issued through a multiple price auction, I-L gilts are auctioned on a uniform price basis. This reflects the higher risks associated with bidding for I-L stock. In an auction for a conventional gilt, a market maker will be able to use the yields of similar maturity stock currently trading in the market to assist in her bid; in addition a long position in the stock can be hedged using exchange-traded gilt futures contracts. There is also a very liquid secondary market in conventional gilts. For these reasons a market maker will be less concerned about placing a bid in an auction without knowing at what level other GEMMs are bidding for the stock. In an I-L gilt auction there is a less liquid secondary market and it is less straightforward to hedge an I-L gilt position. There are also fewer I-L issues in existence, indeed there may not be another stock anywhere near the maturity spectrum of the gilt being auctioned. The use of a uniform price auction reduces the uncertainty for market makers and encourages them to participate in the auction.

11.6.1 Auction procedure

As part of its government financing role, HM Treasury issues an auction calendar just before the start of the new financial year in April. The DMO provides further details on each gilt auction at the start of each quarter in the financial year, when it also confirms the auction dates for the quarter and the maturity band that each auction will cover. For example, the quarterly announcement might state that the auction for the next month will see a gilt issued of between four and six years’ maturity. Announcements are made via Reuters, Telerate and Bloomberg news screens. Eight days before the auction date the DMO announces the nominal size and the coupon of the stock being auctioned. If it is a further issue of an existing stock, the coupon obviously is already known. After this announcement, the stock is listed on the LSE and market makers engage in “when issued” trading (also known as the *grey market*). When issued trading is buying and selling of stock to the forward settlement date, which is the business day after the auction date. As in the Eurobond market, when issued trading allows market makers to gauge demand for the stock amongst institutional investors and also helps in setting the price on the auction day.

Conventional gilts

In conventional gilt auctions bidding is open to all parties.¹⁷ Institutional investors will usually bid via a GEMM. Only GEMMs can bid by telephone directly to the DMO.¹⁸ Other bidders must complete an application form. Forms are made available in the national press, usually the *Financial Times*. Bidding can be competitive or non-competitive. In a competitive bid, participants bid for one amount at one price, for a minimum nominal value of £500,000. If a bid is successful the bidder will be allotted stock at the price they put in. There is no minimum price.

¹⁷ Including private individuals.

¹⁸ When telephoning one’s bid, it is important to quote the correct “big figure” for the stock!

Telephone bidding must be placed by 10.30am on the morning of the auction and in multiples of £1 million nominal. Bidding is closed at 10.30am. In a non-competitive bid, GEMMs can bid for up to ½% of the nominal value of the issue size, while others can bid for up to a maximum of £500,000 nominal, with a minimum bid of £1,000. Non-competitive bids are allotted in full at the weighted-average of the successful competitive bid price. In both cases, non-GEMMs must submit an application form either to the BoE's registrar department or to the DMO, in both cases to arrive no later than 10.30am on the morning of the auction.

The results of the auction are usually announced by the DMO by 11.15am. The results include the highest, lowest and average accepted bid prices, the gross redemption yields for these prices, and the value of non-competitive bids for both GEMMs and non-GEMMs. The DMO also publishes important information on auction performance, which is used by the market to judge how well the auction has been received. This includes the difference between the highest accepted yield and the average yield of all accepted bids, known as the *tail*, and the ratio of bids received to the nominal value of the stock being auctioned, which is known as the *cover*. A well-received auction will have a small tail and will be covered many times, suggesting high demand for the stock. A cover of less than 1.5-times is viewed unfavourably in the market and the price of the stock usually falls on receipt of this news. A cover over 2-times is well received. On rare occasions the cover will be less than one, which is bad news for the sterling bond market as a whole. A delay in the announcement of the auction result is sometimes taken to be as a result of poor demand for the stock. The DMO reserves the right not to allot the stock on offer, and it is expected that this right might be exercised if the auction was covered at a very low price, considerably discounted to par. The DMO also has the right to allot stock to bidders at its discretion. This right is retained to prevent market distortions from developing, for example if one bidder managed to buy a large proportion of the entire issue. Generally the DMO seeks to ensure that no one market maker receives more than 25% of the issue being auctioned for its own book.

Index-linked gilts

Auction bids for I-L gilts are also competitive and non-competitive. Only IG GEMMs may make competitive bids, for a minimum of £1 million nominal and in multiples of £1 million. For I-L gilts there is a uniform price format, which means that all successful bidders receive stock at the same price. A bid above the successful price will be allotted in full. Non-competitive bids must be for a minimum of £100,000, and will be allotted in full at the successful bid price (also known as the "strike" price). IG GEMMs are reserved up to 10% of the issue in the non-competitive bid facility. Non-IG GEMMs must complete and submit an application form in the same way as for conventional gilt auctions.

The DMO reserves the right not to allot stock, and to allot stock to bidders at its discretion. The maximum holding of one issue that an IG GEMM can expect to receive for its own book is 40% of the issue size.

11.6.2 Tap issues

Prior to the adoption of the competitive bid mechanism, taps were the main method of issuing gilts. A tap is a tender for an amount of stock at a minimum price, announced on the morning of the tender. Bids must be received no more than 30 minutes later. An auction procedure has been used for some time now for conventional gilts, but was only adopted in November 1998 for I-L gilts. As such taps are now used only for "market management" purposes, rather than as a form of issue, for all gilts. Market management covers specific one-off conditions such as an excess demand for one stock, or a shortage of stock in the market leading to trading illiquidity. In these circumstances the DMO may "tap" an issue to relieve this condition, by issuing a small amount of the stock into the market. The DMO has stated that taps will not be conducted on stocks a minimum of three weeks either side of an auction for that stock. Tap issues are of small size, usually only £100 million and never more than £400 million. The tap issue result is announced as soon as possible.

11.6.3 Conversions

In the new reformed environment of the gilt market, the emphasis is on having a large, liquid supply of *benchmark* issues in existence. The BoE used conversion offers to switch holdings of illiquid gilts into more liquid benchmark stocks. The DMO has instituted a formal conversion programme. Conversion offers are made to bondholders to enable them to exchange (convert) their gilts into another gilt. The new gilt is usually the benchmark for that maturity. The aim behind conversion offers is to build up the issue size of a benchmark gilt more quickly than would be achieved by auctions alone, which are a function of the government's borrowing requirement. A period of low issuance due to healthy government finances would slow down the process of building up a liquid benchmark. By

the same token, conversions also help to increase the size and proportion of strippable gilts more quickly. Conversions also, in the words of the DMO, concentrate liquidity across the yield curve by reducing the number of illiquid issues and converting them into benchmark issues. Illiquid gilts are usually high coupon issues with relatively small issues sizes. In the past for example, double-dated gilts have been converted into benchmark gilts. Gilts that may be converted are usually medium and long-dated bonds with less than £5 billion nominal outstanding. The current distribution of a gilt is also taken into conversion: issues that are held across a wide range of investors, particularly private retail investors, are less likely to be offered for conversion as the offer will probably not be widely taken up.

The aim of HM Treasury and the DMO is to have liquid benchmark gilts at the 5, 10, 25 and 30-year maturity end of the yield curve. Conversion assists this process. The DMO uses the forward yield curve generated by its yield curve model in setting the terms of the conversion offer. The aim of the process is to make an offer to bondholders of the “source” stock¹⁹ such that a large proportion of the stock is converted at a value fair to both the bondholder and the government. The conversion ratio is calculated using the dirty price of both source and “destination” gilts. Both bond cash flows are discounted to the conversion date using the forward yield curve derived on the date of the conversion offer announcement. This approach also takes into account the “pull-to-par” of both stocks; the running cost of funding a position in the source stock up to the conversion date is not taken into account. Conversion terms are announced three weeks before the conversion date, although an intention to convert will have been announced one or two weeks prior to this.

On announcement of the conversion terms, the DMO holds the fixed terms, in the form of a fixed price ratio of the two stocks, open for the three week offer period. If the terms move in bondholders’ favour they can convert, whereas if terms become unfavourable they may choose not to convert. It is not compulsory to take up a conversion offer. Holders of the source stock may choose to retain it and subsequently trade it, or hold it to maturity. However the source stock will become less liquid and less widely held after the conversion, more so if the offer is taken up in large quantities. If the remaining amount of the source stock is so small that it is no longer possible to maintain a liquid market in it, thereby becoming a rump stock, GEMMs are no longer required to make a two-way price in it. Even if the stock does not end up as a rump issue, the bid-offer spread for it may widen. However these are not issues if the investor is seeking to hold the bond to maturity. The DMO will announce if a stock acquires rump status; it also undertakes to bid for such stocks at the request of a market maker, as assistance towards the maintenance of an orderly market.

In a conversion in October 1999, the DMO offered to convert holdings of 8% 2003 gilts into the new benchmark five-year stock, the 5% 2004. Approximately £1 billion of the older gilt was converted into £1.1 billion of the new benchmark bond. The conversion was very attractive to holders of the source bond and was oversubscribed, being covered more than five times.

11.7 The DMO and secondary market trading

11.7.1 Secondary market operations

As part of its role in maintaining an orderly and efficient market the DMO conducts business in the secondary markets in line with specific requirements and demand. This business generally involves trading with GEMMs in response to particular situations, including the following:

- bidding for rump stock;
- switches of stock;
- bidding for I-L stocks;
- in special situations, bidding for conventional stock;
- intervening to make stock available for *repo* in circumstances a false market has resulted in specific issues being unavailable for borrowing or purchase;
- sale of stock from official portfolios.

These situations are now considered in this section.

¹⁹ This is the stock being converted out of.

- **Bidding for rump stock.** The DMO acts as a buyer of last resort for gilts that have been designated rump stocks, for which GEMMs are not required to quote two-way prices. Although the DMO only deals direct with GEMMs, institutions or private individuals can ask for a bid price via a broker, who will deal with the GEMM.
- **Switches of stock.** GEMMs may request switches out of or into stocks that are held on official portfolios, which the DMO will carry out amongst stocks of the same type. The terms of any switch are set by the DMO.
- **Bidding for I-L stocks.** As part of its role in supporting the liquidity of the market, the DMO will bid for I-L stock at the request of an IG GEMM. Generally any bid will be at the market level or at a discount to the market level.
- **Bidding for conventional stock.** In “exceptional circumstances” the DMO will announce that it accept offers of a specific stock, acceptance of which is at the DMO’s discretion.
- **Supplying stock for use in repo.** In certain circumstances a specific issue will go “tight” in the repo market, meaning that it is difficult to borrow the stock for delivery into short sales. This is typically reflected in the stock going *special* in the repo market.²⁰ On rare occasions the stock may become undeliverable, leading to failed transactions and also failure to deliver into the equivalent gilt futures contract. When this happens the DMO may make the stock available for borrowing, out of official portfolios, or issue a small amount of the stock into the market.
- **Sale of stock from official portfolios.** The DMO will acquire amounts of stock as a result of its secondary market operations, and these holdings may be made available for re-sale into the market. Although it does not actively offer stock to the market, when holdings are made available from its portfolios the DMO will announce via its news screen the stock on offer, and the size available; this is known as the DMO’s “shop window”. No offer price is indicated, but bids must be at the market level or above; acceptance is at the DMO’s discretion. If more than £50 million of one issue is on offer a mini-tender is held, announced via the DMO’s news screens.

11.7.2 Closing reference prices

After the close of business each day the DMO publishes reference prices and the equivalent gross redemption yields for each gilt on its news screens. The final reference price is based on closing two-way prices supplied by each GEMM at the end of the day. The prices, previously referred to as “CGO reference prices” but now following the merger of the CGO with CREST, called DMO or gilt reference prices, are frequently used in the calculation of settlement proceeds in repo and stock loan transactions. The derivation of the reference price is described in Appendix 11.5.

11.8 Settlement

11.8.1 Crest and Central Gilts Office

The Central Gilts Office (CGO) is the computerised book-entry settlement system for gilts. It was first introduced in 1986. The system was upgraded in 1995 to allow for the introduction of new gilt products, namely gilt repo and strips. The CGO service is operated by the Central Gilts Office (CGO), part of the Bank of England. CGO provided facilities for gilt investors to hold stock in *dematerialised* form, and transfer stock electronically. Transfers are processed by an assured payment system based on the principle of *delivery versus payment* (DVP). Thus CGO provides a secure system for the electronic holding and transfer of stocks between members without the need for transfer forms and bond certificates. The service was originally established by the BoE and the London Stock Exchange to facilitate the settlement of specified securities, essentially gilts and certain sterling bonds such as Bulldogs for which the BoE acts as registrar, and was upgraded by the BoE in 1997. This upgrade enhanced the CGO facility to settle gilt repo trading activity, which commenced in January 1996, and to cater for the introduction of the gilt strips facility in December 1997. It also provides a vehicle for the development of real-time Delivery versus Payment (DVP) through links to the Real Time Gross Settlement System (RTGS) for wholesale payments, which was introduced in mid-1996.

In May 1999 responsibility for the CGO service was transferred to CRESTCo Limited, the company that operates the Crest settlement system for London market equities. The gilt settlement service is now operated within Crest, with the merger process completed in July 2000.

²⁰ A *special* repo rate is below the general repo rate. See Chapter 34.

The basic concept of the CGO within Crest remains the same, that is the provision of secure settlement for gilt-edged securities through an efficient and reliable system of electronic book entry transfers in real time against an assured payment. The CGO is a real-time, communication-based system. Settlement on the specified business day (T+1 for normal gilt trades) is dependent on the matching by CGO of correctly input and authenticated instructions by both of the parties and the successful completion of pre-settlement checks on the parties' stock account balances and credit headroom.

The CGO provides facilities for:

- settlement of stock and cash transfers;
- overnight transfer of collateral, known as DBV, to allow CGO members to pass stock against a secured overnight loan;
- automatic reporting of all transactions to the London Stock Exchange;
- matching of instructions between counterparties;
- the movement of stock free of payment;
- processing of stock lending and repo transactions.

In addition the following facilities were added at the upgrade:

- the stripping and reconstitution of gilts, at the request of GEMMs, the DMO and the BoE;
- forward-dated input, useful for the input of gilt repo;
- greater control by settlement banks over the credit risks run on their customers (by means of a debit-capped payment mechanism);
- a flexible membership structure (allowing the names of "sponsored" as well as "direct" members to appear on the register);
- multiple account designations.

All GEMMs as well as most banks and large building societies are members of Crest. Certain institutional investors and brokers are also members. The membership stood at around 300 in 1999, including nominee companies. Over 90% of the total value of gilts was held in dematerialised form in CGO in September 1999. Firms who trade less frequently in gilts often settle through an agent bank. This indirect participation in Crest is usually done via a nominee company, usually a current bank member, or via sponsored membership. In the case of sponsored membership a company opens a Crest account in its name but the sponsor is responsible for conducting the firm's gilt settlement activity.

Part of the reforms in the gilt market through 1998 included a facility for overseas investors to hold gilts in what was then CGO, via either the Euroclear or Cedel clearing systems. This became possible after both Euroclear and Cedel opened accounts at CGO.

11.8.2 Delivery by Value

Delivery by Value (DBV) is a mechanism whereby a CGO member may borrow money from or lend money to another CGO member against overnight gilt collateral. The CGO system automatically selects and delivers securities to a specified aggregate value on the basis of the previous night's CGO Reference Prices; equivalent securities are returned the following business day. The DBV functionality allows the giver and taker of collateral to specify the classes of security to be included within the DBV. The options are: all classes of security held within CGO, including strips and bulldogs; coupon bearing gilts and bulldogs; coupon bearing gilts and strips; only coupon bearing gilts.

DBV repo is a repo transaction in which the delivery of the securities is by the DBV mechanism in CGO; a series of DBV repos may be constructed to form an "open" or "term" DBV repo. The DBV functionality allows repo interest to be automatically calculated and paid.

11.8.3 Merger of CGO, CMO and CREST

In September 1998 the BoE and CRESTCo (the body running the CREST listed equity settlement system) announced that CRESTCo would assume responsibility for the settlement of transactions in gilts and money market instru-

ments. This involved the merger of CREST,²¹ CGO and the Central Money Markets Office (CMO) settlement system. The merger is now complete.

Crest and CGO maintained similar settlement processes before the merger process began. This enabled the merger to commence relatively smoothly. One area of difference was the “circles” cycle run by each system;²² due to the higher number of transactions and the greater value within CGO, there was a higher number of circles operated within CGO. Crest supports multiple runs of circle settlement each day, to enable the CGO mechanism to continue unchanged. As part of the merger, CREST was enhanced to provide identical facilities to CGO to enable the continuing stripping and reconstituting of gilt strips.

There is now one unified securities settlement system (CREST) for all cash capital markets instruments in the United Kingdom. Future initiatives at CREST are intended to result in the construction of cross-border links with securities depositories across Europe.

11.9 Index-linked gilts analytics

Index-linked gilts were first introduced by the UK government in 1981. The authorities commitment to index-linked gilts was emphasised in the DMO’s gilt market review for 1998, which stated that they were to form a higher part of overall gilt sales. A formal auction calendar for index-linked stock was also established. Index-linked gilts have their coupons and principal amounts adjusted to reflect changes in the retail prices index (RPI), with an eight month lag. This time lag is to allow for the next coupon uplift to be calculated and known before the bond is traded. There are currently eleven index-linked gilts in issue, with the longest dated bond maturing in 2030. A formal auction programme for index-linked gilts has been set up by the DMO, with the first auction taking place in October 1998. As part of this, registration of index-linked GEMMs (IG GEMM) was introduced by the DMO. IG GEMMs are recognised as such in the same way as conventional GEMMs; they have access to competitive bidding at IG auctions, and are in contact with the DMO over the type of stock to be scheduled for auction. The requirements for IG GEMMs are the same as for conventional GEMMs, namely active participation in auctions, providing two-way prices in all IG stocks at all times, an additional requirement to maintain a minimum 3% market share and the provision to the DMO of “relevant market information”. As part of the auction programme the DMO will supply a minimum amount of £2.5 billion of stock each year. This figure is a cash figure, not a nominal value.²³

In this section we consider simple measures of inflation expectations using index-linked gilts. An understanding of market inflation expectations is of some use to practitioners and policy makers. For instance, the market-determined yield on an I-L bond, set against a government’s inflation target, gives an indication of the credibility of the target itself. The market’s expected view of inflation is also useful data for financial and industrial sector institutions when they are carrying out forward business planning. Corporate finance and project appraisal analysis that considers future cash flows and expected rates of return also build in an inflation expectation. This expectation estimate is best obtained using yields from the government bond market. Governments use inflation expectations when deciding the make-up of their funding, for example the decision on how much conventional versus index-linked debt to issue.²⁴ This section introduces the concepts involved; the subject of real interest-rate term structure modelling is covered in Part VIII.

Analysts frequently use the yield levels on index-linked bonds and their value relative to conventional yields to infer market expectations of future inflation. In the past information on inflation expectations has been obtained via surveys and theoretical models. These frequently prove unreliable. Therefore an effort has been made from the 1980s onwards to infer expectations from bond prices. In some markets an exchange-traded futures contract on a retail prices index can be traded, which would give a direct indication of the market’s expectation of future inflation

²¹ The CREST settlement system is the clearing system for London market equities and corporate bonds.

²² “Circles” is the term used to describe the daily settlement process.

²³ I am grateful to Gurminder Bhachu at the DMO for pointing this out.

²⁴ Deacon and Derry (1994) cite the example of the Swedish central bank, the Riksbank issuing index-linked bonds for the first time because it believed that the market’s expectations on the future level of inflation was over-stated, which would make the yield on index-linked bonds lower than that on conventional bonds, thus resulting in a saving on borrowing costs for the government.

rates.²⁵ In the UK no contract of this type exists. However because of the existence of index-linked bonds we do have an indication of the market's view on inflation, albeit an indirect one. Several studies have analysed the information content of yields in the I-L gilt market, among them Deacon and Derry (1994) and Woodward (1990).

In analysing I-L gilts we note that the instruments do not offer complete protection from inflation, due to the eight-month lag in the indexation formula. Therefore when we calculate the real redemption yield for I-L gilts we must make an assumption about future inflation, both the level itself and that it remains constant. This enables us to write the expression for the I-L bond price/yield relationship, if we simplify the arrangement to a six-month (rather than eight-month) indexation lag.

$$P = \frac{(1+i)C}{(1+i_a)(1+r)} + \frac{(1+i)C}{(1+i_a)(1+r)^2} + \dots + \frac{(1+i)(C+M)}{(1+i_a)(1+r)^n} \quad (11.7)$$

where

C	is the coupon
M	is the redemption proceeds on maturity (par)
r	is the gross real yield
i	is the known RPI inflation used to set the next coupon payment
i_a	is the assumed average future inflation rate over the term to maturity.

To obtain the *net* real redemption yield we use the net coupon payment, which is calculated using $C \times (1 - t)$, where t is the rate of income tax.

The need to assume a constant future inflation rate becomes significant when we estimate and construct a real yield curve, or term structure of real interest rates (also examined in Part VIII). This is because the level of the assumed inflation rate obviously has an impact on the real yield that is calculated. Figure 11.9 shows the relationship between the assumed inflation rate and the real yield for the 2½% I-L Treasury 2009 in October 1999. The yield calculations were performed on a Bloomberg terminal.



Figure 11.9: Sensitivity of I-L gilt real yield and assumed inflation rate.

11.9.1 Calculating breakeven inflation

Two methods for deriving inflation expectations from gilt prices are described in Deacon and Derry (1994). The first is the “simple” method. In this approach average expected inflation is calculated using the *Fisher identity*, which we encountered in Part I. By the Fisher equation we calculate the expected inflation rate as the spread between the real yield on the I-L gilt, which has been calculated using an assumed constant rate of inflation, and the yield to maturity of a conventional gilt of the same maturity. We observed this earlier when we compared the real yield on the 2½% I-L Treasury 2009 with the redemption yield on the 5¾% Treasury 2009. This yield spread is an estimate of the average expected rate of inflation over the ten years from 1999. A common error is to interpret the yield spread as

²⁵ For example, Deacon and Derry (*ibid*) describe a Consumer Price Index futures contract traded on the New York Coffee, Sugar and Cocoa Exchange in the 1980s.

the expected inflation rate in ten years' time (in our example using the 2½% I-L Treasury 2009), which is incorrect, and not the average rate during the next ten years. However this inflation measure is at best a rough expected approximate measure, with ultimately little value. This is because the real yield measure for an I-L bond is itself calculated using an assumption of an average rate of inflation, which yield is then used to estimate an inflation expectation. Thus this measure uses an inflation expectation to estimate an inflation expectation! Deacon and Derry refer to this as a *lack of internal consistency*.

Another simple method is calculating break-even inflation rates. In this approach an average inflation expectation is estimated by comparing the yield on a conventional gilt to that on an I-L gilt of the same (or similar) maturity, and applying the compound form of the Fisher equation. This means that we decompose the nominal yield on a bond into components of real yield and rate of inflation, using compound interest rules. For the UK market, which uses semi-annual coupon compounding, the expression is given at (11.8):

$$\left(1 + \frac{y}{2}\right) = (1 + i)^{\frac{1}{2}} \cdot \left(1 + \frac{r}{2}\right) \quad (11.8)$$

where

y is the nominal yield
 r is the real yield
 i is the rate of inflation.

The break-even approach implicitly assumes that investors treat conventional and I-L bonds the same and do not demand an inflation or liquidity risk premium for holding either type of bond. That is, they are risk-neutral. Under this assumption a conventional bond and index-linked bond will have the same nominal yield. Under this scenario of equal nominal yields, we can use the conventional bond yield instead of the I-L yield, which enables us to use the I-L bond price/yield formula and the Fisher equation to solve for the real yield and the expected rate of inflation. Such an approach was used in the study by Arak and Kreicher (1985). In their study a 0% tax rate is used as is a zero risk premium, although the base RPI level is that in place at the time of issue, whereas for I-L gilts it is the RPI in place eight months prior to issue that is actually used in calculations.

11.9.2 Duration matching

In the two methods described above we noted that conventional and I-L gilts of roughly the same maturity should be used in the analysis. Another approach is to match the two bond types by duration rather than term to maturity. This appears logical since duration equalises bonds in terms of the weighted-average maturity of their cash flows. Duration (and modified duration) as we saw from Chapter 7 is a measure of the sensitivity of a bond's price to changes in interest rates. This measure can be interpreted also as price sensitivity to changes in expected inflation and the real interest rate, given that the real yield of a bond is a function of the expected rate of inflation.²⁶ For this reason it might be expected that gilts of equal duration should have similar yields. However a drawback to matching gilts by duration arises if the two bonds selected differ significantly in their terms to maturity, so that their yields are dissimilar. For example consider the following measures for two gilts on 9 November 1999²⁷:

2½% IL Treasury 2009

Real yield adjusted duration:	8.386
Inflation adjusted duration:	8.262
Real yield:	1.967% (assuming average inflation of 3%)
Money yield:	4.972%

The conventional gilt with the nearest duration measure is the 9% Treasury 2012.

9% Treasury 2012

Duration:	8.426
Gross redemption yield:	5.109%

²⁶ This follows the explanation given by the expectations hypothesis.

²⁷ Data obtained from Bloomberg analytics screen.

The conventional gilt has a term to maturity three years greater than the I-L bond. A calculation of the break-even inflation rate using the yields from these two bonds could be argued to be the average expected rate of inflation over either 10 years or 13 years.

The same problem associated with calculating the real yield for an I-L gilt arises in duration matching. The formula for calculating duration uses the present value of a bond's cash flows. The cash flows for an I-L gilt can only be known if we assume an average inflation rate for the term to maturity. Just as with the real yield measure, the duration and modified duration of an I-L gilt is sensitive to a change in the assumed inflation rate. Several commentators have suggested that using duration matching should be avoided, including Bootle (1991) and Woodward (1990). The reason for this is because holders of conventional bonds face what is termed nominal interest rate risk, that is the risk of a change in real interest rates or inflation expectations, whereas I-L bond holders are exposed to real interest rate risk only. The two bond types therefore expose investors to unequal risks. Duration matching is only effective for instruments that are exposed to the same risks.

The break-even approach to obtaining a measure of inflation expectation is at best a rough approximation, and the underlying assumptions behind its calculation render it of little use to market practitioners and policy makers. The main issues are:

- the assumption that gilts trade with no risk or liquidity premia is not accurate, as any observation of market yields for specific bonds will confirm;
- to date there have not been a pair of conventional and I-L gilts that have identical maturity dates, forcing analysts to use approximately matching maturities. This leads to inaccuracy in the values for the real interest rate and the expected inflation rate;
- often there is no I-L gilt maturing in a particular year;
- the tax rate used in the calculation of the break-even rate has a significant impact on the result. To produce the net real yield measure for an I-L gilt, analysts assume a marginal level of taxation, usually the basic and the higher rate of tax (20% and 40%).

If we use break-even forward inflation rates we need to apply the appropriate rate of tax in the calculation. Figure 11.10 shows the effect of using different tax rates, using real yields for the 2½% I-L Treasury 2009 stock and the 5¾% Treasury 2009 during 1998/1999. The graph indicates that there is a significant effect on the break-even rate itself of using a different tax rate, but that the differential between the results is uniform irrespective of the tax rate used.

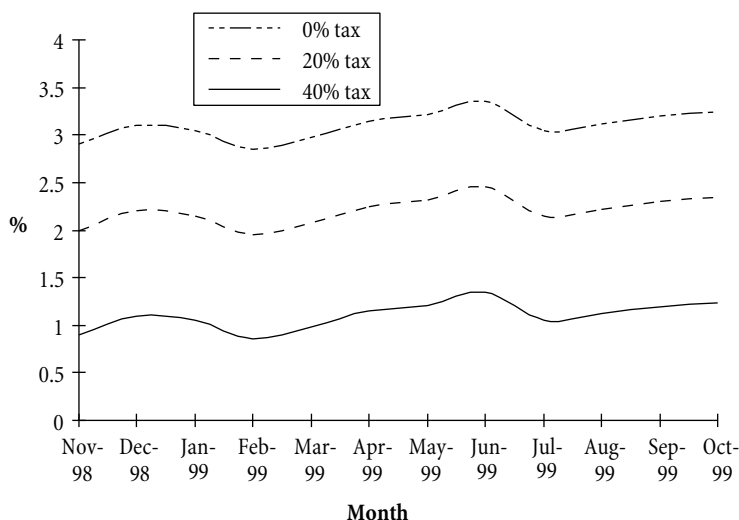


Figure 11.10: Break-even inflation rate and effect of taxation.

There are further issues to consider. Since (as in Figure 11.11) we derive the break-even inflation rate from two specific gilts, the calculation will reflect the idiosyncrasies of these two stocks. The price effect of supply and demand

factors will influence the calculation. Therefore the inflation expectation using this method is peculiar to the two stocks selected and will be a different result for each pair. To illustrate consider Figure 11.11, which compares the break-even inflation rate for the 2½% I-L Treasury 2009 both when compared against the 10-year benchmark bond, the 5¾% Treasury 2009, and another stock of similar maturity the 8% Treasury 2009. The coupon effect of the latter stock is clearly demonstrated.

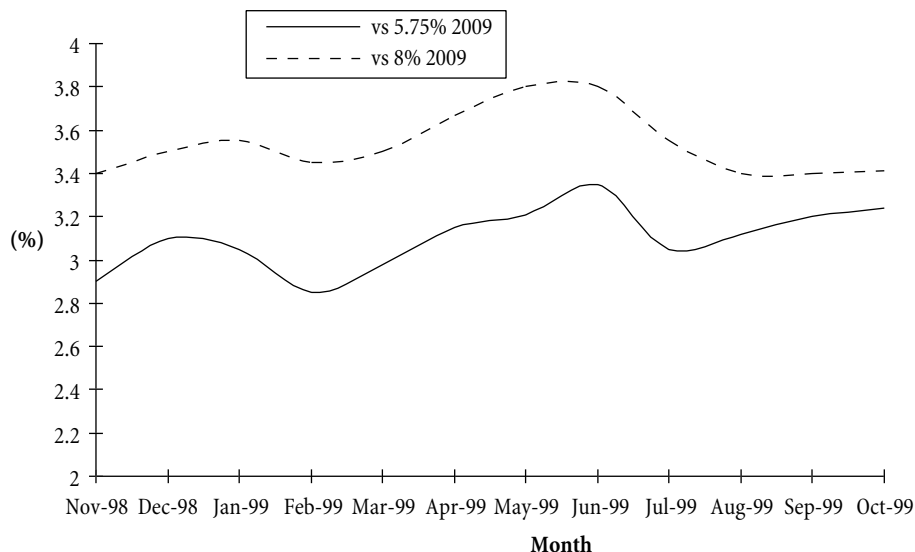


Figure 11.11: Sensitivity of break-even inflation rate to specific conventional stock.

Deacon and Derry (1994) have advocated using double-dated gilts in the analysis when faced with a shortage of gilts for particular maturity dates. This is acceptable because the market convention holds that a gilt trading above par will trade to the earliest maturity date, while if it is below par the bond will be redeemed at the last possible date. However the maturity will change if the bond price moves above or below par; where this is the case using a double-dated gilt will result in distortions in the break-even inflation calculation. The yield on a double-dated bond is also not strictly comparable to that of an equivalent maturity conventional bond (under the maturity assumption for the double-dated bond), since double-dated gilts trade cheap to the yield curve. This reflects the lower liquidity for these bonds and the premium attributable to the call feature attached to the bond, which is against investor interests.

11.9.3 An implied forward inflation rate curve

If we calculate the break-even inflation rate for every I-L gilt for the period 1998/1999, bar the 4.125% IL 2030 (for which there is no matching maturity conventional gilt), we can arrive at an estimate of future inflation expectations over different time periods. This is illustrated in Figure 11.12. The structures are drawn as smooth curves, although with only ten I-L gilts available this approach is problematic. This is one method by which a term structure of future inflation rates can be constructed. However a forward inflation curve derived in this manner is flawed because of the small number of gilts available and because the calculations are strictly applicable only to the specific gilts used. This restricts the use we might make of the curve. Nevertheless although the level of the future inflation expectation is not accurately reported by the curves in Figure 11.12 they can be used to indicate changes in expectations themselves.

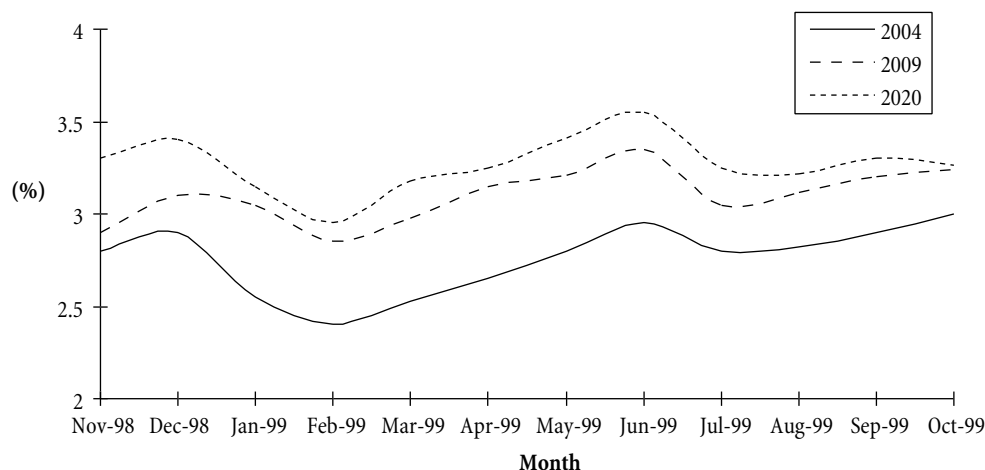


Figure 11.12: Break-even inflation rates over different terms to maturity.

In Part VIII we review approaches to get around the issues we discussed in this section, and the derivation of an implied forward inflation rate curve.

11.10 Gilt strips

The introduction of gilt strips was part of a range of measures developed by the Bank of England (BoE) and HM Treasury in connection with the modernisation and reform of the gilt market. An official market in zero-coupon government bonds had previously been available in the United States from 1985 and France from 1991; among developed economies, strips markets now exist in Canada, Germany, Italy, Spain, Holland, Belgium and New Zealand.

11.10.1 Market mechanics

A strip is a zero-coupon bond, that is a bond that makes no coupon payments during its life and has only one cash flow, its redemption payment on maturity. “Stripping” a bond is the process of separating a standard coupon bond into its individual coupon and principal payments, which are then separately held and traded in their own right as zero-coupon bonds. For example a ten-year gilt can be stripped into 21 zero-coupon bonds, comprised of one bond from the principal repayment and twenty from the semi-annual coupons. Coupon payments due in six, twelve, eighteen and so on months from the stripping date would become six, twelve, eighteen and so on month zero-coupon bonds. In Chapter 6 of this book we explained how the prices of such bonds are related to the yield curve derived from conventional bonds. We also illustrated the general rule that if a yield curve is upward sloping (positive), the theoretical zero-coupon yield curve will lie above the conventional bond yield curve, while if the conventional curve is inverted, the zero-coupon curve will lie below it.

The gilt strips market is a recent development, with trading having commenced only on 8 December 1997. Not all gilts are strippable; only stocks designated as being strippable by the BoE (and subsequently the DMO) may be stripped. The market began quietly with relatively low volumes of trading. After one month’s trading, under 1% of the (then) £82 billion of strippable stock was held in stripped form. In this time turnover in the strips market, for both coupon and principal strips, amounted to 1% of turnover in the conventional gilt market (BoE, *Quarterly Bulletin*, Feb 1998).

Gilts held in Crest can be stripped or reconstituted by gilt-edged market makers (GEMMs). Strips are fully fledged gilts; they remain registered securities and liabilities of HM Government, therefore they have identical credit risk compared to conventional gilts. The minimum strippable amount for all gilts is £10,000 nominal. This can then be increased in units of £10,000 nominal. There is no limit on the amount or proportion of a strippable gilt issue that can be stripped or reconstituted. The Crest system, inherited from CGO, includes a forward input facility for stripping and reconstitution requests to be entered up to one month in advance. GEMMs will include strips trading as part of their general gilt market making obligations.

We have noted that strips are fully-fledged gilts, registered securities and liabilities of the government. They are stripped and reconstituted via the Crest facility, on the direction of GEMMs. It is not possible to remove a strip from Crest and hold it in paper form. Although any investor can hold and trade strips, they are not available via the BoE's Brokerage Service, the facility that allows private investors to buy and sell conventional gilts through the Post Office. Private investors must buy and sell gilts through a stockbroker. This is because when strips were introduced the BoE believed that strips, which carried greater interest-rate risk than conventional gilts of similar maturity, should not be available for purchase by retail investors unless they were aware of their characteristics; therefore strips cannot be purchased through the post.

Stripped coupons from different gilts but with the same coupon dates are fully fungible; this increases their liquidity. At the moment there is no fungibility between coupon and principal strips, although this remains under review and may be possible at a later date. All strippable gilts have the same coupon dates, 7 June and 7 December each year, so that all strips mature on these dates each year. Strips are not deliverable into the LIFFE medium- and long-gilt futures contracts; for these it is necessary to deliver a coupon gilt from amongst those in the futures contract delivery basket.

11.10.2 Strippable gilts

At the end of December 1999 there were 36 conventional gilts in issue, excluding rump status gilts, of which 11 were strippable. At Table 11.3 is a list of the strippable gilts and the amount held in stripped form as at the end of December 1999; for comparison we also show the amount held in stripped form at the end of March 1998, which was one quarter after the introduction of trading. At the beginning of December 1999 the total nominal amount of strippable stock was £116.2 billion, of which £2.82 billion was held in stripped form, representing 2.42% by volume.

Table 11.3 shows that the amount of gilts held in stripped form stayed in the range of 2% to 2.5% of total nominal value during the 1998 and 1999. The BoE stated before the start of the strips market that all new benchmark issues will now be eligible for stripping. All strippable stocks have the same coupon dates (7 June and 7 December), which has allowed all coupon strips to be fungible. The DMO has left open the possibility of making coupon and principal strips fungible, and whether to introduce a second set of coupon dates. There is no limit on the amount or proportion of any strippable gilt that can be stripped.

A complete table of strip prices and yields as at March 1999 is given in Appendix 11.7.

Stock	Redemption date	Amount in issue (£m)	Amount held in stripped form (£m)	Strips as % of issue	Amount in issue (£m)	Amount held in stripped form (£m)
		31/12/1999	31/12/1999	31/12/1999	31/03/1998	31/03/1998
8% Treasury 2000	07-Dec-00	9,800	162	1.65	9,800	314
7% Treasury 2002	07-Jun-02	9,000	390	4.33	9,000	198
6 1/2% Treasury 2003	07-Dec-03	7,987	116	1.45	2,000	27
5% Treasury 2004	07-Jun-04	7,408	2	0.0002	–	–
8 1/2% Treasury 2005	07-Dec-05	10,373	584	5.63	10,373	304
7 1/2% Treasury 2006	07-Dec-06	11,700	259	2.21	11,700	40
7 1/4% Treasury 2007	07-Dec-07	11,000	303	2.75	11,000	163
5 3/4% Treasury 2009	07-Dec-09	8,827	106	1.20	–	–
8% Treasury 2015	07-Dec-15	13,787	167	1.21	13,787	237
8% Treasury 2021	07-Jun-21	16,500	517	3.13	16,500	600
6% Treasury 2028*	07-Dec-28	9,900	333	3.36	2,000	–

*6% Treasury 2028 eligible for stripping following auction on 20 May 1998, which took amount in issue to £5 billion

Table 11.3: List of strippable stocks and amount Stripped as at December 1999. Source: DMO.

11.10.3 Pricing convention

The BoE consulted with GEMMS before the introduction of the strips market on the preferred method for pricing strips. The result of this consultation was announced in May 1997 when the Bank stated that strips would trade on a yield basis rather than a price basis. The day count convention adopted, in both the conventional and strips market, was actual/actual; this was changed from the previous method of actual/365 on 1 November 1997. Those GEMMs who had preferred to quote strips on a price basis had claimed that there may have been difficulties with agreeing a standard formula for converting yields into prices. The yield pricing convention is in line with other zero-coupon bond markets in Europe and North America.

After further consultation the following standard formula was adopted for use in the market:

$$P = \frac{100}{(1 + (r/2))^{\frac{d}{s} + n}} \quad (11.9)$$

where

P	is the price per £100 nominal of the strip
r	is the gross redemption yield (decimal)
d	is the exact number of days from the settlement date to the next quasi-coupon date
s	is exact number of days in the quasi-coupon period in which the settlement date falls
n	is the number of remaining quasi-coupon periods after the current period.

The r and s values are not adjusted for non-working days. A *quasi-coupon* date is a date on which a coupon would be due if the bond was a conventional coupon bond rather a strip. For example a strip maturing on 7 December 2005 would have quasi-coupon dates of 7 June and 7 December each year until maturity. A *quasi-coupon period* is defined to be the period between consecutive quasi-coupon dates. There are always six calendar months, regardless of the nature of the first coupon. For example a gilt settling on its issue date (assuming this is not also a quasi-coupon date) will have a quasi-coupon period which starts on the quasi-coupon date prior to the issue date and ends on the first quasi-coupon date following the issue date.

This method is also used in the US strips market, and is sometimes referred to as the US “street” convention. The formula uses simple interest for the shortest strip, which is consistent with money market convention, and compound interest for all other strips which is then consistent with gilt market convention. The shortest strip is discounted on an actual/actual basis, which is inconsistent with sterling money market convention but is consistent with the convention followed in the gilt market. All other strips are discounted on an actual/actual basis. Using this method results in a jump in price when the shortest-but-one strip becomes the shortest since the formula switches from discounting on a compound interest basis to a simple interest basis.

Expression (11.9) is also used for strips with only one quasi-coupon period remaining.

Yields are quoted to three decimal places; cash prices are calculated to six decimal places. The rounding convention is to round the third decimal place up by one if the fourth (or seventh) decimal place is 5 or above, and then to shorten at the third decimal place.

11.10.4 Interest rate risk for strips

Strips have a longer duration than equivalent maturity conventional bonds. As they are zero-coupon bonds their duration is equal in time to their maturity. The duration of a strip will decline roughly in parallel with time, whereas for a conventional bond the decline will be less. For a given modified duration, a strip will be less convex than a coupon bond. The following points highlight some of the salient points about how duration and convexity of strips compare with those of coupon bonds:

- strips have a Macaulay duration equal to their time in years to maturity;
- strips have a higher duration than coupon bonds of the same maturity;
- strips are *more* convex than coupon bonds of the same maturity;
- strips are less convex than coupon bonds of the same duration.

The reason that a strip is less convex than a conventional coupon bond of the same duration is that the coupon bond will have more dispersed cash flows than the strip. Although strips are less convex than bonds of identical

duration, the highest duration conventional gilt (6% Treasury 2028) had a duration of 15.56 years (modified duration 15.26) in March 1999, whereas the principal strip from that bond had a duration of 29.77 years at that time (modified duration 29.14). Long strips are therefore the most convex instruments in the gilt market.

11.10.5 Uses of strips

Strips are a new instrument in the sterling bond markets. They have a large number of uses for investors and traders. The following properties of strips make them potential attractive investments for market participants:

- simplicity of one cash flow at maturity allows matching to future liabilities;
- more precise management of cash flows;
- no reinvestment risk, as associated with conventional coupon-bearing bonds;
- holdings can be tailored to portfolio sensitivities;
- higher duration and convexity than conventional bonds of the same maturity, which would be useful for either duration weighting a portfolio or for taking risk positions;
- tax advantages in certain jurisdictions.

Strips are arguably the most basic cash flow structure available in the capital markets, being as they are zero-coupon paper. By investing in a portfolio of strips an investor is able to construct a desired pattern of cash flows, one that matches more precisely his investment requirements than that obtained from a conventional bond. In theory therefore there is a high demand for strips in the market. Domestic institutional investors with an interest at the long end of the yield curve will have a demand for long-dated strips, for example, life assurance companies and pension funds who have liabilities up to 30 years or more into the future. While strips can help to meet this demand, due to their providing a known amount at the end of a long-dated investment horizon, they are also attractive in that they present no reinvestment risk. The rate of return gained from buying a strip today and holding to maturity will be a true yield.

Retail investors and their advisers are also interested in strips because they allow them to have investments with cash flows of their choice. Where private investors wish to invest for a known future commitment, they can also hold strips and realise the precise amount required at their investment horizon.

We can summarise the main uses of strips as follows:

- matching long liabilities (pension funds);
- matching cash flows (insurance companies wishing to match their actuary-estimated payments, interested in acquiring forward-starting annuities);
- collateralisation of guarantees (financial companies, for example firms selling products with a guaranteed return, such as a guaranteed equity fund or guaranteed minimum return stock-market linked savings, ISA or other account);
- expressing views on sterling interest rates (commercial and investment banks, foreign investors and money market funds);
- taking advantage of the high duration of strips (hedge funds, proprietary traders).

It is possible to lend and repo strips, as with conventional gilts, and therefore also “short” strips that one does not own and cover such short positions via borrowing or reverse repo. Because of certain properties of strips, namely their longer duration making them more price volatile, certain market participants may prefer not deal in strips repo. However as the gilt repo market includes provision for initial margin and (if required by a participant) daily margin, this should not make strips unattractive as repo collateral. The repo of gilt strips is carried out in exactly the same way as with conventional gilts, with the exception that there is no accrued interest to consider when calculating settlement amounts.

The BoE announced in April 1998 that strips would be eligible in Delivery-by-Value (DBV) collateral in its daily money market operations. Therefore strips have from that date been eligible for use in daily repo operations with the Bank.

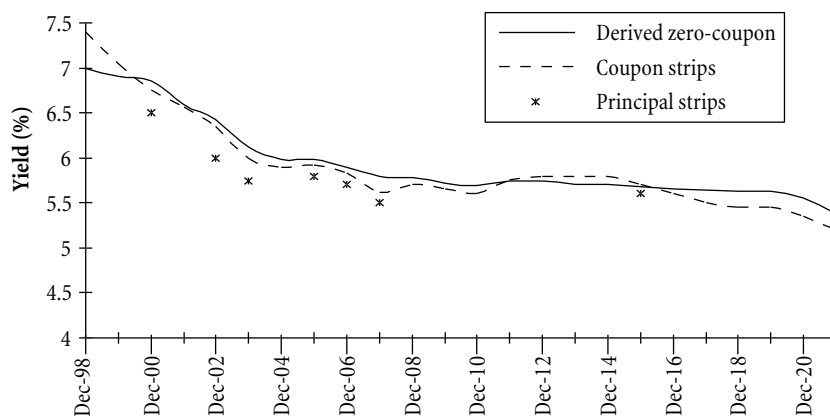


Figure 11.13: Strip yield curves January 1998. Source: Bloomberg, DMO.

11.10.6 Subsequent developments and analysis of market volumes

Since the start of trading in gilt strips, activity has remained low and has built up only gradually. The introduction of the new 30-year benchmark gilt, the 6% Treasury 2028 added a new potentially attractive investment instrument to the market. Although this stock was first auctioned in January 1998 it was not made strippable until May that year, when the total volume outstanding was raised to £5 billion; it was felt by the BoE that the initial amount of £2 billion was not large enough to provide sufficient liquidity. At April 1998 the percentage of strippable stock held in stripped form stood at 2.2% (BoE, QB, May 1998). Turnover remained low in the first four months of trading, at an average of £135 million per week. At the time this figure represented ½% of turnover in the conventional gilt market (*ibid*). The BoE observed that a large proportion of activity involved overseas investors taking a view on sterling and exploiting arbitrage opportunities that existed between the gilt strips market and strips markets in Germany and France. The main customer interest in the early months of trading appeared to be in principal strips at the longer end of the curve, while GEMM activity would appear to be concentrated on trading principal strips against the underlying coupon gilt. This last activity is to be expected, and is the main reason that strips trading originally began in the US market. The profit potential for a government bond market maker who strips a coupon bond is mainly found in any arbitrage opportunity that results from mispricing of the underlying bond or the strip.

There are several reasons behind the slow build-up to trading in strips. A BoE analysis (QB, Feb 1999) has suggested that these reasons include the following:

- because strips are not yet included in any industry benchmarks, there is no particular incentive for fund managers to buy and hold them (and no pressure from actuaries). An “index tracker” bond fund is more likely to buy the conventional long-dated bond;
- as we demonstrated in the section on forward rates, when the benchmark yield curve is negatively sloping, strip yields will lie below coupon bond yields. As the gilt yield curve has been inverted since before the start of trading in strips, strips have traded at yields below the conventional curve, making them look expensive relative to coupon bonds. As strips are zero-coupon instruments their duration will be longer than those of coupon gilts of the same maturity. In this environment therefore, strip yields will be closer to yields of longer-dated coupon gilts, compared to the yield on similar maturity gilts. The shape of the strips yield curve will deter some investors from holding strips because they would view them as dear to the coupon bond curve.
- as client interest in strips has remained low, activity in the professional market (between GEMMs) has also been low; lower liquidity has resulted in less competitive price quotes from GEMMs, which has increased further the cost of buying strips;
- to date the repo market in strips has been limited, which would make the financing of strips positions comparatively more costly. This may turn into something of a vicious circle, as low liquidity in strips will lead to low volumes in strips repo, which will further make the running of strips portfolios less economically attractive. Strips market repo activity should in be helped through the BoE’s incorporation of strips in its daily money

market operation, which made strips eligible for use as collateral via DBV. Strips have been so eligible since April 1998. However strips generally always trade as *special* in the repo and stock loan markets, and coupon strips are very illiquid in these markets.

Private retail investment in strips has been low as a result of the tax treatment for strips, whereby the bonds are taxed annually on any capital gain made, even though no income has been received in the taxation period. The introduction of a new product for retail investors, the Individual Savings Account, which allows tax-free investment in strips (among other securities), should make them more attractive for private investment.

As the market in strips becomes more mature and as more investors become familiar with the instrument, we can expect to see trading volumes increase. In theory interest in trading and holding strips should increase if at any point the yield curve flattens out or becomes positively sloped. In July 1999 the gilt yield curve changed to a positive sloping shape from the two to seven years' maturity range,²⁸ so analysis of strips volumes in the year following this period should indicate higher levels of trading. We hope to cover this in a second edition of this book. In any case the introduction of strips has added a flexible new investment product that should benefit sterling markets generally.

11.11 Zero-coupon bond trading and strategy

As we have noted elsewhere trading in strips began only gradually, and volume and liquidity levels is expected to increase as the market becomes more mature. At the start of trading transparency in the market was not as high as that in the coupon gilt market, a factor that some GEMMs were able to exploit to their advantage. That said, strips are familiar to participants in other markets such as the USA and France, and the principles of strips trading are generally the same across markets. We present here some basic concepts and strategies that are common in strips markets.

11.11.1 Strips versus coupon bonds

Before looking at trading strategies let us reiterate what was said earlier on the properties of strips. The main characteristics of strips are:

- they have a Macaulay duration equal to their maturity;
- they have no reinvestment risk because there are no intermediate cash flows;
- their prices and yields are relatively easy to calculate and require no iteration.

In theory, stripping and reconstituting a bond would be attractive if the sum value of each of the components (coupon and principal strips) is greater or less than the value of the underlying bond. Where this occurs, arbitrage profits are possible. However the market mechanism will ensure that this (almost) never happens, hence strips market participants in the UK are able to deal in an efficient and fairly priced environment.

The higher duration of strips gives them greater leverage. A smaller cash investment is required to buy a certain nominal amount compared to bonds. As strips have a much higher duration for the same calendar maturity they offer greater flexibility in matching specific cash flows. The higher convexity of strips is also an attractive feature, as convexity in a bond is a desired property. It means that the bond will appreciate more in price for a given decline in yield, than it will depreciate for the same back-up in yield. Table 11.4 is a convexity example from the German bund strips market.

Yield = 5.47%	Modified duration	Convexity	% price change – 100bp yield	% price change + 100bp yield
Bund:				
■ 6.25% 2024	11.2	2.4	13.9	–11.5
■ Principal Strip 2024	24.9	6.4	28.4	–22.0
■ Coupon Strip 2010	11.2	1.6	13.1	–11.5

Table 11.4: Bund market strips example. Data source: Bloomberg.

²⁸ Although this was short-lived, with the curve reverting to negatively sloping after the 3-year maturity area shortly after.

Strips offer the opportunity to create strategies for a given duration with higher convexity than the equivalent coupon bond, which means strips can benefit from market volatility. This is shown in Table 11.4.

There is a further potential attraction and this is that strips can reduce currency exposure for non-sterling based investors. The increase duration and lower price (in a positive sloping yield curve environment) of a strip relative to a similar maturity coupon gilt means that an investor can buy a smaller nominal amount of zero-coupon bonds and still maintain their desired interest rate exposure.

11.11.2 Determining strip value

It is always important to understand how strips behave under different yield curve conditions and over time; this will help the investor to identify relative value. We have already shown that the yield on a strip is predominantly a function of the shape of the yield curve, as well as related factors such as supply and demand, liquidity, market sentiment, future interest rate expectations and tax issues.

The three most common ways to calculate the value of a strip are:

- valuation using the bond curve;
- equivalent duration method;
- theoretical zero-coupon curve construction.

Valuation using the bond curve

The spread between a strip and a bond with the same maturity is often used as an indicator of strip value. It is essentially a rough-and-ready approach; its main drawback is that two instruments with different risk profiles are being compared against each other. This is particularly true for longer maturities.

Equivalent duration method

When measuring relative value, aligning the strip and coupon bond yields on the basis of modified duration will allow for a better comparison. From Figure 11.13 it would appear that at this time coupon strips are cheap at the short end and particularly expensive at the longer end. A proper analysis will require us to construct a curve of yields against modified duration, and check for value on such a curve. However we would have no indication as to how strip values will change for various yield curve moves.

Theoretical zero-coupon curve

This is the most common way of determining the value of a strip, via the derivation of a theoretical zero-coupon curve. We saw in Chapter 6 how the actual strip yield curve can differ from a spot curve derived from the benchmark yield curve. When such anomalies occur there may be opportunity for profitable relative value trading. For example, a trader may wish to examine the shapes of the theoretical and actual zero-coupon curves and where they have differed from each other over time. A spread trade can be put on where an anomaly is detected, for instance where there is a greater than usual divergence of an actual yield from a theoretical yield. When the curves converge again the spread is unwound and the trader takes his profit.

A theoretical spot curve is usually calculated from a theoretical bond par yield curve (the par curve is often constructed by plotting the existing bond yields against their respective maturities, and then fitting a curve through these points using predetermined criteria). We examined in the forward rates case study how the zero-coupon or spot yield curve is then derived from a coupon curve. Strips will often trade at a spread to the theoretical zero-coupon bond value; this indicates which maturities are cheap and which expensive. And as we noted just now, in this way anomalies can be identified. In the gilt strips market, the zero-coupon curve tends to be well arbitrated on forward rates. This means that while there may well appear to be arbitrage opportunities available when analysing the spot yield curve, special repo conditions will make actual arbitrage almost impossible. This is usually taken into account in the shape of the forward curve.

11.11.3 Slope of the yield curve

We illustrated the relationship between coupon and zero-coupon yield curves in Chapter 6. When the curve is flat, the spot curve will also be flat. When the yield curve is negative, the theoretical zero-coupon curve must lie below the coupon yield curve. This is because the yield on coupon-bearing bonds is affected by the fact that the investor receives part of the cash flow before the maturity of the bond; the discount rates corresponding to these earlier

payment dates are higher than the discount rate corresponding to the final payment date on redemption. In addition the spread between zero-coupon yields and bond yields should increase negatively with maturity, so that zero-coupon bonds always yield less than coupon bonds.

In a positively shaped yield curve environment the opposite is true. The theoretical zero-coupon curve will lie above the coupon curve. It is interesting however to observe the overall steepness of the curves. In general the steeper the coupon curve is, the steeper the zero-coupon curve will be. It should be remembered that each yearly value of the coupon curve is considered in the derivation of the zero-coupon curve. Hence a yield curve could for example, have exactly one-year, 10-year and 30-year yields, while the theoretical zero-coupon 30-year yield could be substantially higher or lower. The derived yield level would depend on whether the points on the term structure in between these maturity bands were connected by a smooth curve or straight line. This argument is sometimes cited as a reason for not using the bootstrapping method, in that the theoretical zero-coupon yields that are obtained are too sensitive for real-world trading. Bond analysts often use sophisticated curve smoothing techniques to get around this problem, and produce theoretical values that are more realistic. This issue becomes more important when there are few bonds between points on the term structure, such that linear interpolation between them produces inaccurate results. For example between the ten-year and thirty-year maturities there are eight liquid gilts; between twenty and thirty years there are only two gilts. This is a key reason for analysts to use more sophisticated curve-fitting techniques.

11.12 Strips market anomalies

From the start of gilt strips trading the market has observed some long-standing anomalies that mirror observations from other strips markets, such as those in the USA and France. These include the following:

- **Final principal trades expensive.** It might be expected that the strip yield curve should behave in a similar fashion to the coupon curve. However due to supply and demand considerations more weight is always given to the final principal strip, and this is indeed so in the gilt strips market.
- **Longest maturity is the most expensive.** A characteristic seen in all well-developed strip markets is that maturities with the longest duration and the greatest convexity trade expensive relative to theoretical values. Conversely, intermediate maturities tend to trade cheap to the curve. This can be observed when looking at the gilt strips and coupon curves.
- **Principal strips trade at a premium over coupon strips.** Principal strips reflect the premium investors are prepared to pay for greater liquidity and, in some markets, regulatory and tax reasons. This rule is so well established that principal strips will sometimes trade more expensive relative to coupon strips even when their outstanding nominal amount is lower than that of coupon strips.
- **Intermediate maturity coupons are often relatively cheap.** Market makers in the past have often found themselves with large quantities of intermediate maturity coupon strips, the residue of client demand for longer maturities. This has occurred with gilt strips where at certain times coupon strips of 3–8 years' maturity have traded cheap to the curve.
- **Very short coupon strips trade expensive.** In a positively sloped yield curve environment short strips are often in demand because they provide an attractive opportunity to match liabilities without reinvestment risk at a higher yield than coupon bonds of the same maturity. We have not observed this in the gilt strips market to date, as the yield curve has been inverted from before the start of trading. However in France for example the short end up to three years is often well bid.

11.13 Trading strategy

11.13.1 Bond replication through strips

This is the theoretical strategy and the one that first presents itself. The profit potential for a GEMM who strips a gilt lies in arbitrage resulting from a mispricing of the coupon bond. Due to the market mechanism requiring that there be no arbitrage opportunity, the bid price of a gilt must be lower than the offer price of a synthetic gilt (a gilt reconstituted from a bundle of coupon and principal strips); equally the offer price of the gilt must be higher than the bid price of a synthetic gilt. Of course if the above conditions are not satisfied, a risk-free profit can be obtained by trading the opposite way in both instruments simultaneously and earning the difference between the two prices.

The potential profit in stripping a gilt will depend on actual gilt yields prevailing in the market and the theoretical spot rate yield curve. To illustrate how a GEMM might realise a profit from a coupon stripping exercise, consider a hypothetical five-year, 8 per cent gilt selling at par (and therefore offering a yield to maturity of 8%), trading in a market that consists of the set of gilts shown in Table 11.5.

Maturity date	Years to maturity	Coupon (%)	Yield to maturity	Price
1-Sep-99	0.5	0.0	6.00	97.0874
1-Mar-00	1.0	10.0	6.30	103.5322
1-Sep-00	1.5	7.0	6.40	100.8453
1-Mar-01	2.0	6.5	6.70	99.6314
1-Sep-01	2.5	8.0	6.90	102.4868
1-Mar-02	3.0	10.5	7.30	108.4838
1-Sep-02	3.5	9.0	7.60	104.2327
1-Mar-03	4.0	7.3	7.80	98.1408
1-Sep-03	4.5	7.5	7.95	98.3251
1-Mar-04	5.0	8.0	8.00	100.0000

Table 11.5: Hypothetical gilt prices and yields.

Assume a settlement date of 1 March 1999, which is one of our coupon dates, so that each of the bonds has precisely 0.5, 1, 1.5 and so on years to maturity. The prices and gross redemption yields of our gilts are stated for each bond. Using these yields and prices we can compute the theoretical or *implied* zero-coupon (spot) interest rates for each period. This is shown in Table 11.6. Let us imagine that the GEMM buys the gilt at par and strips it, with the intention of selling the resulting zero-coupon bonds at the yields indicated for the corresponding maturity shown in Table 11.6. The theoretical spot rates shown have been calculated using the coupon yields and the *bootstrapping* method.

The gross redemption and spot yield curves for the set of hypothetical gilts are shown in Figure 11.14.

Maturity date	Years to maturity	Yield to maturity (%)	Theoretical spot rate (%)
1-Sep-99	0.5	6.00	6.000
1-Mar-00	1.0	6.30	6.308
1-Sep-00	1.5	6.40	6.407
1-Mar-01	2.0	6.70	6.720
1-Sep-01	2.5	6.90	6.936
1-Mar-02	3.0	7.30	7.394
1-Sep-02	3.5	7.60	7.712
1-Mar-03	4.0	7.80	7.908
1-Sep-03	4.5	7.95	8.069
1-Mar-04	5.0	8.00	8.147

Table 11.6: Hypothetical gilt spot yields.

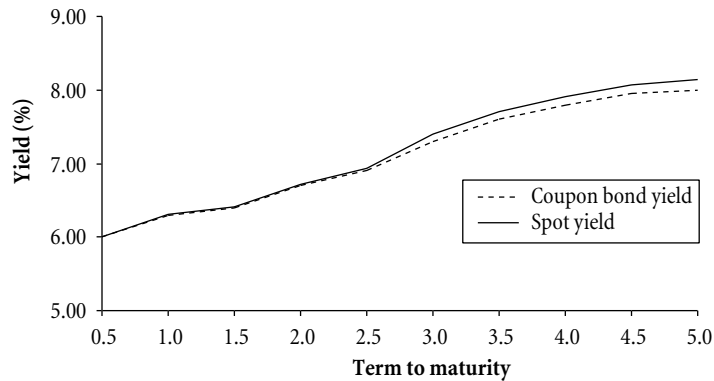


Figure 11.14: Hypothetical gilt yield curves.

Table 11.7 shows the price that the GEMM would receive for each strip that was created from the 8% five-year gilt. As we saw in Part I the price of a coupon bond is the discounted total present value of all its cash flows using the required market interest rate. Here we can equate it to the total present value of all the cash flows from the strips, each discounted at the yield corresponding to its specific maturity (from Table 11.6). The proceeds received from selling the strips come to a total of £100.4913 per £100 of par value of the gilt originally bought by the GEMM.

Maturity date	Years to maturity	Cash flow	Present value at 8%	Yield to maturity (%)	Present value at yield to maturity
1-Sep-99	0.5	4	3.8462	6.00	3.8835
1-Mar-00	1.0	4	3.6982	6.30	3.7594
1-Sep-00	1.5	4	3.5560	6.40	3.6393
1-Mar-01	2.0	4	3.4192	6.70	3.5060
1-Sep-01	2.5	4	3.2877	6.90	3.3760
1-Mar-02	3.0	4	3.1613	7.30	3.2258
1-Sep-02	3.5	4	3.0397	7.60	3.0809
1-Mar-03	4.0	4	2.9228	7.80	2.9453
1-Sep-03	4.5	4	2.8103	7.95	2.8164
1-Mar-04	5.0	104	70.2587	8.00	70.2587
			100.0000		100.4913

Table 11.7: Theoretical profit from gilt coupon stripping.

We can see why the GEMM has had the opportunity to realise this profit. Consider the fourth column in Table 11.7. This shows us how much the GEMM paid for each of the cash flows by buying the entire package of cash flows, that is, by buying the bond at a yield of 8%. For instance, consider the £4 coupon payment due in three years. By buying the five-year gilt priced to yield 8%, the GEMM pays a price based on 8% (4% semi-annual) for that coupon payment, which is £3.1613. However if we accept the assumptions in this illustration, investors are willing to accept a lower yield, 7.30% (3.65% semi-annual) and purchase a strip with three years to maturity at the price marked. Note that the present value calculation uses equation (11.10) which is the standard discounted cash flow equation, adjusted for bonds paying semi-annual coupons.

$$PV = \frac{FV}{(1 + (r/2))^{nm}} \quad (11.10)$$

where n is the number of years in the term (years to maturity), r is the bond yield to maturity and m is the number of compounding periods per year. Thus investors here are willing to pay £3.2258. On this one coupon payment (now of

course a strip versus a coupon payment) the GEMM realises a profit equal to the difference between £3.2258 and £3.1613, or £0.0645. From all the strips the total profit is £0.4913 per £100 nominal.

Maturity date	Years to maturity	Cash flow	Present value at 8%	Theoretical spot rate (%)	Present value at spot rate
1-Sep-99	0.5	4	3.8462	6.000	3.8835
1-Mar-00	1.0	4	3.6982	6.308	3.7591
1-Sep-00	1.5	4	3.5560	6.407	3.6390
1-Mar-01	2.0	4	3.4192	6.720	3.5047
1-Sep-01	2.5	4	3.2877	6.936	3.3731
1-Mar-02	3.0	4	3.1613	7.394	3.2171
1-Sep-02	3.5	4	3.0397	7.712	3.0693
1-Mar-03	4.0	4	2.9228	7.908	2.9331
1-Sep-03	4.5	4	2.8103	8.069	2.8020
1-Mar-04	5.0	104	70.2587	8.147	69.7641
			100.0000	~ 100.0000	

Table 11.8: Theoretical versus fair value strip pricing.

Let us now imagine that instead of the observed yield to maturity from Table 11.7, the yields required by investors is the same as the theoretical spot rates also shown. Table 11.8 shows that in this case the total proceeds from the sale of zero-coupon gilts would be approximately £100, which being no profit would render the exercise of stripping uneconomic. This shows that where strips prices deviate from theoretical prices, there may be profit opportunities. We have shown elsewhere that there are differences between observed strip yields and theoretical yields, indicating that there are (often very small) differences between derived prices and actual prices. Do these price differences give rise to arbitrage opportunities? Due to the efficiency and transparency of developed country bond markets, the answer is usually no. It is the process of coupon stripping that prevents the price of a gilt from trading at a price that is *materially* different from its theoretical price based on the derived spot yield curve. And where discrepancies arise, any arbitrage activity will cause them to disappear very quickly. As the strips market becomes more liquid, the laws of supply and demand will eliminate obvious arbitrage opportunities, as has already happened in the US Treasury market and is usually the norm in the gilts market. However there will remain occasional opportunities to exploit differences between actual market prices of strips and the theoretical price given by the benchmark (coupon) gilt yield curve.

11.13.2 Tracking theoretical spreads

Anomalies can sometimes be detected by tracking the spreads of strips against the theoretical yield curve. As noted earlier, supply and demand considerations make a strips that is not in demand trade at cheaper levels. This may induce an investor to buy the strip. Conversely a strip that is in demand will become expensive and thus, might signal a selling opportunity when compared to the average levels it has traded at historically.

11.13.3 Rolling down the yield curve

In a positive yield curve environment strips will give a superior return due to a greater “rolldown” effect, since the zero-coupon curve will stand higher than the par curve, especially at short maturities. On the other hand when a change in monetary policy is anticipated, switching into long duration strips will provide greater leverage and price performance.

11.13.4 Duration weighted switches

It is important to ensure that the risk profile of a portfolio or position remains the same when switching from bonds into strips (assuming that the current interest rate risk exposure is what is desired). This can be achieved by duration-weighting the new portfolio made up of the strips and cash. Table 11.9 gives three hypothetical examples of duration weighted switches for gilts and strips with assumed prices and yield values in a positive yield curve environment.

	Nominal	Cash	Deposit	Yield
(1) Bond into strip: same maturity				
Sell Gilt 5.75% 2009	100	100	0	5.75
Buy Principal strip 2009	129	75	25	5.98
(2) Bond into strip: longer maturity				
Sell Gilt 5.75% 2009	100	100	0	5.75
Buy Principal strip 2021	356	59	41	7.14
(3) Bond into strip: shorter maturity				
Sell Gilt 5.75% 2009	100	100	0	5.75
Buy Coupon strip June 2002	288	262	−162	3.83
			(borrow)	

Table 11.9: Duration-weighted switching.

11.13.5 Barbell strategies

A *barbell* strategy involves selling an intermediate maturity coupon bond and using the proceeds to buy a duration-weighted combination of both shorter- and longer-duration bonds. The opposite to this position is called a *butterfly*. Barbell strategies have two advantages:

- barbells increase the holding period return;
- barbells may increase the convexity of the portfolio.

To achieve a possible yield pick-up in a positive yield curve environment, an investor could sell the long gilt (2028 maturity) and buy both the five-year gilt and the long strip to pick up yield while maintaining the same exposure to the market with the help of the high duration strip. However in a negative yield curve environment, which prevailed in the gilt market at the start of trading in strips until July 1999 (and changed to being upward sloping only up to around the three-year maturity), this strategy is not possible. We have illustrated this strategy using a set of yields that existed in the German bund and strips market in July 1998, shown in Table 11.10 below.

Sell 26-year bond, buy 5-year bond and 25-year strip			
	Cash	Mod Dur	Yield
Sell 6.25% Jan 2024	1.000	12.2	6.47
Buy 6% September 2003	0.603	4.9	4.96
Buy July 2023 strip	0.397	23.4	9.27
Duration weighted yield pick-up			14 bp

Table 11.10: Bund strips barbell strategy.

The relative performance of the barbell is evidently subject to second-order yield curve risk. If the shape of the curve changes the performance of the position will be affected. For example if the yield curve steepens, that is the yield spread between the short-dated maturity and long-dated maturity widens, the barbell would benefit; if the curve flattens the butterfly would probably outperform.

11.13.6 Portfolio optimisation

Strips are attractive for cash flow matching of assets with specific liabilities, or for enhancing the portfolio yield through duration matching. This can be used both at the short and the long-end of the yield curve, amongst both money market participants and longer-dated investors.

11.13.7 Cross-currency spreads

As we previously noted, the high duration characteristic of strips means that less cash is needed to invest in a market to retain the same exposure to yield moves, giving less currency exposure compared to holding coupon bonds. Table 11.11 shows an hypothetical example, in a positive yield curve environment, of a US dollar-based investor who is holding the 5¾% Treasury 2009 gilt, but wishes to reduce his currency risk. By switching out of the bonds into a

duration-weighted amount of 15-year strips and cash (surplus funds are deposited in short-term cash accounts), the investor has reduced his currency risk while still being able to benefit from a fall in sterling interest rates. Part of the currency risk has been exchanged for yield curve exposure.

	Cash	Nominal	Mod Dur	Yield
5.75% 2009	100	100	7.0	5.68
\$ cash (1year)	50	53	0.0	6.00
Strip 2015	50	120	14.1	6.33
Weighted yield pick up				48 bp

Table 11.11: Cross-currency trade example.

11.13.8 Currency protection

During periods of high currency volatility, investors may wish to reduce their exposure to a certain currency but maintain an interest rate exposure in that currency's bond market. This can be achieved by switching from bonds into strips, and releasing the cash from this switch into assets of a currency that is perceived to be more stable. The position can be switched back when the period of currency volatility passes. The ability to take a long position in a market while limiting currency exposure will also reduce the hedging cost of the position.

11.14 Illustration: Yield and cash flow analysis

The following examples illustrate the yield analysis and cash flows for the 5¾% Treasury 2009, which matures on 7 December 2009, its principal strip and a coupon strip maturing on 7 December 2009. The 5¾% 2009 was the ten-year benchmark gilt during 1999. The market information reflects the position as at February 1999, for settlement date 11 February 1999. Interest rate and price data was obtained from Bloomberg.

Table 11.12 below shows the cash flows paid out to a bondholder of £1 million nominal of the 5¾% 2009. On the trade date (10 February 1999, for settlement on 11 February) this bond traded at 113.15, with a corresponding yield of 4.2224%. The convexity of this bond at this time was 0.820. The relevant implied spot interest rates at each of the cash-flow dates are shown alongside.

The cash flow for the December 2009 principal strip is shown at Table 11.13. Note that if one is calling up this security on the Bloomberg® system, the “ticker” is UKTR, comprising the standard UKT (for “United Kingdom Treasury”) and the suffix R (from “residual”, the Bloomberg term for principal strips). The Bloomberg ticker for coupon strips is UKTS.

Pay date	Cash flow	Spot	Pay date	Cash flow	Spot
7-Jun-99	28,750.00	5.1474	7-Dec-04	28,750.00	4.2746
7-Dec-99	28,750.00	4.9577	7-Jun-05	28,750.00	4.3168
7-Jun-00	28,750.00	4.8511	7-Dec-05	28,750.00	4.3599
7-Dec-00	28,750.00	4.7376	7-Jun-06	28,750.00	4.3738
7-Jun-01	28,750.00	4.6413	7-Dec-06	28,750.00	4.3881
7-Dec-01	28,750.00	4.5481	7-Jun-07	28,750.00	4.3603
7-Jun-02	28,750.00	4.4614	7-Dec-07	28,750.00	4.3326
7-Dec-02	28,750.00	4.3962	7-Jun-08	28,750.00	4.2942
7-Jun-03	28,750.00	4.3307	7-Dec-08	28,750.00	4.2548
7-Dec-03	28,750.00	4.2654	7-Jun-09	28,750.00	4.2153
7-Jun-04	28,750.00	4.2696	7-Dec-09	1,028,750.00	4.1759
Nominal	1,000,000		Previous coupon date 7-Dec-98		
Duration	8.33		Accrued interest 10,425.82		
Total cash flow	1,632,500.00		Present value 1,141,925.84		

Table 11.12: Cash flow analysis Treasury 5¾% 2009, yield 4.2224%.
Source: Bloomberg.

The yield on the principal strip at this time was 4.1482%, which corresponds to a price of 64.13409 per £100 nominal. Given that the yield curve is inverted at this time, this is what is expected, a yield lower than the gross redemption yield for the coupon gilt. For a holding of £1 million nominal there is only one cash flow, the redemption payment of £1 million on the redemption date. The convexity for the principal strip was 1.175, which illustrates the higher convexity property of strips versus coupon bonds. Comparing the tables we can see also that duration for the strip is higher than that for the coupon gilt. Note the analysis for the principal strip gives us a slightly different spot curve.

Pay date	Cash flow	Spot	Pay date	Cash flow	Spot
7-Jun-99	0.00	5.1835	7-Dec-04	0.00	4.2683
7-Dec-99	0.00	4.9577	7-Jun-05	0.00	4.3102
7-Jun-00	0.00	4.8509	7-Dec-05	0.00	4.3529
7-Dec-00	0.00	4.7373	7-Jun-06	0.00	4.3672
7-Jun-01	0.00	4.6411	7-Dec-06	0.00	4.3821
7-Dec-01	0.00	4.5480	7-Jun-07	0.00	4.3571
7-Jun-02	0.00	4.4613	7-Dec-07	0.00	4.3322
7-Dec-02	0.00	4.3952	7-Jun-08	0.00	4.2937
7-Jun-03	0.00	4.3290	7-Dec-08	0.00	4.2543
7-Dec-03	0.00	4.2629	7-Jun-09	0.00	4.2148
7-Jun-04	0.00	4.2651	7-Dec-09	1,000,000.00	4.1753
Nominal		1,000,000	Previous coupon date 7-Dec-98		
Duration		10.82	Accrued interest 0.00		
Total cash flow		1,000,000.00	Present value 641,340.87		

Table 11.13: Cash flow analysis Treasury 5¾% 2009 Principal Strip, yield 4.1482%. Source: Bloomberg.

Finally we show at Table 11.14 the cash flow analysis for a coupon strip maturing on 7 December 2009. The yield quote for this coupon strip at this time was 4.4263%, corresponding to a price of 62.26518 per £100 nominal. This illustrates the point on strip prices we referred to earlier; according to a strict interpretation of the law of one price, all strips maturing on the same date should have the same price (because, why should an investor have a different yield requirement for the same nominal future cash flow depending on whether the £100 nominal she receives on maturity was sourced from a coupon payment or from the principal payment?). However the liquidity differences between principal and coupon strips makes the former easier to trade and also more sought after by investors, which explains the difference in yield between principal and coupon strips. The more liquid instrument trades at the lower yield.

Pay date	Cash flow	Spot	Pay date	Cash flow	Spot
7-Jun-99	0.00	5.2025	7-Dec-04	0.00	4.3217
7-Dec-99	0.00	5.0151	7-Jun-05	0.00	4.3672
7-Jun-00	0.00	4.8927	7-Dec-05	0.00	4.4136
7-Dec-00	0.00	4.7633	7-Jun-06	0.00	4.4353
7-Jun-01	0.00	4.6881	7-Dec-06	0.00	4.4576
7-Dec-01	0.00	4.6108	7-Jun-07	0.00	4.4299
7-Jun-02	0.00	4.5138	7-Dec-07	0.00	4.4023
7-Dec-02	0.00	4.4450	7-Jun-08	0.00	4.3608
7-Jun-03	0.00	4.3761	7-Dec-08	0.00	4.3183
7-Dec-03	0.00	4.3074	7-Jun-09	0.00	4.2758
7-Jun-04	0.00	4.3141	7-Dec-09	1,000,000.00	4.2333
Nominal		1,000,000	Previous coupon date 7-Dec-98		
Duration		10.82	Accrued interest 0.00		
Total cash flow		1,000,000.00	Present value 641,340.87		

Table 11.14: Cash flow analysis, December 2009 Coupon Strip, yield 4.4263%. Source: Bloomberg.

A graph of the spot yield curves, drawn from the rates for the coupon gilt, principal and coupon strips is shown at Figure 11.15.

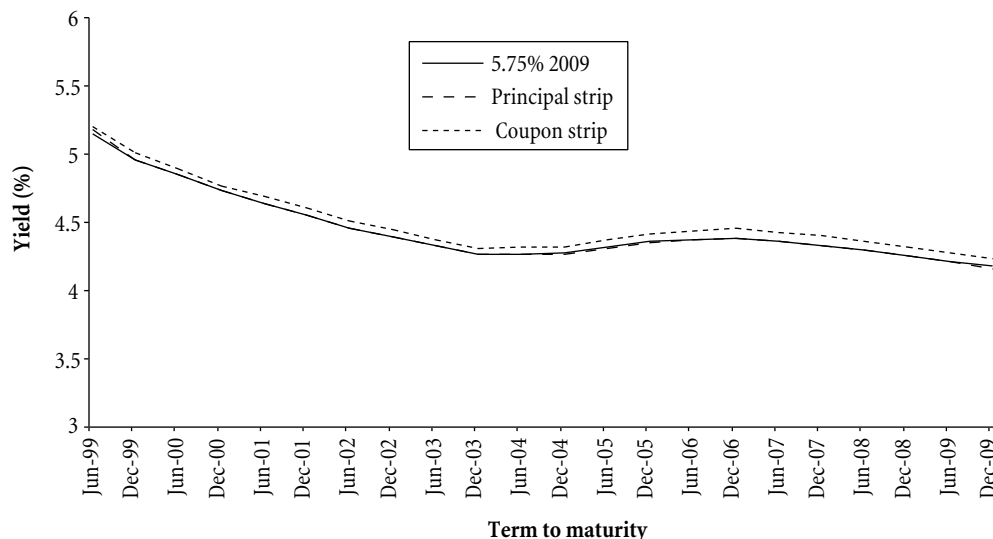


Figure 11.15: Spot yield curves for 5.75% 2009 gilt, principal and coupon strips, 10 February 1999.
Source: Bloomberg.

11.15 Future developments in strips

The gilt strips market is a relatively new market and there remains the possibility of new products being introduced at some point in the future. The BoE and DMO are in regular consultation with the market as part of their normal business activity in gilts, and should there be a large demand for new developments then new instruments may be introduced. Products that have already been mooted include deferred payment gilts, which would not make coupon payments for a set period after issue, and annuities, which would comprise a stream of coupon payments, with no principal repayment at maturity. The DMO has stated that investors can create these instruments themselves from within the existing strips facility. For example investors could use strips to create a synthetic deferred payment gilt or annuity by acquiring strips that provide cash flows for specific points in the future. Alternatively investors could purchase the entire term of cash flows from a stripped gilt and then sell the strips that were not required. For similar reasons the BoE originally (and now the DMO) decided against direct issuance of strips, alongside the strips facility. At present time it is uncertain to what extent demand exists for such direct issue, and what the pattern of demand would be. In any case the current arrangements are sufficient to meet demand, rendering it unnecessary for the central authorities to analyse and identify what this demand is.

Before the start of trading, the BoE consulted with the market on the need for a second pair of coupon dates in addition to the planned and subsequently introduced pair of 7 June and 7 December. An extra set of cash flow dates would increase investor choice. However it was felt at the time that it may reduce available volume and hence liquidity, and therefore was not introduced. This issue remains under consideration, as either a possibility for only the short end of the market, or along the yield curve, and may be introduced should liquidity build up and sufficient demand be deemed to exist.

In the BoE's original consultation paper on the strips market, the possibility of issuing index-linked strips was raised. Certain investment institutions (generally long-dated bond investors such as pension funds, who currently also invest in coupon index-linked gilts) have expressed interest in such instruments. Allowing the market to strip indexed bonds would enable them to create inflation-linked products that are more tailored to clients' needs, such as indexed annuities or deferred payment indexed bonds. In overseas markets where stripping of indexed government bonds does take place, the resulting strip is an individual uplifted cash flow. An interesting strips facility exists in the New Zealand market, where the cash flows are separated into three components: the principal, the principal inflation

adjustment and the set of inflation-linked coupons (that is, an indexed annuity). In the UK however it is felt that the small issue size of index-linked gilts and their coupons would result in a strips market of very low liquidity. The authorities have stated that the case for index-linked gilts will be reviewed in the light of experience gained with conventional strips; indeed all possibilities are under constant review and may be introduced if the market develops a significant demand for these types of instruments. Hence as the gilt strips market develops we can expect to see new developments and possibly new structures being introduced. If this contributes to maintaining the attraction of gilts to domestic and overseas investors it will be to the advantage of sterling markets as a whole.

11.16 HM Treasury and the remit of the Debt Management Office

In May 1997 the BoE was granted independent powers for the setting of interest rates. At the same time the government began the process of transferring the Bank's responsibility for debt and cash management to the newly-created Debt Management Office, which process was completed in April 1998. From that date the responsibility for gilt issuance was taken over by the DMO. The role of the DMO covers all official operational decision-making in the gilt market. From the last quarter of 1999 the DMO also assumed responsibility for cash management for HM Government.

The DMO is an executive agency of HM Treasury. Its main objectives are to meet the annual remit for the sale of gilts, with an emphasis on minimising the cost to HM Treasury; and to promote a liquid market for gilts and gilt trading. To facilitate this the DMO attempts to conduct its operations in as transparent a way as possible, again with a view to keeping costs to the Treasury as low as possible. The DMO has also set itself further objectives that include responding to the demand for new products and providing quality customer service. As part of its remit for 1998/99 the DMO published an auction calendar, and increased the proportion of index-linked gilts that are issued as part of the total funding requirement. Although the DMO is part of HM Treasury, it operates as a separate agency and at arm's length from the government. Policy though is ultimately set by the Chancellor.

In December 1998 the DMO published its framework for the future of UK government cash management, for which it will assume responsibility in 1999. The main objective for the DMO will be to cover Exchequer cash flows that are anticipated from its forecasts; this will be accomplished through a structured Treasury Bill programme and also through daily money market operations. The BoE will conduct its daily money market operations in the normal manner, which involves bill and repo tenders at 09.45 and 14.30 each day. The DMO has stated that it will work to avoid clashes with the Bank's operations.

11.16.1 Structure of debt management

A review of debt management policy²⁹ in 1995 resulted in significant reform of the gilt market and its structure. The most visible result of these reforms was the introduction of the gilt repo and strips markets. Other changes were made with a view to enhance the transparency and liquidity of the market, as well as to increase the attraction of the market for overseas investors. There was also a new debt management ethos that included the following:

- an emphasis on building up large benchmark issues of gilts along the five, ten, 20 and 30-year points on the yield curve. This has resulted in fewer gilt issues but more of a larger nominal size. For example the number of conventional issues outstanding has fallen from 96 in 1992 to 68 in March 1999 (of these, 20 were rump issues that trade infrequently). There are also more large size issues; in March 1999 there were 22 gilts with over £5 billion nominal outstanding in issue, whereas in 1992 there were only five such gilts. In 1999 one gilt issue, the 8% Treasury 2021, was the second largest bond issue in the world, with £16.5 billion nominal outstanding;³⁰
- gilt issuance is in accordance with an auction calendar that has been released at the start of the fiscal year;
- the government's borrowing forecast is announced at the start of the fiscal year, in a Debt Management Report (DMR) issued by HM Treasury. The DMR includes a forecast of the amount, type and maturity of gilts that are to be issued in the year. GEMMs and institutional investors are involved in a consultation process with the DMO on the formulation of these plans.

²⁹ *Report of the Debt Management Review*, July 1995, published jointly by HM Treasury and the Bank of England.

³⁰ The world's largest issue in 1999 was a US Treasury bond, coincidentally also an 8% 2021, with \$32.8 billion nominal outstanding.

The new revised debt management structure also states that the DMO will consult with market participants to ensure the continued liquidity and transparency of the market.

11.17 Gilt derivatives and repo markets

The gilt market forms the cornerstone of the sterling asset markets. Therefore the gilt repo market and the exchange-traded market in gilt derivatives are both important features of the debt capital markets as a whole. In terms of derivatives both the LIFFE and MATIF futures exchanges trade standard futures and option contracts on gilts. Futures contracts are reviewed in detail in Chapter 41 and this section presents an overview of the main exchange-traded gilt contract, the LIFFE Long Gilt futures contract.

A futures contract is a financial instrument is a legally binding obligation to make or take delivery of an underlying specified asset at a fixed date in the future, at a price agreed at the time the contract is entered into. The asset can be a tangible one such as wheat or oil or a non-tangible one such as an equity index. Futures contracts written on non-tangible assets such as financial instruments are known as *financial futures*. Both commodity and financial futures are used for hedging and speculative purposes.

11.17.1 LIFFE long gilt contract

LIFFE's long gilt futures contract was the first government bond futures contract listed in Europe, in 1982. A detailed contract specification is given in Chapter 41. In 1998 the contract's specifications were changed to reflect changes in the gilt cash market structure, and one *lot* of the contract now represents £100,000 nominal of a gilt of a *notional* 7% coupon and maturing in from 8¾ to 13 years. The price quotes is, like cash gilts, in decimal units (£0.01) and the bid-offer spread is usually £0.01. During 1998 the average daily trading volume in the long gilt was 64,000 lots³¹ and the number of *open interest* contracts ran at an average level of 164,000 during the year. Open interest represents the number of contracts that are run overnight and not closed out before the end of the trading day; the level of open interest is one reflection of the size of the hedging demand for gilts and other sterling bonds.

In the early 1990s a medium-gilt future was also traded on LIFFE, but was later discontinued due to low trading volumes. In January 1998 the contract was re-launched, with the same terms as the long gilt future but with the underlying gilt specified as one of between 4–7 years' maturity. The delivery cycles of both contracts follow market convention for exchange-traded contracts, with expiry dates in March, June, September and December each year.

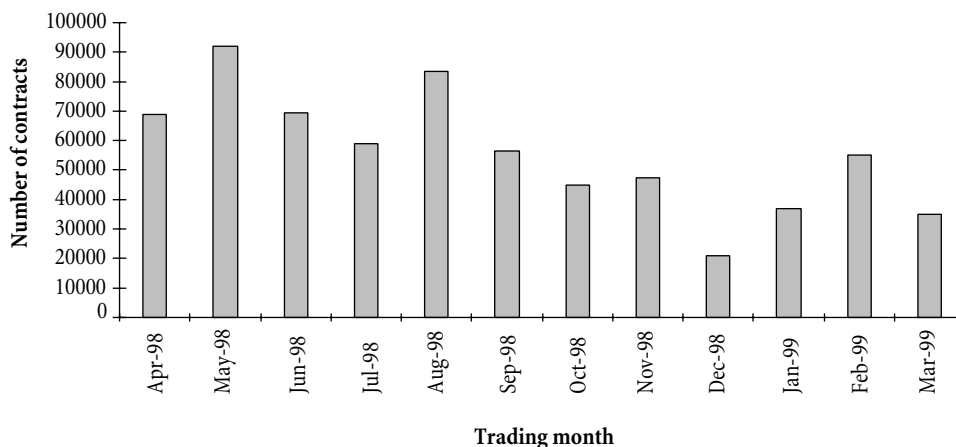


Figure 11.16: LIFFE long gilt future average daily trading volume. Source: LIFFE.

There are a range of users of gilt futures contracts. These include:

- GEMMs and market makers of sterling denominated bonds;
- market-makers in sterling interest rate swaps;

³¹ Source: LIFFE.

- institutional investors in the gilt market including fund managers, pension funds and life companies;
- issuers of sterling bonds including Eurobonds and Bulldogs;
- speculators and arbitrageurs such as securities houses and hedge funds.

The market participants noted above use gilt futures for a range of purposes. A GEMM will be concerned with hedging its cash gilt book, institutional investors will also be concerned with hedging as well as investing future cash flows, portfolio duration adjustment, portfolio insurance and income enhancement.³² Issuers of sterling bonds use futures to hedge their underwriting positions. Finally traders such as speculators and arbitrageurs will trade futures as part of a directional play on the market, for yield curve trades and as part of volatility trading.

The delivery process for a futures contract is connected to its pricing, as this provides the link to (and convergence with) the cash market. However only a small percentage of futures contracts traded are actually taken to delivery. It is the party that is the seller of the contract (and who runs the position into the delivery month) that may choose to deliver a gilt from a list of deliverable gilts that meet the contract's specifications. Therefore the buyer of a contract (again, who runs the position into the delivery month) will anticipate receiving the particular gilt that will create the maximum profit or minimum loss for the delivering seller when assessing the fair price of the contract. This bond is known as the *cheapest-to-deliver* (CTD) gilt. To identify the CTD gilt, and therefore the fair price of the futures contract, it is necessary to look at the potential profit or loss from what is known as a *cash-and-carry* arbitrage. This is a position consisting of a long position in the underlying bond and an equivalent short position in the futures contract. Both positions must be put on simultaneously. The cash-and-carry arbitrage profit/loss calculation is given by:

Cash inflow less cash outflow = short futures – long underlying:

$$\begin{aligned} & (\text{futures price} \times \text{price factor} + \text{accrued interest at delivery} + \text{coupon income}) \\ & \text{less} \\ & (\text{long gilt clean price} + \text{accrued interest at purchase} + \text{financing cost}). \end{aligned}$$

A discussion on the *price factor* (also known as the *conversion factor*) will be delayed until the chapter on bond futures. To consider our arbitrage trade then, since an arbitrage gain can be made if the price of the futures contract and the corresponding price of the CTD gilt are out of line, the market forces the price of the futures contract closely tracks the price of the underlying gilt that is the CTD gilt at the time. The close relationship between the long gilt futures price and the price of the CTD gilt means that the future contract provides a flexible hedging mechanism for both gilts and other long-dated sterling bonds. The same consideration applies for the medium gilt futures contract and medium-dated gilts and sterling bonds.

11.17.2 LIFFE gilt options

The popularity and liquidity of the gilt futures contract led to the listing of gilt options on LIFFE in March 1986. Average daily volume in this contract was more than 12,000 lots during 1998. The contract is an option on the futures contract and not a cash gilt. The options are “American” style options, which means they can be exercised by the long at any time between trade date and expiry date. They are agreements under which the buyer acquires the right (but not the obligation) to take (*call*) or make (*put*) delivery of the underlying futures contract, at the price agreed at the time of dealing. On LIFFE “serial expiry months” are available for its exchange-traded options; serial options are expiry months other than the traditional quarterly months of March, June, September and December. Gilt option expiry months are listed such that the two nearest serial months and the two nearest quarterly months are always available for trading.

Traditionally gilt futures and options have been traded by open-outcry in designated pits on the floor of the LIFFE exchange building. In April 1999 the exchange launched an electronic dealing platform known as Connect for Futures. The long gilt contract was the first to be traded on it. The system is a screen based dealing platform that also enables traders to observe the depth of the market, as orders above and below the current bid and offer level are listed on the screen. Although floor trading in individual pits was retained, LIFFE announced in October 1999 that

³² These last two functions actually employ options on gilt futures.

all futures and options trading was to move to an electronic screen-based platform by the end of the year, and trading had moved off the floor by the end of November 1999.

11.17.3 Gilt repo market

The term repo comes from the expression “sale and repurchase” agreement. Repo is a short-term money market instrument and is typically a loan secured with bonds as collateral. Although repo is a well-established instrument and has existed in the US Treasury market from 1918, it was introduced only recently in the gilt market, in January 1996. Prior to this there was no facility to borrow gilts via a repo transaction. Gilt-edged market makers were able to borrow stock from designated Stock Exchange Money Brokers (SEMBs) in order to deliver into a short sale. The SEMBs obtained the stock from institutional investors with large holdings of gilts, and stock was lent to GEMMs in return for collateral, usually posted as other gilts or cash. The introduction of an open market in gilt repo enabled any market participant to borrow gilts and also to fund gilt positions at a lower rate (the *repo rate*) than the interbank lending rate. This contributed to improved cash market liquidity. The market quickly grew to over £50 billion of repo outstanding, developing alongside the existing unsecured sterling money market. Market had size stood at over £105 billion by the end of March 1999.³³

The BoE is involved in the repo market as part of its daily operations in the sterling money markets. From the first quarter of 2000 the DMO also used gilt repo as part of its cash management operations on behalf of the government, which are designed to smooth the net daily cash flows between central government and the private sector. Gilt repo is reviewed in detail in Chapter 34.

11.18 The Minimum Funding Requirement

In May 1997, shortly after a general election that brought the Labour party to power for the first time since 1979, the responsibility for the conduct of monetary policy was transferred to the Bank of England. Shortly after that the gilt yield curve inverted, an occurrence explained by bond analysts as reflecting the new 2.5% long-term, inflation target set for the BoE and the prospect of eventual sterling entry into the European Union’s euro currency. The curve remained inverted, changing slightly in July 1999 to being positive sloping out to seven years before inverting for the remaining term to maturity.³⁴ The continued inverted nature of the curve, especially the historically very low yields at the very long end of the curve, reflect high demand for long-dated gilts and a relative lack of supply at this end of the curve. The high demand has been explained partly as a result of the Minimum Funding Requirement (MFR) and its effect on pension funds’ investment. In this section we review the MFR and the impact of the inverted yield curve on future funding.

11.18.1 The Minimum Funding Requirement

The MFR was the term given to government reform of the regulation of occupational pension schemes.³⁵ The reforms introduced an MFR as part of pensions reform, to be introduced in a phased scheme from April 1997 through to 2002. The MFR is not a requirement for pension funds to hold gilts but rather a notice on the discount rate that a pension fund should use when determining the value of its liabilities with respect to different members of the fund scheme. The liabilities for pensioners must be calculated using a discount rate relating to the current yield on long-dated gilts, either conventional or I-L gilts as appropriate to the fund. Long-dated gilts are taken to be those of greater than 15 years to maturity. Subsequent analysis by some market commentators has stated that this has increased the importance of long-dated gilts to pension funds, and implied that holdings of these instruments have increased as a direct result.

11.18.2 Impact on the gilt market

Although the MFR will influence a pension funds view on long-dated gilt yields, the requirement itself is not an instruction to hold the instruments themselves. There are several factors that have contrived to depress long-dated gilt yields however, which we can consider here.

In the first instance the reduction in government borrowing has resulted in a supply shortage of gilts generally, not just at the long end of the curve. From a net gilt sales requirement of over £35 billion in 1993/1994, the figure fell

³³ Source: DMO.

³⁴ This positively-sloping curve was short-lived; the shape reverted to negatively-sloping from the 2-year maturity shortly after.

³⁵ This formed part of the Pensions Act (1995).

to a surplus of nearly £10 billion in 1998/1999 and a projected net requirement of less than £5 billion for the year 1999/2000. The decreasing supply has contributed to low long-dated gilt yields. For pension funds a further issue from the year 2000 was that £14 billion of long gilts were re-classified as “mediums” because they mature in 2015.

The historically low yields on gilts also reflect the economic performance of the UK economy and the benign future inflation expectations in place. This has more of an impact than any perceived shortage of stock at the long end. In fact the UK gilt stock has a higher average maturity than most developed country bond markets, and this exposes the government to relatively higher interest rate risk. The demand for gilts stretches across the maturity spectrum and only one class of investor (the pension funds) has a comparatively high interest at the very long-end. Other institutional investors concentrate on the short (banks, building societies) and medium (life companies) end of the yield curve. The authorities do not believe the MFR itself to be solely behind the shape of the yield curve from 1997 onwards, which can be explained largely by the historically low level of supply, mature pension fund requirement for long-dated bonds and historically low inflation expectations. Nevertheless, they instituted a review of the MFR, to be known as the Myners Report, which was due to be delivered in November 2000. It was expected to recommend greater holding by institutional investors of highly-rated corporate bonds, which would relieve the demand at the long end of the gilt yield curve.

11.19 Developments in electronic trading

The Debt Management Office (DMO) issued a consultation paper in January 2000 on its relationship with the gilt-edged market makers (GEMMs), which sought views on the likely impact on this relationship as a result of changes in the trading environment, once electronic trading platforms were introduced into the gilt market. The consultation period ran during the first quarter of 2000. Currently GEMMs carry out their market making obligations via the telephone. The introduction of electronic trading platforms in bond markets around the world is an indicator that the gilt market will also switch to an electronic platform for secondary market trading in due course. A number of new electronic brokerages are already, or are planning to begin, operating in the euroland government bond markets, including the systems introduced during 1999 by the London Clearing House. Although sterling bonds were not traded on these systems in 1999, it was planned to introduce them soon as possible. When this occurs there will be electronic matching and execution of trades. It is realistic to expect their introduction in gilts over the short term, and in response to this the DMO invited comment on whether its existing relationship with the GEMMs, and via them, its contact with the secondary market in gilts, remained suitable. In effect the DMO may have been concerned that secondary market obligations may not have been carried out as effectively over an electronic system, however the likely impact will probably be similar to the equity market, where liquid stocks are highly transparent to investors but illiquid ones are still difficult to trade in. It remains to be seen if an electronic trading platform would make dealing at the long end of the curve, which became slightly less liquid to deal in as demand outstripped supply during 1998 and 1999, any more transparent.

Appendices

APPENDIX 11.1 List of gilt stocks outstanding, 31 December 1999

Conventional gilts	Redemption date	Amount in issue (£m)	Amount held in stripped form (£m)
9% Conversion 2000	3 March 2000	5,358	–
13% Treasury 2000	14 July 2000	3,171	–
8% Treasury 2000	7 December 2000	9,800	119
10% Treasury 2001	26 February 2001	4,406	–
11% 1/2 Treasury 2001/2004	19 March 2001	1,620	–
Floating Rate 2001	10 July 2001	3,000	–
7% Treasury 2001	6 November 2001	12,750	–
7% Treasury 2002	7 June 2002	9,000	280
9 3/4% Treasury 2002	27 August 2002	6,527	–
8% Treasury 2002/2006	5 October 2002	2,050	–
8% Treasury 2003	10 June 2003	7,600	–
10% Treasury 2003	8 September 2003	2,506	–
6 1/2% Treasury 2003	7 December 2003	7,987	145
5% Treasury 2004	7 June 2004	7,408	2
3 1/2% Funding 1999/2004	14 July 2004	543	–
6 3/4% Treasury 2004	26 November 2004	6,500	–
9 1/2% Conversion 2005	18 April 2005	4,842	–
8 1/2% Treasury 2005	7 December 2005	10,373	584
7 3/4% Treasury 2006	8 September 2006	4,000	–
7 1/2% Treasury 2006	7 December 2006	11,700	259
8 1/2% Treasury 2007	16 July 2007	7,397	–
7 1/4% Treasury 2007	7 December 2007	11,000	303
9% Treasury 2008	13 October 2008	5,621	–
5 3/4% Treasury 2009	7 December 2009	8,827	106
6 1/4% Treasury 2010	25 November 2010	4,750	–
9% Conversion 2011	12 July 2011	5,273	–
7 3/4% Treasury 2012/2015	26 January 2012	800	–
9% Treasury 2012	6 August 2012	5,361	–
5 1/2% Treasury 2008/2012	10 September 2012	1,000	–
8% Treasury 2013	27 September 2013	6,100	–
8% Treasury 2015	7 December 2015	13,787	167
8 3/4% Treasury 2017	25 August 2017	7,550	–
8% Treasury 2021	7 June 2021	16,500	517
6% Treasury 2028	7 December 2028	9,900	333
2 1/2% Treasury	Undated	474	–
3 1/2% War Loan	Undated	1,909	–

Index-linked gilts	Redemption date	Amount in issue (£m)	Nominal value including inflation uplift (£m)
2 1/2% I-L Treasury 2001	24 September 2001	2,150	4,538
2 1/2% I-L Treasury 2003	20 May 2003	2,700	5,663
4 3/8% I-L Treasury 2004	21 October 2004	1,300	1,584
2% I-L Treasury 2006	19 July 2006	2,500	5,944
2 1/2% I-L Treasury 2009	20 May 2009	2,625	5,506
2 1/2% I-L Treasury 2011	23 August 2011	3,475	7,700
2 1/2% I-L Treasury 2013	16 August 2013	4,200	7,778
2 1/2% I-L Treasury 2016	26 July 2016	4,495	9,098
2 1/2% I-L Treasury 2020	16 April 2020	3,800	7,566
2 1/2% I-L Treasury 2024	17 July 2024	4,450	7,527
4 1/8% I-L Treasury 2030	22 July 2030	2,150	2,629

“Rump” gilts	Redemption date	Amount in issue (£m)
8 1/2% Treasury 2000	28 January 2000	109
13 3/4% Treasury 2000/2003	25 July 2000	53
9 1/2% Conversion 2001	12 July 2001	3
9 3/4% Conversion 2001	10 August 2001	35
10% Conversion 2002	11 April 2002	21
9 1/2% Conversion 2002	14 June 2002	2
9% Exchequer 2002	19 November 2002	83
11 3/4% Treasury 2003/2007	22 January 2003	234
9 3/4% Conversion 2003	7 May 2003	11
12 1/2% Treasury 2003/2005	21 November 2003	152
13 1/2% Treasury 2004/2008	26 March 2004	95
10% Treasury 2004	18 May 2004	20
9 1/2% Conversion 2004	25 October 2004	307
10 1/2% Exchequer 2005	20 September 2005	23
9 3/4% Conversion 2006	15 November 2006	6
8% Treasury 2009	25 September 2009	560
12% Exchequer 2013/2017	12 December 2013	57
2 1/2% Annuities	Undated	3
3% Treasury	Undated	55
3 1/2% Conversion	Undated	108
2 1/2% Consolidated	Undated	275
2 3/4% Annuities	Undated	1
4% Consolidated	Undated	358

Table 11.15: List of gilt stocks outstanding, 31 December 1999. Source: DMO.

APPENDIX 11.2 The UK Retail Prices Index

	1982	1983	1984	1985	1986	1987	1988	1989	1990	1991	1992	1993	1994	1995	1996	1997	1998	1999
January	78.73	82.61	86.84	91.2	96.25	100	103.3	111.0	119.5	130.2	135.6	137.9	141.3	146.0	150.2	154.4	159.5	163.4
February	78.76	82.97	87.2	91.94	96.6	100.4	103.7	111.8	120.2	130.9	136.3	138.8	142.1	146.9	150.9	155.0	160.3	163.7
March	79.44	83.12	87.48	92.8	96.73	100.6	104.1	112.3	121.4	131.4	136.7	139.3	142.5	147.5	151.5	155.4	160.8	164.1
April	81.04	84.28	88.64	94.78	97.67	101.8	105.8	114.3	125.1	133.1	138.8	140.6	144.2	149.0	152.6	156.3	162.6	165.2
May	81.62	84.64	88.97	95.21	97.85	101.9	106.2	115.0	126.2	133.5	139.3	141.1	144.7	149.6	152.9	156.9	163.5	165.6
June	81.85	84.84	89.2	95.41	97.79	101.9	106.6	115.4	126.7	134.1	139.3	141.0	144.7	149.8	153.0	157.5	163.4	165.6
July	81.88	85.3	89.1	95.23	97.52	101.8	106.7	115.5	126.8	133.8	138.8	140.7	144.0	149.1	152.4	157.5	163.0	165.1
August	81.9	85.68	89.94	95.49	97.82	102.1	107.9	115.8	128.1	134.1	138.9	141.3	144.7	149.9	153.1	158.5	163.7	165.5
September	81.85	86.06	90.11	95.44	98.3	102.4	108.4	116.6	129.3	134.6	139.4	141.9	145.0	150.6	153.8	159.3	164.4	166.2
October	82.86	86.36	90.67	95.59	98.45	102.9	109.5	117.5	130.3	135.1	139.9	141.8	145.2	149.8	153.8	159.5	163.5	
November	82.66	86.67	90.95	95.92	99.29	103.4	110.0	118.5	130.0	135.6	139.7	141.6	145.3	149.8	153.9	159.6	164.4	
December	82.51	86.89	90.87	96.05	99.62	103.3	110.3	118.8	129.9	135.7	139.2	141.9	146.0	150.7	154.4	160.0	164.4	

Table 11.16: United Kingdom Retail Price Index. Source: ONS.

APPENDIX 11.3 Summary statistics on the gilt portfolio (September 1999)

Nominal value	£291.3 billion
Market value	£326.4 billion
Weighted average market yield	5.66%
Average maturity	9.9 years
Average modified duration	7.3
Average convexity	0.992
Average nominal value outstanding (Largest 20 issues)	£9.3 billion

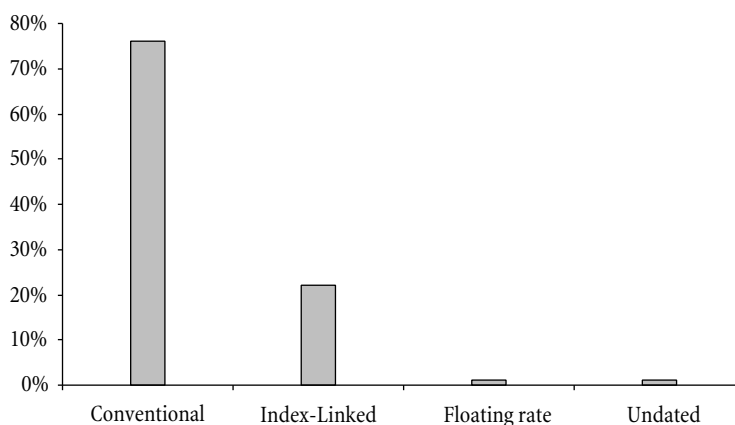


Figure 11.17: Composition of gilt stock in September 1999. Source: DMO 1999.

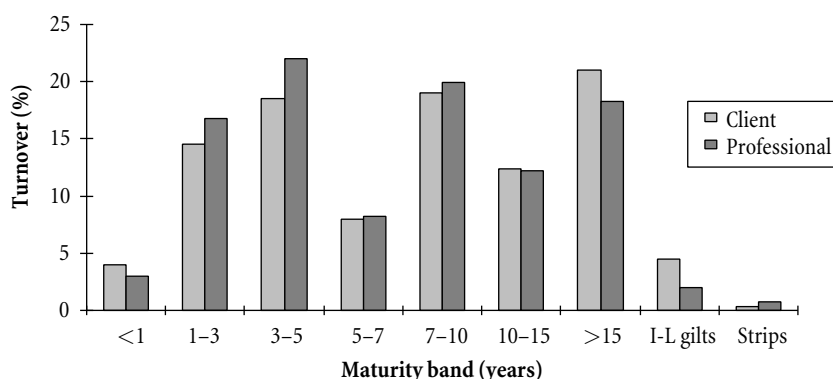


Figure 11.18: Percentage of GEMM market turnover by maturity band (September 1999). Source: BoE.

APPENDIX 11.4 Gilt Edged Market Makers as at September 1999

ABN Amro Bank NV	Lehman Brothers International (Europe) *
Barclays Capital *	Merrill Lynch International *
Credit Suisse First Boston Gilts Limited	Morgan Stanley & Co International Limited
Deutsche Morgan Grenfell	Greenwich NatWest *
Dresdner Kleinwort Benson *	Salomon Smith Barney
Goldman Sachs International Limited	Societe Generale
HSBC Greenwell *	Warburg Dillon Read
JP Morgan Securities Limited	Winterflood Securities

* indicates IG GEMM status

Table 11.17: GEMMs as at September 1999. Source: DMO.

APPENDIX 11.5 DMO reference prices

All GEMMs supply closing bid and offer clean prices supply for conventional gilts, except for those gilts with less than £250 million nominal outstanding. IG GEMMs also supply prices for I-L gilts. In the case of gilt strips the market makers supply mid-market strip yields, which are converted into prices using the standard DMO formula. The prices of rump stocks are set by the DMO using the prices of liquid gilts as a reference. The basic procedures followed by the DMO is the calculate the median price for each gilt from the range of prices; outlying prices are excluded before an arithmetic average price is calculated from the remaining prices. For conventional gilts, which includes double-dated and undated gilts, the margin for outlying prices is £0.15, for floating-rate gilts it is £0.03, for I-L gilts it is £0.20 and for strips it is £0.30. The averaged prices are rounded to two decimal places, and are then adjusted to dirty prices using the interest accrued for that gilt to the next settlement day. Final dirty prices and strip prices are calculated to six decimal places.

The reference gross redemption yields are then calculated from the reference prices. For I-L gilts the yield calculation assumes an inflation rate of 3%. Yields are rounded to three decimal places.

The rounded average clean price, the rounded dirty price and the gross redemption yield for each gilt are displayed on the DMO's news screens.

EXAMPLE 11.3 Calculation of DMO reference price

- The stock is the 7% Treasury 2002, closing prices on 20 October 1999, with settlement on 21 October 1999. The clean prices contributed from nine GEMMs are set out below.

101.39, 101.42, 101.47, 101.48, 101.55, 101.58, 101.61, 101.65, 101.71.

The median price is therefore 101.55. The "outlier" limit for the stock is £0.15, therefore we reject any prices greater than this amount away from the median. This leaves us with the remaining prices:

101.42, 101.47, 101.48, 101.55, 101.58, 101.61, 101.65.

The arithmetic average of these prices is 101.537142857. This is rounded to two decimal places to give us 101.54. The accrued interest to the settlement date for this stock is 2.608219. The rounded dirty price is therefore 101.54 + 2.608219 or £104.148219. The gross redemption yield at this price, rounded to three decimal places is 5.285%.

APPENDIX 11.6 Formula for the calculation of accrued interest using the actual/actual day-count basis

From 1 November 1998 the day-count basis for calculating accrued interest on gilts was changed from the actual/365 market convention to the actual/actual basis. For conventional gilts the accrued interest calculation uses the following formula and is rounded to six decimal places.

Standard dividend periods

Where the settlement date falls on or before the ex-dividend date:

$$AI = \frac{t}{n} \times \frac{C}{2}.$$

Where the settlement date falls after the ex-dividend date:

$$AI = \left(\frac{t}{n} - 1 \right) \times \frac{C}{2}$$

where

- C is the coupon rate
- t is the number of days from the last dividend date to the settlement date
- n is the number of days in the full coupon period in which the settlement date occurs.

Short first dividend periods

Where the settlement date falls on or before the ex-dividend date:

$$AI = \frac{t^*}{n} \times \frac{C}{2}$$

Where the settlement date falls after the ex-dividend date:

$$AI = \left(\frac{t^* - s}{n} \right) \times \frac{C}{2}$$

where

- t^* is the number of days from the issue date to the settlement date
- s is the number of days from the issue date to the next short coupon date.

Long first dividend periods

In the case of a new-issue stock with a long first coupon period the accrued interest calculation uses the following formulae. The procedure involves splitting the period between the issue date and the dividend payment date into the bond's coupon periods. This leads to a change in the rate of accrual on the theoretical coupon payment between the issue date and the long first dividend period

Where the settlement date falls during the first coupon period:

$$AI = \frac{t}{n_1} \times \frac{C}{2}.$$

Where the settlement date falls during the second coupon period on or before the ex-dividend date:

$$AI = \left(\frac{s_1}{n_1} + \frac{s_2}{n_2} \right) \times \frac{C}{2}.$$

Where the settlement date falls during the second coupon period after the ex-dividend date:

$$AI = \left(\frac{s_2}{n_2} - 1 \right) \times \frac{C}{2}$$

where

- t is the number of days from the issue date to the settlement date in the first coupon period (this applies only if the gilt settles in the first coupon period).
- n_1 is the number of days in the full coupon period in which the issue date falls
- n_2 is the number of days in the full coupon period after the coupon period in which the issue date falls
- s_1 is the number of days from the issue date to the next (theoretical) coupon date
- s_2 is the number of days from the (theoretical) coupon date after the issue date to the settlement date in the coupon period after the coupon period in which the issue date occurs (this applies only if the gilt settles in the second coupon period).

APPENDIX 11.7 List of gilt strip prices and yields, March 1999

Strip	Price	Yield	Modified duration	Convexity	Strip	Price	Yield	Modified duration	Convexity
Jun-99 C	98.692	5.068	0.26	0.002	Jun-13 C	51.427	4.714	13.950	2.013
Dec-99 C	96.355	4.918	0.75	0.009	Dec-13 C	50.278	4.706	14.440	2.156
Jun-00 C	93.930	5.009	1.23	0.021	Jun-14C	49.128	4.706	14.930	2.301
Dec-00 P	91.586	5.036	1.72	0.038	Dec-14 C	48.012	4.705	15.420	2.452
Dec-00 C	91.685	4.973	1.72	0.038	Jun-15C	46.886	4.708	15.900	2.607
Jun-01 C	89.462	4.976	2.21	0.060	Dec-15 P	46.100	4.670	16.390	2.767
Dec-01 C	87.348	4.949	2.7	0.086	Dec-15 C	45.836	4.705	16.390	2.766
Jun-02 P	85.467	4.867	3.19	0.117	Jun-16 C	44.865	4.693	16.880	2.932
Jun-02 C	85.350	4.910	3.19	0.117	Dec-16 C	43.889	4.686	17.370	3.102
Dec-02C	83.426	4.861	3.68	0.154	Jun-17 C	42.922	4.682	17.860	3.277
Jun-03 C	81.581	4.824	4.17	0.194	Dec-17 C	41.986	4.676	18.350	3.457
Dec-03 P	80.158	4.691	4.66	0.240	Jun-18 C	41.049	4.673	18.840	3.640
Dec-03 C	79.805	4.786	4.66	0.240	Dec-18C	40.165	4.665	19.330	3.830
Jun-04 C	77.897	4.798	5.15	0.290	Jun-19 C	39.311	4.658	19.820	4.023
Dec-04 C	76.067	4.798	5.63	0.345	Dec-19 C	38.486	4.648	20.310	4.225
Jun-05 C	74.355	4.783	6.12	0.405	Jun-20 C	37.737	4.632	20.800	4.428
Dec-05 P	72.819	4.740	6.61	0.470	Dec-20 C	36.835	4.638	21.290	4.635
Dec-05 C	72.661	4.774	6.61	0.469	Jun-21 P	36.204	4.612	21.780	4.849
Jun-06 C	70.960	4.776	7.1	0.539	Jun-21 C	36.023	4.635	21.770	4.848
Dec-06 C	69.278	4.781	7.59	0.613	Dec-21 C	35.426	4.608	22.270	5.067
Dec-06 P	69.304	4.775	7.59	0.613	Jun-22 C	34.699	4.599	22.760	5.290
Jun-07 C	67.747	4.765	8.08	0.692	Dec-22 C	34.005	4.587	23.250	5.518
Dec-07 C	66.237	4.753	8.57	0.776	Jun-23C	33.321	4.578	23.740	5.750
Dec-07 P	66.408	4.723	8.57	0.776	Dec-23 C	32.678	4.565	24.230	5.988
Jun-08 C	64.798	4.733	9.06	0.866	Jun-24 C	32.043	4.553	24.720	6.231
Dec-08 C	63.287	4.735	9.55	0.959	Dec-24 C	31.432	4.538	25.220	6.482
Jun-09 C	61.852	4.731	10.04	1.056	Jun-25 C	30.892	4.519	25.710	6.734
Dec-09 P	61.804	4.518	10.53	1.161	Dec-25 C	30.343	4.502	26.200	6.991

Strip	Price	Yield	Modified duration	Convexity	Strip	Price	Yield	Modified duration	Convexity
Dec-09 C	60.479	4.723	10.52	1.159	Jun-26 C	29.818	4.484	26.690	7.252
Jun-10 C	59.167	4.711	11.01	1.266	Dec-26 C	29.314	4.466	27.180	7.519
Dec-10 C	57.798	4.711	11.5	1.379	Jun-27 C	28.816	4.448	27.670	7.790
Jun-11 C	56.460	4.713	11.99	1.496	Dec-27 C	28.316	4.432	28.160	8.067
Dec-11 C	55.152	4.713	12.48	1.618	Jun-28 C	27.853	4.413	28.650	8.350
Jun-12 C	53.881	4.713	12.97	1.745	Dec-28 C	27.339	4.402	29.140	8.637
Dec-12 C	52.640	4.713	13.46	1.877	Dec-28 P	27.635	4.365	29.150	8.640
P–Principal strip					C–Coupon strip				

Table 11.18: Strip prices and yields, March 1999. Source: HSBC, Barclays Capital, Bloomberg.

APPENDIX 11.8 Growth of the UK national debt

Year	National Debt (£ million)	Year	National Debt (£ million)
1694	1.2	1975	45,260
1697	15	1980	95,314
1727	53	1985	158,029
1783	232	1988	197,430
1815	900	1989	197,320
1855	805	1992	214,528
1914	651	1993	248,839
1918	5,872	1994	306,871
1940	7,900	1995	349,163
1945	21,365	1996	390,683
1955	26,934	1997	419,546
1965	30,441	1998	418,431

Table 11.19: UK national debt. Source: DMO 1999.

APPENDIX 11.9 Related gilt market web sites and screens

UK Debt Management Office	http://www.dmo.gov.uk
HM Treasury	http://www.hm-treasury.gov.uk
Bank of England	http://bankofengland.co.uk
LIFFE	http://www.liffe.com
London Stock Exchange	http://www.londonstockex.co.uk

Subject	Reuters	Telerate	Bloomberg	Topic3
Index	DMO/INDEX		DMO <GO>	
Announcements	DMO/GILTS1 to 7	22550 TP 2256		44715
Shop windows	DMO/GILTS 8 to 11			44715 – 44718
Reference prices	GEMMA01 to 06	47216 to 47221		
Reference prices (I-L)	GEMMA08	47223		
Reference prices (Strips)	GEMMA13 to 19	21291 to 21297		

Table 11.20: Debt Management Office news screens.

Selected bibliography

- Arak, M., Kreicher, L., "The real rate of interest: inferences from the new UK indexed gilts", *International Economic Review*, Vol. 26, No. 2, June 1985, pp. 399–408.
- Bank of England, *The Official Gilt Strips Facility*, October 1997.
- Bank of England, *Quarterly Bulletin*, February 1998, February 1999.
- Battley, N., Wedd, P., (ed.), *The European Bond Markets, An Overview and Analysis for Issuers and Investors*, McGraw-Hill, 1997.
- Belchamber, C., *The UK Government Bond Market*, CSFB Gilts, 1988.
- Bootle, R., *Index-Linked Gilts – A Practical Investment Guide*, 2nd edition, Woodhead-Faulkner, 1991.
- Choudhry, M., *The Gilt Strips Market*, SI (Services) Publishing, 1999.
- Choudhry, M., "The impact of demand and liquidity on the information content of the government bond yield curve: an example from the United Kingdom gilt market", unpublished working paper, Birkbeck College, University of London, 1999.
- Debt Management Office, *The future of UK government cash management*, December 1998.
- Debt Management Office, *Gilt Review*, 1997–1998.
- DMO, Debt Management Report, March 1999.
- DMO, *The Gilt Review*, 1998, 1999.
- DMO, *Quarterly Review*, Q2 1999.
- Deacon, M., Derry, A., *Inflation Indexed Securities*, FT Management, 1998.
- Deacon, M., Derry, A., "Deriving Estimates of Inflation Expectations from the Prices of UK Government Bonds", *Bank of England Working Paper* No. 23, July 1994.
- The Economist, *Admiring those shapely curves*, 4 April 1998, p.117.
- Gray, S., *The Management of Government Debt*, Bank of England, May 1996.
- Higson, C., *Business Finance*, Butterworths, 1995.
- McConnachie, R., *Primary Dealers in Government Securities Markets*, Bank of England, May 1996.
- Pearce, D., "Short-term inflation expectations: evidence from a monthly survey", *Journal of Money Credit and Banking*, Vol. 19, No. 3, August 1987, pp. 388–395.
- Phillips, P., *The Merrill Lynch Guide to the Gilt-Edged and Sterling Bond Market*, The Book Guild, 1996.
- The Times, *Switch auction is first step in gilts reform*, 22 October 1999.
- Woodward, G., "The real thing: a dynamic profile of the term structure of real interest rates and inflation expectations in the United Kingdom, 1982–1989", *Journal of Business*, Vol. 63, No. 3, 1990, pp. 378–398.

Questions and exercises

- Define the following terms:
 - a benchmark issue
 - the grey market
 - gilt-edged market maker
- Calculate the accrued interest for the following gilts for settlement on 21 October 1999 (refer to Appendix 11.1 for the maturity date and hence coupon dates):
 - 8% Treasury 2000, £1 million nominal
 - 7% Treasury 2002, £10 million nominal
 - 5% Treasury 2004, £949,000 nominal
 - 6% Treasury 2028, £215 million nominal
- The following is an extract from the DMO's Gilts quarterly review for September 1999:

Auction Date	Issue amount	Stock	Cover	Tail (bp)	Lowest accepted price	Yield at lowest accepted price (%)
28-Apr-99	£0.5 bn	4.125% IL 2030	0.94	–	179.34	1.97
26-May-99	£2.5 bn	6% 2028	2.24	2	120.2	4.72
22-Jun-99	£2.5 bn	5% 2004	2.01	2	98.7	5.30
28-Jul-99	£0.375 bn	2.5% IL 2011	1.93	–	225.5	2.19
28-Sep-99	£2.75 bn	5.75% 2009	2.54	1	100.3	5.71

Figure 11.19: Auction results, DMO Quarterly Review Q3, September 1999.
Used with permission.

What is meant by the auction “cover” and “tail”? Why is there no tail for index-linked stock auctions? There is no report of the average yield at each auction; if that was reported, what would it be referring to? If a particular auction had been reported with a tail higher than 2, what would that imply?

4. What reasons would you put forward as explanation for the low trading volumes in gilt strips in the two years since their introduction?
5. The BoE designates certain gilt issues as strippable. Suggest a practical reason why all gilts could not be made strippable. Liquidity reasons aside, why would double-dated gilts not be suitable strippable securities?
6. What is the commonly accepted proxy for a “risk-free” interest rate in the sterling markets?
7. Sakyia is a private investor in gilts. She decides on a purchase of £5000 nominal 7% Treasury 2002 gilt, which is offered at £101.65. How much total consideration will she pay (ignoring stockbroker’s commission) if she deals for settlement:
 - (a) 10 days after the ex-dividend date
 - (b) seven business days after the ex-dividend date
 - (c) 100 days after the ex-dividend date?
8. The yield on a certain gilt is 5.771%, while the yield on a sterling Eurobond of the same maturity is 5.924%. Which bond is offering the better return?
9. What is the “break-even” inflation rate? How is it used in the gilt market?
10. Consider the Retail Prices Index table in Appendix 11.2. If an index-linked gilt with a 2½% coupon is issued by the DMO on 1 November 1997, calculate the first three coupon payments.
11. The gross redemption yield on a ten-year conventional gilt is 5.771%, while the real yield on an index-linked gilt of similar maturity, and assuming an average inflation rate of 3%, is 2.726%. What is the break-even inflation rate? If the expected rate of inflation is 3.5%, which gilt is the better buy?
12. Write a short paper attempting to explain the shape of the gilt yield curve from July 1997 to July 1999, and the subsequent change to a positive sloping curve out to the seven-year term. In October 1999 the yield curve was inverted out to the long-end, with the 30-year maturity trading at a yield of 4.771% against a one-year yield of 5.972%.
13. You are a director at the DMO charged with investigating at what end of the term structure the next gilt issue should be. What are the key considerations in deciding where the market should be tapped?

12 The US Treasury Bond Market

The United States Treasury market is the largest, most liquid and most transparent bond market in the world. Although during 1999 and the first half of 2000 a shortage of supply resulted in lower levels of liquidity, especially at the long end of the curve, generally trading in Treasuries comes quite close to an economist's world of frictionless markets, perfect competition and freely available information. The market is closely observed and analysed by capital markets participants around the world, not just those in the US, reflecting the importance of US Treasury yields as the benchmark interest rates for all global debt capital. Treasuries are backed by the faith and credit of the US government and so are viewed as risk-free investments. The yield on a Treasury of a particular maturity is the benchmark rate for that term in the domestic market and for dollar-denominated instruments globally. The size and liquidity of the market give it a leading role in the economy, while the influence of the US economy on the global economy is reflected in the liquidity and trading volume of the Treasury market. As in the UK gilt market, the bid-offer price spread is very narrow and almost non-existent at the short end. Treasury yields influence other government markets and corporate markets and a move in Treasury rates is often mirrored in foreign debt markets. Treasury securities are held by a wide range of investors, in the US this includes the US Federal Reserve, investment and commercial banks, local authorities (known as *municipal* authorities), institutional investors and private individuals. Outside of the domestic market investors include foreign governments, central banks, investment banks and institutional investors.¹

In this chapter we describe the market in US Treasuries, their method of issue and market structure and conventions.

12.1 The US Treasury

United States government bonds are issued by the US Department of the Treasury, which is responsible for revenue collection, funding central government (known as *federal government*) expenditure and borrowing on behalf of the federal government. Each month the US Treasury publishes data for general circulation on the federal debt, including the terms of each bond issue. This is known as the *Monthly Statement of the Public Debt*. The large size of the Treasury market reflects the large federal deficits run by the US government in recent years; for example through the 1980s and early 1990s the annual budget deficit ran at over \$100 billion.² The total debt of the government was over \$4 trillion in 1998.³ As such the US Treasury is the largest single issuer of debt in the world. A factor contributing to the high liquidity of the Treasury market is its large size. The market had outstanding nominal value of over \$2.3 trillion in 1998, the sum of 210 different Treasury securities. Individual securities have large issue sizes, which makes them highly liquid in the secondary market. Compare this with the situation in other government and corporate markets: Treasury issues of over \$10 billion dollars are common, whereas corporate bond issues of even \$1 billion are quite rare. In the US domestic market the average corporate bond issue in 1998 was just under \$200 million in size.⁴ Hence the large size of individual issues and the size of the market as a whole implies a high volume of trading as well as a liquid market.

The US Treasury issues both coupon and discount securities. Coupon securities are conventional coupon bonds of between one and 30 years' maturity, which pay on a semi-annual interest basis and repay principal on maturity. Coupon bonds issued with between two and 10 years to maturity are known as *Treasury notes*, and those issued with original maturities of between 10 and 30 years are known as *Treasury bonds*. A bond issued originally with 30 years to maturity over 20 years ago is still known as a bond, not a note. There is no distinction between the way notes and bonds are traded in the market and in practice they are simply referred to as "Treasuries" or "bonds". The market is

¹ The United States Treasury market is as old as the United States itself. The first loan was authorised by Continental Congress in June 1775, and was denominated in pounds sterling! The second loan in October 1776, and all subsequent issues, were denominated in dollars. (Ryan 1997, p. 4).

² \$1 was equal to approximately £0.61 in November 1999.

³ Source: *The Economist*.

⁴ *Ibid.*

a plain vanilla one and all bonds are conventional bullet maturities, although certain bond issues are callable within five years to maturity. Bonds issued since 1985 have no call feature.

Debt issues of less than one year's maturity, which are known as *Treasury bills*, are discount securities which means that they are issued at discount and repay par (\$100 per cent) on maturity. These are money market instruments. The Department of the Treasury also issues non-marketable bonds known as *savings bonds*. These are non-negotiable fixed interest certificates with a fixed term to maturity, although they may be redeemed early with an interest penalty. As they are aimed at the retail market, they make up a small proportion of the total government debt.

12.2 The Federal Reserve

The central bank of the United States is the Federal Reserve. It is made up of a Board of Governors and the twelve Federal Reserve District banks and branches. The Reserve banks are private organisations owned by the commercial banks that are the members of the Federal Reserve in that district. The "Fed" as it is known is an independent central bank and its Open Market Committee is responsible for setting monetary policy. The duties of the Federal Reserve fall into four general areas, which are:

- conducting the nation's monetary policy by influencing the money and credit conditions in the economy in pursuit of full employment and stable prices;
- supervising and regulating banking institutions to ensure the safety and soundness of the nation's banking and financial system and to protect the credit rights of customers;
- maintaining the stability of the financial system and containing systemic risk that may arise in financial markets;
- providing certain financial services to the US government, to the public, to financial institutions and to foreign financial institutions, including playing a major role in operating the nation's payment system.

However it is in the area of monetary policy where the Fed has the most impact on the bond markets. It carries out its duties in this regard in the following way:

- via *open market operations*, which is the buying and selling of US government securities (mainly Treasuries) in the market, in order to influence the level of reserves in the depository system. It targets the level of the overnight lending rate, the *Federal Funds* rate through open market operations;
- through its reserve requirements, which state that commercial and other banks and depository institutions must hold in reserve an amount of cash in line with the level of their assets;
- through the *discount* rate, the interest rate that commercial banks and other depository institutions are charged when they borrow funds from a Federal Reserve bank.

A key component of the system is the *Federal Open Market Committee* (FOMC), which is made up of the Board of Governors, and the presidents of five of the Federal Reserve banks. The FOMC oversees open market operations and also approves any change in the discount rate initiated by any Federal Reserve bank. Speeches and testimonies to Congress by FOMC members are therefore closely observed and analysed by the market for any indications of policy direction.

The current chairman of the Federal Reserve is Alan Greenspan, who was appointed for a fourth successive four-year term by President Clinton during 1999. Under his chairmanship the Federal Reserve has built a sound reputation for conducting monetary policy, and its record of combining strong economic growth with low inflation has been closely scrutinised by central banks around the world as they gain independence for setting interest rates.

The appointment of the Federal Reserve chairman is made by the President of the United States.

12.3 Market convention

Treasury bond prices are quoted in various newspapers in the US and abroad, including *The Wall Street Journal*. The market itself is an "over-the-counter" market, meaning that dealing is conducted over the telephone between transaction participants. Hence no central record of actual trades exists. The newspaper price listings are representative bid and offer prices. In the US market the term "ask" is usually used for offer price. Figure 12.1 shows

bond quotations for 9 November 1999, taken from the next day's issue of *The Wall Street Journal (Europe)*. Note that the maturity date for each issue is shown as the last two digits of the maturity year and the month. For example a bond maturing in November 2015 is shown as "Nov 15". Investors would need to check from another source the precise maturity date, although US Treasury notes and bonds tend to mature on the 15th of the month. A government publication, the *Monthly Statement of the Public Debt of the United States* is an authoritative source of information on US Treasury securities. Figure 12.2 shows strip prices, from the same day.

The interest basis for US Treasury bonds is semi-annual. Thus a 15 February maturity bond makes coupon payments on 15 February and 15 August each year until maturity. Coupon interest from Treasuries is liable to federal income tax but is exempt from state and local income taxes.

U.S. TREASURY ISSUES											
Tuesday, November 9, 1999											
Representative and Indicative Over-the-Counter quotations based on transactions of \$1 million or more.											
Treasury bond, note and bill quotes are as of mid-afternoon. Colons in bond and note bid-and-asked quotes represent 32nds; 10:01 means 101 1/32. Net changes in 32nds. Treasury bill quotes in hundredths, quoted on terms of a one-day discount. Days to maturity calculated from settlement date. All yields are based on a one-day settlement and calculated on the offer quote. Current 13-week and 26-week bills are boldfaced.											
For bonds callable prior to maturity, yields are computed to the earliest call date for issues quoted above par and to the maturity date for issues quoted below par. n-Treasury note, w-When issued; daily change is expressed in basis points.											
Source: Dow Jones TeleRate/Canfor Fitch/Perle											
U.S. Treasury strips as of 3 p.m. Eastern time, also based on transactions of \$1 million or more. Colons in bid-and-asked quotes represent 32nds; 10:01 means 101 1/32. Net changes in 32nds. Treasury bill quotes in hundredths, quoted on terms of a one-day discount and calculated on the offer quote. Current 13-week and 26-week bills are boldfaced.											
For bonds callable prior to maturity, yields are computed to the earliest call date for issues quoted above par and to the maturity date for issues below par.											
Source: Bear, Stearns & Co. via Street Software Technology Inc.											
GOVT. BONDS & NOTES						Maturity					
Rate	Mo/Yr	Bid	Asked	Chg.	Yld.	Rate	Mo/Yr	Bid	Asked	Chg.	Yld.
5 th Nov 99n	99:31	100:01	...	3.47		6 th Nov 99n	101:18	101:20	...	5.80	
5 th Nov 99n	99:31	100:01	...	3.47		6 th Nov 99n	101:18	101:20	...	5.82	
5 th Nov 99n	100:00	100:02	...	3.15		3 rd Jul 02n	99:12	99:12	...	3.86	
5 th Nov 99n	100:00	100:02	...	3.15		6 th Jul 02n	100:13	100:13	...	5.81	
5 th Nov 99n	100:04	100:06	...	4.17		6 th Aug 02n	101:09	101:11	...	5.84	
5 th Dec 99n	100:00	100:02	...	3.07		6 th Aug 02n	100:00	101:00	...	5.85	
5 th Dec 99n	100:00	100:02	...	3.07		6 th Sep 02n	100:00	101:00	...	5.84	
5 th Dec 99n	100:10	100:12	...	4.89		6 th Oct 02n	99:23	99:25	...	5.83	
6 th Jan 00n	100:04	100:06	...	5.21		11 th Nov 02	115:12	115:14	...	5.93	
6 th Jan 00n	99:31	100:01	...	5.16		6 th Dec 02n	99:23	99:23	...	5.85	
6 th Jan 00n	100:16	100:18	...	5.09		6 th Dec 02n	99:09	99:11	...	5.85	
5 th Feb 00n	100:04	100:06	...	5.18		5 th Jan 03n	98:26	98:28	...	5.88	
6 th Feb 00n	100:00	100:03	...	5.13		6 th Feb 03n	98:25	98:27	...	5.89	
5 th Feb 00n	100:16	100:18	...	5.18		6 th Feb 03n	98:25	98:27	...	5.89	
5 th Mar 00n	100:01	100:03	...	5.22		5 th Apr 03n	99:16	99:18	...	5.89	
5 th Mar 00n	100:17	100:19	...	5.27		10 th May 03	114:24	114:28	...	5.99	
5 th Apr 00n	99:31	100:01	...	5.40		5 th May 03n	98:22	98:24	...	5.89	
5 th Apr 00n	100:01	100:03	...	5.41		6 th Jun 03n	98:10	98:12	...	5.88	
5 th Apr 00n	100:18	100:20	...	5.38		6 th Jun 03n	98:10	98:12	...	5.88	
5 th May 00n	100:14	100:16	...	5.37		6 th Aug 03n	97:26	97:28	...	5.88	
5 th May 00n	101:23	101:25	...	5.31		5 th Aug 03n	99:13	99:15	...	5.91	
5 th May 00n	99:31	100:01	...	5.44		11 th Aug 03	116:25	116:29	...	6.03	
6 th May 00n	100:12	100:14	...	5.43		6 th Nov 03n	94:05	94:07	...	5.89	
5 th Jun 00n	99:28	99:30	...	5.46		6 th Nov 03n	100:10	100:12	...	5.89	
5 th Jun 00n	100:06	100:08	...	5.46		6 th Nov 03n	95:22	95:24	...	5.89	
5 th Jul 00n	99:27	99:29	...	5.50		6 th Nov 03n	100:00	100:00	...	5.87	
5 th Jul 00n	100:12	100:14	...	5.48		6 th Nov 03n	97:13	97:14	...	5.90	
6 th Aug 00n	100:09	100:11	...	5.52		6 th Nov 03n	100:01	100:01	...	5.89	
6 th Aug 00n	100:10	100:12	...	5.52		6 th Nov 03n	100:14	100:15	...	5.85	
6 th Aug 00n	99:19	99:21	...	5.56		6 th Nov 03n	100:14	100:15	...	5.85	
6 th Aug 00n	100:15	100:17	...	5.57		6 th Nov 03n	100:14	100:15	...	5.85	
6 th Sep 00n	99:00	99:02	...	5.59		6 th Nov 03n	100:14	100:15	...	5.85	
6 th Sep 00n	100:13	100:15	...	5.57		6 th Nov 03n	100:14	100:15	...	5.85	
6 th Oct 00n	98:14	98:16	...	5.60		6 th Nov 03n	100:14	100:15	...	5.85	
6 th Oct 00n	100:03	100:05	...	5.58		6 th Nov 03n	100:14	100:15	...	5.85	
6 th Nov 00n	100:02	100:04	...	5.62		6 th Nov 03n	100:14	100:15	...	5.85	
6 th Nov 00n	100:24	100:26	...	5.61		6 th Nov 03n	100:14	100:15	...	5.85	
6 th Nov 00n	99:30	99:30	...	5.61		6 th Nov 03n	100:14	100:15	...	5.85	
6 th Nov 00n	99:30	99:30	...	5.62		6 th Nov 03n	100:14	100:15	...	5.85	
6 th Dec 00n	98:26	98:28	...	5.65		6 th Nov 03n	100:14	100:15	...	5.85	
6 th Dec 00n	99:25	99:27	...	5.64		6 th Nov 03n	100:14	100:15	...	5.85	
6 th Jan 01n	98:29	98:31	...	5.64		6 th Nov 03n	100:14	100:15	...	5.85	
6 th Jan 01n	99:15	99:17	...	5.64		6 th Nov 03n	100:14	100:15	...	5.85	
5 th Feb 01n	99:19	99:21	...	5.65		6 th Nov 03n	100:14	100:15	...	5.85	
6 th Feb 01n	100:14	100:16	...	5.67		6 th Nov 03n	100:14	100:15	...	5.85	
11 th Feb 01n	100:08	100:10	...	5.67		6 th Nov 03n	100:14	100:15	...	5.85	
5 th Feb 01n	99:04	99:06	...	5.65		6 th Nov 03n	100:14	100:15	...	5.85	
6 th Feb 01n	99:29	99:31	...	5.64		6 th Nov 03n	100:14	100:15	...	5.85	
6 th Mar 01n	99:27	99:29	...	5.68		6 th Nov 03n	100:14	100:15	...	5.85	
6 th Mar 01n	100:23	100:25	...	5.69		6 th Nov 03n	100:14	100:15	...	5.85	
6 th Apr 01n	100:23	100:25	...	5.69		6 th Nov 03n	100:14	100:15	...	5.85	
8 th May 01n	100:28	100:30	...	5.68		6 th Nov 03n	100:14	100:15	...	5.85	
13 th May 01n	110:17	110:21	...	5.68		6 th Nov 03n	100:14	100:15	...	5.85	
6 th May 01n	99:09	99:11	...	5.69		6 th Nov 03n	100:14	100:15	...	5.85	
6 th May 01n	101:03	101:05	...	5.71		6 th Nov 03n	100:14	100:15	...	5.85	
6 th Jun 01n	101:10	101:12	...	5.72		6 th Nov 03n	100:14	100:15	...	5.85	
6 th Jun 01n	99:17	99:19	...	5.75		6 th Nov 03n	100:14	100:15	...	5.85	
6 th Jul 01n	101:10	101:14	...	5.73		6 th Nov 03n	100:14	100:15	...	5.85	
7 th Aug 01n	103:14	103:16	...	5.75		6 th Nov 03n	100:14	100:15	...	5.85	
13 th Aug 01n	112:16	112:20	...	5.74		6 th Nov 03n	100:14	100:15	...	5.85	
5 th Aug 01n	99:17	99:19	...	5.69		6 th Nov 03n	100:14	100:15	...	5.85	
6 th Aug 01n	101:06	101:08	...	5.75		6 th Nov 03n	100:14	100:15	...	5.85	
5 th Sep 01n	99:23	99:24	...	5.76		6 th Nov 03n	100:14	100:15	...	5.85	
6 th Sep 01n	101:01	101:02	...	5.77		6 th Nov 03n	100:14	100:15	...	5.85	
5 th Oct 01n	100:27	100:29	...	5.74		6 th Nov 03n	100:14	100:15	...	5.85	
7 th Nov 01n	103:14	103:16	...	5.77		6 th Nov 03n	100:14	100:15	...	5.85	
6 th Nov 01n	118:21	118:25	...	5.74		6 th Nov 03n	100:14	100:15	...	5.85	
6 th Nov 01n	100:06	100:08	...	5.74		6 th Nov 03n	100:14	100:15	...	5.85	
6 th Dec 01n	100:19	100:21	...	5.79		6 th Nov 03n	100:14	100:15	...	5.85	
6 th Jan 02n	117:20	117:24	...	5.77		6 th Nov 03n	100:14	100:15	...	5.85	
6 th Feb 02n	100:29	100:31	...	5.79		6 th Nov 03n	100:14	100:15	...	5.85	
6 th Mar 02n	101:23	101:25	...	5.81		6 th Nov 03n	100:14	100:15	...	5.85	
6 th Apr 02n	101:26	101:28	...	5.80		6 th Nov 03n	100:14	100:15	...	5.85	
7 th May 02n	103:26	103:30	...	5.82		6 th Nov 03n	100:14	100:15	...	5.85	

TREASURY BILLS											
Rate	Mo/Yr	Bid	Asked	Chg.	Yld.	Rate	Mo/Yr	Bid	Asked	Chg.	Yld.
12 th Nov 99n	99:12	99:12	...	4.61		12 th Nov 99n	99:12	99:12	...	4.61	
18 th Nov 99n	8	4.62	5.44	...	0.14	18 th Nov 99n	8	4.62	5.44	...	0.14
26 th Nov 99n	16	4.57	4.49	...	0.11	26 th Nov 99n	16	4.57	4.49	...	0.11
Dec 02n	22	4.55	4.47	...	0.05	Dec 02n	22	4.55	4.47	...	0.05
Dec 02n	22	4.55	4.47	...	0.05	Dec 02n	22	4.55	4.47	...	0.05
Dec 16n	36	4.43	4.29	...	0.07	Dec 16n	36	4.43	4.29	...	0.07
Dec 23n	43	4.61	4.57	...	0.04	Dec 23n	43	4.61	4.57	...	0.04
Jan 06n	50	4.57	4.51	...	0.04	Jan 06n	50	4.57	4.51	...	0.04
Jan 13n	57	4.95	4.91	...	0.04	Jan 13n	57	4.95	4.91	...	0.04
Jan 20n	64	5.14	5.12	...	0.06	Jan 20n	64	5.14	5.12	...	0.06
Jan 27n	71	5.11	5.09	...	0.03	Jan 27n	71	5.11	5.09	...	0.03
Feb 03n	78	5.07	5.05	...	0.04	Feb 03n	78	5.07	5.05	...	0.04
Feb 10n	85	5.02	5.00	...	0.03	Feb 10n	85	5.02	5.00	...	0.03
Feb 17n	92	5.03	5.02	...	0.03	Feb 17n	92	5.03	5.02	...	0.03
Feb 24n	99	5.08	5.07	...	0.02	Feb 24n	99	5.08	5.07	...	0.02
Mar 02n	106	5.11	5.09	...	0.02	Mar 02n	106	5.11	5.09	...	0.02
Mar 09n	113	5.01	4.99	...	0.02	Mar 09n	113	5.01	4.99	...	0.02
Mar 16n	120	5.07	5.05	...	0.02	Mar 16n	120	5.07	5.05	...	0.02
Mar 23n	127	4.96	4.94	...	0.02	Mar 23n	127	4.96	4.94	...	0.02
Mar 30n	134	4.99	4.97	...	0.04	Mar 30n	134	4.99	4.97	...	0.04
Apr 06n	141	4.99	4.97	...	0.03	Apr 06n	141	4.99	4.97	...	0.03
Apr 13n	148	5.07	5.05	...	0.02	Apr 13n	148	5.07	5.05	...	0.02
Apr 20n	155	5.10	5.08	...	0.01	Apr 20n	155	5.10	5.08	...	0.01
Apr 27n	162	5.13	5.11	...	0.02	Apr 27n	162	5.13	5.11	...	0.02
May 04n	169	5.14	5.12	...	0.01	May 04n	169	5.14	5.12	...	0.01
May 11n	176	5.14	5.14	...	0.01	May 11n	176	5.14	5.14	...	0.01
May 18n	183	5.14	5.14	...	0.01	May 18n	183	5.14	5.14	...	0.01
May 25n	190	5.16	5.14	...							

MONDAY'S MARKET DATA

Figure 12.1: US Treasury page, *Wall Street Journal*, 10 November 1999.
© Dow Jones & Co Inc. Used with permission.

U.S. GOVERNMENT STRIPS													
Mat.	Type	Bid	Asked	Chg.	Ask	Yld.	Mat.	Type	Bid	Asked	Chg.	Ask	Yld.
Nov 99	ci	99:30	99:30		4.78		Nov 07	ci	61:07	61:12	-5	6.18	
Nov 99	np	99:30	99:30		4.74		Feb 08	ci	59:29	60:01	-4	6.27	
Feb 00	ci	98:22	98:22		5.08		May 08	ci	58:30	59:03	-4	6.28	
Feb 00	np	98:21	98:21		5.18		Aug 08	ci	58:00	58:05	-5	6.28	
May 00	ci	97:11	97:11		5.28		Nov 08	ci	57:03	57:07	-5	6.29	
May 00	np	97:08	97:08		5.48		Feb 09	ci	56:02	56:06	-4	6.32	
Aug 00	ci	95:29	95:29	-1	5.53		May 09	ci	55:05	55:10	-4	6.32	
Aug 00	np	95:27	95:28	-1	5.60		Aug 09	ci	54:08	54:13	-4	6.33	
Nov 00	ci	94:15	94:16	-1	5.66		Nov 09	ci	53:13	53:18	-4	6.34	
Nov 00	np	94:14	94:15	-1	5.70		Nov 09	bp	52:24	52:29	-4	6.46	
Feb 01	ci	93:07	93:08		5.61		Feb 10	ci	52:13	52:18	-4	6.36	
Feb 01	np	93:07	93:08		5.62		May 10	ci	51:18	51:23	-4	6.37	
May 01	ci	91:27	91:27		5.69		Aug 10	ci	50:23	50:28	-4	6.38	
May 01	np	91:27	91:28		5.69		Nov 10	ci	49:28	50:01	-5	6.39	
Aug 01	ci	90:17	90:18		5.71		Feb 11	ci	48:31	49:05	-5	6.41	
Aug 01	np	90:15	90:16		5.73		May 11	ci	48:05	48:10	-5	6.42	
Nov 01	ci	89:06	89:08		5.74		Aug 11	ci	47:11	47:16	-5	6.43	
Nov 01	np	89:06	89:08		5.74		Nov 11	ci	46:19	46:24	-5	6.43	
Feb 02	ci	87:27	87:29	-2	5.78		Feb 12	ci	45:26	45:31	-2	6.44	
May 02	ci	86:19	86:20	-2	5.79		May 12	ci	45:01	45:06	-2	6.45	
May 02	np	86:20	86:21	-2	5.78		Aug 12	ci	44:09	44:14	-2	6.45	
Aug 02	ci	85:09	85:11	-2	5.82		Nov 12	ci	43:17	43:22	-2	6.46	
Aug 02	np	85:13	85:14	-2	5.77		Feb 13	ci	42:26	43:00	-1	6.47	
Nov 02	ci	84:01	84:03	-2	5.83		May 13	ci	42:04	42:09	-1	6.47	
Feb 03	ci	82:21	82:23	-2	5.90		Aug 13	ci	41:12	41:18	-1	6.48	
Feb 03	np	82:27	82:29	-2	5.82		Nov 13	ci	40:22	40:27	-1	6.49	
May 03	ci	81:12	81:15	-3	5.92		Feb 14	ci	40:00	40:05	-1	6.50	
Aug 03	ci	80:07	80:10	-3	5.91		May 14	ci	39:11	39:17	-1	6.50	
Aug 03	np	80:12	80:15	-3	5.86		Aug 14	ci	38:23	38:28	-1	6.50	
Nov 03	ci	79:02	79:05	-2	5.91		Nov 14	ci	38:03	38:08	-1	6.51	
Feb 04	ci	77:22	77:25	-3	5.98		Feb 15	ci	37:15	37:20		6.51	
Feb 04	np	78:05	78:08	-3	5.84		May 15	bp	37:23	37:29		6.46	
May 04	ci	76:16	76:19	-3	6.00		Nov 15	ci	36:28	37:01		6.51	
May 04	np	76:21	76:25	-3	5.94		Aug 15	ci	36:09	36:15		6.51	
Aug 04	ci	75:16	75:19	-3	5.96		Nov 15	bp	36:10	36:16		6.50	
Aug 04	np	75:17	75:20	-3	5.95		Nov 15	ci	35:23	35:28		6.51	
Nov 04	ci	74:00	74:04	-3	6.07		Nov 15	bp	35:24	35:29		6.50	
Nov 04	bp	73:27	73:30	-4	6.11		Feb 16	ci	35:04	35:09	-1	6.51	
Nov 04	np	74:07	74:10	-4	6.01		Feb 16	bp	35:08	35:13	-1	6.49	
Feb 05	ci	72:27	72:30	-4	6.09		May 16	ci	34:18	34:23	-1	6.51	
Feb 05	np	73:04	73:08	-4	6.01		May 16	bp	35:02	35:07	-1	6.42	
May 05	ci	71:22	71:26	-4	6.10		Aug 16	ci	34:00	34:06	-1	6.51	
May 05	bp	71:19	71:23	-4	6.12		Nov 16	ci	33:17	33:23	-1	6.49	
May 05	np	72:01	72:05	-4	6.01		Nov 16	bp	33:27	34:01	-1	6.44	
Aug 05	ci	70:18	70:22	-4	6.11		Feb 17	ci	32:31	33:05	-1	6.50	
Aug 05	bp	70:15	70:18	-4	6.14		May 17	ci	32:14	32:19	-2	6.50	
Aug 05	np	70:27	70:31	-4	6.04		May 17	bp	32:15	32:21	-2	6.49	
Nov 05	ci	69:17	69:21	-4	6.11		Aug 17	ci	31:30	32:04	-1	6.50	
Nov 05	np	69:29	70:01	-4	6.01		Aug 17	bp	32:00	32:05	-1	6.49	
Feb 06	ci	68:09	68:13	-4	6.15		Nov 17	ci	31:15	31:21		6.49	
Feb 06	bp	68:17	68:21	-4	6.10		Feb 18	ci	30:30	31:03	-2	6.50	
Feb 06	np	68:26	68:30	-4	6.03		May 18	ci	30:15	30:20	-1	6.49	
May 06	ci	67:08	67:12	-4	6.16		May 18	bp	30:16	30:21	-1	6.49	
Aug 06	ci	66:07	66:11	-4	6.16		Aug 18	ci	29:31	30:04	-2	6.50	
Nov 06	ci	65:07	65:11	-4	6.16		Nov 18	ci	29:16	29:22	-1	6.49	
Feb 07	ci	64:02	64:06	-4	6.20		Nov 18	bp	29:17	29:22	-1	6.49	
Feb 07	np	64:18	64:22	-4	6.09		Feb 19	ci	29:02	29:07	-1	6.49	
May 07	ci	63:01	63:05	-5	6.21		Feb 19	bp	29:04	29:09	-1	6.48	
May 07	np	63:14	63:19	-5	6.12		May 19	ci	28:20	28:25	-1	6.49	
Aug 07	ci	62:02	62:07	-5	6.21		Aug 19	ci	28:07	28:13	-2	6.47	
Aug 07	np	62:19	62:23	-5	6.10		Aug 19	bp	28:10	28:15	-1	6.46	
Nov 19	ci	27:26	27:31		6.47		Nov 19	ci	27:26	27:31		6.47	
Feb 20	ci	27:08	27:13	-2	6.49		Feb 20	ci	27:08	27:13	-2	6.49	
Feb 20	bp	27:11	27:16	-2	6.47		Feb 20	bp	27:11	27:16	-2	6.47	
May 20	ci	26:27	27:00	-1	6.48		May 20	ci	26:27	27:00	-1	6.48	
May 20	bp	26:28	27:01	-1	6.48		May 20	bp	26:28	27:01	-1	6.48	
Aug 20	ci	26:15	26:20	-2	6.48		Aug 20	ci	26:15	26:20	-2	6.48	
Aug 20	bp	26:15	26:21	-2	6.47		Aug 20	bp	26:15	26:21	-2	6.47	
Nov 20	ci	26:02	26:08	-2	6.47		Nov 20	ci	26:02	26:08	-2	6.47	
Feb 21	ci	25:22	25:27	-2	6.46		Feb 21	ci	25:22	25:27	-2	6.46	
Feb 21	bp	25:27	26:00	-2	6.43		Feb 21	bp	25:27	26:00	-2	6.43	
May 21	ci	25:11	25:16	-2	6.45		May 21	ci	25:11	25:16	-2	6.45	
May 21	bp	25:13	25:18	-2	6.44		May 21	bp	25:13	25:18	-2	6.44	
Aug 21	ci	24:29	25:02	-2	6.46		Aug 21	ci	24:29	25:02	-2	6.46	
Aug 21	bp	25:01	25:06	-2	6.43		Aug 21	bp	25:01	25:06	-2	6.43	
Nov 21	ci	24:21	24:26	-2	6.43		Nov 21	ci	24:21	24:26	-2	6.43	
Nov 21	bp	24:22	24:27	-2	6.43		Nov 21	bp	24:22	24:27	-2	6.43	
Feb 22	ci	24:08	24:13	-4	6.44		Feb 22	ci	24:08	24:13	-4	6.44	
May 22	ci	23:30	24:03	-4	6.42		May 22	ci	23:30	24:03	-4	6.42	
Aug 22	ci	23:20	23:25	-5	6.41		Aug 22	ci	23:20	23:25	-5	6.41	
Aug 22	bp	23:27	24:00	-2	6.37		Aug 22	bp	23:27	24:00	-2	6.37	
Nov 22	ci	23:13	23:18	-4	6.38		Nov 22	ci	23:13	23:18	-4	6.38	
Nov 22	bp	23:14	23:19		6.37		Nov 22	bp	23:14	23:19		6.37	
Feb 23	ci	23:02	23:07	-4	6.38		Feb 23	ci	23:02	23:07	-4	6.38	
Feb 23	bp	23:10	23:15	-4	6.33		Feb 23	bp	23:10	23:15	-4	6.33	
May 23	ci	22:25	22:30	-4	6.36		May 23	ci	22:25	22:30	-4	6.36	
Aug 23	ci	22:16	22:21	-4	6.35		Aug 23	ci	22:16	22:21	-4	6.35	
Aug 23	bp	23:02	23:07	-4	6.24		Aug 23	bp	23:02	23:07	-4	6.24	
Nov 23	ci	22:08	22:13	-4	6.33		Nov 23	ci	22:08	22:13	-4	6.33	
Feb 24	ci	22:00	22:05	-4	6.31		Feb 24	ci	22:00	22:05	-4	6.31	
May 24	ci	21:23	21:28	-4	6.30		May 24	ci	21:23	21:28	-4	6.30	
Aug 24	ci	21:13	21:18	-4	6.29		Aug 24	ci	21:13	21:18	-4	6.29	
Nov 24	ci	21:05	21:10	-4	6.28		Nov 24	ci	21:05	21:10	-4	6.28	
Nov 24	bp	21:05	21:10	-4	6.28		Nov 24	bp	21:05	21:10	-4	6.28	
Feb 25	ci	21:01	21:06	-4	6.24		Feb 25	ci	21:01	21:06	-4	6.24	
Feb 25	bp	21:01	21:06	-4	6.24		Feb 25	bp	21:01	21:06	-4	6.24	
May 25	ci	20:21	20:26	-4	6.25		May 25	ci	20:21	20:26	-4	6.25	
Aug 25	ci	20:12	20:17	-4	6.24		Aug 25	ci	20:12	20:17	-4	6.24	
Nov 25	ci	20:11	20:16	-4	6.25		Nov 25	ci	20:11	20:16	-4	6.25	
Nov 25	bp	20:02	20:07	-4	6.24		Nov 25	bp	20:02	20:07	-4	6.24	
Feb 26	ci	19:20	19:25	-4	6.26		Feb 26	ci	19:20	19:25	-4	6.26	
Feb 26	bp	19:30	20:02	-4	6.21		Feb 26	bp	19:30	20:02	-4	6.21	
May 26	ci	19:11	19:16	-4	6.26		May 26	ci	19:11	19:16	-4	6.26	
Aug 26	ci	19:03	19:08	-4	6.26		Aug 26	ci	19:03	19:08	-4	6.26	
Aug 26	bp	19:05	19:09	-4	6.24		Aug 26	bp	19:05	19:09	-4	6.24	
Nov 26	ci	18:24	18:29	-4	6.26		Nov 26	ci	18:24	18:29	-4	6.26	
Nov 26	bp	18:29	19:01	-4	6.24		Nov 26	bp	18:29	19:01	-4		

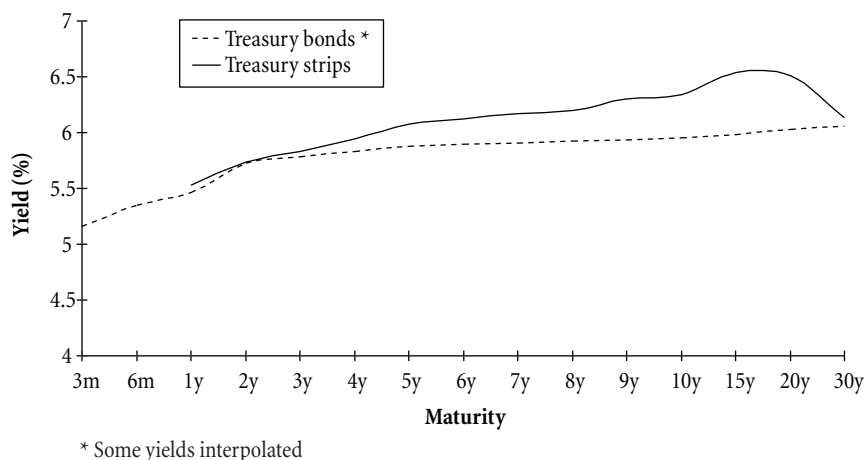


Figure 12.3: US Treasury yield curve, 9 November 1999. Source: Bloomberg.

12.4 The Primary Market

All Treasury securities are issued using the auction system. There is a weekly auction for three-month and six-month bills, while one-year bills are auctioned in the third week of each month. Notes and bonds are issued in accordance with a pre-announced auction calendar. Auction bids may be competitive or non-competitive. In a non-competitive bid no bid price is specified and the bidder states the nominal value of stock required. The bidder is then allotted stock at the average price of the accepted competitive bids, provided enough stock is available. If the auction has been covered many times non-competitive bidders are scaled down. For competitive bids the Treasury accepts the highest bids until the stock has been allotted in full and then rejects the remaining bids. The weighted average price of the accepted competitive bids determines the price for the non-competitive bids. In allotting stock, the Treasury first strips out the amount required for non-competitive bidders; the remaining stock is then allotted to competitive bidders. Dealers with the highest bids are allotted securities at that yield, and lower bids are then allotted paper until the supply is exhausted. This is known as a *multiple-price auction* and is the convention for Treasury issues. Short-dated notes however, which are maturities up to five years, are auctioned on a *single-price auction* basis. In a single-price auction, all bidders are allotted paper at the highest yield of all accepted competitive bids. As in the UK gilt market, the auction results also include details of the auction *cover* and *tail*. In the US market the term *stop yield* is used to refer to the highest accepted yield (that is, the lowest price) at which securities were allotted. The difference between the stop yield and the average yield of all accepted yields is of course the tail. As we noted in Chapter 11, the success of an auction is measured using its cover and tail. A large tail will indicate a relatively poorly received issue, because it indicates that the average price at which dealers received stock was significantly lower than the highest price at which bonds were allotted. Generally a tail of 2–3 basis points or less is regarded as a well-received new issue.

Note that unlike in the gilt market, auction bids are on a yield basis. The primary dealer therefore submits the yield at which it will accept securities. This does not apply to non-competitive bids of course, for which a nominal value required is submitted. The maximum size of a non-competitive bid is \$1 million nominal. Bids are submitted electronically via a computerised auction system.

A set auction calendar exists for notes and bonds. The Treasury issues two-, three-, five-, seven-, 10- and 30-year bonds on a regular basis. Up until April 1986 there was a regular issue of 20-year bonds, and four-year bonds were discontinued in December 1990. The different maturities are auctioned at set times during the year, as summarised in Table 12.1 below. There is a *when-issued* market in securities whose details have been announced by the Treasury but have not yet been issued. Trades in when-issued bonds are for value on the working day after the auction.

Bond	Auction month
2-year	Every month
3-year	February, May, August, November
5-year	Every month
7-year	March, May, September, December
10-year	February, May, August, November
30-year	February, May, August, November

Table 12.1: US Treasury auction calendar.

The precise details of each auction are announced by the Treasury on the last Wednesday of the preceding month, including the issue nominal size and coupon if it is a new stock. It also states how much of the issue will be used to repay existing bonds that are being redeemed and how much represents new government funds.

Market makers in Treasury securities are registered as *primary dealers* with the Federal Reserve Bank. A list of primary dealers is given at Appendix 12.1. Primary dealers have a direct dealing relationship with the Federal Reserve and may also bid at auctions. Their competitive bids may include amounts for securities for both their own book as well as for securities on behalf of customers' accounts. Broker-dealers in the market may also bid on behalf of their customers.

12.5 The Secondary Market

12.5.1 Notes and bonds

The secondary market for US Treasury securities is extremely liquid and transparent. Transactions are conducted by multiple dealers on an OTC basis and take place around-the-clock during the working week, although around 95% of trading occurs during New York business hours (Fleming 1997).⁵ The main market makers are the primary government dealers, who deal directly with the Federal Reserve Bank during its open market operations. During the first half of 1996 primary dealers traded an average of \$190 billion of Treasury securities each day, of which \$67 billion was with customers and the remainder was inter-dealer.⁶ The newest issue for a certain maturity is the benchmark bond, known in the Treasury market as the *on-the-run* or the *current coupon* issue. Such bonds would take over benchmark status from issues then known as *off-the-run*. Current issues are in greater demand amongst institutional investors and therefore are more liquid than off-the-run issues.

Primary dealers transact most inter-dealer business via Inter-dealer Brokers (IDB), of which there are six registered in the market. A list of IDBs is given at Appendix 12.2. Primary dealers post prices anonymously on IDB screens, which are only available to other primary dealers, at which they are prepared to deal. The screens therefore display live prices. Trading via an IDB is conducted on the telephone and the final counterparty remains anonymous after the transactions has taken place. In the IDB market there is no minimum or maximum price limit and price quote spreads for on-the-run issues can be as low as one-quarter of a tick or 1/128th of a point. For less liquid issues the bid-ask spread is wider, and can be up to 1/16th of a point or more.⁷

Dealing prices are displayed on "GovPX", a joint venture set up by the primary dealers and IDBs in 1991. The idea behind GovPX was to improve transparency of Treasury prices. GovPX uses feeds from IDBs to display real-time prices to the market via on-line vendor services. Users must subscribe to the service. The display includes the current best bid and ask prices and the price and size of each trade. As the quotes are obtained directly from the broker screens they may be considered firm, live prices.

For illustration purposes Table 12.2 shows the current 10-year on-the-run issue yield and accrued interest details for settlement on 10 November 1999. The settlement proceeds are for \$1 million nominal of the bond. The price is a mid-price quote obtained from a primary dealer, the analysis is obtained using Bloomberg®. The mid-price of the bond is listed as "100-12+". This means it is \$100 and 12½ ticks (that is, 25/64th) or \$100.390625.

⁵ Fabozzi (1997) states that a small amount of US Treasury trading also takes place on the New York Stock Exchange.

⁶ Fleming/Remonola (1999).

⁷ This is fairly wide. For short-dated bonds it's wide enough to drive a bus through!

US Treasury 6% 15 August 2009

Price	100-12+	Nominal	1,000,000
		Settlement date	10 November 1999
Yield to maturity	%	Settlement proceeds (\$):	
Street	5.945	Principal	1,003,906.25
Treasury	5.944	Accrued Interest	14,184.78
Equivalent annual compounding	6.034	(87 days)	
		Total	1,018,091.03
Duration	7.431		
Modified duration	7.217		
Basis point value	0.07347		
Convexity	0.654		
Yield value of a 32nd	0.00425		

Table 12.2: Treasury 6% 2009 yield analysis, 10 November 1999.

There are two yield to maturity measures that are used, the “street” and the “treasury” yield conventions. The two measures differ in the way they discount the first coupon payment when it is not exactly six months away. The treasury convention, which is also known as the “Fed” method, assumes simple interest over the period from the value date to the next coupon date. The street convention, also known as the “Securities Industry Association” method, assumes compound interest over the period from value date to the next coupon date. A first coupon may not have precisely six months to payment on two occasions, basically when the first coupon is over six months away, known as a *long first coupon* and when it is less than six months away, known as a *short first coupon*. Long or short first coupons will arise depending on the issue day and month of the bond. A short coupon date also arises in the secondary market when a bond is traded in between coupon dates. For this reason the street convention is normally always used in the secondary market.⁸ To illustrate, consider if the next coupon date is T days from the value date or previous coupon date, and there are t days between the bond’s issue date or previous coupon date and the first coupon date or next coupon date. The treasury convention discounts the coupon payment using a simple interest method as follows:

$$\frac{C/2}{(1 + \frac{T}{t}r)}$$

while the street convention uses compound interest discounting as shown:

$$\frac{C/2}{(1 + r)^{\frac{T}{t}}}.$$

Treasury securities accrued interest on an actual/actual basis, which we discussed in Chapter 3. Interest accrues from the previous coupon date (inclusive) to the settlement date (exclusive). This means that the value date is the same as the settlement date. Under the actual/actual method, to calculate accrued divide the actual number of days passed since the last coupon date by ($2 \times$ the actual number of days in the coupon period). This sometimes gives rise to “368 day” years!

12.5.2 Treasury bills

Treasury bills, also called T-Bills, are quoted differently in the market as they are money market instruments. Bills are *discount* instruments and so the price quote is on a discount basis. To calculate the yield on a bill we use (12.1). Note that the US money markets assume a 360-day count basis, similar to the euro money markets but different to sterling money markets, which use a 365-day basis.

⁸ Prior to January 1991 the five-year Note was always issued with a long first coupon, as on issue there was eight months to the first coupon. There is also a short first coupon when the auction date falls on a non-business day, in which case the auction takes place on the next business day. The first coupon date will be on the nominal issue date of the coupon month, which will be less than six months forward (Fabozzi, 1997).

$$rd = \frac{D_d}{M} \times \frac{360}{n} \quad (12.1)$$

where

- rd is the discount rate
- D_d is the cash discount, the difference between the par value and the price
- M is the maturity or par value
- n is the number of days to maturity, known as the *tenor*.

EXAMPLE 12.1 Bill discount rate

- A \$100 Treasury bill with 90 days to maturity is offered for sale at \$98.69. What in fact is the quote for the discount rate?

D_d is 100 minus 98.95, which is 1.05. Therefore the discount rate is:

$$rd = \frac{1.31}{100} \times \frac{360}{90} = 0.0524.$$

The discount rate is therefore 5.24%.

As T-Bills are quoted on a discount basis, we need to calculate the actual price payable for a bill traded in the market. This is done using (12.2) which is simply (12.1) rearranged for the cash discount.

$$D_d = rd \times M \times \frac{n}{360}. \quad (12.2)$$

We then use $M - D_d$ to calculate the price of the bill.

In comparing yields across different money market instruments and short-dated bonds it is important to compare like-for-like. This requires adjusting yields where necessary. Bill discount rates must be converted to an actual cash return, which is the interest earned on the actual cash amount invested. The resulting yield then needs to be converted to a 365-day basis if it is compared with short-dated bond yields. To compare the bill yield to other money market instruments the market uses what is known as a “CD-equivalent” yield, given at (12.3):

$$r_{CD} = \frac{360rd}{360 - n(rd)}. \quad (12.3)$$

A comprehensive review of money market instruments and yields is contained in Chapter 31.

12.6 Treasury strips

The first ever zero-coupon government bonds were developed in the US Treasury market. They date from 1982 when two primary dealers, Merrill Lynch and Salomon Brothers (now part of Travelers Citigroup) “stripped” coupons from conventional Treasury bonds that they had purchased and placed in secure custody accounts. As such the first strips were synthetic securities. The cash flows from a Treasury security were sold individually as zero-coupon bonds; the customer was therefore buying a receipt representing a specific cash flow of a Treasury bond, rather than an actual Treasury security itself. This process was called “coupon stripping” and was adopted by other primary dealers.⁹

Zero-coupon bonds represent attractive investments for certain investors in their own right; the original motivation behind their issue however was to exploit an arbitrage mispricing between the total value of the strips that made up the coupon bond and the value of the source coupon bond itself. The primary dealers profited from this mispricing. To illustrate the stripping process, consider a ten-year coupon bond. If the bond has a coupon of 6%, \$100 million nominal of this bond will have cash flows of £3 million payable every six months, plus a final payment of \$103 million on maturity (representing the redemption payment and the final coupon payment). Each of

⁹ The first strips were given other names. Fabozzi (1997) cites “TIGRs” (Treasury Income Growth Receipts), CATS (Certificates of Accrual on Treasury Securities), LIONs (Lehman Investment Opportunities Notes), GATORs, COUGARs and DOGS among others.

these cash flows represent zero-coupon bonds, which can then be sold at a discount to investors. The original strips market suffered from liquidity problems because bonds could not be settled electronically, and were not fungible with another primary dealer's receipts.

The official strips market was introduced in 1985. The Treasury announced its Separate Trading of Registered Interest and Principal Securities programme, from where the name "strips" originates. Some primary tax legislation was enacted beforehand to remove tax advantages of holding strips over coupon bonds.¹⁰ Under the Treasury structure, all new notes and bonds of over ten years' maturity are eligible for stripping, and as in the gilt market strips created from coupon bonds remain the direct obligation of the US government. The liquidity in the official market is also comparable with that in the coupon bond market since strips now settle through the same Federal Reserve book-entry system as cash gilts. Trading in strips is now entirely in the official market and there is no facility to trade in synthetic strips created by primary dealers from a coupon bond they have bought.

Figure 12.2 is an extract from *The Wall Street Journal* for 10 November 1999, showing Treasury Strips information for the day before. It shows strips trading in the market together with their bid and ask prices, as well as the ask yield.

<HELP> for explanation, <MENU> for similar functions. DL24 Govt PX1
Hit PAGE FWD for off-the-run Bills, Notes, and Bonds.
14:55 CURRENTS/WHEN ISSUED Bloomberg
GENERIC

TREASURY BILLS				
13Mo	3/22/01	↓	5.76/75	5.92 --
26Mo	6/21/01	↓	5.68/67	5.92 --
31Yr	11/29/01	↓	5.31/30	5.59 --

NOTES/BONDS				
4 6	9/02	↑	1101-00 /01	5.38 - 02
5 5 ¹ / ₂	10/02	↑	1100-19+/20+	5.38 - 01+
6 5 ³ / ₈	11/02	2yr	1100-15+/16	5.35 - 02
7 5 ⁷ / ₈	11/04	↑	1102-04+/05+	5.25 - 05
8 6 ¹ / ₂	5/05	↑	1105-30+/31+	5.21 - 05+
9 5 ¹ / ₂	11/05	2yr	1102-19 /19+	5.14 - 06+
10 6	8/09	↑	1104-30+/00	5.27 - 11
11 6 ¹ / ₂	2/10	↑	1108-24 /25+	5.27 - 12
12 5 ¹ / ₂	8/10	10yr	1103-30+/31+	5.22 - 12+

OTHER MARKETS				
19US Long(CBT)	14:45	↑	104-09	- 12
2010Y Fut(CBT)	14:45	↑	104-16	- 09
215Yr Fut(CBT)	14:45	↓	103-00+	- 05
22EURO\$ (IMM)	yd		93.4637	--
23Fed Funds	14:51	↓	6.5000	--
24S&P 500 Ind	14:55	↑	1326.90	+4.16
25NASDAQ Comp	14:55	↑	2629.77	+5.25
26DowJones Ind	14:55	↑	10660.20	+14.78
27Gold (CMX)	14:22	↓	271.70	- .30
28Crude Oil	yd		29.76	--

Copyright 2000 BLOOMBERG L.P. Frankfurt 69-200410 Hong Kong 2-977-6000 London 202-330-7500 New York 212-318-2000
Princeton 609-279-3000 Singapore 65-212-1000 Sydney 2-977-8886 Tokyo 3-201-6900 Sao Paulo 11-2048-4500
1432-212-1 19-Dec-00 14:55:38

Bloomberg
PROFESSIONAL

Figure 12.4: Bloomberg treasury prices screen PX1. Source: Bloomberg L. P.
Reproduced with permission.

12.7 Inflation-protected Treasury bonds

The US Treasury has only recently issued bonds with cash flows linked to inflation. The first such bonds were auctioned in January 1997. The market suffered from relatively low levels of liquidity in the first year but the market has since picked up. Institutional investors were slow to see the attractions of the bonds, indeed the Treasury initially directed its publicity at individual retail investors. The market has since picked up in both activity and volume.

Index-linked bonds have traditionally been viewed as attractive instruments during a period of high inflation, or in countries with a history of high inflation rates. For this reason certain commentators in the US markets questioned the need for such bonds when inflation was running at or below a 3% annual level. However the view was that, depending on where the market priced the bonds, there would be advantages to certain types of investors with long-dated savings horizons. This was recognised by most analysts who suggested that inflation-protected bonds filled a long-standing gap in the US domestic bond market.

¹⁰ Essentially the synthetic market had enabled investors to avoid income tax on the return on strips, as this was presented as capital gain. Subsequently the tax legislation was amended to bring return from strips into the domain of income tax.

12.7.1 Market structure

Inflation-protected Treasury bonds are available in denominations of \$1,000. Settlement is by book-entry transfer, in the same manner as conventional Treasury securities, so that no paper certificates are issued. Dividends therefore are paid directly into bondholders' bank accounts.

An auction of inflation-protected bonds is held every three months. Investors may make purchases from broker-dealers or directly on application at the Treasury. Treasury inflation-protected bonds differ from index-linked bonds in some other markets in that the principal amount of the bond is adjusted upwards in line with the level of inflation, while the interest rate remains constant. Therefore as the inflation rate rises in the economy, the principal amount of the bond is adjusted in line with this level, resulting in a higher dividend payment, although the coupon rate itself remains constant. The bonds are linked to the Consumer Price Index for urban consumers, known as CPI-U.

As an example, consider an investor holding \$10,000 nominal of a 4% inflation-protected bond. The interest payment in the first year is a nominal \$400. Now assume an inflation rate during the year of 3%. At the end of year 1 the value of the bond is adjusted upwards to reflect the level of inflation, so that the nominal value of the bond is now \$10,300. The coupon of 4% is now payable on this higher principal amount, which is adjusted again at the end of the second year. As the principal amount has risen from \$10,000 to \$10,300 the interest payment would rise from \$400 to \$412 in the second year. For individual retail investors of course the rise in principal value is taxable as current income, so this limits the instrument's attraction.

The Treasury is currently considering issuing inflation-adjusted zero-coupon bonds, whose principal is adjusted at maturity, in addition to bonds that would make periodic payments of both principal and interest.

12.7.2 Market developments

Inflation-protected bonds were among the highest performing Treasuries during 1999. Liquidity has improved and narrowed bid-ask spreads, and market volume in April 1999 stood at \$70 billion. As the inflation level itself has remained both low and steady, inflation-protected bonds were considered to be "cheap" during the year. The relative value of the instruments is a function of the investor's view on future inflation levels. For example in April 1999 the yield on 10-year inflation-protected bonds was 3.91%, against the yield on the conventional 10-year Treasury benchmark was 5.19%. This assumed a forward average inflation rate of 1.28% (sometimes referred to in the market as the "break-even inflation rate").¹¹ Therefore investors who believed that inflation would rise above this level would be potentially interested in the bonds. During much 1998 this break-even rate had fallen below 1%, which was considered unrealistic by most US domestic market analysts. This would make inflation-protected Treasuries attractive to institutional investors. The CPI-U index was 1.6% during 1998.

It is important to remember however that prices and yields in the inflation-protected market reflect the lower liquidity of that market. The majority of trading activity in the bonds is concentrated around the auction date. This should be less of a concern over time, as issue sizes are built up. At the time of writing the Treasury planned to auction \$8 billion of inflation-protected bonds each quarter. It also announced that it would not issue five-year notes, so the inflation-protected Treasury market is limited to only 10-year and 30-year bonds.

12.8 Treasury repo market

The repo market¹² in US Treasuries is, not surprisingly given the cash market's size, the largest repo market in the world. There is also an associated repo market in money market instruments including T-Bills, in federal agency securities and in asset-backed and mortgage-backed securities. Repo is used for funding positions by primary dealers, as well as for investment purposes by institutions and banks. The Federal Reserve also uses Treasury repo in its daily open market operations, as well in the operations it carries out on behalf of foreign central banks. As in all repo markets there is a range of repo rates, starting with the Treasury security *general collateral* repo rate to specific repo rates tradeable for specific securities. Note that the repo rate lies below the *Federal Funds* interest rate. The Fed funds is an interbank interest rate representing unsecured loans and deposits, which is why it is higher than the secured rate represented by repo.

In Chapter 34 there is a comprehensive review of repo markets and the role of the repo instrument in the money markets.

¹¹ Rates obtained from Fox Market Wire (Web site is noted in bibliography).

¹² The term *Repo* comes from "sale and repurchase" agreement.

12.9 Federal Agency bonds

There are a number of government agencies and government-sponsored agencies that issue debt capital as part of their operations. Bonds issued by government agencies are the direct obligations of the agencies themselves, and are not obligations of the federal government. In theory they are not risk-free (or default-free) in the manner of Treasury securities. In practice however it is extremely unlikely that any government agency would be allowed to go bankrupt or default on a loan. For this reason agency bonds trade at a very low spread to Treasuries, and sometimes with no spread at all. The only practical difference is that agency securities are smaller size issues than equivalent-maturity Treasuries, which might be expected to impact on liquidity.

Agencies are of two types, government-sponsored agencies and federal agencies, so that the agency bond markets is also divided into two. So-called *government sponsored enterprises* are private entities with borrowing powers exercisable on behalf of certain types of people, such as farmers, homeowners and students. *Federally-related institutions* are federal government agencies; they do not issue bonds directly but borrow funds via the Federal Financing Bank. The institutions include the Export-Import Bank, Farmers Housing Administration, Government National Mortgage Association, Tennessee Valley Authority and Washington Metropolitan Area Transit Authority amongst others. Most (but not all) federal agency bonds are backed explicitly by the federal government.

Government-sponsored agency bonds are not backed explicitly by the federal government.¹³ Therefore there is an element of credit risk associated with these bonds, reflected in a higher yield. However the yield differential is in practice more a reflection of the liquidity of this type of paper rather than the credit risk. There is a smaller amount of paper available and daily trading volumes can be as low as 10% of that of Treasuries. This still amounts a \$20 billion average trading volume however and the extent of the yield spread historically has not been great. There is a difference among individual issues however, some of which trade as Treasuries while others are less liquid and have fewer market makers supporting them. Towards the end of 1999 and during the first quarter of 2000 the market experienced increased volatility, due partly to the reduced supply of Treasury stocks as a result of the US budget surplus, and partly due to the debate on the extent of government backing for agency securities being re-ignited (see box).

The agency market is a large and diverse market and interested readers may wish to consult specialist texts, some of which are noted in the bibliography.

THE TREASURY YIELD CURVE

During the first quarter of 2000 the US Treasury announced that it would begin to pay off the country's debt, via a programme of buy-backs across all maturities. The market perceived that this would result in a scarcity of stock that was most acute at the very long end, resulting in an inversion of the yield curve. The belief that Treasuries will be in short supply has led market participants to consider whether other instruments should be used as benchmarks. Other anomalies also arose, for example certain investment banks priced 30-year US dollar corporate bonds against the 10-year Treasury benchmark rather than the 30-year issue (*IFR*, 19 February 2000). Another alternative that is being actively considered is the use of the interest-rate swap curve as the primary benchmark for corporate bonds. This raises additional issues, for example the effect on bid-offer spreads of excessive demands for certain maturities or if a majority of participants are all one way, for example they all wish to pay fixed. There are advantages however, as swaps are also widely used (as well as Treasuries) to hedge corporate bond positions, and mortgage-backed bonds are usually valued on a Libor floating interest-rate basis.

However the primary contender for an alternative benchmark to the Treasury market is the agency debt market. Certain market participants believe that the agency market, primarily bonds issued by Fannie Mae and Freddie Mac, are an attractive benchmark and hedging instrument, as most issues exist in large size, so that as the number and volume of Treasuries decreases, agency issues can be used to fill the gap. However the government guarantee for agency securities has always been implicit, rather than explicit. In March 2000 the Treasury Undersecretary, Mr Gensler, while speaking at the (US House of Representatives) House Banking subcommittee stated that investors should not expect the government to guarantee their investments, or provide protection against loss. Although the market had been aware of the Treasury position vis-à-vis agency securities,

¹³ One government-sponsored agency, the Farm Credit Financial Assistance Corporations, *does* have its bonds backed by the full faith and credit of the US government.

this caused a loss of investor confidence and a widening of yield spreads. The spread on 10-year Freddie Mac issues increased from 92 basis points over, the day before the comments made by the Treasury official, to 113 basis points over (*IFR*, 25 March 2000). The swap curve also rose, with 10-year swap spreads widening from 110 basis points to 126 basis points over. The agency and swap spreads later fell back to 105 and 118 basis points over respectively, one week after the comments had been made. On the other hand bonds issued by the GNMA (“Ginnie Mae”), which carry an explicit government guarantee, outperformed Fannie Mae and Freddie Mac securities during the week following Mr Gensler’s remarks, as investors embarked on a “flight to quality”. The prices of GNMA mortgage-backed securities were between 16/32nds to 20/32nds higher, compared to agency issues, at the end the week in question. In Figures 12.5 and 12.6 we show the yield curves for Treasury securities and agency securities in April 2000 and April 1995. The change in yield spreads is apparent.

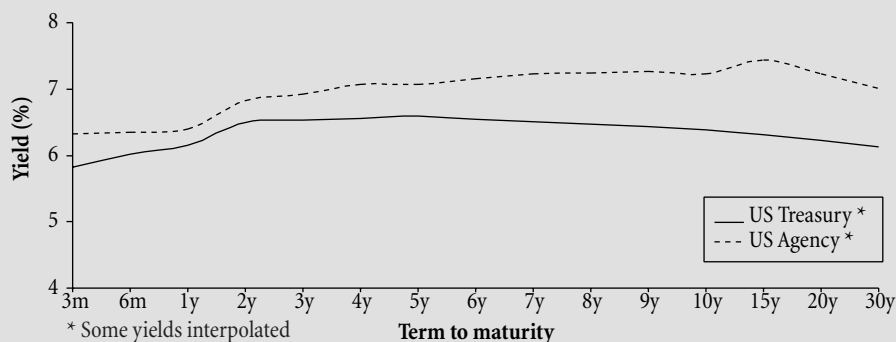


Figure 12.5: Yield curves and spreads for 4 April 2000. Source: Bloomberg.

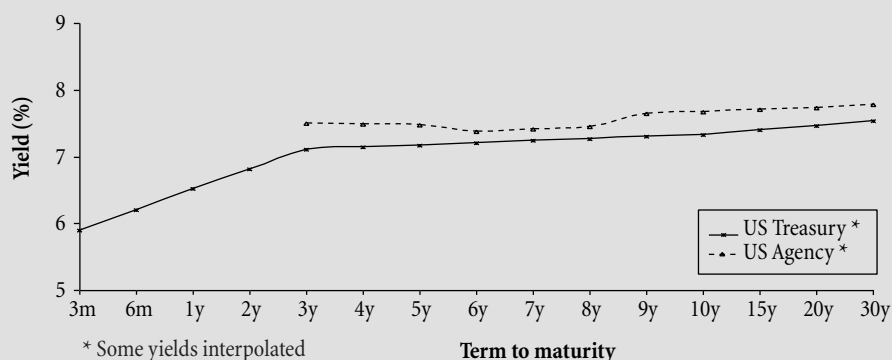


Figure 12.6: Yield curves and spreads for 5 April 1995. Source: Bloomberg.

This story highlights two important points. First, the government guarantee of agency securities has always been implicit, and in times of investor unease or loss of confidence will not be regarded as highly as it is in times of investor confidence. The view of the Treasury – that agency bonds are not explicitly government guaranteed – has been known for some time, whereas the government itself, as represented by say, certain members of the US Congress, may feel that the guarantee is more explicit. Secondly, it shows that both alternative benchmarks to the government bond, agency securities and the swap curve, react in a volatile fashion to market events in a way that indicates that the market does not believe them to be wholly satisfactory benchmarks. In a way though, the market’s reaction can also be taken to indicate the enhanced role of agency securities in the US debt market, at a time when Treasury securities are in ever shorter supply.

The issue of an orderly benchmark in debt capital markets is a topical one in countries around the world, as decreasing budget deficits result in a lower supply of government bonds. The gilt yield curve in the United Kingdom has been inverted during the three years from July 1997 and this is mainly ascribed to a shortage of stock at the long end. As well as suggesting alternative benchmarks such as agency securities and the swap curve,

some commentators recommended that governments issue bonds irrespective of whether there is a borrowing requirement, in order to maintain an orderly capital market. The extra funding raised should then be used to invest in capital and infrastructure projects. At the time of writing, no government had adopted this line of thinking, and the economic consensus in place remains that paying off public sector debt, and running budget surpluses, is a desirable macroeconomic objective.

12.10 Derivatives markets

Trading in cash Treasury securities takes place around the world, although the centre of the cash market could be said to be in New York where the primary dealers have their offices. Alongside the cash market is a large and liquid derivatives market, centred in Chicago. The main derivatives exchanges where Treasury securities are traded are the Chicago Board of Trade (CBOT), where the principal contract is the Treasury bond future (known as the T-bond contract), and the Chicago Mercantile Exchange (CME) where short-term US dollar interest rate futures are traded (known as Eurodollar futures). The same contracts are also traded in other exchanges, for example on SIMEX in Singapore. However the LIFFE exchange in London abandoned its T-bond futures contract after experiencing low volumes and liquidity, and the long bond contract on the Tokyo Stock Exchange was de-listed during 1999.

One contract of the T-bond future on CBOT represents \$100,000 nominal of a notional underlying bond. Any Treasury with a maturity of 15 years or longer is eligible for delivery into the contract. As with the cash instrument, prices are quoted in ticks, giving a tick value for the contract of \$31.25. There is a daily price limit of three points or \$3000 per contract above or below the previous day's closing price. Limits are removed on the second business day preceding the first day of the delivery month.

Contracts mature in the conventional delivery months of March, June, September and December. The last trading day is the seventh business day preceding the last business day of the delivery month, and the last delivery day is the last business day of the delivery month. Trading hours are 07:20 to 14:00 local time, with evening and overnight trading hours on an electronic dealing system.

The CBOT also lists the following other derivative contracts:

- options on T-bond futures;
- futures and options on two-year, five-year and 10-year Treasury notes;
- futures and options on a long-dated municipal bond index;
- futures on 30-day Fed funds.

The Eurodollar contract is reviewed in the chapter on money market derivative instruments.

12.10.1 Treasury futures contracts

The CBOT lists futures contracts on the Treasury long bond, 10-year, five-year and two-year notes. The description of the first two of these contracts is given at Table 12.3 below. The notional coupon on both contracts was changed from 8% to 6% from the March 2000 contract. Bonds that are eligible for delivery into the long bond contract are those that, if not callable, have a maturity of a minimum of 15 years from the first day of the delivery month of the contract. If a bond is callable, it is eligible provided that it is not callable for at least 15 years from the first day of the delivery month. Eligible bonds for the 10-year note contract must have a maturity of between 6½ and 10 years from the first day of the delivery month. Table 12.4 shows the delivery basket for the June 2000 long bond futures contract on 4 April 2000. For a description of the technical terms and values shown in the table, readers should refer to Chapter 41; however a brief explanation is given here. As a futures contract specified a *notional* coupon for the bond that is delivered into it, any bond that meets the maturity specifications of the contract is eligible for delivery into the contract. Therefore to equalise each deliverable bond in terms of the futures contract, a *conversion factor* is used. This is the approximate price (in decimals, irrespective of the contract being discussed) at which the deliverable bond would have a yield-to-maturity equal to the notional coupon, which in the case of Treasury contracts is 6.00%. In other words, it is the approximate price at which the deliverable bond would trade if it had a yield of 6.00%.

Conversion factors are calculated by the futures exchange and remain the same throughout the life of the contract; they are unique to each bond. A bond with an actual coupon higher than the notional coupon will have a conversion factor higher than 1.00, while bonds with actual coupons lower than the notional coupon will have conversion factors lower than 1.00. The conversion factor for the same bond steadily decreases over successive

futures contracts, for bonds with coupons higher than the notional, reflecting the *pull to par* of the bond's price towards par as it approaches maturity. The opposite occurs for bonds with coupons lower than the notional coupon. Conversion factors remain unchanged for each specific contract however. Note that the conversion factor is not a "hedge ratio", and we show why in Chapter 41.

The calculation of conversion factors is shown in Appendix 12.4.

US Long Bond	
Notional bond	US Treasury 20-year 6%
Contract size	\$100,000
Value of one point	\$1,000
Tick size	0-01 (32nds)
Tick value	\$31.25
Trading hours	07:20 – 14:00 14:15 – 16:30 18:00 – 05:00
Bloomberg ticker	US [futures letter and year] <CMDTY>
US 10-year note	
Notional bond	US Treasury 10-year 6%
Contract size	\$100,000
Value of one point	\$1,000
Tick size	0-00+ (64ths)
Tick value	\$15.625
Trading hours	07:20 – 14:00 14:15 – 16:30 18:00 – 05:00
Bloomberg ticker	TY [futures letter and year] <CMDTY>

Table 12.3: CBOT Treasury long bond and 10-year note futures contracts.

Trade date	4-Apr-00					
Settlement	5-Apr-00					
Futures price	97-26					
Repo rate	6.05%					
Treasury Bond	Price	Yield %	Conversion factor	IRR %	Gross basis	Net basis
8.875% 2017	128-00+	6.214	1.3038	5.21	15.6	8.2
9.125% 2018	131-13+	6.210	1.3383	5.13	16.6	9.3
9% 2018	130-19+	6.203	1.3298	5.01	17.2	10.5
8.75% 2017	126-16+	6.211	1.2879	4.96	17.4	10.6
8.875% 2019	129-17+	6.196	1.3187	4.91	18.0	11.3
8.5% 2020	126-07	6.188	1.2851	4.89	16.6	11.3
9.875% 2015	136-01+	6.229	1.3835	4.87	23.1	12.4
8.125% 2019	121-23+	6.183	1.2390	4.69	17.4	12.8
8.75% May 2020	129-09	6.185	1.3156	4.68	19.2	13.7
8.75% Aug 2020	129-16+	6.181	1.3178	4.64	19.8	14.1
7.5% 2016	113-13+	6.194	1.1542	4.52	16.9	13.3
7.875% 2021	119-27	6.170	1.2195	4.50	18.0	14.3
10.625% 2015	143-03+	6.225	1.4533	4.49	30.7	17.2
8.125% 2021	123-01+	6.168	1.2518	4.43	19.4	15.3

7.25% 2016	110-22	6.191	1.1261	4.36	17.3	14.4
9.25% 2016	130-13+	6.207	1.3250	4.34	26.3	17.2
8.125% 2021	122-29+	6.168	1.2502	4.30	20.4	16.6
8% 2021	121-29	6.153	1.2383	3.73	25.1	21.8
7.25% 2022	113-11+	6.142	1.1516	3.64	23.0	21.0
7.625% 2022	117-30	6.146	1.1980	3.64	24.3	21.9
7.125% 2023	112-00+	6.139	1.1379	3.59	22.9	21.2
6.25% 2023	101-17	6.125	1.0310	3.23	22.0	21.0
7.5% 2024	117-16+	6.114	1.1902	2.34	35.2	33.5
7.625% 2025	119-09	6.107	1.2702	2.09	38.5	36.4
6.875% 2025	110-00+	6.094	1.1126	1.64	38.1	37.4
6% 2026	99-01	6.074	1.0000	0.83	39.0	39.8
6.75% 2026	108-27+	6.071	1.0981	0.55	46.4	46.2
6.5% 2026	105-20+	6.069	1.0656	0.45	45.2	45.5
6.625% 2027	107-13	6.062	1.0824	0.12	49.1	49.1
6.375% 2027	104-08+	6.053	1.0498	-0.30	50.6	51.0
6.125% 2027	101-04+	6.039	1.0166	-1.09	54.5	55.6
5.5% 2028	92-29+	6.023	0.9326	-1.80	54.5	56.2
5.25% 2028	89-21	6.011	0.8984	-2.54	57.0	59.3
5.25% 2029	89-26	5.996	0.8982	-3.31	62.6	64.8
6.125% 2029	102-07	5.963	1.0171	-5.23	87.5	88.8
6.25% 2030	105-29	5.831	1.0344	-12.80	151.3	153.7

Table 12.4: Delivery basket, June 2000 CBOT Treasury long bond contract, 4 April 2000.
Source: Bloomberg.

Trade date	4-Apr-00					
Settlement	5-Apr-00					
Futures price	98-08+					
Repo rate	6.05%					
Treasury Bond	Price	Yield %	Conversion factor	IRR %	Gross basis	Net basis
6.125% 2007	99-09	6.247	1.0071	4.72	10.2	10.1
6.625% 2007	102-01+	6.262	1.0342	4.62	13.5	11.2
5.5% 2008	95-18+	6.218	0.9702	4.60	7.7	10.7
4.75% 2008	90-17+	6.181	0.9195	4.24	6.1	12.6
6.25% 2007	100-01	6.243	1.0133	4.22	14.7	14.1
5.625% 2008	96-11+	6.202	0.9769	4.13	11.6	14.2
5.5% 2009	95-16+	6.149	0.9662	3.14	18.3	21.4
6% 2009	99-00+	6.139	1.0000	2.80	24.0	24.8
6.5% 2010	103-23+	5.992	1.0358	-1.66	62.4	61.6

Table 12.5: Delivery basket, June 2000 CBOT Treasury 10-year note contract, 4 April 2000.
Source: Bloomberg.

12.11 Historical long-bond yields

For historical interest, we present at Figure 12.7 a graph of yield levels in the US and UK from the earliest recorded data. An early form of UK government bonds were undated (that is, perpetual) gilts issued under the name *consolidated* or “consols”. The earliest recorded figures date from 1756. In the United States federal government bonds were

issued to help finance railway building, with the earliest yield recorded in 1857. The first long-dated Treasury bonds were issued in 1921. The chart is a fascinating picture of long-dated government yields in the two countries since the eighteenth century.

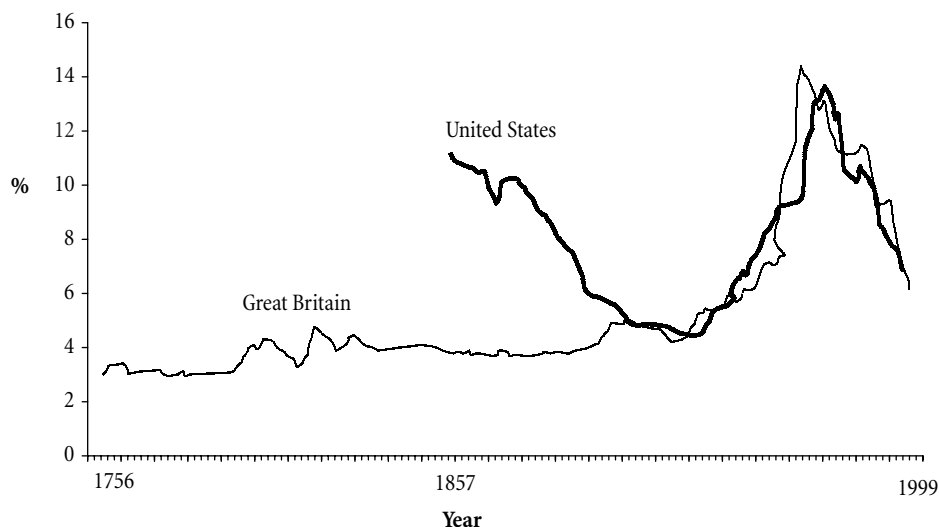


Figure 12.7: Long-dated government yields in the US and UK.

Source: BoE, US Dept of Commerce, Strata Consulting.

Appendices

APPENDIX 12.1 Registered US Treasury primary dealers as at January 2000

ABN AMRO Incorporated	Greenwich Capital Markets, Inc.
Banc of America Securities LLC	HSBC Securities (USA) Inc.
Banc One Capital Markets, Inc.	J. P. Morgan Securities, Inc.
Barclays Capital Inc.	Lehman Brothers Inc.
Bear, Stearns & Co., Inc.	Merrill Lynch Government Securities Inc.
Chase Securities Inc.	Morgan Stanley & Co. Incorporated
CIBC World Markets Corp.	Nesbitt Burns Securities Inc.
Credit Suisse First Boston Corporation	Nomura Securities International, Inc.
Daiwa Securities America Inc.	Paine Webber Incorporated
Deutsche Bank Securities Inc.	Paribas Corporation
Donaldson, Lufkin & Jenrette Securities Corporation	Prudential Securities Incorporated
Dresdner Kleinwort Benson North America LLC.	SG Cowen Securities Corporation
Fuji Securities Inc.	Salomon Smith Barney Inc.
Goldman, Sachs & Co.	Warburg Dillon Read LLC.
	Zions First National Bank

Source: Federal Reserve Bank of New York.

APPENDIX 12.2 US Treasury Inter-Dealer Brokers

Cantor Fitzgerald Securities	Exco USA Securities Inc.
Garban GuyButler Inc.	Hilliard Farber & Co.
Liberty Brokerage Inc.	Tullet & Tokyo Securities Inc.

APPENDIX 12.3 Treasury notes and bonds

Bond	Price	Yield %	Duration	Bond	Price	Yield %	Duration
UST 6% 15 Aug 2000	99-30+	6.05	0.43	UST 5.25% 15 May 2004	95-07	6.57	3.73
UST 8.75% 15 Aug 2000	101-04	5.96	0.43	UST 13.75% 15 Aug 2004	126-18	6.70	3.55
UST 5.125% 31 Aug 2000	99-17	6.12	0.47	UST 7.25% 15 Aug 2004	102-12	6.61	3.85
UST 6.125% 30 Sep 2000	100-00+	6.06	0.53	UST 6% 15 Aug 2004	97-23+	6.59	3.93
UST 4.5% 30 Sep 2000	99-02	6.23	0.54	UST 11.625% 15 Nov 2004	119-14+	6.69	3.71
UST 4% 31 Oct 2000	98-20	6.22	0.62	UST 7.875% 15 Nov 2004	104-30+	6.61	3.92
UST 5.75% 31 Oct 2000	99-22	6.22	0.62	UST 5.875% 15 Nov 2004	97-07	6.57	4.07
UST 8.5% 15 Nov 2000	101-16	6.14	0.65	UST 7.5% 15 Feb 2005	103-20	6.62	4.20
UST 5.75% 15 Nov 2000	99-21+	6.22	0.66	UST 12% 15 May 2005	122-28	6.68	4.00
UST 4.625% 30 Nov 2000	98-27+	6.24	0.70	UST 8.25% 15 May 2005	100-12	8.15	4.21
UST 5.625% 30 Nov 2000	99-18	6.22	0.70	UST 6.5% 15 May 2005	99-17	6.60	4.38
UST 4.625% 31 Dec 2000	98-22	6.30	0.79	UST 10.75% 15 Aug 2005	118-03+	6.70	4.33
UST 5.5% 31 Dec 2000	99-11+	6.30	0.79	UST 6.5% 15 Aug 2005	99-14+	6.61	4.63
UST 5.25% 31 Jan 2001	99-02	6.33	0.87	UST 5.875% 15 Nov 2005	96-16	6.62	4.80
UST 4.5% 31 Jan 2001	98-14	6.32	0.87	UST 9.375% 15 Feb 2006	112-27+	6.70	4.73
UST 11.75% 15 Feb 2001	104-27	6.25	0.90	UST 5.625% 15 Feb 2006	95-06+	6.61	5.08
UST 5.375% 15 Feb 2001	99-04	6.34	0.91	UST 6.875% 15 May 2006	101-06	6.63	5.03
UST 7.75% 15 Feb 2001	101-07	6.35	0.91	UST 7% 15 July 2006	101-28	6.62	5.19
UST 5% 28 Feb 2001	98-24	6.34	0.95	UST 6.5% 15 Oct 2006	99-13+	6.60	5.32
UST 5.625% 28 Feb 2001	99-10+	6.33	0.95	UST 7.625% 15 Feb 2007	101-16	7.34	5.49
UST 6.375% 31 Mar 2001	100-00	6.37	1.00	UST 6.25% 15 Feb 2007	98-03+	6.59	5.69
UST 4.875% 31 Mar 2001	98-16	6.34	1.01	UST 6.625% 15 May 2007	100-03	6.60	5.70
UST 5% 30 Apr 2001	98-16+	6.34	1.10	UST 6.125% 15 Aug 2007	97-09+	6.58	6.03
UST 13.125% 15 May 2001	107-15+	6.35	1.09	UST 7.875% 15 Nov 2007	102-18+	7.42	5.79
UST 8% 15 May 2001	101-24	6.39	1.12	UST 5.5% 15 Feb 2008	93-15+	6.56	6.45
UST 5.625% 15 May 2001	99-03+	6.39	1.13	UST 5.625% 15 May 2008	94-02	6.57	6.48
UST 6.5% 31 May 2001	100-02	6.41	1.17	UST 8.375% 15 Aug 2008	104-23	7.60	6.23
UST 5.25% 31 May 2001	98-20	6.41	1.18	UST 8.75% 15 Nov 2008	106-05+	7.75	6.17
UST 5.625% 30 Jun 2001	100-07	6.41	1.25	UST 4.75% 15 Nov 2008	88-04+	6.56	6.97
UST 5.75% 30 Jun 2001	99-04	6.43	1.26	UST 9.125% 15 May 2009	108-10	7.83	6.35
UST 5.5% 31 Jul 2001	98-23+	6.44	1.34	UST 5.5% 15 May 2009	92-28	6.54	7.11
UST 6.625% 31 Jul 2001	100-07	6.43	1.34	UST 6% 15 Aug 2009	96-16+	6.49	7.26
UST 13.375% 15 Aug 2001	109-08	6.45	1.34	UST 10.375% 15 Nov 2009	114-01+	8.23	6.37
UST 7.875% 15 Aug 2001	101-28	6.45	1.37	UST 11.75% 15 Feb 2010	120-14	8.64	6.42
UST 5.5% 31 Aug 2001	98-20+	6.46	1.42	UST 6.5% 15 Feb 2010	101-02	6.35	7.45
UST 6.5% 31 Aug 2001	100-01	6.45	1.42	UST 10% 15 May 2010	113-22+	8.00	6.66
UST 6.375% 30 Sep 2001	99-26	6.48	1.46	UST 12.75% 15 Nov 2010	127-20+	8.71	6.44
UST 5.625% 30 Sep 2001	98-23	6.48	1.47	UST 13.875% 15 May 2011	135-16	8.81	6.49
UST 5.25% 31 Oct 2001	99-19+	6.48	1.54	UST 14% 15 Nov 2011	138-14	8.69	6.67
UST 5.875% 31 Oct 2001	99-00+	6.49	1.55	UST 10.375% 15 Nov 2012	121-15	7.69	7.62
UST 15.75% 15 Nov 2001	114-10	6.54	1.48	UST 12% 15 Aug 2013	133-16	7.90	7.78
UST 7.5% 15 Nov 2001	101-16	6.51	1.57	UST 13.25% 15 May 2014	144-05	7.98	7.68
UST 5.875% 30 Nov 2001	98-30+	6.50	1.63	UST 12.5% 15 Aug 2014	139-26	7.84	8.06
UST 6.125% 31 Dec 2001	99-10+	6.50	1.71	UST 11.75% 15 Nov 2014	135-22	7.65	8.10
UST 6.375% 31 Jan 2002	99-23+	6.50	1.75	UST 11.25% 15 Feb 2015	144-07	6.56	8.71
UST 6.25% 31 Jan 2002	99-16	6.51	1.80	UST 10.625% 15 Aug 2015	139-06	6.55	8.98

Bond	Price	Yield %	Duration	Bond	Price	Yield %	Duration
UST 14.25% 15 Feb 2002	113-24+	6.50	1.75	UST 9.875% 15 Nov 2015	132-12+	6.54	9.02
UST 6.5% 28 Feb 2002	100-00	6.50	1.87	UST 9.25% 15 Feb 2016	126-21	6.53	9.40
UST 6.25% 28 Feb 2002	99-17	6.49	1.88	UST 7.25% 15 May 2016	107-15+	6.49	9.80
UST 6.625% 31 Mar 2002	100-06	6.51	1.89	UST 7.5% 15 Nov 2016	110-04+	6.49	9.90
UST 6.625% 30 Apr 2002	100-06	6.51	1.98	UST 8.75% 15 May 2017	122-30+	6.51	9.75
UST 7.5% 15 May 2002	101-28+	6.53	2.00	UST 8.875% 15 Aug 2017	124-15	6.50	9.97
UST 6.5% 31 May 2002	99-29	6.52	2.06	UST 9.125% 15 May 2018	127-25	6.49	9.97
UST 6.25% 30 Jun 2002	99-12	6.53	2.15	UST 9% 15 Nov 2018	126-29	6.49	10.15
UST 6% 31 Jul 2002	98-27	6.51	2.24	UST 8.875% 15 Feb 2019	125-27	6.48	10.43
UST 6.375% 15 Aug 2002	99-19+	6.53	2.21	UST 8.125% 15 Aug 2019	118-05+	6.47	10.76
UST 6.25% 31 Aug 2002	99-10	6.54	2.32	UST 8.5% 15 Feb 2020	122-19	6.46	10.81
UST 5.875% 30 Sep 2002	98-14+	6.53	2.34	UST 8.75% 15 May 2020	125-15+	6.47	10.63
UST 5.75% 31 Oct 2002	98-04	6.52	2.43	UST 8.75% 15 Aug 2020	125-23+	6.46	10.89
UST 11.625% 15 Nov 2002	112-02+	6.60	2.32	UST 7.875% 15 Feb 2021	116-09	6.44	11.25
UST 5.75% 30 Nov 2002	98-00+	6.54	2.51	UST 8.125% 15 May 2021	119-10+	6.44	11.06
UST 5.625% 31 Dec 2002	97-21+	6.53	2.60	UST 8.125% 15 Aug 2021	119-14	6.44	11.32
UST 5.5% 31 Jan 2003	97-08	6.55	2.69	UST 8% 15 Nov 2021	118-07	6.43	11.23
UST 10.75% 15 Feb 2003	110-23	6.64	2.59	UST 7.25% 15 Aug 2022	109-29	6.41	11.84
UST 6.25% 15 Feb 2003	99-06	6.56	2.71	UST 7.625% 15 Nov 2022	114-09+	6.42	11.58
UST 5.5% 28 Feb 2003	97-05+	6.55	2.77	UST 7.125% 15 Feb 2023	108-18+	6.40	12.00
UST 5.5% 31 Mar 2003	97-02+	6.56	2.77	UST 6.25% 15 Aug 2023	98-10+	6.39	12.45
UST 5.75% 30 Apr 2003	97-22+	6.65	2.85	UST 7.5% 15 Nov 2024	113-27	6.38	12.09
UST 10.75% 15 May 2003	111-15	6.66	2.71	UST 7.625% 15 Feb 2025	115-15+	6.37	12.31
UST 5.5% 31 May 2003	96-28+	6.57	2.94	UST 6.875% 15 Aug 2025	106-10+	6.37	12.66
UST 5.375% 30 Jun 2003	96-16+	6.55	3.03	UST 6% 15 Feb 2026	95-21	6.34	13.13
UST 5.25% 15 Aug 2003	96-00	6.56	3.16	UST 6.75% 15 Aug 2026	105-03+	6.35	12.92
UST 11.125% 15 Aug 2003	113-13	6.67	2.95	UST 6.5% 15 Nov 2026	101-31+	6.34	12.86
UST 5.75% 15 Aug 2003	97-15+	6.57	3.14	UST 6.625% 15 Feb 2027	103-19	6.34	13.07
UST 11.87% 15 Nov 2003	116-19+	6.68	3.03	UST 6.375% 15 Aug 2027	100-15+	6.33	13.27
UST 4.25% 15 Nov 2003	92-19	6.53	3.38	UST 6.125% 15 Nov 2027	97-10	6.33	13.21
UST 5.875% 15 Feb 2004	97-20	6.56	3.54	UST 5.5% 15 Aug 2028	89-08+	6.31	13.85
UST 4.75% 15 Feb 2004	93-26	6.55	3.60	UST 6.125% 15 Aug 2029	98-00+	6.27	13.77
UST 12.375% 15 May 2004	120-09+	6.70	3.35	UST 6.25% 15 May 2030	101-10	6.15	13.75
UST 7.25% 15 May 2004	102-08	6.61	3.60				

Table 12.6: United States Treasury Notes and Bonds as at 10 March 2000. Source: Bloomberg.

APPENDIX 12.4 Formula for calculating the conversion factor: CBOT Treasury contracts

The conversion factor for a bond, also known as the *price factor*, is given by (12.4):

$$CF = a \times \left(\frac{C}{2} + c + d \right) - b \quad (12.4)$$

where

C is the bond's coupon

$$a = \frac{1}{1.03^{\frac{v}{6}}} \quad b = \frac{C}{2} \times \frac{6-v}{6}$$

$$c = \begin{cases} \frac{1}{1.03^{2n}} & \text{if } z < 7 \\ \frac{1}{1.03^{2n+1}} & \text{otherwise} \end{cases} \quad d = \frac{C}{0.06} \times (1 - c)$$

n is the number of (whole) years from the first day of the delivery month to the maturity date of the bond

$$v = \begin{cases} z & \text{if } z < 7 \\ 3 & \text{if } z \geq 7 \text{ (bonds and 10-year note)} \\ (z - 6) & \text{if } z \geq 7 \text{ (five-year and two-year note)} \end{cases}$$

z is the number of months between n and the maturity date of the bond, rounded to quarter-years for Treasury bond and 10-year note and to nearest month for five-year and two-year notes.

Selected bibliography

- Balduzzi, P., Elton, E., Green, T.C., "Economic news and the yield curve: Evidence from the US Treasury market", Working paper, New York University, 1997.
- Bhattacharya, A., Fabozzi, F., (eds.), *Asset-backed securities*, FJF Associates, 1996.
- Burghardt, G., Belton, T., Lane, M., Papa, J., *The Treasury Bond Basis*, McGraw-Hill, 1994.
- Dougal, H.E., Gaumnitz, J.E., *Capital Markets and Institutions*, 5th edition, Prentice Hall, 1986.
- Essex, M., Pitchford, R., *World Bond Markets*, John Wiley and Sons, 1996.
- Fabozzi, F., (ed.), *The Handbook of Fixed Income Securities*, 5th edition, Chapter 8, McGraw-Hill, 1997.
- Fabozzi, F., *Fixed Income Securities*, FJF Associates, 1997.
- Fabozzi, F., Modigliani, F., *Capital markets: institutions and instruments*, Prentice-Hall, 1996.
- Fleming, M.J., "The round-the-clock market for US Treasury securities", *Economic Policy Review*, Federal Reserve Bank of New York, July 1997, pp. 9–32.
- Fleming, M.J., Remolana, E.M., "What moves the bond market?", *Economic Policy Review*, Federal Reserve Bank of New York, December 1997, pp. 31–50.
- Fleming, M.J., Remolana, E.M., "Price formation and liquidity in the US Treasury market: the response to price formation", *Journal of Finance* 54:(5), October 1999, pp. 1901–1915.
- The Federal Reserve System, Purposes and Functions*, 7th edition, US Government Printing Office, 1984.
- FOX Market Wire, <http://www.foxmarketwire.com/032399/inflation/>, 15 November 1999.
- Jordan, B.D., Kuipers, D.R., "Negative option values are possible: The impact of treasury bond futures on the cash US Treasury market", *Journal of Finance Economics* 46:(1) October 1997, pp. 67–102.
- Livingston, M., *Money and Capital Markets*, 2nd edition, New York Institute of Finance, 1993.
- Mishkin, F.S., *Money, Banking and Financial Markets*, 3rd edition, Harper Collins, 1992.
- Ryan, R. (ed.), *Yield Curve Dynamics*, Glenlake Publishing, 1997.
- Stigum, M., *The Money Market*, Dow Jones Books, 1990.
- US Government Printing Office, *Treasury Bulletin* 1998, 1999.

Questions and exercises

1. Discuss what types of investors may be interested in the US Treasury market.
2. How does the US Treasury issues bonds? Is there any risk to a participant in the issue process?
3. The Treasury announces an auction of \$10 billion nominal of ten-year bonds. \$1 billion nominal of non-competitive bids are received. The following competitive bids are received:

Yield	Amount
6.002%	\$3 billion
6.007%	\$3 billion
6.008%	\$4 billion
6.009%	\$2 billion
6.011%	\$6 billion

Calculate the price paid by non-competitive bidders for the stock they are allotted. What is the auction cover? What is the tail?

4. A new issue auction result is stated as follows: "all bids were accepted at the average yield or better, that is with no tail". What is the average yield? If there is no tail in an auction, what does this imply about the market reaction to the auction?
5. A 90-day bill is offered for sale at a discount rate of 5.36%. What is the price of the bill?
6. What Treasury contracts are traded on the Chicago Board of Trade?
7. What is the relevance of the conversion factor for a futures-deliverable bond?
8. A long bond futures contract deliverable bond has a coupon of 6.75% and on the first day of the delivery month for the September 2000 contract had 19 years, 9 months and 21 days to maturity. Calculate the conversion factor for the bond for this contract.
9. How do government agency bonds trade in the market compared to US Treasuries?
10. Where does the Federal Funds rate trade in relation to the repo rate?
11. What is a primary dealer? An off-the-run issue?

13 International Bond Markets

In this chapter we briefly describe the market structure in place in selected international government bond markets. In any debt capital market it is typically government bonds that receive the highest credit rating; indeed the established ratings agencies will not assign a credit rating to a borrower that is higher than that of the borrower's domicile country. Therefore government bonds are the natural benchmark for all other interest rates, with borrowers paying a credit risk premium over the equivalent maturity government debt. The first debt market in any country has always been the government market, and this is usually the largest as well. The government markets in countries such as the United States, Japan and the other G7 countries are the largest in the world. Table 13.1 shows the nominal value of public and private sector bond markets for selected countries at the end of 1998. Table 13.2 shows the size of foreign currency borrowing in debt markets for selected countries.

Net issues Amount outstanding (1997)																		
	In \$billion	% of GDP	In billions of US dollars				1997				In per cent of total for all countries				1997			
Public sector			1994	1995	1996	1997	Q1	Q2	Q3	Q4	1994	1995	1996	1997	Q1	Q2	Q3	Q4
France	647.4	46.4	69.3	74.4	61.0	45.6	14.2	31.7	2.8	-3.1	5.4	6.7	5.5	6.0	5.2	15.5	2.8	-1.8
Germany	777.5	37.3	89.9	13.1	40.8	38.3	15.6	17.0	-0.3	9.7	7.0	1.2	3.7	5.1	5.7	8.3	-2.9	5.0
Italy	1,123.40	98.1	145.9	61.1	69.9	12.2	13.9	1.6	6.2	-9.6	11.4	5.5	6.2	1.6	5.1	0.8	6.1	-5.5
Japan	3,116.80	74.3	277.9	309.9	257.8	203.8	99.4	83.2	-5.5	26.6	21.7	27.9	23.2	27.0	36.3	40.7	-5.4	15.3
Netherlands	177.5	49.2	9.4	22.7	13.4	5.0	2.4	4.1	0.4	-1.8	0.8	2.0	1.2	0.7	0.9	2.0	0.4	-1
United Kingdom	465.4	36.1	31.2	61.3	15.2	11.2	5.3	7.0	1.6	-2.7	2.4	5.5	1.4	1.5	1.9	3.4	1.6	-1.5
United States	7,337.1	90.8	389.8	342.6	378.7	264.6	80.5	20.0	52.6	111.5	30.4	30.8	34.0	35.1	29.4	9.8	51.7	63.9
Total	15,993.9		1,281.0	1,112.6	1,112.4	754.4	273.8	204.3	101.8	174.4	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0
Private sector																		
France	465.8	33.4	-23.3	-19.3	-14	-17.3	0.2	-12.2	5.0	-10.3	-5.3	-2.7	-1.7	-2.5	0.1	-7	2.8	4.9
Germany	952.5	45.7	40.1	95.5	79.1	67.5	35.0	9.9	23	-0.3	9.1	13.4	9.6	9.6	25.2	5.7	12.8	-0.1
Italy	348.3	30.4	12.3	21	42.1	-9.9	0.9	-0.3	-6.2	-4.3	2.8	2.9	5.1	-1.4	0.6	-0.2	-3.5	-2
Japan	1,316.9	31.4	11.4	80	116.0	0.2	4.3	44.5	-5.5	54.5	2.6	11.2	14.0	0.0	-3.1	-25.7	-3.1	25.9
Netherlands	50.3	13.9	2.4	5.0	-2.4	-0.1	-0.3	1.5	-1.9	0.6	0.5	0.7	-0.3	0.0	-0.2	0.9	-1.1	0.3
United Kingdom	302.4	23.5	27.5	18	52.8	47.6	10.2	14.8	20.3	2.3	6.3	2.5	6.4	6.9	7.3	8.5	11.3	1.1
United States	5,077.5	62.8	241.5	417.7	461.4	529.7	96.3	154.6	113.8	165.0	55.1	58.6	55.7	75.5	69.4	89.3	63.5	78.5
Total	9,738.3		438.3	712.7	828.0	701.3	138.8	173.1	179.3	210.1	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0

Table 13.1: Selected Industrial Countries: Domestic Debt Securities by Nationality of Issuers.

Sources: Bank for International Settlements, International Banking and Financial Market Developments (various issues); and International Monetary Fund, World Economic Outlook database.

In contrast to the corporate bond market, the majority of government debt is denominated in the domestic currency of the issuer. This reflects the fact that governments have access to the pool of domestic savings and can usually meet their financing requirements without recourse to overseas investors.¹ In recent years it has been recognised as something of an advantage for a significant proportion of national debt to be held by foreign investors, and central authorities have often reformed market structures in attempt to facilitate this. There is wide variation in the level of sophistication across markets. It is common for government markets to be made up of essentially

¹ There are notable exceptions to this. The government of Italy historically has had a significant level of foreign currency debt, issuing bonds in overseas markets, as has Canada. Countries such as Sweden and Austria regularly issue debt in the Euro-bond market. Certain regional governments (local authority) such as Quebec and public utilities also issue international bonds.

conventional plain vanilla instruments, whilst usually being the most liquid and transparent. More sophisticated instruments are usually observed in corporate bond markets.

Net issues Amount outstanding (1997)			In billions of US dollars				1997				In per cent of total for all countries				1997			
	In \$billion	% of GDP	1994	1995	1996	1997	Q1	Q2	Q3	Q4	1994	1995	1996	1997	Q1	Q2	Q3	Q4
Public sector	750.6		100.0	78.0	96.4	71.4	23.8	29.7	26	-8.1	100.0	100.0	100	100	100.0	100.0	100.0	100.0
France	18.9	1.4	2.0	1.0	7.3	5.6	1.2	0.9	0	3.4	2	1.3	7.6	7.8	5.0	3.0	0.0	-42
Germany	8.6	0.4	3.8	1.0	-2.6	1.9	0.3	0.3	2.9	-1.6	3.8	1.3	-2.7	2.7	1.3	1.0	11.2	19.8
Italy	54.8	4.8	9.6	8.1	5.0	3.7	0.8	1.4	1.9	-0.4	9.6	10.4	5.2	5.2	3.4	4.7	7.3	4.9
Japan	23.9	0.6	1.3	1.6	2.0	1.9	0.3	0.7	0.7	0.1	1.3	2.1	2.1	2.7	1.3	2.4	2.7	-1.2
Netherlands	0.5	0.1	0.2	0.0	0.0	0.1	0.0	0.0	0.0	0.0	0.2	0	0	0.1	0.0	0.0	0.0	0.0
United Kingdom	12.8	1.0	-0.5	-0.5	0.0	-3.2	-0.1	0.0	0.0	-3.1	-0.5	-0.6	0	4.5	-0.4	0.0	0.0	38.3
United States	49.4	0.6	3.0	12.1	15.8	15.2	4.1	7.1	4.6	-0.6	3	15.5	16.4	21.3	17.2	23.9	17.7	7.4
Private sector	2,475.20		175.5	218.3	421.9	494.1	120.7	125	149.4	98.9	100	100	100	100	100.0	100.0	100.0	100.0
France	201.1	14.4	18.5	9.9	12.9	19	5.6	0.3	7.9	5.1	10.5	4.5	3.1	3.8	4.6	0.2	5.3	5.2
Germany	383.6	18.4	54.1	71.5	95.3	85.4	31.8	24.5	14.5	14.6	30.8	32.8	22.6	17.3	26.3	19.6	9.7	14.8
Italy	42.7	3.7	2.2	-0.9	0.5	6.1	0.2	2.5	0.6	2.8	1.3	-0.4	0.1	1.2	0.2	2.0	0.4	2.8
Japan	295.8	7.1	-1.1	6.6	14.2	-1.1	2.6	-2.2	7.6	-9.1	-0.6	3.0	3.4	-0.2	2.2	-1.8	5.1	-9.2
Netherlands	139.9	38.8	22.0	19.8	24.6	34.2	8.2	9.2	10.1	6.7	12.5	9.1	5.8	6.9	6.8	7.4	6.8	6.8
United Kingdom	294.3	22.9	17.3	14.4	38.5	47.7	11.4	7.2	13.4	15.6	9.9	6.6	9.1	9.7	9.4	5.8	9.0	15.8
United States	506.0	6.3	23.6	47.2	116.1	162.9	29.2	38.8	50.3	44.7	13.4	21.6	27.5	33	24.2	31	33.7	45.2

Table 13.2: Selected Industrial Countries: International Debt Securities by Nationality of Issuers.

Sources: Bank for International Settlements, International Banking and Financial Market Developments (various issues); and International Monetary Fund, World Economic Outlook database.

	Japan	Italy	Netherlands	United States	Germany
1983	6.0			35.3	0.0
1984	2.5			29.8	0.0
1985	6.4	11.2		26.1	0.0
1986	3.2	9.0	-4.3	16.1	0.0
1987	2.5	4.0	-2.9	18.0	0.0
1988	1.7	8.5	4.1	13.0	0.0
1989	2.2	7.7	-7.2	31.1	-0.2
1990	5.5	5.4	6.9	51.5	0.0
1991	6.1	-0.1	9.1	28.8	0.0
1992	4.4	1.2	1.8	23.6	0.0
1993	14.2	0.2	3.4	48.6	0.0
1994	10.4	1.0	-2.5	33.5	0.0
1995	4.6	0.2	-0.1	18.2	0.1
1996	7.1	-6.5	12.0	10.6	0.0
1997	11.5	-2.9		27.7	0.0

For Germany, does not include international issues of bonds. For Italy and the Netherlands does not include commercial paper.

Table 13.3: Debt Securities Financing by Non-Financial Firms in Selected Industrial Countries (As a percentage of total funds raised in financial markets). Sources: Organization for Economic Cooperation and Development (OECD), Financial Statistics; Non-Financial Enterprises Financial Statements (Part III), and Deutsche Bundesbank, KapitalMarkt Statistik.

Tables 13.1 and 13.2 highlight some interesting features. Note that the size of the domestic debt market increases with the size of the country's economy, and that the US corporate debt market is significantly larger than other corporate markets, in both absolute and relative terms. The US corporate market is considerably larger than the combined markets in the major European countries and Japan. The fact that US companies rely on international markets to a lesser extent than European borrowers also reflects the size of the domestic market in that country.

There is no doubt that the corporate bond market in the US is highly developed, and this is perhaps best illustrated by the level of new issues from non-financial sector firms. Outside the US and the United Kingdom there is less of a tradition for non-financial institutions to raise funds in the bond markets. This is illustrated in Table 13.3.

From 1983 an average of 28.5% of annual bond issuance in the US markets has been by non-financial firms. Contrast this with the equivalent level in Japan, at approximately 6%, while the figure in Germany has been negligible. This is further illustrated in Table 13.4. Historically the US corporate market has been an important source of corporate finance, while debt markets outside the US are still capable of further development. The introduction of the euro in eleven countries of the European Union should facilitate this, as yield spreads have practically disappeared in EU government markets. This should translate into a demand for higher yielding bonds, which should attract non-financial institutions in Europe.

	United States	Germany	Japan
1985	134.6	0.4	27.6
1991	121.0	0.3	28.5
1992	117.9	0.3	32.2
1993	119.0	0.3	34.7
1994	107.0	0.2	33.9
1995	97.7	0.2	37.1
1996	89.4	0.2	40.1
1997	81.5	0.2	41.3

Notes: Non-financial sector debt securities includes corporate bonds and commercial paper outstanding, and financial sector debt securities includes bonds and short-term paper. Ratio of outstanding amounts as a percentage.

Table 13.4: Debt Securities of Non-Financial Corporate Sector Relative to Financial Sector: United States, Germany, and Japan. Source: Federal Reserve Bulletin, various issues; Deutsche Bundesbank Monthly Report, various issues; and Bank of Japan, Economic Statistics Monthly, various issues.

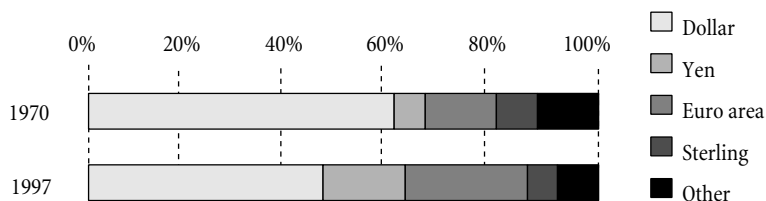


Figure 13.1: World bond market by currency. Source: World Bank, BIS.

13.1 Overview of government markets

13.1.1 The primary market in government bonds

The size of government budget deficits has resulted in a steady increase in the size of public sector tradeable debt over the last twenty years or so. During this time we have witnessed the emergence of a global, integrated capital market which has resulted in a greater proportion of government bonds being held by foreign investors. The global investor base has increasingly been targeted by borrowing authorities, who see this as means of lowering borrowing

costs. For instance in 1998 foreign investors held just over 40% of US government debt. This is a significant increase from the level under fifteen years previously. This is by far the highest level among developed countries, although it has been increasing across all developed countries. The US figure also reflects that the US dollar is a reserve currency. In recent years central authorities have implemented reforms to their market structure to help make their bonds more attractive to overseas investors. Reforms have typically been such things as paying of gross coupons, allowing international book-entry settlement (usually via institutions such as Euroclear and Clearstream), instituting pre-announced auction calendars and greater transparency and liquidity in the secondary market.

One of the main reforms in government markets has been the introduction of *auctions* as the main method by which bonds are issued. Auctions are used in many developed and emerging government markets to issue debt, while corporate debt markets normally employ an underwriting *syndicate* to issue and place debt. The general understanding is that the auction method maximises revenue for the government. To facilitate greater transparency the issuing authorities release an *auction calendar* at the start of the fiscal year, which lists the government's proposed borrowing level and the approximate dates on which funds will be released. An auction calendar lowers the level of uncertainty for institutional investors, who are then in a better position to structure the maturity of their portfolios in line with the issuing calendar. The removal of uncertainty for market participants contributes to the success of a government bond auction.

Another innovation in government markets has been the concentration on *benchmark* issues. A specific large-issue stock set as the benchmark issue will retain liquidity in the secondary market. If the market has a whole is comprised essentially of large volume benchmark bonds it will be more liquid, and this helps to make it more transparent. The demand among institutional investors for benchmark issues may be observed in the yield curve; benchmark issues tend to trade around 10–20 basis points below the curve. For example Figure 13.2 and the table alongside show how the five-year, 10-year and 30-year benchmark gilts traded below other yields.

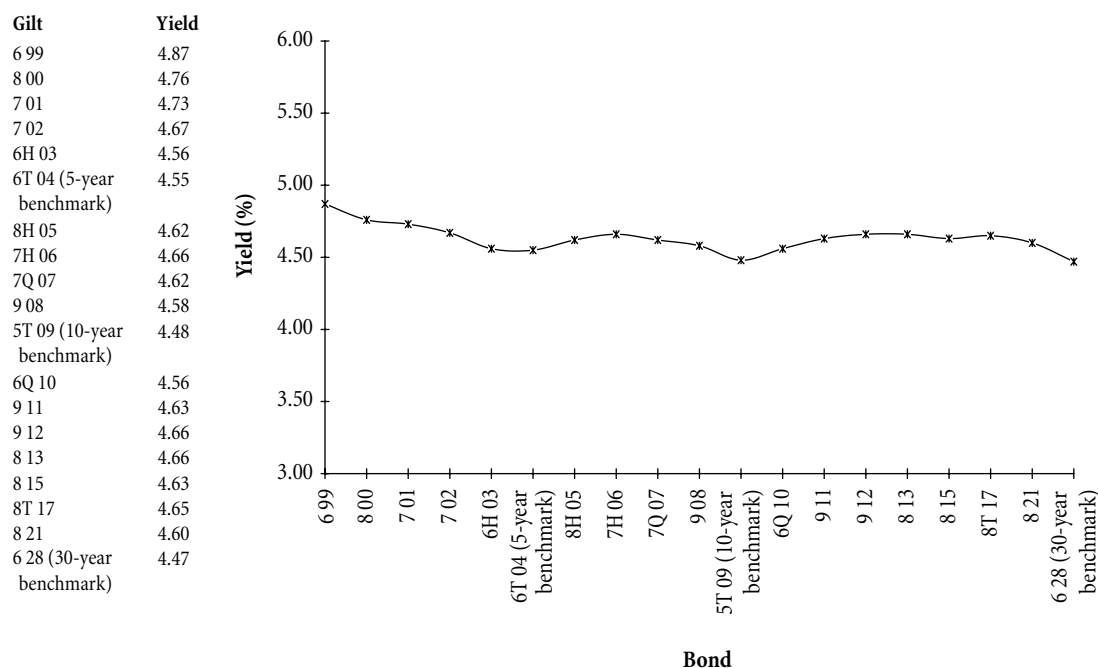


Figure 13.2: Gilt yield curve 23 June 1999, showing “expensive” benchmark bonds.

Investors prefer to hold benchmark bonds because the higher volumes in issue tend to make them liquid and have smaller bid-offer price spreads. The government also gains as the yield on benchmark issues is lower than other issues. The size of individual issues is important for smaller government bond markets, where there may not be enough investor demand to spread across a wide range of issues. This is a topical issue for emerging market economies.

Benchmark bonds can be increased in size by auctioning further tranches at later dates, rather than issuing a new bond.

Another trend in government markets across the world has been an increase in their “user-friendly” level. Authorities frequently consult with market participants including market makers on issues such as what part of the yield curve to tap for further borrowing and the timing of auctions. New instruments and structures are often introduced as a result of such consultation, such as zero-coupon bonds (*strips*) and a repurchase facility. Different classes of investors have an interest in different parts of the yield curve, for example banks concentrate on the short-end of the curve while most fund managers will have more interest in the long-end. The issuing authorities usually attempt to meet the needs of all classes of investor. A market structure that minimises transaction costs will also be relatively more attractive to foreign investors, so market participants recommend that taxes such as withholding taxes and stamp duty be removed. This has indeed been observed in developed markets. If taxes are retained it is usually in the government’s interest to institute a system that enables them to be repaid to non-resident investors as soon as possible.

13.1.2 The secondary market in government bonds

The typical structure in government markets is for a group of designated banks and securities houses, to be registered with the central authority, as the market makers or *primary dealers* in government bonds. These firms are required to maintain liquidity by making continuous two-way prices in all government bonds at all times that the market is open. They are usually also required to provide market intelligence to the issuing authority. In return for performing their market making function, primary dealers often are granted certain privileges, which usually include the right to participate at auctions, access to anonymous live-price dealing screens (usually as part of an *inter-dealer broker service*) and bond borrowing facilities at the issuing authority.

Although developed government markets frequently contain bonds of up to 30 years’ maturity, in emerging economies a market in short-dated securities develops before one in longer-dated bonds. Hence a *money market* in government Treasury-bills often develops first.

A common mechanism in bond and money markets is a repurchase or *repo* market. Repo is a secured borrowing instrument, and a repo market in government bonds helps participants to fund their bond holdings by borrowing funds against security in the form of the bond holdings themselves. An *open* market in repo enables any market participant to fund a long position or borrow bonds to deliver into a short position. Where no repo market exists, firms must obtain funding in the unsecured money markets. The existence of an open market in repo facilitates liquidity in the main *cash* market.

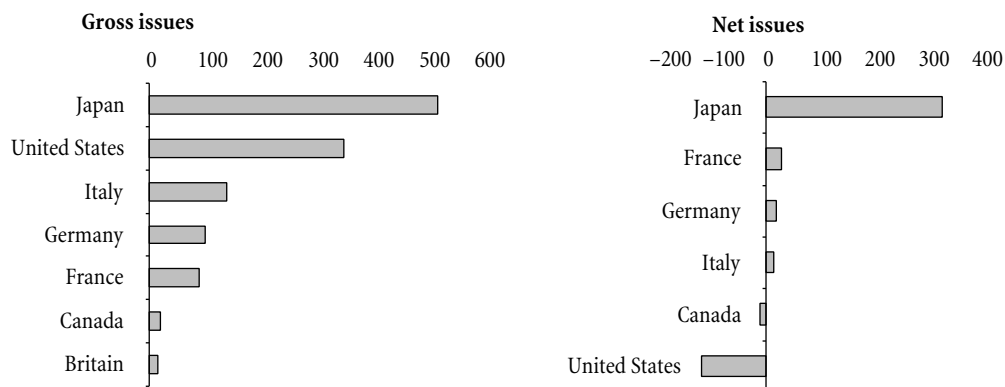


Figure 13.3: Selected government bond issuance in 1999, \$billion. Source: Strata Consulting.

In the descriptions that follow the emphasis is essentially on the government bond market. Following the adoption of a single currency, the *euro* in January 1999 by eleven countries of the European Union, the markets in these countries now trade as a unified market, with identical trading conventions.² Bonds in some of the European

² The countries in the euro area as of May 2000 are France, Germany, the Netherlands, Belgium, Luxembourg, Italy, Spain, Portugal, Finland, Ireland and Austria.

markets described below now trade as euro currency bonds; however the size of the respective domestic markets is given prior to the introduction of the euro. We also describe for comparison three developing bond markets; issues concerned with investing in such economies are covered in greater detail in the section on emerging markets.

Appendix 13.4 lists the structures and conventions in a wider range of selected bond markets around the world, with data correct as at the end of 1998. For euro currency countries the data is correct as at June 1999. Appendix 13.5 lists the sovereign credit ratings for selected countries, as assigned by the Standard & Poor's rating agency.

13.2 Germany

13.2.1 Introduction

Although Germany no longer offers a *deutschmark* as the anchor currency for Europe, the German bond market is the continent's safe haven for global investors. This reflects the size and strength of the German economy and its superior performance in maintaining relatively stable inflation levels in the post-war period. Recently however with the introduction of the euro, the structures and institutions have been changing in Germany. For example although it built up a solid reputation in its handling of monetary policy and for keeping inflation at low levels, the Bundesbank lost much of its status with the introduction of the euro currency in 1999. Monetary policy is now the responsibility of the European Central Bank (ECB), which is based in Frankfurt. As such there is no longer the close scrutiny of the Bundesbank's repo rates and its open market operations that there once was. The Bundesbank targeted inflation mainly through controlling money supply, whereas the ECB does not have an explicit money supply target for the euro area.

The steady increase in the level of the public debt through the 1990s, initially triggered by unification with East Germany, led to relatively large issues in the bond market. At the beginning of 1996 the public sector debt in Germany was approximately DEM 2015 billion, which was just under 58% of GDP.³

13.2.2 Market structure

The rise in budget deficits through the 1990s resulted in Germany becoming a net importer of capital. Much of this was achieved through bond market sales. The foreign holding of government bonds was DEM 478 billion at the beginning of 1996, which was over 25% of total debt outstanding. The need to attract foreign investors contributed to reform of the bond market. Previously the market had been geared towards domestic investors, for example the privately placed notes issue programme known as *Schuldscheine*. Following reunification, the central authorities introduced market reforms including the issue of liquid benchmark bonds. For example from 1989 to 1996 the value of publicly placed federal government bonds rose to DEM 865 billion from DEM 375 billion, while the amount of *Schuldscheine* outstanding remained relatively static at around DEM 100 billion. In the beginning of 1996 the market size stood at the following levels, as reported by the Bundesbank:

Bunds	DEM 369.5 billion
Bobls	DEM 186 billion
Schätze	DEM 71.85 billion
Federal savings bonds	DEM 59.4 billion
Treasury discount paper	DEM 17 billion

As well as federal government bonds, the German *Länder* and municipalities also issue publicly placed bonds. These are more popular with domestic investors however, as overseas investors prefer to deal in federal government bonds that are linked to futures contracts traded on London's LIFFE and the Deutsch Termminbörse in Frankfurt (DTB). The DTB merged with futures exchanges in Austria and Switzerland in 1999 to form Eurex, the European futures exchange.

The two main types of German federal government debt are bunds and Bobls. Bonds pay annual coupon and are book-entry securities listed on German stock exchanges. The following section reviews the main features.

³ The mark was fixed to the euro at a level of 1.96 on 1 January 1999. See Appendix 13.2 for all euro currency fixed exchange rates. The source of the data above is the OECD.

13.2.3 Market volume

German government bonds have been issued by the federal government as well as public sector bodies such as the federal railway, the post office and so on. The bund market is a large and liquid market and essentially the benchmark yield indicator for European government bonds. The most common maturity for new bonds is 10 years. As at February 1998 there was approximately DEM673 million nominal outstanding, and bunds accounted for around 58% of this total.

The typical size of a new issue is between DEM10–15 billion. Bonds of between 8 and 30 years' maturity are normally issued. Seven days after issue the bonds are listed and trade on all eight of the domestic stock exchanges. The Frankfurt bourse is the largest and most important exchange. Issues are generally fixed coupon, bullet maturities. Trading is on a clean price basis. Banks and brokers quote prices on screens and these usually move in increments of DEM0.01. The typical transaction size is around DEM25 million, with price quotes of around DEM0.05 for liquid issues and DEM0.08–0.10 for less liquid issues.

New issues of stock take place through a closed shop of banks and financial institutions, including foreign banks, known as the Federal Bond Syndicate (Konsortium). Around 100 firms make up this syndicate. Until 1990 the procedure for new issues was for all members of the consortium to be allocated a fixed percentage of the total issue size irrespective of the issue terms. From 1990 a new procedure was introduced and this involves splitting the issue into two tranches which combines the traditional method together with a competitive auction. The first tranche has fixed terms, including issue price, and is allocated among syndicate members. The second tranche is auctioned with bids made via consortium banks in DEM0.01 increments for bonds with the same conditions, except issue price, as the first tranche. Bids can be made until the morning after the launch date, and the Bundesbank allocates the stock within two hours after the auction close. In the secondary market bunds settle via the domestic Kassenverein system, although for investors who require custody outside of Germany can clear trades through Euroclear and Cedel. Domestic settlement takes place two business days after trade date, although international settlement follows normal Eurobond practice and clears on T+3.

Interest on bunds is paid on an annual basis, and accrues from the previous coupon date (inclusive) to the settlement date (exclusive). The value date is always the same as the settlement date. There has been no provision for ex-dividend trading since 1994. Prior to the introduction of the euro at the start of 1999, bunds accrued interest on a 30/360 year day count basis. However in line with all euroland bonds they now accrue on an actual/actual basis. Yields can be calculated by any one of three methods, the ISMA method, Fangmeyer and Moosmuller. The differences between these methods are based on their assumptions about how to calculate the compound interest in a broken-year period. The ISMA method is used internationally. In the Fangmeyer and Moosmuller methods, simple interest is calculated for partial coupon periods while compound interest is calculated for full periods. The former calculation uses annual compounding while the latter compounds at the same frequency as coupon payments on the bond. So in fact for bunds the two methods produce identical yields.

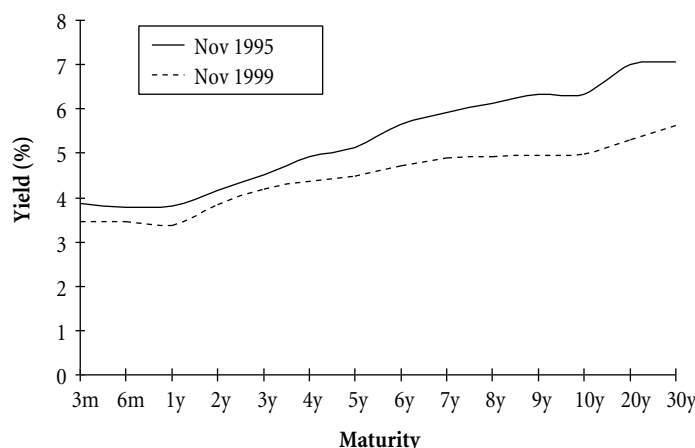


Figure 13.4(i): German government bond yield curves, November 1995 and November 1999. Source: Bloomberg.

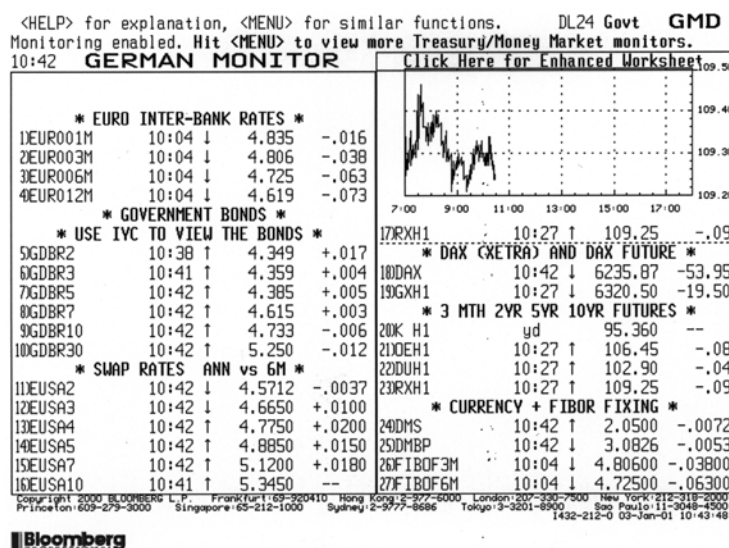


Figure 13.4(ii): German market monitor, screen GMD. Source: Bloomberg L. P.

Reproduced with permission.

Bundesobligationen (Bobl) are five-year federal notes. They were originally intended to promote the formation of financial capital by different social groups in the population. Foreign investors were permitted to purchase the notes after 1988. New issues range in size from DEM500–10,000 million. They are vanilla bonds with fixed coupon and bullet payment on maturity. As with bunds a book-entry system is used and there are no physical bond certificates. Since 1995 the Bundesbank has held a quarterly auction of Bobls, and also discontinued issue of its four-year *Schätze* notes. In all other respects bobls trade in a similar fashion to bunds.

13.3 Italy

The size of the Italian public sector deficit, built up in the post-war period and extending into the 1970s and 1980s partly as a result of the country's large-scale subsidising of nationalised industries and generous social welfare system, has resulted in Italy having the world's third largest government bond market. Part of the economic criteria for countries planning to entering EMU in 1999 was a commitment to stabilising and reducing public debt. The Italian government met the budget deficit criteria for EMU in part by levying a special one-off "EMU tax".⁴ Nevertheless Italian government bonds should continue to play an important part in the global debt markets. Foreign investors held approximately 16% of total Italian government debt at the beginning of 1998.⁵

The authorities have engaged on major reforms of the debt market over the last 15 years; the most notable was the separation of the central bank, the Bank of Italy, from the Treasury. A screen-based secondary market has also been introduced, as has the payment of gross coupons. The reforms were aimed at increasing the attraction of the government debt market for overseas investors. The Treasury also concentrated on a funding programme concentrated more on long-dated fixed-rate bonds, which are the instruments of greatest interest to foreign investors, and less on short-dated securities. The domestic investor base had traditionally been more attracted to short-dated instruments, a reflection of the inflation-prone Italian economy in the 1970s and 1980s. For example in 1981 over 66% of the government borrowing requirement had been met through the issue of short-term bonds known as "BOTs", compared to 8% of the debt raised through the long-dated bonds known as "BTPs". In 1994 the figures were 4% and 77% respectively, a significant turn-around. The longest-dated maturity was a 30-year bond.

⁴ The criteria for countries entering EMU included a budget deficit no larger than 3% of GDP and gross public debt no larger than 60% of GDP. However certain criteria were relaxed, for example the Italian and Belgian gross public debt levels were at over 100% of GDP at the time of entry to EMU.

⁵ Source: Banca d'Italia.

Nevertheless the average maturity of Italian government debt was three-years in 1995, the lowest figure in Europe, although this was a higher figure than in 1982 when the average maturity was just one year.

At the end of 1995 the outstanding government debt stood at L2058 trillion. Budget deficits are financed almost entirely through the public issue of securities. Roughly a quarter of the publicly placed debt is still comprised of *Buoni Ordinari del Tesoro* or BOTs. These are discount securities issued in three-, six-, and 12-month maturity bands at fortnightly auctions. Unlike other European government markets there was virtually no demand for longer-dated paper, *Buoni del Tesoro Poliennali* or BTPs. This reflected the high inflation economy that existed for many years. However these became more popular in the 1990s. BTPs originally paid a semi-annual coupon (although quoted as an annualised yield) but this was altered when markets were harmonised in preparation for EMU in 1999. The ten-year BTP contract on LIFFE was a major market instrument prior to EMU. The other instrument introduced in an attempt to lengthen the average maturity of the debt stock were *Certificati di Credito del Tesoro* or CCTs, which are floating-rate notes first issued in 1977. Although longer-dated paper, they were linked to a moving index so therefore more attractive to domestic investors. Until 1994 CCTs paid a spread over the average yield of a range of BOT auctions; they also paid semi-annual coupons. Since 1995 CCT coupons are indexed to a single auction of the six-month BOT auction, subsequently changed to the 12-month BOT auction.

The newest instrument is the *Certificati del Tesoro a Zero Coupon* or CTZ, which are two-year zero-coupon notes. These have proved popular with domestic retail investors. There is also a secondary market in *Certificati del Tesoro con Opzione* or CTOs, although these have not been issued since 1992 and account for only a small part of overall debt. They are similar to BTPs but contain an option which allows the bondholder to sell them back to the Treasury half-way through their nominal life.

Prior to introduction of the euro the Treasury, in line with other governments in the EU, issued ECU notes. These were known as *Certificati del Tesoro in Ecu* or CTEs. From 1993 the Treasury issued five-year CTEs, with fixed-rate annual coupon.

13.3.1 Market structure

The Italian government bond market is the third largest in the world, with nominal outstanding totalling L1723 trillion (\$996 billion) in January 1998. This debt is made up of the different bonds noted above. Roughly a third of the nominal outstanding are floating rate notes known as *Certificati di Credito del Tesoro* (CCTs); another third is made up of fixed-rate notes called *Buoni del Tesoro Poliennali* (BTPs). Other types of bonds include putable fixed-rate notes called *Certificati del Tesoro con Opzione* (CTOs) and Treasury bills (BOTs). Two-year zero-coupon bonds known as CTZs have been issued since 1995. Government bonds are issued in book-entry form however they can be converted into physical form on request. CTZs are available only in book-entry form.

All bonds trade on a clean price basis. Settlement is three business days after the trade date and clearing is available through Euroclear and Cedel. Government bonds used to pay interest on an annual and semi-annual basis, depending on the security, however all debt now pays an annual coupon. BTPs and CTOs, as well as CCTs with original maturities of seven years or less originally paid coupons semi-annually, while all other types paid annually. Interest traditionally accrued from the previous coupon date (inclusive) to the settlement date (inclusive), so consequently the Italian accrual rules added one extra day of interest to comparable calculations in other markets. With the introduction the single currency however this arrangement has been changed to bring it into line with other euro bond markets. In the same way the day count basis was changed from 30/360 to actual/actual as part of euro harmonisation. Bonds do not trade ex-dividend. Actual/365 is used for discounting.

Italian government bond yields are quoted gross of a withholding tax of 12.5%, introduced in September 1987. There was a gap of six years before a procedure was introduced that enabled foreign investors to claim reimbursement of this tax. The procedure itself was time-consuming and cumbersome. In 1994 a new computerised system was instituted that resulted in foreign investors receiving gross coupons promptly, and reimbursement of tax took 30 days. From June 1996 overseas bondholders received gross coupons, removing the issue of reimbursement completely.

13.3.2 Primary and secondary markets

The Bank of Italy advises the Treasury on funding issues and organises the auctions at which government bonds are sold. All new issue of paper is via an auction system, managed by the central bank. The Treasury announces annually the dates on which auctions will be held, and announces quarterly the bonds and minimum issue sizes that will be offered in the following three months. Auctions are held at the beginning and in the middle of the month. A

new BTP series is normally issued every three months. A “Dutch auction” system is used, which means that bonds are allotted to the highest bidders first, but all successful bidders pay the same price, known as the marginal price, which is that of the lowest accepted bid. The exception is for BOTs, which are US-style, with successful bidders paying the price at which they bid. The Treasury does not set a base price but it does calculate an exclusion price based on the average level of offers, in order to deter highly speculative or non-market bids. Auctions are automated using the national interbank network. Settlement takes place three business days after the auction date for all notes except BTPs and CCTs which settle two days after the auction date.

Secondary market settlement is on a T+3 basis. The smallest denomination prior to EMU was L5 million, which enables the accommodation of private investors. Individuals generally trade on the Milan stock exchange, however the professional market is over-the-counter both in London and also through the official market, the Mercato dei Titoli di Stato (MTS), known as the Telematico. The MTS was introduced by the Treasury in 1988 and has contributed to higher market liquidity. It is a screen-based wholesale market, with a minimum dealing size (pre-EMU) of L5 billion. It is open from 09:00 to 17:00 local time. The average daily turnover in 1998 was over L15.5 trillion, with an average bid-offer spread of 29 basis points.⁶

From 1994 there has been a three-tier dealing system in the professional market. The primary dealers are known as government bond *specialists* who are market-makers in all issues. They must undertake to buy a minimum of 3% of the nominal value of each auction. They are entitled to take up extra paper at the marginal price immediately after successful auctions. A curious arrangement in the Italian market is a second tier of market makers known as *primary dealers*, who have less onerous market-making responsibilities than specialists. The third tier of the structure is made up of ordinary dealers who fulfil a broker-dealer role. Specialists traded more than 60% of cash market volume in 1998. Market supervision is shared between the Treasury, the bank of Italy and Consob, the stock exchange commission.

	L trillion		%	
	Dec 1994	Dec 1995	Dec 1994	Dec 1995
BOT	394	416	27.1	25.5
CCT	508	546	34.9	33.7
BTP	405	517	27.8	31.9
CTO	63	60	4.3	3.7
CTE	50	55	3.4	3.4
Other	36	29	2.4	1.8
Total	1455	1621	100	100

Table 13.5: Stock of Italian government debt in December 1996.

Source: Banca Italia, 1996.

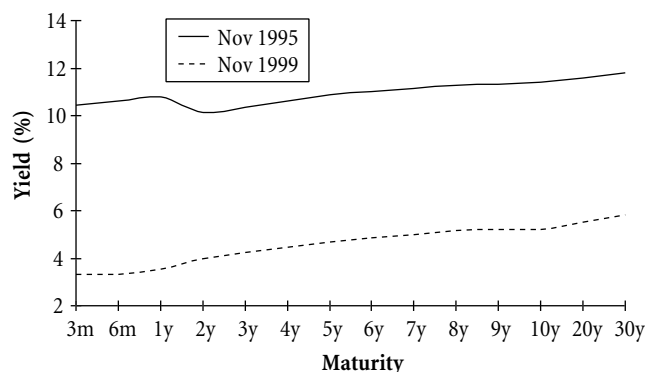


Figure 13.5(i): Italian government bond yield curves November 1995 and November 1999. Source: Bloomberg.

⁶ Source: Reuters.

PXIT
Screen Printed
14:59 ITALY - GOVERNMENT BONDS

DGB Govt PXIT

PAGE 1 / 1

	Time	Bid	Ask	Bid	Ask	Yld	Yld	Chg	Yesterd
BENCHMARKS									
1) BTPS 5 ³ / ₄ 09/02 _{2YR}	14:58	101.64	101.67	4.78	4.76	+0.1			101.68
2) BTPS 4 10/03 _{3YR}	14:52	98.04	98.07	4.81	4.80	+0.3			98.11
3) BTPS 4 07/04 _{4YR}	14:53	97.38	97.41	4.87	4.86	+0.4			97.50
4) BTPS 4 ³ / ₄ 07/05 _{5YR}	14:57	99.58	99.62	4.91	4.90	+0.4			99.75
5) BTPS 7 ³ / ₄ 11/06 _{6YR}	14:58	113.99	114.03	5.02	5.01	+0.4			114.21
6) BTPS 6 11/07 _{7YR}	14:58	105.49	105.53	5.10	5.09	+0.3			105.71
7) BTPS 5 05/08 _{8YR}	14:57	99.43	99.47	5.15	5.14	+0.4			99.67
8) BTPS 4 ¹ / ₄ 11/09 _{9YR}	14:52	93.25	93.29	5.27	5.26	+0.4			93.58
9) BTPS 5 ¹ / ₂ 11/10 _{10YR}	14:58	102.20	102.26	5.27	5.26	+0.5			102.63
10) BTPS 6 05/31 _{30YR}	14:58	102.95	103.01	5.87	5.87	+0.4			103.63

<PAGE FWD> FOR MORE BONDS

White = Benchmark Bonds

EUR 1 .8904 -.0055
MIB3143667 --

Copyright 2000 BLOOMBERG L.P. Frankfurt: 69-220410 Hong Kong: 2-977-6000 London: 202-330-7500 New York: 212-318-2000
Princeton: 609-279-3000 Singapore: 65-212-1000 Sydney: 2-977-8886 Tokyo: 3-3201-6930 Sao Paulo: 11-3048-4550
1432-212-1 19-Sep-00 14:59:22

Bloomberg
PROFESSIONAL

Figure 13.5(ii): Bloomberg Italian benchmark bond price screen, PXIT on 21 December 2000. Source: Bloomberg L. P. Reproduced with permission.

13.4 France

The French government bond market is one of the most liquid markets in the world. The central authorities have pursued a policy of targeting overseas investors, and the level of foreign holdings of French government debt rose from virtually nil in 1986 to over 35% by 1996. This was achieved in part by following the US model of concentrating on large, liquid conventional bonds, and a regular pre-announced timetable of auctions of medium- and long-dated bonds. It also introduced zero-coupon bonds in 1991, before either the UK or Germany. The French market is a transparent and efficient one and is an important sector of the euro-denominated bond market, although benchmark status in euro-government paper tends to be allocated to German government bonds.

The Treasury has simple issuance and settlement procedures which ensures high transparency and liquidity. The total nominal outstanding was FF2130 billion in December 1997. Of this about 63% was made up of long-term debt, and the average maturity of total debt outstanding was 6½ years.

13.4.1 The primary market

The government's Ministry of the Economy is the central authority in government debt. The Ministry has publicly stated its commitment to maintaining an orderly and transparent market that is keen to attract foreign investors. The market is made up three different instruments, which are *Obligation Assimilable du Trésor* (OATs), *Bons du Trésor à Taux Fixe et à Intérêt Annuel* (BTANs) and *Bons du Trésor à Taux Fixe et à Intérêt précompté* (BTFs). OATs are the standard means of financing the central budget deficit; they are bonds with maturities of up to 30-years. BTANs are fixed-rate bonds of maturities of between two and five years, while BTFs are short-dated Treasury bills with a maximum maturity of one year. Like bills in other markets they are discount instruments. The Treasury issues one 13-week BTF each week and alternately a six-month BTF and a one-year BTF.

Market liquidity is enhanced by a policy of issuing fungible debt, so that outstanding issues are frequently increased in size through regular issues. Debt management is carried out through the *Fonds de Soutien des Rentes* or FSR, which is the government debt management fund.

Government auctions take place twice a month, with long-dated OATs auctioned on the first Thursday of each month and medium-term BTANs auctioned on the third Thursday. There is a weekly auction of BTFs every Monday, and every two months there is an additional issue of BTANs on the second Wednesday of the month. Details such as the security and amount of the auction are released two business days before the auction date. At the auction sealed bids may be handed in directly to the Bank of France, the central bank, although most market participants use a

computerised remote bidding system known as Telsat. All bonds are listed on the Paris Stock Exchange, but trading also takes place in an active OTC market that is managed by primary dealers known as *Spécialistes en Valeurs du Trésor* or SVTs. The primary dealers are responsible for ensuring Treasury auctions are a success and are required to make continuous prices in the secondary market. They account for almost 90% of the securities sold at each auction. The auction system is on the US model, with highest bids being served first. Lower bids are served in quantities decided at the discretion of the Treasury. Auctions on the first Thursday of each month are for payment on the 25th or following business day. Bonds will trade in a grey market until this date and SVTs quote two-way prices on a when-issued basis up to one week before the auction date.

OATs are issued as existing tranches of existing bonds, so that the current size of some OATs is as large as FFr 100 billion. The Treasury announces an issuance calendar at the start of each year. Typically the 10-year bond as well as the long bond (30-year) are re-opened every month. Bonds are issued in book-entry form so physical paper is unavailable. Most OATs are fixed coupon bonds with bullet maturities, paying interest on an annual basis. There are some esoteric older issues in existence however, with special features. For example the variable rate January 2001 issue is based on the average yield of long-term government bonds over the twelve-month period preceding the coupon payment.

BTANs are fixed interest Treasury notes with maturities of up to five years. They are the main instrument the Treasury uses for short-term government financing, together with bills (BTFs). There is little distinction in the secondary market however, where BTANs trade like medium-dated OATs. The paper is issued in book-entry form without certificates. The Treasury uses a fungible issuance procedure for BTANs; two-year and five-year notes are issued every month. Like OATs the paper trades in the OTC market, which is managed by the SVTs. The key difference between the two types of bonds is in the basis of trading: BTANs trade on a yield basis, with annualised yields quoted to three decimal places. The bid-offer spread is around 2 basis points on average. In most other respects the bonds have similar terms and arrangements as OATs.

13.4.2 The secondary market

OATs are tradeable on the Paris stock exchange but the majority of market dealing is on an OTC basis. There is a screen-based price system for OATs, with the bid-offer spread generally around 5–15 centimes for liquid issues. Prices are quoted net of tax and costs. OATs trade on a clean price basis. Coupon is paid gross to non-residents.

Clearing takes place three days after trade date for domestic and international settlement. Bonds are cleared internationally through Euroclear and Cedel or domestically through the Paris stock exchange clearing system, known as SICOVAM. Settlement takes place on T+1 or T+3 for bonds traded domestically and on T+3 for international trades. Transactions between primary market dealers take place on a delivery versus payment basis.

French government bond futures are traded on the *Marché à Terme International de France*, or MATIF which was founded in 1986 and is one of the largest futures and options exchanges in the euro area. Prior to EMU its key contracts were the futures and options on a notional seven-to-10-year French government bond known as the “*notionnel*”. It now trades futures and options on a notional 10-year euro denominated bond.

BTANs and BTFs are only tradeable on an OTC basis. The SVTs announce bid and offer prices together with the volume available for trading at those prices. BTANs are quoted on a yield basis and the rate of return is expressed as an annualised rate over 365 days, while BTFs are quoted on a money market straight-line yield expressed as an annual percentage over 360 days. BTANs and BTFs are delivered and settled through the “*Saturne*” system run by the Bank of France. The market is very liquid and transparent; BTANs are among the most liquid short term securities in Europe. There is a large and liquid repurchase (repo) market in government securities; unusually, repo market makers are registered at the Bank of France.

French government bonds have a slightly esoteric arrangement for the calculation of accrued interest, which depends on the type of settlement. Bonds settled T+3 in the domestic market via SICOVAM accrue from the previous coupon date (inclusive) to the trade date (exclusive). All other domestic settlement and international settlement results in accrual from the previous coupon date (inclusive) to the settlement date (exclusive). There is no facility for ex-dividend trading in OATs. The daycount basis has always been actual/actual in the French market, which mean that no adjustment was necessary after the introduction of the euro.

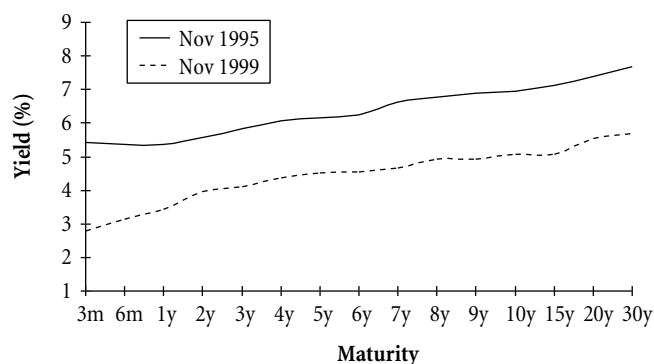


Figure 13.6(i): French government bond yield curves November 1995 and November 1999. Source: Bloomberg.

PXFR
Screen Printed
14:58 FRANCE - GOVERNMENT BONDS
DGB Govt PXFR
PAGE 1 / 1

	Time	Bid	Price	Ask	Bid	Yield	Ask	Yld	Ysterday's	Close
BENCHMARKS										
1) BTNS 4 1/2 07/02 _{2y}	14:54	99.650	99.710	4.721	4.680	+0.32	99.700			
2) BTNS 4 1/2 07/03 _{3y}	14:57	99.580	99.640	4.667	4.642	+0.27	99.640			
3) BTNS 3 1/2 07/04 _{4y}	14:52	96.190	96.250	4.681	4.662	+0.38	96.310			
4) BTNS 5 01/06 _{6y}	14:57	101.390	101.450	4.684	4.670	+0.38	101.560			
5) FRTR 6 1/2 10/06 _{6y}	14:57	108.890	108.950	4.719	4.708	+0.30	109.050			
6) FRTR 5 1/2 10/07 _{7y}	14:57	103.830	103.890	4.826	4.816	+0.38	104.050			
7) FRTR 8 1/2 10/08 _{8y}	14:57	122.860	122.920	4.913	4.904	+0.35	123.120			
8) FRTR 4 10/09 _{9y}	14:57	92.990	93.050	4.998	4.989	+0.44	93.290			
9) FRTR 5 1/2 10/10 _{10y}	14:57	103.680	103.740	5.015	5.007	+0.51	104.070			
10) FRTR 8 1/2 10/19 _{20y}	14:57	135.910	136.040	5.411	5.402	+0.56	136.720			
11) FRTR 5 1/2 04/29 _{30y}	14:57	99.850	99.980	5.508	5.499	+0.62	100.740			

White = Benchmark Bonds <Page Fwd> for more bonds

MNA 89.20 -.29 FRF 7.3678 +.0403 CAC 15952.35+64.86
FJA 95.186 -- EUR 1.8901 -.0058 MIA 2461 --

Copyright 2000 Bloomberg L.P. Frankfurt 1-69-265410 Hong Kong 852-2-277-6000 London 207-235-7900 New York 212-318-3000
Princeton 609-279-3000 Singapore 65-212-1000 Sydney 2-977-8686 Tokyo 3-3201-6900 Sao Paulo 11-3048-4500
1432-212-1 19-Dec-00 14:58:04

Bloomberg
PROFESSIONAL

Figure 13.6(ii): PX screen on Bloomberg, French government benchmarks. Source: Bloomberg L. P. Reproduced with permission.

13.4.3 OAT strips

After the US, the French government market was one of the first to introduce zero-coupon bonds or “strips”. Subsequently both Belgium and the Netherlands introduced strips. In France the Treasury has allowed primary dealers to strip long-dated OATs since June 1991. The range of bonds which may be stripped has been more restricted than in the US market but all but two of the 10-year and longer-dated OATs are now strippable. Essentially all OATs maturing on 25 April and 25 October may be stripped. The first bond to be designated was the 30-year bond, the 8.5% 2019. The mechanics of the stripping process are:

- all SVTs are allowed to strip and reconstitute bonds eligible for stripping. The minimum amount to be stripped is Ffr 20 million (pre-euro);
 - stripping and reconstitution is carried out via SICOVAM, and is free of charge.
- As at November 1999, 22 bonds could be stripped. These are listed in Table 13.6.

The market has been popular with institutional and retail investors. Table 13.7 shows stripping volumes for the five stocks that were strippable in June 1996.

Bond	Maturity date	Bond	Maturity date
9.5 % 00	25 Apr 2000	5.5% 07	25 Oct 2007
6.75% 02	25 Apr 2002	5.25% 08	25 Apr 2008
8.5% 03	25 Apr 2003	8.5% 08	25 Oct 2008
6.75% 03	25 Oct 2003	4% 09	25 Apr 2009
5.5% 04	25 Apr 2004	4% 09	25 Oct 2009
6.75% 04	25 Oct 2004	6.5% 11	25 Apr 2011
7.5% 05	25 Apr 2005	8.5% 19	25 Oct 2019
7.75% 05	25 Apr 2005	8.25% 22	25 Apr 1922
7.25% 06	25 Apr 2006	8.5% 23	25 Apr 1923
6.5% 06	25 Oct 2006	6% 25	25/10/1925
5.5% 07	25 Apr 2007	5.5% 29	25/04/1929

Table 13.6: OAT strippable issues.

Bond	Strippable from	Nominal stripped FFr bn	Percentage stripped
6.75% Oct 2003	Jun 1993	0.4	0.5
8.5% Apr 2003	Sep 1992	4.4	4.6
8.5% Oct 2008	Jun 1992	6.2	15.5
8.5% Oct 2019	Jun 1991	40.8	66.7
8.5% Apr 2023	May 1992	30.8	43.1

Table 13.7: Stripping volumes June 1997. Source: HSBC Markets.

Strippable bonds have one of two coupon dates, the 25 April or 25 October, so only coupon strips from bonds with the same maturity month are fungible. The minimum denomination of principal strips is €500 which is the same as the minimum denomination for OATs. Over 60% of the 2019 and 2023 bonds have been stripped. The French strips market is the most liquid in the world after the US market, and the bid-offer quote for trades in the professional market is around 2 basis points. Table 13.8 lists the principal strips trading in the market in November 1999, and at Figure 13.7 we show the principal strip yield curve compared to the coupon bond yield curve, also as at November 1999.

Source bond	Maturity	Yield (%)	Source bond	Maturity	Yield (%)
9.5 % 00	25 Apr 2000	3.45	5.5% 07	25 Oct 2007	5.02
6.75% 02	25 Apr 2002	4.13	5.25% 08	25 Apr 2008	5.12
8.5% 03	25 Apr 2003	4.44	8.5% 08	25 Oct 2008	5.16
6.75% 03	25 Oct 2003	4.47	4% 09	25 Apr 2009	5.16
5.5% 04	25 Apr 2004	4.51	4% 09	25 Oct 2009	5.17
6.75% 04	25 Oct 2004	4.6	6.5% 11	25 Apr 2011	
7.5% 05	25 Apr 2005	4.68	8.5% 19	25 Oct 2019	4.35
7.75% 05	25 Oct 2005	4.67	8.25% 22	25 Apr 1922	
7.25% 06	25 Apr 2006	4.7	8.5% 23	25 Apr 1923	
6.5% 06	25 Oct 2006	4.81	6% 25	25 Oct 2025	6.06
5.5% 07	25 Apr 2007	4.93	5.5% 29	25 Apr 2029	5.97

Table 13.8: OAT principal strips November 1999. Source: Bloomberg.

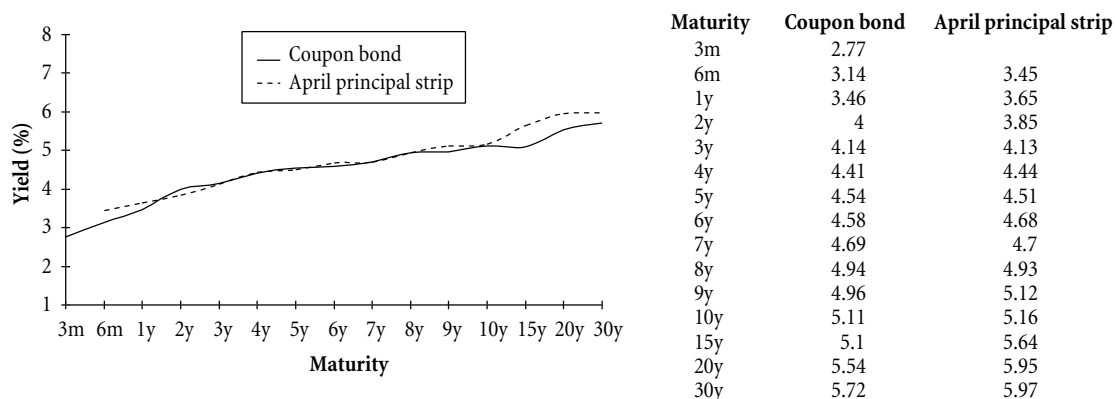


Figure 13.7: OAT coupon bond and principal strips yield curve November 1999. Source: Bloomberg.

Trading OAT Strips

OAT strips trade in a liquid secondary market on a yield basis. The price/yield relationship is given by (13.1):

$$P = \frac{100}{(1 + r)^{\text{years to maturity}}}, \quad (13.1)$$

$$r = \left(\left(\frac{100}{P} \right)^{1/\text{years to maturity}} - 1 \right) \cdot 100.$$

Stripping of OATs is carried out by delivering a government bond to the central bank, in return for which a complement of strips representing the issue's total cash flows is received. This is similar to the operation in the US and UK markets. The reverse operation is carried out when reconstituting a bond. Any coupon strip may be delivered, however principal strips are not fungible. The value of strips derived from a coupon bond is related to the market price of the underlying bond; however the yield on different maturity strips will of course reflect the spot rate for the specific maturity. This was reviewed in Chapter 6. OAT strips exhibit the typical zero-coupon bond market anomalies that was discussed in the chapter on UK gilt strips.

Figure 13.8 shows the spread between the 30-year principal strip and the equivalent coupon OAT during 1995 and 1996.

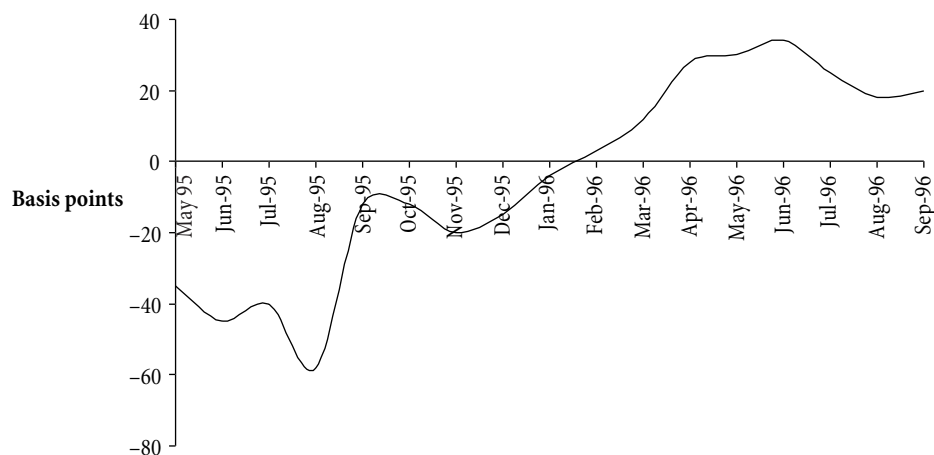


Figure 13.8: OAT versus OAT strip 2023. May 1995–September 1996. Source: HSBC.

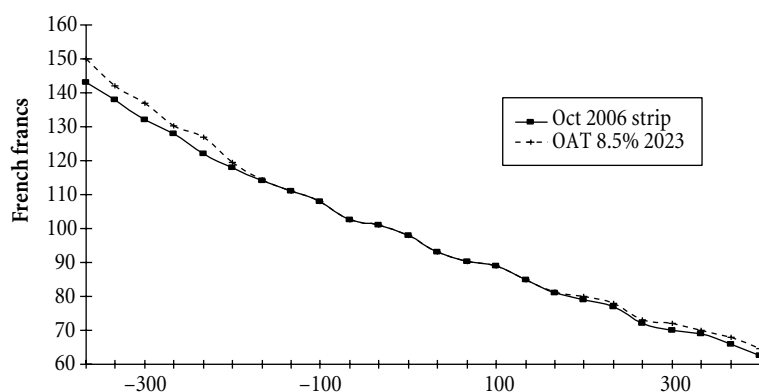


Figure 13.9: Convexity of OAT strip versus coupon OAT.

13.5 Japan

The world's largest net external creditor nation, Japan has a highly advanced and well-diversified economy and the second largest bond market in the world. The market itself has not attracted overseas investor interest to any significant extent however. There was JPY 272 trillion (US\$2.72 trillion) of Japanese Government Bonds (JGBs) outstanding in December 1997.⁷ The Japanese market is the only developed market that has a structure and arrangements that are substantially different to that found in Europe and North America. The Japan Institute for Securities Information and Public Relations has hinted that the securities industry needs to reform itself so that foreign investors view the market in a more positive light; specifically the Institute, whose members are Japanese securities dealers, the country's stock exchanges and the domestic bond underwriters association, suggested that areas such as transparency and secondary market trading could be improved. This reflected the widespread feeling by overseas investors during the 1990s that the Japanese Government Bond (JGB) market was illiquid at any one time, with the exception of the benchmark 10-year issue.

The view that the JGB market is illiquid is not accurate however, and it is more likely that the trading and settlement arrangements are factors that put off overseas investors from becoming bondholders. Bonds that are deliverable into the Tokyo Stock Exchange's JGB futures contract are sufficiently liquid; the delivery is comprised of seven- to 11-year bonds. Outside this however trading is concentrated on benchmark issues, much more so than in Europe or the US, and there is lower liquidity in say, two-, four- and six-year issues. Trading is also light at the very long-end of the curve. At certain times up to 90% of trading can be concentrated in the current 10-year benchmark. These factors contribute to a under-representation of overseas investors compared to the levels in say France and Italy, in what must be considered a major and important bond market.

13.5.1 The primary market

Although the JGB market is probably of more interest to domestic investors than international investors, its size guarantees that it is closely observed and analysed around the world. There are three types of JGBs but there is no difference as regards trading or investing in the bonds. The Bank of Japan breaks down issues into interest-bearing long-dated government bonds, medium-dated bonds, five-year discount government bonds and Treasury bills. Long-dated bonds made up nearly half of the total issue in 1996, nearly Y24 trillion. An unusual arrangement at the short-end of the market is that Treasury bills are split into Treasury financing bills and food financing bills, and foreign exchange funding bills. There is also a local government market and a public utility market; both types of debt are guaranteed by the central government.

Bonds are issued through a combination of a bank syndicate underwriting and a competitive auction. The coupon and size of each new issue are announced on the day of the auction after consultation with the syndicate, and the average auction price determines the price of the syndicated offering. The only departure from this procedure is for 20-year bonds, which are issued wholly via a competitive auction. There is a monthly auction of four-year bonds and 10-year bonds; outside these maturities there is no pre-arranged or pre-announced schedule. An interesting

⁷ Source: JP Morgan.

difference in the market structure is numbering of bonds. All JGBs are numbered, the number being used rather than the coupon and maturity to identify issues. The Ministry of Finance sets the terms of an auction, which detail the terms of the issue based on market intelligence gathered the previous day. In an auction 60% of the issue is allotted via a US style auction, with the remainder placed via an underwriting syndicate of licensed institutions. Individual syndicate members are restricted to bidding for a maximum of 18% of the total issue amount. If a new issue has the same coupon and maturity as an outstanding issue, the new tranche will be fungible with the existing issue from the first coupon date.

Bonds are generally issued in registered form but they may be converted into bearer form (or vice versa) within two days of issue. The bonds are plain vanilla fixed coupon issues with bullet maturities. In theory JGBs can be called early at par, although this rarely happens in practice. Not all bonds are listed; 10-year and 20-year JGBs are traded on the Tokyo Stock Exchange (TSE). Although trading volume on the exchange is low, the TSE closing prices serve an important purpose as a public pricing source for JGBs, and most marking-to-market uses these closes. Other JGBs are not listed. The TSE itself has only a small role in the JGB market, beyond providing these official closes, and most trading occurs in the OTC market both directly and via brokers.

13.5.2 The secondary market

JGBs are traded on a simple yield basis, which takes no account of reinvestment income, although overseas investors are usually quoted the equivalent semi-annual yield instead. The difference is worth noting, for example the simple yield of 2.83% for the benchmark 174th JGB issue in September 1999 was equal to a semi-annual yield of 2.98%. Bonds trade in the secondary market on a yield basis rather than a price basis. Among developed economies only Sweden uses this method for non money-market instruments. Coupons on JGBs are expressed in terms of an annual amount of interest paid per yen of nominal debt, and are paid on a semi-annual basis on specified dates. A coupon is fixed close to market yields at the time of the bond's auction. Coupon on JGBs is paid semi-annually, usually on the 20th of the month. Interest accrued from the previous coupon date (inclusive) to the settlement date (exclusive). The daycount basis is actual/365, although the 366th day in a leap year is counted. There is no ex-dividend trading arrangement. There is a 20% withholding tax payable on coupons, which are paid net. This is halved for overseas investors, and may be reduced to 0% in special circumstances and on application. A transfer tax of 0.03% is payable by the seller of a JGB if the buyer is domiciled in Japan.

The simple yield to maturity is given by (13.2):

$$\text{Yield (\%)} = \frac{(C + (100 - P)/\text{Remaining life})}{P/100} \quad (13.2)$$

where

C is the coupon
 P is the bond price.

Trading in the secondary market is mainly on an OTC basis. Note that there is no market making obligation. Dealing is conducted by investment banks, securities houses and other financial institutions licensed to deal in public bonds. In 1999 over 500 such institutions were so licensed. Inter-dealer broker firms are referred to as *brokers' brokers*. The bid-offer spread is typically as low as half a basis point for benchmark issues and 2–3 basis points for other issues.

Settlement for JGBs varies according to the type of trade. Bargains on the TSE settle four days after trade date. Settlement for OTC trades take place on set days of each month. The dates are specified by the central bank, within a general rule that there must be a minimum of seven business days between trade date and settlement. Another rule states that settlement cannot take place within 14 days of a coupon date, in which case settlement takes place on the coupon date itself. JGBs are not delivered abroad and cannot therefore be cleared through Euroclear and Cedel. Foreign investors must appoint a local custodian to handle settlement, which is not on a payment versus delivery basis, as bonds and cash settle in different accounts.

13.5.3 The Bank of Japan

Among the major central banks in the world, the *Nippon Ginko* or Bank of Japan (BoJ) has a relatively low level of independence. This is because in legal terms the constitution of the country does not explicitly state that the BoJ is responsible for monetary policy. In practice however the Ministry of Finance delegates policy to it. The BoJ was

founded in 1882. The bank's objectives are the regulation of the currency, overseeing the provision of credit and finance, and the supervision of the banking system itself. The governor of the BoJ is appointed by the government and serves for five years; at the end of a five-year term the incumbent may be re-appointed to serve another term.

13.5.4 The derivatives market⁸

JGB futures are traded electronically on the Tokyo Stock Exchange (TSE), a contract is also traded on LIFFE in London, which moved from an open-outcry floor-based trading system to an electronic system in November 1999. The TSE also lists a 20-year government bond future and a five-year contract launched only in 1996. The three-month interest rate contract, called the Euroyen future is traded on the Tokyo International Financial Futures Exchange, which also lists a one-year Euroyen contract.

The TSE's JGB future is based on a notional 10-year bond with a face value of ¥100 million and a notional coupon of 6%. It trades from 09:00 to 11:00 and from 12:30 to 15:30. The contract is priced in hundredths, so one tick movement is 0.01. There is a limit on price movement of two yen either side of the previous day's closing price. The contract is deliverable on the 20th day of the delivery month or the next business day, with contracts trading for expiry in the conventional delivery months of March, June, September and December. The final trading day in any contract is nine business days before the delivery date. All JGBs with maturities of between seven and 11 years are in the delivery basket. For the 20-year bond contract the delivery basket is made up of bonds with maturities of between 15 and 21 years. The conversion factor for the ten-year bond futures contract is given at Appendix 13.3.

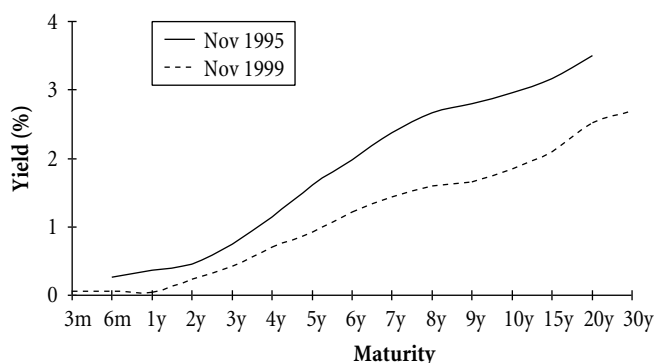


Figure 13.10: Japanese government bond yield curves November 1995 and November 1999. Source: Bloomberg.

13.6 Australia

The Australian government bond market attracts a level of overseas interest that might be considered to be out of proportion to its size. This reflects the traditional view that the market is relatively high yielding for a developed economy, yet offering the protection of a AAA credit rating. The government bond market in Australia is comparatively small. There is an added interest from outsiders in the form of foreign companies wishing to issue corporate bonds in Australian dollars and swap the proceeds into their domestic currency, to take advantage of any cheapness in the A\$ swap curve. The size of the market in Commonwealth Government Bonds (CGBs) was A\$103 billion in January 1998.

13.6.1 Market structure

Of the total outstanding volume of government debt, approximately 75% is in the form of fixed-rate bonds and Treasury Indexed bonds known as TIBs. The remainder is in the form of floating-rate bonds and Treasury Adjustable Rate bonds or TABs. The conventional fixed-rate bonds pay a semi-annual coupon. The authorities have targeted liquidity levels by consolidating the issues into larger size benchmark bonds. There were 20 such benchmarks, known as "hot stocks" in 1999, with the longest maturity conventional paper expiring in 2011.⁹ Index-

⁸ For a detailed review of bond futures contracts please see Chapter 41.

⁹ This is the 5¾% 2011 bond, maturing on June 2011. The next-longest maturity stock is the 7½% 2009. There is an index-linked bond that matures in 2020.

linked maturities go out to 2020. Issue sizes are sufficient to provide liquidity, with sizes ranging from A\$2 billion to A\$6 billion. Foreign holdings were estimated at nearly A\$30 billion at the beginning of 1998, which is a relatively high proportion of total outstanding marketable debt.

The Australian government market is one of only five major markets to offer inflation-indexed bonds.¹⁰ The first such bond was issued in 1985. These bonds, known as TIBs attract the same type of institutional investor as those in other markets, namely long-dated investors such as pension fund managers. In Australia the market is relatively small and generally most TIBs have longer maturities than conventional bonds, limiting the scope for comparison and analysis that one observes in say, the UK gilt market. Generally the authorities do not pre-announce issues of TIBs as supply is tailored to meet demand for specific maturities.

Floating-rate bonds or TABs, which have maturities of three years or five years, were introduced in 1994. The coupon on TABs is linked to the Bank Bill Index. Initially the bonds were issued via an underwriting syndicate group but they are now issued via competitive tender.

Both bonds and short-term paper are issued via tender, both new stock and further tranches of existing issues. Bonds are generally registered securities and are listed on the Australian Stock Exchange. Tenders for bonds are submitted via the Reserve Bank of Australia's electronic dealing system, known as the Information and Transfer System or RITS. The tender is a US-style auction, with successful bidders paying the prices they bid. More paper is therefore allotted to the higher bidders. There is no set timetable for fixed-rate bond issues and in the past bonds have been issued periodically on an "as required" basis through the competitive tender. However from August 1995 the Treasury has held tenders on a roughly regular basis, with tenders taking place every four to six weeks. The details of an issue are announced the day before the auction and settlement takes place two days later. Generally a tender is held on a Tuesday.

In the secondary market the liquidity is generally high. Average daily turnover was over A\$5 billion in 1998. Trading is on an OTC basis via the telephone. Liquidity was substantially enhanced through the development of futures contracts on the Sydney Futures Exchange, which introduced a 90-day bank bill contract in 1979 and a 10-year bond futures contract as early as 1984. There is also a three-year bond futures contract. This concentrates liquidity in the cash market at these maturities. Fixed-rate bonds pay coupon semi-annually, on the 15th of the month. Interest accrues from the previous coupon date (inclusive) to the settlement date (exclusive). An ex-dividend arrangement is in place, bonds trade ex-div 7 days prior to the coupon date. This means that as coupon is paid on the 15th, accrued interest will be negative for any settlement date falling on or after the 8th of the month and before the payment date in the months coupon is paid. The daycount basis is actual/actual.

CGBs are unusual amongst bond markets (especially Commonwealth bond markets) in trading on a yield basis. The bid-offer spread varies between 1–4 basis points depending on the liquidity of the bond in question.

Settlement for all bonds of over six months' maturity takes place three days after trade date, where bonds are to be redeemed in under six months settlement occurs on the same day (if dealt before 12.00 noon) or the next business day. Settlement is generally via RITS, which is an automated clearing system serviced by the Reserve Bank. Clearing details are arranged by the parties to the transaction with the Reserve Bank, which is responsible for registering transfer of ownership. International settlement via Euroclear and Clearstream is available, while foreign investors who wish to settle trades domestically need to appoint a local custodian.

Note that there is also an active market in state government bonds, known as *semis*. The term *semis* refers to "semi-government" bonds, which is the term for bonds issued by the six Australian states. There is no formal credit guarantee of these bonds by the Commonwealth government, hence these bonds trade at a slight spread over the central government bonds.

13.6.2 The Reserve Bank of Australia

Australia's central bank (RBA) acts as agent and adviser to the Treasury on debt management, it also deals in government bonds in the secondary market for market-smoothing and monetary policy purposes. It has an informal target of limiting underlying inflation to an average of between 2% and 3%. The underlying inflation rate is calculated by the Treasury. The RBA's main instrument of monetary policy is the overnight cash rate; any change in rates is announced at 09:30 on the day of the rate change. It also carries out daily open market operations using via

¹⁰ The other major government bond markets with inflation-linked bonds are the UK, US, Canada and Sweden. They also trade in New Zealand. Previously index-linked bonds have traded in France, Brazil, Finland and Israel.

repo or outright purchase of federal government securities. There is also a facility for market participants to sell Treasury notes of less than 90 days' maturity to the RBA.

The RBA has the following formal objectives:

- maintaining currency stability;
- maintaining full employment;
- economic prosperity and welfare of the country's population.

Note that these objectives do not contain an explicit inflation target, and unlike say the Bank of England, the RBA is also tasked with attempting to maintain full employment.

13.6.3 Derivatives on the Sydney Futures Exchange

Contracts traded on the Sydney Futures Exchange (SFE) have conventional maturity dates in March, June, September and December each year. The trading hours of the exchange are 08:30–12:30 and 14:00–16:30. Computerised screen-based trading takes place from 16:40 to 06:00. Exchange-traded option contracts are American-style options, meaning they can be exercised on any business day up to and including the day of expiry. For the three-year and 10-year bond contracts, the future represents a notional bond of 12% coupon. The face value is A\$100,000. The price quotation is 100 minus the yield (quoted as annual per cent rate), in multiples of 0.005% for the 10-year contract and 0.01% for the three-year contract. Settlement is in the middle of the delivery month.

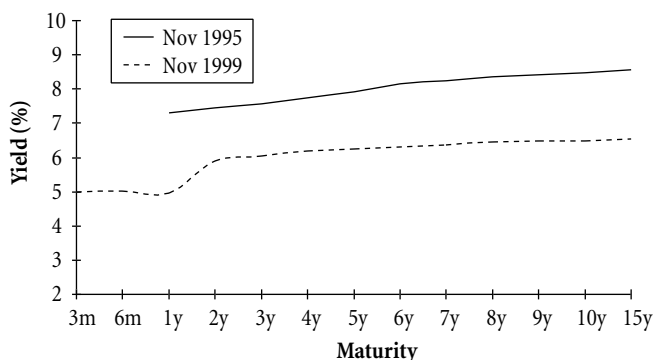


Figure 13.11(i): Australian government yield curves November 1995 and November 1999. Source: Bloomberg.

PXAU Screen Printed 14:57 DGB Govt PXAU

AUSTRALIAN-GOVERNMENT BONDS PAGE 1 / 1

	Yesterday's Close	Yield Bid	Yield Ask	Price Bid	Price Ask	Price Change	Time
1) ACGB 8 $\frac{3}{4}$ 01/01.1yr	5.990	5.57	5.54	100.150	100.152	--	5:55
2) ACGB 12 11/01	5.460	5.44	5.41	105.571	105.598	-.030	5:55
3) ACGB 9 $\frac{3}{4}$ 03/02 $\frac{1}{2}$	5.280	5.30	5.27	105.153	105.189	-.035	5:55
4) ACGB 10 10/02	5.270	5.31	5.28	107.937	107.991	-.045	5:55
5) ACGB 9 $\frac{1}{2}$ 08/03.1yr	5.270	5.31	5.28	110.159	110.237	-.071	5:55
6) ACGB 9 09/04.1yr	5.290	5.34	5.31	112.173	112.280	-.096	5:55
7) ACGB 7 $\frac{1}{2}$ 07/05.1yr	5.310	5.37	5.34	108.487	108.613	-.136	5:55
8) ACGB 10 02/06.1yr	5.340	5.39	5.36	120.409	120.558	-.143	5:55
9) ACGB 6 $\frac{3}{4}$ 11/06.1yr	5.350	5.41	5.38	106.653	106.809	-.159	5:55
10) ACGB 10 10/07.1yr	5.420	5.47	5.44	125.410	125.604	-.183	5:55
11) ACGB 8 $\frac{3}{4}$ 08/08.1yr	5.440	5.50	5.47	120.051	120.259	-.252	5:55
12) ACGB 7 $\frac{1}{2}$ 09/09.1yr	5.450	5.51	5.48	113.629	113.852	-.263	5:55
13) ACGB 5 $\frac{3}{4}$ 06/11.1yr	5.450	5.51	5.48	101.864	102.103	-.305	5:55
White = Benchmark Bonds							
ATAs 94.515	-.045	IRAs 94.20	--	AS313203.80	+19.50		
YBAs 94.740	-.040						

Copyright 2000 BLOOMBERG L.P. Frankfurt 49-360410 Hong Kong 2-877-6000 London 207-300-7500 New York 212-318-2000
 Paris 1-609-279-2000 Singapore 65-212-1000 Sydney 2-977-8888 Tokyo 3-3201-6900 Sao Paulo 11-3048-5000
 1432-212-1 19-Dec-00 14:57:19

Bloomberg PROFESSIONAL

Figure 13.11(ii): PX screen on Bloomberg, Australian government bonds. Source: Bloomberg L. P. Reproduced with permission.

13.7 New Zealand

The New Zealand bond markets often serves as an alternative to the Australian market for foreign investors. The superior performance of the Reserve Bank in keeping inflation levels stable through the 1990s also caught the attention of overseas investors, and the independence of the central bank, together with its explicit inflation target, has served as a model for other countries. The result is that foreign bondholders hold nearly 30% of the country's marketable government debt, which is a very high figure in its own right, more so when one considers that the overall level of debt is relatively low for a developed economy.

Government bonds in New Zealand are formally issued by the Queen in Right of New Zealand. An anomaly is that most tenders are re-openings of existing issues. The total outstanding debt in January 1997 was approximately NZ\$21 billion, or around \$13 billion.

13.7.1 Market structure

From 1991 government bonds have been issued through a regular auction programme; usually more than one issue is auctioned at one time, although never more than three. The average tender size is NZ\$400 million. Bonds with benchmark status have a larger issue size, typically around NZ\$2.5–3 billion. Before 1991 most of the liquidity was concentrated in the three-to-five-year sector. From 1991 the government began lengthening the average maturity of the national debt, and the first 10-year bond was issued in March that year. The responsibility for overseeing the government's debt programme is held by the Debt Management Office (DMO). As part of its effort to improve liquidity the DMO ran a conversion programme from 1992 onwards aimed at allowing holders of older illiquid issues to switch into new liquid bonds.

Government bonds exist in registered form only, with a minimum denomination of NZ\$10,000. Although all bonds are listed on the New Zealand stock exchange, trading is conducted on an OTC basis. Bonds are conventional bullet bonds paying coupon on a semi-annual basis. Index-linked stocks were discontinued in 1984, but were re-issued from 1995 onwards. The index-linked bonds in the New Zealand market are capital indexed bonds similar to those issued in Australia. Yields are quoted on a semi-annual basis and the market is unusual in that trading is conducted on a yield basis. The bid-offer yield spread is typically three or four basis points. Turnover and liquidity is highest in the five- and 10-year area of the yield curve. A feature of the bonds is that they pay coupon on the 15th of the dividend month. Coupon is paid semi-annually. Interest accrued from the previous coupon date (inclusive) up to but excluding the settlement date. This means that the value date is the same as the settlement date. One feature worth noting is that odd-dated new bonds and re-openings of existing bonds are issued with full accrued interest from the previous normal coupon date. This allows new tranches to carry the same first coupon as the existing issue and so be fungible with it. The interest accrual basis is actual/actual.

Like UK gilts, bonds in New Zealand have an ex-dividend trading rule. Bonds trade ex-dividend 10 days before the coupon payment date. This means that on the 5th of the coupon month accrued interest is negative for any settlement date falling on or after the 6th of the month up to and including the 15th of the month.

Bonds are issued on an "as needed" basis through an auction. Non-competitive bidding was discontinued in 1989 and now auctions are by competitive bid only. Bids are submitted on a yield basis, with a minimum bid size of NZ\$1 million. There is a range of benchmarks, with two-, three-, four-, five-, eight- and 10-year bonds designated as benchmarks. A futures market trades contracts representing underlying three-year and 10-year bonds. There is also a liquid open repo market.

13.7.2 Taxation and settlement

A non-resident withholding tax (NRWT) of 10–15% applies to interest earned on New Zealand fixed interest securities. This can be avoided by overseas bondholders who register with the Reserve Bank of New Zealand Registry department, under the Approved Issuer Levy (AIL) regime. Registered investors will receive gross coupon.

Settlement for domestic transactions takes place on T+2 and is delivery versus payment. For non-resident counterparties settlement takes place on T+3. Settlement is arranged by the parties to the transaction, although as registered securities ownership transfer is recorded at the Reserve Bank registry. Overseas investors may appoint a local custodian to handle settlement, although government bonds can be cleared through Euroclear and Clearstream. A domestic clearing system known as Austraclear is also available and is used by most domestic banks.

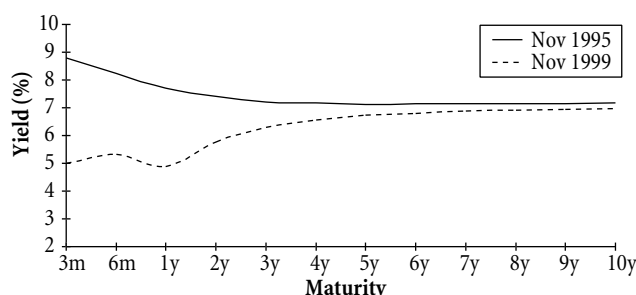


Figure 13.12: New Zealand government bond yield curves November 1995 and November 1999. Source: Bloomberg.

13.8 Canada

Traditionally the bond market in Canada provided, along with Australia, New Zealand and South Africa, an alternative investment profile for investors looking to diversify out of sterling assets. In today's global integrated investment environment it still attracts attention, and the government debt market, modelled closely along the lines of the US market, is a liquid and transparent one. This has made it popular with overseas investors, notwithstanding the downgrade in credit rating it suffered in 1995, who cite its relatively impressive inflation performance.

At the start of 1998 outstanding Canadian government debt was in the order of C\$436 billion. Table 13.9 carries greater detail. Roughly one-third of the total debt was made up of Treasury bills. The yield curve is long-dated with maturities extending out to 30 years. Most bonds are conventional bonds paying a semi-annual coupon, although some bonds were issued in the past that had a call feature attached. The government also issues inflation-linked or *real return* bonds, whose cash flows are linked to the consumer price index. For indexed bonds interest is calculated on the principal value adjusted for inflation to date, and a lump sum to compensate for inflation throughout the bond's life is paid along with the nominal principal at maturity. Unusually for a central bank, the Bank of Canada uses interest-rate swaps where necessary to enable the government to receive fixed-rate interest and pay floating rates, at a spread below the three-month Bankers' Acceptance rate. The bank publishes details of its dealing in the swap market, and total notional value of these swaps was C\$8 billion in January 1998.

	C\$ billion
Treasury bills	159.55
Conventional bonds	226.19
US pay – Canada bills	5.65
US pay – Notes, Bonds	7.89
Non-marketable bonds	35.91
Savings bonds, etc	

Table 13.9: Canadian government securities volumes January 1998. Source: Bridge-Telerate.

13.8.1 The primary market

The primary market is administered by the Bank of Canada on behalf of the Department of Finance. Bond issue is by an auction system, which has been in place since 1992. The government publishes quarterly calendars containing auction and settlement dates, plus details of the maturity of the bonds to be issued. There is an emphasis on issuing bonds in the benchmark maturities of two-, three-, five-, 10- and 30-year bonds. Further details including amounts to be issued are released on the Thursday before the auction, which takes place on the following Wednesday. On new issues the coupon is not known until the release of the auction results, and is set at the nearest $\frac{1}{4}\%$ increment below the auction average. The Bank of Canada accepts both competitive and non-competitive bids from primary distributors, although only one non-competitive bid up to a maximum of C\$2 million is allowed. The total amount of competitive bids from one dealer may not exceed 20% of the total amount of stock being auctioned. The delivery date is usually between 10 days and two weeks after the auction date. New issue bonds settle in book-entry form but may be registered on request.

The primary distributors, a syndicate of investment dealers and banks, are eligible to bid at bill and bond auctions and to trade with the central bank. The dominant group in the syndicate are the *jobbers*, who fulfil a market making function. They are obliged to provide firm two-way price quotes for all bonds and bills. They are also required to ensure that between them all auctions are covered and to provide market intelligence to the central bank. Jobbers take part in open market operations and deal in repo with the central bank. Jobbing companies that are investment dealers are able to finance their bond holdings through regular repos. Bank jobbers do not have this facility, because they already possess overdraft facilities at the central bank. All jobbers may take part in “special” repo transactions initiated by the central bank, often in order to provide an indication on views on monetary policy. Investment dealer jobbers are members of the Investment Dealers Association, while bank jobbers are supervised by the Office of the Superintendent of Financial Institutions.

13.8.2 The secondary market

The government bond market in Canada is very liquid. Foreigners hold over 30% of government debt, an impressively high figure. Bonds trade on a clean price basis, with a bid-offer spread of C\$0.05 or less. For very long-dated bonds the spread is between C\$0.05 and C\$0.10. The two-to-10-year sector of the yield curve is the most liquid although the benchmark bonds in the 10-to-30-year area are also actively traded. The most liquid issues have outstanding size of C\$1 billion or more. Small issues which are not re-opened do not trade actively; these are known as “orphaned” issues.

All government bonds pay semi-annual coupon. New issues frequently have short first coupons but never a long first coupon. Generally two-year bonds have coupon dates of 15 March and 15 September, the three-year pays on 1 November and 1 May and the five-year pays on 1 March and 1 September. Both the 10-year and the 30-year bonds pay coupon on 1 June and 1 December. Amongst older issues coupon dates vary. Accrued interest is calculated on an actual/365 day count basis. There is no ex-dividend trading. The yield calculation is the standard US “street” method, although if there is only one coupon payment remaining a simple yield is used instead. Coupon is paid gross to foreign investors.

Settlement of government bonds takes place on a T+2 basis for short-dated maturities (up to three-year bonds), otherwise settlement is on T+3. Settlement is through the Book Based System, a computerised book-entry system, between market participants via the Canadian Depository for Securities (CDS). Bonds may be settled via Euroclear or Clearstream if a counterparty wishes it.

13.8.3 The T-bill market

The raising of debt in Canada probably relies on short-term funding to a greater extent than other developed economies. The T-bill rate is an important indicator in the market of monetary policy, as it was previously linked to the bank base rate. Bond market jobbers are obliged to quote two-way prices in T-bills as well. Issuance of T-bills rose significantly in the 1980s as the government relied on them increasingly for funding. This was partly a result of the Borrowing Authority Act, which states that if the government exceeds the limit for financing set by Parliament it must raise the difference through issuing debt that has a maturity of no more than six months. Due to its size and liquidity there is an active foreign presence in the bill market.

The Bank of Canada auctions three-, six- and 12-month T-bills on a weekly basis, every Tuesday. Settlement is on Thursdays. Banks, investment dealers and the central bank submit competitive bids, which are on a yield basis, direct to the Ministry of Finance. The auctions are US style, with bidders paying what they bid. Those with the highest bid (lowest yield) are allotted the first tranche of bills. Those bidding at the highest accepted bid, which is the cut-off yield, will often receive only a proportion of the bills they bid for.

13.8.4 The Bank of Canada

The Canadian central bank was set up in 1935 and was modelled on the Bank of England. It is tasked with a broad range of objectives including regulating banking and controlling the money supply. It has an explicit inflation target, which is a range of 1–3%. A key measure is the Monetary Conditions Index (MCI), which is a short-hand measure of the combined impact of short-term interest rates and the exchange rate. Observing and analysing the MCI assists the Bank in deciding whether it needs to adjust monetary policy in pursuit of its target inflation range. The MCI is calculated using the 90-day commercial paper rate and the trade-weighted exchange rate against the G10 industrial countries. The nominal level of the MCI has no official impact, but is used as a method of tracking trends in monetary conditions. The Bank does not target a precise MCI level.

The Bank of Canada influences the money markets mainly via the overnight call loan interest rate. It also employs two other mechanisms to affect overnight rates. It can control the money supply to 12 major credit institutions as part of their daily cash settlement, encouraging them to borrow or lend funds overnight. It also uses overnight repo directly in the market. The overnight rate affects other interest rates because money market dealers usually finance their inventories using overnight funds. The Bank Rate, the overdraft rate at which the central bank lends to the 12 clearing firms, was previously linked to the three-month T-bill rate, but in February 1996 it was pegged to the ceiling of the overnight rate target range. Finally the Bank uses a drawdown/re-deposit mechanism to withdraw or re-deposit funds from the accounts that the federal government holds with the clearing banks. This also influences the overnight rate. If the overnight rate moves away from the central bank's target range, the Bank will use repos or reverse repos to affect its level.

13.8.5 Derivatives market

Futures and options in Canadian government securities are traded on the Montreal Stock Exchange. There are contracts on short-term instruments including one-month and three-month bankers acceptances as well as contracts on five-year and 10-year government bonds. The bond contracts mature in the conventional expiry months of March, June, September and December and have a face value of C\$100,000. The minimum price move is C\$10. Trading hours are 08:20–15:00 local time. The Chicago Board of Trade also offers similar contracts on 10-year government bonds.

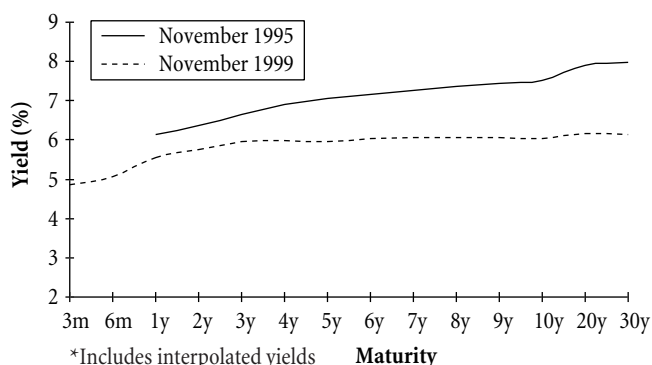


Figure 13.13(i): Canadian government bond yield curves November 1995 and November 1999. Source: Bloomberg.

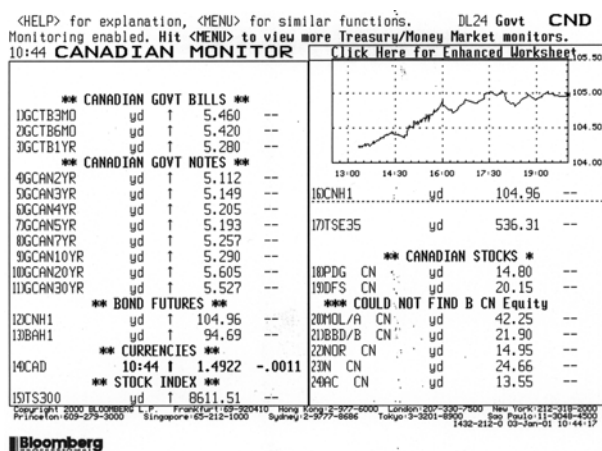


Figure 13.13(ii): Canadian market monitor screen CND on Bloomberg, 21 December 2000. Source: Bloomberg L. P. Reproduced with permission.

13.9 Hungary

An example of an “emerging economy”, the debt capital market in Hungary is comparatively well developed and relatively liquid when matched against other markets in the same category. In November 1999 the longest-dated bond was just under ten years in maturity (the 9½% February 2009). Foreign investors are limited to bonds which were originally issued with a maturity of one year or longer. However bonds with original maturities of more than one year but with less than one year remaining to redemption can be held by overseas investors. The debt market is composed primarily of government securities, issued through a system of regular auctions. The nominal size outstanding of roughly \$10 billion in November 1999 means that the trading is relatively liquid up to the five-year part of the curve. Bills and bonds are issued, with bonds being the government’s primary long-term funding instrument. Bonds are available to foreign investors via local primary dealers. Two-, 3-, 5- and 7-year bonds are issued twice a month in an auction in which the price range is pre-announced. The government has also begun issuing 10-year bonds. The 7-year bonds are unusual in that they pay a floating-rate coupon linked to the consumer price index; all other bonds pay fixed semi-annual coupons.

The majority of trading takes place on an OTC basis, with limited trading on the Budapest Stock Exchange. Foreign investors must participate via a local custodian, who will manage securities and transfer cash on behalf of the investor via accounts on the exchange’s KELLER clearing system. Settlement is on a T+2 basis. An unusual feature is that it is possible to settle in the OTC market on a T+0 basis if dealing is undertaken by 11.00 hours.

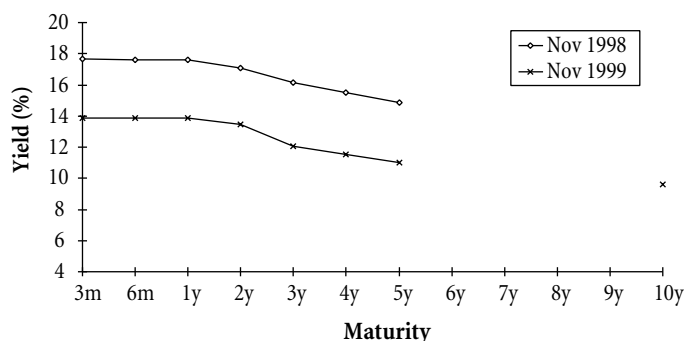


Figure 13.14: Hungarian government bond yield curves November 1998 and November 1999. Source: Bloomberg.

13.10 South Africa

The capital markets in South Africa are labelled as an “emerging market”, however they offer reasonable liquidity and a developed infrastructure. They also include a wide array of instruments, all of which are available to foreign investors. The emerging description is more a function of the investor views of the macro-economy rather than the liquidity of the government bond market. For example, among emerging markets it is rare in offering a liquid interest-rate swap market. The decision to invest in the market can therefore be more of an economic and political one compared to other emerging markets, where the mechanics of the market may hinder trading.

13.10.1 Market structure

The debt capital market comprises a large bond market and a smaller but still liquid money market. Large-size transactions are difficult to transact in the money market however. The main instruments in the money market are negotiable certificates of deposit (NCDs), bankers acceptances (BAs) and Treasury bills. Negotiable certificates of deposit are the most liquid money market instruments in South Africa. They are issued by all major local banks with maturities up to five years, however the highest liquidity is in maturities up to one year. A typical transaction size is ZAR5–10 million (roughly \$1–2.1 million). Typically NCDs trade on a yield basis and the average bid-offer spread is 10 basis points.

	Volume outstanding (ZAR billion)
NCDs	50
BAs	8
T-bills	18
Bonds	343

Table 13.10: South African debt capital markets
volume outstanding November 1999. Source: JP Morgan.

Bankers acceptances are the second most liquid money market instrument, they are mainly issued by the four largest banks. The highest liquidity (and the highest area of issuance) is in the three-month sector. Trading size is similar to NCDs. Bankers acceptances trade on a discount basis and the average bid-offer spread is roughly 10 basis points. Treasury bills are issued by the South African Reserve Bank on behalf of the government. They are offered via a tender every Friday. The auction is bid-price style, at which foreign investors are free to submit bids. Bills are issued in 91-day and 182-day maturities. The weekly tender size is usually ZAR800 million for the 91-day bill and ZAR300 million for the 182-day bill. Unlike the NCD and BA market, the secondary market in T-bills is largely illiquid because local banks use them to meet reserve requirements and also because investors tend to hold them to maturity.

The domestic bond market is large and liquid. Bonds are traded on the South African Bond Exchange as well as on an OTC basis. In the first half of 1999 the monthly nominal value of turnover ranged from approximately ZAR275 billion to ZAR375 billion.¹¹ A large proportion of this was repo trading however. A limited number of bond futures and options contracts are traded on the South African Futures Exchange (SAFEX). The government is the largest issuer of bonds; government bonds are known as “gilts”.¹² Public bodies known as *parastatals* also issue bonds. The four parastatals are Eskom (electricity utility), Transnet (transport utility), Telkom (telecommunications) and the Trans-Caledon Tunnel Authority (TCTA, the special water projects). These issuers make up nearly 95% of the market. Issues of Transnet and Telkom, as well as certain issues of TCTA carry an explicit government guarantee, while Eskom bonds are implicitly guaranteed. Note that Telkom was privatised in 1997 so its issues after that year are not government guaranteed.

	Volume outstanding (ZAR billion)
Government	292
Eskom	26
Transnet	16
Telkom	5
TCTA	4

Table 13.11: Government and parastatals debt outstanding November 1999.

Most of the turnover is in 16 liquid bonds, which range in maturity from one to 27 years. The longest-dated bond is the 10½% 2026 issue, which matures in December 2026. The second longest stock after that is the 13½% 2015 issue. There is a zero-coupon issue that matures in 2020. Bonds are distributed across the entire yield curve. Currently eight bonds have benchmark status. In 1998 the central bank introduced a new primary dealer system, which serves as an exclusive distribution system for government bonds. Coupon is payable semi-annually, with an ex-dividend trading period beginning one month before the coupon date. Accrued interest is calculated on an actual/365 day count basis. The South African government is unusual in issuing zero-coupon bonds directly; in the major government strips market such as the those in the US, UK, France and Germany the issuing authorities designate certain bonds as being strippable, and then leave it to the market to carry this out.

¹¹ Source: JP Morgan.

¹² As well as the United Kingdom, South Africa shares this name for its government bonds with India.

13.10.2 Taxation and settlement

There are no local or withholding taxes on money market and bond market instruments for overseas investors. Transactions in the bond market settle on T+3. Clearing is via either physical delivery or electronic settlement, for which a central depository has been established. Foreign investors usually appoint an authorised local bank as their agent and custodian.

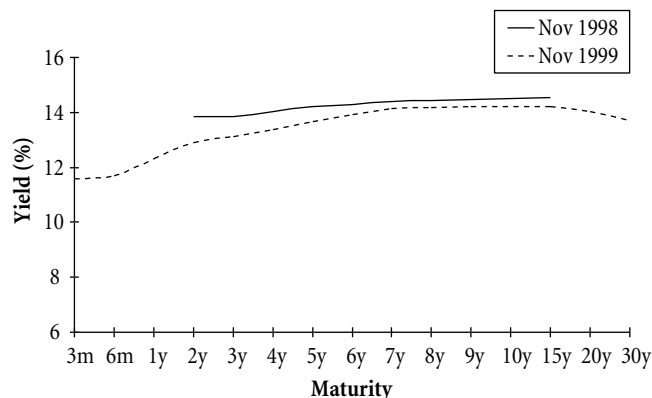


Figure 13.15(i): South African government bond yield curves November 1998 and November 1999. Source: Bloomberg.

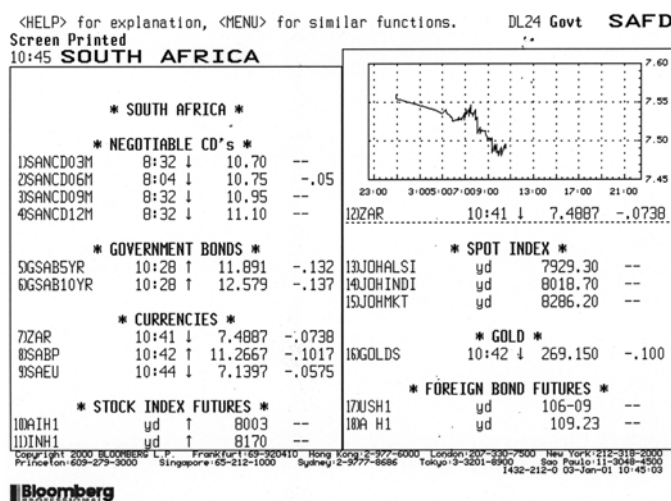


Figure 13.15(ii): South African market monitor on Bloomberg, screen SAFD. Source: Bloomberg L. P. Reproduced with permission.

13.11 Egypt

The Egyptian fixed interest market is open to foreign investors, but liquidity is available only in the T-Bill market. Although the volume of public debt outstanding is fairly large at \$45 billion in November 1997, the secondary market in bonds is limited. The government regularly issues 91-day and 182-day T-Bills, but the bond market is still developing.

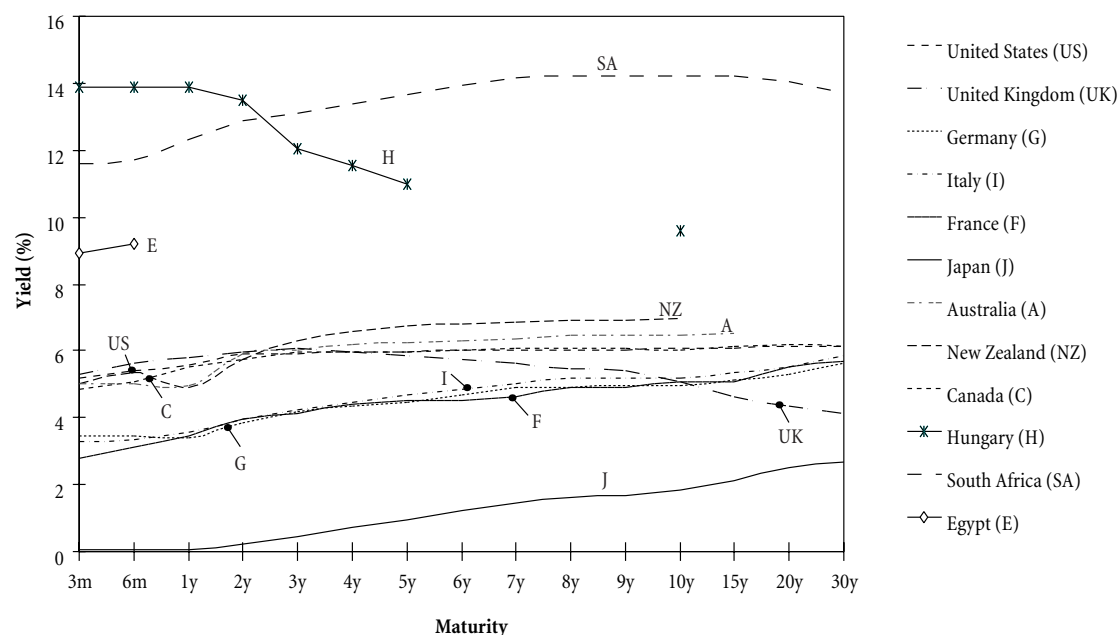
In November 1999 there were seven bonds trading, with maturities ranging from one year to ten years. All bonds are listed on the Cairo Stock Exchange. They pay semi-annual coupon, with accrued interest calculated on an actual/365 day count basis. The longest-dated bond at this time was the 10% 2009, maturing on 15 April 2009, which was rated A- by the Standard & Poor's rating agency. The bonds are plain vanilla issues except for two which are

callable.¹³ An anomaly for foreign investors is that the market is closed on Fridays and open on Sundays, reflecting the different week-end structure that exists in certain Middle Eastern, African and Asian countries.

T-Bills are auctioned weekly on Mondays (91-day) and Wednesdays (182-day). Auctions are on a competitive basis, with the central bank setting a price range within which bids must fall. Bills are issued in physical bearer form, with delivery to the custodian taking place 14 days after the auction date. The secondary market in bills trades both OTC and on the Cairo Stock Exchange, with foreigners holding about 20% of the outstanding paper in November 1999. The yield on 91-day T-bills was 8.895% in November 1999.

Maturity	United States	United Kingdom	Germany	Italy	France	Japan	Australia	New Zealand	Canada	Hungary	South Africa	Egypt
3m	5.2	5.27	3.45	3.3	2.79	0.06	4.99	4.99	4.85	13.89	11.60	8.895
6m	5.429	5.62	3.45	3.33	3.14	0.06	5.01	5.33	5.06	13.89	11.7	9.2
1y	5.577	5.78	3.38	3.56	3.43	0.05	4.96	4.89	5.54	13.88	12.3	
2y	5.899	5.99	3.84	3.96	3.97	0.23	5.89	5.77	5.76	13.47	12.89	
3y	5.929	6.06	4.2	4.25	4.11	0.43	6.03	6.28	5.97	12.06	13.11	
4y	5.959	5.94	4.36	4.45	4.38	0.7	6.18	6.56	5.99	11.525	13.39	
5y	5.989	5.83	4.47	4.7	4.53	0.92	6.26	6.73	5.97	10.99	13.67	
6y	6.0016	5.74	4.71	4.86	4.54	1.22	6.32	6.805	6.04		13.91	
7y	6.0142	5.61	4.88	5.01	4.65	1.44	6.37	6.88	6.06		14.15	
8y	6.0268	5.49	4.92	5.16	4.92	1.6	6.44	6.91	6.055		14.19	
9y	6.0394	5.4	4.96	5.2	4.93	1.65	6.47	6.94	6.05		14.2	
10y	6.052	5.06	4.97	5.2	5.08	1.85	6.48	6.97	6.04	9.58	14.21	
15y	6.0975	4.65	5.13	5.36	5.07	2.1	6.54		6.105		14.21	
20y	6.12025	4.36	5.29	5.52	5.53	2.52			6.17		14.04	
30y	6.143	4.13	5.64	5.84	5.7	2.69			6.14		13.7	

Interpolated yields in **bold**



* Includes interpolated yields for some curves

Figure 13.16: Selected government bond yield curves, November 1999. Source: Bloomberg.

¹³ The source for volume data on Egyptian government bonds is noted as a Web site in the bibliography.

Appendices

APPENDIX 13.1 Country credit ratings

Germany

Moody's and Standard & Poor's (S&P) award Germany their highest ratings, Aaa/AAA for long-term and P1/A1+ for short-term foreign currency debt. However Germany never used any of these ratings prior to EMU, as the Bundesbank believed that it was inappropriate for a country with a reserve currency (the mark) to issue public-sector debt in foreign currencies.

Italy

The Moody's rating for Italy is A1, which is the lowest sovereign rating of any of the major bond markets. Both long-term debt denominated in Lira (now euro-denominated debt) and foreign-currency debt is awarded this grade. The S&P rating is higher at AA for foreign currency debt. Euro-denominated bonds are awarded a AAA rating by S&P.

France

Both Moody's and S&P award France the highest rating possible Aaa/AAA for long-term debt and P1/A1+ for short-term debt.

Japan

Sovereign domestic and foreign currency debt were until recently both awarded Aaa/AAA and P1/A1+ ratings by Moody's and S&P respectively. However in November 1998 Moody's downgraded Japan's debt to Aa1. In February 2000 the country's yen-denominated debt was placed under review for a possible downgrade, again by Moody. The agency cited the rising levels of public sector debt to fund fiscal programmes to encourage economic growth as the key reason behind the review. At the time of the review public debt stood at around 120% of GDP, and was expected to grow to 130% by the end of 2001.

Australia

The Moody's credit rating is Aa2 for foreign currency debt and Aaa for domestic currency debt. S&P award an AA rating for foreign currency debt (downgraded from AAA in 1989) and AAA for domestic currency debt.

New Zealand

Both Moody's and S&P award their highest rating to government debt, Aaa and AAA.

Canada

Moody's award a Aa2 rating for foreign-currency debt and Aa1 for domestic currency debt, while S&P ratings are AA+ and AAA.

Hungary

Government stock is not currently rated by Moody's. The bonds are rated A by S&P.

South Africa

Moody's rating is Baa1, the S&P rating is BBB+.

Egypt

Moody's rating is Baa1, S&P rating is BBB-.

APPENDIX 13.2 Fixed euro exchange rates on 1 January 1999

Austrian schilling to Euro	13.7603
Belgian franc to Euro	40.3399
Finnish markka to Euro	5.94573
French franc to Euro	6.55957
German mark to Euro	1.95583
Irish punt to Euro	0.787564
Italian lira to Euro	1936.27
Luxembourg franc to Euro	40.3399
Netherlands guilder to Euro	2.20371
Portuguese escudo to Euro	200.482
Spanish peseta to Euro	166.386

Table 13.12: Euro exchange rates.**APPENDIX 13.3 Japanese Government Bond Futures contract conversion factor**

The JGB contract is based on a bond with notional 6% coupon. The conversion factor (also known as *price factor*) is given by (13.3) and (13.4).

1. When time from delivery to maturity is 10 years or less:

$$CF = \frac{\frac{C}{0.06} \times (1.03^{\frac{m}{6}} - 1) + 100}{1.03^{\frac{m}{6}} \times 100} - \frac{C(6 - t)}{1200}. \quad (13.3)$$

2. When time from delivery to maturity is greater than 10 years:

$$CF = \frac{(C/0.06) \times (1.03^{\frac{m}{6}} - 1) + 100}{1.03^{\frac{m}{6}} \times 100} - \frac{C(6 - (t - 6))}{1200} \quad (13.4)$$

where

- C is the underlying bond coupon
- t is the number of months from delivery day to the next coupon
- n is the number of remaining half-years from the next coupon date to the final maturity date
- m is the number of months from delivery to the final maturity date.

APPENDIX 13.4 Selected country bond markets

The market conventions for the bond markets in selected countries are given in Table 13.13 below. The data is believed correct as at June 1999, however structures are often modified or replaced and we apologise for any detail errors. The table is reproduced with kind permission from Patrick Brown's book, *Bond Markets, Structures and Yield Calculations*, courtesy of Fitzroy Dearborn publishers.

Price quotation

The most common price convention is a cash price per cent nominal of bonds, quoted "clean" or without accrued interest. There are a number of variations on this which are detailed below, the terms for which are used in the table.

- Clean The clean price quoted as a percentage of 100 nominal
- Dirty The gross price, also known as "dirty" price, which is the clean price plus accrued interest
- CP The clean price per bond
- P The actual gross price per bond, including accrued interest
- rm A yield quote
- rmm A money market yield
- rd A discount rate

Settlement period

Bonds may settle on the day they are traded or a number of days after the trade date; the convention differs across countries. Settlement delays due to public holidays differ across countries. Business days in the Euro markets are defined as any day when the cash market for the currency and Euroclear and Clearstream are open. The two clearing systems are closed on 1 January and 25 December each year.

In the table, settlement is stated as the number of days after the trade date on which a transaction is cleared. Where this is different in the international markets, a second number appears.

Accrual basis

The standard market convention is for coupon interest on bonds to accrue from the issue date or last coupon date (inclusive) up to but excluding the settlement date. There are variations on this however, the most common being the inclusion of both last coupon and settlement dates in the day count.

Ex-dividend period

In certain markets there is a short period just preceding the coupon date when bonds are traded *ex-coupon*, that is without the next coupon payment. This is known as an ex-dividend or ex-coupon period. There is no ex-dividend period in the international bond market. During the ex-dividend period accrued interest is usually calculated backwards from the interest payment date to the settlement date. The number of days in the ex-dividend period is shown in the table.

Yield basis

The market standard convention in bond markets is to calculate return as a gross redemption yield or yield-to-maturity (YTM). In certain markets, for bonds that have less than one year to maturity a money-market equivalent yield is used (YTM/MM). The difference between the two measures is considered in Chapter 4, as are the different yield formulae. For instruments such as T-Bills the convention is to use a money market yield (MM). In the majority of European markets the redemption yield is quoted using annual interest compounding, whereas in the US and UK it is quoted using semi-annual compounding. The formula for converting one measure to the equivalent of the other is given in Chapter 4.

Settlement

An increasing number of bond types may be settled now in the international clearing systems Euroclear and Cedel. The domestic systems used in the various countries are listed below.

AUC	Austraclear
BOJ	Bank of Japan
BoE	Bank of England Registrars, for private clients
C	Cedel
CDS-BBS	Canadian Depository for Securities – Book base system
CDS-DCS	Canadian Depository for Securities – Debt clearing system
CREST	The UK Crest book-entry system
CVB	Central de Valores Mobiliarios
DC	Deutsche Börse Clearing (formerly Kassenverein)
DOM	Domestic
E	Euroclear
IGSO	Irish Gilt Settlement Office, operated by the Central Bank of Ireland
ISEC	Istanbul Stock Exchange Clearing & Settlement Bank
KDPW	Polish domestic clearing house
NBB	National Bank of Belgium
OKB	Central Bank of Austria
SEGA	Swiss Securities Clearing Corporation
SICO	Sicovam
SLOVK	Slovakian Securities Centre
STN	Saturne
TURK	Central Bank of Turkey
VN	Russian Vneshtorbank
VP	Danish Securities Centre

Withholding tax

Markets in which bond coupon payments are paid net of a withholding tax are indicated with the relevant tax rate. The symbol “*” after a tax rate indicates that non-resident foreigners are not liable to the withholding tax.

	Price quotation	Settlement period (days)	Accrual basis	Ex-dividend period	Coupon frequency	Yield basis	Yield compounding period	Settlement	Withholding tax (%)	Money market accrual basis
AUSTRALIA										act/365 ³²
<i>Government bonds</i>	rm	3	act/365	7	2	YTM	2	AUC,C,E	10	
AUSTRIA										act/360
<i>Fixed interest</i>	Clean	3 or 5	30E/360	¹⁵	1	YTM	1	C,E,OKB	22*	
BELGIUM										act/365
<i>All except OLO strips</i>	Clean	3	30E/360	none	1	YTM/MM	1	C,E,NBB	15*	
<i>OLO strips</i>	rm	3	30E/360 ¹¹	none	–	YTM/MM ²³	1	C,E,NBB	–	
CANADA										act/365 ³²
<i>Treasury bills</i>	rmm	Trade date	act/365	–		MM	–	CDS-DCS,C,E	none	
<i>Government</i>	Clean	2 or 3 ⁷	act/365	none	2	YTM/MM	2	CDS-DCS,C,E	none	
<i>Provincial/Municipal</i>	Clean	3	act/365	none	2	YTM/MM	2	CDS-BBS,C,E	none	
<i>Corporate</i>	Clean	3	act/365	none	2	YTM/MM	2	CDS-BBS,C,E	0/25	
CZECH REPUBLIC										act/360
<i>All bonds</i>	Clean	3	30E/360	30	1	YTM	1	²⁹	0/25 ³⁰	
DENMARK										act/360
<i>Govt, T-notes, mortgage bonds</i>	Clean	3	30E/360	30	²¹	YTM	1	VP,C,E	none	
<i>Government FRNs</i>	Clean	3	30E/360	30	4	²⁴	–	VP,C,E	none	
<i>Zero-coupon T-bills</i>	P ¹	3	–	–	–	YTM/MM ²⁵	1	VP,C,E	none	
FINLAND										act/365 ³²
<i>All bonds</i>	rm	3	30E/360	none	1	YTM/MM	1	C,E,DOM	28*	
FRANCE										act/360
<i>BTF</i>	rmm	1 or 3	act/360	none	–	MM	–	STN	none	
<i>BTAN,BMTN,TCN</i>	rm	1 or 3	act/act	none	1	YTM	1	C,E,STN	none	
<i>OAT & fixed-rate bonds</i>	Clean	3	act/act	none	1	YTM/MM	1	C,E,SICO,Relit	none	
<i>Post-determined variable bonds</i>	Clean	3	act/year	none	1	–	1	C,E,SICO	none	
<i>Pre-determined FRNs, incl TEC</i>	Clean	3	act/year	none	4	–	1	C,E,SICO	none	
<i>Convertible bonds</i>	P	3		none	1	YTM	1	SICO, E, C	none	
GERMANY										act/360
<i>Fixed-rate bonds</i>	Clean	2 or 3	act/act	none	1	YTM/MM ²⁶	1	C,E,DC	30*	
<i>Floating-rate notes</i>	Clean	2 or 3	act/360	none	2 or 4	–	–	C,E,DC	30*	
GREECE										act/365
<i>FRNs, T-bills</i>	Clean	2 or 3	act/365	none	1	–	–	DOM, E, C	none	
HUNGARY										act/360
<i>Government</i>	Clean	2	act/365	¹⁶	1 or 2	YTM/MM	1	DOM	0/10	
IRELAND										act/365
<i>Government (gilt-edged)</i>										
<i>Fixed-rate – 365 day²</i>	Clean	1	act/act	¹⁷	2	YTM	2	C,E,IGSO	none	
<i>Fixed-rate – 360 day²</i>	Clean	1	30E/360	¹⁷	1	YTM	2	C,E,IGSO	none	
<i>Fixed-rate – act/act²</i>	Clean	1	act/act	¹⁷	1	YTM	2	C,E,IGSO	none	
<i>Variable-rate</i>	Clean	1	act/year ¹²	¹⁷	4	MM	2	C,E,IGSO	none	
ITALY										act/365 ³²
<i>BOT, CTZ</i>	Dirty	2 (CTZ - 3)	act/act	–	–	MM	–	C,E,DOM	12.5	
<i>Other bonds</i>	CP	3	30E/360 ¹³	none	1	YTM	1	CE,DOM	12.5*	

JAPAN										act/365 ³²
Treasury bills	<i>rmm</i>	2	act/365	–	–	MM	–	BOJ	18	
Government (JGB)	Clean	3	act/365	¹⁸	2	YTM	–	BOJ	20*	
Other bonds	Clean	⁸	act/365	¹⁸	2	YTM	–	DOM	20*	
LUXEMBOURG										act/365
All bonds	Clean	3	act/act	none	1	YTM/MM	1	C	none	
NETHERLANDS										act/360
All bonds	Clean	3	act/act	none	1	YTM	1	C,E,Necigef	none	
NEW ZEALAND										act/365 ³²
Treasury bills	<i>rmm</i>	2	act/365	–	–	MM	–	AUC	none	
Government bonds	<i>rm</i>	2 ⁹	act/365	10	2	YTM	2	AUC,CE	none	
NORWAY										³³
All bonds	Clean	3	act/365	14	1 or 2 ²²	YTM	1	C,E	none	
POLAND										
Fixed-rate notes	Clean	2 or 3	act/act	5	1		1	KDPW	none	
Floating-rate notes	Clean	2 or 3	act/360	10	–	–	–	KDPW		
PORTUGAL										act/365
All bonds	Clean ³	4 or 3	act/act	none	1	YTM ²⁷	1	C,E,CVB	20/25 ³¹	
RUSSIA										act/year
Minfin bonds	Clean	7 calendar	30E/360	none ¹⁹	1	YTM/MM	1	VN	none	
Federal loan bonds (OF'Z)	Clean	Trade date	act/365	–	–	MM	–	MICEX	–	
Treasury, acceptances (GKO)	Dirty	Trade date	act/360	–	–	MM	–	MICEX	–	
SLOVAKIA										
Government	Clean	3 or 5	30E/360 ¹⁴	7	2	YTM	2	SLOVK		
SPAIN										act//360
All bonds	Clean	3	act/act	none	1	YTM/MM	1	C,E,Expaclear	25	
SWEDEN										act/365 ³²
All bonds	<i>rm</i>	3	30E/360	5	1	YTM	1	VPC	none	
SWITZERLAND										30E/360 ³²
Fixed-rate bonds	Clean	3	30E	none	1	YTM	1	SEGA	35*	
Floating-rate notes	Clean	3	act/360	none	2	–	–	SEGA	35*	
TURKEY										act/365
Revenue-sharing certs	⁴	Trade date	act/365	none	1	–	1	TURK	none	
Corporate bonds	<i>rm</i>	Trade date	act/365	none	1	YTM	1	ISEC	11/22	
FRN-indexed	⁴	Trade date ¹⁰	act/365	none	1	–	–	ISEC,TURK	none	
Asset-backed securities etc., ⁵	<i>rm</i>	Trade date	act/365	none	–	MM	–	ISEC	11/22	
Treasury bills/FRNs	<i>rm</i>	Trade date ¹⁰	act/365	none	1	MM	–	ISEC,TURK	none	
Privatization bills	⁶	Trade date	act/365	none	1	MM	–	TURK	none	
UNITED KINGDOM										act/365
Government (gilt-edged)										
Fixed-rate	Clean	1	act/act	7 ²⁰	1	YTM	2	CREST,C,E,BoE	none	
Index-linked	Clean	1	act/act	7 ²⁰	2	²⁸	2	CGO,C,E	none	
Floating-rate notes	Clean	1	act/year	none	2 or 4	–	–	CGO,C,E	none	
Strips	<i>rm</i>	1	act/act	–	–	YTM	2	CGO,C,E	–	
Bulldogs (foreign)	Clean	1	30E/360	none	2	YTM	1 or 2	CREST,C,E	none	
Corporate bonds	Clean	1,3, or 5	act/365	varies	2	YTM	2	CREST	25	
UNITED STATES										act/365 ³²
Treasury bills	<i>rd</i>	1	act/360	–	–	MM	–			
Treasury notes & bonds	Clean	1	act/act	none	2	YTM/MM	2	Fedwire	none	
Other bonds	Clean	3	30U/360	none	2	YTM/MM	2			
EURO DENOMINATED BONDS										act/360
Fixed-rate bonds	Clean	3	act/act	–	1	YTM	1	C,E,DOM		
Floating-rate notes	Clean	3	act/360	–	2 or 4	–	–	CE,DOM		

INTERNATIONAL BONDS

34

<i>Straights & Convertibles</i>	Clean	3	30E/360	none	1	YTM	1	C,E	none
<i>Floating-rate notes</i>	Clean	3	act/360	none	2 or 4	–	–	C,E	none

Abbreviations used in Table 13.13

BOT	Buoni Ordinari del Tesoro	LIBOR	London Interbank Offered Rate
BMTN	Bons a Moyen Terme Negociables	OAT	Obligations Assimilables du Tresor
BTAN	Bons du Tresor a Interet Annuel	OLO	Obligation Lineaire
BTF	Bond du Tresor a Taux Fixe	OT	Obrigacoes do Tesouro
CTZ	Certificati del Tesoro (zero-coupon)	OTC	Over-the-counter market
FRN	Floating Rate Note	TCN	Titres de Creance Negociables
JGB	Japanese Government Bond	TEC	Taux de l'Echeance Constante

Table 13.13: Bond market conventions. Country data reproduced from Patrick Brown, *Bond Markets, Structures and Yield Calculations*. Fitzroy Dearborn, 1998. Used with permission. Additional sources: Bloomberg, Reuters, ISMA, BoE, IMF, London Stock Exchange, Embassy of the Republic of Slovakia.

Notes to Table 13.13

1. The actual price is for an amount of DKK1 million.
2. Irish government fixed-coupon bonds issued before 14 June 1993 accrue interest on an actual/365 basis and pay semi-annual coupon. Stock issued between June 1993 and August 1997 uses actual/360 basis with annual coupon, bonds issued after that date accrue on an act/act basis.
3. In the domestic market Treasury OT bonds are priced per bond, to which accrued interest is added.
4. There is no secondary market in these instruments, and therefore no price quote convention.
5. The instruments include asset-backed securities, commercial paper and bank bills.
6. The price P is given by (13.5):

$$P = \frac{1}{1 + (LIBOR + s) \times n / 36500} \times rate \quad (13.5)$$

where

- s is the margin over LIBOR
- n is the number of days to maturity
- $rate$ is the central bank foreign currency buying rate.

7. Bonds with a remaining life of over three years settle after three days. Bonds with a maturity below this settle in two days.
8. Settlement is on the 10th, 20th and last day of each month, but scheduled to be changed to T+7 days during the second half of 1999.
9. International settlement is seven calendar days.
10. Domestic settlement is on the trade date, international settlement is T+2.
11. The accrual basis is actual/365 when the strip has a remaining maturity of under one year.
12. The coupon rate for each stock is declared by the authorities two days before the start of the coupon period to which the rate will apply. The coupon is based on the average Dublin interbank rate (DIBOR) over the previous 10 business days, together with the margin for the individual bond.
13. 13. The number of days accrued includes *both* the last coupon date and the settlement date, that is, one day more than market convention.
14. This is calculated to the trade date and not the settlement date.
15. For coupon dates between 10th and 24th, ex-dividend date is the first Monday of the month; for coupon dates between 25th and 9th of next month, ex-dividend date is the third Monday of the month.
16. This applies to OTC trades.

17. Bonds trade ex-dividend from the nearest Wednesday that is three weeks before the coupon date.
18. Japanese bonds have periods during which they cannot be traded. The last trading day for JGBs is eight business days ahead of the coupon date; for non-government stock the period is three weeks.
19. The instrument is not tradeable during the 30 days before the coupon date.
20. The ex-dividend period is 7 business days, except for 3½% War Loan irredeemable stock, which has an ex-dividend period of 10 days.
21. Convention is 1 for government bonds and Treasury bills, and 2 or 4 for mortgage bonds.
22. Certain FRNs pay quarterly coupons.
23. OLO strips with a maturity of over one year have a price calculated from the standard redemption yield formula, on a 30E/360 day-count basis. For strips with maturity under one year, price is calculated from the money market-equivalent yield on an actual/365 basis.
24. The coupon rate is calculated from the average yield rm on government fixed-rate bonds that have a maturity of between three months and three years. The coupon rate C on FRNs is given by (13.6):

$$C = \left((1 + rm)^{0.25} - 1 \right) \times 100. \quad (13.6)$$

25. Yields are calculated using both money market and bond market conventions.
26. The standard ISMA method is used although certain domestic institutional investors use the Moosmüller method (see Chapter 4).
27. Yields are compounded with the fractional period being calculated as act/365, although bonds accrued interest on a act/act day basis.
28. The gross redemption yield is a “real” yield, which is calculated using a constant level of inflation for the life of the bond.
29. Bonds can be settled on the Prague Stock Exchange and the Securities central registry.
30. Government bonds are tax-free. Bonds issued by banks, local authorities and corporates are subject to 25% tax. Foreign residents may reclaim this tax.
31. Most bonds are subject to 20% withholding tax, although some OT’s pay gross coupons. Corporate Eurobonds have a 25% withholding tax if there is no domestic paying agent. Non-resident foreigners may reclaim this tax.
32. Domestic market accrues on an actual/365 basis, Euro market accrued basis is actual/360.
33. The money market uses more than one convention, depending on instrument the basis is 30E/360, act/360, act/365 and act/act.
34. In the international markets the convention is to calculate interest on an act/360 day basis. Certain currencies calculate interest differently for domestic markets and international markets, for example interest on Japanese yen lent in Japan is calculated on an act/365-day basis while Euro-yen uses act/360 basis. Countries that calculate interest on an act/365 basis include sterling, Canadian dollars and Irish pounds.

APPENDIX 13.5 Rating agencies sovereign credit ratings

Sovereign	Moody's	Standard & Poor's	Fitch IBCA	Thomson Financial Bank Watch	Sovereign	Moody's	Standard & Poor's	Fitch IBCA	Thomson Financial Bank Watch
Argentina	B1	BB	BB	BB	Kuwait	Baal	A	A	A
Australia	Aa2	AA+	AA	AA	Latvia	Baa2	BBB	BBB	BB+
Austria	Aaa	AAA	AAA	AAA	Lebanon	B1	BB-	BB-	B+
Azerbaijan				B	Liechtenstein		AAA		
Bahamas	A3				Lithuania	Bal	BBB-	BB+	BB+
Bahrain	Ba1		BBB+	BB+	Luxembourg		AAA	AAA	AAA
Bangladesh				CCC	Macau	Baal			BBB+
Barbados	Ba1	A-		BB+	Malaysia	Baa3	BBB	BBB	BBB-
Belarus				C	Malta	A3	A	A	
Belgium	Aa1	AA+	AA-	AA+	Mauritius	Baa2			
Belize	Ba2				Mexico	Bal	BB	BB	BB-
Bermuda	Aa1	AA	AA		Moldova	B2		B-	
Bolivia	B1	BB-		B	Morocco	Bal	BB		BB
Brazil	B2	B+	B	B	Mongolia		B		
Bulgaria	B2	B	B+	B	Netherlands		AAA	AAA	AAA
Canada	Aa2	AA+	AA	AA+	New Zealand	Aa2	AA+		
Cayman Islands	Aa3				Nicaragua	B2			B-
Chile	Baa1	A-	A-	A	Norway	Aaa	AAA	AAA	AAA
China	A3	BBB	A-	BBB+	Oman	Baa2	BBB-		BBB-
Colombia	Ba2	BB+		BB+	Pakistan	Caal	B-		C
Cook Islands		B-			Panama	Baal	BB+	BB+	BB+
Costa Rica	Bal	BB		BB-	Papua New Guinea	B1	B+		
Croatia	Baal	BBB-	BB+	BB+	Paraguay	B2	B		
Cuba	Caal				Peru	Ba3	BB	BB	B+
Cyprus	A2	A		A	Philippines	Bal	BB+	BB+	BB
Czech Rep.	Baal	A-	BBB+	BBB	Poland	Baal	BBB	BBB+	BBB
Denmark	Aaa	AA+	AA+	AA+	Portugal	Aa2	AA	AA	AA
Dominican Rep.	B1	B+		B	Qatar	Baa2	BBB		BBB
Ecuador	Caa3			D	Romania	B3	B-	B-	B-
Egypt	Bal	BBB-	BBB-	BB+	Russia	B3	SD	CCC	CCC
El Salvador	Baa3	BB+		BB+	Saudi Arabia	Baa3			A-
Estonia	Baal	BBB+	BBB	BBB+	Singapore	Aal	AAA	AA+	AA
Fiji	Bal				Slovakia	Bal	BB+	BB+	BB
Finland	Aaa	AA+	AAA	AA+	Slovenia	A3	A	A	A-
France	Aaa	AAA	AAA	AAA	South Africa	Baa3	BB+	BB	BB+
Germany	Aaa	AAA	AAA	AAA	South Korea	Baa2	BBB	BBB	BBB

Sovereign	Moody's	Standard & Poor's	Fitch IBCA	Thomson Financial Bank Watch	Sovereign	Moody's	Standard & Poor's	Fitch IBCA	Thomson Financial Bank Watch
Greece	A2	A-	BBB+	BBB+	Spain	Aa2	AA+	AA+	AA+
Guatemala	Ba2			B-	Sri Lanka				B-
Hong Kong	A3	A	A+	A-	Surinam		B-		
Honduras	B2			B-	Sweden	Aa1	AA+	AA	AA+
Hungary	Baa1	BBB	BBB+	BBB+	Switzerland	Aaa	AAA	AAA	AAA
Iceland	Aa3	A+	AA-		Taiwan	Aa3	AA+		AA
India	Ba2	BB		BB	Thailand	Ba1	BBB-	BBB-	BBB-
Indonesia	B3	CCC+	B-	CCC	Trinidad & Tobago	Ba1	BBB-		BB+
Iran	B2				Tunisia	Baa3	BBB-	BBB-	BB+
Ireland	Aaa	AA+	AAA	AA	Turkey	B1	B	B+	BB-
Israel	A3	A-	A-	A-	Turkmenistan	B2		B-	
Italy	Aa3	AA	AA-	AA-	UAE	A2			A+
Jamaica	Ba3	B			Ukraine	Caal			CCC
Japan	Aa1	AAA	AA+	AAA	UK	Aaa	AAA	AAA	AAA
Jordan	Ba3	BB-		B+	USA	Aaa	AAA	AAA	AAA
Kazakhstan	B1	B+	BB-	B	Uruguay	Baa3	BBB-	BBB-	BBB-
Kenya				B	Venezuela	B2	B	BB-	B
					Vietnam	B1			CCC

Table 13.14: Sovereign foreign currency long-term credit ratings February 2000.
Source: Moody's, S&P, Chase Manhattan, IFR. Used with permission.

APPENDIX 13.6 Benchmark government bonds, 25 January 2000**WORLD BOND PRICES****BENCHMARK GOVERNMENT BONDS**

Jan 24	Red Date	Coupon	Bid Price	Bid Yield	Day chg yield	Wk chg yield	Month chg yld	Year chg yld
Australia	11/01	12.000	109.0171	6.56	+0.01	+0.02	+0.40	+1.80
	09/09	7.500	101.4222	7.29	+0.01	+0.05	+0.41	+2.22
Austria	07/01	5.500	101.7200	4.21	+0.01	-0.02	+0.05	+1.22
	01/10	5.500	97.7717	5.80	-0.01	+0.02	+0.37	+2.01
Belgium	06/02	8.750	109.2200	4.58	-	+0.03	+0.12	+1.67
	03/09	3.750	85.6900	5.81	-	+0.02	+0.33	+1.95
Canada	12/01	5.250	98.4600	6.14	-0.05	-0.01	+0.20	+1.36
	06/09	5.500	92.7500	6.55	-0.05	+0.06	+0.32	+1.62
Denmark	11/01	8.000	105.4400	4.74	-0.03	+0.06	+0.16	+1.30
	11/09	6.000	100.5400	5.92	-0.02	+0.04	+0.33	+1.94
Finland	09/01	10.000	108.7800	4.26	+0.01	-0.11	+0.08	+1.39
	04/09	5.000	94.5860	5.77	-	+0.03	+0.35	+1.96
France	07/01	3.000	98.3800	4.17	-0.01	+0.01	+0.07	+1.32
	04/06	7.250	110.7200	5.19	-	+0.04	+0.38	+1.81
	10/09	4.000	87.5426	5.70	-0.01	+0.02	+0.35	+2.02
	04/29	5.500	90.9982	6.17	-0.02	-0.04	+0.29	+1.60
Germany	09/01	3.500	98.9083	4.20	+0.01	+0.02	+0.12	+1.30
	04/06	6.250	104.7400	5.33	-	+0.05	+0.35	+1.87
	01/10	5.375	98.4594	5.58	-0.01	+0.02	+0.34	+1.93
	07/28	4.750	81.6028	6.13	-0.02	-0.05	+0.27	+1.55
Greece	01/02	7.600	101.7100	6.64	-0.07	+0.07	-0.33	-3.48
	01/09	6.300	97.6200	6.66	-	+0.13	+0.21	+0.34
Ireland	10/01	6.500	103.4600	4.36	+0.02	-0.04	+0.08	+1.34
	04/10	4.000	86.1800	5.81	-	+0.02	+0.36	+1.98
Italy	11/01	3.500	98.5700	4.35	+0.01	+0.02	+0.12	+1.39
	07/04	4.000	95.2100	5.22	+0.02	+0.07	+0.33	+1.97
	11/09	4.250	88.8900	5.75	-0.01	+0.03	+0.34	+1.89
	11/29	5.250	86.4900	6.26	-	-0.03	+0.34	+1.47
Japan	12/01	6.000	110.7690	0.27	-0.04	-0.03	-0.07	-0.29
	12/04	4.500	116.8610	0.95	-0.06	-0.06	-0.06	-0.29
	09/09	1.900	102.6350	1.60	-0.07	-0.10	-0.11	-0.14
	09/19	2.900	110.2210	2.25	-0.09	-0.12	-0.18	-0.35
Netherlands	02/02	3.000	97.1700	4.48	+0.01	+0.02	+0.13	+1.59
	07/09	3.750	85.9100	5.72	-	+0.02	+0.36	+1.98
New Zealand	02/01	8.000	101.5736	6.38	-0.01	+0.10	+0.27	+1.41
	07/09	7.000	96.6491	7.50	-	-0.03	+0.20	+2.00
Norway	05/01	7.000	101.1200	6.06	+0.03	+0.01	+0.16	+0.69
	05/09	5.500	93.8500	6.39	-0.03	+0.01	+0.31	+1.44
Portugal	03/01	8.750	106.0500	3.25	-0.01	-0.35	-0.44	+0.13
	07/09	3.950	86.3900	5.86	-0.01	+0.01	+0.32	+1.99
Spain	01/01	5.000	101.0200	3.94	+0.02	+0.03	+0.08	+1.03
	01/10	4.000	86.6254	5.80	-0.01	+0.02	+0.35	+1.98
Sweden	06/01	13.000	110.7250	4.69	-0.01	+0.06	+0.04	+1.44
	04/09	9.000	120.5960	6.02	-0.01	+0.07	+0.38	+2.10
Switzerland	10/01	5.500	104.6000	2.66	+0.02	+0.12	+0.18	+1.30
	02/09	3.250	97.1500	3.63	+0.01	+0.09	+0.11	+1.38
UK	11/01	7.000	100.7200	6.56	-	-	+0.12	+2.07
	11/04	6.750	101.5200	6.38	-0.01	+0.04	+0.31	+2.21
	12/09	5.750	99.8000	5.78	-0.01	+0.06	+0.36	+1.64
	12/28	6.000	118.9100	4.78	-0.02	+0.16	+0.20	+0.58
US	10/01	5.875	99.0000	6.48	-	-	+0.24	+1.85
	11/04	5.875	96.9844	6.62	-0.03	+0.04	+0.31	+2.00
	08/09	6.000	94.8438	6.74	-0.05	+0.06	+0.33	+2.01
	08/29	6.125	92.6875	6.70	-0.01	+0.01	+0.22	+1.52

London closing * New York closing.

Source: Interactive Data/FT Information

Yields: Local market standard/Annualised yield basis. Yields shown for Italy exclude withholding tax at 12.5 per cent payable by non residents.

©The Financial Times, 25 January 2000. Used with permission.

Selected bibliography

Bank of Canada, Monetary Policy Report, May 1997.

Brown, P., *Bond Markets, Structures and Yield Calculations*, Fitzroy Dearborn, 1998.

Deane, M., Pringle, R., *The Central Banks*, Hamish Hamilton, 1994.

Direzione Generale del Tesoro, *A Guide to the Government Bond Market*, July 1994.

European Bond Commission, *European Bond Markets*, Probus, 1993.

Goldstein, M., Folkerts-Landau, D., *International Capital Markets: Developments, Prospects and Key Policy Issues*, International Monetary Fund, 1994.

J.P. Morgan, *Guide to Local Markets*, November 1997.

Moody's Investors Service, *Sovereigns, Supranationals Credit Opinions*, September 1996.

OECD, *Economic Outlook* 61, December 1998.

Smith, R.T., "Markets for Corporate Debt Securities", *IMF Working Paper* No. 95/67, 1995.

Reserve Bank of Australia Bulletin, November 1993, December 1997.

Web site (data on Egyptian government bonds): <http://www.egyptianstocks.com>

Part III Corporate Debt Markets

The market for corporate debt is large and diverse. In fact it is in the corporate markets that the most exciting innovations and the most exotic products are observed, as government markets are fairly plain vanilla in nature. In Part III we review the most important instruments that form part of this market. There is also a chapter dealing with credit analysis.

The bonds described here are not exclusively corporate instruments; many sovereign governments for example issue Eurobonds. Not surprisingly the quality of paper issued ranges from triple-A risk-free, such as World Bank bonds, to high-yield and un-rated bonds. Bonds that feature embedded options, such as callable bonds and convertibles, are now traded under a much more rigorous analytical regime than even five years previously; most banks will use a binomial valuation model to price these bonds. This serves to illustrate that the corporate debt markets continuously benefit from the latest developments in financial engineering, and it is here that one will observe the latest structures and innovations.

14 Corporate Debt Markets

In this and the next 16 chapters we review the corporate bond markets. The corporate markets cover a wide range of instruments and issuer currencies. There is a great variety of structures and products traded in the corporate markets, many of which easily could be the subject of separate books in their own right. All instruments serve the same primary purpose however, of serving as an instrument of corporate finance. The exotic structures that exist have usually been introduced in order to attract new investors, or retain existing investors, in what is an extremely competitive market.

Generally the term “corporate markets” is used to cover bonds issued by non-government borrowers. The bonds issued by regional governments and certain public sector bodies, such as national power and telecommunications utilities, are usually included as “government” debt, as they almost always are covered by an explicit or implicit government guarantee. All other categories of borrower are therefore deemed to be “corporate” borrowers. The combined market is a large one, as we indicated in the previous chapter. Table 14.1 shows non-government international bond issuance from 1996, split by currency. The majority of bonds are denominated in US dollars, euros, and Japanese yen.

	US\$	Sterling	Euro	Other	Total
1996	261	51	153	108	573
1997	334	63	148	86	631
1998	342	78	209	65	693
1999*	249	52	260	38	599
* January-June 1999					
Volumes are \$ billion					

Table 14.1: Non-government international bond issuance.
Source: CapitalData Bondware; Bank of England.

The euro, introduced in eleven countries of the European Union in January 1999, has already become a popular currency for corporate bond issuers. This is indicated in Figure 14.1 which shows that the volume of issues in the euro currency was already over twice that in the first eight months of the year compared to issue volumes for the eleven currencies separately during the whole of 1998. Corporate bond issues in sterling are detailed in Figure 14.2, split into maturity bands of short-, medium- and long-dated bond issues.

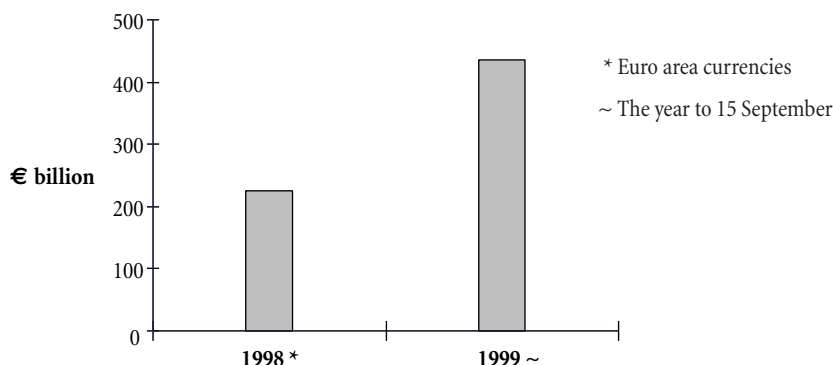


Figure 14.1: International bonds issued in euro. Source: *The Economist*.

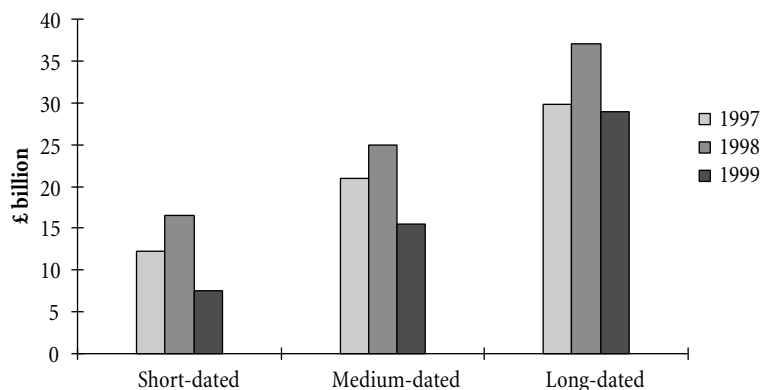


Figure 14.2: Sterling non-government bond issuance. Source: BoE.

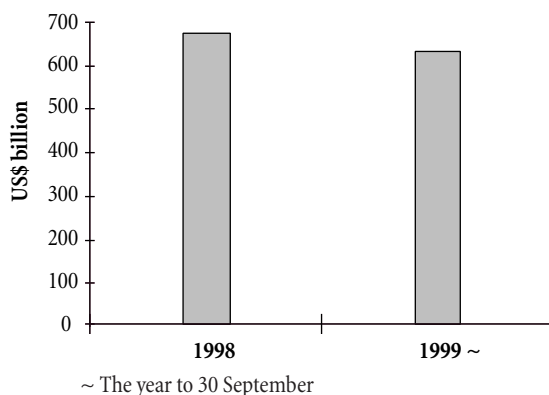


Figure 14.3: US dollar corporate bond issuance. Source: BoE.

14.1 Introduction

Corporate bonds are tradeable debt instruments issued by non-government borrowers. The majority of corporate bonds are plain vanilla instruments, paying a fixed coupon and with a fixed term to maturity. Corporate debt covers a wide range of instruments traded in the primary and secondary markets. Corporate issuers will use the debt capital markets to raise finance for short-, medium- and long-term requirements and projects. Corporate borrowers will raise both *secured* and *unsecured debt*, with the former type of borrowing secured on assets of the company. The ease with which a corporation can raise unsecured finance in the public market is a function of its *credit rating*. Often corporate bonds are classified by the type of issuer, for example banks and financial services companies, utility companies and so on. Issuer classification is then broken down further, for example utility companies will be subdivided into electricity companies, water companies and gas distribution companies.

Under the terms of any corporate bond issue, the borrower is obliged to pay the periodic interest on the loan, represented by the coupon rate, as well as repay the principal on maturity of the debt. If the borrower fails to pay interest on the loan as it is due, or to repay the principal on maturity, this is known as *default*¹ and the borrower will be in breach of the terms of the issue. This is a serious event. In the event of default¹ bondholders are entitled to enforce payment via the legal process and the courts. As providers of debt finance, bondholders rank higher than preferred and ordinary shareholders in the event of a winding-up of the company. Depending on the level of the *security* associated with the particular bond issue, bondholders may rank ahead of other creditors. The issue of the *credit risk* of a specific issuer is the key concern for corporate bond investors.

¹ The term *technical default* is used to refer to non-payment or delay of a coupon as it becomes due, when the issuer is still otherwise financially solvent.

The yield on corporate bonds is set by the market and reflects the credit quality of the issuer of the paper, as well as the other considerations relevant to price setting such as level of liquidity, yield on similar bonds, supply and demand and general market conditions. Yields are always at a *spread* above the similar maturity government bond. A well-received corporate bond will trade at a relatively low spread over the government yield curve. Although the common perception is that bonds are held by investors for their income, and are less price volatile than equities, this is an over-generalisation. Certain bonds are highly volatile, while others are held for their anticipated price appreciation.

Certain corporate issues are aimed at only the professional (institutional) market and have minimum denominations of \$10,000 or £10,000. Generally however they are issued in denominations of \$1,000 (or £1,000, €1,000, etc). In the US domestic market the par value for corporate bonds is sometimes taken to be \$1,000 instead of the more conventional \$100. A market order to purchase “100 bonds” (or simply “100”) would therefore mean \$100,000 nominal of the bond. In other markets though the standard convention is followed of using 100 as the par value of the bond.

14.1.1 Basic provisions

The majority of corporate bonds are term bonds, that is, they will have a fixed term to maturity, at which point they must be redeemed by the borrower. Only companies of the highest quality can issue bonds of maturity much greater than 10 years at anything less than prohibitive cost; however it is common to see long-dated bonds of up to 30 years maturity or longer issued by high credit-quality companies. Let us look at some other relevant features.

- **Bond security.** A corporate that is seeking lower cost debt, or that does not have a sufficiently high credit rating, may issue secured debt. As security the issuer will pledge either fixed assets such as land and buildings, or personal property. The security offered may be *fixed*, in which case a specific asset is tied to the loan, or *floating*, meaning that the general assets of the company are offered as security for the loan, but not any specific asset. A *mortgage debenture* gives bondholders a charge over the pledged assets, called a *lien* (a lien is a legal right to sell mortgaged property to satisfy unpaid obligations to creditors). Where companies do not own fixed assets or other real property they often offer as collateral securities of other companies that they hold. A *debenture* bond is secured not by a specific pledge of property but bondholders have a claim over company assets in general, often ahead of other creditors.
- **Provisions for paying off bonds.** In some cases a corporate issue will have a call provision that gives the issuer the option to buy back all or part of the issue before maturity. The issuer will find a call option useful as it means that debt can be refinanced when market interest rates drop below the rate currently being paid, without having to wait until the bond's maturity. At the same time such a provision is disadvantageous to the bondholder, who will require a higher yield as compensation. Call provisions can take various forms. There may be a requirement for the issuer to redeem a pre-determined amount of the issue at regular intervals. Such a provision is known as a *sinking fund* requirement. This type of provision for repayment of corporate debt may be designed to retire all of a bond issue by the maturity date, or it may be arranged to pay off only a part of the total by the end of the term. If only a part is paid off, the remaining balance is known as a *balloon maturity*. The purpose of a sinking fund is to reduce the credit risk attached to the bond. Investors will derive comfort from the fact that provisions have been made regarding the final redemption of the bond, and will be more willing to buy the bond. Clearly this may be necessary for borrowers of lower credit quality.

In most cases the issuer will satisfy any sinking fund requirement by either making a cash payment of the face amount of the bonds to be redeemed to the bond trustee, who will call the bonds for repayment by drawing serial numbers randomly, or by delivering to the trustee bonds with a total face value equal to the amount that must be retired from bonds purchased in the open market. The sinking fund call price, as with callable bonds generally, is the par value of the bonds, although in a few cases there may be a set percentage of par that is redeemable.

These are only a sample of the features that can be attached to corporate bonds. We will consider them and certain others in the remainder of this chapter and in the following chapters.

14.2 Determinants of the development of a corporate market

Corporate bond markets generally develop after a functioning market in government securities has been established. A study by the International Monetary Fund² has found that six general areas of the debt markets, concerned with the infrastructure and regulation of the markets, are generally required to be in place before corporate markets can develop. Some of these six areas are described below.

- **Money markets.** The IMF study found that an already established and liquid money market will facilitate the development of a corporate market. This is essentially because in order to develop a yield curve against which corporate debt can be priced, the short-end of the yield curve must be established first, as short-date rates form the cornerstone for longer-dated benchmark interest rates (Schinasi states that the short-end “anchors” the rest of the yield curve). A liquid market in short-term instruments serves as a benchmark for corporate bonds that offer different levels of liquidity, credit quality and terms to maturity. A well-established money market, particularly a repo market, also enables market participants to finance both long and short positions in other bond instruments.
- **Regulatory infrastructure.** A common observation is that regulatory policies have an influence in either encouraging or inhibiting the development of a corporate market. Generally the essential pre-requisite is that *supervision* provision should be well established in order to provide comfort to investors that the market is well policed. However regulation should not be unnecessarily cumbersome or bureaucratic as this will drive business elsewhere. Overly strict financial regulations often result in the growth of “offshore” markets, witness the development of the Eurobond market in London and the growth of Singapore and Hong Kong as financial trading centres for the products of several Asian countries. Capital controls in the United States are said to have been the prime factor behind the development of the “Eurodollar” market, the offshore market in US dollars.³ This is a topical issue, for instance towards the end of 1999 the European Union, as part of an effort to harmonise tax policies across its member countries, was debating the introduction of a withholding tax for cross-border investments. The UK government was resisting the implementation of such a tax, on the grounds that this would drive the Eurobond market away from London, with a consequent loss in trading income and employment opportunities. Excessive regulation and taxation policies are considered to be partly behind the slow growth of the money markets in countries such as France, Germany, Italy and Japan; for example the *commercial paper* market in these countries has grown only slowly.⁴ Taxation policies also impact the development of markets, and may serve to force debt issuance offshore. It is common for developed markets to allow gross payment of coupon to overseas investors, while corporate accounting policy often enables companies to offset the interest payable on debt against their income tax liability.
- **Investor base.** Another ingredient that is believed to be important in the development of corporate debt markets is a professional and diversified investor base. In advanced economies it is institutional investors such as life companies and pension funds that are the largest investing firms. At the shorter end of the yield curve, banks and corporates are important investors. In the US, money market mutual funds are also heavily invested in short-dated instruments. An institutional investor base will lead to a demand for corporate debt, which will facilitate the market’s development

Other areas cited by the IMF study (1998) include the concentration of market power amongst financial firms and the culture of corporate finance in the domestic market.

14.3 The primary market

The issue of corporate debt in the capital markets require a primary market mechanism. The first requirement is a collection of merchant banks or investment banks that possess the necessary expertise. Investment banks provide advisory services on corporate finance as well as *underwriting* services, which is a guarantee to place an entire bond

² Schinasi and Todd Smith, IMF Working Paper 98/173, 1998. The full reference is stated in the bibliography.

³ The term “Eurodollar” originally referred to a deposit of US dollars outside the United States, originally in Europe. A Eurodollar deposit can of course be in any country outside of the US, not just in Europe.

⁴ See Alworth and Borio (1993).

issue into the market in return for a fee.⁵ As part of the underwriting process the investment bank will either guarantee a minimum price for the bonds, or aim to place the paper at the best price available. The IMF study (1998) notes that investment banking expertise is something that is acquired over time. This is one reason why the major underwriting institutions in emerging economies are often branch offices of the major integrated global investment banks.

Small size bond issues may be underwritten by a single bank. It is common however for larger issues, or issues that are aimed at a cross-border investor base, to be underwritten by a *syndicate* of investment banks. This is a group of banks that collectively underwrite a bond issue, with each syndicate member being responsible for placing a proportion of the issue. The bank that originally won the *mandate* to place the paper invites other banks to join the syndicate. This bank is known as the *lead underwriter*, *lead manager* or *book-runner*. An issue is brought to the market simultaneously by all syndicate members, usually via the *fixed price re-offer* mechanism. This is designed to guard against some syndicate members in an offering selling stock at a discount in the grey market, to attract investors, which would force the lead manager to buy the bonds back if it wished to support the price. Under the fixed price re-offer method, price undercutting is not possible as all banks are obliged not to sell their bonds below the initial offer price that has been set for the issue. The fixed price usually is in place up to the first settlement date, after which the bond is free to trade in the secondary market.

A corporate debt issue is priced over the same currency government bond yield curve. A liquid benchmark yield curve therefore is required to facilitate pricing. The extent of a corporate bond's yield spread over the government yield curve is a function of the market's view of the credit risk of the issuer (for which formal credit ratings are usually used) and the perception of the liquidity of the issue. The pricing of corporate bonds is sometimes expressed as a spread over the equivalent maturity government bond, rather than as an explicit stated yield, or sometimes as a spread over another market reference index such as Libor. If there is no government bond of the same maturity as the corporate bond, the issuing bank will price the bond over an interpolated yield, obtained from the yields of two government bonds with maturities lying either side of the corporate issue. If there is no government bond that has a maturity beyond the corporate issue, the practice in developed economies is to take a spread over the longest dated government issue. In developing markets however, the bond would probably not be issued.

Formal credit ratings are important in the corporate markets. Investors usually use both a domestic rating agency in conjunction with an established international agency such as Moody's or Standard & Poor's. As formal ratings are viewed as important by investors, it is in the interest of issuing companies to seek a rating from an established agency, especially if it is seeking to issue foreign currency debt and/or place its debt across national boundaries.

14.4 The secondary market

Corporate bonds virtually everywhere are traded on an over-the-counter (OTC) basis, that is, directly between counterparties over the telephone. Bonds are usually listed on an exchange though, as many institutional investors have limitations on the extent to which they may hold non-listed instruments. Eurobonds for example are usually listed on the London Stock Exchange or Luxembourg Stock Exchange, while those issued by Asian borrowers are frequently also listed on the Hong Kong or Singapore exchanges. In the United States there are more corporate bond issues listed on the New York Stock Exchange (NYSE) than there are equities, and the dollar value of daily bond trading is at least as high as that in equities. On the NYSE a low volume of trading in bonds does take place on the exchange itself, but this dwarfed by the volume of trading in the OTC market.

The level of liquidity varies greatly for corporate bonds, ranging from completely liquid (for example, a World Bank global bond issue) to completely illiquid, which is common when investors have bought the entire issue of a bond and held it to maturity. The number of market makers in a particular issue will also determine its liquidity. In return for providing liquidity, market makers also retain a major market privilege because they have exclusive access to inter-dealer broker price screens.

Another factor that is important to secondary market liquidity is the clearance and settlement system. In Japan for example, the settlement system until very recently was a decentralised, paper-based system, which acted as a

⁵ If the bank cannot sell an entire issue to its customers or other institutions in the market, it will take the remaining stock onto its own books. The fee payable by the borrower is compensation to the bank for taking on this underwriting risk.

barrier to market liquidity. A computerised, de-materialised “book-entry” system for settling corporate bonds, as represented by Euroclear and Clearstream, contributes to liquidity because it assists market participants to trade without being exposed to delivery or payment risk.

Total issues	1985	1990	1991	1992	1993	1994	1995	1996	1997	1998
	165.7	299.9	389.8	471.1	646.6	498	573.2	n.a.	n.a.	n.a.
By type of offering:										
Public, domestic	119.6	189.3	286.9	379.1	486.9	365.2	408.8	386.3	397.2	374.1
Private placement, domestic	46.2	87	74.9	65.9	116.2	76.1	87.5	n.a.	n.a.	78.4
Foreign sales and other issues *	–	23.6	28.0	27.1	43.5	56.7	76.9	74.8	79.2	n.a.
By industry:										
Manufacturing	52.1	53.1	86.6	82.1	88.0	43.4	61.1	42.0	58.5	55.4
Commercial and miscellaneous	15.1	40.0	36.7	43.1	60.4	40.7	50.7	34.1	41.3	38.0
Transportation	5.7	12.7	13.6	9.8	10.8	6.9	8.4	5.1	6.9	5.5
Public utility	13.0	17.5	23.9	49.1	56.3	13.3	13.8	8.2	9.7	8.9
Communication	10.5	6.7	9.4	15.4	32.0	13.3	23.0	13.3	18.5	15.3
Real estate and financial	69.3	169.3	219.6	272.9	394.1	380.4	416.3	358.5	386.4	378.1

* Includes foreign sales only.

Note: Includes all debt security issues with a maturity greater than one year.

Table 14.2: Bond market financing by US Firms (in US\$ billion).
Source: Federal Reserve Bulletin.

	Households	Foreign	Banks	Insurance	Private pensions	Public pensions	Mutual funds	Brokers dealers	Others
1985	9.4	14.3	11.7	35.6	11.3	12.2	2.9	2.6	0
1992	12.6	12.8	11.3	38.5	8.8	9.9	4.5	1.7	0
1993	16.2	12.5	10.7	37.1	8.8	6.9	5.7	2.2	0
1994	14.5	12.3	10.4	36.9	9.2	7.2	6.9	2.5	0
1995	14.6	11.8	9.6	35.5	9.1	6.8	8.7	3.2	0.7
1996	13.7	12.7	9	36.1	9.5	6.6	8.2	2.6	1.5
1997	15.1	13.3	8	35.2	9.7	5.8	8.3	2.8	1.8
1998	14.4	13.7	7.7	35.2	9.8	6	8.6	2.7	1.9

Table 14.3: Holders of US Corporate and Yankee bonds (in per cent of total).
Source: Flow of Funds, Board of Governors of the Federal Reserve Board; IMF.

14.5 Fundamentals of corporate bonds

The market generally classifies corporate bonds by credit rating and by sector of issuer. In the US for example issuers are classified as public utilities, transport companies, industrial companies, banking and financial institutions, and international (or *Yankee*) borrowers. Within these broad categories issuers are broken down further, for example transportation companies are segmented into airlines, railway companies and road transport companies. In other respects corporate bonds have similar characteristics to those described in Chapter 1, although it is often in the corporate market that exotic or engineered instruments are encountered, compared to the generally plain vanilla government market.

14.5.1 Term to maturity

In the corporate markets, bond issues usually have a stated term to maturity, although the term is often not fixed because of the addition of call or put features. The convention is for most corporate issues to be medium- or long-dated, and rarely to have a term greater than 20 years. In the US market prior to the Second World War it was once common for companies to issue bonds with maturities of 100 years or more, but this is now quite rare. Only the highest rated companies find it possible to issue bonds with terms to maturity greater than 30 years; during the 1990s such companies included Coca-Cola, Disney and British Gas.

Investors prefer to hold bonds with relatively short maturities because of the greater price volatility experienced in the markets since the 1970s, when high inflation and high interest rates were common. A shorter-dated bond has lower interest rate risk and price volatility compared to a longer-dated bond. There is thus a conflict between investors, whose wish is to hold bonds of shorter maturities, and borrowers, who would like to fix their borrowing for as long a period as possible. Although certain institutional investors such as pension fund managers have an interest in holding 30-year bonds, it would be difficult for all but the largest, best-rated companies, to issue debt with a maturity greater than this.

14.5.2 Bond interest payment

Corporate bonds pay a fixed or floating-rate coupon. Floating-rate bonds were reviewed in Chapter 5. Zero-coupon bonds are also popular in the corporate market, indeed corporate zero-coupon bonds differ from zero-coupon bonds in government markets in that they are actually issued by the borrower, rather than simply being the result of a market-maker stripping a conventional coupon bond.

The fixed interest rate payable by a conventional bond is called the bond *coupon*, and we used this term when describing bonds in Chapter 1. The term originates from the time when bonds were *bearer* instruments, and were issued with coupons attached to them. The bondholder would tear off each coupon and post it to the issuer as each interest payment became due. These days bonds are *registered* instruments, and the investor receives the interest payment automatically from the issuer's registrar or paying agent. Therefore it is technically incorrect to refer to a bond's "coupon" but the convention persists from earlier market infrastructure. Many bond issues including Eurobonds and US Treasuries are held in "de-materialised" form, which means only one "global" certificate is issued, which is held with the clearing or custody agent, and investors receive a computer print-out detailing their bond holding.

Zero-coupon bonds are issued in their own right in the corporate markets, but are otherwise similar to zero-coupon bonds in government markets. Note that the term "strip" for a zero-coupon bond is usually used only in the context of a government bond strip. In the US market zero-coupon bonds or "zeros" were first issued in 1981 and initially offered tax advantages for investors, who avoided the income tax charge associated with coupon bonds.⁶ However the tax authorities in the US implemented legislation that treated the capital gain on zeros as income, thus wiping out the tax advantage. The tax treatment for zeros is similar in most jurisdictions. Zeros are still popular with investors however because they carry no reinvestment risk. The lack of reinvestment risk is appreciated more by investors in a declining interest rate environment, whereas in a rising interest rate environment investors may prefer to have coupon to reinvest. Zeros are also preferred during a period of relatively high interest rates, as the compounding effect is greater.

As a zero-coupon bond is issued at a discount to its face value, and then repaid at par, there is a significant liability for the borrower on maturity. For a long-dated bond this liability can be very large. This may be a concern for bondholders, so it is usually only highly-rated borrowers that are able to place zero-coupon bonds. Lower-rated borrowers in the US domestic market have issued *deferred-interest bonds*, also known as *zero/coupon bonds*. With a deferred-interest bond the investor receives no interest payments for a set period after issue, say the first five years of the bond's life. At the end of the deferred interest period, the accumulated interest is paid out. The rate payable is usually considerably higher than the market level. This feature makes the bond attractive for lower-rated companies or start-up companies who might be expected to suffer from cash-flow problems in their early years. The thinking behind deferred-interest bonds is that, after the no coupon period, the issuer will be in a financially stable state and able to pay off the accumulated interest, and indeed redeem the bond and refinance at a lower rate of interest.

⁶ The first offer of zero-coupon bonds was by J.C. Penney Co, Inc in the domestic US market in 1981.

In the case of default, bankruptcy laws in the US for example, allow the bondholder to claim back the value of the purchase price plus “accrued” interest up to the time the issuer when into liquidation. This value is essentially the value of the bond at the time. The face value of the bond of course, cannot be claimed.

14.6 Bond security

International bonds and bonds sold in the Euro markets are unsecured. It is sometimes said that the best type of security is a company in sound financial shape that is able to service its debt out of its general cash flow, or a company with a high credit rating. While this is undoubtedly true, a secured bond is sometimes preferred by investors. In domestic markets, corporate bonds are often issued with a form of security attached in order to make them more palatable to investors. The type of security that is offered varies and can be either *fixed* or *floating*, that is a specific or general charge on the assets of the borrowing company. The type of security sometimes defines the market that the bond trades in, for example a mortgage-backed bond or a debenture. We consider the main types of security below.

14.6.1 Mortgage bonds

A bond that is secured with a *lien* over some or all of the issuer’s mortgaged properties is known as a *mortgage bond*. A lien is the legal right to sell mortgaged property in the event that the borrower is unable to meet obligations arising from its bond issue. The coupon payable is lower than would otherwise be payable if the bond was issued without a first charge on the issuer’s properties. In the US market such an unsecured bond is called a *debenture*, although confusingly a debenture in the UK domestic market is a bond that *is* secured, although on a floating rather than fixed asset.

An example of a bond with collateral in the form of a charge over property is the Annington Finance No. 4 plc issue (AF4), which dates from December 1997. This is a sterling bond and is described in Example 14.1 below.

EXAMPLE 14.1 Annington Finance No. 4 plc

- Annington Finance is a public limited company incorporated in England, whose share capital is owned by Annington Homes Limited, a subsidiary of Annington Holdings plc. The holding company was established in 1996 to purchase housing stock from the United Kingdom’s Ministry of Defence (MoD), who had previously run the housing for the benefit of members of the country’s armed forces and their families. This housing was known as the Married Quarters Estate (MQE). Annington Holdings plc purchased the housing stock from the MoD and then leased it back to it.

As part of the purchase proceeds, Annington Holdings plc issued bonds secured on the assets themselves. The bonds consisted of:

- £1.24 billion Class A Zero-coupon notes due December 2022
- £1 billion Class B Zero-coupon notes due January 2023
- £900 million Secured floating-rate notes due January 2023

The floating-rate notes pay interest at Libor plus 40 basis points up to 2008, after which the interest steps up to Libor plus 100 basis points. The issuer has an option to redeem the floating-rate notes after 2008.

The collateral for the issue consists of a first fixed security on the housing stock of the MQE, which is made up of 760 sites throughout the UK and approximately 55,000 housing units. The MQE was leased back to the MoD who pay rental fees on it. The cash flows for the bond issue are financed by these rental proceeds, as well as the proceeds from the occasional disposal of individual residential properties. On the basis of this security the bonds were rated “A2” by Moody’s and “A” by Fitch IBCA. Duff & Phelps rated the class A notes as “AAA” and the other notes as “A”. The class A notes rank ahead of the class B notes. On maturity the issuer is required to repay the interest and principal outstanding on all the notes.

Mortgage bonds may be issued as part of a *series*. This often happens when companies wish to finance their operations continuously through the issue of long-dated debt. This means that as one bond issue matures, another series is issued under the same mortgage.

There are restrictions on the amount of debt that may be issued with a charge over the same property. Generally the terms of a mortgage bond issue state that property purchased by the borrower after the issue of the bonds is still

covered by the lien attached to those bonds. This is known as an *after-acquired clause*. Additional borrowing secured by the properties is usually set as a percentage of the value of the after-acquired assets.

14.6.2 Other collateral

Debt is often issued by firms that do not possess fixed assets such as property or factories. These companies may offer security for bonds they issue in the form of other collateral that they own, such as government bonds and bonds and equity of other companies. In the US market bonds that have been secured with collateral of this form are known as *collateral trust bonds*. Assuming the bonds are issued at par, the nominal value of an issue is always lower than the market value of the collateral pledged, to provide an element of safety for investors. Under the terms of the issue the borrower delivers the collateral that has been pledged into the safe custody of the bond's trustee or paying agent, which will hold them for the term of the bond's life. In the event of default, bondholders are entitled to ownership of the collateral; if part of this is equity, the voting rights of the equity are transferred to the bondholders at the same time.

Terms of issue of a collateral trust bond include certain provisions to protect investors. The main one is that the value of the collateral must remain at a specified level above the nominal value of the bonds, and if there is decline in value of the collateral the issuer must provide additional collateral to make up the shortfall. This is similar to a margin call in a repo transaction.

14.6.3 Debentures

In the US market a debenture bond is one that is not secured by a pledge on a specific fixed asset or property. Bondholders have only a claim on the assets of the issuer in the event of default in line with general creditors. In the UK market a *debenture* is a secured bond, one that is secured on the general assets of the issuing company rather than a specified fixed asset.

US market debentures generally are of two types, those that are issued by highly-rated borrowers who do not need necessarily to provide security for investors, and those issued by borrowers who have already issued bonds secured on their assets, such as mortgage bonds. This second type of debenture pays a higher coupon than both the other debenture as well as the secured bonds of its issuer. A company that has debenture bonds outstanding but no secured bonds may provide a *negative pledge clause*, which states that the debenture issue will rank equally with any secured bonds that may be issued subsequently. This protects existing bondholders. Other terms under which the bonds are issued include a limitation on the amount of dividends that the company can pay, or the proportion of current earnings that can be used to pay a dividend. With share dividends themselves becoming increasingly unfashionable in the US, these restrictions are seen less in the domestic bond market.

14.6.4 Subordinated bonds

Debt issued by companies that ranks behind both secured debt and debentures is known as *subordinated* debt. A subordinated bond therefore has the highest credit risk of any domestic bond issue, because it is not backed with security of any kind. As well as ranking behind secured bonds and debentures, subordinated bonds may also in some cases rank after other creditors such as trade creditors. For this reason subordinated bonds pay the among the highest yields in a domestic market. They are often issued with other features that are designed to make them more attractive. Such features may include an option to convert the bond into the ordinary shares of the issuing company at a specified date or dates. A bond with such a feature is then a *convertible*. Convertible bonds are reviewed in Chapters 16–17. Another common feature is a *step-up* feature, which states that the bond will pay a higher coupon after a set period of time, say ten years if it is not redeemed at that point. The higher coupon is often set at a punitive rate, say 100 basis points higher than the initial coupon, which acts as an incentive to the issuer to redeem the bond after the initial period. Step-up subordinated bonds are common in the sterling market, in both fixed-date and perpetual form, and have been issued by smaller borrowers such as independent merchant banks.⁷

The US market has also been tapped by borrowers issuing subordinated bonds that are convertible into the ordinary shares of another company. These are known as *exchangeable bonds*.

⁷ For example in 1994 perpetual step-up sterling notes were issued by banks such as NM Rothschild, Barings and Robert Fleming.

14.7 Redemption provisions

Highly-rated corporate borrowers often are able to issue bonds without indicating specifically how they will be redeemed (by implication, maturity proceeds will be financed out of the company's general cash flows or by the issue of another bond). This luxury is not always available to borrowers with low ratings. To make their debt issue more palatable to investors, they may make specific provisions for paying off a bond issue on its maturity date. We consider the most common provisions in this section.

14.7.1 Sinking funds

The term *sinking fund* originally referred to a ring-fenced sum of cash that was put aside to form the proceeds used in the repayment of a fixed-term bond. Although the operation of sinking funds have since changed, the term is still used. Essentially a sinking fund facility is where a set proportion of a bond issue is redeemed every year, say 5% of the nominal value, until the final year when the remaining outstanding amount is repaid. This outstanding amount is known as a *balloon maturity*. Although it is still possible to encounter the original type of sinking fund, the majority of them are those that pay off a small part of an issue each year during its life. Sinking funds that are set up for and apply to a specific issue are known as *specific* sinking funds. There are also sinking funds that are set up by issuers and apply to their entire range of bonds. These are known as *blanket* or *aggregate* sinking funds.

In most cases the issuer will pass the correct cash proceeds to the bond's trustee, who will use a lottery method to recall bonds representing the proportion of the total nominal value outstanding that is being repaid. The trustee usually publishes the serial numbers of bonds that are being recalled in a newspaper such as *The Wall Street Journal* or the *Financial Times*. Another method by which bonds are repaid is that the issuer will purchase the required nominal value of the bonds in the open market; these are then delivered to the trustee, who cancels them. There are two types of sinking funds, those that pay off the same amount of the issue each year, say 5% of the outstanding nominal value, or those that pay a progressively greater amount each year, say 5% in the first year and then 1% more each year until maturity, when the entire amount is redeemed. It is common to find utility and energy companies in the US that have bond sinking funds that incorporate this variable provision. A *doubling option* in a sinking fund entitles the issuer to repay double the amount that was originally specified as going to be repaid, while a provision that allows the issuer to repay an amount larger than the specified value is known as an *accelerated provision*.

The price at which bonds are redeemed by a sinking fund is usually par. If a bond has been issued above par, the sinking fund may retire the bonds at the issue price and gradually decrease this each year until it reaches par. Sinking funds reduce the credit risk applying to a bond issue, because they indicate to investors that provision has been made to repay the debt. However there is a risk associated with holding them, in that at the time bonds are paid off they may be trading above par due to a decline in market interest rates. In this case investors will suffer a loss if it is their holding that is redeemed.

14.7.2 Redemption and asset sales

Bonds that are secured through a charge on fixed assets such as property or plant often have certain clauses in their offer documents that state that the issuer cannot dispose of the assets without making provision for redemption of the bonds, as this would weaken the collateral backing for the bond. These clauses are known as *release-of-property* and *substitution-of-property* clauses. Under these clauses, if property or plant is disposed, the issuer must use the proceeds (or part of the proceeds) to redeem bonds that are secured by the disposed assets. The price at which the bonds are retired under this provision is usually par, although a special redemption price other than par may be specified in the repayment clause.

14.7.3 Call and refund provisions

A large number of corporate bonds, especially in the US market, have a call provision on dates ahead of the stated maturity date. Borrowers prefer to have this provision attached to their bonds as it enables them to refinance debt at cheaper levels when market interest rates have fallen significantly below the level they were at at the time of the bond issue. A call provision is a negative feature for investors, as bonds are only paid off if their price has risen above par. Although a call feature indicates an issuer's interest in paying off the bond, because they are not attractive for investors callable bonds pay a higher yield than non-callable bonds of the same credit quality and maturity.

In general callable bonds are not callable for the first five or 10 years of their life, a feature that grants an element of protection for investors. Thereafter a bond is usually callable on set dates up to the final maturity date. In the US

market another restriction is the *refunding redemption*. This prohibits repayment of bonds within a set period after issue with funds obtained at a lower interest rate or through issue of bonds that rank with or ahead of the bond being redeemed. A bond with refunding protection during the first five or 10 years of its life is not as attractive to a borrower as a bond with absolute call protection. Bonds that are called are usually called at par, although it is common also for bonds to have a call schedule that states that they are redeemable at specified prices above par during the call period.

There are also bonds that are callable in part rather than in whole. In this case the issuer or trustee will select the bonds to be repaid on a random basis, with the serial numbers of the bonds being called published in major news publications.

14.8 Corporate bond risks

The market risks associated with holding corporate bonds are identical to those reviewed in Chapters 7–10. Corporate bonds however hold additional risk for investors, unlike the developed market government bonds that were (implicitly) the subject of the earlier chapters. Although we cover this topic in greater detail in the chapter on risk management, in this section we introduce the additional risks to investors of holding corporate debt.

14.8.1 Credit risk

Unlike a (developed economy) government bond, holding a corporate bond exposes investors to *credit risk*, also known as *default risk*. This is the risk that the issuing company will default on its bond's coupon payments or principal repayment, resulting in a loss to the bondholders. Default may occur due to general financial difficulties, which may turn out to be short-term in nature, or a prelude to bankruptcy and liquidation. In the most extreme case it may be many years, if ever, before investors receive some of their money back.

The price of a corporate bond reflects the market's view of the credit risk associated with holding it. If the credit risk is perceived to be low, the *spread* of the issuer's bonds over the equivalent-maturity government bond will be low, while if the credit risk is deemed to be high, the yield spread will be correspondingly higher. This yield spread is sometimes referred to as the *credit spread* or *quality spread*. A 10-year bond issued by a highly rated corporate borrower will have a lower spread than a 10-year bond issued by a borrower with a lower credit rating. The higher yield on the bond issued by the lower-rated borrower is the compensation required by investors for holding the paper. Bond issuers are rated for their credit risk by both investment houses' internal credit analysts and by formal rating agencies such as Standard & Poor's, Moody's, Duff and Phelps and FitchIBCA. This topic is reviewed in Chapter 30.

Figure 14.5 illustrates an hypothetical "credit structure of interest rates". The credit structure is dynamic and will fluctuate with changes in market conditions and the general health of the economy.

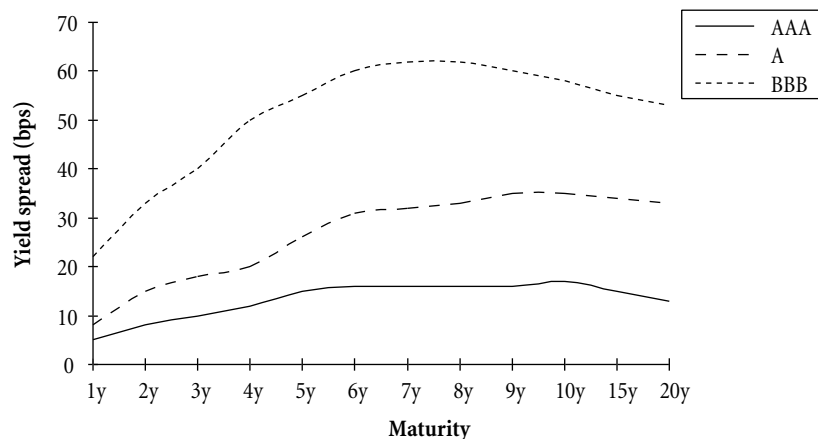


Figure 14.4: Credit structure of interest rates.

Credit spreads over government yields and between corporate borrowers of different credit quality fluctuate with market conditions and in line with the business cycle. Spreads are highest when an economy is in a recession and corporate health is relatively weak. A significant downward market correction also tends to widen credit spreads as investors embark on a “flight to quality” that depresses government bond yields. At the height of an economic boom spreads tend to be at their narrowest, not only because corporate balance sheets are in healthy shape but also because investors become less risk averse in times of a strong economy. This affects low-rated borrowers as well as highly-rated ones, so that spreads fall generally during an economic boom. In Figure 14.4 note how the credit spread for the hypothetical BBB-rated bonds falls as maturity increases; this is because analysts view the credit spread as being more liable to improve over time, as a company is upgraded, while a bond issued by a company that is already well-rated can only stay where it is or suffer a downgrade.

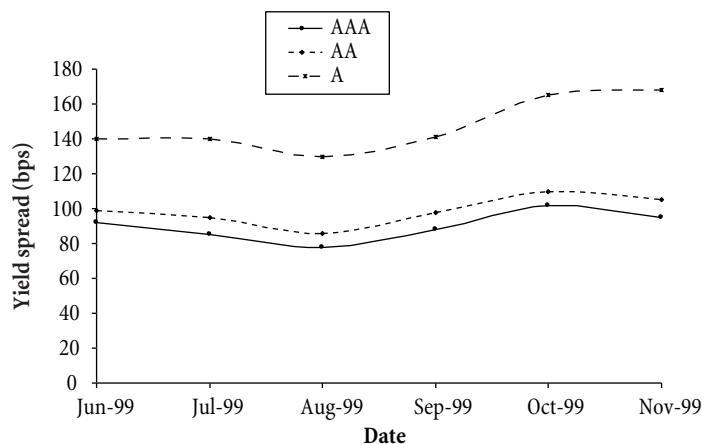


Figure 14.5: Sterling bonds 10-year yield spreads over gilts. Source: Halifax plc.

14.8.2 Liquidity risk

A bond with a ready market of buyers and sellers is always more attractive to investors than one that is difficult to trade. Such a bond is called a *liquid* bond. Liquidity is a function of the ease with which market participants may buy or sell a particular bond, the number of market makers that are prepared to quote prices for the bond, and the spread between the buying and selling price (the bid-offer spread). Illiquid bonds will have wide spreads and perhaps a lack of market makers willing to quote a bid price for them. Although some government bonds can sometimes be illiquid, liquidity risk, which is the risk that a bond held by an investor becomes illiquid, is primarily a corporate bond market risk. The yield on a bond that is, or about to become, illiquid, will be marked up by the market in compensation for the added risk of holding it. The best gauge of an issues liquidity is the size of its bid-offer spread. Government bonds frequently have a spread of 0.03 per cent or less, which is considered very liquid. Corporate spreads are wider but a spread of 0.10 to 0.25, up to 0.50 of one per cent is considered liquid. A bid-offer spread wider than this is considered illiquid, while a spread of say 1% or more is virtually non-tradeable. Some bonds are not quoted with a bid price or an offer price, indicating they are completely illiquid.

14.8.3 Call risk

In the previous section we stated that many domestic bond issues in the US market have call or refunding features attached to them. Such callable bonds have an added risk associated with holding them, known as *call risk*. This is the risk that the bond will be called at a time and price that is disadvantageous to bondholders or exposes them to loss. For example consider a 10-year bond with a call provision that states that the bond may be called at any time after the first five years, at par. For the last five years of the bond's life, if market interest rates fall below the bond's coupon rate, the price of the bond will not rise above par by as much as that of a similar maturity (and similar credit quality) bond that has no call feature. The price/yield relationship of a bond callable at par differs from a conventional bond, as the price will be less responsive to downward moves in yield once the price is at par.

14.8.4 Event risk

Event risk is peculiar to corporate bonds. This is the risk that, as a result of an unexpected corporate event, the credit risk of a bond increases greatly, so that the yield of the bond rises very quickly to much higher levels. The events can be external to the company, such as a natural disaster or regulatory change, or internal such as a merger or acquisition. A natural disaster may be an earthquake or flood, while regulatory change might be a change in the accounting or tax treatment of certain types of corporate debt. Event risk can effect companies across an industry. An example of this was the fall of Barings Bank in 1995. The owners of Barings subordinated perpetual bonds suffered a loss as a result of the bankruptcy of the company, however holders of bonds issued by similar companies also suffered losses as the market marked the yields on these bonds up as well. Figure 14.6 shows the change in average yield spread for bonds issued by N.M. Rothschild and Robert Fleming at the time of the Barings crash.

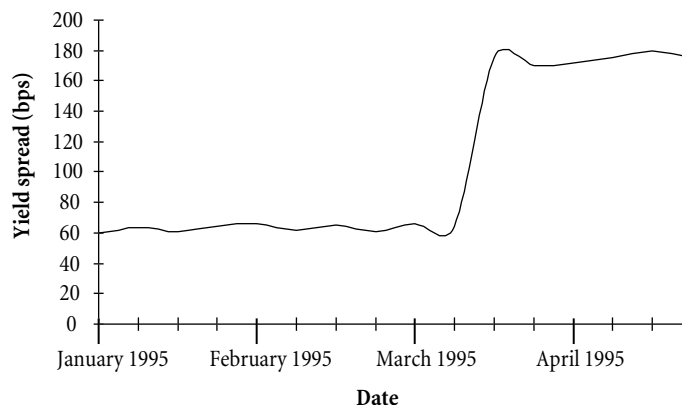


Figure 14.6: Impact of Barings Bank crisis on yield spread of bonds issued by similar companies. Source: ABN Amro Hoare Govett Sterling Bonds Limited.

A takeover is another example of an event that may result in loss for bondholders. For example if, as a result of a merger or takeover the debt of the amalgamated company is downgraded, bondholders will suffer a loss in capital value. To protect against event risk, bonds may have provisions in them that require an acquiring company to repurchase them, under specified conditions. Such provisions are known as *poison puts*. Other features that may be included in the terms of corporate bonds are *maintenance of net worth* and *offer to redeem* clauses. The former provision states that if an issue falls below a certain level of net worth, the borrower must redeem the bond at par. The latter clause is similar except that it does not apply to an entire issue, but only to those bonds whose holders wish to redeem.

14.8.5 Spens clause

In corporate markets a *Spens clause* is sometimes included in a bond's issue terms. This relates to the redemption proceeds of the bond. Under the terms of the clause, if there is a change in control of the issuing company, the company is required to redeem the notes within 45 days of the change of ownership. If the bonds are not redeemed in this time, bondholders may give notice to the issuer that the bond must be retired. The redemption amount is determined in accordance with (14.1) below, which essentially states that the yield at which the bond is redeemed is a spread over the benchmark government bond against which the corporate bond is priced. This spread is negotiable between the issuer and the bondholders, unless it is specified in the Spens clause itself.

$$P_{red} = \sum_{t=N_{red}}^{t=N_{mat}} \frac{(C + M)}{(1 + ry)^{((t - N_{red})/B)}} \quad (14.1)$$

where

P_{red} is the redemption proceeds
 N_{mat} is the maturity date of the bond
 N_{red} is the redemption date

C	is the coupon
M	is the principal repayment
ry	is $\left(1 + \frac{rm}{2}\right)^2 - 1$
rm	is the yield to maturity of the benchmark bond over which the bond is priced
t	is each coupon date and the number of days between N_{red} and N_{mat}
$t - N_{red}$	is the number of days between t and N_{red}
B	is the day-count base (360 or 265).

14.9 High-yield corporate bonds

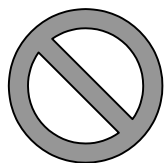
High-yield bonds were developed in the US corporate bond market, and have not as yet emerged as a significant investment outside the dollar market, except perhaps in the UK. The bonds were previously known as *junk bonds*, and this term is still used occasionally. They refer to bond issues with very low credit ratings, typically below BBB (S&P rating). Bonds in this category may not necessarily have started life as high-yield debt; they may have been rated as investment grade when originally issued and then suffered successive rating downgrades until rated as non-investment grade. The majority of high-yield bonds however were rated as below investment grade at the time of issue. Some of the bond structures are quite sophisticated examples of financial engineering, as when they have been issued as part of a “leveraged buy-out” finance package or as part of a re-capitalisation that results in higher levels of debt. For example a high-yield bond structure may include deferred coupons, with no coupon payments for a number of years after issue. This recognises the fact that the debt burden of the issuer may result in severe cash flow problems in the early years, so that coupons are deferred. Where there is a deferred coupon structure, the bond may include a *step-up* feature, so that after a period of time the coupon rate is increased, to compensate for the low (or zero) coupon rate at the start of the bond’s life.

The high-yield bond sector is introduced in Chapter 29.

14.10 Corporate bond offering circular

The following is an example of a typical bond issue offering circular, relating to a plain vanilla US dollar bond issued by an hypothetical company in the Euro markets. The bond has been underwritten by an hypothetical syndicate of banks, with two hypothetical banks involved as joint lead-managers, Jones Brothers International (Europe) and John Paul George Corporation. The content of the circular is fairly representative of a bond issue in the international markets, and there will be differences where for example, the bond structure is not plain vanilla. These will only be detail differences however; the following can be taken to be a standard and fairly representative document.

OFFERING CIRCULAR



JACKFRUIT CORPORATION

(INCORPORATED IN SURREY, ENGLAND)

U.S. \$500,000,000

6¾% Senior Notes due 7 June 2009

Issued pursuant to its

EURO MEDIUM-TERM NOTE PROGRAM

The U.S. \$500,000,000 67/8 % Senior Notes due on 7 June 2009 (the “Notes”) of Jackfruit Corporation (the “Issuer”) will be issued pursuant to the Issuer’s Euro Medium-Term Note Program (the “Program”). Interest on the Notes will be payable annually on 7 June of each year, commencing on 7 June 1999, and at maturity. The Notes will not be redeemable prior to maturity except in certain circumstances affecting taxation, as more

fully described in the Information Memorandum referred to below under the heading “Terms and Conditions of the Notes.”

Applications have been made for listing of and permission to deal in the Notes on The Stock Exchange of Singapore Limited and The Stock Exchange of Hong Kong Limited, which are expected to be effective on or about 23 June 1998 and 24 June 1997, respectively. Application will be made to The London Stock Exchange Limited (the “London Stock Exchange”) for the Notes to be issued under the Issuer’s Program, which will be amended in order to, among other things, increase the maximum principal amount of medium-term notes outstanding at any one time. An Information Memorandum and Information Memorandum Addendum, each expected to be dated 26 June 1998, together with the Issuer’s Annual Report on for the year ended 31 December 1997 and its Quarterly Report for the quarter ended 31 March 1998, will comprise the listing particulars expected to be approved by the London Stock Exchange (the “London Listing Particulars”). Draft copies of such documents expected to comprise the London Listing Particulars may be inspected during normal business hours on any weekday (excluding Saturdays) at the principal place of business of the Issuer and at the offices of the Fiscal Agent in London. It is expected that the Notes will be admitted to the Official List of the London Stock Exchange, subject to the satisfaction of certain conditions, with effect on or about 2 July 1998.

The Notes will initially be represented by a temporary global Note without coupons which will be deposited with a common depository acting on behalf of Cedel Bank and Euroclear on or about 20 June 1998 and will be exchangeable for definitive Notes in bearer form with coupons not earlier than 30 July 1998, upon certification of non-U.S. beneficial ownership.

This Offering Circular (including the Pricing Supplement relating to the Notes set forth herein) is supplemental to, and should be read in conjunction with, the Information Memorandum and Information Memorandum Addendum, each dated 19 June 1997, as may be amended or superseded from time to time (collectively, the “Information Memorandum”), which are annexed hereto. Terms defined in the Information Memorandum shall have the same meaning when used in this Offering Circular. References herein to “Dollars,” “\$” and “U.S.\$” are to the lawful currency of the United States of America.

JONES BROTHERS INTL (EUROPE)
NATIONAL CREDIT BANK
AKERS MARKETS
STEGAS SECURITIES LIMITED
THE ABC BANK plc

JOHN PAUL GEORGE CORPORATION
SANBIN INTERNATIONAL
HENRY MARSHALL SECURITIES LIMITED
CAPE LIMITED
CLAXTON CAPITAL MARKETS (EUROPE) LIMITED

The date of this document is 18 June 1998.

TERMS OF THE NOTES

The Terms and Conditions of the Notes will include the information set forth in the Information Memorandum under the heading “Terms and Conditions of the Notes,” as supplemented and modified by the Pricing Supplement set forth below.

Pricing Supplement Dated 18 June, 1998

Jackfruit Corporation
U.S. \$500,000,000
67/8% Senior Notes due 7 June 2009 (the “Notes”)
Issued pursuant to its
Euro Medium-Term Note Program

Terms used herein shall be deemed to be defined as such for the purposes of the Conditions; items in the form of pricing supplement included in the Information Memorandum that are not applicable to the Notes are omitted. The terms for the issue of the Notes are as follows:

TYPE OF NOTE

Form of Notes to be initially issued:	Temporary global Note in bearer form. Temporary Globe Note is exchangeable for a Permanent Global Note on 30 days' notice
Ranking	The Notes are Senior Notes and will constitute direct unconditional and unsecured obligations of the Issuer and will rank pari passu in right of payment with all outstanding unsecured senior debt of the Issuer
Fixed Rate/Floating Rate/Zero Coupon/ Original Issue Discount/Indexed Redemption Amount/Indexed Interest/Dual Currency/Partly Paid/ Installment/Extendible/Renewable/ combination/other:	Fixed Rate
Convertible automatically or at the option of the Issuer and/or Noteholders into Note(s) of another Interest Basis:	No

DESCRIPTION OF THE NOTES

Provisions for exchange of Notes:	Interests in the temporary global Note in bearer form are exchangeable for definitive Notes in bearer form on or after the Exchange Date following certification as to the non-U.S. beneficial ownership thereof.
Talons for future Coupons to be attached to definitive Notes in bearer form:	No
Series Number:	10
Nominal Amount (Face Amount) of Notes to be issued:	U.S. \$500,000,000
Specified Currency:	United States Dollars
Specified Denomination (s):	U.S. \$1,000, U.S. \$10,000 and U.S. \$100,00
Issue Price:	99.264%
Issue Date:	20 June 1998
Proceeds to Issuer:	U.S. \$488,254,800

PROVISIONS RELATING TO INTEREST PAYABLE

Interest Basis:	Fixed Rate
Fixed Rate of Interest:	6¾%
Fixed Interest Dates:	7 June of each year, beginning 7 June 1999

PROVISIONS REGARDING REDEMPTION/MATURITY

Maturity Date:	7 June 2009
Redemption at Issuer's option:	No, except in certain circumstances affecting taxation, as set forth in Condition 10(b)
Redemption at Noteholder's option:	No
Final Redemption Amount for each Note:	100%

GENERAL PROVISIONS APPLICABLE TO THIS ISSUE OF NOTES

Additional sales restrictions:

In addition to the sales restrictions set forth in the Information Memorandum under the heading "Subscription and Sale," the following restrictions apply:

United Kingdom: With regard to the United Kingdom, references to Article 11 (3) of the Financial Services Act 1986 (Investment Advertisements) (Exemptions) Order 1995 in the Information Memorandum shall be deemed to refer to Article 11 (3) of the Financial Services Act 1986 (Investment Advertisements) (Exemptions) Order 1996 (as amended).

Hong Kong: Each Manager has represented and agreed that it has not offered or sold and will not offer or sell in Hong Kong by means of any document, any Notes other than to persons whose ordinary business it is to buy or sell shares or debentures (whether as principal or agent) or in circumstances which do not constitute an offer to the public within the meaning of the Companies Ordinance of Hong Kong.

Each Manager has represented and agreed that until such time as the listing of the Notes on The

Stock Exchange of Hong Kong Limited has been formally approved or unless it is a person permitted to do so under the securities laws of Hong Kong, it has not issued or had in its possession for the purpose of issue, and will not issue or have in its possession for the purpose of issue, in Hong Kong, any advertisement, invitation or document relating to the Notes, other than with respect to Notes intended to be disposed of to persons outside Hong Kong or to be disposed of in Hong Kong only to persons whose business involves the acquisition, disposal or holding of securities, whether as principal or agent.

Singapore: No prospectus in connection with the offer of the Notes will be registered with the Registrar of Companies in Singapore and the Notes will be offered in Singapore pursuant to an exemption invoked under Section 106C of the Companies Act, Chapter 50 of Singapore (the “Singapore Companies Act”). Accordingly each Manager has represented and agreed that documents or material prepared in connection with the offer of the Notes may not be, and have not been, circulated, and the Notes may not be, and have not been, offered or sold, directly or indirectly to the public or any member of the public in Singapore other than (i) to an institutional investor or other person specified in Section 106C of the Singapore Companies Act, (ii) to a sophisticated investor, and in accordance with the conditions, specified in Section 106D of the Singapore Companies Act or (iii) otherwise pursuant to, and in accordance with the conditions of, any other applicable provision of the Singapore Companies Act.

Method of distribution:

Syndicated
 Lead Managers:
 Jones Brothers International (Europe)
 John Paul George Corporation
 Co-Lead Managers:
 National Credit Bank
 Sanbin International
 Akers Securities
 Henry Marshall Securities Limited
 Stegas Securities
 The ABC Bank plc
 Claxton Capital Markets (Europe) Limited

Commissions:	0.275% Management and Underwriting 1.35% Selling
Stabilizing Dealer/Manager:	In connection with the issue of the Notes, Jones Brothers International (Europe) may over-allot or effect transactions which stabilize or maintain the market price of the Notes at a level which might not otherwise prevail. Such stabilizing, if commenced, may be discontinued at any time. Such stabilizing shall be in compliance with all relevant laws and regulations.
Notes to be listed:	Yes
Stock Exchange(s):	The London Stock Exchange Limited The Stock Exchange of Singapore Limited The Stock Exchange of Hong Kong Limited With regard to the London Stock Exchange listing of this issue of Notes, see "Other terms or special conditions" below.
Common Code for Euroclear and Cedel Bank and ISIN:	Common Code: 001234554321 ISIN: XS001234554321

Selected bibliography

Alworth, J.S., Borio, C.E.V., "Commercial Paper Markets: A Survey," BIS Economic Papers No. 37, Bank for International Settlements, 1993.

Bank of England, *Quarterly Bulletin*, August 1999.

Economist, "Much indebted to EU", 18 September 1999.

Fabozzi, F.J., *Bond Markets, Analysis and Strategies*, 2nd edition, Prentice Hall, 1993.

Schinasi, G.J., Todd Smith, R., "Fixed Income Markets in the United States, Europe, and Japan: Some Lessons for Emerging Markets," a Working Paper of the International Monetary Fund, No. 98/173, December 1998.

Wilson, R.S., Fabozzi, F.J., *Corporate Bonds; Structures and Analysis*, FJF Associates, 1996.

Questions and exercises

1. What is a callable bond? What are the disadvantages for a bondholders of holding such a bond?
2. Erin plc issues a subordinated bond backed with a sinking fund. Does this make the bond more attractive to investors? Are there any disadvantages for investors?
3. Describe event risk. List some possible examples of event risk for investors, including those that are external to the issuing company.
4. Consider the following hypothetical corporate bond issues:

Issuer	Rating	Yield	Spread (bp)	Government benchmark
Global Telecom plc	AA	6.87%	40	10
United Petroleum plc	AAA	6.77%	30	10
National PowerStation plc	AAA	7.61%	62	20
General Foods Inc	AA	7.66%	67	20
JPGR Ltd	BBB	8.44%	145	20

- (a) Explain the “rating” behind each bond.
 - (b) Which bond has the highest level of credit risk?
 - (c) What does the “spread” refer to? What is the spread a function of? How does measure the spread at any one time? Why is their a spread for each of the bonds?
 - (d) What is meant by “government benchmark”?
5. Consider the same hypothetical bonds discussed in the previous question. For these bonds:
 - (a) Should a AAA-rated bond trade at a higher or lower yield than a AA-rated bond of the same maturity? Give reasons for your answer.
 - (b) What is the spread between the Global Telecom plc and the
 - (c) United Petroleum plc issue?

Assume now that the United Petroleum plc bond is callable. What effect would you expect this to have on the yield of the bond, and its spread against the Global Telecom bond?
6. What are the additional risks associated with holding corporate bonds, compared to those associated with holding government bonds?
7. Discuss the main requirements of an efficient primary market in corporate bonds.
8. What is the main method by which corporate bonds are issued in the primary market. How does this differ from the main method of issuing government securities?
9. What is liquidity risk?
10. America Inc. issues both a debenture and a subordinated bond in the US domestic market. Both bonds have five-year fixed maturities. Which bond will have the higher yield?

15 Analysis of Bonds With Embedded Options

In Chapter 4 we reviewed the yield to maturity calculation, the main measure of bond return used in the fixed income markets. For conventional bonds the yield calculation is relatively straightforward because the issue's redemption date is known and fixed. This means that the future cash flows that make up the total cash flows of the bond are known with certainty. As such the cash flows that are required to calculate the yield to maturity are easily ascertained. Callable, put-able and sinking fund bonds, generally termed bonds with *embedded options*, are not as straightforward to analyse. This is because some aspect of their cash flows, such as the timing or the value of their future payments, are not certain. The term *embedded* is used because the option element cannot be separated from the bond itself. Since callable bonds have more than one possible redemption date, the collection of future cash flows contributing to their overall return is not clearly defined. If we wish to calculate the yield to maturity for such bonds, we must assume a particular redemption date and calculate the yield to this date. The market convention is to assume the first possible maturity date as the one to be used for yield calculation if the bond is priced above par, and the last possible date if the bond is priced below par. The term *yield-to-worst* is sometimes used to refer to a redemption yield calculation made under this assumption; this is the Bloomberg term. If the actual redemption date of a bond is different to the assumed redemption date, the measurement of return will be meaningless and irrelevant.

The market therefore prefers to use other measures of bond return for callable bonds. The most common method of return calculation is something known as *option-adjusted spread analysis* or OAS analysis; a very good account of OAS analysis is contained in Windas (1994). In this chapter we present one of the main methods by which callable bonds are priced. Although the discussion centres on callable bonds, the principles apply to all bonds with embedded option elements in their structure.

15.1 Understanding embedded option elements in a bond

Consider an hypothetical sterling corporate bond issued by ABC plc with a 6% coupon on 1 December 1999 and maturing on 1 December 2019. The bond is callable after five years, under the schedule shown at Table 15.1. We see that the bond is first callable at a price of 103.00, after which the call price falls progressively until December 2014, after which the bond is callable at par.

Date	Call Price	Date	Call Price
01-Dec-2004	103.00	01-Dec-2009	101.75
01-Dec-2005	102.85	01-Dec-2010	101.25
01-Dec-2006	102.65	01-Dec-2011	100.85
01-Dec-2007	102.50	01-Dec-2012	100.45
01-Dec-2008	102.00	01-Dec-2013	100.25
		01-Dec-2014	100.00

Table 15.1: Call schedule for “ABC plc” 6% bond due December 2019.

Although our example is hypothetical, this form of call provision is quite common in the corporate debt market. The basic case can be stated quite easily; in our example the ABC plc bond pays a fixed semi-annual coupon of 6%. If the market level of interest rates rises after the bonds are issued, ABC plc effectively gain because it is paying below-market financing costs on its debt. If rates decline however, investors gain from a rise in the capital value of their investment, but in this instance their upside is capped by the call provisions attached to the bond.

The difference between the value of a callable bond and that of an (otherwise identical) non-callable bond of similar credit quality is the value attached to the option element of the callable bond. This is an important relationship and one that we will consider, but first a word on the basics of option instruments.

15.1.1 Basic features of options

An option is a contract between two parties. The buyer of an option has the right, but not the obligation, to buy or sell an underlying asset at a specified price during a specified period or at a specified time (usually the expiry date of the option contract). The price of an option is known as the *premium*, which is paid by the buyer to the seller or *writer* of the option. An option that grants the holder the right to buy the underlying asset is known as a *call* option; one that grants the right to sell the underlying asset is a *put* option. The option writer is short the contract; the buyer is long. If the owner of the option elects to *exercise* her option and enter into the underlying trade, the option writer is obliged to execute under the terms of the option contract. The price at which an option specifies that the underlying asset may be bought or sold is known as the exercise or *strike* price. The expiry date of an option is the last day on which it may be exercised. Options that can be exercised anytime from the time they are struck up to and including the expiry date are called *American* options. Those that can be exercised only on the expiry date are known as *European* options.

The profit/loss profiles for option buyers and sellers are quite different. The buyer of an option has her loss limited to the price of that option, while her profit can in theory be unlimited. The seller of an option has her profit limited to the option price, while her loss can in theory be unlimited, or at least potentially very substantial.

The value or price of an option is comprised of two elements, its *intrinsic value* and its *time value*. The intrinsic value of an option is the value to the holder of an option if it were exercised immediately. That is, it is the difference between the strike price and the current price of the underlying asset. The holder of an option will only exercise it if there is underlying intrinsic value. For this reason, the intrinsic value is never less than zero. To illustrate, if a call option on a bond has a strike price of £100 and the underlying bond is currently trading at £103, the option has an intrinsic value of £3. An option with intrinsic value greater than zero is said to be *in-the-money*. An option where the strike price is equal to the price of the underlying is said to be *at-the-money* while one whose strike price is above (call) or below (put) the underlying is said to be *out-of-the-money*.

The time value of an option is the difference between the intrinsic value of an option and its total value. An option with zero intrinsic value has value comprised solely of time value. That is,

$$\text{Time value of an option} = \text{Option price} - \text{Intrinsic value}$$

The time value reflects the potential for an option to move into the money during its life, or move a higher level of being in-the-money, before expiry. Time value diminishes steadily for an option up to its expiry date, when it will be zero. The price of an option on expiry is comprised solely of intrinsic value.

Later in this chapter we will illustrate how the price of a bond with an embedded option is calculated by assessing the value of the “underlying” bond and the value of its associated option. The basic issues behind the price of the associated option are considered here.¹ The main factors influencing the price of an option on an interest-rate instrument such as a bond are:

- the strike price of the option;
- the current price of the underlying bond, and its coupon rate;
- the time to expiry;
- the short-term risk-free rate of interest during the life of the option;
- the expected volatility of interest rates during the life of the option.

The effect of each of these factors will differ for call and put options and American and European options. There are a number of option pricing models used in the market, the most well-known of which is probably the Black–Scholes model. Market participants often use their own variations of models or in-house developed varieties. The fundamental principle behind the Black–Scholes model is that a synthetic option can be created and valued by taking a position in the underlying asset and borrowing or lending funds in the market at the risk-free rate of interest. Although it is the basis for certain subsequent option models and is still used widely in the market, it is not necessarily appropriate for certain interest-rate instruments. For instance Fabozzi (1997) points out the unsuitability of the Black–Scholes model for certain bond options, based on its underlying assumptions. As a result a number of other methods have been developed for callable bonds analysis.

¹ For a technical review of option pricing, see Chapters 43–46.

15.1.2 The call provision

A bond with early redemption provisions essentially is a portfolio containing an underlying conventional bond, with the coupon and maturity date of the actual bond, and a put or call option on this underlying issue. Analysis therefore is interest-rate dependent, it must consider the possibility of the option being exercised when valuing the bond. The value of a bond with an option feature is the sum of the values of the individual elements, that is the underlying bond and the option component. This is expressed at (15.1):

$$P_{bond} = P_{underlying} + P_{option} \quad (15.1)$$

Expression (15.1) above states simply that the value of the actual bond is composed of the value of the underlying conventional bond together with the value of the embedded option(s). The relationship would hold for a true conventional bond, as the option component value would be zero. For a put-able bond, an embedded put option is an attractive feature for investors as the put feature contributes to its value by acting as a floor on the bond's price. Thus the greater the value of the put, the greater the value of the actual bond. We can express this by re-writing (15.1) as (15.2):

$$P_{bond} = P_{underlying} + P_{put} \quad (15.2)$$

The expression at (15.2) above states that the value of a put-able bond is equal to the sum of the values of the underlying conventional bond and the embedded put option. If any of the components of the total price were to increase in value, then so would the value of the out-able bond itself.

A callable bond is viewed as a conventional bond together with a short position in a call option, which acts as a cap on the actual bond's price. This "short" position in a call option reduces the total value of the actual bond, so we present the bond price in the form (15.3):

$$P_{bond} = P_{underlying} - P_{call} \quad (15.3)$$

Equation (15.3) above states that the price of a callable bond is equal to the price of the underlying conventional bond less the price of the embedded call option. Therefore if the value of the call option were to increase, the value of the callable bond would decrease. That is, when a bondholder of such a bond sells a call option, they receive the option price; the difference between the price of the option-free bond and the callable bond at any time is the price of the embedded call option. The precise nature of the behaviour of the attached option element will depend on the terms of the callable bond issue. If the issuer of a callable bond is entitled to call the issue at any time after the first call date, the bondholder has effectively sold the issuer an American call option. However the call option price may vary with the date the option is exercised; this occurs when the call schedule for a bond has different call prices according to which date the bond is called. The underlying bond at the time the call is exercised is comprised of the remaining coupon payments that would have been received by the bondholder had the issue not been called. However for ease of explanation the market generally analyses a callable bond in terms of a long position in a conventional bond and a short position in a call option, as stated by (15.3) above. Note of course that the option is embedded; it does not trade in its own right. Nevertheless it is clear that embedded options are important elements not only in the behaviour of a bond but in its valuation as well.

15.2 The Binomial tree of short-term interest rates

In an earlier chapter we illustrated how a coupon bond yield curve could be used to derive spot (zero-coupon) and implied forward rates. A forward rate is defined as the one-period interest rate for a term beginning at a forward date and maturing one period later. Forward rates form the basis upon which a *binomial* interest-rate tree is built. For an introduction to a binomial process, see Appendix 15.1.

An option model that used implied forward rates to generate a price for an option's underlying bond on a future date would implicitly assume that interest rates implied by the yield curve today for a date in the future would occur with certainty. Such an assumption would essentially repeat the errors associated with yield-to-worst analysis, which would be inaccurate because interest rates do not remain unchanged from a future today to a future pricing date. To avoid this inaccuracy, a binomial tree model assumes that interest rates do not remain fixed but fluctuate

over time. This is done by treating implied forward rates, sometimes referred to as *short rates*,² as outcomes of a binomial process. In a binomial interest-rate process we construct a binomial tree of possible short rates for each future time period. In the binomial tree we model two interest rates as the possible outcomes of a previous time period, when the interest rate was known.

EXAMPLE 15.1 An introduction to arbitrage-free pricing

- Consider a hypothetical situation. Assume that the short-term yield curve describes the following environment:

Six-month rate: 5.00%

One-year rate: 5.15%

Assume further that in six months' time the then six-month rate will be either 5.01% or 5.50%, and that the probability of either new rate is equal at 50% each. Our capital market is a semi-annual one, that is convention is for bonds to pay a semi-annual coupon, as in the US and UK domestic markets. We can illustrate this state in the following way:

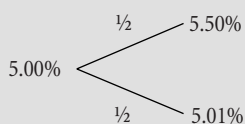


Figure 15.1

Figure 15.1 is a *binomial interest rate tree* or *lattice* for the six-month interest rate. The tree is called “binomial” because there are precisely two possibilities for the future level of the interest rate. Using this lattice, we can calculate the tree for the prices of six-month and one-year zero-coupon bonds. The six-month zero-coupon bond price today is given by $100 / (1 + (0.05/2))$ or 97.56098. The price tree is given at Figure 15.2.

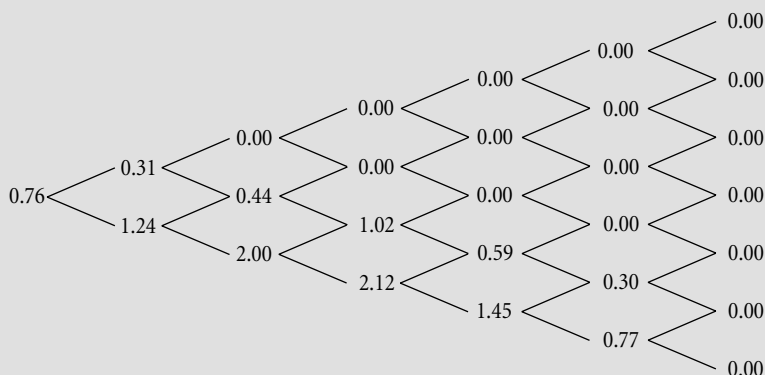


Figure 15.2

Period 0 is today; period 1 is the point precisely six months from today. Given that we are dealing with a six-month zero-coupon bond, it is apparent that there is only one *state of the world* whatever the interest rate is in period 1; the maturity value of the bond, which is 100.

The binomial lattice for the one-year zero-coupon bond is given in Figure 15.3.

² Although it is not strictly correct to refer to forward rates in this way.

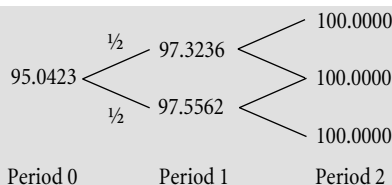


Figure 15.3

At period 0 the price of the one-year zero-coupon bond is $100/(1 + (0.0515/2)^2)$ or 95.0423. The price of the bond at period 1, at which point it is now a six-month piece of paper, is dependent on the six-month rate at the time, shown in the diagram. At period 2 the bond matures and its price is 100. The model at Figure 15.3 demonstrates that the average or *expected* value of the price of the one-year bond at period 1 is $((0.5 \times 97.3236) + (0.5 \times 97.5562))$ or 97.4399. This is the expected price at period 1, therefore using this the price at period 0 is $97.4399/(1 + 0.05/2)$ or 95.06332.

However, we know that the market price is 95.0423. This demonstrates a very important principle in financial economics, that markets do not price derivative instruments on the basis of their expected future value. At period 0 the one-year zero-coupon bond is a more risky investment compared to the shorter-dated bond; in the last six months of its life it will be worth either 97.32 or 97.55 depending on the direction of six-month rates. Investors' preference is for a bond that has a price of 97.4399 at period 1 with certainty. The price of such a bond at period 0 would be $97.4399/(1 + (0.05/2))$ or 95.0633. In fact the actual price of the one-year bond at that date, 95.0423, indicates the *risk premium* that the market places on the bond.

We can now consider the pricing of an option. What value should be given to a six-month call option maturing in six months' time (period 1) written on 100 nominal of the six-month zero at a strike price of 97.40? The binomial tree for this option is given at Figure 15.4. This shows that at period 1 if the six-month rate is 5.50% the call option has no value, because the price of the bond is below the strike price. If on the other hand the six-month rate is at the lower level, the option has a value of $97.5562 - 94.40$ or 0.1562.

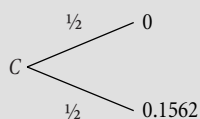


Figure 15.4

How do we calculate the price of the option? Option pricing theory states that to do this one must construct a *replicating portfolio* and find the value of this portfolio. In our example we must set up a portfolio of six-month and one-year zero-coupon bonds today that will have no value at period 1 if the six-month rate rises to 5.50%, but will have a value of 0.1562 if the rate at that time is 5.01%. If we let the value of the six-month and one-year bonds in the replicating portfolio be C_1 and C_2 respectively at period 1, we may set the following equations:

$$C_1 + 0.973236C_2 = 0 \quad (15.4)$$

$$C_1 + 0.975562C_2 = 0.1562. \quad (15.5)$$

The value of the six-month zero-coupon bond in the replicating portfolio at period 1 is 100 as it matures. In the case of an interest-rate rise the value of the one-year bond (now a six-month bond) at period 1 is 97.3236. The total value of the portfolio is given by the first expression above, which states that this value must also be equal to the value of the option. The second expression gives the value of the replicating portfolio in the even that rates decrease, when the option value is 0.1562.

Solving the expressions above gives us $C_1 = -65.3566$ and $C_2 = 67.1539$. What does this mean? Basically to construct the replicating portfolio we purchase 67.15 of one-year zero-coupon bonds and sell short 65.36 of the six-month zero-coupon bond. However the original intention behind the replicating portfolio was because we wished to price the option: the portfolio and the option have equal values. The value of the portfolio is a known quantity, as it is equal to the price of the six-month bond at period 0 multiplied by C_1 together with the price of

the one-year bond multiplied by C_2 . This is given by

$$(0.9756 \times -65.3566) + (0.950423 \times 67.1539) = 0.0627.$$

That is, the price of the six-month call option is 0.06. This is the *arbitrage-free* price of the option; below this price a market participant could buy the option and simultaneously sell short the replicating or portfolio and would be guaranteed a profit. If the option was quoted at a price above this, a trader could write the option and buy the portfolio. Note how the probability of the six-month rate increasing or decreasing was not part of the analysis. This reflects the arbitrage pricing logic. That is, the replicating portfolio must be equal in value to the option whatever direction interest rates move in. This means probabilities do not have an impact in the construction of the portfolio. This is not to say that probabilities do not have an impact on the option price, far from it. For example, if there is a very high probability that rates will increase (in our example), intuitively we can see that the value of an option to an investor will fall. However this is accounted for by the market in the value of the option or callable bond at any one time. If probabilities change, the market price will change to reflect this.

Let us now turn to the concept of *risk neutral* pricing. Notwithstanding what we have just noted about how the market does not price instruments using expected values, there exist risk-neutral probabilities for which the discounted expected value does give the actual price at period 0. If we let p be the risk-neutral probability of an interest rate increase and $(1-p)$ be the probability of a rate decrease, we may set p such that

$$\frac{97.3236p + 97.5562(1-p)}{1 + \frac{1}{2}0.05} = 95.0423.$$

That is, we can calculate a value for p such that the discounted expected value, using the probability p rather than the actual probability of $\frac{1}{2}$ provides the true market price. The above expression solves to give $p = 0.5926$.

In our example from the option price tree in Figure 15.5, given the risk-neutral probability of 0.5926 we can calculate the option price to be

$$\frac{(0.5926 \times 0) + (0.4074 \times 0.1562)}{1 + \frac{1}{2}0.05} = 0.0621.$$

This is virtually identical to the 0.062 option price calculated above. Put very simply risk-neutral pricing works by first finding the probabilities that produce prices of the replicating or *underlying* security equal to the discounted expected value. An option on the security is valued by discounting this expected value under the risk-neutral probability.

We can now turn to binomial trees. In the description above we had a two-period tree, moving to period 2 we might have Figure 15.6

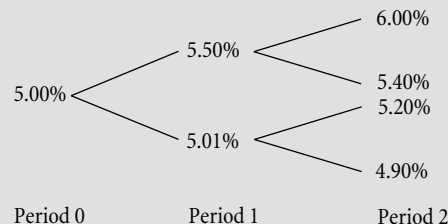


Figure 15.5

This binomial tree is known as a *non-recombining tree*, because each node branches out to two further nodes. This might seem more logical, and such trees are used in practice in the market. However implementing it requires a considerable amount of computer processing power, and it is easy to see why. In period 1 there are two possible levels for the interest rate, and at period 2 there are four possible levels. After N interest periods there will be 2^N possible values for the interest rate. If we wished to calculate the current price a 10-year callable bond that paid semi-annual coupons, we would have over 1 million possible values for the last period set of nodes. For a 20-year bond we would have over 1 trillion possible values. (Note also that binomial models are not

used with a six-month time step between nodes in practice, but have much smaller time steps, further increasing the number of nodes).

For this reason certain market practitioners prefer to use a *recombining* binomial tree, where the upward-downward state has the same value as the downward-upward state. This is shown below.

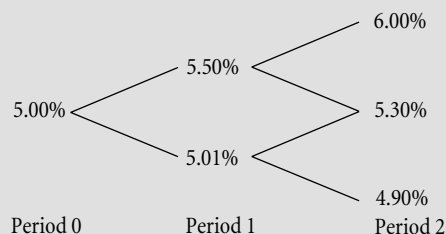


Figure 15.6

The number of nodes and possible values at the latest time step is much reduced in a recombining tree. For example the number of nodes used to price a 20-year bond that was being priced with one-week time steps would be $(52 \times 20) + 1$ or 1041. Implementation is therefore more straightforward with a recombining tree.

15.3 Pricing callable bonds

We can now consider a simple pricing method for callable bonds. We will assume a binomial term structure model. The background to this is given in the box above, *Introduction to arbitrage-free pricing*. It is well worth reading this box, especially if one is not familiar with binomial models or the principle of the arbitrage-free pricing of financial instruments. For a background on the binomial process itself see Appendix 15.1. Using the binomial model we can derive a *risk-neutral* binomial lattice, where each lattice carries an equal probability of upward or downward moves, for the evolution of the six-month interest rate. The time step in the lattice is six months. This model is then used to price an hypothetical semi-annual coupon bond with the following terms:

Coupon	6%
Maturity	Three years
Call schedule:	
Year 1	103.00
Year 1.5	102.00
Year 2	101.50
Year 2.5	101.00
Year 3	100.00

Table 15.2

The tree is shown at Figure 15.7.

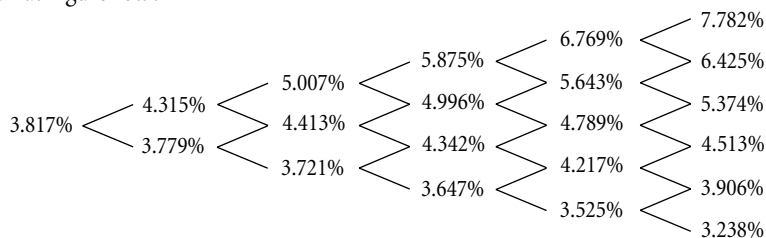


Figure 15.7

In the first instance we construct the binomial tree that describes the price process followed by the bond itself, if we ignore its call feature. This is shown at Figure 15.8. Note that the maturity value of the bond on the redemption

date is given as 100.00, that is, we perform the analysis on the basis of the bond's ex-coupon value. The final cash flow would of course be 103.00.

We construct the tree from the final date backwards. At each of the nodes at year 3, the price of the bond will be 100.00, the (ex-coupon) par value. At year 2.5 the price of the bond at the highest yield will be that at which the yield of the bond is 7.782%. At this point, the price of the bond after six months will be 103.00 in both the "up" state and the "down" state. Following risk-neutral pricing therefore the price of the bond at this node is

$$P_{bond} = \frac{0.5 \times 103 + 0.5 \times 103}{1 + (0.07782/2)} = 99.14237.$$

The same process is used to obtain the prices for every node at year 2.5. Once all these prices have been calculated, we repeat the process for the prices at each node in year 2. At the highest yield, 6.769%, the two possible future values are

$$99.14237 + 3.0 = 102.14237 \text{ and } 99.79411 + 3.0 = 102.79411.$$

Therefore the price of the bond in this state is given by

$$P_{bond} = \frac{0.5 \times 102.14237 + 0.5 \times 102.79411}{1 + (0.06769/2)} = 99.11374.$$

The same procedure is repeated until we have populated every node in the lattice. At each node the ex-coupon bond price is equal to the sum of the expected value and coupon, discounted at the appropriate six-month interest rate. The completed lattice is shown at Figure 15.8

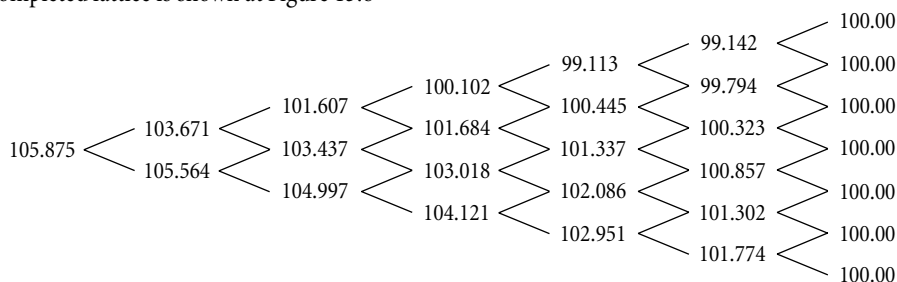


Figure 15.8

Once we have calculated the prices for the conventional element of the bond, we can calculate the value of the option element on the callable bond. This is shown in Figure 15.9. On the bond's maturity date the option is worthless, because it is an option to call at 100, which is the price the bond is redeemed at in any case. At all other node points a valuation analysis is called for.

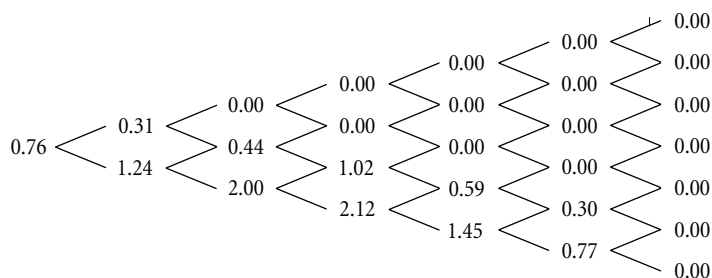


Figure 15.9

The holder of the option in the case of a callable bond is the issuing company. At any time during the life of the bond, the holder will either exercise the option on the call date or elect to hold it to the next date. The option holder must consider:

- the value of holding the option for an extra period, denoted by P_{CB} ;
- the value of exercising the option straight away, denoted P_C .

If the value of the former exceeds that of the latter, the holder will elect to not exercise, and if the value at the exercise date is higher the holder will exercise immediately. At year 2.5 call date for example, there is no value in holding the option because it will be worthless at year 3. Therefore at any point where the option is in-the-money the holder will exercise.

We can express the general valuation as follows. The value of the option for immediate exercise is V_t ; the value if one is holding on to the option for a further period is V_T . Additionally let P be the value of the bond at any particular node, S the call option price, and V_h and V_l the values of the option in the up-state and down-state respectively. The value of the option at any specified node is V . The six-month interest rate at any specified node point is r . We have

$$V_T = \frac{0.5V_h + 0.5V_l}{1 + \frac{1}{2}r} \quad (15.6)$$

$$V_t = \max(0, P - S)$$

while the expression for V is $V = \max(V_T, V_t)$.

The rule is as demonstrated above, to work backwards in time and apply the expression at each node, which produces the option value binomial lattice tree.

The general rule with an option is as they have more value “alive than dead”. This means that it is sometimes it is optimal to run an in-the-money option rather than exercising straight away. The same is true for callable bonds. There are a number of factors that dictate whether an option should be exercised or not. The first is the asymmetric profile resulting when the price of the “underlying” asset rises; option holders gain if the price rises, but will only lose the value of their initial investment if the price falls. Therefore it is optimal to run with the option position. There is also time value, which is lost if the option is exercised early. In the case of callable bonds, it is often the case that the call price decreases as the bond approaches maturity. This is an incentive to delay exercise until a lower exercise price is available. The issue that may influence the decision to exercise sooner is coupon payments, as interest is earned sooner.

To return to our hypothetical example, we can now complete the price tree for the callable bond. Remember that the option in the case of a callable bond is held by the issuer, so the value of the option is subtracted from the price of the bond to obtain the actual value. We see from Figure 15.10 that the price of the callable bond today is 105.875 – 0.76 or 105.115. The price of the bond at each node in the lattice is also shown. By building a tree in this way, which can be programmed into a spreadsheet or as a front-end application, we are able to price a callable or puttable bond.

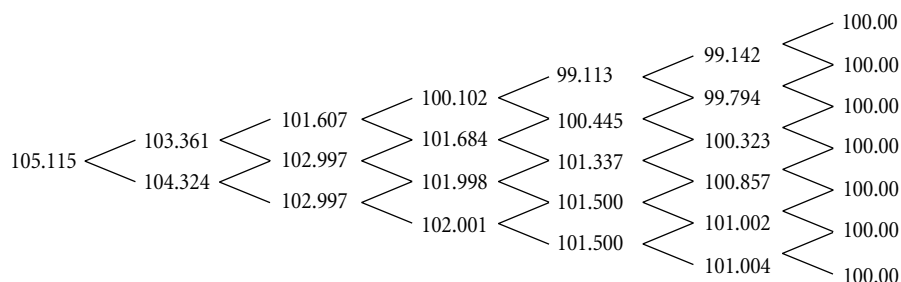


Figure 15.10

15.4 Price and yield sensitivity

As we saw in Chapter 7, the price/yield relationship for a conventional vanilla bond is essentially convex in shape, while for a bond with an option feature attached this relationship changes as the price of the bond approaches par, at which the bond is said to exhibit *negative convexity*. This means that the rise in price will be lower than the fall in price for a large change in yield of a given number of basis points. We summarise the price/yield relationship for both conventional and option-feature bonds in Table 15.3.

Change in yield	Value of price change for	
	Positive convexity	Negative convexity
Fall of 100 bp	X%	Lower than Y%
Rise of 100 bp	Lower than X%	Y%

Table 15.3

The price/yield relationship for a callable bond exhibits negative convexity as interest rates fall. Option-adjusted spread analysis is used to highlight this relationship for changes in rates. This is done by effecting a parallel shift in the benchmark yield curve, holding the spread level constant and then calculating the theoretical price along the nodes of the binomial price tree. The average present value then becomes the projected price for the bond. General results for a hypothetical callable bond, compared to a conventional bond are shown at Figure 15.11. In our example once the market rate falls below the 10% level, the bond exhibits negative convexity. This is because it then becomes callable at that point, which acts as an effective cap on the price of the bond.

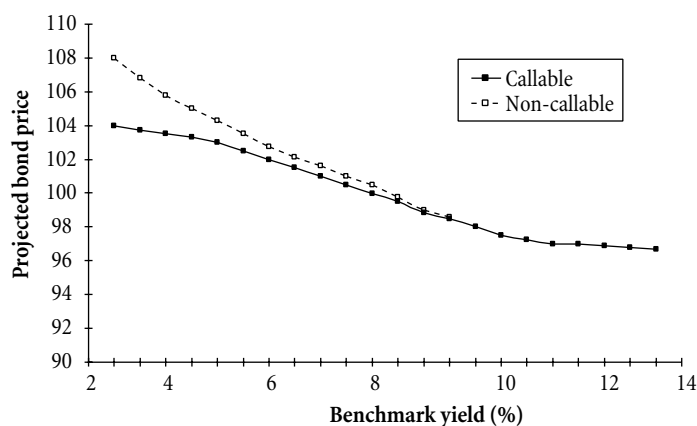


Figure 15.11: Projected prices for callable and conventional bonds with identical coupon and final maturity dates

The market analyses bonds with embedded options in terms of a yield spread, with a “cheap” bond trading at a higher yield spread and a “dear” bond trading at a lower yield spread. The usual convention is to quote yield spreads as the difference between the redemption yield of the bond being analysed and the equivalent maturity government bond. This is not accurate because the redemption yield is in effect a meaningless number – there is not a single rate at which all the cash flows comprising either bond should be discounted but a set of spot or forward rates that are used for each successive interest period. The correct procedure for discounting therefore is to determine the yield spread over the spot or forward rate curve. With regard to the binomial tree what we require then, is the constant spread that, when added to all the short-rates on the binomial tree, makes the theoretical (model-derived) price equal to the observed market price. The constant spread that satisfies this requirement is known as the *option-adjusted spread* (OAS). The spread is referred to as an “option-adjusted” spread because it reflects the option feature attached to the bond. The OAS will depend on the volatility level assumed in running the model. For any given bond price, the higher the volatility level specified, the lower will be the OAS for a callable bond, and the higher for a puttable bond. Since the OAS is calculated usually relative to a government spot or forward rate curve, it reflects the credit risk and any liquidity premium between the corporate bond and the government bond. Note that OAS analysis reflects the valuation model being used, and its accuracy is reflection of the accuracy of the model itself.

15.4.1 Measuring bond yield spreads

The binomial model evaluates the return of a bond by measuring the extent to which its return exceeds the returns determined by the risk-free short-rates in the tree. The difference between these returns is expressed as a spread and may be considered the *incremental return* of a bond at a specified price. Determining the spread involves the following steps:

1. the binomial tree is used to derive a theoretical price for the specified bond;
2. the theoretical price is compared with the bond's observed market price;
3. if the two prices differ, the rates in the binomial model are adjusted by a user-specified amount, which is the estimate of the spread;
4. using the adjusted rates a new theoretical price is derived and compared with the observed price;
5. the last two steps are repeated until the theoretical price matches the observed price.

The process can be carried out in a straightforward fashion using a software application.

15.5 Price volatility of bonds with embedded options

In Chapter 7 we reviewed traditional duration and modified duration measures for bond interest-rate risk. Modified duration is essentially a predictive measure, used to describe the expected percentage change in bond price for a 1% change in yield. The measure is a snap-shot in time, based on the current yield of the bond and the structure of its expected cash flows. In analysing a bond with an embedded option, the bondholder must assume a fixed maturity date, based on the current price of the bond, and calculate modified duration based on this assumed redemption date. However under circumstances where it is not exactly certain what the final maturity is, modified duration may be calculated to the first call date and to the final maturity date. This would be of little use to bondholders in these circumstances, since it may be unclear which measure is appropriate. The problem is more acute for bonds that are continuously callable (or put-able) from the first call date up to maturity.

15.5.1 Effective duration

To recap from Chapter 7, the duration for any bond is calculated using (15.7) which assumes annualised yields.

$$D = \frac{\sum_{t=1}^n \frac{tC_t}{(1+rm)^t}}{P} \quad (15.7)$$

Fabozzi (1997) describes how the measure can be approximated using (15.8):

$$D_{\text{approx}} = \frac{P_- - P_+}{2P_0 (\Delta rm)} \quad (15.8)$$

where

- P_0 is the initial price of the bond
- P_- is the estimated price of the bond if the yield falls by Δrm
- P_+ is the estimated price of the bond if the yield rises by Δrm
- Δrm is the change in the yield of the bond.

The drawbacks of the traditional measure are overcome to a certain extent when OAS analysis is used to measure the *effective duration* of a bond. Whereas traditional duration seeks to predict a bond's price changes based on a given price and assumed redemption date, effective duration is solved for from actual price changes resulting from specified shifts in interest rates. Applying the analysis to a bond with an embedded option means that the new prices resulting from yield changes reflect changes in the cash flow. Effective duration may be thought of as a duration measure which recognises that yield changes may change the future cash flow of the bond. For bonds with embedded options the difference between traditional duration and effective duration can be significant; for example for a callable bond the effective duration is sometimes half that of its traditional duration measure. For mortgage-backed securities the difference is sometimes greater still.

To calculate effective duration using the binomial model and (15.8) we employ the following procedure:

- calculate the OAS spread for the bond;
- change the benchmark yield through a downward parallel shift;
- construct an adjusted binomial tree using the new yield curve;
- add the OAS adjustment to the short-rates at each of the node points in the tree;

- use the modified binomial tree constructed above to calculate the new value of the bond, which then becomes P_+ for use in equation (15.8).

To determine the lower price resulting from a rise in yields we follow the same procedure but effect an upward parallel shift in the yield curve.

Effective duration for bonds that contain embedded option is often referred to as *option-adjusted spread duration*. There are two advantages associated with using this measure. These are that, by incorporating the binomial tree into the analysis, the interest-rate dependent nature of the cash flows is taken into account. This is done by holding the bond's OAS constant over the specified interest-rate shifts, in effect maintaining the credit spread demanded by the market at a constant level. This takes into account the behaviour of the embedded option as interest rates change. The second, and possibly more significant advantage is that OAS duration is calculated based on a parallel shift in the benchmark yield curve, which gives us an indication of the change in bond price with respect to changes in market interest rates rather with respect to changes in its own yield.

The derivation of the expression for option-adjusted modified duration is given at Appendix 15.2.

15.5.2 Effective convexity

In the same way that we calculate an effective duration measure for bonds with embedded options, the standard measure of bond convexity we reviewed in Chapter 9 may well be inappropriate for such bonds, for the same reason that the measure does not take into account impact of a change in market interest rates on a bond's future cash flows. The convexity measure for any bond may be approximated using (15.9), described in Fabozzi (1997).

$$CV = \frac{P_+ + P_- - 2P_0}{P_0 (\Delta rm)^2}. \quad (15.9)$$

If prices input to (15.9) are those assuming that remaining cash flows for the bond do not change when market rates change, the convexity value is that for an option-free bond. To calculate a more meaningful value for bonds with embedded options, the prices used in the equation are derived by changing the cash flows when interest rates change, based on the results obtained from the binomial model. This measure is called *effective convexity* or *option-adjusted convexity*. The derivation of this measure is given at Appendix 15.3.

15.6 Sinking funds

In some markets corporate bond issuers set up *sinking fund* provisions. They are more widely used in the US corporate market. For example consider the following hypothetical bond issue:

Issuer	ABC plc
Issue date	01-Dec-99
Maturity date	01-Dec-19
Nominal	£100 million
Coupon	8%
Sinking fund provision	£5 million 1 December, 2009 to 2018

Table 15.4

In the example of the ABC plc 8% 2019 bond, a proportion of the principal is paid out over a period of time. This is the formal provision. In practice the actual payments made may differ from the formal requirements.

A sinking fund allows the bond issuer to redeem the nominal amount using one of two methods. The issuer may purchase the stipulated amount in the open market, and then deliver these bonds to the Trustee³ for cancellation. Alternatively the issuer may call the required amount of the bonds at par. This is in effect a *partial call*, similar to a callable bond for which only a fraction of the issue may be called. Generally the actual bonds called are selected

³ Bond issuers appoint a Trustee that is responsible for looking after the interests of bondholders during the life of the issue. In some cases the Trustee is appointed by the underwriting investment bank or the Issuer's solicitors. Specialised arms of commercial and investment banks carry out the Trustee function, for example Chase Manhattan, Deutsche Bank, Bank of New York, Citibank and others.

randomly by certificate serial numbers. Readers will have noticed however that the second method by which a portion of the issue is redeemed is actually a call option, which carries value for the issuer. Therefore the method by which the issuer chooses to fulfil its sinking fund requirement is a function of the level of interest rates. If interest rates have risen since the bond was issued, so that the price of the bond has fallen, the issuer will meet its sinking fund obligation by direct purchase in the open market. However if interest rates have fallen, the issuing company will call the specified amount of bonds at par. In the hypothetical example given at Table 15.4, in effect ABC plc has ten options embedded in the bond, each relating to £5 million nominal of the bonds. The options each have different maturities, so the first expires on 1 December 2009 and subsequent options maturing on 1 December each following year until 2018.⁴ The decision to exercise the options as they fall due is made using the same binomial tree method that we discussed earlier.

Appendices

APPENDIX 15.1 An illustration of the binomial process

In general we observe that bonds that contain embedded options trade in the market with the option element possessing little or no *intrinsic* value. If a callable bond is not immediately exercisable, by definition the option has no intrinsic value. This will hold regardless of the difference between the option exercise price and the price of the underlying bond. Since the embedded options in most callable and put-able bonds have no intrinsic value, the value that is attached to them is composed entirely of *time* value. It is important to measure this time value when analysing option-embedded bonds. There are several valuation methodologies that are used in the market to facilitate this. Whatever the model that is used, the objective is to derive a price distribution of the underlying security at a future date. This enables the analyst to measure the future intrinsic value and then calculate its present value today, which enables her to determine if the option will be exercised at that future date.

The binomial model is a commonly encountered valuation model. This is best illustrated using the example of tossing a coin, where the probabilities of the two expected outcome from tossing a coin are precisely equal. To illustrate the principles involved, assume a coin-toss process whereby an outcome of “heads” results in a payout of £20, while an outcome of “tails” results in a payout of £0. There is an equal probability of achieving heads or tails for every coin toss. On a single toss of the coin the only possible outcome is a payout of £20 or £0. Over a large number of coin tosses we would expect the outcome to be heads 50% of the time and tails 50% of the time. In general the expected payoff of one toss is calculated by weighting the payoff associated with each possible outcome by the probability that the outcome will occur. This is expressed as (15.10):

$$E(w) = \sum_{i=1}^n (w_i \times p(i)) \quad (15.10)$$

where

$E(w)$	is the expected payoff from one coin toss
w_i	is the payout received from an outcome i
p_i	is the probability that an outcome i will occur
n	is the number of possible different outcomes
i	is 1, 2, 3, ... m .

In our case of a £20 gain from achieving a “heads” the expected payoff from one toss of a coin would therefore be calculated as follows:

$$\begin{aligned} &= (w_1 \times p(1)) + (w_2 \times p(2)) = ((20) \times (50\%)) + ((0) \times (50\%)) \\ &= £10. \end{aligned}$$

Note that the expected outcome is £10, despite the fact that the only outcome from a single toss of the coin is £20 or £0. The way to interpret the result is that the payoff from a large number of tosses is an average of £10 per toss. In

⁴ The “options” are European options, in that they can only be exercised on the expiry date.

other words the payoff from n coin tosses is n multiplied by the expected payoff from one coin toss. This is expressed formally as (15.11):

$$E(nw) = nE(w). \quad (15.11)$$

From (15.11) we would calculate that the expected payoff from 10 coin tosses would be £100.

As our hypothetical process has an expected payoff of £10 per coin toss, it is described as having value. To quantify the extent of this value, consider the following situation, where we are given the following choice:

- paying today for 10 tosses of the coin, to take place one year from today;
- buying today a *risk-free* one-year Treasury bill with a nominal value of £100, at a price of £95.18.

The payoff from 10 tosses of the coin is not certain. Although the highest probability outcome is a payoff of £100, it is of course possible to receive considerably more or less than this. This is because there is a probability that the outcome is more or less than five “heads”. The outcome in the second case is certain: the Treasury bill will pay out £100 on maturity. The difference between these two choices is that the coin toss outcome has an element of risk, whereas there is no risk associated with the T-bill.

The gain achieved from purchasing the T-bill is the difference between the redemption amount and the current price, which is £4.82. This gain is risk-free, in that if we pay £95.18 today we will receive £100 in one year's time, a gain of £4.82. This gain is 5.064%, or an approximate bond-equivalent return of 5.00%. This is referred to as the *risk-free rate of return*. Given that this is the risk-free rate of return available to anyone in the market, it is also the minimum return we would accept from investing in the coin-toss, which means that the maximum we would pay for that investment is £95.18. A rational investor would not pay more than this because at that price a risk-free return is available in the shape of the T-bill. Therefore £95.18 represents the maximum present value of the expected outcome of 10 coin tosses, as determined by the 5% risk-free rate. A feature of the market is that the present value of an expected future payoff is determined by the risk-free rate of return during the intervening period.

The coin toss illustration is an example of a *binomial* process. In a binomial process each action has only two possible outcomes. A *trinomial* process would have precisely three possible outcomes.

The Binomial distribution

If any process meets the following conditions:

- there is a specified discrete number of trials;
- there are only two possible outcome possible from each trial (for example, heads, tails; or defective, not defective);
- the probability of the outcomes in each trial does not change;
- the trials are independent, that is, the outcome of the previous trial has no bearing on the outcome of the current trial;

then it said to be a Binomial distribution. Under these conditions we may use the binomial formula to calculate directly the probabilities of any process. The formula is given at (15.12):

$$p(r) = {}^nC_r p(r)q^{(n-r)} \quad (15.12)$$

where

- ${}^nC_r$ is the term for determining the number of different ways an event can occur
- p is the probability of the outcome from the trial under investigation
- q is $(1 - p)$, the probability of the specified outcome from the trial not occurring
- n is the number of trials
- r is the specified number of outcomes.

The term ${}^nC_r$ is calculated as follows:

$${}^nC_r = \frac{n!}{r!(n - r)!}$$

where ! is a factorial.

It is not always necessary to make the calculation of the probability directly as Binomial tables exist; however the tables typically show only the probabilities for certain combinations of n and p . Binomial tables are contained in standard statistics textbooks and are not reproduced here.

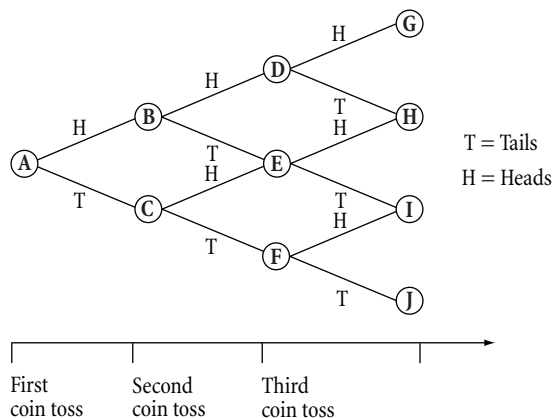


Figure 15.12: Binomial tree of outcome of tossing a coin three times.

The binomial tree diagram illustrates the properties of a binomial process. These properties form the basis for the model used to evaluate options embedded in bonds. It is usually referred to as the *binomial tree of interest rates* and is demonstrated in the chapter.

APPENDIX 15.2 Deriving the modified duration expression for a bond with an embedded option

Consider a callable bond, the price of which may be expressed as (15.13):

$$P_{cbond} = P_{underlying} - P_{call}. \quad (15.13)$$

Differentiating (15.13) with respect to the bond yield rm we obtain (15.14):

$$\frac{dP_{cbond}}{drm} = \frac{dP_{underlying}}{drm} - \frac{dP_{call}}{drm}. \quad (15.14)$$

Dividing both sides of (15.14) by the price of the callable bond results in (15.15):

$$\frac{dP_{cbond}}{drm} \frac{1}{P_{cbond}} = \frac{dP_{underlying}}{drm} \frac{1}{P_{cbond}} - \frac{dP_{call}}{drm} \frac{1}{P_{cbond}}. \quad (15.15)$$

We then multiply the numerator and denominator of the right-hand side of (15.15) by the price of the underlying bond, shown as (15.16):

$$\frac{dP_{cbond}}{drm} \frac{1}{P_{cbond}} = \frac{dP_{underlying}}{drm} \frac{1}{P_{underlying}} \frac{P_{underlying}}{P_{cbond}} - \frac{dP_{call}}{drm} \frac{1}{P_{underlying}} \frac{P_{underlying}}{P_{cbond}}. \quad (15.16)$$

The components of (15.16) measure duration and modified duration of the underlying and callable bond, as shown by (15.17) and (15.18):

$$\frac{dP_{cbond}}{drm} \frac{1}{P_{cbond}} = -MD_{cbond} \quad (15.17)$$

$$\frac{dP_{underlying}}{drm} \frac{1}{P_{underlying}} = D_{underlying}. \quad (15.18)$$

This relationship was demonstrated in Chapter 7. We omit the negative sign in (15.17) as it does not have any impact on how the option-adjusted duration is derived.

Therefore we may state that:

$$MD_{cbond} = D_{underlying} \frac{P_{underlying}}{P_{cbond}} - \frac{dP_{call}}{drm} \frac{1}{P_{underlying}} \frac{P_{underlying}}{P_{cbond}}. \quad (15.19)$$

The change in the value of the embedded call for a change in yield is expressed by (15.20):

$$\frac{dP_{call}}{drm}. \quad (15.20)$$

Any change in the value of the embedded call is dependent on the change in the price of the underlying bond for a specified yield change. This may be expressed as (15.21):

$$\begin{aligned} P_{call} &= fn(P_{underlying}) \\ P_{underlying} &= g(rm). \end{aligned} \quad (15.21)$$

By applying calculus equation (15.20) may be expressed as (15.22):

$$\frac{dP_{call}}{drm} = \frac{dP_{call}}{dP_{underlying}} \frac{dP_{underlying}}{drm}. \quad (15.22)$$

The change in the value of the embedded call resulting from a change in price of the underlying is contained in the first term of the right-hand side of (15.22). This is the definition of the *Delta* of an option. Therefore we may state the following:

$$\frac{dP_{call}}{drm} = \text{Delta} \times \frac{dP_{underlying}}{drm}. \quad (15.23)$$

Equation (15.23) may be used to obtain the expression for the call-adjusted duration of a callable bond; by substituting it into (15.19) and rearranging it we obtain (15.24), which is the duration of a callable bond, adjusted for the call feature.

$$MD_{cbond} = D_{underlying} \times \frac{P_{underlying}}{P_{cbond}} \times (1 - \text{Delta}). \quad (15.24)$$

APPENDIX 15.3 Option-adjusted convexity measure

This derivation is undertaken again for a callable bond, broken down as shown by (15.13). The convexity measure is the second derivative of the price/yield function, obtained by multiplying the second derivative by the reciprocal of the price of the bond at that point. Equation (15.14) above is the first derivative of the price of a callable bond. Applying calculus allows us to express (15.14) as (15.25):

$$\frac{dP_{cbond}}{drm} = \frac{dP_{underlying}}{drm} - \frac{dP_{call}}{dP_{underlying}} \frac{dP_{underlying}}{drm}. \quad (15.25)$$

The second derivative of this expression is (15.26):

$$\frac{d^2 P_{cbond}}{drm^2} = \frac{d^2 P_{underlying}}{drm^2} - \left(\frac{d^2 P_{call}}{dP_{underlying}^2} \left(\frac{dP_{underlying}}{drm} \right)^2 + \frac{dP_{call}}{dP_{underlying}} \frac{d^2 P_{underlying}}{drm^2} \right). \quad (15.26)$$

The right-hand side of (15.26) contains the terms for establishing the value of the first and second derivatives. The first term on the right-hand side is the second derivative of the price of the underlying bond with respect to change in yield. Multiplying this term by the price of the underlying bond results in (15.27):

$$\frac{d^2 P_{\text{underlying}}}{drm^2} \times \frac{P_{\text{underlying}}}{P_{\text{underlying}}} \quad (15.27)$$

The expression at (15.27) is in fact simply the convexity of the underlying bond multiplied by its price, which we can express as (15.28):

$$CV_{\text{underlying}} \times P_{\text{underlying}} \quad (15.28)$$

Consider next the term below:

$$\frac{d^2 P_{\text{underlying}}}{dP_{\text{underlying}}^2} \left(\frac{dP_{\text{underlying}}}{drm} \right)^2$$

The first term is the expression for the rate of change of the delta of the call option component with respect to the change in the price of the underlying. This is in fact the definition of the *gamma* of an option, so that we may write:

$$\text{Gamma} \times \left(\frac{dP_{\text{underlying}}}{drm} \right)^2, \text{ which may also be expressed as:}$$

$$\text{Gamma} \times \left(\frac{dP_{\text{underlying}}}{drm} \right)^2 \times \frac{P_{\text{underlying}}^2}{P_{\text{underlying}}^2} \text{ This is equivalent to (15.29):}$$

$$\text{Gamma} \times (D_{\text{underlying}})^2 \times P_{\text{underlying}}^2 \quad (15.29)$$

We may also set the second derivative of the price of the underlying bond in the following terms:

$$\text{Delta} \times CV_{\text{underlying}} \times P_{\text{underlying}} \quad (15.30)$$

Substituting (15.28), (15.29) and (15.30) into (15.25) results in (15.31):

$$\begin{aligned} \frac{d^2 P_{\text{bond}}}{drm^2} = & CV_{\text{underlying}} \times P_{\text{underlying}} \\ & - \left(\text{Gamma} \times (D_{\text{underlying}})^2 \times P_{\text{underlying}}^2 + \text{Delta} \times CV_{\text{underlying}} \times P_{\text{underlying}} \right) \end{aligned} \quad (15.31)$$

If we multiply the right-hand side of (15.30) by the reciprocal of the price of the callable bond, and then rearrange the terms for the convexity, we obtain the expression for the option-adjusted convexity of a callable bond, shown as (15.32):

$$CV_{\text{bond}} = \frac{P_{\text{underlying}}}{P_{\text{bond}}} \left(CV_{\text{underlying}} \times (1 - \text{Delta}) - P_{\text{underlying}} \times \text{Gamma} \times (D_{\text{underlying}})^2 \right) \quad (15.32)$$

Selected bibliography

- Bodie, Z., Taggart, R., "Future Investment Opportunities and the Value of the Call Provision on a Bond", *Journal of Finance* 33, 1978, pp. 1187–2000.
- Fabozzi, F.J., *Fixed Income Mathematics: Analytical and Statistical Techniques*, 3rd edition, McGraw-Hill, 1997, Chapter 16.
- Kalotay, A., Williams, G.O., Fabozzi, F.J., "A model for the Valuation of Bonds and Embedded Options", *Financial Analysts Journal*, May–June 1993, pp. 35–46.
- Kish, R., Livingstone, M., "The Determinants of the Call Feature on Corporate Bonds", *Journal of Banking and Finance* 16, 1992, pp. 687–703.
- Mitchell, K., "The Call, Sinking Fund, and Term-to-Maturity Features of Corporate Bonds: An Empirical Investigation", *Journal of Financial and Quantitative Analysis* 26, June 1991, pp. 201–222.

Narayanan, M.P., Lim, S.P., “On the Call Provision on Corporate Zero-Coupon Bonds”, *Journal of Financial and Quantitative Analysis* 24, March 1989, pp. 91–103.

Tuckman, B., *Fixed Income Securities*, Wiley, 1996, Chapter 17.

Van Horne, J.C., *Financial Management and Policy*, Prentice Hall, 1986.

Windas, T., *An Introduction to Option-Adjusted Spread Analysis*, Bloomberg Publications, 1994.

Questions and exercises

1. How is the price of a callable bond expressed in terms of a conventional bond and an option?
2. Why does the convexity measure become inappropriate for bonds that contain embedded options? What is negative convexity?
3. In using option-adjusted spread analysis, what provision is made for the credit risk of the corporate callable bond?
4. The 5% 2004 gilt is trading at 94.74 and a yield of 6.327%. It has precisely five years to maturity. Its modified duration is 3.979. Calculate its effective duration (assume a parallel shift of 20 basis points). How well does the effective duration approximate the modified duration?
5. How does the market break down a callable bond in terms of analysing it?
6. A call option has a delta of 0.15. Explain this statement.
7. A bond has a 20-year maturity and is callable after ten years. A second bond has a nine-year maturity and is callable after three years. At a price level close to par, which bond's duration will be most sensitive to a change in market yields?
8. An investor is analysing a 7% coupon callable corporate bond currently trading at 102. The bond has five years to maturity and is callable at par. Assume that the call option value is calculated to be 2.87. What is the option-adjusted yield of the bond? What would happen to the call option value and the call-adjusted yield if the assumed volatility level increased?
9. A callable bond is trading in the market at 101.35. The value of the embedded option is calculated as 0.85. The duration of the underlying bond is 4.13, and the delta is 0.24. What is the call-adjusted duration of the bond?
10. Rashid Asset Management is reviewing its portfolio. The market has been rallying recently, and the fund manager feels that if the market rallies further, she should take her profits, switching out of €20 million of 10-year government bonds into high coupon BBB-rated corporate bonds that are callable on set dates after five years until they mature in seven years. The fund manager believes the market is unlikely to rally further however. The corporate bonds have a 90% chance of being called in five years and are treated as five-to-seven year paper when calculating the duration of the portfolio. Consider the following:
 - (a) why is modified duration an inappropriate measure for a high-coupon corporate bond?
 - (b) what is a more accurate measure than modified duration?
 - (c) why would switching out of the government bonds and into high-coupon callable bonds lower the portfolio's duration?

16

Convertible Bonds I

A *convertible* bond is a corporate security that gives the bondholder the right, without imposing an obligation, to convert the bond into another security under specified conditions, usually the ordinary shares of the issuing company. The decision to convert is solely at the discretion of the bondholder.¹ Once converted into ordinary shares, the shares cannot be exchanged back into bonds. As a result of their structure and option feature, in the market convertibles display the characteristics of both debt and equity instruments; as such they are often referred to as *hybrid* instruments. In addition to convertible bonds, there are also *convertible preferred stock*, which are essentially preference shares that are convertible into ordinary shares, again under terms specified at the time of issue. Convertibles are an important element of corporate finance and have benefited from the development of complex valuation models as applied in the option markets. Due to their hybrid nature, convertibles have presented some issues in their analysis and valuation in the past, but modern techniques have essentially resolved this and issue volumes have been steadily increasing during the 1990s, with volumes usually highest during rising stock markets. The sustained bull-run in global equities markets during the second half of the 1990s, led by Wall Street but also observed in other markets, has also witnessed growth in convertibles volumes. This reflects the fact that issuing a right to share in future equity price growth is most attractive during times of rising markets, and this allows corporates to issue convertibles on favourable terms.

In this chapter we review the basic characteristics of convertible securities, and the advantages and disadvantages of the bonds for both issuers and investors. The following chapter reviews the valuation and analysis of convertible bonds.

Convertible bond issues in 1998 by currency	\$billion
US dollars	96
Sterling	16
French francs	7
Other currencies	7

Table 16.1: Convertible bond issuance in 1998. Source: Strata Consulting.

16.1 Basic description

Convertible bonds are typically fixed coupon securities that are issued with an option to be converted, at the bondholder's discretion, into the equity of the issuing company under specified terms and conditions. They are usually subordinated securities, and may only be issued by companies with a strong enough credit rating to tap the markets. The view of the market on the performance of the issuing company's shares is also a key factor, because investors are buying into the right to subscribe for the shares at a later date and, if exercised, at a premium on the open market price. For this reason the price of a convertible bond at any time will reflect changes in the price of the underlying ordinary shares; it also reflects changes in interest rates. Convertibles are typically medium- to long-dated instruments, and are usually issued with maturities of 10 to 20 years. The coupon on a convertible is always below the level payable on the same issuer's non-convertible bond of the same maturity. The bonds are usually convertible into ordinary shares of the issuing company under a set ratio and a specified price.

In addition to the basic fixed coupon convertible there is a range of other instruments available to corporate borrowers. These include the *zero-coupon convertible*, which trades similarly to a zero-coupon vanilla bond and is

¹ Although generally a corporate financing instrument, convertibles have also been issued by governments, where the bond is usually convertible into another debt instrument. Certain corporate issues are also convertible into the ordinary shares of another company, these are known as *exchangeable* bonds.

issued at a deep discount. There is usually a low possibility of conversion with these bonds. A similar instrument to this is the *discount convertible*.

Some convertibles are also callable by the issuer, under pre-specified conditions. These are known as *convertible calls* and remove one of the advantages of the straight convertible – that conversion is at the discretion of the bondholder – because by calling a bond the issuer is able to force conversion, on potentially unfavourable terms. Put-able convertible bonds are the opposite of this and allow bondholders to redeem the bond as well as effect conversion at their discretion. The *premium put convertible* may be converted on only one date during its life, compared to the *rolling put convertible* which may be converted on a series of dates during its life; it is generally issued with a lower coupon than a conventional convertible. The addition of a put feature in a convertible is regarded as an extra inducement for investors, as they offer downside price protection. The *exchangeable* security is a bond that is issued by one company, but is convertible into the shares of another company, usually one in which the issuer holds a substantial interest. The *bond with warrant* is a convertible bond that is issued with an attached warrant, which may be detached and traded separately in the secondary market. Bonds with warrants are issued at a lower coupon than those issued as traditional convertibles. *Step-up convertible* bonds and preference shares are a more recent innovation. These pay a fixed coupon for a specified first part of their life, say the first five years, and then pay a higher coupon until they mature or are converted.

Convertible bonds have a long history in the capital markets, having been issued by utility companies in the United States in the 19th century. In 1997 the global convertibles market was estimated at over \$360 billion. The US is the largest issuer of convertibles, and historically the biggest issuers were utility and transport companies. Unlike domestic and international securities, the market is essentially an exchange-traded one, with bonds listed on an exchange such as the New York Stock Exchange or the London Stock Exchange. This provides an added advantage in that the instruments may be more liquid than OTC market domestic bonds, and are also more transparent. However liquidity is also a function of the number of market makers in the stock and the amount of an issue available for trading, so that some convertibles will be more illiquid than conventional bonds.

EXAMPLE 16.1 The LYON

One innovation in the convertible market is the *liquid yield option note* or LYON. The LYON was introduced by Merrill Lynch in 1985 and is a zero-coupon, convertible, callable and put-able bond. More than \$13 billion nominal of such bonds were issued in the US market between 1985 and 1992 (Bhattacharya/Zhu 1997). The bonds are deep discount securities, with maturities typically between 15 and 20 years. The provision of a put feature in the bonds affords an element of downside protection for investors not available to conventional convertible bondholders; the put is usually exercisable on a range of pre-set dates. In other respects however the bonds are analysed in the same way as conventional convertibles.

16.1.1 Terms and conditions

Consider a standard convertible bond issued by our hypothetical borrower ABC plc. In the example ABC plc has issued a bond that gives the right, but not the obligation, to the bondholder to convert into the underlying shares of ABC plc at a specified price during the next ten years (see Table 16.2 below).

Issuer	ABC plc
Coupon	10%
Maturity	December 2009
Issue size	£50,000,000
Face value	£1,000
Number of bonds	50,000
Issue price	£100
Current price	£103.50
Conversion price	£8.50
Dividend yield	3.50%

Table 16.2: ABC plc 10% Convertible 2009.

For our hypothetical issue the conversion right or *option* may or may not be taken up at any time during the life of the bond, known as an *American* option, as opposed to a *European* option convertible which may only be converted on the maturity date of the bond. The majority of convertibles are American-style.

The ratio of exchange between the convertible bond and the ordinary shares can be stated either in terms of a *conversion price* or a *conversion ratio*. Bonds are always issued at a premium, which is the amount by which the conversion price, also known as the *exercise* or *strike* price, lies above the current share price.

The ABC plc bond has a face value of £1000, and a conversion price of £8.50, which means that each bond is convertible into 117.64 ordinary shares. To obtain this figure we have simply divided the face value of the bond by the conversion price to obtain the conversion ratio, which is $£1000/£8.50 = 117.64$ shares. This is also known as the *conversion ratio* and is given by (16.1):

$$\text{Conversion ratio} = \text{Bond denomination} / \text{Conversion price.} \quad (16.1)$$

The terms and conditions under which a convertible is issued, and the terms under which it may be converted into the issuer's ordinary shares, are listed in the offer particulars or *prospectus*. The legal obligations of issuers and the rights of bondholders are stated in the *indenture* of the bond.

The conversion privilege can be stated in terms of either the conversion price or the conversion ratio. Conversion terms for a convertible do not necessarily remain constant over time. In certain cases convertible issues will provide for increases or *step ups* in the conversion price at periodic intervals. A £1000 denomination face value bond may be issued with a conversion price of say, £8.50 a share for the first three years, £10 a share for the next three years and £12 for the next five years, and so on. Under this arrangement the bond will convert to fewer ordinary shares over time which, given that the share price is expected to rise during this period, is a logical arrangement. The conversion price is also adjusted for any corporate actions that occur after the convertibles have been issued, such as rights issues or stock dividends. For example, if there was a 2 for 1 rights issue, the conversion price would be halved. This provision protects the convertible bondholders and is known as an *anti-dilution* clause.

The *parity* or *intrinsic value* of a convertible refers to the value of the underlying equity, expressed as a percentage of the nominal value of the bond. Parity is given by (16.2):

$$\text{Parity} = \text{Share price} / \text{Conversion price or} \quad (16.2)$$

$$\text{Share price} / \text{Conversion ratio.}$$

The bond itself may be analysed – in the first instance – as a conventional fixed income security, so using its coupon and maturity date we may calculate a current yield (running yield) and yield to maturity. The *yield advantage* is the difference between the current yield and the *dividend yield* of the underlying share, given by (16.3):

$$\text{Yield advantage} = \text{Current yield} - \text{Dividend yield.} \quad (16.3)$$

In this case the current yield of the bond is 9.66%, which results in a yield advantage of 6.16%. Equity investors also use another measure, the *break-even* value which is given by (16.4):

$$\text{Break-even} = (\text{Bond price} - \text{Parity}) / \text{Yield advantage.} \quad (16.4)$$

16.1.2 Investor analysis

The analytical requirements of the market investor are slightly different to those of the market maker or trader. In this section we consider the former. When evaluating convertible securities the investor must consider the expected performance of the underlying shares, the future prospects of the company itself and the relative attraction of the bond as a pure fixed income instrument in the event that the conversion feature proves to be worthless. In addition the assessment of the bond will take into account the credit quality of the issuer, the yield give-up suffered as a result of purchasing the convertible over a conventional bond, the conversion premium ratio, and the fixed income advantage gained over a purchase of the underlying shares in the first place.

We have already referred to the conversion ratio, which defines the number of shares of common stock that is received when the bond is converted. The conversion price is the actual price paid for the shares when conversion occurs.

$$\text{Conversion price} = \frac{\text{Par value of bond}}{\text{Conversion ratio}}. \quad (16.5)$$

The *conversion premium* is the percentage by which the conversion price exceeds the current share price. Using our previous illustration for ABC plc, the convertible has a conversion ratio of 117.64 (that is, 117.64 shares are received in return for the bond with a par value of £1000) and therefore a conversion price of £8.50. If the current price of the share is £6.70, then we have:

$$\begin{aligned}\text{Percentage conversion premium} &= \frac{\text{Conversion price} - \text{Share price}}{\text{Share price}} \\ &= \frac{£8.50 - £6.70}{£6.70} \\ &= 26.87\%.\end{aligned}$$

The *conversion value* of the bond shows the current value of the shares received in exchange for the bond. It is given by:

$$\text{Conversion value} = \text{Share price} \times \text{Conversion ratio}. \quad (16.6)$$

As the current share price is £6.70, then the current conversion value would be:

$$\text{Conversion value} = £6.70 \times 117.64 = £788.19.$$

Assume that the bond is trading at 103.50 (per 100), then the *percentage conversion price premium*, or the percentage by which the current bond price exceeds the current conversion value is given by (16.7) below.

$$\text{Percentage conversion price premium} = \frac{\text{Price of bond} - \text{Conversion value}}{\text{Conversion value}} \quad (16.7)$$

$$\text{In our example the premium value is } \frac{1035 - 788.19}{788.19} = 31.32\%.$$

The premium value in a convertible may be illustrated as shown in Figure 16.1. Investors are concerned with the point at which the ratio of the parity of the bond to the investment value moves far above the bond floor. At this point the security trades more like equity than debt. The opposite to this is when the equity price falls to low levels, to the point at which it will need to appreciate by a very large amount before the conversion option has any value; at this point the convertible trades like a pure fixed income instrument. Investors sometimes view a convertible bond price in terms of where it is standing in terms of the premium line in Figure 16.1, and assess its chances of moving into the “hybrid” part of the chart. This analysis has no value however.

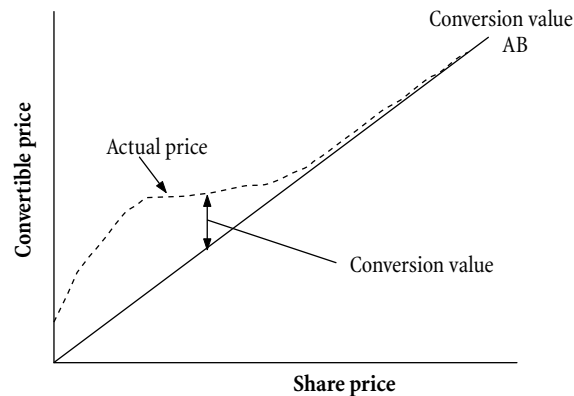


Figure 16.1: Convertible bond and conversion premium.

As we reviewed in Chapter 15, the value of a convertible bond may be broken down and viewed as the sum of the value of two instruments, the straight bond of the issuer and a call option on the issuer's equity. Considering Figure 16.1, the value of the convertible bond minus the conversion feature is represented by the line AB. It is sometimes referred to as the *straight line* value and is the conventional redemption yield measure. As we noted above the value

of a convertible if it is converted straight away is the conversion value. Therefore the minimum value of a convertible bond is the higher of its straight line value and conversion value.

As a convertible is viewed as a conventional bond with a warrant attached, its fair price is a combination of the price of a vanilla bond and the price of a call option, notwithstanding the dilution effect of the new shares that are issued and the coupon payments that are saved as a result of conversion.

The proportionate increase in the number of shares outstanding if all the bonds were to be converted, referred to as p , is given by (16.9):

$$p = \frac{\text{Number of convertible bonds} \times \text{Conversion ratio}}{\text{Number of shares outstanding before conversion}}. \quad (16.8)$$

The fair value of a convertible is given by (16.9):

$$\text{Price of convertible} = \text{Price of vanilla bond} + \frac{P_C}{1 + p} \times \text{Conversion ratio}. \quad (16.9)$$

where P_C is the value of an American call option with an exercise price equal to the conversion price and an expiry date equal to the maturity of the bond. The price of the vanilla bond is calculated in exactly the same way as a standard bond in the same risk class.

Equation (16.9) will give us the fair price of a convertible if the bond is not callable. However some convertibles are callable at the issuer's option prior to the final maturity date. The issuer can therefore effect conversion when the share price has risen to the point where the value of the shares received on conversion equals the call price of the bond. As the firm has an incentive to call the bond when this occurs, the call price puts an effective ceiling on the price of the convertible given by (16.9).

We can calculate a realistic call date for a callable convertible by using the expression at (16.10):

$$\text{Call price} = \text{Current share price} \times (1 + g)^t \times \text{Conversion ratio} \quad (16.10)$$

where g is the expected growth rate in the share price and t is the time in years. Let us then assume that the call price of the hypothetical ABC plc bond is £100 (that is, par per face value of £1000), the conversion ratio is 117.64 and the current share price is £6.70 and is growing at 8.25 per cent per year. Given that the right-hand side of expression (16.10) is the conversion value of the convertible in t years' time, we can calculate at what point the conversion value will equal the call price.

If we look again at (16.9) and using our assumed terms and values, we can see that the price of the convertible depends on the price of a vanilla bond with identical years to maturity (and a terminal value of £100) and a call option with the same years to expiry. The value of the option component is determined using equity option models and is considered in Chapter 17.

The attraction of a convertible for a bondholder lies in its structure being one of a combined vanilla bond and option. As we shall see in our introductory discussion of options, option pricing theory tells us that the value of an option increases with the price variance of the underlying asset. However bond valuation theory implies that the value of a bond decreases with the price variance of the issuer's shares, because the probability of default is increased. Therefore attaching an option to a bond will act as a kind of hedge against excessive downside price movement, while simultaneously preserving the upside potential if the firm is successful, since the bondholder has the right to convert to equity. Due to this element of downside protection convertible bonds frequently sell at a premium over both their bond value and conversion value, resulting in the premium over conversion value that we referred to earlier. The conversion feature also leads to convertibles generally trading at a premium over bond value as well; the higher the market price of the ordinary share relative to the conversion price, the greater the resulting premium.

EXAMPLE 16.2

- Consider the following Euro convertible bond currently trading at 104.80.

Denomination:	£1000
Coupon:	4.50%
Maturity:	15 years
Conversion price:	£25 per share

The issuer's shares are currently trading at £19.50

1. Number of shares into which the bond is convertible:

$$\text{Conversion ratio} = £1000 / 25 = 40.$$

2. Parity or conversion value:

$$\frac{\text{Current share price}}{\text{Conversion price}} \times 100 = 78\%.$$

3. Effective conversion price:

$$\frac{\text{Price of convertible}}{\text{Conversion ratio}} = 2.62 \text{ (that is, £26.20 per £1000).}$$

4. Conversion premium:

$$\frac{((3.) - \text{Current share price})}{\text{Current share price}} \times 100 = 34.36\%.$$

5. Conversion premium – alternative formula:

$$\text{Convertible price} / \text{Parity} = 34.36\%.$$

6. What do the following represent in terms of intrinsic value and time value?

Parity: parity value reflects the equity value of the bond

Premium: the ratio of the bond price divided by parity value; this includes a measure of time value

16.2 Advantages of issuing and holding convertibles

16.2.1 Borrowers' advantages

The main advantage to a borrowing company in issuing convertible bonds is that the cost of the loan will be lower than a straight issue of debt. This is because, as a result of providing an equity option feature with the instrument, the coupon payable is lower than would be the case with a conventional bond. The bondholder accepts a lower coupon as the price for being able to share in the success of the company during the life of the bond, without having the direct exposure to the equity market that a holding in the ordinary shares would entail. The yield spread below which a convertible may be sold varies over time and with the quality of the issuer. Credit rating agencies generally rate convertible issues one grade below the straight debt of the issuer, although this would appear to reflect the price volatility of convertibles more than credit concerns. The second advantage to an issuer is that, under certain circumstances, it may be able to sell ordinary shares at a more favourable price via conversion than through a direct issue in the market. This may occur when, for example, the price of shares in a direct offer is lower because the shares represent investment in a project that is not expected to show returns until a period into the future. The company can issue callable convertibles with an exercise price above the direct market price, and then call the bond at a later date, forcing conversion at the higher price.

A disadvantage of issuing convertibles is where the company experiences a significant rise in its share price; in this case the interest cost may turn out to have been prohibitive and the company would have gained if it had issued shares directly. This however is only known in hindsight. The same occurs if there is a substantial drop in the share price after convertibles have been issued; here there is no incentive to for bondholders to convert and the company is left with debt on its balance sheet until maturity, when it might have expected to have converted this to equity capital.

16.2.2 Investors' advantages

The advantages to an investor in holding convertible bonds centres on the ability to participate in the fortunes of the company without having to have a direct equity holding. The bondholder has a fixed coupon income stream, together with the advantages of senior debt (so it ranks above equity but below secured debt). If the underlying share price rise, the value of the convertible will rise as well, reflecting the increase in value of the embedded option,

and if the conversion premium disappears the investor is able to realise an instant gain. This is the upside advantage. There is also downside advantage, because if the price of the underlying share falls, the convertible price will fall only to the point at which it represents fair value for an equivalent conventional fixed interest security. Although the coupon available with a convertible is lower than that available on a conventional vanilla bond, it will be higher than the dividend yield available from holding the share directly. If there is a rise in interest rates, there is further downside protection available in the time value of the embedded option, which may also add a floor to the price. Therefore in theory convertibles offer the downside protection of a debt instrument as well as the upside potential of an equity instrument.

The disadvantages to an investor in holding convertibles mirror the effects of the advantages: the main one is that the investor must accept a lower yield compared to bonds of identical maturity and credit quality. If a convertible is also callable, then this is an additional disadvantage for the investor, as the issuer may force conversion of the bond at its choosing, under potentially unfavourable conditions for the investor. The other disadvantages of holding convertibles are only apparent in hindsight: if the issuer's share price does not appreciate, the investor will have accepted a below-market coupon level for the life of the bond, and possibly a drop in the price of the bond below its issue price.

There a range of investor classes that may be interested in holding convertibles at one time or another. These include equity fund managers who are currently bearish of the market: purchasing convertibles allows them an element of downside market protection, whilst still enabling them to gain from upside movements. Equity managers who wish to enhance the income from their portfolios may also be interested in convertibles. For bond fund managers, convertibles provide an opportunity to obtain a limited exposure to the growth potential and upside potential associated with an option on equities.

Selected bibliography and references

Calamos, J., *Convertible Securities*, McGraw-Hill, 1998.

Connolly, K., *Pricing Convertible Bonds*, Wiley, 1998.

Fabozzi, F., *Bond Markets, Analysis and Strategies*, Prentice Hall, 1991.

Kitter, G., *Investment Mathematics for Finance and Treasury Professionals*, Chapter 6, John Wiley and Son, 1999.

17

Convertible Bonds II

In the previous chapter we reviewed convertible bond instruments and the features that differentiated them from conventional fixed interest bonds. In this chapter we consider the pricing and valuation of these securities.

17.1 Traditional valuation methodology

Let us consider another hypothetical security issued by ABC plc, a 20-year convertible bond with a coupon of 8%. “One” bond has a nominal amount of £100 and may be converted into 10 ordinary shares of ABC plc. In November 1999 the bond is trading at £102 and the underlying shares at £2.50. In 1999 the company paid a dividend of £0.08 per share, a dividend yield of 3.20%.

If an investor buys just £100 nominal of the convertible, the premium paid over a direct purchase of the ordinary shares is equal to $(100 \times 102\%) - (2.50 \times 10)$ or £77 per bond. The compensation for this premium is the cash flow differential between the convertible and the underlying shares, which is calculated as $(£100 \times 8\%) - (25 \times 3.20\%)$ or £7.20. The number 25 above is the parity value for the convertible obtained by multiplying the current share price by the conversion ratio. The annual cash flow differential measure implies that the investor receives £7.20 per annum more income from holding the convertible than would be received in the form of dividends from a holding of 10 ordinary shares in the company. This gives us a *payback period* measure of $£77 / 7.20$ or 10.694 years. Using the above analysis we may derive the formula at (17.1) below as an expression for the payback period for a convertible bond.

$$\text{Payback} = \frac{\frac{\text{premium}}{1 + \text{premium}}}{rc - \frac{rd}{1 + \text{premium}}} \quad (17.1)$$

where *rc* is the running yield on the convertible bond and *rd* is the dividend yield on the underlying ordinary shares. Note that (17.1) assumes a fixed coupon stream and therefore can only be used for bonds that pay a constant coupon. Bonds that have variable coupon payments, such as step-up convertibles, as well as bonds with no cash flow such as zero-coupon convertibles, cannot be analysed using (17.1). If we input the same parameters to (17.1) we will obtain the same value for the payback period of 10.694 years.

The concept of payback analysis for convertible securities is similar to payback or *breakeven* analysis in corporate finance: the length of time after which the initial outlay in a capital project is recouped as a result of investment returns from the project. However much corporate finance analysis now uses net present value and the concept of the internal rate of return for project appraisal. The assumptions behind payback period are also restrictive; if we consider (17.1) we see that for the measure to be meaningful it requires a constant dividend yield and a flat yield curve. It also does not discount cash flows and ignores cash flows beyond the payback period. Although it is possible to introduce certain variations into the analysis, for example a (constant) dividend growth rate and discounting of cash flows, the measurement does not take into account the option element of the convertible and the downside price floor available to investors. The value of a convertible must consider not only its premium over the underlying share price but also the fact that investors have the right, but not the obligation, to convert their holding and conversion is at their discretion. The decision to exercise is closely related to the future level of the issuer's share price, and therefore cannot be predicted. As well as holding a call option on the issuer's ordinary shares, bondholders are also holding in effect a put option, because if the share price falls and makes conversion uneconomic, they will elect to hold the bond to maturity and receive the redemption payment instead. Any fair valuation of a convertible should therefore ideally consider both option features of the bond.

The remainder of this chapter reviews a convertible valuation model that incorporates the likelihood of the option feature being exercised during the life of the bond.

17.2 Fair value of a convertible bond

The current analytical approach to convertible bond valuation is to consider the instrument as a conventional vanilla bond and an embedded option(s). That is, the value of a convertible is the sum of its bond value and the value of the embedded option. The bond element is valued using the standard redemption yield method described in Chapter 4. The option element is valued using a binomial option pricing model, which we review in this section.

17.2.1 The binomial model

The first option pricing model was developed by Fischer Black and Myron Scholes and was described in the *Journal of Political Economy* in 1973. Another approach, later termed the *binomial model* was described by Cox, Ross and Rubinstein (1979). Convertible bond analysis, like callable and put-able bonds analysis, frequently uses the binomial model.

The fair price of a convertible bond is the one that provides no opportunity for arbitrage profit, that is, it precludes a trading strategy of running simultaneous but opposite positions in the convertible and the underlying equity in order to realise a profit. There is no need to take a view on the price expectations of the equity to arrive at such a fair price. In this section we consider an application of the binomial model to value a convertible security. Under the conditions of an option pricing model such as Black–Scholes or the binomial model, we assume no dividend payments, no transaction costs and a risk-free interest rate with no bid-offer spreads.

In similar fashion to the description in Chapter 15, application of the binomial model requires the binomial tree detailing the price outcomes from the start period. In the case of a convertible bond this will refer to the prices for the underlying asset, which is the ordinary share of the issuing company. This is shown in Figure 17.1 below.

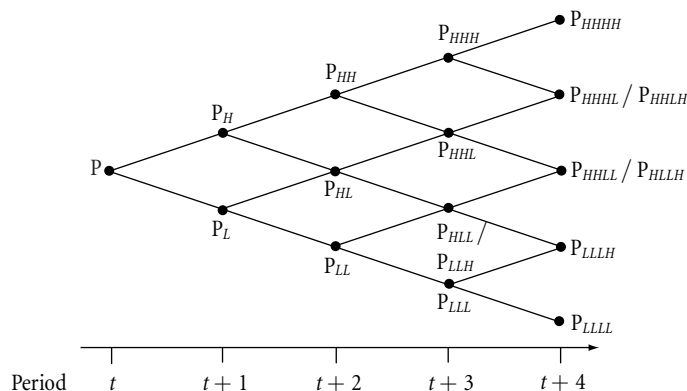


Figure 17.1: Underlying equity price binomial tree.

If we accept that the price of the equity follows such a path, we assume that it follows a *multiplicative binomial process*. This is a geometric process which accelerates as the share price increases and decelerates as the share price falls. This assumption is key to the working of the model and has been the subject of some debate. Although it is not completely accurate, by assuming that market returns follow this pattern we are able to model a series of share price returns that in turn enable us to calculate the fair value of an instrument containing an embedded option.

In Figure 17.1 the current price of the underlying share is given as P , in period 1 at time t . In the next period, time $t + 1$, the share price can assume a price of P_H or P_L , with P_H higher than P and P_L lower than P . If the price is P_H in period 2, the price in period 3 can be P_{HH} or P_{HL} , and so on. The tree may be drawn for as many periods as required.

The value of a convertible bond is a function of a number of variables; for the purposes of this analysis we set the different parameters required as shown below.

P_{conv}	is the price of the convertible bond
P_{share}	is the price of the underlying equity
C	is the bond coupon
r	is the risk-free interest rate

N	is the time to maturity
d	is the risk of default
σ	is the annualised share price volatility
c	is the call option feature
p	is the put option feature
rd	is the dividend yield on the underlying share

In the first instance we wish to calculate the value of a call option on the underlying shares of a convertible bond. For Figure 17.1 if we state that the probability of a price increase is 50%, this leaves the probability of a price decrease as $1 - p$ or 50%. If we were to construct a portfolio of δ shares, funded by M pounds sterling, which mirrored the final payoff of the call option, we can state that the call option must be equal to the value of the portfolio, to remove any arbitrage possibilities. To solve for this we set the following constraints:

$$\delta P_H + rM = P_H - X \quad (17.2)$$

$$\delta P_L + rM = 0 \quad (17.3)$$

where X is the strike price of the option. If we say that the exercise price is 100 then it will be higher than P_L and lower than P_H . This is illustrated by Figure 17.2.

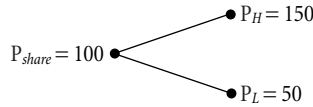


Figure 17.2: Binomial price outcome for call option.

From equation (17.3) we set:

$$rM = -\delta P_L \quad (17.4)$$

$$M = \frac{-(\delta P_L)}{r}. \quad (17.5)$$

Substituting (17.5) into (17.2) gives us:

$$\begin{aligned} \delta P_H - \delta P_L &= P_H - X \\ \Rightarrow \delta &= \frac{P_H - X}{P(H - L)}. \end{aligned} \quad (17.6)$$

The number of shares that would be held in the portfolio is δ , which is known as the *delta* or *hedge ratio*.

We use the relationships above to solve for the value of the borrowing component M , by substituting (17.6) into (17.3), giving us:

$$\begin{aligned} \frac{P_H - X}{P(H - L)} \cdot P_L + rM &= 0, \text{ and rearranging for } M \text{ we obtain:} \\ M &= \frac{-L(P_H - X)}{r(H - L)}. \end{aligned} \quad (17.7)$$

Given the relationships above then we may set the fair value of a call option as (17.8):

$$c = \delta P + r. \quad (17.8)$$

Following Black-Scholes, an estimate of the extent of the increase and decrease in prices from period 1 is given by:

$$H = e^{r+\sigma} \quad (17.9)$$

$$L = e^{r-\sigma} \quad (17.10)$$

where r is the risk-free interest rate and σ is the volatility of the share price at each time period.

We are now in a position to apply this analysis to a convertible security. Table 17.1 sets the terms and parameters for an hypothetical convertible bond and underlying share price.

Equity price	100
Conversion price	100
Coupon	5% s/a
Time to maturity	5 years
Volatility	10%
r	4%
" H "	1.095138
" L "	0.950716

Table 17.1

Using the parameters above we may calculate the value of the underlying equity, which is priced at 100 in period 1. The volatility is assumed constant at 10%, and the maturity of the bond is five years or 1825 days. The price is calculated over ten periods, with each period equal to a half-year or 180 days. The risk-free interest rate is 4%, however this must be continuously compounded, which is calculated as follows:

$$r = \ln(1.04) = 0.039220$$

and over one time period or 180 days this is 0.02017 or 2.017%. This is therefore the risk-free interest rate. The volatility level of 10% is an annualised figure, following market convention. This may be broken down per time period as well, and this is calculated by multiplying the annual figure by the square root of the time period required. This is shown below:

$$\sigma \times \sqrt{t/N} = 0.10 \times \sqrt{0.5} = 0.07071.$$

This allows the calculation of H and L which are given below:

$$H = e^{0.012518 + 0.1424} = 1.095138$$

$$L = e^{0.012518 - 0.1424} = 0.950716.$$

Using these parameters the price tree for the underlying equity, from time periods t to $t+10$ under the assumptions given is shown at Table 17.2, with $t+10$ representing the last period five years from now.

Period	0	1	2	3	4	5	6	7	8	9	10
0	100.00	114.89	124.51	144.08	179.06	205.12	235.69	269.07	294.45	341.25	398.74
1		89.02	100.05	115.94	125.86	155.86	174.60	189.98	245.49	276.48	315.68
2			79.32	88.05	101.04	118.59	136.98	154.87	181.46	207.63	237.87
3				70.14	81.13	90.89	103.74	120.46	138.37	168.74	192.58
4					57.43	67.63	79.03	90.65	103.58	128.41	142.35
5						49.06	56.90	69.05	84.14	94.18	106.41
6							40.14	50.05	61.29	74.85	83.34
7								35.89	45.20	55.04	67.32
8									31.24	42.03	51.26
9										27.67	33.49
10											22.07

Table 17.2: Underlying share price tree.

The valuation now proceeds to the conventional bond element of the convertible. The bond has a coupon of 5.0% payable semi-annually and a maturity of five years. We assume a "credit spread" of 200 basis points above the risk-free interest rate, so a discount rate of 6.00% is used as the market rate required to value the bond. This credit spread is a subjective measure based on the perceived credit risk of the bond issuer. On maturity the bond must be

priced at par or 100, while the final coupon is worth 2.50. Therefore the value of the bond on maturity in the final period (at $t + 10$) must be 102.50. This is shown at Table 17.3. The value of the bond at earlier nodes along the binomial tree is then calculated using straightforward discounting, using the 6% discount rate. For example at $t + 9$ the valuation is obtained by taking the maturity value of 102.5 and discounting at the credit-adjusted interest rate, as shown below:

$$P = (102.5/1.03) + 2.5 = 102.01.$$

This process is continued all the way to time t .

Period	0	1	2	3	4	5	6	7	8	9	10
0	98.23	98.60	98.98	99.38	99.79	100.21	100.64	101.08	101.54	102.01	102.50
1		98.60	98.98	99.38	99.79	100.21	100.64	101.08	101.54	102.01	102.50
2			98.98	99.38	99.79	100.21	100.64	101.08	101.54	102.01	102.50
3				99.38	99.79	100.21	100.64	101.08	101.54	102.01	102.50
4					99.79	100.21	100.64	101.08	101.54	102.01	102.50
5						100.21	100.64	101.08	101.54	102.01	102.50
6							100.64	101.08	101.54	102.01	102.50
7								101.08	101.54	102.01	102.50
8									101.54	102.01	102.50
9										102.01	102.50
10											102.50

Table 17.3: Conventional bond price tree.

We may take the analysis further for a conventional convertible bond plus embedded option. Table 17.4 shows the price tree for the conventional bond where the share price and conversion price is equal to 100 in the current time period. If we assume the share price in period $t + 9$ is 94.18, then in period $t + 10$ the share can assume only one of two possible values, 106.41 or 83.34 (see Table 17.2). In these cases the value of the call option c_H and c_L will be equal to the higher of the bond's conversion value or its redemption value, which is 106.41 if there is a rise in the price of the underlying or 102.50 if there is a fall in the price of the underlying. These are the range of possible final values for the bond, however we require the current (present) value, which must be discounted at the appropriate rate. To determine the correct rate to use, consider the corresponding price of the conventional bond when the share price is 27.67 at period $t + 9$. The price of the bond is calculated on the basis that on maturity the bond will be redeemed irrespective of what happens to the share price. Therefore the appropriate interest rate to use when discounting a conventional bond is the credit-adjusted rate, as this is a corporate bond carrying credit risk. However this does not apply at a different share price; consider the corresponding conventional bond price when the underlying share price is 341.25, in the same time period. The hedge ratio at this point on the binomial price tree is calculated using:

$$\delta = \frac{(c_H - c_L)}{P(H - L)} = 1.$$

The position of a bondholder at this point is essentially long of the underlying stock and also receiving a coupon. A position equivalent to a risk-free bond may be put on synthetically by holding the convertible bond and selling short one unit of the underlying equity. In the event of default the position is hedged, therefore in this case the correct discount rate to use is the risk-free interest rate. The correct rate to use is dependent on the price of the underlying share and how this affects the behaviour of the convertible, and will be either the risk-free rate or a credit-adjusted rate. The adjusted rate can be obtained using (17.11) and indeed all the convertible prices at period $t + 9$ are obtained using (17.11). The process is then carried out "backwards" to complete the entire price tree. At period t with the share price at 100 then the fair value of the convertible is seen to be 113.91.

$$r_{adjusted} = \delta \cdot r + (1 - \delta) \cdot \text{credit adjustment}. \quad (17.11)$$

Period	0	1	2	3	4	5	6	7	8	9	10
0	113.91	120.21	134.70	158.87	188.08	207.34	237.69	270.61	295.12	341.25	398.74
1		106.32	108.73	126.95	135.73	159.09	178.05	190.65	246.67	276.48	315.68
2			104.50	107.12	118.40	125.40	142.28	155.87	182.43	207.63	237.87
3				102.47	105.39	112.58	118.79	122.86	138.98	168.74	192.58
4					101.05	103.41	106.08	108.43	112.79	128.41	142.35
5						100.21	102.76	103.36	103.08	106.07	106.41
6							100.64	101.98	101.75	102.01	102.50
7								101.08	101.54	102.01	102.50
8									101.54	102.01	102.50
9										102.01	102.50
10											102.50

Table 17.4: Convertible bond price tree.

17.2.2 Analysis for call and put features

Convertible bonds are sometimes issued with call and/or put provisions embedded within them. Such features may be analysed using the same binomial tree as was used for valuing the straight convertible bond. Using the same hypothetical bond considered in the previous section, assume now that the bond additionally is callable at set dates, as specified below.

Call provision	
Year 2	115
Year 3	110
Year 4	105
Year 5	100

As specified above, the bond is not callable for the first two years after issue, after which it is callable at specified prices on each anniversary of issue until maturity. The call feature is academic after year 4 because the bond is redeemable at par on the maturity date in any case. As part of the analysis we assume that the issuer will act in its interests, and will call the bond if there is material advantage to be gained from so doing. This then enables us to introduce the call feature into the binomial tree.

The issuer will effect *forced conversion* at nodes on the binomial tree (that occur on bond issue anniversary dates from year 2) where the share price or parity value is greater than the exercise price. It is worthwhile for the issuer to call the bond in such cases because the option is worth more “alive” than “dead”. We may consider forced conversion using the price tree at Table 17.5.

Period	0	1	2	3	4	5	6	7	8	9	10
0	109.34	118.69	130.46	158.87	187.98	206.34	236.89	268.56	294.45	341.25	398.74
1		102.35	110.60	125.95	134.63	158.67	177.27	190.07	245.49	276.48	315.68
2			102.80	105.68	117.64	123.85	141.08	155.28	181.46	207.63	237.87
3				100.76	105.56	108.55	115.75	121.05	138.37	168.74	192.58
4					102.57	101.21	103.45	105.43	103.58	128.41	142.35
5						100.21	100.91	102.06	103.08	106.07	106.41
6							100.64	101.98	101.75	102.01	102.50
7								101.08	101.54	102.01	102.50
8									101.54	102.01	102.50
9										102.01	102.50
10											102.50

Table 17.5: Convertible bond with call feature, binomial price tree.

The callable bond price at $t + 8$ of 294.45 corresponds with the underlying share price at this time period. The holding period return of the convertible at this point may be calculated from the corresponding value at this point for the straight convertible bond, which is 295.12. It is worthwhile for the issuer to call the bond at this point (at a call price of 105) because this value is higher than the value of the bond if it is converted. Therefore we assume the issuer will call the bond at 105 and pay the accrued interest to date. Note that this is academic however because the bondholder will elect to convert and receive the higher of the parity value or 294.45, albeit at the cost of accrued interest.

If the underlying share price is lower than the conversion price, for example at 31.24 in period $t + 8$, the holding value of the convertible is of course the straight bond price of 101.54. There is no value to the company of calling the bond at this point.

Essentially if the holding value of the bond plus accrued interest is higher than the call price, then the fair value of the bond will be equal to the higher of the call price plus accrued or parity value; otherwise the bond value will be equal to the higher of the holding value or the call price. In our illustration the straight convertible bond had a value today of 113.91. The addition of a call feature has reduced the price of the bond (that is, increased the yield, as we would expect) to 109.34, so the call provision is worth 4.57.

17.2.3 Model parameters

The binomial model reviewed in the previous two sections will calculate the fair value for a convertible where certain parameters have been specified. It is immediately apparent that altering any of the inputs to the model will have an impact on the price calculation. In this section we consider the effect of changing one of these parameters.

Share price

The price of the underlying share is a key parameter of the model. A change in the value of the underlying share will result in a change in the value of the convertible; specifically a rise in the underlying will result in a rise in the price of the convertible, and a fall in the price of the underlying will result in a fall in the price of the convertible. The *delta* of an option instrument measures the extent of this change. Figure 17.3 depicts a graph drawn from the values in Table 17.2. It shows that when the share price is at low levels relative to the conversion price, the sensitivity of the convertible to movements in the share price is low. However when the share price is high, this sensitivity or *delta* approaches unity, so that a unit move in the price of the share is matched by a unit move in the price of the bond. In the former case the option is said to be “out-of-the-money”, while in the latter case the option is “in-the-money”. Where the delta approaches unity the option is said to be “deeply in-the-money”.

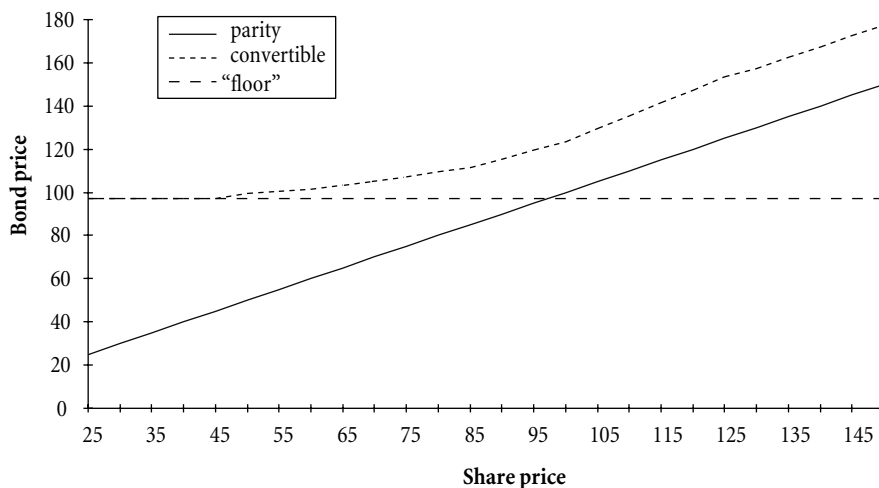


Figure 17.3: Convertible bond price sensitivity.

The measure of the change in the price of the convertible with respect to a change in the price of the underlying equity is given by the delta of the convertible, that is

$$\delta = \frac{\text{change in convertible price}}{\text{change in underlying price}}. \quad (17.12)$$

As we will note again in the introductory chapter on options the delta is defined as the first derivative of the price of an instrument with respect to the price of the underlying asset. Here we are concerned with the delta being the measure of the change in the price of the convertible bond with respect to change in the price of the underlying share. The value of delta in our illustration is given by the gradient of the convertible price line as shown in Figure 17.3. When the option feature of the convertible is deeply in-the-money, the convertible behaves more or less in the same way as the underlying share itself, but one that pays a coupon rather than a dividend.

The second derivative of the price change function, *gamma*, is the measure of the rate of change of delta with respect to the rate of change of the underlying share price, and is given by (17.13):

$$\gamma = \frac{\Delta\delta}{\Delta P_{\text{share}}}. \quad (17.13)$$

At low price levels for the underlying share, gamma is very low. This reflects the fact that at this price the delta is close to zero and the convertible behaves more like a conventional bullet bond, so that for small changes in the share price the delta will be unchanged. This pattern is repeated at high share price levels, where the delta is unity and gamma is close to zero: when a convertible is deep in-the-money and the delta is one, small increases in the share price will not affect the delta.

Volatility

A change in the volatility of the underlying share price will affect the price of the convertible, given that under the binomial model its value has been calculated based on a volatility level for the share. The measure of sensitivity of the convertible price to a change in the volatility of the underlying share is given by *vega*. Put simply, an increase in the volatility of the underlying has the effect that there is a greater probability of nodes on the binomial price tree being reached. This has the effect of increasing the value of the convertible.

Interest rate

A change in the risk-free interest rate will have an impact on the price of the convertible. The measure of sensitivity of a convertible bond's price to changes in the level of interest rates is given by *rho*. The key factor is that the price of deeply out-of-the-money convertibles is sensitive to changes in interest rates, because at this point the convertible behaves similarly to a conventional straight bond. Deep in-the-money convertibles behave more like the underlying equity and are hardly affected by changes in interest rates.

17.3 Further issues in valuing convertible bonds

17.3.1 Dilution

Up to now the discussion has focused on treating the convertibility feature of the convertible bond much as a call option on the underlying stock. There are practical reasons why a convertible is not precisely identical to a call option however. In the first instance when a convertible bond is converted into the underlying shares, this results in the issue of new shares. This does not happen in the exercise of an equity call option, for which already existing shares are delivered. The effect on existing shareholders of a further issue of ordinary shares is termed *dilution*. In practice the effect of dilution is not often accounted for in the pricing of a convertible, although it is relevant.

The basic effect is given as (17.14) below, which we illustrate for an equity warrant. The exercise of a warrant results in the issue of new shares, upon which it may be shown that:

$$P_{\text{warrant}} = \max((P_{\text{share}} - E)/(1 + \lambda), 0) \quad (17.14)$$

where E is the exercise price and λ is the dilution factor resulting from the exercise of the warrant. This means that the holder of the warrant will receive the maximum of the diluted difference between the share price and exercise price, and zero. However the share price will reflect the effect of the approaching exercise date, that is it will incorporate the impact of the anticipated dilution. Logically this should have an impact on the price of the warrant or convertible, but is a function of how much in-the-money the convertible is. In a binomial model the probabilities can be adjusted to one and zero for the final price path, if the convertible is deeply in- or out-of-the-

money. With certain probabilities entered, the analyst may work backwards along tree to arrive at the probability of conversion at each node. The effect of dilution could then be ignored for deeply out-of-the-money convertibles.

A new issue of convertible bonds can be expected to have an effect on the volatility of the share price and its likely path along a binomial tree. As the bond approaches maturity a commonly observed phenomenon is of it being “pulled to the money”. The share price is effectively capped as a significant rise would result in share selling as the hedge ratio approached 100%, with selling by market makers who are long the convertible.

17.3.2 Volatility level

The pricing model reviewed in the previous section assumed a constant volatility level for the price returns of the underlying equity. This is clearly unrealistic over periods of time, although it is accepted by the market for short periods; this is because changes in volatility levels appear to occur quite slowly. Such changes also appear to be largely random rather than systematic, that is volatility appears to be *stochastic*. There is a considerable literature on incorporating changing volatility levels into pricing models, although it is common for banks to use the static model described in this chapter. This is because the binomial model appears to perform well in practice. Instruments such as convertible bonds have relatively long maturities, and are less sensitive to jumps in short-term volatility levels. That is, the impact of a change in volatility is viewed as not material when pricing long-dated instruments.

There are a number of issues that it is worthwhile to consider however. One of the most important is the concept of *mean reversion*. This is the term for a commonly observed behaviour pattern of volatility levels, whereby volatility levels return to a mean level after a period of corrections or market crashes. For example after the 1987 stock market crash or “Black Monday” in the sterling market in 1992, individual volatility levels achieved very high figures (certain stock volatilities exceeded 100%), but it was only a matter of time before they reverted to a mean level. The same has been observed for commodity prices, for example the crude oil price in the aftermath of the Gulf crisis of 1990/91.

The *implied volatility* levels of longer-dated instruments such as convertible bonds reflect the fact that the market anticipates mean reversion. Observation of market behaviour also confirms that the volatility levels of individual shares tend to move in the same direction. This is logical, in the same way that share prices themselves reflect an element of specific company issues and an element that is attributed to the market in general. Another observation from the market is that share prices tend to be inversely related to changes in volatility levels. This is common for both individual shares as well as for market indices. The common explanation for this behaviour is that when share prices fall (rise), the debt/equity ratio rises (falls), which implies a higher (lower) risk level for equities. This indicates *skewed* distribution of returns. So far we have not discussed the distribution of stock price returns, which is generally assumed to follow a *normal* distribution, which is an assumption of the Black–Scholes and binomial models.¹ The market also accepts that this is not completely accurate, and that the distribution of returns tends to have fatter tails than a conventional normal distribution, a phenomenon known as *leptokurtosis*.² The existence of fatter tails will have an impact on pricing models, as deeply in or out-of-the-money options will tend to be mispriced.

These issues are reviewed in Part V.

17.4 Convertible bond default risk

Virtually all convertible bonds are issued by corporates and therefore are expose their holders to an element of *default risk*.³ This is defined as the risk of a corporate failing to meet its contractual obligations regarding the payment of coupon or redemption proceeds. Although shareholders are at risk of the issuing company going bankrupt, strictly speaking this is not default risk because shares do not promise to pay a fixed interest cash flow for a set period of time in the future. The presence of default risk introduces further issues in the valuation of convertible bonds, which are considered in this section.

¹ Strictly speaking for the Black–Scholes model the assumption is that market returns follow a *geometric Brownian motion*.

² This is discussed in Chapter 37.

³ Certain developed country governments have issued convertible bonds, including the United Kingdom, but these are convertible into another debt issue.

17.4.1 Credit ratings

The most commonly quoted ratings are those issued by Standard & Poor's and Moody's. Within domestic markets there are other local agencies that may be more influential or more frequently used. Rating agencies assign a formal credit rating to the individual issue of a corporate, although it is common for the market to refer to say, a "double A-rated company". The ratings fall into two main categories, *investment grade* and *speculative grade*. There is also a third category, companies in default. The role of the formal rating agencies is reviewed in Chapter 30; the main areas that they assess include the debt/equity ratio, the asset base, volatility of earnings per share, and the level of subordination of debt. The agencies analyse published accounting data as well as qualitative data such as the credibility and strength of senior management, and publish forecasts on company performance.

The credit rating of a company is a major determinant of the yield that will be payable by that company's bonds. This is because, in an increasingly integrated and global capital market, *name recognition* (the traditional major determinant) plays a decreasing role. The yield spread of a corporate bond over the risk-free bond yield is known as the *default premium*. In practice the default premium is composed of two elements, the compensation element specific to the company and the element related to market risk. This is because, in an environment where the default of one company was completely unrelated to the default of other companies, the return from a portfolio of corporate bonds would equal that of the risk-free bond, as the gains from bonds of companies that did not default compensated for the loss from those that did default. The additional part of the default premium, the *risk premium* is the compensation for risk exposure that cannot be diversified away in a portfolio, known as *systematic* or *non-diversifiable* risk. Observation of the market tells us that in certain circumstances the default patterns of companies are related, for example in a recession there are more corporate defaults, and this fact is reflected in the risk premium.

17.4.2 The credit spread

Earlier in this chapter we reviewed a binomial pricing model for convertible bonds. One of the parameters of the model is the credit-risk adjusted discount rate. The appropriate rate to use for this parameter is not always clear cut, as it is a subjective matter as to what credit spread to apply to the risk-free rate. The analysis is less problematic if there is an equivalent-maturity conventional bond from the same issuer in existence. The yield spread between such a bond and the convertible bond can be said, simplistically, to reflect the market's appetite for the convertible over the straight bond. If this is not available, it may be appropriate to consider the conventional bond issued by a company that is in the same industry as the issuer, and has a similar capital structure. For deeply out-of-the money convertibles, the yield may be taken to be the equivalent of a conventional bond, and therefore it is possible to view the theoretical spread directly.

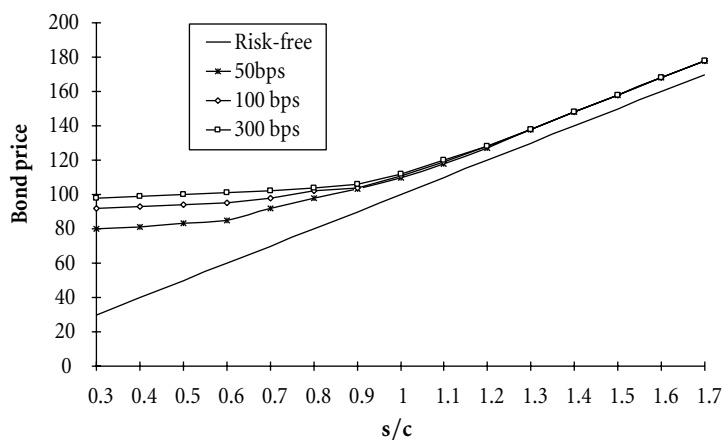


Figure 17.4: Price sensitivity of credit spread.

The sensitivity of the theoretical yield spread of a convertible bond to moves in the underlying equity price varies with credit rating. This is to be expected, since, in addition to their yields, lower-rated bonds have higher price volatility levels. This sensitivity is at its highest during times of extreme volatility in the equity market and during

times of recession, during which yield spreads widen most significantly. It is possible to plot this sensitivity, shown as Figure 17.4.

The graph at Figure 17.4 shows the theoretical price profile of the hypothetical 1.50% coupon convertible bond introduced earlier, with different credit spreads selected. The risk-free interest rate is 1.50%, and the constant volatility level is 20% as before. The price profile is then plotted for credit-adjusted spreads of 50 basis points, 100 basis points and 300 basis points. When a convertible is deeply in-the-money, default risk is less significant, and at a delta value approaching unity, the risk-free rate becomes the relevant rate at which to value the bond; at this point, a change in the credit spread has little or no impact on the price of the convertible. The opposite is true when the convertible is deeply out-of-the-money and/or when the delta approaches zero.

Appendices

APPENDIX 17.1 Capital asset pricing model

We briefly discuss the capital asset pricing model because of the relevance of the underlying equity value in the valuation of convertibles. The capital asset pricing model (CAPM), attributable to Sharpe (1964), is a cornerstone of modern financial theory and originates from analysis on the cost of capital. The cost of capital of a company may be broken down as shown by Figure 17.5.

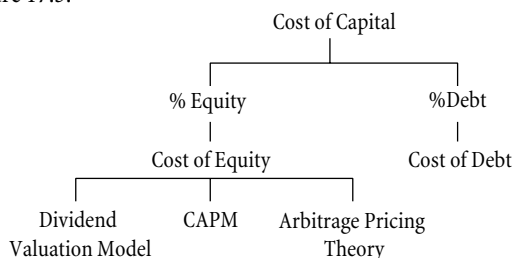


Figure 17.5: Components of the cost of capital.

The three most common approaches used for estimating the cost of equity are the *dividend valuation model*, CAPM and the *arbitrage pricing theory*. CAPM is in a class of market models known as *risk premium* models which rely on the assumption that every individual holding a risk-carrying security will demand a return in excess of the return they would receive from holding a risk-free security. This excess is the investor's compensation for her risk exposure. The risk premium in CAPM is measured by *beta*, it is known as *systematic*, *market* or *non-diversifiable* risk. This risk is caused by macroeconomic factors such as inflation or political events, which affect the returns of all companies. If a company is affected by these macroeconomic factors in the same way as the market (usually measured by a stock index), it will have a beta of 1, and will be expected to have returns equal to the market. Similarly if a company's systematic risk is greater than the market, then its capital will be priced such that it is expected to have returns greater than the market. Essentially therefore beta is a measure of volatility, with a company's relative volatility being measured by comparing its returns to the market's returns. For example if a share has a beta of 2.0, then on average for every 10% that the market index has returned above the risk-free rate, the share is expected to have returned 20%. Conversely for every 10% the market has under-performed the risk-free rate, the share is expected to have returned 20% below. Beta is calculated for a share by measuring its variance relative to the variance of a market index such as the FTSE All Share or the S&P 500. The most common method of estimating beta is with standard regression techniques based on historical share price movements over say, a five-year period.

To obtain the CAPM estimate of the cost of equity for a company, two other pieces of data are required, the risk-free interest rate and the equity risk premium. The risk-free rate represents the most secure return that can be achieved in the market. It is theoretically defined as an investment that has no variance and no covariance with the market; a perfect proxy for the risk-free rate therefore would be a security with a beta equal to zero, and no volatility. Such an instrument does not, to all intents and purposes, exist. Instead the market uses the next-best proxy available, which in a developed economy is the government issued Treasury bill, a short-dated debt instrument guaranteed by the government.

The equity risk premium represents the excess return above the risk-free rate that investors demand for holding risk-carrying securities. The risk premium in the CAPM is the premium above the risk-free rate on a portfolio assumed to have a beta of 1.0. The premium itself may be estimated in a number of ways. A common approach is to use historical prices, on the basis that past prices are a satisfactory guide to the future⁴ and use these returns over time to calculate an arithmetic or geometric average. Research has shown that the market risk premium for the US and UK has varied between 5.5% and 11% historically (Mills 1994), depending on the time period chosen and the method used.

Once the beta has been determined, the cost of equity for a corporate is given by CAPM as (17.15):

$$k_e = r_f + (\beta \times r_e) \quad (17.15)$$

where

k_e	is the cost of equity
r_f	is the risk-free interest rate
r_e	is the equity risk premium
β	is the share beta.

The primary assumption behind CAPM is that all the market-related risk of a share can be captured in a single indicator, the beta. This would appear to be refuted by evidence that fund managers sometimes demand a higher return from one portfolio than another when both apparently are equally risk, having betas of 1.0. The difference in portfolio returns cannot be due to differences in specific risk, because diversification nearly eliminates such risk in large, well-balanced portfolios. If the systematic risk of the two portfolios were truly identical, then they would be priced to yield identical returns. Nevertheless the CAPM is often used by analysts to calculate cost of equity and hence cost of capital.

If we consider the returns on an individual share and the market as positively sloping lines on a graph plotting return, beta is usually given by (17.16):

$$r_s = \alpha_{sI} + \beta_{sI}r_I + \varepsilon_{sI} \quad (17.16)$$

where

r_s	is the return on security s
r_I	is the return on the market (usually measured for a given index)
α_{sI}	is the intercept between s and I , often termed the “alpha”
β_{sI}	is the slope measurement or beta
ε	is a random error term.

Selected bibliography

Bhattacharya, M., Zhu, Y., “Valuation and Analysis of Convertible Securities”, in Fabozzi, F., *The Fixed Income Handbook*, 5th edition, McGraw-Hill, pp. 791–817.

Bierman, H., “The cost of warrants”, *Journal of Financial and Quantitative Analysis*, June 1973, pp. 499–503.

Black, F., Scholes, M., “The pricing of options and corporate liabilities”, *Journal of Political Economy* 81, 1973, pp. 637–659.

Brennan, M., Schwartz, E., “Analysing convertible bonds”, *Journal of Financial and Quantitative Analysis* 15, November 1980, pp. 907–929.

Calamos, J.P., *Investing in Convertible Securities*, Longman, 1988.

Cooper, I., *The relationship between two methods of valuing convertible bonds*, London Business School, 1988.

Cowan, A.R., Nandkumar N., Singh, A.K., “Calls and out of the money convertible bonds”, *Financial Management* 22, 1993, pp. 105–116.

Cox, J., Ross, S., Rubinstein, M., “Option pricing: a simplified approach”, *Journal of Financial Economics* 7(3), September 1979, pp. 229–263.

⁴ Not that any self-respecting independent financial advisor would say this!

- Dunn, K., Eades, K., "Voluntary conversion of convertible securities and the option call strategy", *Journal of Financial Economics* 23, 1989, pp. 273–301.
- Gemmell, G., *Options Pricing*, McGraw-Hill International, 1993.
- Ho, T.S.Y., Lee, S., "Term structure movements and pricing interest rate contingent claims", *Journal of Finance*, December 1986, pp. 1–22.
- Ingersoll, J., Jr., "An examination of convertible call policies on convertible securities", *Journal of Finance* 32, May 1977, pp. 463–478.
- Kim Yong-Cheol, Stultz, R., "Is there a global market for convertible bonds?", *Journal of Business* 65, 1992, pp. 75–92.
- King, R., "The effect of convertible bond equity values on dilution and leverage", *The Accounting Review* 59, 1984, pp. 419–431.
- Liebowitz, M., "Understanding convertible securities", *Financial Analysts Journal* 30, 1974, pp. 57–67.
- Lewis, C., "Convertible debt: valuation and conversion in complex capital structures", *Journal of Banking and Finance* 15, 1991, pp. 665–682.
- McGuire, S., *Convertibles*, Woodhead-Faulkner, 1990.
- Mills, R., *Strategic Value Analysis*, Mars Business Associates, 1994.
- Sharpe, W., "Capital Asset Prices: A Theory of Market Equilibrium Under Conditions of Risk", *Journal of Finance*, September 1964.
- Walter, J., Que, A., "The valuation of convertible bonds", *Journal of Finance* 28, 1973, pp. 713–732.

Questions and exercises

- Anwar Investment Limited announces that part of its portfolio will be invested in high-yield convertible bonds, BB- and BBB-rated. The fund manager stated to the financial press that she was interested in this sector as the bonds trade at discounts and are unlikely to be called.
 - what sort of premium over their "straight value" would these bonds trade at?
 - why is the fund manager interested in lower-rated bonds, and what is the significance of her belief that the bonds are unlikely to be called?
- Consider a convertible bond issued with the following terms and conditions:

Par value:	£1,000
Coupon:	8.00%
Market price:	£100
Conversion ratio:	50
Estimated value of straight bond:	£590
Yield to maturity of straight bond:	15.8%

The current price of the ordinary share is £15.00, and the last dividend was £0.50 per share. Calculate the following:

- conversion value
- market conversion price
- conversion premium per share
- conversion premium ratio
- premium over straight value
- yield advantage of straight bond
- premium payback period

Assume that the price of the ordinary share rises to £30. What will be the approximate return gained from investing in the convertible? What would the return be if £15 had been invested directly in the share? Why would the return on investing in the ordinary shares be higher than investing in the convertible?

Assume that the price of the ordinary share now falls to £5. Re-calculate the answers to the questions above. In this case, why would the return from investing in the shares be lower than investing in the convertible bond?

3. A fund manager is searching for opportunities to invest in convertible bonds that are trading at low levels as a result of the issuer's equity having fallen in value. The manager prefers convertibles with 10- to 15-year maturity ranges, trading at yields that are similar to conventional bonds, issued by companies that are believed to be good long-term prospects. Comment on the fund manager's strategy.
4. What are the drawbacks of using the payback period measure to analyse convertible bonds?
5. Farhana works in the research department of a stockbroker. She is preparing an analysis on ABC plc, which has issued conventional bonds and convertibles. A number of investment banks make markets in OTC options on ABC plc ordinary shares. Farhana is considering two possible portfolios:

Portfolio 1: €100,000 nominal of ABC plc convertible bonds with a conversion ratio of 25

Portfolio 2: €100,000 nominal of ABC plc conventional bonds, and a call option on 100 ABC plc ordinary shares

The portfolios are similar in that both consist of ABC plc bonds and a further interest in ABC plc ordinary shares. The terms of the bonds are as follows:

	<i>Convertible</i>	<i>Conventional</i>
Coupon	6%	9%
Maturity (years)	5	5
Current price	94.00	100
Redemption yield	7.482%	9%

Call option terms:

Strike price	€50.00
Expiry	5 years
Premium	€10.00

Compare the two portfolios, and suggest how Farhana's report should assess them with respect to:

- (a) the specific transactions that are required to change the holding from one of bonds to one of ordinary shares;
 - (b) the parties involved in each of the transactions;
 - (c) the risks associated with the transactions.
6. Complete the following table, using 4.1558 as the present value of an annuity of €1 payable annually for five years, discounted at the risk-free rate of 6.50%.

	<i>Convertible</i>	<i>Conventional</i>
Cost of bonds:		
Cash amount of coupon:		

Calculate the present value of the difference in the cash flows. Which portfolio should Farhana recommend be purchased, based on the analysis carried out above?

18

The Eurobond Market I

Virtually nowhere has the increasing integration and globalisation of the world's capital markets been more evident than in the Eurobond market. It is an important source of funds for many banks and corporates, not to mention central governments. The Eurobond market has benefited from much of the advances in financial engineering, and has undergone some innovative changes in the debt capital markets. It continues to develop new structures, in response to the varying demands and requirements of specific groups of investors. The range of innovations have customised the market to a certain extent and often the market is the only opening for certain types of government and corporate finance. Investors also often look to the Eurobond market due to constraints in their domestic market, and Euro securities have been designed to reproduce the features of instruments that certain investors may be prohibited from investing in in their domestic arena. Other instruments are designed for investors in order to provide tax advantages. The traditional image of the Eurobond investor, the so-called "Belgian dentist", has changed and the investor base is both varied and geographically dispersed.

The key feature of Eurobonds is the way they are issued, internationally across borders and by an international underwriting syndicate. The method of issuing Eurobonds reflects the cross-border nature of the transaction, and unlike government markets where the auction is the primary issue method, Eurobonds are typically issued under a "fixed price re-offer" method or a "bought deal". There is also a regulatory distinction as no one central authority is responsible for regulating the market and overseeing its structure.

This chapter reviews the Eurobond market in terms of the structure of the market, the nature of the instruments themselves, the market players, the issuing process and technical aspects such as taxation and swap arrangements. A subsequent chapter reviews the secondary market.

18.1 Eurobonds

A Eurobond is a debt capital market instrument issued in a "Eurocurrency" through a syndicate of issuing banks and securities houses, and distributed internationally when issued, that is sold in more than one country of issue and subsequently traded by market participants in several international financial centres. The Eurobond market is divided into sectors depending on the currency in which the issue is denominated. For example US dollar Eurobonds are often referred to as *Eurodollar* bonds, similar sterling issues are called *Eurosterling* bonds. The prefix "Euro" was first used to refer to deposits of US dollars in continental Europe in the 1960s. The Euro-deposit now refers to any deposit of a currency outside the country of issue of that currency, and is not limited to Europe. For historical reasons and also due to the importance of the US economy and investor base, the major currency in which Eurobonds are denominated has always been US dollars. The volume of non-sovereign Eurobond issues from 1996 to 1999 is shown at Figure 18.1.

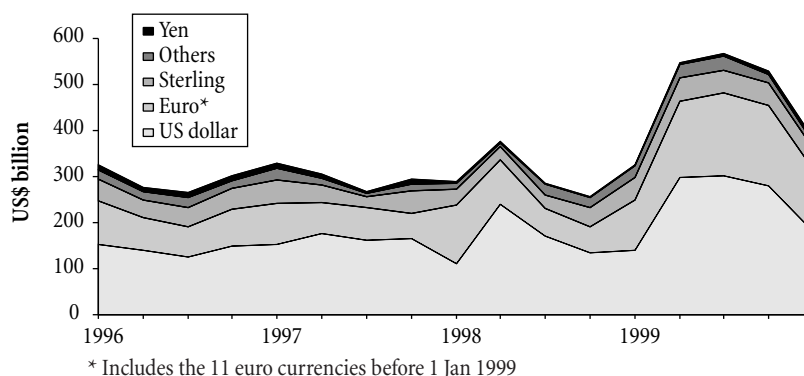


Figure 18.1: Non-government international bond issuance, 1996–1999. Source: BoE.

The first ever Eurobond is generally considered to be the issue of \$15 million nominal of ten-year 5½% bonds by Autostrada, the Italian state highway authority, in July 1963.¹ The bonds were denominated in US dollars and paid an annual coupon in July each year. This coincides with the imposition in the United States of the Interest Equalisation Tax, a withholding tax on domestic corporate bonds, which is often quoted as being a prime reason behind the establishment of overseas deposits of US dollars.

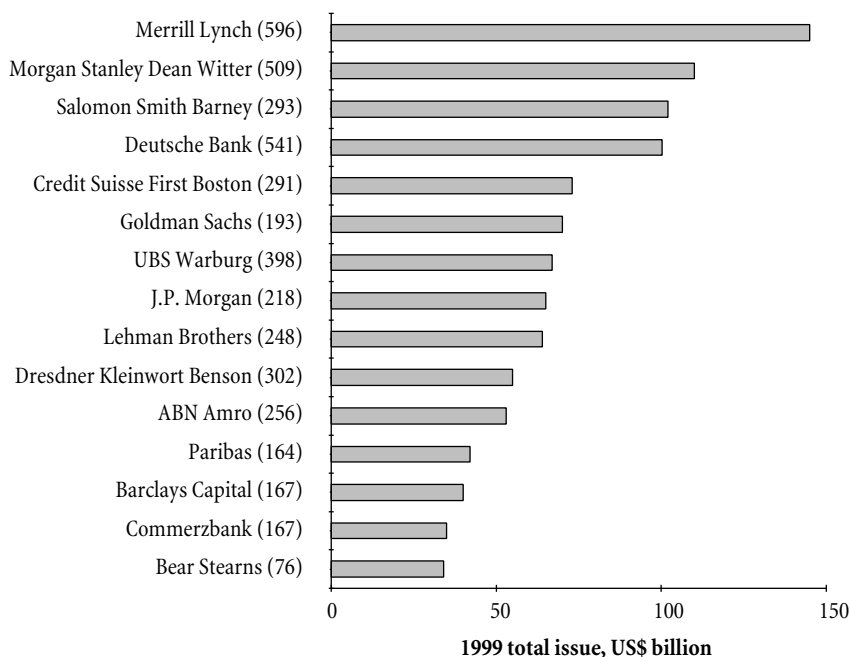


Figure 18.2: International Bond Issues in 1999, with leading book-runners (the number in brackets is the number of issues). Source: *The Economist*.

18.2 Foreign bonds

At this stage it is important to identify “foreign bonds” and distinguish them from Eurobonds. Foreign bonds are debt capital market instruments that are issued by foreign borrowers in the domestic bond market of another country. As such they trade in a similar fashion to the bond instruments of the domestic market in which they are issued. They are usually underwritten by a single bank or a syndicate of domestic banks, and are denominated in the currency of the market in which they are issued. For those familiar with the sterling markets the best example of a foreign bond is a *Bulldog* bond, which is a sterling bond issued in the UK by a non-UK domiciled borrower. Other examples are *Yankee* bonds in the United States, *Samurai* bonds in Japan, *Rembrandt* bonds in the Netherlands, *Matador* bonds in Spain, and so on. Hence a US company issuing a bond in the UK, denominated in sterling and underwritten by a domestic bank would be issuing a *Bulldog* bond, which would trade as a gilt, except with an element of credit risk attached. In today’s integrated global markets however, the distinction is becoming more and more fine. Many foreign bonds pay gross coupons and are issued by a syndicate of international banks, so the difference between them and Eurobond may be completely eroded in the near future.

The most important domestic market for foreign bond issues has been the US dollar market, followed by euros, Swiss francs and Japanese yen. There are also important markets in Canadian and Australian dollars, and minor markets in currencies such as Hong Kong dollars, Kuwaiti dinars and Saudi Arabian riyals.

¹ Decovny (1998) states that the first Eurobond issue was in 1957, but its identity is not apparent.

18.3 Eurobond instruments

There is a wide range of instruments issued in the Eurobond market, designed to meet the needs of borrowers and investors. We review the main types in this section.

18.3.1 Conventional bonds

The most common type of instrument issued in the Euro markets is the conventional vanilla bond, with fixed coupon and maturity date. Coupon frequency is on annual basis. The typical face value of such Eurobonds is \$1000, €1000, £1000 or so on. The bond is unsecured, and therefore depends on the credit quality of its issuer in order to attract investors. Eurobonds have a typical maturity of five to ten years, although many high quality corporates have issued bonds with maturities of thirty years or even longer. The largest Eurobond market is in US dollars, followed by issues in Euros, Japanese yen, sterling and a range of other currencies such as Australian, New Zealand and Canadian dollars, South African rand and so on. Issuers will denominate bonds in a currency that is attractive to particular investors at the time, and it is common for bonds to be issued in more “exotic” currencies, such as East European, Latin American and Asian currencies.

Eurobonds are not regulated by the country in whose currency the bonds are issued. They are typically registered on a national stock exchange, usually London or Luxembourg. Listing of the bonds enables certain institutional investors, who are prohibited from holding assets that are not listed on an exchange, to purchase them. The volume of trading on a registered stock exchange is negligible however; virtually all trading is on an over-the-counter (OTC) basis directly between market participants.

Interest payments on Eurobonds are paid gross and are free of any withholding or other taxes. This is one of the main features of Eurobonds, as is the fact that they are “bearer” bonds, that is there is no central register. Historically this meant that the bond certificates were bearer certificates with coupons attached; these days bonds are still designated “bearer” instruments but are held in a central depository to facilitate electronic settlement.

Issuer	Rating	Coupon	Maturity	Volume e mn	Launch spread (benchmark) bps
Pearson	Baa1/BBB+	4.625%	July 2004	400	82
Lafarge	A3/A	4.375%	July 2004	500	52
Mannesmann	A2/A	4.875%	September 2004	2500	75
Enron	Baa2/BBB+	4.375%	April 2005	400	90
Swissair	–	4.375%	June 2006	400	78
Renault	Baa2/BBB+	5.125%	July 2006	500	88
Continental Rubber	–	5.25%	July 2006	500	100
Yorkshire Water	A2/A+	5.25%	July 2006	500	75
British Steel	A3/A-	5.375%	August 2006	400	105
International Paper	A3/BBB+	5.375%	August 2006	250	105
Hammerson	Baa1/A	5%	July 2007	300	92
Mannesmann	A2/A	4.75%	May 2009	3000	70

Table 18.1: Selected euro-denominated Eurobond issues in 1999. Source: Bloomberg.

18.3.2 Floating rate notes

An early innovation in the Eurobond market was the floating rate note (FRN). They are usually short- to medium-dated issues, with interest quoted as a spread to a reference rate. The reference rate is usually the London interbank offered rate (Libor), or the Singapore interbank offered rate for issues in Asia (Sibor). The euro interbank rate (Euribor) is also now commonly quoted. The spread over the reference rate is a function of the credit quality of the issuer, and can range from 10 to 150 basis points over the reference rate or even higher. Bonds typically pay a semi-annual coupon, although quarterly coupon bonds are also issued. The first FRN issue was by ENEL, an Italian utility company, in 1970. The majority of issuers are financial institutions such as banks and securities houses.

There are also perpetual, or undated, FRNs, the first issue of which was by National Westminster Bank plc in 1984. They are essentially similar to regular FRNs except that they have no maturity date and are therefore

“perpetual”. Most perpetual FRNs are issued by banks, for whom they are attractive because they are a means of raising capital similar to equity but with the tax advantages associated with debt. They also match the payment characteristics of the banks assets. Traditionally the yield on perpetuals is higher than both conventional bonds and fixed-term FRNs.

18.3.3 Zero-coupon bonds

An innovation in the market from the late 1980s was the zero-coupon bond, or *pure discount* bond, which makes no interest payments. Like zero-coupon bonds initially in government markets, the main attraction of these bonds for investors was that, as no interest was payable, the return could be declared entirely as capital gain, thus allowing the bondholder to avoid income tax. Most jurisdictions including the US and UK have adjusted their tax legislation so that the return on zero-coupon bonds now counts as income and not capital gain.

18.3.4 Convertible bonds²

Another instrument that is common in the Eurobond market is the convertible bond. A Eurobond is convertible if it may be exchanged at some point for another instrument, usually the ordinary shares (equity) of the issuing company. The decision to elect to convert is at the discretion of the bondholder. Convertibles are analysed as a structure comprised of a conventional bond and an embedded option.

The most common conversion feature is an *equity convertible*, which is a conventional bond that is convertible into the equity of the issuer. The conversion feature allows the bondholder to convert the Eurobond, on maturity or at specified times during the bond's life, into a specified number of shares of the issuing company at a set price. In some cases the bond is convertible into the shares of the company that is guaranteeing the bond. The issuing company must release new shares in the event of conversion. The price at which the bond is convertible into shares, known as the exercise price, is usually set at a premium above the market price of the ordinary shares in the market on the day the bond is issued. Investors will exercise their conversion rights only if the market price has risen sufficiently that a gain will be realised by converting. The incorporation of a conversion feature in a bond is designed to make the bond more attractive to investors, as it allows them to gain from a rise in the issuing company's share price. The conversion feature also acts as a floor for the bond price. The advantages of convertibles for borrowers include the following:

- as the bond incorporates an added attraction in the form of the conversion feature, the coupon payable on the bond is lower than it otherwise would be; this enables the borrower to save on interest costs;
- issuing convertibles is one method by which companies can broaden the geographical base of their equity holders;
- companies are usually able to raise a higher amount at one issue if the bond is convertible, compared to a conventional bond.

Against these factors must be weighed certain disadvantages associated with convertibles, which include the following:

- the investor's insurance against the volatility of share price movements, an attraction of the convertible, is gained at the cost of a lower coupon than would be obtained from a conventional bond;
- convertibles are often issued by companies that would have greater difficulty placing conventional paper. Convertibles are usually subordinated and are often viewed more as equity rather than debt. The credit and interest-rate risk associated with them is consequently higher than for conventional bonds.

There have been variations on the straight convertible bond in the Eurobond market. This includes the *convertible preference share*. This is a combination of a perpetual debt instrument cash flow with an option to convert into ordinary shares. Sometimes these issues are convertible not into shares of the issuer, but rather into the equity of a company in which the issuer has a significant shareholding.

Another variation is the *equity note*, which is a bond that is redeemed in shares and not in cash. The equity note is not a true convertible, since the conversion feature is not an option for the bondholder but a condition of the bond

² Convertible bonds are reviewed in Chapters 16–17.

issue, and is guaranteed to take place. A more accurate description of an equity note would be an “interest bearing equity future” note.

Eurobonds have also been issued with a feature that allows conversion into other assets such as crude oil or gold, or into other bonds with different payment characteristics. These are known as *asset convertibles*. Examples of such bonds include FRNs that are convertible under specified circumstances into fixed-rate bonds. One version of this was the *drop-lock* bond, which was first introduced in the early 1980s during a period of high interest rates. Drop-lock bonds are initially issued as FRNs but convert to a fixed-rate bond at the point that the reference rate falls to a pre-set level. The bond then pays this fixed rate for the remainder of its life. During the 1990s as interest rate volatility fell to relatively lower levels, drop-locks fell out of favour and it is now rare to see them issued.

Currency convertibles are bonds that are issued in one currency and are redeemed in another currency or currencies. Often this at the discretion of the bondholder; other currency convertibles pay their coupon in a different currency to the one they are denominated in. In certain respects currency convertibles possess similar characteristics to a conventional bond issued in conjunction with a forward contract. The conversion rate is specified at the time of issue, and may be either a fixed-rate option or a floating-rate option. With a fixed-rate option the exchange rate between the currencies is fixed for the entire maturity of the bond at the time it is issued; with a floating-rate option the exchange rate is not fixed and is the rate prevailing in the market at the time the conversion is exercised. Initially most currency convertibles offered a fixed-rate option, so that the foreign exchange risk resided entirely with the issuer. Floating-rate options were introduced in the 1970s when exchange rates began to experience greater volatility.

18.3.5 Eurowarrants

The Eurobond warrant or *Eurowarrant* is essentially a call option attached to a conventional bond. The call option is convertible into either ordinary shares or other bonds of the issuing company, or rarely, another company. A typical Eurobond warrant will be comprised of a conventional bond, issued in denominations of \$1,000 or \$10,000, paying a fixed coupon. The attached warrant will entitle the bondholder to purchase shares (or bonds) at a specified price at set dates, or a set time period, up until maturity of the warrant, whereupon the warrant expires worthless. Warrants are often detached from their host bond and traded separately.

The exercise price of a warrant is fixed at a premium over the market price of the equity at issue. This premium is separate to the premium associated with a warrant in the secondary market, which is the total premium cost connected with buying the warrant and immediately exercising it into the equity, and not the cost associated with a purchase of the equity in the open market.

There are several advantages that Eurobond warrants hold for investors. They are composed of two assets that are usually traded separately in the secondary market; indeed warrants are often attached to bonds as a “sweetener” for investors. Investors have an interest in the performance of the shares of the issuer without having a direct exposure to them. Should the intrinsic value of the warrant fall to zero, there is still time value associated with the warrant up until the maturity of the bond. Warrants typically possess high *gearing*, which is defined as the ratio of the cost of the warrant to the cost of the shares that the warrant holder is entitled to purchase. Borrowers may also gain from attaching warrants to their bond issues. The advantages include being able to pay a lower coupon than might otherwise have been the case. The exercise of a warrant results in the issuer receiving cash for the shares that are purchased (albeit at a below-market rate), compared with a convertible bond where the issuer receives only bonds that are subsequently cancelled. This is a feature of the warrant’s gearing, as the value of the warrant is always less than the price at which the company guarantees to issue new equity to the warrant holder. The disadvantage at the time the warrant is exercised is that the company is receiving a below-market price for its shares at a time when they are trading at a historically high level; however there is a form of compensation for this since the company would have issued the bonds at a lower coupon rate than would have been the case had the warrants not been attached.

18.4 The issuing process: market participants

When a company raises a bond issue its main concerns will be the success of the issue, and the interest rate that must be paid for the funds borrowed. An issue is handled by an international syndicate of banks. A company wishing to make a bond issue will invite a number of investment banks and securities houses to bid for the role of lead manager. The bidding banks will indicate the price at which they believe they can get the issue away to

investors, and the size of their fees. The company's choice of lead manager will be based on the bids, but also the reputation and standing of the bank in the market. The lead manager when appointed, will assemble a syndicate of other banks to help with the issue. This syndicate will often be made up of banks from several different countries. The lead manager has essentially agreed to underwrite the issue, which means that he guarantees to take the paper off the issuer's hands (in return for a fee). If there is an insufficient level of investor demand for the bonds the lead manager will be left holding ("wearing") the issue, which in addition to being costly will not help its name in the market. When we referred to an issuer assessing the reputation of potential lead managers, this included the company's view on the "placing power" of the bank, its perceived ability to get the entire issue away. The borrowing company would prefer the issue to be over-subscribed, which is when demand outstrips supply.

In many cases the primary issue involves a *fixed price re-offer* scheme. The lead manager will form the syndicate which will agree on a fixed issue price, a fixed commission and the distribution amongst themselves of the quantity of bonds they agreed to take as part of the syndicate. The banks then re-offer the bonds that they have been allotted to the market, at the agreed price. This technique gives the lead manager greater control over a Eurobond issue. It sets the price at which other underwriters in the syndicate can initially sell the bonds to investors. The fixed price re-offer mechanism is designed to prevent underwriters from selling the bonds back to the lead manager at a discount to the original issue price, that is "dumping" the bonds.

Before the bond issue is made, but after its basic details have been announced, it is traded for a time in the *grey market*. This is a term used to describe trading in the bonds before they officially come to the market, mainly market makers selling the bond short to other market players or investors. Activity in the grey market serves as useful market intelligence to the lead manager, who can gauge the level of demand that exists in the market for the issue. A final decision on the offer price is often delayed until dealing in the grey market indicates the best price at which the issue can be got away.

Let us now consider the primary market participants in greater detail.

18.4.1 The borrowing parties

The range of borrowers in the Euromarkets is very diverse. From virtually the inception of the market, borrowers representing corporates, sovereign and local governments, nationalised corporations, supranational institutions, and financial institutions have raised finance in the international markets. The majority of borrowing has been by governments, regional governments and public agencies of developed countries, although the Eurobond market is increasingly a source of finance for developing country governments and corporates.

Governments and institutions access the Euromarkets for a number of reasons. Under certain circumstances it is more advantageous for a borrower to raise funds outside its domestic market, due to the effects of tax or regulatory rules. The international markets are very competitive in terms of using intermediaries, and a borrower may well be able to raise cheaper funds in the international markets. Other reasons why borrowers access Eurobond markets include:

- a desire to diversify sources of long-term funding. A bond issue is often placed with a wide range of institutional and private investors, rather than the more restricted investor base that may prevail in a domestic market. This gives the borrower access to a wider range of lenders, and for corporate borrowers this also enhances the international profile of the company;
- for both corporates and emerging country governments, the prestige associated with an issue of bonds in the international market;
- the flexibility of a Eurobond issue compared to a domestic bond issue or bank loan, illustrated by the different types of Eurobond instruments available.

Against this are balanced the potential downsides of a Eurobond issue, which include the following:

- for all but the largest and most creditworthy of borrowers, the rigid nature of the issue procedure becomes significant during times of interest and exchange rate volatility, reducing the funds available for borrowers;
- issuing debt in currencies other than those in which a company holds matching assets, or in which there are no prospects of earnings, exposes the issuer to foreign exchange risk.

Generally though the Euromarket remains an efficient and attractive market in which a company can raise finance for a wide range of maturities.

The nature of the Eurobond market is such that the ability of governments and corporates to access it varies greatly. Access to the market for a first-time borrower has historically been difficult, and has been a function of global debt market conditions. There is a general set of criteria, first presented by van Agtmael (1983) that must be fulfilled initially, which for corporates include the following:

- the company should ideally be domiciled in a country that is familiar to Eurobond issuers, usually as a result of previous offerings by the country's government or a government agency. This suggests that it is difficult for a corporate to access the market ahead of a first issue by the country's government;
- the borrowing company must benefit from a level of name recognition or, failing this, a sufficient quality credit rating;
- the company ideally must have a track record of success, and needs to have published financial statements over a sufficient period of time, audited by a recognised and respected firm, and the company's management must make sufficient financial data available at the time of the issue;
- the company's requirement for medium-term or long-term finance, represented by the bond issue, must be seen to fit into a formal strategic plan.

Generally Eurobond issuers are investment-grade rated, and only a small number, less than 5%³, are not rated at all.

18.4.2 The underwriting lead manager

Issuers of debt in the Eurobond market select an investment bank to manage the bond issue for them. This bank is known as the underwriter because in return for a fee, it takes on the risk of placing the bond amongst investors. If the bond cannot be placed in total, the underwriting bank will take on the paper itself. The issuer will pick an investment bank with whom it already has an existing relationship, or it may invite a number of banks to bid for the mandate. In the event of a competitive bid, the bank will be selected on the basis of the prospective coupon that can be offered, the fees and other expenses that it will charge, the willingness of the bank to support the issue in the secondary market, the track record of the bank in placing similar issues and the reach of the bank's client base. Often it is a combination of a bank's existing relationship with the issuer and its reputation in the market for placing paper that will determine whether or not it wins the mandate for the issue.

After the mandate has been granted, and the investment bank is satisfied that the issuer meets its own requirements on counterparty and reputational risk, both parties will prepare a detailed financing proposal for the bond issue. This will cover topics such as the specific type of financing, the size and timing of the issue, approximate pricing, fees and so on. The responsibilities of the lead manager include the following:

- analysing the prospects of the bond issue being accepted by the market; this is a function of both the credit quality of the issuer and the market's capacity to absorb the issue;
- forming the *syndicate* of banks to share responsibility for placing the issue. These banks are co-lead managers and syndicate banks;
- assisting the borrower with the prospectus, which details the bond issue and also holds financial and other information on the issuing company;
- assuming responsibility for the legal issues involved in the transaction, for which the bank's in-house legal team and/or external legal counsel will be employed;
- preparing the documentation associated with the issue;
- taking responsibility for the handling of the fiduciary services associated with the issue, which is usually handled by a specialised agent bank;
- if deemed necessary, establishing a pool of funds that can be used to stabilise the price of the issue in the *grey market*, used to buy (or sell) bonds if required.

³ Source: IMF.

These duties are usually undertaken jointly with other members of the syndicate. For first-time borrowers the prospectus is a very important document, as it is the main communication media used to advertise the borrower to investors. In a corporate issue, the prospectus may include the analysis of the company by the underwriters, financial indicators and balance sheet data, a detailed description of the issue specifications, the members of the underwriting syndicate, and details of placement strategies. In a sovereign issue, the prospectus may cover a general description of the economy of the country's, including key economic indicators such as balance of payments figures and export and import levels, the state of the national accounts and budget, a description of the political situation (with an eye on the stability of the country), current economic activity, and a statement of the current external and public debt position of the country.

18.4.3 The co-lead manager

The function of the co-lead manager in Eurobond issues developed as a consequence of the distribution of placing ability across geographic markets. For example, as the Eurobond market developed, underwriters who were mainly US or UK banks did not have significant client bases in say, the continental European market, and so banking houses that had a customer base there would be invited to take on some of the issue. For a long time the ability to place \$500 000 nominal of a new Eurobond issue was taken as the benchmark against a potential co-lead manager.

The decision by a lead manager to invite other banks to participate will depend on the type and size of the issue. Global issues such as those by the World Bank, which have nominal sizes of \$1 billion or more, have a fairly large syndicate. The lead manager will assess whether it can place all the paper or it, in order to achieve geographic spread (which may have been stipulated by the issuer) it needs to form a syndicate. It is common for small issues to be placed entirely by a single lead manager.

18.4.4 Investors

The structure of the Eurobond market, compared to domestic markets, lends a certain degree of anonymity, if such is desired, to end-investors. This is relevant essentially in the case of private investors. The institutional holders of investors are identical to those in the domestic bond markets, and include institutional investors such as insurance companies, pension funds, investment trusts, commercial banks, and corporations. Other investors include central banks and government agencies; for example the Kuwait Investment Office and the Saudi Arabian Monetary Agency both have large Eurobond holdings. In the United Kingdom, banks and securities houses are keen holders of FRN Eurobonds, usually issued by other financial institutions.

18.5 Fees, expenses and pricing

18.5.1 Fees

The fee structure for placing and underwriting a Eurobond issue are relatively identical for most issues. The general rule is that fees increase with maturity and decreasing credit quality of the issuer, and decrease with nominal size. Fees are not paid directly but are obtained by adjusting the final price paid to the issuer, that is, taken out of the sale proceeds of the issue. The allocation of fees within a syndicate can be slightly more complex, and in the form of an *underwriting allowance*. This is usually paid out by the lead manager.

Typical fees will vary according to the type of issue and issuer, and also whether the bond itself is plain vanilla or more exotic. Fees range from 0.25% to 0.75% of the nominal of an issue. Higher fees may be charged for small issues

18.5.2 Expenses

The expenses associated with the launch of a Eurobond issue vary greatly. Table 18.2 illustrates the costs associated with a typical Eurobond transaction. Not every bond issue will incur every expense, however these elements are common.

Printing (prospectus, certificates, etc)	Clearing and bond issuance
Legal counsel (Issuer and investment bank)	Paying agent
Stock exchange listing fee	Trustee
Promotion	Custodian
Underwriters expenses	Common depositary

Table 18.2: Expense elements, Eurobond issue.

The expense items in Table 18.2 do not include the issuer's own expenses with regard to financial accounting and marketing. The reimbursement for underwriters is intended to cover such items as legal expenses, travel, delivery of bonds and other business expenses.

In general Eurobonds are listed on either the London or Luxembourg stock exchanges. Certain issues in the Asian markets are listed on the Singapore exchange. To enable listing to take place an issuer will need to employ a listing agent, although this is usually arranged by the lead manager. The function of the listing agent is to (i) provide a professional opinion on the prospectus, (ii) prepare the documentation for submission to the stock exchange and (iii) make a formal application and conduct negotiations on behalf of the issuer.

18.5.3 Pricing

One of the primary tasks of the lead manager is the pricing of the new issue. The lead manager faces an inherent conflict of interest between its need to maximise its returns from the syndication process and its obligation to secure the best possible deal for the issuer, its client. An inflated issue price invariably causes the yield spread on the bond to rise as soon as the bond trades in the secondary market. This would result in a negative impression being associated with the issuer, which would affect its next offering. On the other hand, too low a price can permanently damage a lead manager's relationship with the client.

For Eurobonds that are conventional vanilla fixed income instruments, pricing does not present too many problems in theory. The determinants of the price of a new issue are the same as those for a domestic bond offering, and include the credit quality of the borrower, the maturity of the issue, the total nominal value, the presence of any option feature, and the prevailing level and volatility of market interest rates. Eurobonds are perhaps more heavily influenced by the target market's ability to absorb the issue, and this is gauged by the lead manager in its preliminary offering discussions with investors. The credit rating of a borrower is often similar to that granted to it for borrowings in its domestic market, although in many cases a corporate will have a different rating for its foreign currency debt compared to its domestic currency debt.

In the grey market the lead manager will attempt to gauge the yield spread over the reference pricing bond at which investors will be happy to bid for the paper. The reference bond is the benchmark for the maturity that is equivalent to the maturity of the Eurobond. It is commonly observed that Eurobonds have the same maturity date as the benchmark bond that is used to price the issue. As lead managers often hedge their issue using the benchmark bond, an identical maturity date helps to reduce basis risk.

18.6 Issuing the bond

The three key dates in a new issue of Eurobonds are the announcement date, the offering day and the closing day. Prior to the announcement date the borrower and the lead manager (and co-lead managers if applicable) will have had preliminary discussions to confirm the issue specifications, such as its total nominal size, the target coupon and the offer price. These details are provisional and may well be different at the time of the closing date. At these preliminary meetings the lead manager will appoint a fiscal agent or trustee, and a principal paying agent. The lead manager will appoint other members of the syndicate group, and the legal documentation and prospectus will be prepared.

On the announcement date the new issue is formally announced, usually via a press release. The announcement includes the maturity of the issuer and a coupon rate or range in which the coupon is expected to fall. A telex is also sent by the lead manager to each prospective underwriter, which is a formal invitation to participate in the syndicate. These banks will also receive the preliminary offering circular, a timetable of relevant dates for the issue, and documentation that discloses the legal obligations that they are expected to follow should they decide to participate in the issue. The decision to join is mainly, but not wholly, a function of the bank's clients interest in the issue, which the bank needs to sound out.

The *pricing day* signals the end of the subscription period, the point at which the final terms and conditions of the issue are agreed between the borrower and the syndicate group. If there has been a significant change in market conditions, the specifications of the bond issue will change. Otherwise any required final adjustment of the price is usually undertaken by a change in the price of the bond relative to par. The ability of the lead manager to assess market conditions accurately at this time is vital to the successful pricing of the issue.

Once the final specifications have been determined, members of the syndicate have roughly 24 hours to accept or reject the negotiated terms; the bonds are then formally offered on the *offering day*, the day after the pricing day,

when the issuer and the managing group sign the subscription or underwriting agreement containing the final specifications of the issue. The underwriting syndicate then enters into a legal commitment to purchase the bonds from the issuer at the price announced on the pricing day. A final offering circular is then produced, and the lead manager informs the syndicate of the amount of their allotments. The lead manager may wish to either over-allocate or under-allocate the number of available bonds, depending on its view on future levels and direction of interest rates. There then begins the *stabilisation period*, when the bonds begin to trade in the secondary market, where Eurobonds trade in an over-the-counter market. About 14 days after the offering day, the *closing day* occurs. This is when syndicate members pay for bonds they have purchased, usually by depositing funds into a bank account opened and run by the lead manager on behalf of the issuer. The bond itself is usually represented by a *global note*, held in Euroclear or Clearstream, initially issued in temporary form. The temporary note is later changed to a *permanent* global note. Tranches of an issue targeted at US investors may be held in the Depository Trust Corporation as a registered note.

18.6.1 The Grey market

The subscription period of a new Eurobond issue is characterised by uncertainty about potential changes in market conditions. After the announcement of the issue, but before the bonds have been formally issued, the bonds trade in the *grey market*. The existence of the grey market, where bonds are bought and sold, for settlement on the first settlement date after the offering day. Grey market trading enables the lead manager to gauge the extent of investor appetite for the issue, and make any adjustment to coupon if required. A grey market that functions efficiently will at any time, reflect the market's view on where the bond should trade, and what yield the bond should be offered. It enables investors to trade in the primary market possessing information as to the likely price of the issue in the secondary market.

Another principal task of the lead manager is to stabilise the price of the bond issue for a short period after the bond has started trading in the secondary market. This is known as the stabilisation period, and the process is undertaken by the lead manager in concert with some or all of the syndicate members. A previously established pool of funds may be used for this purpose. The price at which stabilisation occurs is known as the *syndicate bid*.

18.6.2 Alternative issue procedures

In addition to the traditional issue procedure where a lead manager and syndicate offer bonds to investors based on a price set, on pricing day, based on a yield over the benchmark bond, there are a number of other issue procedures that are used. One of these methods includes the *bought deal*, where a lead manager or a managing group approaches the issuer with a firm bid, specifying issue price, amount, coupon and yield. Only a few hours are allowed for the borrower to accept or reject the terms. If the bid is accepted, the lead manager purchases the entire bond issue from the borrower. The lead manager then has the option of selling part of the issue to other banks for distribution to investors, or doing so itself. In a volatile market the lead manager will probably parcel some of the issue to other banks for placement. However it is at this time that the risk of banks dumping bonds on the secondary market is highest; in this respect lead managers will usually pre-place the bonds with institutional investors before the bid is made. The bought deal is focused primarily on institutional rather than private investors. As the syndicate process is not used, the bought deal requires a lead manager with sufficient capital and placement power to enable the entire issue to be placed.

In a *pre-priced offering* the lead manager's bid is contingent on its ability to form a selling group for the issue. Any alterations in the bid required for the formation of the group must be approved by the borrower. The period allocated for the formation of the group is usually 2–4 days, and after the group has been formed the process is identical to that for the bought deal.

Yet another approach is the *auction issue*, under which the issuer will announce the maturity and coupon of a prospective issue and invite interested investors to submit bids. The bids are submitted by banks, securities houses and brokers and include both price and amount. The advantages of the auction process are that it avoids the management fees and costs associated with a syndicate issue. However the issuer does not have the use of a lead manager's marketing and placement expertise, which means it is a method that can only be employed by very high quality, well-known borrowers.

18.7 Covenants

Eurobonds are unsecured and as such the yield demanded by the market for any particular bond will depend on the credit rating of the issuer. Until the early 1980s Eurobonds were generally issued without covenants, due to the high quality of most issuers. Nowadays it is common for covenants to be given with Eurobond issues. Three covenants in particular are frequently demanded by investors:

- a negative pledge;
- an “event risk” clause;
- a gearing ratio covenant.

Negative pledge

A negative pledge is one that restricts the borrowings of the group which ranks in priority ahead of the debt represented by the Eurobond. In the case of an unsecured Eurobond issue this covenant restricts new secured borrowings by the issuer, as well as new unsecured borrowings by any of the issuer’s subsidiaries, since these would rank ahead of the unsecured borrowings by the parent company in the event of the whole group going into receivership.

Disposal of assets covenant

This sets a limit on the amount of assets that can be disposed of by the borrower during the *tenor* (term to maturity) of the debt. The limit on disposals could be typically, a cumulative total of 30 per cent of the gross assets of the company. This covenant is intended to prevent a break-up of the company without reference to the Eurobond investors.

Gearing ratio covenant

This places a restriction on the total borrowings of the company during the tenor of the bond. The restriction is set as a maximum percentage say, 150–175 per cent of the company’s or group’s net worth (share capital and reserves).

18.8 Trust services

A Eurobond issue requires an agent bank to service it during its life. The range of activities required are detailed below.

18.8.1 Depositary

The depositary for a Eurobond issue is responsible for the safekeeping of securities. In the Euromarkets well over 90% of investors are institutions, and so as a result issue are made in dematerialised form, and are represented by a global note. Trading and settlement is in computerised book-entry form via the two main international clearing systems, Euroclear and Clearstream. Both these institutions have appointed a group of banks to act on their behalf as depositaries for book-entry securities; this is known as *common depositaries*, because the appointment is common to both Euroclear and Clearstream. Both clearing firms have appointed separately a network of banks to act as specialised depositaries, which handled securities that have been issued in printed note or *definitive* form.

As at February 2000 there were 21 banks that acted as common depositaries on behalf of Euroclear and Clearstream, although the majority of the trading volume was handled by just three banks, Citibank NA, Chase Manhattan and Deutsche Bankers Trust. The common depositary is responsible for:

- representing Euroclear and Clearstream, and facilitating delivery-versus-payment of the primary market issue by collecting funds from the investors, taking possession of the temporary global note (which allows securities to be released to investors), and making a single payment of funds to the issuer;
- holding the temporary global note in safe custody, until it is exchanged for definitive notes or a permanent global note;
- making adjustments to the nominal value of the global note that occur after the exercise of any options or after conversions, in line with instructions from Euroclear or Clearstream and the fiscal agent;
- surrendering the cancelled temporary global note to the fiscal agent after the exchange into definitive certificates or a permanent global note, or on maturity of the permanent global note.

A specialised depository will hold definitive notes representing aggregate investor positions held in a particular issue; on coupon and maturity dates it presents the coupons or bond to the paying agent and passes the proceeds on to the clearing system.

18.8.2 Paying agent

Debt issuance in the Euromarkets requires a fiscal or principal paying agent, or in the case of a programme of issuance (for example a Euro-MTN programme) an issuing and paying agent. The responsibility of the paying agent is to provide administrative support to the issuer throughout the lifetime of the issue. The duties of a paying agent include:

- issuing securities upon demand in the case of a debt programme;
- authenticating definitive notes;
- collecting funds from the issuer and paying these out to investors as coupon and redemption payments;
- in the case of global notes, acting on behalf of the issuer to supervise payments of interest and principal to investors via the clearing systems, and in the case of definitive notes, paying out interest and coupon on presentation by the investor of the relevant coupon or bond to the paying agent;
- transferring funds to sub-paying agents, where these have been appointed. A security that has been listed in Luxembourg must have a local sub-paying agent appointed for it;
- maintaining an account of the cash flows paid out on the bond;
- arranging the cancellation and subsequent payment of coupons, matured bonds and global notes, and sending destroyed certificates to the issuer.

A paying agent will act solely on behalf of the issuer, unlike a Trustee who has an obligation to look after the interests of investors. For larger bond issues there may be a number of paying agents appointed, of which the *principal paying agent* is the coordinator. A number of *sub-paying agents* may be appointed to ensure that bondholders in different country locations may receive their coupon and redemption payments without delay. The term *fiscal agent* is used to describe a paying agent for a bond issue for which no trustee has been appointed.

18.8.3 Registrar

The role of the registrar is essentially administrative and it is responsible for keeping accurate records of bond ownership for registered securities. As most Eurobonds are issued in bearer form, there is not a great deal of work for registrars in the Euromarket, and the number of holders of registered notes is normally quite low.

The responsibilities of the registrar include:

- maintaining a register of all bondholders, and records of all transfers of ownership;
- coordinating the registration, transfer or exchange of bonds;
- issuing and authenticating new bonds should any transfer or exchange take place;
- maintaining a record of the outstanding principal value of the bond;
- undertaking administrative functions relating to any special transfers.

18.8.4 Trustee

An issuer may appoint a trustee to represent the interests of investors. In the event of default, the trustee is required to discharge its duties on behalf of bondholders. In certain markets a trustee is required by law, for instance in the United States a trustee has been a legal requirement since 1939. In other markets an issuer may appoint a trustee in order to make the bond issue more attractive to investors, as it means that there is an independent body to help look after their interests. This is particularly important for a secured issue, where the trustee sometimes holds collateral for the benefit of investors. Assets that are held by the trustee can be protected from the creditors of the issuer in the event of bankruptcy. A trustee has a variety of powers and discretion, which are stated formally in the issue trust deed, and these include its duties in relation to the monitoring of covenants, and duties to bondholders.

18.8.5 Custodian

A custodian provides safekeeping services for securities belonging to a client. The client may be an institutional investor such as a pension fund, that requires a portfolio of securities in many locations to be kept in secure custody on their behalf. As well as holding securities, the custodian usually manages corporate actions such as dividend payments.

18.9 Form of the bond

Eurobonds are issued in temporary global form or permanent global form. If issued in temporary form, the note is subsequently changed into either permanent global form or *definitive* form, which may be either a bearer note or registered.

18.9.1 Temporary global form

On issue the majority of Eurobonds are in the form of a single document known as a temporary global bond. This document represents the entire issue, executed by an officer of the issuer and certified by the fiscal agent or principal paying agent. After a period of time the temporary global bond, as its name suggests, is exchanged for either a permanent global bond or bonds in definitive form, which are separate certificates representing each bond holding.

The main reason bonds are issued in temporary form is because of time constraints between the launch of issue, when the offer is announced to the market, and closing, when the bonds are actually issued. This period differs according to the type of issue and instrument, for example for a plain vanilla issue it can be as little as two weeks whereas for more exotic issues (such as a securitisation) it can be a matter of months. The borrower will be keen to have the periods short as possible, as the financing is usually required quickly. As this results in there being insufficient time to complete the security printing and authentication of the certificates, which represent the final definitive form, a temporary bond is issued to enable the offering to be closed and be placed in a clearing system, while the final certificates are produced. Bonds are also issued in temporary form to comply with certain domestic selling regulations and restrictions, for example a US regulation that definitive bonds cannot be delivered for a 40-day period after issue. This is known as the *lock-up* period.

18.9.2 Permanent global bond

Like the temporary bond the permanent global bond is a word-processed document and not a security printed certificate, issued on the closing date. It represents the entire issue and is compiled by the underwriter's legal representatives. In most cases it is actually held for safe-keeping on behalf of Euroclear and Clearstream by the trust or clearing arm of a bank, known as the *common depositary*. Borrowers often prefer to issue notes in permanent global form because this carries lower costs compared to definitive notes, which are security printed.

18.9.3 Definitive form

Under any circumstances where it is required that investors have legal ownership of the debt obligation represented by a bond issue they have purchased, a borrower is obliged to issue the bond in definitive form. The situations under which this becomes necessary are listed on the permanent global bond document, and include the following:

- where an investor requires a definitive bond to prove legal entitlement to the bond (s)he has purchased, in the case of any legal proceedings undertaken concerning the bond issue;
- in the event of default, or if investors believe default to have occurred;
- where for any reason the bonds can no longer be cleared through a clearing system, in which case they must be physically delivered in the form of certificates.

Bonds issued in definitive form may be either *bearer* or *registered* securities. A bearer security has similar characteristics to cash money, in that the certificates are documents of value and the holder is considered to be the beneficiary and legal owner of the bond. The bond certificate is security printed and the nature of the debt obligation is detailed on the certificate. Transfer of a bearer security is by physical delivery. Some of the features of traditional bearer securities include:

- *coupons*, attached to the side of the certificate, and which represent each interest payment for the life of the bond. The holder is required to detach each coupon as it becomes due and send it to the issuer's paying agent;⁴
- a *promise to pay*, much like a bank note, which confirms that the issuer will pay the bearer the face value of the bond on the specified maturity date;
- in some cases, a *talón*; this is the right for the bond holder to claim a further set of coupons once the existing set has been used (this only applies to bonds that have more than 27 interest payments during their lifetime, as IPMA rules prohibit the attachment of more than 27 coupons to a bond on issue).

The administrative burdens associated with bearer securities is the main reason why the procedures associated with them are carried out via the clearing systems and paying agents, rather than individually by each investor.

18.9.4 Registered bonds

Bonds issued in registered form are transferred by an entry on a *register* held by the issuer or its agent; the promise to pay is made to those names that appear on the register. Most Eurobonds are issued in bearer form for ease in clearing. Issues that are placed wholly or partly in the United States do however include an option allowing investors to take the bonds in registered form. This is done as most issues in the US are sold under *private placement*, in order to be exempt from SEC selling restrictions, and private placement in that country requires that the bonds are in registered form. In such cases the issuer will appoint a New York *registrar* for the issuer, usually the trust arm of a bank.

18.9.5 Fiscal agent

A Eurobond issuer will appoint either a fiscal agent or a Trustee; both perform similar roles but under differing legal arrangements. The fiscal agent is appointed by and is the representative of the issuer, so unlike a Trustee it does not represent the bondholders. The main responsibilities of the fiscal agent are to pay the principal and interest payments, and it performs a number of administrative roles as well, such as the publication of financial information and notices to investors.

18.9.6 Listing agent

Issuers must appoint a listing agent if they wish to list the bond on the London or Luxembourg stock exchanges, as this is a requirement of the rules of the exchange. The listing agent communicates with the exchange on behalf of the issuer, and lodges the required documentation with it. In the UK the listing agent must be authorised under financial regulatory legislation (at the time of writing, the Financial Services Act 1986, although this in the process of being updated with new legislation covering the new Financial Services Authority regulatory body) and is usually the lead manager for the issue, although it is also common for a fiduciary service provider to be appointed to this role.

18.10 Clearing systems

The development of the international bond market has taken place alongside the introduction of specialised clearing systems, which are responsible (among other things) for the settlement and safekeeping of Eurobonds. The two main clearing systems are Euroclear and Clearstream.⁵

Euroclear was created by the Morgan Guaranty Trust Company of New York in 1968. Ultimately ownership passed to a consortium of banks and it is now run by Euroclear Clearing Systems plc, and operated by a co-operative company in Brussels.

The original Cedel was created in 1970 in Luxembourg and is owned by a consortium of around 100 banks, no one of which may hold more than 5% of the company. The two clearing systems do not restrict their operations to the settlement and custody of Eurobonds.

Both clearing systems exist to avoid the physical handling of bearer instruments, both on issue and in the secondary market. This means that on issue the actual bond certificates, which may be in *definitive bearer* or *global* form are passed on to a "trust" bank, known as the *depository* for safekeeping. The clearing system will track

⁴ This is the origin of the term "coupon" to refer to the periodic interest payments of a bond. There is a marvellous line in the film *Mission Impossible* when the character played by Tom Cruise, discussing terms in the back of a car with the character played by Vanessa Redgrave, demands payment in the form of US Treasury securities "with coupons attached." This is wonderfully out-of-date, but no less good fun for it!

⁵ Clearstream was previously known as Cedel Bank.

holdings via a book entry. To participate in the clearing system set up, an investor must have two accounts with it, which may be its own accounts or accounts held by their bank who will act as a nominee on their behalf; these are a *securities clearance* account, to which a security is credited, and a *cash* account, through which cash is received or paid out.

The clearing system will allocate a unique identification code, known as the International Securities Identification Number (ISIN) to each Eurobond issue, and a “Common Code” is derived from the ISIN. The Common Code is essentially the identification used for each bond issue whenever an instruction is sent to the clearing agent to deal in it. The ISIN will be in addition to any number issued by a domestic clearing agent, for example the Stock Exchange number (SEDOL) for London listed securities. Both clearing systems have specific roles in both the primary and secondary markets. In the primary market they accept a new issue of Eurobonds, and on *closing* the required number of bonds are credited to the securities clearance account of the banks that are part of the issue syndicate. Securities are then transferred (electronic book entry) to securities accounts of investors.

The clearance systems keep a record on the coupon payment and redemption dates for each bond, and “present” the bonds for payment on each appropriate date. Investors therefore do not need to present any coupons or certificates themselves, which is why the system is now paperless.

18.11 Market associations

18.11.1 International Securities Market Association

The International Securities Market Association (ISMA) is a self-regulatory body based in Zurich whose membership (from over 60 countries) consists of firms dealing in the international securities markets. It was originally known as the Association of International Bond Dealers. The body provides a regulatory framework for the international markets and has established uniform practices that govern nearly all transactions in Eurobonds between members. ISMA has also:

- introduced a standard method of calculating yields for Eurobonds;
- contributed towards the harmonisation of procedures for settling market transactions, and co-operation between the two main settlement institutions, Euroclear and Cedel;
- introduced TRAX, a computerised system for matching and reporting transactions in the market.

Dealers in the international markets must cooperate with national governments and ensure that market practice is consistent with national laws. ISMA provides a point of contact between the markets and government bodies. The ISMA centre at the University of Reading in England has also established itself as a leading research body, concentrating on the financial and securities markets, as well as offering Masters degrees in a range of capital markets subjects.

18.11.2 International Primary Market Association

The International Primary Market Association (IPMA) is a trade association of the leading underwriters in the primary international capital markets. It is also a self-regulatory associations that has issued practical guidelines it expects members to follow. Its specific interest is with new issues, the co-operation between underwriters in a syndicate and standardised documentation. Members of IPMA are required to belong to ISMA. The IPMA recommendations are applicable to new issues and relate to matters such as:

- early disclosure of the terms of an issue;
- underwriting commitments;
- allotments of securities with investors;
- payment of commissions;
- delivery of bond/share certificates;
- fixed price offerings and re-offer schemes.

The IPMA has also issued statements of best practice concerned with topics on a range of issuance procedures, for example in 1985 it published guidelines on the process of stabilisation. These guidelines emphasised the importance of correct pricing by lead managers.

18.12 Secondary market

In Chapter 19 we consider some detailed aspects of secondary market trading; here we present the basic features. Most Eurobonds are tradeable. Although in theory transfer is by physical delivery because the bonds are bearer instruments, the great majority of bonds will settle by the Euroclear or Clearstream International ("Clearstream") settlement systems.⁶ Liquidity in the market varies over time and for individual issues will be a function of:

- size of issue;
- level of investor demand for the paper;
- commitment of market makers to support the issue.

A large number of Eurobonds are illiquid and market makers will quote a bid price only. No offer price is made because the market maker (unless he actually owns some of the issue) will be unable to find bonds to deliver to the buyer if it is illiquid. Many Eurobonds issued in the second tier currencies, such as Greek drachma, will have been issued and then immediately asset swapped, and hence there will be no paper available to trade (many large issuers will issue Eurobonds in a currency other than that which they require, in order to meet a specific customer demand for paper in that currency; after issue the proceeds are swapped into the desired currency. In the meantime the bonds will be held to maturity by the investors and usually not traded in the secondary market).

High-quality Eurobond issues will trade almost as government paper. For example issues by the World Bank or the European Investment Bank (EIB) trade at very low spreads above the same currency government bonds and are highly liquid. For example at times EIB sterling Eurobonds have traded at only 7–9 basis points above the same maturity gilt.

18.13 Settlement

Settlement of Eurobond transactions takes place within 28 days for primary market issues and T+3 days for secondary market trades. Virtually all trades settle within the two main clearing systems, Euroclear and Clearstream. Euroclear was established in Brussels in 1968 by an international group of banks, the original entity known as Cedel was established in Luxembourg in 1970. Both clearing systems will settle in T+3 days, however the facility exists to settle trades in T+1 if both parties to a trade wish it.

In the Euroclear system bonds are placed in the custody of the clearing system, through a Europe-wide network of depository banks. The transfer of bonds on settlement is undertaken by means of a computer book-entry. This was the basic concept behind the introduction of Euroclear, the substitution of book entries for the physical movement of bonds. The actual physical securities to which a trading party has title are not identified in the majority of transactions made through Euroclear. The clearing system is made possible because the terms and conditions of any Eurobond issue are objectively specified, so that all bonds of a particular issue are standardised, and so fungible for one another. There is no requirement to assign a specific bond serial number to an individual holder, which occurs with registered bonds. Clearstream operates on much the same basis. Participants in either system must be institutions with their own account (they may have an agent settle for them). Settlement takes place through the simultaneous exchange of bonds for cash on the books of the system. An "electronic bridge" connecting the two systems allows transfer of securities from one to the other.

Selected bibliography

Andersen, T., "How the grey market became respectable", *Euromoney*, May 1982.

Crawford, A., "Stabilization brings the jitters", *Euromoney*, April 1987, p. 277.

Decovny, S., *Swaps*, FT Prentice Hall, 1998, p. 68.

Kerr, I., *A History of the Eurobond Market*, Euromoney Publications, 1984.

Van Agtmael, A., "Issuance of Eurobonds: Syndication and Underwriting Techniques and Costs", in George, A., Giddy, I. (eds.), *International Financial Handbook*, Section 5.2, Wiley, 1983.

⁶ In 1999 Cedel Bank and Deutsche Terminbourse merged their operations, and the resulting entity was named Clearstream International or simply Clearstream. Cedel Bank had originally been known as Cedel.

19

Eurobonds II

In this chapter we consider briefly some of the issues of Eurobond issuance and trading. First though we review the legal aspects of the Eurobond market. This is an area that is always under the scrutiny and review of national governments, so the facts behind legal and taxation issues do not remain static. However there are some general issues that we can present here. The last part of the chapter reviews the impact and influence of swap transactions in the market.

19.1 Legal and tax issues

Investor and borrowers in the Eurobond market may at any one time fall under the auspices of a number of countries laws and regulations. These relate to the withholding tax on the bond coupons, income tax, disclosure and prospectus requirements and restrictions on sales to certain classes of investor. The most important legal considerations for professional participants relate to (i) the possibility that the bonds are eventually distributed to residents in the United States, which is prohibited, and (ii) London, as the principal financial centre where the sale and trading of bonds takes place. The first consideration means that the market is subject to legislation in the US that dates from 1933¹ and Federal income tax regulations. The second consideration means that the market comes under certain aspects of English law. With regard to taxation, the key features of Eurobonds are that:

- the bonds are “bearer” rather than registered securities;
- interest and principal payments are not subject to withholding tax at source in the country where the issuer is resident for tax purposes.

The fact that payments of interest and principal on Eurobonds are not subject to any form of withholding tax at source in the country where the borrower is deemed to be resident for tax purposes is the primary feature of Eurobonds for investors, generally cited to be of key importance in making the market attractive for investors across a range of countries. Non-resident investors in Eurobonds are usually subject to the withholding tax requirements of the resident country of the bond issuer when that party repays interest or principal on bonds held by these non-residents. The tax advantages to an investor from the absence of withholding tax (combined with the fact that the bonds are issued in bearer form) are significant. A large proportion of Eurobonds are held by private investors, and much of this is made anonymously by means of external discretionary accounts, such as those run by Swiss banks. This is a source of some frustration to tax authorities in certain countries. The absence of withholding tax also confers a certain benefit to issuers of Eurobonds. Where a bond issue was subject to withholding tax, an issuer would need to make the terms of the issue more attractive, that is a higher coupon, in order to make the bond as attractive as the Eurobond issuer. This will carry higher associated costs for the issuer.

19.2 The secondary market

The market in trading Eurobonds is conducted on an over-the-counter basis. In 1998 a number of automated electronic trading system were also introduced. The pre-eminence of London as the main trading centre for the Eurobond market is well-established, although Brussels, Frankfurt, Zurich and Singapore are also important trading centres. The advantages of London as a trading centre are generally regarded as being:

- a low level of regulatory interference in the functioning of the market;
- the presence of well-established infrastructure and institutions, as well as experienced human resources;
- the use of the English language as the market’s main language of communication.

There are over 40 different market makers registered with ISMA, and although in theory they are all required to make two-way prices in their chosen markets, the level of commitment is very varied. The bid-offer spread can be as

¹ The US Securities Act of 1933.

low as 0.10 for very liquid issues such as World Bank and EIB bonds, to no offer price quoted for illiquid issues. In between there are a range of spread sizes. The normal market size also varies, from £100,000 nominal to £500,000.

The valuation of Eurobonds is usually done on the basis of a yield spread over the relevant government bond yield curve. This yield spread is a function of the credit quality of the bond, its liquidity in the market and the level of supply and demand. The bonds also move in line with general moves in interest rates, so that if there is a change in the gilt yield curve, a sterling Eurobond will change in yield, irrespective of whether the bond's issuer was perceived as being a weaker or a stronger credit. A market maker wishing to hedge a position in Eurobonds will usually use either the benchmark government against which the bond is priced or, if a non-cash option is preferred, will use bond futures contracts to hedge the position. These topics are all covered in chapters elsewhere.

19.3 Eurobonds and swap transactions

In Chapters 39 and 40 we discuss interest-rate and other swap instruments. Readers who are unfamiliar with swap instruments may wish to review these chapters first. The issue of new Eurobonds and the use of "asset swaps" in conjunction with issues is a vital part of the market, with investment banks keeping a close observation of the asset swap curve to spot any opportunities that may arise that makes a new issue of paper more attractive. New issues of Eurobonds are often launched to facilitate a swap which has been arranged in advance.

The existence of the currency swap and asset swap market is one of the key reasons for the growth and popularity of the Eurobond market. A borrower can issue bonds in virtually any liquid and convertible currency, according to where there is demand and what the yield curve looks like, and swap the proceeds into the currency that it requires. The cost of borrowing is usually significantly lower than if the borrower had issued bonds in the required currency. Swap driven issues are very common in the Eurobond market, and the key motivator is that borrowing costs will be cheaper. If this cheap borrowing opportunity is not available, it is unlikely that the bond will be issued, because entering into a swap exposes the issuer to additional credit risk. Swap financing will require a borrower to obtain debt initially that has undesirable currency and/or coupon characteristics. If the counterparty to a swap defaults, the borrower will be left with a risk exposure on the original debt. However swap financing remains attractive because of the opportunity to obtain cheaper borrowing costs, despite the additional exposure to credit risk entailed in the transaction.

The market in swaps is governed by the International Swap Dealers Association (ISDA). In the market the majority of transactions are plain vanilla in nature, and involve one of the following:

- cross-currency fixed-rate swaps, usually referred to as *currency swaps*;
- interest-rate swaps;
- cross-currency hybrid swaps;
- basis swaps.

Currency swaps are very common in the market. Under the plain vanilla version, two counterparties issue fixed-rate debt denominated in different currencies. They then exchange the interest (and sometimes) the principal repayments on their respective debt obligations. Under the conventional pattern the amounts exchanged remain fixed at maturity. We will not cover the mechanics of a currency swap here as this will be reviewed later; likewise interest-rate swaps and the concepts of comparative advantage and the fixed- versus floating-rate legs of an interest-rate swap. Swap agreements do not always involve the exchange of debt repayment streams. In certain cases one of the revenue streams exchanged in a swap can represent the income interest stream on an asset, or conventional security such as a corporate bond. Eurobond issues are frequently brought to the market primarily for the purpose of such "asset swapping". For the investment bank, swapping asset base interest payments is one means by which bond issues can be re-packaged.

Other instruments used include *basis swaps*, which involve the exchange of two floating-rate payments streams, each of which is based on a short-term interest rate. The most common of these instruments have the following reference rates:

- Libor versus the US commercial paper rate;
- Libor versus the Prime rate.

Basis swaps are not the primary motivators of Eurobond issues, but are often included in more complex swap agreements which may involve Eurobond borrowing.

20 Warrants

20.1 Introduction

A warrant entitles its holder to purchase a specified asset at a set price at a specified date or dates. The terms defining a warrant usually remain unchanged during its entire life, and the asset may be bonds, equities, an index, commodities or other instruments. Hence a warrant is an option issued by a firm to purchase a given number of shares in that firm (*equity warrant*) or more of the firm's bonds (*bond warrant*), at a given exercise price at any time before the warrant expires. If the warrant is exercised, the firm issues new shares (or bonds) at the exercise price and so raises additional finance. Warrants generally have longer maturities than conventional options (five years or longer, although there is usually a liquid market in very long-dated over-the-counter equity options), and some warrants are perpetual.

Warrants are usually attached to bonds (*host bond*) to start with, in most cases such warrants are detachable and can be traded separately. Equity warrants do not carry any shareholders rights until they are exercised, for example they pay no dividends and have no voting rights. Bond warrants can either be exercised into the same class of bonds as the host bond or into a different class of bond. In valuing a warrant it is important to recognise that exercising the warrant (unlike exercising an ordinary call option) will increase the number of shares outstanding. This will have the effect of diluting earnings per share and hence reducing the share price. Hence although equity warrants are often valued in the same way as an American call option, the pricing must also take into account this dilution effect. Warrants are often used in conjunction with a new bond issue, to act as a "sweetener", and are common instruments in the Japanese bond and equity markets. If the issuing company performs well, the investor can eventually exercise the warrant purchase the company's equity at the exercise price fixed at the time the warrant was issued. During this time, in the same way as for a convertible bondholder, the investor has the security of holding the company's fixed interest debt, which acts as a type of security in the event that the company's share price declines.

In the UK some companies use warrants to obtain a steady flow of new investment. For example, every year from 1988 the London-listed company BTR has issued bonus warrants free to its shareholders, with the exercise price set just out-of-the-money. From the investor's viewpoint warrants may be used as a means of having an exposure to a company's shares but with a relatively low capital outlay at the start. They also allows the investor already holding shares to sell them while still maintaining an equity stake. This is known as *cash extraction* and is a straightforward strategy. The investor sells the shares, uses some of the proceeds to buy warrants representing the same number of shares, and invests the remaining cash in interest-bearing instruments. The price of warrants, just like convertibles, do not move one-for-one with the underlying equity unless they are deep in-the-money however, so a rise in the share price will not be matched by the same rise in the warrant price. In the short term therefore an investor following a cash extraction strategy may miss out on share price performance, in addition to any dividend payments.

EXAMPLE 20.1 Tate & Lyle warrants

- Tate & Lyle plc issued 5¾% 10-year bonds in March 1991, with 37,200 share warrants attached. The bond was issued with a face value of £5,000. Each share warrant entitles the holder to subscribe for 866 ordinary shares up to March 2001. The exercise price was £3,968.90 per warrant, or 458.3p per share until 20 March 1993, rising in annual increments to £5,143.75 per warrant (594p per share) from 20 March 2000. Alternatively a warrant may be exercised by surrendering one bond for the same number of shares, rather than making a cash payment.

20.2 Analysis

As with ordinary options the value of a warrant has two components, an intrinsic value (which in the warrant market is known as *formula value*) and a time value (*premium* over formula value). Although the term *premium* is used in the options market to refer to the price paid for an option, in the warrant market the conventional term *price*

is used. The *warrant premium* is usually used to refer to the amount by which the warrant price plus the exercise price exceeds the current underlying share price.

The formula value is determined by equation (20.1):

$$\text{Formula value} = (\text{Share price} - \text{Exercise price}) \times \text{Number new shares issued on exercise.} \quad (20.1)$$

If the exercise price exceeds the share price, the formula value is zero and the warrant is said to be “out-of-the-money”. If the share price exceeds the exercise price the warrant is in-the-money and the formula value is positive. The time value is always positive up until expiry of the warrant. As with options the time value declines as the expiry date approaches and on the expiry date itself the time value is zero.

The fair price of a warrant is given by (20.2):

$$\text{Warrant value} = \frac{P_C}{1 + p} \times \text{number of new shares issued if warrant is exercised} \quad (20.2)$$

where p is the proportionate increase in the number of shares outstanding if all the warrants were exercised, and P_C is the value of an American call option with the same exercise price and expiry date as the warrant.

If a company issues new shares in a rights issue at a price below the market price or issues convertible bonds or new warrants, the value of any existing warrants already in issue will be affected. It is usual therefore for companies to issue warrants with a provision that allows for the exercise price to be reduced in the event of any corporate action that adversely affects the current warrant price.

A warrant is attractive to investors because if the firm is successful and its share price rises accordingly, the warrant can be exercised and the holder can receive higher-value shares at the lower exercise price. Virtually all warrants are issued by corporations and are equity warrants. In the late 1980s the Bank of England introduced *gilt warrants* which could be exercised into gilts; however none are in existence at present. However it is of course possible to trade in OTC call options on gilts with a number of banks in the City of London.

Covered warrants are issued by a securities house or investment bank rather than the company itself. The aim of the securities house is to create an active and liquid market in the warrants, and to earn profit from making a market in them. When covered warrants are exercised there is no recourse to the company, and the company does not issue new shares. There is thus no dilution effect. The securities house must settle the warrant holder's application to subscribe for shares, either by providing shares already in issue or by the payment of sufficient cash to allow the warrant holder to buy shares in the market.

20.3 Bond warrants

Very occasionally bonds are issued with debt warrants attached. For example an issue of bonds with a coupon of 5% could be made with warrants that give the holder the right to subscribe at a future date for more 5% bonds at a fixed price. The warrants would be attractive to investors who expect interest rates to fall in the future; a fall in rates could result in the warrants being exercised or sold on at a profit, as the lower rates would now make them more valuable.

Say that a bond warrant entitles its holder to purchase bonds with a face value of M at a price of E . The bonds issued on exercise of the warrants may be either a further tranche of an existing issue or a new issue. The exercise cost of purchasing the underlying bond via the warrant is given by (20.3):

$$\text{Cost} = \left(P_w + \frac{E \times M}{100} \right) \times \frac{100}{M} \quad (20.3)$$

where

P_w	is the price of the warrant
E	is the exercise price
M	is the par value of the underlying bonds which may be purchased per warrant.

When a bond warrant is exercised, the cost to the purchaser includes the accrued interest up to the exercise date.

If the warrant entitles the holder to the right to purchase a bond which is already in existence, a premium or discount resulting from purchasing the bond via the warrant, as opposed to directly in the market, may be calculated using (20.4):

$$\text{Premium \%} = \left(\frac{\text{Exercise cost}}{P_{\text{bond}}} - 1 \right) \times 100 \quad (20.4)$$

where P_{bond} is the clean price of the underlying bond.

EXAMPLE 20.2 ABC plc bond warrant

- ABC plc warrant with the right to subscribe for 8% 2005 bonds. Each warrant gives the holder the right to subscribe for £1,000 nominal amount of the company's 8% bond due 2005, at the exercise price of 100 per cent of the nominal amount plus accrued interest from the previous coupon date (payable 7 June and 7 December each year). The warrants are exercisable up to and including 7 June 2001.

If the price of the warrant is £24, the exercise cost of purchasing the £1,000 nominal amount of the 8% bonds is:

$$(24 + (100 \times 1000) / 100) \times 100 / 1000$$

which is 102.4% in addition to any accrued interest.

If the bonds are trading in the market at 98.00, as the exercise cost of the bonds via the warrant is 102.4%, the premium is $(102.4/98) - 1$ which is 4.490%.

20.4 Comparison of warrants and convertibles

Warrants and convertibles are both hybrid instruments, both are issued by companies in the international markets. They are essentially similar in many respects, including:

- **valuation:** the theoretical value for a warrant is often calculated using the Black–Scholes or a similar model. In practice however investors are willing to pay only a fraction of the theoretical price of a warrant compared to convertibles, which trade near to or at fair value. Investors often pay more for the conversion premium on an issue of convertibles than they will pay for warrants in an issue of bonds with warrants attached. It is often the case therefore that companies are able to raise more capital by issuing convertibles rather than by issuing bonds with warrants attached;
- **investor base:** although warrants can have a long term to maturity, often they are held by short-term investors who buy them in the expectation of re-selling them at a profit when the company's share price rises by a sufficient amount. Until the exercise date approaches, many investors do not intend to hold the warrants in order to subscribe for shares in the future. The opposite is usually true for convertible bonds, whose investors are more likely to hold convertibles until conversion or redemption;
- **call flexibility:** bonds with warrants attached are often non-callable. The company cannot therefore force warrant holders into an immediate decision whether or not to subscribe for shares. A call option is only rarely a feature of a warrant bond;
- **maturity:** the maturity of a bond with warrants attached is predictable (unless a call feature is also included), as the bond portion remains outstanding until maturity. In contrast a convertible could remain outstanding for a proportionately much shorter time of its life, and be converted into equity. It is more common for convertibles to have call and/or put features attached. If a company required certainty of redemption dates for its financial planning, warrant bonds would be preferable to convertibles for this reason.

Generally however warrants are issued as a “sweetener” attached to a main issue, whereas convertibles are important corporate finance instruments in their own right. In certain markets though, for example in Japan, warrants are an important financing instrument.

Selected bibliography and references

- Bird, A., “Evaluating warrants”, *The Investment Analyst*, December 1971.
- Chen, A.H.Y., “A model of warrant pricing in a dynamic market”, *Journal of Finance* 25, December 1970, pp. 1041–1060.
- Connolly, K.B., Phillips, G.A., *Japanese Warrant Markets*, Macmillan, 1992.
- Crouhy, M., Galai, D., *Warrant valuation and equity volatility*, HEC, 1988.

- Emmanuel, D., "Warrant valuation and exercise strategy", *Journal of Financial Economics* 12, August 1983, pp. 211–236.
- Galai, D., Schneller, M., "Pricing warrants and the value of the firm", *Journal of Finance* 33, December 1978, pp. 1333–1342.
- Giguere, G., "Warrants: a mathematical model of evaluation", *Analysts Journal* 14, November 1958.
- Green, R., "Investment incentives, debt and warrants", *Journal of Financial Economics* 13, 1984, pp. 115–136.
- Lauterbach, B., Schultz, P., "Pricing warrants: an empirical study of the Black–Scholes model and its alternatives", *Journal of Finance* 45(4), 1990, pp. 1181–1209.
- Noreen, E., Wolfson, M., "Equilibrium warrant pricing models and accounting for executive stock options", *Journal of Accounting Research* 19, Autumn 1981, pp. 384–398.
- Schulz, G., Trautman, S., "Valuation of warrants: theory and empirical tests for warrants written on German stocks", unpublished paper, University of Stuttgart, 1990.
- Schwartz, E., "The valuation of warrants: implementing a new approach", *Journal of Financial Economics* 4, January 1977, pp. 79–93.
- Shelton, J., "The relation of the pricing of a warrant to the price of its associated common stock", *Financial Analysts Journal* 23, 1967, pp. 88–99.
- Spatt, C., Sterbenz, F., "Warrant exercise, dividends and re-investment policy", *Journal of Finance* 43, 1988, pp. 493–506.

21

Medium-term Notes

Medium-term notes (MTNs) are corporate bonds that have evolved into an important source of corporate funding. They are not exclusively corporate instruments however and have been issued by sovereigns, supra-nationals and federal and local authorities. The first MTN was issued by the General Motors corporation in 1972, and was sold directly to investors rather than via an agent bank. During the 1970s the MTN market was largely illiquid, and in 1981 the volume of outstanding issues was less than \$1 billion. In that year Merrill Lynch issued an MTN for Ford Motor Credit and also undertook to make a secondary market in the paper. Since then the MTN market has grown into a major corporate finance instrument, traded both domestically and internationally, and at the end of 1998 the outstanding volume of MTN issues around the world was approaching \$1 trillion (see Table 21.1).

Market	Nominal outstanding (\$ bln)
United States	
Domestic corporate issues	275
Federal agency	174
Other	46
Sub-total	495
International markets	
Euro-MTNs	475
Domestic markets	26
Sub-total	501
Total	996

Table 21.1: Size of the global MTN market, year-ending 1998.
Source: Strata Consulting.

A medium-term note is essentially a plain vanilla debt security with a fixed coupon and maturity date. The term “medium-term” is something of a misnomer, as the bonds range in maturity from nine months to 30 years or more; however the first MTNs generally had maturities of five years or less. They were originally designed to bridge the gap between *commercial paper* and long-dated bonds. An MTN is an unsecured debt, therefore the majority of MTNs are investment grade quality. In terms of the way they trade in the market, MTNs are virtually identical to conventional corporate bonds, and the main difference between an MTN and a corporate bond is the manner in which it is issued in the primary market. The unique characteristic of MTNs is that they are offered to investors continually over a period of time by an agent of the issuer, as part of an MTN *programme*. MTNs are usually offered in the market by investment banks acting as agents, and sold on a “best efforts” basis. The issuing bank does not act as an underwriter of the bonds, unlike with a conventional bond issue, and therefore the borrowing company is not guaranteed to place all its paper. As MTNs are usually offered as part of a continuous programme, they are issued in smaller amounts than conventional bonds, which are generally sold in larger amounts at one time. Notes can be issued either as *bearer* securities or *registered* securities. A Euromarket in MTNs developed in the mid-1980s. Euro MTNs (EMTNs) trade in a similar fashion to Eurobonds; they are debt securities issued for distribution across markets internationally.

The majority of MTNs are conventional bonds with fixed coupon rate and single maturity date. There is a wide range of structures available however, and MTNs have been issued with floating-rate coupons, call and put features, amortising nominal amounts, multi-currency structures or as part of more exotic structures such as asset swaps. Certain MTN issues are underwritten by investment banks as well, making them indistinguishable from conventional corporate bonds.

21.1 Introduction

The first MTN issue was made by General Motors Acceptance Corporation (GMAC) in the United States in 1972. At that time the instrument was seen as a longer-dated version of commercial paper, which for regulatory reasons in the US may not have a longer maturity than nine months or 270 days. GMAC and (shortly after) other motor car manufacturers were interested in sources of finance that matched the maturity of their car loans to consumers, which were cheaper to issue than conventional bond offerings. For this reason the first MTNs were issued to investors directly, thus avoiding underwriting fees. However this resulted in a lack of liquidity in the secondary market. A requirement of the Securities and Exchange Commission (SEC) that regulatory approval be obtained for any change to a registered public offering also resulted in higher transaction costs for borrowers, some of whom issued MTNs via private placement. In the 1980s the market began to grow in volume after investment banks issued MTNs as agent and acted as market makers in the secondary market.¹

A new regulation instituted by the SEC, Rule 415, also assisted market development as it allowed for *shelf registration* of an MTN programme. Under regulations in the United States, any corporate debt issued with a maturity of more than 270 days must be registered with the SEC. In 1982 the SEC adopted Rule 415 which permitted shelf registration of new corporate debt issues. A continuous MTN programme can be registered in this way. The issuer must file with the SEC details on (i) historical and current financial information and (ii) the type and amount of securities it plans to issue under the programme. This is known as “filing a shelf”. The adoption of Rule 415 made the administration of continuously offered notes relatively straightforward for the issuer. Under shelf registration, bonds may be sold for up to two years after the registration date without requiring another registration statement every time there is a new offer. Borrowers are able to issue paper at short notice in response to favourable market conditions, such as a drop in market interest rates or investor demand for their paper, without having to register each new offering, as long as it is within the two year period. For issues outside a shelf registration there is a delay between the filing with the SEC and the actual date of public offering, which may be up to five days and potentially prevent the borrower from taking advantage of market conditions.

The adoption of financial engineering techniques has also contributed to the growth of the market. Companies are able to publish a single prospectus that encompasses the entire MTN programme, and within the programme issue bonds with a variety of structures and in different currencies, to suit specific conditions and requirements. Individual issues within a single programme may have a range of coupons, maturity dates and other structures. For example, MTNs have been issued as zero-coupon bonds, floating-rate bonds, with step-up or step-down coupons, denominated in a foreign currency or indexed to an exchange-rate or commodity; they are frequently issued in conjunction with a swap structure. Floating-rate MTNs pay coupons linked to a reference rate such as LIBOR, but have been linked to the commercial paper rate, the T-bill rate, Federal funds rate and the *prime* rate. Floating-rate MTNs often pay monthly or quarterly coupons, compared to conventional FRNs which usually pay a semi-annual coupon. The larger borrowers issue debt as part of global MTN programmes, which enable them to place debt in their domestic market and internationally in the Euro market, in any liquid currency they wish. The MTN market is flexible and individual programmes may be adapted to suit the requirements of borrowers and investors alike; some of the most innovative structures have been observed in the MTN market before their introduction in the conventional corporate bond market.²

21.2 The primary market

MTN issues are arranged within a programme. A continuous MTN programme is established with a specified issue limit, and sizes can vary from \$100 million to \$5,000 million or more. Within the programme MTNs can be issued at any time, daily if required. The programme is similar to a revolving loan facility; as maturing notes are redeemed, new notes can be issued. The issuer usually specifies the maturity of each note issue within a programme, but cannot exceed the total limit of the programme.

¹ Initially the banks would only quote prices for their own issues.

² In July 1993 Walt Disney issued an MTN with a 100-year maturity as part of its rolling global Euro-MTN programme. This is equal to the longest-dated maturity conventional debt instrument issued in recent years.

EXAMPLE 21.1 ABC plc

- ABC plc establishes a five-year \$200 million MTN programme and immediately issues the following notes:
 \$50 million of notes with a one-year maturity
 \$70 million of notes with a five-year maturity

ABC plc can still issue a further \$80 million of notes; however in one year's time when \$50 million of notes mature, it will be able to issue a further \$50 million if required. The total amount in issue at any one time never rises above \$200 million.

The first step for the borrower is to arrange shelf registration with the SEC. This ensures the widest possible market for the programme; there are also no re-sale or transfer restrictions on the bonds themselves. The shelf registration identifies the investment bank or banks that will be acting as agents for the programme and who will distribute the paper to the market. A domestic programme may have only one agent bank, although two to four banks are typical. Global and Euro-MTN programmes usually have more agent banks. Once registration is complete the borrower issues a prospectus supplement detailing the terms and conditions of the programme. Often a draft prospectus is issued first, and only issued in final once the issuing bank has gauged market reaction. A draft prospectus is known as a *red herring*. Within a programme a borrower may also issue conventional corporate bonds, underwritten by an investment bank, but there is none of the flexibility available compared to an MTN issue, which can be arranged at very short notice, so conventional bond offerings within a programme are rare.

The agent banks sometimes publicise the maturities and yield spreads that are to be offered as part of the programme; a typical example for an hypothetical programme in the sterling market is given at Table 21.2.

Maturity	Yield spread (bps)	Benchmark bond	Current yield
9 months	20	13% 2000	5.80
12 – 24 months	25	7% 2001	6.25
2 – 3 years	35	7% 2002	6.37
3 – 4 years	45	6.5% 2003	6.38
4 – 5 years	50	6.75% 2004	6.34
5 – 6 years	55	8.5% 2005	6.36
9 – 11 years	50	5.75% 2009	5.77
15 – 20 years	30	8% 2021	5.02

Table 21.2: MTN programme offer, October 1999.

The exact date of a particular maturity issue is not always known at the time the programme is announced, so yields are often given as a spread over the equivalent maturity government bond. If the borrower has a particular interest to tap the market at specific points of the yield curve, the spread offered at that point is increased in order to attract investors. Once the required funds have been raised, offer spreads are usually reduced. If the full amount stated in the registration details is raised, US domestic market borrowers need to file a new registration with the SEC. The size of individual issues within a programme varies with the funding strategy of the borrower. Certain companies have a preference to raise large amounts at once, say \$100 to \$200 million, and raise funds using fewer issues. This also maintains a “scarcity value” for their paper compared to borrowers who tap the market more frequently. Other companies adopt the opposite approach, with small size issues of between \$5 to \$10 million spread over more dates.

Domestic market MTNs are primarily offered on an agency basis, although issues within a programme are sometimes sold using other methods. Agent banks sometimes acquire the paper for their own book, trading it later in the secondary market. Specific issues may be underwritten by an agent bank, or sold directly to investors by the corporate treasury arm of the borrowing company.

The main issuers of MTNs in the US market are:

- general finance companies, including automobile finance companies, business credit institutions and securities houses;
- banks, both domestic and foreign;
- governments and government agencies;
- supranational bodies such as the World Bank;
- domestic industrial and commercial companies, primarily motor car and other industrial manufacturing companies, telecommunications companies and other utilities;
- savings and loan institutions.

During the 1980s the MTN market was dominated by financial institutions, accounting for over 90% of issue volume. This share was reduced to approximately 70% by 1992 (Crabbe 1992), the remainder of the issues being accounted for by other categories of borrower.

There is a large investor demand in the US for high-quality corporate paper, much more so than in Europe where the majority of bonds are issued by financial companies. This demand is particularly great at the short- to medium-term maturity end. As the market has a large number of issuers, investors are able to select issues that meet precisely their requirements for maturity and credit rating. The main investors are:

- investment companies;
- insurance companies;
- banks;
- savings and loan institutions;
- corporate treasury departments;
- state institutions.

It can be seen that the investor base is very similar to the issuer base!

All the main US investment banks make markets in MTNs, including Merrill Lynch, Goldman Sachs, Morgan Stanley, CSFB and Salomon Smith Barney. In the UK active market makers in MTNs include RBS Financial Markets and Barclays Capital.

21.3 MTNs and corporate bonds

A company wishing to raise a quantity of medium-term or long-term capital over a period of time has the choice of issuing MTNs or long-dated bonds. An MTN programme is a series of issues over time, matching the issuer's funding requirement, and therefore should be preferred over a bond by companies that do not need all the funding at once, nor for the full duration of the programme. Corporate bonds are preferred where funds are required immediately. They are also a better choice for issuers that expect interest rates to rise in the near future and wish to lock in a fixed borrowing rate for all the funds required. The decision on whether to raise finance using MTNs or corporate bonds will be taken after consideration of the interest cost and flexibility offered by each instrument. That MTNs offer financing advantages over conventional bonds under certain circumstances is reflected in the growth and current size of the market; however the same borrowers are evident in both markets, which implies that both instruments possess advantages over the other under specific conditions.

The main difference between MTNs and corporate bonds is the process by which they are sold and distributed. There are other differentiating features however. MTNs are almost invariably sold at par on issue, whereas conventional bonds are usually offered at a slight discount to par. The proceeds on the day of issue are settled on the same day for MTNs (making them similar to money market instruments in this respect), while the settlement for new issue traditional bonds is the following day, or T+3 for international issues. In the US market, corporate bonds pay semi-annual coupon on either the 1st or the 15th of the month; the latter is identical to Treasury securities. MTNs however have coupon dates payable on a fixed cycle basis, irrespective of their issue or maturity date. This payment convention means that MTNs have a long or short first coupon, and a short final coupon, whereas conventional bonds would always have a regular final coupon. MTNs pay interest on a 30/360 day-count basis, similar to Eurobonds and US domestic corporate bonds.

EXAMPLE 21.2

- An MTN programme pays semi-annual interest on 1 June and 1 December each year and on maturity of the individual note. An issue within the programme of £10 million 6.75% bonds with a two-year maturity on 1 July would pay a short first coupon of £281,250 on the first coupon date in December, regular coupons of £337,500 on the next three coupon dates and a short final coupon of £56,250 on the maturity date.

21.3.1 Issue size and liquidity

The size of an issue has the most significant impact on the relative cost of an MTN issue versus a straight bond. For large issues, which are regarded as nominal amounts of over \$400 million borrowed over a medium or long term, the all-in cost of a straight bond issue is generally lower than the all-in cost of an MTN programme. This reflects the economies of scale that may be achieved when issuing such an amount on one date, as well as the greater secondary market liquidity of larger-sized issues. For this reason borrowers who have a heavy funding requirement for a specific period in time will usually prefer to raise the funds with a straight bond issue. The liquidity premium associated with large volume issues is not known with certainty, but is estimated at around 5 to 10 basis points (Kitter 1999); for large amounts this saving would be substantial. However this premium is indicative of the improved liquidity in the MTN market.

Another factor that borrowers consider is the cost saving associated with the distribution process for MTNs. To fully place a large bond issue, perhaps because the whole issue has not been taken up by customers, the bond may need to be offered at a higher yield, which raises the coupon for the borrower. If an individual bond within an MTN programme is not fully placed,³ borrowers have the option of raising the remaining sum by offering another bond at a different maturity, or as part of a different structure to another group of investors. Since all the funding from an MTN issue need not be priced at the coupon required by the marginal buyers, and may be raised at slightly different times, the financing costs for MTNs are often below those of a straight bond issue.⁴

21.3.2 MTN issue options

The flexibility afforded by an MTN programme is often behind the corporate treasurer's decision to employ them as funding instruments, irrespective of the interest cost advantage of straight bonds. A major flexibility of MTNs is their term to maturity. It is common for MTNs to be issued with non-standard terms to maturity, such as 15 months, 3.5 years and so on. This contrasts with straight bonds which are usually issued with maturities of 2, 5, 7, 10 and 30 years in the US market and often just 5 or 10 years in the Euro market. This makes MTNs the preferred instrument when exact maturity terms are required, for example when a borrower wishes to precisely match assets with liabilities. The cost of a bond underwriting makes small issues prohibitively expensive, and it is rare to see a bond offered with total nominal value outstanding of less than \$100–150 million. If a corporate has a requirement for a smaller amount, it is more practical to issue an MTN. Some individual issues within MTN programmes have been for as little as \$5 million; again, this flexibility allows companies to meet their funding requirements more precisely.

A continuous programme of MTN issues has the potential benefit of a lower average interest cost, compared to a single straight bond issue. For example, over a six-month period, five MTN issues of \$20 million each may have a lower average interest cost than a single issue of \$100 million in the same period. This may compensate for the lower interest cost of straight bonds, mentioned earlier, and is more likely during periods of relatively high interest rate volatility.

Once a programme has secured shelf registration, the process of issuing an MTN can be very quick, often less than half a day. This enables agent banks to issue debt on behalf of a borrower in response to specific investor requirements, or to changes in the yield or swap curves. In fact a substantial amount of MTN issues originate as a result of *reverse enquiry*. This is when investors have a requirement for debt products of a certain maturity and credit quality. For example a bond fund manager may be interested in 10-year paper with an A-rating, paying at least 50 basis points above the government yield. This is detailed to their investment bank, who is also an MTN agent

³ For example the lead manager may have client orders for \$460 million of a \$500 million issue; in order to attract customer interest for the remaining paper, the coupon may need to be raised by 10–25 basis points.

⁴ Another consideration is the commission to the agent bank, generally around 0.125% to 0.75% of the nominal value. Underwriting fees in a conventional bond offering range from 0.25% to 1.50%, and may be higher for international issues.

bank, and if the requirements suit the borrower, there will be an issue of bonds from within the borrower's MTN programme. Bonds issued in response to reverse enquiry are often the most exotic instruments in the MTN market, due to investor requirements. This includes some of the example bonds described later in this chapter. This flexibility again makes MTNs an attractive option for borrowers.

Finally a significant volume of MTNs are placed privately with investors, directly or via an agent bank. An advantage of this distribution method is that it avoids publicity, as the transaction details may be known only to the investor and borrower (and agent). Companies may wish to avoid raising funds in the bond market, and the publicity associated with this, during times of market correction or volatility, or if they are in a state of financial distress. This makes the MTN market particularly attractive during recessions and market downturns. The private placement market is also used by overseas borrowers that seek to place paper in the US market, as SEC approval is only required for a public offering. However generally the financing costs are lower in the public market than the private placement market, with its lower liquidity, so the majority of domestic borrowers use the public offering method.

21.4 Issue mechanism

21.4.1 The issue process

Issuers of MTNs usually specify an Issuing and Paying Agent (IPA) responsible for providing investors with the ability to present interest coupons and notes in various locations around the world. The IPA function required for medium term note programmes is usually viewed as a processing and administration function, and is therefore normally of most interest to the settlements and processing areas of an issuing organisation.

Once an agent has been selected as the IPA for a new programme, and draft legal documentation is available, it will allocate the transaction to a documentation department. This department will review the documentation from a legal perspective, and often calls in external legal firms to assist in the review. However the primary functions that the IPA performs is to receive an issuer's instructions, arrange for the issuance of the security to the relevant dealer via the international clearing systems, and then to service the security throughout its term. Generally Euro-MTN transactions are represented by a single security, known as the *global note*. When an issuer and underwriting bank agree a new transaction, both parties will advise the IPA of the transaction details, such as the currency, amount, issue date, interest basis, maturity, issue price and so on. Although the dealer is not obligated to advise the IPA of the trade details, market practices are such that this has now become the norm. The IPA issues the security after receiving the instructions of the issuer together with an authorised pricing supplement, which is the term sheet listing the issue details.

Once trade details have been received, the IPA will contact Euroclear and Clearstream and advise them of the trade information. The clearing systems will then advise the IPA of the unique security codes, known as International Securities Identification Number (ISIN) and the Common Code, which are used to identify the security during its term. The IPA will then input the trade and settlement information into a "new issuance account", while the dealer will input its instructions into the relevant clearing system. The IPA's instruction will be a securities delivery versus payment instruction, while the dealer's will be a securities receipt versus payment instruction. Processing the transaction on this basis means that all parties are protected and that the securities will never be issued unless the correct issue proceeds are paid. On the actual issuance date, the IPA will receive the cash proceeds from the dealer and will make onward payment to the issuer; it also creates the global note that represents the issue and delivers this to the *common depositary* for Euroclear and Clearstream. The common depositary is usually called the *custodian*.

21.4.2 Servicing the issue

The IPA is responsible for servicing the MTN during its life. Approximately ten business days before an interest payment date, or before the maturity date of an issue, it will advise the issuer of the forthcoming interest payment and provide them with payment details for the repayment amount due. On the instalment due date, the IPA will pay the clearing system(s) the amount due to the investors holding via their computer systems, and will also credit the proceeds to the relevant investor's account. Investors holding securities outside of a clearing system have to physically present their EMTNs (and Eurobonds as well) for payment at one of the designated paying agents for the issue.

The activities described above summarise the core function that is performed by an IPA; in addition throughout the life of a programme the IPA also performs numerous other activities on behalf of the issuer. Such activities include:

- being responsible for the safekeeping of the master global notes;
- the submission of any reports required by regulatory entities, such as the Bank of England, Japanese Ministry of Finance and the Bundesbank;
- acting as the calculation agent service for cash flows paid by floating rate, indexed linked, and dual-currency note issues;
- arranging for the listing of the note issues at a relevant stock exchange;
- arranging for the publication of notices in the financial press;
- maintaining comprehensive details of all transactions on its computer systems; and
- responding to external enquiries, such as requests made by auditors.

The process followed by IPAs is very similar to that used in the Eurobond market (see Chapters 18 and 19).

21.5 The secondary market

A liquid secondary market in MTNs was first established in the US market by Merrill Lynch which undertook to quote bid prices to any investor wishing to sell MTNs before maturity, provided that the investor had originally bought the notes through them. In other words Merrill Lynch was guaranteeing a secondary market to borrowers that issued notes through it. This undertaking was repeated by other banks, resulting in a market that is now both large and liquid. That said, MTNs are not actively traded and market makers do not quote real-time prices on dealing screens. The relatively low volume of secondary market trading stems from a disinclination of investors to sell notes they have bought, rather than a lack of market liquidity.

There is a wide range of maturities available for MTNs in the secondary market. The maturity of individual issues reflects the funding requirements of their issuers; for example bonds issued by motor car finance companies usually match the duration of loans to their customers, so they tend to have three-year to five-year maturities. Bonds issued by industrial and manufacturing companies have longer maturities. The maturity profile of MTNs in the US market is shown at Figure 21.1.

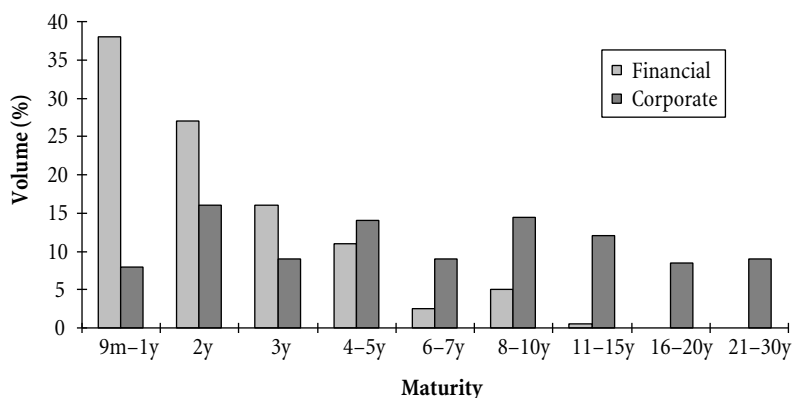


Figure 21.1: Maturity profile of US MTN market, 1998. Source: Chase Manhattan.

The yield on MTNs is a function of the credit quality of the issuer, as well as the liquidity of the paper in the secondary market and market maker's support. In the US domestic market MTNs are quoted on a yield basis, often as a spread over the equivalent maturity Treasury security. The highest yield spread is observed on the lowest rated bonds. Spreads vary according to market conditions and the business cycle (for example, they are at their widest during times of recession and after a market correction), as illustrated by Figure 21.2.

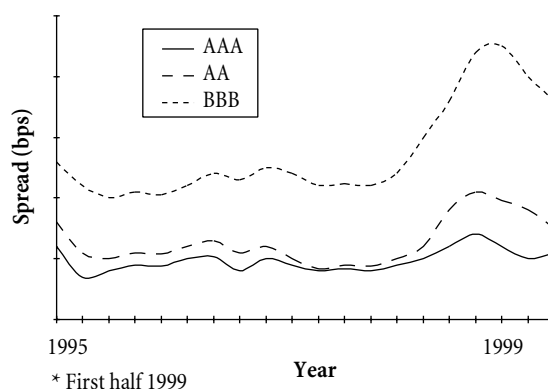


Figure 21.2: Credit structure of yield spreads, 10-year sterling MTNs, 1995–1999.

Source: Greenwich NatWest.

The MTN market is a major instrument of corporate finance in the US market, and is growing in importance in the Euro market. An indication of its significance is given by its growing share of total investment-grade debt issuance. For instance in the US debt market, MTNs accounted for 47% of total investment grade debt issued in 1995, where total debt comprised MTNs and straight bond issues. This is an increase from around 12% in the early 1980s (Roland 1999).

21.5.1 Credit rating

MTNs are unsecured debt. A would-be issuer of MTNs will usually seek a credit rating for its issue from one or more of the main rating agencies such as Moody's or Standard & Poor's. The rating is given by the agency for a specific amount of possible new debt; should the issuer decide to increase the total amount of MTNs in issue it will need to seek a review of its credit rating. As MTNs are unsecured paper only the higher rated issuers, with an "investment grade" rating for their debt, are usually able to embark on a revolving facility. Although there are "junk" rated MTNs, there is no liquid market in them, and they account for less than 1% of outstanding volume.

Companies issuing MTNs generally have high ratings within the "investment grade" category. During 1995, \$65 billion of the \$99 billion of MTNs issued in the US domestic market represented debt with a rating of "A" or higher, while at the end of that year approximately 98% of the outstanding debt in the US market was rated at investment grade level.

EXAMPLE 21.3 Reverse FRN with swap

- A subsidiary of a global integrated banking house that engages in investment activity requires US dollar funding. A proportion of its funds are raised as part of a \$5 billion Euro-MTN programme. Due to demand for sterling assets they issue a five-year pounds sterling reverse floating rate medium-term note as part of the MTN programme, with the following details.

Issue size	£15 million
Issue date	20 January 1998
Maturity	21 January 2003
Rate payable	9% from 20/1/98 to 20/7/98 19% — $(2 \times \text{LIBOR6mo})$ thereafter to maturity
Price at issue	99.92
Proceeds	£14,988,000

As the issuer requires US dollar funding, it swaps the proceeds into dollars in the market in a cross-currency swap, and pays US dollar three-month Libor for this funding. On termination the original currencies are swapped back and the note redeemed.

21.6 The Euro-MTN market

The development of a market in offshore or internationally-traded MTNs was originally due to US companies seeking sources of finance overseas. The Euro-MTN market has since expanded dramatically and it is now an important source of corporate funding for US, European, Japanese and Asian domiciled companies. Euro-MTNs trade essentially as Eurobonds, that is they are international bonds that can be bought and sold across international boundaries. There is also a domestic market in MTNs in the UK, France, Germany and several other European countries, as well as Japan.⁵ The main trading centre is in London, where most of the major underwriters and market making banks trade out of. The growth of the Euro market has been even more rapid than the US one, rising from approximately \$10 billion in 1990 to just under \$500 billion in 1998 (Roland 1999). The flexibility of an MTN programme, which was behind much of the growth in the US domestic market, is the key reason behind the expansion of the Euro market.

Euro-MTNs are essentially identical to MTNs in the US domestic market, with the key exception that they are not subject to national regulations or national registration requirements. The issuer base in the Euro market is much more concentrated among financial institutions and banks, and there is a lower appetite for lower-grade credit quality paper. In 1998 over 65% of Euro-MTNs were rated at AAA or AA, compared to just 13% of domestic US MTNs (Roland 1999). Another slight difference is in the maturity structure; most Euro-MTNs have maturities of 5 to 10 years, and it is rare to encounter maturities of 30 years. However there is a diverse range of structured Euro-MTNs in the market, according to Roland (1999) structured transactions account for up to 60% of Euro-MTN issues, compared to under half of that in the US market. As one might expect, currency swap structures such as those described in Example 21.3 are more common in the Euro market; note that the bonds in that example are part of the Euro-MTN programme of a major integrated banking house.

21.7 Structured MTNs

The application of financial engineering techniques has resulted in the introduction of exotic MTN structures. As a result both borrowers and investors have had their requirements met precisely through the use of tailor-made bond structures. Put simply, in a structured MTN, the borrowing company issues an MTN, which may or may not be a plain vanilla instrument itself, that is part of a swap agreement that changes the nature of the interest payments that the borrower makes. The first structured notes involved the issue of a conventional MTN in conjunction with an interest-rate swap. If the MTN was a fixed-coupon bond, the issuer would enter into an interest-rate swap whereby it received fixed interest and paid floating-rate interest; the end result would be that the issuer now had a floating-rate interest rate liability and not a fixed-rate one. The relevant swap terms are identical to the MTN ones, that is the fixed-rate payments are on the same date as the MTN coupon dates, and on the same interest basis. The borrower might do this because such an arrangement saves it interest payments not available through the issue of a straight floating-rate MTN. For borrowers, the primary motivation for entering into structured note arrangements is because a reduction in interest costs can be achieved. The interest savings must be sufficient to offset the increased transaction costs of structured deals, because these frequently require additional tax, accounting and legal advice, which may be supplied by the agent bank or by a separate advisory firm.

The flexibility of the MTN market has resulted in many structured transactions being created as a result of a reverse enquiry. An investor who has an interest in acquiring an instrument with specific terms, such as a link to an exotic exchange rate, equity index or commodity, may not be able to meet their requirements in the conventional market. If, via an agent bank, a borrower is able to issue an instrument that meets the specific needs of the enquiry, the investor will be able to purchase an instrument that fulfils its requirements precisely. The establishment of the inverse-floater MTN market in the US in the early 1990s was in response to investor needs; the issuer's of inverse floaters usually hedged their interest-rate risk exposure in the swap market.

The other drivers of structured deals are the investment banks themselves, who may present an idea for a particular deal to their investor clients. Often this occurs when the structured finance team at the bank has spotted

⁵ Note that there is a cross-over in terminology, and the terms "international" and "Euro" are frequently interchanged. A bond issued in a domestic market in Europe is not a Eurobond, equally there are domestic MTN programmes in several European countries. In this chapter the term "Euro-MTN" is used to refer to MTNs that trade across international boundaries, and can be in any currency.

an area of the market where value may be obtained for the client, or a price anomaly may be exploited. According to Crabbe (1993), structured deals in the US market accounted for between 20% to 30% of MTN issue volume in the first six months of 1993, from a figure of under 5% ten years previously. The growth of structured deals is further evidence of the flexibility of the MTN market, although of course many of the structures have also been observed in the conventional bond market. In the remainder of this section we present examples of structured MTNs that have been issued as part of a global US dollar Euro-MTN programme by an investment banking group. They illustrate the wide range of features available to investors; in fact it is probably accurate to say that the range of arrangements available is limited only by market participants' imagination.

In Example 21.4 we present a description of some of the structured MTN deals that have taken place during 1998 and 1999. The issuer and counterparty banks are all large investment banking groups.

EXAMPLE 21.4 Medium-Term Notes issued as part of a global \$5 billion Euro-MTN programme

- The issuer is an integrated banking house, a subsidiary of which is an investment vehicle in the United States. As such the subsidiary's funding requirement is exclusively in US dollars, however it issues paper wherever there is customer demand. Foreign currency issues are swapped into dollars, on which the issuer pays floating-rate interest. The subsidiary has a AAA-rating.

Japanese yen MTN

Issue size	JPY500 million
Maturity	5 years
Rate payable	0.1%
Issue price	100 per cent
Proceeds	JPY500 million

This bond was swapped into US dollars, the equivalent amount of which was \$3.56 million. During the life of the bond the issuer pays floating-rate interest on the US dollars, while the yen interest payments are made by the swap bank; on maturity the exact start proceeds are swapped back, enabling the issuer to redeem the bond. The structure is illustrated at Figure 21.3.

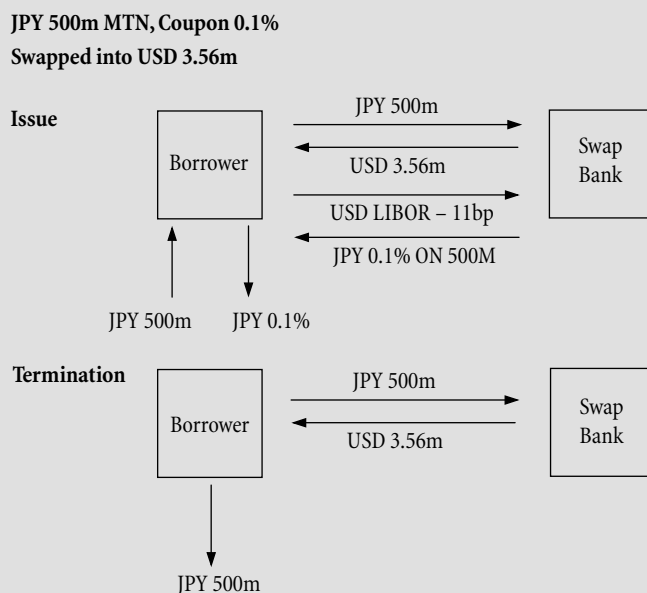


Figure 21.3: Structured MTN issue.

Swiss franc step-up notes

Issue date	25/3/1997	
Maturity	25/3/2002	
Issue size	CHF 15 million	
Issue price	100 per cent	
Coupon	2.40% to 25/3/1999	Callable 25/3/1999
	2.80% to 25/3/2000	25/3/2000
	3.80% to 25/3/2001	25/3/2001
	4.80% to 25/3/2002	

This bond was issued in conjunction with a currency swap, that is the CHF15 million was swapped into \$10.304 million, on which the issuer pays floating-rate interest. These amounts were to be swapped back on termination, although the bond was in fact called at the first call date and the swap cancelled. The structure is illustrated at Figure 21.4.

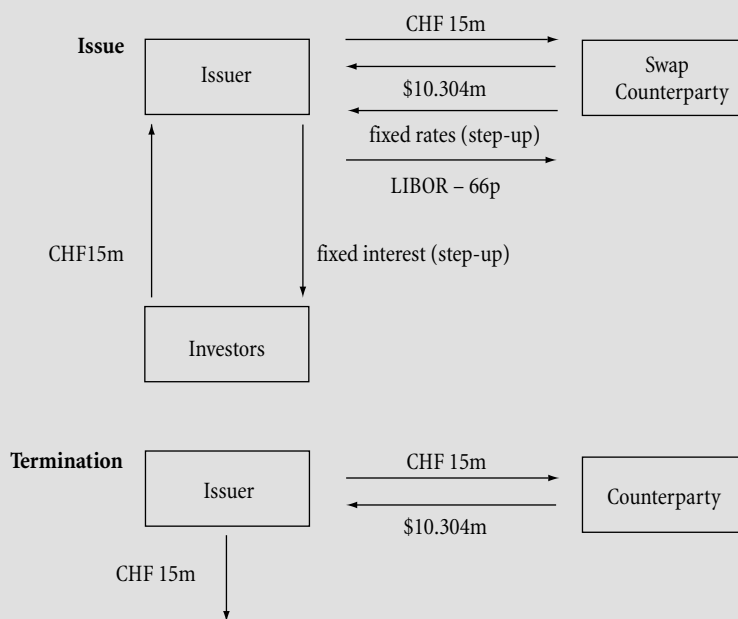


Figure 21.4: Structured MTN issue.

€6.15 million zero-coupon equity basket notes due January 2004

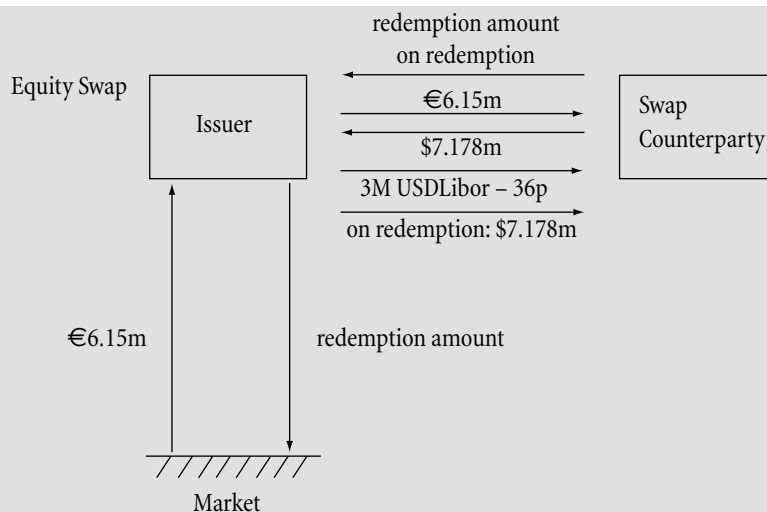
Issue date	22/1/1999
Maturity	22/1/2004
Issue price	100 per cent
Interest basis	Zero coupon
Final redemption amount:	

The greater of

- (i) 100%, or
- (ii) $100\% + (50\% \times (X_m - 2704/2704))$,

subject to a maximum of 133%, where X_m is the level of the FTSE Eurotop 100 stock index.

The proceeds are swapped into dollars, an equivalent amount of \$7.178 million, on which the issuer pays floating-rate interest of three-month LIBOR minus 3 basis points, during the life of the bond. This is illustrated in Figure 21.5.



Issuer pays 3m \$ LIBOR - 3bp during life of swap on notional of \$7.178m.

Figure 21.5: Structured MTN equity swap issue.

Belgian franc “DEM LIBOR Accrual” note

Issue date 13/7/1998

Maturity 17/7/1999

Issue size BEF300 million

Coupon $(3\text{-month BEF-LIBOR} + 0.90)\% \times (\text{Accrual factor})$, where the coupon rate will accrue for each day on which the DEM-LIBOR rate is within a range of 3.42% - 4.03%

The terms of this issue included an unusual feature, a collar within which interest can accrue. The collar was the Deutschmark LIBOR rate; if the rate moved outside this stated range, no interest was payable on the note. In return for this the investor received a relatively high interest-rate from what was in effect a triple-A risk. The structure is illustrated as Figure 21.6.

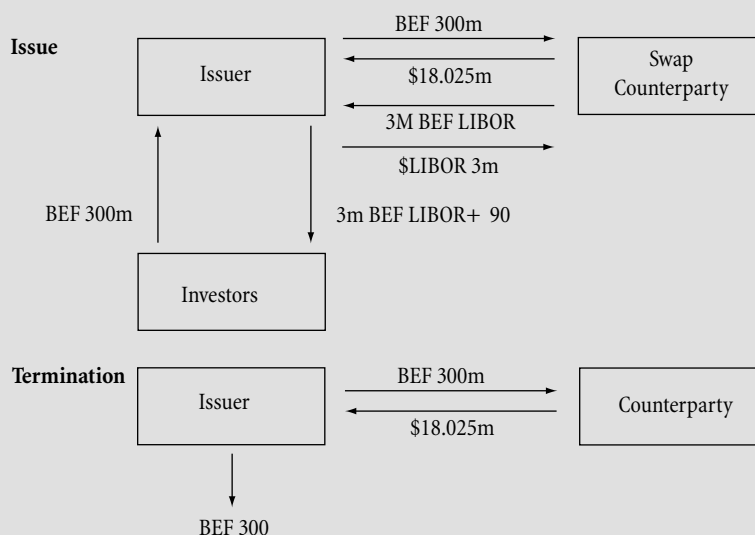


Figure 21.6: Structured MTN issue.

USD Brazil credit-linked notes

Issue date	11/3/1998
Maturity	11/9/2000
Issue size	\$30 million
Issue price	100 per cent
Interest basis	Six-month USD-LIBOR + 430 basis points, not payable if a “Brazilian Credit Event” has occurred

The redemption amount for this note is contingent on there being any relevant credit events occurring during the bond’s life. A “credit event” is defined in the MTN issue terms and conditions and is related to Brazilian corporate credit quality.

Japanese yen step-down notes due 2000

Issue date	4/1/1995
Maturity	29/3/2000
Issue size	JPY2.5 billion
Coupon	31% to 29/3/1995; thereafter 2.2% to redemption
Swap proceeds	\$14 million

This is a straightforward MTN issue with yen proceeds swapped into dollars. Note however the very high initial coupon. The structure is illustrated in Figure 21.7.

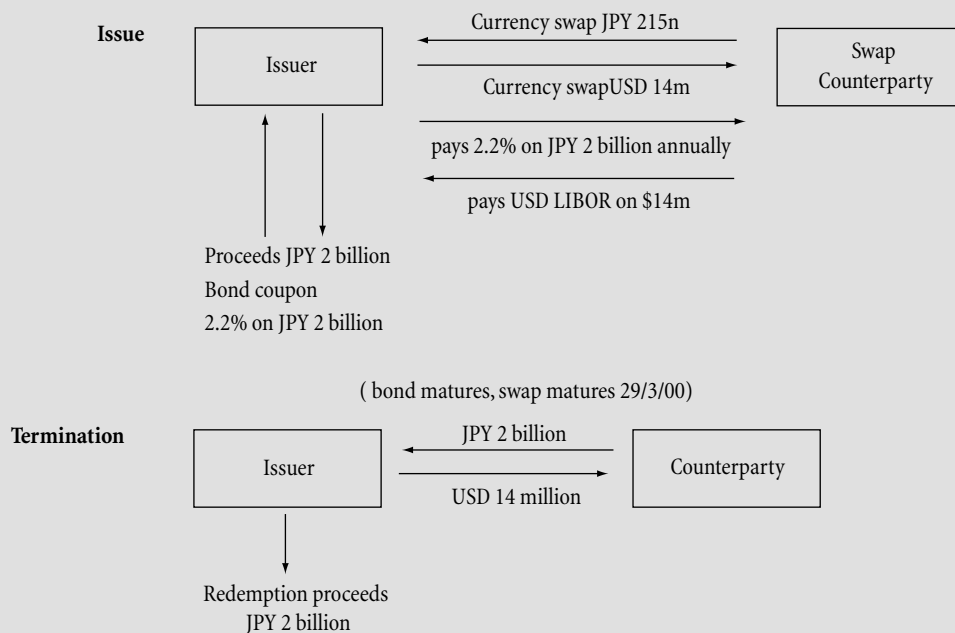


Figure 21.7: Structured MTN issue.

ECU 12 million Korea Development Bank credit-linked zero-coupon notes due June 2000

Issue date	16/06/1998
Maturity	16/06/2000
Issue price	83.40%
Interest basis	Zero-coupon

The bonds are redeemable at par, or in the event of a “credit event” (defined in the issue prospectus) prior to maturity are redeemable at a “revised redemption amount”, which is the greater of (a) zero and (b) a product of the Future Value percentage, Recovery percentage and Accreted Notional, subject to a maximum of 100% (that

is, redemption will be a maximum of ECU 12 million). The terms quoted here are defined in the issue prospectus.

US dollar zero-coupon basket-linked notes due 2001

Issue	1/07/98
Maturity	1/07/2000
Issue price	100 per cent

Redemption amount:

The bond maturity value is $95\% + (82\% \times (X_m - 100/100))$, where X_m is the arithmetic mean of the values of a basket of global equities (the securities that constitute the equity basket are specified in the issue term sheet).

Selected bibliography and references

BPP Financial Publishing, *Corporate Debt Markets*, BPP Holdings plc, 1993.

Crabbe, L., "Anatomy of the Medium-Term Note Market", *Federal Reserve Bulletin*, August 1993, pp. 751–768.

Crabbe, L., "Corporate Medium-Term Notes", *The Continental Bank Journal of Applied Corporate Finance*, Winter 1992, pp. 90–102.

Post, M., "The Effect of SEC Amendments to Rule 2a-7 on the Commercial Paper Market", *Finance and Economics Discussion Series 1992* (Board of Governors of the Federal Reserve System, May 1992).

Fabozzi, F., *Bond Markets, Analysis and Strategies*, 2nd edition, Prentice Hall 1993, pp. 135–137.

Roland, M., "Sterling corporate debt and the MTN market", unpublished paper, University of London, 1999.

Questions and exercises

1. What is the main determinant of the yield at which an MTN trades?
2. What is a structured MTN? Give an example of one, drawing a diagram of the cash flows for the structure you have chosen.
3. Assess how the development of the MTN market has affected the way that corporates may raise finance.
4. An AAA-rated multinational corporation is considering ways in which to finance a re-investment programme of its plant and machinery in several production centres. It's operating currency is euros. How might an MTN programme be used to fund this requirement, given that most of the investors that might be interested in lending to the company are located in the United States?

22 Commercial Paper

Strictly speaking *commercial paper* is a money market product, and this chapter could be placed within the section on money markets. However CP is an important corporate finance instrument and it is worthwhile to review the subject now, as part of our discussion on corporate debt markets.

Companies fund part of their medium- and long-term capital requirements in the debt capital markets, through the issue of bonds. Short-term capital and *working* capital is usually sourced directly from banks, in the form of bank loans. An alternative short-term funding instrument however is commercial paper (CP), which is available to corporates that have a sufficiently strong credit rating. Commercial paper is a short-term unsecured promissory note. The issuer of the note promises to pay its holder a specified amount on a specified maturity date. CP normally has a zero coupon and trades at a *discount* to its face value. The discount represents interest to the investor in the period to maturity. CP is typically issued in bearer form, although some issues are in registered form.

Outside of the United States CP markets were not introduced until the mid-1980s, and in 1986 the US market accounted for over 90% of outstanding commercial paper globally.¹ In the US however, the market was developed in the late nineteenth century, and as early as 1922 there were 2200 issuers of CP with \$700 million outstanding. In 1998 there was just under \$1 trillion outstanding, as shown in Table 22.1. CP was first issued in the United Kingdom in 1986, and subsequently in other European countries.

	\$bln	1985	1991	1992	1993	1994	1995	1996	1997	1998
Financial firms		213.9	414.7	395.5	398.1	399.3	430.7	486.6	590.8	765.8
Non-financial firms		87.2	147.9	133.4	147.6	155.7	164.6	188.3	184.6	200.9
All Issuers		301.1	562.6	528.9	545.7	555	595.3	674.9	775.4	966.7

Table 22.1: The US Commercial Paper Market. Source: Federal Reserve Bulletin.

Originally the CP market was restricted to borrowers with high credit rating, and although lower-rated borrowers do now issue CP, sometimes by obtaining credit enhancements or setting up collateral arrangements, issuance in the market is still dominated by highly-rated companies. The majority of issues are very short-term, from 30 to 90 days in maturity; it is extremely rare to observe paper with a maturity of more than 270 days or nine months. This is because of regulatory requirements in the US,² which states that debt instruments with a maturity of less than 270 days need not be registered. Companies therefore issue CP with a maturity lower than nine months and so avoid the administration costs associated with registering issues with the SEC.

	US CP	Eurocommercial CP
Currency	US dollar	Any Euro currency
Maturity	1 – 270 days	2 – 365 days
Common maturity	30 – 50 days	30 – 90 days
Interest	Zero coupon, issued at discount	Usually zero-coupon, issued at discount
Quotation	On a discount rate basis	On a discount rate basis or yield basis
Settlement	T + 0	T + 2
Registration	Bearer form	Bearer form
Negotiable	Yes	Yes

Table 22.2: Comparison of US CP and Eurocommercial CP.

¹ OECD (1989).

² This is the Securities Act of 1933. Registration is with the Securities and Exchange Commission.

As with MTNs there are two major markets, the US dollar market with outstanding amount in 1998 just under \$1 trillion as noted above, and the Eurocommercial paper market with outstanding value of \$290 billion at the end of 1998.³ Commercial paper markets are wholesale markets, and transactions are typically very large size. In the US over a third of all CP is purchased by money market unit trusts, known as mutual funds; other investors include pension fund managers, retail or commercial banks, local authorities and corporate treasurers.

Although there is a secondary market in CP, very little trading activity takes place since investors generally hold CP until maturity. This is to be expected because investors purchase CP that match their specific maturity requirement. When an investor does wish to sell paper, it can be sold back to the dealer or, where the issuer has placed the paper directly in the market (and not via an investment bank), it can be sold back to the issuer.

22.1 Commercial Paper programmes

The issuers of CP are often divided into two categories of company, banking and financial institutions and non-financial companies. The majority of CP issues are by financial companies, as noted in Table 22.1. Financial companies include not only banks but the financing arms of corporates such as General Motors, Ford Motor Credit and Chrysler Financial. Most of the issuers have strong credit ratings, but lower-rated borrowers have tapped the market, often after arranging credit support from a higher-rated company, such as a *letter of credit* from a bank, or by arranging collateral for the issue in the form of high-quality assets such as Treasury bonds. CP issued with credit support is known as *credit-supported commercial paper*, while paper backed with assets is known naturally enough, as *asset-backed commercial paper*. Paper that is backed by a bank letter of credit is termed *LOC paper*. Although banks charge a fee for issuing letters of credit, borrowers are often happy to arrange for this, since by so doing they are able to tap the CP market. The yield paid on an issue of CP will be lower than a commercial bank loan.

Although CP is a short-dated security, typically of three- to six-month maturity, it is issued within a longer term programme, usually for three to five years for euro paper; US CP programmes are often open-ended. For example a company might arrange a five-year CP programme with a limit of \$100 million. Once the programme is established the company can issue CP up to this amount, say for maturities of 30 or 60 days. The programme is continuous and new CP can be issue at any time, daily if required. The total amount in issue cannot exceed the limit set for the programme. A CP programme can be used by a company to manage its short-term liquidity, that is its working capital requirements. New paper can be issued whenever a need for cash arises, and for an appropriate maturity.

Issuers often roll over their funding and use funds from a new issue of CP to redeem a maturing issue. There is a risk that an issuer might be unable to roll over the paper where there is a lack of investor interest in the new issue. To provide protection against this risk issuers often arrange a stand-by line of credit from a bank, normally for all of the CP programme, to draw against in the event that it cannot place a new issue.

There are two methods by which CP is issued, known as *direct-issued* or *direct paper* and *dealer-issued* or *dealer paper*. Direct paper is sold by the issuing firm directly to investors, and no agent bank or securities house is involved. It is common for financial companies to issue CP directly to their customers, often because they have continuous programmes and constantly roll-over their paper. It is therefore cost-effective for them to have their own sales arm and sell their CP direct. The treasury arms of certain non-financial companies also issue direct paper; this includes for example British Airways plc corporate treasury, which runs a continuous direct CP programme, used to provide short-term working capital for the company. Dealer paper is paper that is sold using a banking or securities house intermediary. In the US, dealer CP is effectively dominated by investment banks, as retail (commercial) banks were until recently forbidden from underwriting commercial paper. This restriction has since been removed and now both investment banks and commercial paper underwrite dealer paper.

Although CP is issued within a programme, like MTNs, there are of course key differences between the two types of paper, reflecting CP's status as a money market instrument. The CP market is issuer-driven, with daily offerings to the market. MTNs in contrast are more investor-driven; issuers will offer them when demand appears. In this respect MTNs issues are often "opportunistic", taking advantage of favourable conditions in the market.

³ Source: BIS.

22.2 Commercial paper yields

Commercial paper is a discount instrument. There have been issues of coupon CP, but this is very unusual. Thus CP is sold at a discount to its maturity value, and the difference between this maturity value and the purchase price is the interest earned by the investor. The CP day-count base is 360 days in the US and euro markets, and 365 days in the UK. The paper is quoted on a discount yield basis, in the same manner as Treasury bills. The yield on CP follows that of other money market instruments and is a function of the short-dated yield curve. The yield on CP is higher than the T-Bill rate; this is due to the credit risk that the investor is exposed to when holding CP, for tax reasons (in certain jurisdictions interest earned on T-Bills is exempt from income tax) and because of the lower level of liquidity available in the CP market. CP also pays a higher yield than Certificates of Deposit (CD), due to the lower liquidity of the CP market.

Although CP is a discount instrument and trades as such in the US and UK, euro currency Eurocommercial paper trades on a yield basis, similar to a CD. The discount rate for an instrument was discussed in Chapter 2. The expressions below are a reminder of the relationship between true yield and discount rate.

$$P = \frac{M}{1 + r \times \frac{\text{days}}{\text{year}}} \quad (22.1)$$

$$rd = \frac{r}{1 + r \times \frac{\text{days}}{\text{year}}} \quad (22.2)$$

$$r = \frac{rd}{1 - rd \times \frac{\text{days}}{\text{year}}} \quad (22.3)$$

where M is the face value of the instrument, rd is the discount rate and r the true yield.

EXAMPLE 22.1

1. A 60-day CP note has a nominal value of \$100,000. It is issued at a discount of $7\frac{1}{2}$ per cent per annum. The discount is calculated as:

$$\begin{aligned} \text{Dis} &= \frac{\$100,000(0.075 \times 60)}{360} \\ &= \$1,250. \end{aligned}$$

The issue price for the CP is therefore \$100,000 – \$1,250, or \$98,750.

The money market yield on this note at the time of issue is:

$$\left(\frac{360 \times 0.075}{360 - (0.075 \times 60)} \right) \times 100\% = 7.59\%.$$

Another way to calculate this yield is to measure the capital gain (the discount) as a percentage of the CP's cost, and convert this from a 60-day yield to a one-year (360-day) yield, as shown below.

$$\begin{aligned} r &= \frac{1,250}{98,750} \times \frac{360}{60} \times 100\% \\ &= 7.59\%. \end{aligned}$$

Note that these are US dollar CP and therefore have a 360-day base.

2. ABC plc wishes to issue CP with 90 days to maturity. The investment bank managing the issue advises that the discount rate should be 9.5 per cent. What should the issue price be, and what is the money market yield for investors?

$$Dis = \frac{100(0.095 \times 90)}{360}$$

$$= 2.375.$$

The issue price will be 97.625.

The yield to investors will be:

$$\frac{2.375}{97.625} \times \frac{360}{90} \times 100\% = 9.73\%.$$

Selected bibliography

Alworth, J., Borio, C., "Commercial Paper Markets: A Survey", *BIS Economic Papers No.37*, 1993, Bank for International Settlements.

Corporate Finance, *A Guide to Commercial Paper in Europe 1991*, Euromoney Publications, September 1991.

Corporate Finance, *Guide to International Commercial Paper*, Euromoney Publications, January 1993.

OECD, *Competition in Banking*, OECD 1989.

Stigum, M., *The Money Market*, Dow-Jones Irwin, 1990.

Questions and exercises

1. Why is commercial paper an alternative to short-term bank borrowing for a corporation?
2. What is the difference between directly-placed paper and dealer-placed paper?
3. A money market fund purchases euro Eurocommercial with the following terms:

Purchase date	6 January 2000
Maturity date	6 April 2000
Yield	4.91%
Nominal amount	€100 million

 What cash amount is paid for the paper?
4. If you wish to achieve a yield of 6.25% on £10 million 30-day CP, what would be the discount rate and the amount of the discount?
5. If the discount rate in the previous question was in fact 6.25%, what would the yield and the amount be?
6. What does the yield spread between commercial paper and equivalent maturity Treasury bills reflect?
7. A 180-day US dollar CP is purchased at a discount rate of 5.75%, and sold at a discount rate of 5.62% after 30 days. What yield has been achieved on an annualised basis?

23

Preference Shares and Preferred Stock

Preference shares, or *preferred stock* as they are known in the United States, are a class of shares that entitle the holder to preferences over those of the company's ordinary shares. The most usual preference concerns dividend rights, but other provisions may sometimes be included. They are non-equity shares, but are also described sometimes as equity. They are not debt instruments although they trade similar to certain types of debt, and often the preference share market making desk is located within the bond division of a bank. Preference shares rank below debt instruments in the event of a wind-up of a company, but above ordinary shares. They have a long history; the market in preference shares was well established in both the United Kingdom and United States in the nineteenth century. The main types of preference share are fixed-dividend, adjustable-rate and auction market preference shares. These main variations will be reviewed later in the chapter.

Preference shares may be defined as shares which provide their holders an entitlement to receive a dividend, but only up to a specific limit, which is usually a fixed amount every year. They may also give their holders a limited right to participate in any surplus in the event of a winding up, should there be a liquidation and sale of the company's assets. Preference shares may also be redeemable on fixed terms or on terms dictated by the issuing company. Despite their name however preference shares are similar to debt capital and this is why it is necessary to review their characteristics here. However preference shares are not debt, but are a form of ownership in a company, despite the fact that most forms of preference stocks do not grant their holders a voting right. The instruments might be fairly described as a peculiar cross between shares and bonds, and share some but not all characteristics of both. For example certain preference shares are unlike ordinary shares in that if a dividend is not paid in one year, it will *accumulate* and must be paid before ordinary share dividends. Unlike bonds however a failure to pay dividends is not a default, although there are several negative implications associated with such an action. They are similar to bonds in that they do not entitle holders a vote in the company (usually, as long as the dividend is paid), although voting rights are usually granted if a dividend is not paid. The preference share market making desk in a bank is usually situated in the fixed interest division, rather than in the equity division. This reflects that the valuation of preference shares fluctuates with the yield curve.

In this chapter we introduce the different types of preference shares, and how they differ from conventional fixed income securities. In the US and UK there is wide variety of preference shares in the market and we are only able to review them here; interested readers may wish to consult the references listed in the bibliography at the end of the chapter.

23.1 The size of the market

The majority of borrowers in the US domestic market are financial institutions. In the UK domestic market the instrument has been more popular with non-financial corporates, although banks have also been large-volume issuers. During the period 1983–1993 over \$81 billion was raised in the US preferred market, of which around 62% was by financial companies.¹ Table 23.1 shows the level of issuance in the US market during this period.

Preference shares are purely a corporate financing instrument and credit ratings for individual share issues are as important as they are for corporate bonds. In the US domestic market, ratings are issued by four ratings firms, known as “nationally recognised statistical rating organisations” or NRSROs, which are Standard & Poor's, Moody's, Duff and Phelps and Fitch Investors Service. The ratings issued by the NRSROs, although outwardly similar to the ratings for corporate bonds in some cases, need to be assessed differently however; this is because preferred stock should be analysed within the range of other preference shares, distinct from debt issues.

¹ Wilson (1997).

US preferred stock market	1983	1984	1985	1986	1987	1988	1989	1990	1991	1992	1993
Volumes (\$ million)											
Fixed dividend	2,300	1,600	463	1,060	2,260	2,900	1,120	3,720	3,415	3,295	11,430
Variable dividend	2,800	4,375	3,180	5,480	6,290	5,325	5,980	2,480	1,370	6,740	3,990
Number of issues by industry sector											
Banks	19	32	26	54	73	62	49	21	14	28	31
Investment companies	–	–	–	–	–	–	20	7	12	25	24
Utility companies	49	42	21	13	32	42	16	11	16	23	54
Industrial/transportation companies	6	7	4	18	14	8	15	8	5	3	15

Table 23.1: US preferred stock market, 1983–1993. Source: IFC.

23.2 Description and definition of preference shares

23.2.1 Introduction

An annual preference dividend is payable, usually on a semi-annual basis, as a fixed percentage amount of the nominal value of the share. Unlike the interest payable on a conventional bond though, which is paid out of the pre-tax profits of the issuing company, preference share dividends are paid out post-tax profits. The dividend on a preference share must be paid before any dividend can be paid to ordinary equity shareholders. The primary differentiating feature of preference shares compared to ordinary shares is the treatment of the dividend that is payable by them, which are at a fixed rate or variable rate. The dividend paid by a conventional preference share is at a fixed rate of the face value, or a fixed cash value per share. The dividend must be paid before any can be paid on ordinary shares; for the majority of preference shares the dividend is in cash. During the bull market of the mid-1980s up to the crash of October 1987, some preference shares in the US market paid dividends in the form of more shares. These were known as *PIKs* preferred stock.

There are also preference shares that pay a variable rate dividend in the US market. Generally these pay a dividend that is adjusted or re-set quarterly at a fixed spread to the Treasury yield curve. The spread, known as the dividend reset spread, will lie above or at the level of one of three points on the yield curve, which are the three-month maturity point, the 10-year point and the 30-year point. Instead of using the redemption yield at these points, the adjusted dividend is based on the Treasury constant maturity (TCM) yield, which is calculated by the Federal Reserve. This means that the adjusted dividend is not a pure short- or long-term yield but a composite of the two. Variable rate preferred stock in the US is known as adjustable-rate preferred stocks or ARPS. They are singular instruments in that they are neither money market paper nor long-dated bonds. They possess some of the attributes of ordinary shares however, in that they do not exhibit the “pull to par” effect of bonds with a fixed maturity. However they are related to the debt market yield curve and their value will fluctuate with this.

In the United Kingdom the coupon rate of preference dividend is shown net of tax, as the amount of dividend that shareholders will receive in cash. The gross pre-tax dividend is this net amount plus a *tax credit* for the amount of income tax deducted. If the tax credit is 25%, the gross dividend is multiplied by 100/75 of the net dividend.

EXAMPLE 23.1

- A company issues 6% preference shares of £1 and the tax credit is 25%. The gross dividend received by shareholders is £0.08, given by the net amount of £0.06 and the tax credit of $0.06 \times 25/75$. The gross dividend is therefore $£0.06 \times (£1.00 \times 100/75)$, which is £0.08.

The tax credit on United Kingdom dividends was reduced from 25% to 20% in 1993. In the first budget of the Labour government (elected in May 1997) the right to re-claim this tax credit was removed. This has had profound effects on the preference share market in the UK, described in an extract from *The Times*, which is reproduced with permission at the end of this chapter.

23.2.2 Rights in a liquidation

In the event of a liquidation of the company, the order of entitlement to payment from the proceeds of selling off the company's assets is that preference shareholders rank behind trade creditors but ahead of ordinary shareholders.

The amount receivable in a winding-up however is limited to the nominal value of the shares, and possibly also any unpaid dividend. Certain preference shares have issue terms that allow them to participate with the ordinary shares in the event of liquidation, which may seem to some to be a somewhat dubious advantage. For example, the preference share may entitle a holder to receive upon the company's liquidation, twice the amount per share distributed on each ordinary share. This is more common in the US market.

23.2.3 Voting rights

Preference shares do not, as a rule, entitle holders to a vote in the running of the company. This is perhaps the chief difference between preference shares and ordinary shares. The general understanding is that, because preference shareholders receive a regular and fixed dividend, they do not require voting rights. This is not always the case, and certain issues do entitle holders to a vote. For example the preferred stock of Southern California Edison, a US utility company, carries with it varying levels of voting rights. The cumulative preferred stockholders each have six votes per share, while the \$100 cumulative preferred stock entitles its holders to two votes per share. The votes may be used cumulatively in the case of the election of company directors.

The general exception is when preference shares are in arrears after the non-payment of a dividend. In the US market if more than four dividends have not been paid, which in most cases would be over a two-year period, preference shares are allotted voting rights, sometimes just to be used in the election or re-election of directors. Where this provision is included in the terms of the share issue, they are known as *contingent voting stock*, because the right to vote is contingent on the preference shareholders not receiving the dividends to which they are entitled. When a dividend arrear(s) has been paid off, the voting rights will cease. Such provisions are common, sometimes because of regulatory requirements. The New York Stock Exchange for example, states that non-voting preferred stock must be contingent voting stock otherwise it cannot be listed on the exchange.

The other type of voting power that is often carried by preference shares relates to specific corporate actions that may affect the standing or value of the shares themselves. For instance preference shareholders may be entitled to vote on proposals to increase the authorised amount of any class of stock that would rank ahead of the preference shares, in terms of dividends or rights to assets in the event of a liquidation. The shareholders may also be entitled to give their approval on a merger or consolidation, the results of which might adversely affect the rights and preferences of the preference shares. Shares that have this privilege written into their terms are known as *vetoing stock*. However this veto power is usually available only if the preference shares are in dividend arrears, and will be removed if the arrears are paid off.

23.2.4 Types of preference shares

A "straight" or conventional preference share has the following characteristics:

- interest is paid at a fixed annual rate, but only if profits are sufficiently high for the company to afford the dividend;
- when any due dividend is unpaid, the shareholders do not have the right to payment in arrears at a later date;
- the share is *perpetual*, that is it is irredeemable or "permanent".

In fact the majority of preference shares both in the US and the UK are not conventional, rather they incorporate one or more of the following features; they may be *cumulative*, *participating*, *redeemable* or *convertible*. Preference shares may have one or more, or all, of these features.

- **Cumulative preference shares** entitle their holder to a fixed rate of dividend, and if any dividend is not paid on the due date, the arrears remains payable and will accumulate. The preference shareholders must receive their arrears of dividend before any ordinary share dividend can be paid to equity shareholders. This provision is a significant restriction to the management of a company and would result in the ordinary shares being marked down; it is rare for such shares not to receive a dividend unless the company is in serious financial difficulty. Certain cumulative preference shares impose other restrictions in the event that a dividend is not paid. For example the issuer may not be able to redeem any stock that ranks below the preference shares. Shares that have a sinking fund attached to them may have their sinking payments suspended, with no funds being allowed to redeem preferred or ordinary shares.

- **Participating preference shares** have additional dividend rights. The holders of participating preference shares, in addition to receiving their fixed dividend, are also entitled to a share of the company's surplus profits. This extra *participating* dividend is usually set at a specified percentage of the dividend paid on the ordinary shares. In the US market virtually all the preferred stock in the market is non-participating. Holders of such shares will not benefit from the company posting ever-increasing after-tax profits after every year, except indirectly in that the rating of their preference shares will receive a boost.
- **Redeemable preference shares** are shares that either will be redeemed at a specified future date, or could be redeemed at a specified future date at the option of the company or the preference shareholders. The shares are redeemed at par or at a premium to par value. In the US market preferred stock with no redemption provisions are rare, and are true perpetual securities. Most issues are callable during a time period after their initial issue, in whole or in part, at the option of the issuer and at pre-specified prices. The redemption value must include accrued dividend interest at the time of the call. The point at which the preferred stock is callable is often a medium- to long-term after issue, and may not be for 10 or 20 years after the issue date. Callable issues are often protected from being called for a set period after the issue of debt or equity that ranks with or ahead of the preferred shares. This is to protect shareholders from capital loss arising from a new issue of stock, when the price would be expected to fall and the company could then cash in by redeeming the stock.
- **Convertible preference shares** entitle their holder to a right to convert the preference shares into ordinary shares of the company at a specified future date or between specified future dates, and at a specified rate of conversion. Convertible preference shares are usually redeemable; if they are not converted into equity they will eventually be redeemed. They are similar in many respects to convertible bonds, although the method of quoting prices differs according to which market they are traded in. In domestic markets the shares the convention is to quote them in the same manner as ordinary shares; in international markets (Euromarkets) they are quoted in a similar way to bonds, that is as a percentage of nominal or par value.

EXAMPLE 23.2

- A company with £10 million 6% cumulative preference shares of £1 nominal in issue was unable to pay the dividend on the last payment date, to both preference and ordinary shareholders. It would now like to resume dividend payments in the current year, beginning with an interim dividend. The company in fact is not able to pay an interim dividend to ordinary shareholders until the preference shareholders have been paid their dividend. This is £900,000, comprised of the arrears of the last dividend (6% of £10 million) and the interim dividend of half this amount (£300,000). If the preference shares had been non-cumulative, their holders would not be entitled to the arrears of the dividend and in the current year they would receive only the interim dividend of £300,000.

Another instrument that is an example of the close similarities between convertible bonds and convertible preference shares are *convertible capital bonds*. These were first issued in the euroconvertibles market in 1989 by J. Sainsbury plc, a UK supermarket group. The bonds can be converted into redeemable convertible preference shares at the option of the company.

It is common for companies to have many classes of preference share in issue at any one time. The terms of each class will have been determined separately at the time of their respective issuance.

EXAMPLE 23.3 (i)

- The Rank Organisation plc, a UK leisure group, has an issue of 8¼% convertible redeemable preference shares of 20p each, convertible in any year from 1993–2003 into ordinary shares of 25p nominal value. The conversion rate is 10.6383 ordinary shares for every 100 of the preference shares. The company also has a call option and can redeem the shares at £1 per share at any time after 30 April 2003, and the shares will be redeemed in any event at £1 per share on 31 July 2007. The shares are listed on the London Stock Exchange are priced in the same way as ordinary shares, that is in pence per share.

23.3 (ii)

- Tate & Lyle plc has convertible cumulative preference shares in issue. They are convertible in any year up to 2008 into ordinary shares at a conversion ratio of 11.299 ordinary shares for every 25 of the preference shares. The preference shares have a nominal value of 12.5p each and are redeemable at £1 per share on the maturity date of 28 February 2013, but at the issuer's option at any time from 1 September 2008.

23.3 (iii)

- Thorn EMI (another UK leisure group) issued redeemable convertible preference shares in the Euromarket in 1989. The shares are convertible into the company's ordinary shares and had a maturity of ten years. Although the bonds were described as preference shares they had most of the characteristics of convertible bonds, and in fact were referred to as "bonds" by most market analysts. The price of the shares was quoted as an amount per cent, for example 129 per 100 nominal. The bonds offered a put option that entitled holders to redeem them at 130.22% on 2 February 1994 and 180.64% on 2 February 1999, when they matured. The bonds were also callable.

23.2.5 Sinking funds

A *sinking fund* provision in a preference share issue will operate in a similar fashion to a sinking fund with a bond. It will provide for the periodic redemption of a proportion of the issue, most commonly on an annual basis. For example, the sinking fund may pay off 5% of the original number of shares each year. Preference shares that are callable and also have sinking funds may set different dates when the two features are applicable, so that the sinking fund may come into operation after the stock ceases to be callable. Sinking fund payments may be made in shares of stock purchased in the open market or by the call of the required number of shares at the sinking fund call price, which is normally par or another stated value. As with the payment of dividends, the failure to make a sinking fund payment is not considered a default, unlike with a bond, and the company could not be placed in bankruptcy.

Certain preferred issues have *purchase funds*. These are essentially optional on the part of the issuer because it will have to use its best efforts to retire a portion of the shares at periodic intervals if they can be purchased in the open market, or otherwise through a tender, at below the redemption price. If the shares are trading above the purchase price, the purchase fund cannot operate. A purchase fund can act as a floor for the price of the shares, at a time of high dividend yields, but would not operate in a low yield environment.

23.3 Cost of preference share capital

The gross yield that investors expect from preference shares and bonds of the same issuing company was historically the same, although liquidity considerations meant that the yield on preference shares would be slightly higher. The yield on preference shares in the UK market is now considerably higher than in the bond market, as a result of tax changes introduced by the Labour government in 1997.² This has resulted in there being no real advantage to a UK company in raising capital in the preference share market, which may well become illiquid in the near future. For the purposes of analysis, let us consider an historical example that illustrates the considerations involved, which would apply in other markets.

EXAMPLE 23.4

- In March 1993 General Accident plc, an UK insurance group that subsequently merged with another insurer (Commercial Union) and is now known as CGU plc,³ issued 110 million 7.875% cumulative irredeemable preference shares of £1 each at an issue price of 100.749p per share. The tax credit on UK dividends at that time was 25%, or 25/75 of the net cash dividend. The rate of interest payable on long-dated corporate bonds was then approximately 10.50%. What is the cost of the capital?

The dividend if 7.875% is net of tax (at that time). The gross dividend is treated as an advance payment of corporation tax by the company, and the annual profit that the company needs to cover the dividend payments is £8,662,500, obtained by multiplying the nominal value of the shares and multiplying this by the coupon. If the company pays corporation tax at a rate of say 33%, a bond issuing paying a gross yield of 10.50% would have an

² The legislation did not take effect until the following year.

³ It subsequently purchased Norwich Union plc and is now CGNU plc.

after-tax cost of approximately 7%, obtained by multiplying the coupon rate of 10.50% by $(100 - 33)\%$. The higher cost of the preference shares therefore might be attributable to the premium required for holding them because they are irredeemable.

Companies that are regarded as higher risk will pay a higher yield on their preference shares compared to their bonds. The yield on preference shares is usually shown as a current yield or, in the case of redeemable shares, a redemption yield. Preference shares that have an embedded option feature may be analysed using a yield-to-call or yield-to-put. The yield calculation uses an actual/360 or 30/360 basis in the US market and an actual/365 basis in the UK market. It is important to remember that the yield calculation is in effect a net yield, so that the value must be “grossed up” if returns are being compared to those of bond instruments. This is because the convention is to quote gross yields for bonds.

23.4 The preference share market

In the UK domestic market preference shares are traded in the same way as ordinary shares that is, they are listed on an exchange. Trading is on the exchange via its dealing system. Domestic issues of preference shares can be relatively small size, and they are often issued as part of a company financial re-structuring or through a private placement to a small group of investors. In 1996 the Bank of Ireland issued conventional preference shares to raise part of the financing required in its take-over of the Bristol & West building society. The preference shares were also used to pay some of the membership of the building society, who were entitled to a share of the society’s reserves in return for giving up ownership of it. In the US under half of all publicly issued preference shares are listed on the NYSE or the American Stock Exchange, with the remainder trading in the OTC market. The normal unit of trading is 100 shares but this is not universal and some issue trade in lots of 10 shares. It is possible to deal in odd-lots of shares. Listing an issue may lead to a lower dividend yield for an issuer if it improves its marketability. This is more significant if the issue itself is a relatively large size one. Dealing via an exchange system also makes an issue more transparent in the market.

In international markets companies usually issue redeemable convertible preference shares, which are regarded as part of the Euroconvertibles market, together with convertible bonds and equity warrant bonds.

23.5 Auction market preferred stock (Amps)

Auction market preferred stock (Amps) is a particular type of preference share. Its main distinguishing feature is that the dividend payable on the shares is determined by auction at regular intervals. The Amps market was developed in the US and the instrument has been issued by a number of foreign borrowers in the domestic US market.

Amps are placed through an investment bank or securities house that acts as lead manager for the issue. They are sold to investors through the auction process. They cannot be offered for sale to the general public, nor may they be traded on a stock exchange. A panel of investors is invited to participate in the auction, and they submit bids for the dividend rate that they will accept in return for purchasing a quantity of the stock. The stock is sold at a fixed price to the investors bidding at the lowest dividend level.

EXAMPLE 23.5

- A US company is issuing Amps and a group of investment institutions is invited to bid for the stock. The size of the issue is \$100 million. Bids are received for \$60 million at a dividend yield of 6%, \$90 million at a yield of $6\frac{1}{4}\%$ and \$150 million at a yield of $6\frac{1}{2}\%$. The shares will be sold to the investors bidding the lowest dividend yield, which is paid on the entire issue. Given the ratio of the bidding amount, the yield payable in our simple example is $6\frac{1}{4}\%$. The investors who bid 6% will receive the full quantity of stock they bid for, on a yield of $6\frac{1}{4}\%$, while the remaining \$40 million will be allotted to investors bidding at this level on a pro-rate basis. The stock is issued at par.

Auctions occur at regular intervals, sometimes as frequently as every four weeks. Investors wishing to buy Amps at an auction may submit a bid to receive a dividend yield of their choice; at subsequent auctions existing holders may sell part or all of their holding to bidders. After the primary issue existing holders can opt to hold their stock,

bid for more of the shares or sell to another bidder. When all orders have been submitted an “auction agent” is responsible for deciding the allocation.

The number of Amps for which bid orders have been received is compared with the volume of Amps available. If the volume of orders exceeds the quantity of stock available the dividend rate is set at the lowest rate at which all the Amps will be taken up, and holders will subsequently receive this dividend level. At this point Amps must be sold by existing holders who bid at a higher rate to successful new purchasers who bid at a lower rate. At each auction there is a maximum rate of dividend that will be paid on the stock. This level is a function of the credit rating of the stock at the time of the auction and is determined by a pre-set formula. If the credit rating is downgraded, the maximum rate payable will be raised, and this new rate of dividend is used as a default rate. The dividend rate payable until the next auction, which must not be held for a minimum of 28 days, is the default rate. There may be an insufficient number of bidders in an auction for the quantity of Amps that are available below the maximum (default) rate. When this happens, existing holders who wish to sell will be unable to do so.

EXAMPLE 23.6

- ABC Inc, a US company, has an issue of \$500 million of Amps. In an auction bids are submitted for \$300 million of stock at yields ranging from $6\frac{1}{2}\%$ to 7% and there are hold orders for \$150 million. The maximum rate is 7%. Therefore there are insufficient bids for the stock. Bidders for \$300 million will be able to buy the stock, and existing holders of \$150 million of stock will retain their holding. Holders of a further \$50 million who wished to sell their stock will be unable to do so. They must hold their paper until at least the next auction. The dividend rate payable to the next auction is the default rate of 7%.

Dividends on Amps are paid at pre-determined fixed intervals and sometimes match the auction cycle. For example British Aerospace (now known as BAE Holdings) Amps pay a dividend every 49 days. Although they are known as “preferred stock” Amps are similar in many respects to bonds. The dividend on Amps is usually around 100 basis points above the yield on straight short-term fixed debt, but the advantages are that there are no covenants of any kind attached to Amps issues and they are redeemable only at the issuer’s option. The cost of Amps to the issuer will increase however if its credit rating is downgraded. The default rate of dividend that applies if an auction is unsuccessful could be raised at the next auction.

Issues of Amps vary in size, although in the mid-1990s UK company issues in the US domestic market ranged in size from \$100 million to \$700 million.

Amps are only redeemable at the option of the issuer and in theory therefore are perpetual. They are issued without any form of covenant and a failure to pay a dividend on an Amps does not constitute a default (as is the case for all preference shares). Holders of Amps are not able to demand redemption in the event of non-payment of a dividend. The inability of shareholders to demand redemption was used at one point to argue that Amps should be regarded as equity finance. This was not accepted by the UK authorities. Amps may not be “perpetual” finance, as a company, having issued Amps could find that its credit rating was downgraded. The dividend that would be payable would then rise at subsequent auctions, and if the downgrade were sufficiently great, the cost of paying dividends on the Amps might then become so high as to cause the issuer to redeem them. In the UK in the early 1990s rising dividend costs on Amps made redemption an attractive option. For example BET plc announced a £200 million rights issue in July 1992 in order to finance a redemption of most of its outstanding Amps. The cost of the dividend payment on the Amps had risen from a level just below the US commercial paper rate in 1991 to a premium of over 21% above CP rates in mid-1992. The extract below from the *Financial Times* for 15 July 1992 suggested that domestic US investors were not sufficiently familiar with the name to believe it was worthwhile holding the paper without substantial rise in the dividend yield to compensate for whatever trouble they thought the company was getting into.

“The history of suspicion and misunderstanding of these instruments in the UK may seem odd to a US observer. But the market for Amps consists wholly of US investors who are interested primarily in big, familiar, triple-A US corporations like Coca-Cola and Exxon, rather than unknown foreign minnows.”

(FT, 15 July 1992)

During this period other UK companies also redeemed their Amps issues. For example Rank Organisation plc redeemed its \$200 million Amps issue in April 1991 after its credit rating was downgraded from A- to BBB- by Standard & Poors.

A lingering death in the corporate debt market

Richard Miles, *The Times*, 15 September 1999

© *The Times*, © News International, 15 September 1999. Reproduced with permission.

The investment community could soon be mourning the death of the £5 billion market in preference shares, a form of corporate debt that was highly popular in the Seventies. Fund managers and marketmakers say the trade in preference shares has all but stopped. Many believe “prefs” will be allowed to wither on the vine, as issuing companies and investors switch to more tax-efficient types of debt. Prefs differ from corporate bonds in that the income is distributed in the form of a dividend rather than interest. Should the issuer collapse, prefs rank below other classes of secured and unsecured debt, but ahead of ordinary stock. Some preference shares have limited voting rights.

The reasons for the market’s decline are multiple. Changes to company taxes and the rapid emergence of a market in sterling-denominated corporate bonds have both played their part. In addition, problems of illiquidity have been compounded by the generally depressed state of the UK fixed-interest market. Without doubt, the biggest blow to prefs came with the Chancellor of the Exchequer’s decision to abolish the dividend tax credit when he reformed advance corporation tax in his maiden Budget in 1997. Overnight, the value of preference shares dropped 20 per cent for one of the stock’s biggest group of buyers: gross investors. Prefs thus lost much of their appeal for pension funds which criticised the ACT changes as a Maxwell-style raid on the savings of the elderly. Retail investors, however, were given a stay of execution: they can still obtain a dividend credit of 10 per cent for the next five years so long as they hold the stock through a personal equity plan (Pep) or individual savings account (Isa). The ACT changes triggered a gradual move to corporate bonds, a market now conservatively valued at £150 billion in the UK. Many managers of unit trusts have already adjusted the balance of their portfolios to reflect this.

“We have not bought prefs for two years,” said Ian Dickson, manager of the Henderson Preference & Bond unit trust, which at £228 million is one of the biggest retail funds in the sector. “Preference shares were the great story of the late 1970s, but they no longer stack up against corporate bonds. You are not getting paid for the additional risk.” Mr Dickson said that his fund was 80 per cent invested in prefs just a couple of years ago. Today, that weighting has shrunk to 20 per cent. By way of illustration, Mr Dickson points to the yield on the preference shares of Bank of Scotland, one of the bigger issuers in this market. The yield is 6.8% with the 10 per cent dividend tax credit, falling to 6 per cent for a net investor. The bank’s own long-dated bonds pay 6.6 per cent, effectively on a net basis.

Issuers have also begun to snub the market. Hugh Everitt, director of corporate bonds at CGU, the insurance and fund management group, said the number of issues on the market was 400 two years ago. Today, there are only 250. A two-tier market has evolved, with only the larger issuing companies – typically financial services companies – offering reasonable levels of liquidity to investors. The last issue of any significance was when Halifax paid £750 million for Birmingham Midshires Building Society earlier this year. Members received a package of preference shares, although most have cashed in the stock already.

The initial contraction of the market was triggered by the desire of non-financial issuers to buy back their preference shares in an exercise aimed at tidying up their balance sheets. Philip Roantree, manager of the £40 million Aberdeen Sterling Bond unit trust (formerly the Proficil Preferred and Fixed Interest Fund), said there had been 49 such buybacks in the past 12 months. A typical example is Burmah Castrol, the oil company, which last year repurchased all four classes of its preference shares at a slight premium. Par value for the shares totalled £19 million in 1998, a tiny sum when compared with the group’s total net assets of £874 million.

How quickly the market expires depends to a large extent on the action of the financial companies. Changes to international banking laws mean that banks can now raise money elsewhere in the debt markets with greater tax efficiency. Should they start buying back their prefs, then the collapse could be rapid because they represent 40 per cent of the market.

All eyes are on Abbey National, a leading player in this market. Mr Everitt said the bank had already obtained permission from shareholders to buy back its preference shares. The feeling is that, if Abbey National moves to do so, then the other financial companies will quickly fall into line, effectively wiping out prefs. Any remaining stock would be highly illiquid.

It is not all doom and gloom for pref investors. Given the tight holding of the stock by fund managers, any issuer wanting to buy back stock is expected to pay a premium. This partly explains the lack of trade in the market over the past year or so: investors are holding on to stock in the hope of a capital gain.

Sue Whitbread, an associate director of Chartwell, an independent investment adviser, said small investors should welcome the prospect of buybacks after a year of losses. Other advisers have also urged their clients to hold on to their prefs in expectation of such a windfall.

It is possible that the prefs market might stage a dramatic recovery, but the odds are against such a comeback. Certainly, that is the feeling among the buyers of prefs. In pursuit of higher yields, fund managers are now looking to the junk bond market. Stalwarts of the retail market such as M&G and Fidelity are now packing their portfolios with sub-investment grade bonds. Whether private investors fully understand the risks involved is a moot point – and another story.

Selected bibliography

Brealey, R., Myers, S., *Principles of Corporate Finance*, McGraw-Hill, 1996.

Finnerty, J.D., “Financial Engineering in Corporate Finance: An Overview”, *Financial Management* 17, Winter 1988, pp. 14–31.

Ross, S., Westerfield, R., Jaffe, J., *Corporate Finance*, Irwin/McGraw-Hill, 1998.

Wilson, R., “Nonconvertible Preferred Stock”, in Fabozzi, F., *The Handbook of Fixed Income Securities*, 5th edition, McGraw-Hill, 1997, Chapter 14, pp. 263–286.

24 The US Municipal Bond Market

Municipal bonds in the United States are securities issued by state and local governments as well as institutions created by them. Until recently the municipal bond market was viewed as being akin to the US Treasury market, ranking only slightly behind Treasuries in terms of credit quality. Although in theory this is an accurate view, investors nowadays view municipal securities as more similar to corporate bonds, with significantly higher credit risk than Treasury bonds. This is considered to be case irrespective of whether individual bonds are rated at investment-grade or not. Since municipal bonds trade at yields that are closer to corporate bonds than Treasury bonds, they are reviewed here in the section on corporate debt markets.¹ The principal difference between municipal bonds and Treasury bonds, aside from the credit considerations, is that municipal bonds are tax-exempt, that is interest is exempt from federal income taxation. The bonds may not necessarily be exempt from state or local income taxes, however as Treasuries are liable to federal taxes, municipal bonds are known as tax-exempt securities. However although there are both tax-exempt and taxable municipal bonds, the sector is sometimes referred to as the tax-exempt market. More than 50,000 entities have issued municipal bonds in the United States. They are issued for a variety of reasons, along the entire length of the yield curve. Short-dated paper is often issued to cover local authority cash flows, ahead of the receipt of taxation proceeds. Long-dated paper is issued for capital and other projects, for example to assist in the financing of say, schools, hospitals, and communication and infrastructure facilities such as airports.

The exemption from federal taxation makes municipal bonds particularly attractive to individual retail investors, approximately 75% of all outstanding paper is held by individuals. The remaining bonds are held by banks, mutual funds and property companies.² Banks are attracted to municipal bonds for various reasons; they include the tax advantage but also for liquidity purposes³, for which municipal bonds are often preferred to Treasury bonds. Municipal bonds are eligible to be used as collateral when commercial banks borrow funds from the Federal Reserve. Additionally banks may serve as underwriters of municipal bond issues and will frequently emerge from such activity holding an inventory of the paper. Property companies are attracted to municipal bonds because of the tax advantages and also because their profits follow a highly cyclical pattern; municipal bonds are purchased during times of high profit, to lower their income tax liability.

24.1 Description of municipal bonds

The municipal bond in the United States is essentially a plain vanilla bond market. The majority of municipal securities are conventional fixed interest bonds; there are also a considerable number of floating-rate securities. Floating-rate municipal bonds will pay a spread over a specified reference interest rate. The reference rate may be a Treasury rate, the prime rate, or a municipal index such as the Municipal Bond Buyer index or the Merrill Lynch Index. The municipal market has also seen the issue of inverse floating-rate bonds, whose coupon is adjusted in the opposite direction to the move in the reference rate. Municipal zero-coupon bonds are known as *original-issue discount bonds* or *OIDs*. These bonds are similar to government zero-coupon securities, but the return to the investor is exempt from federal tax. There is also an almost unique type of zero-coupon municipal bond known as a *municipal multiplier* or *compound interest bond*. This bond is issued at par and pays interest. However the interest payments themselves are not distributed to the bondholder until the maturity date; in the meantime the issuer reinvests the

¹ The municipal bond markets were also affected by several high profile instances of local authorities getting into financial difficulty, which resulted in the widening of spreads on bonds issued by them. These included the financial crisis in the City of New York in 1975, as well as the default of Orange County, California twenty years later. In addition to this, federal bankruptcy law effective from 1979 makes it more straightforward for municipal bond issuers to seek protection from bondholders by filing for bankruptcy.

² Feldstein /Fabozzi 1997.

³ By liquidity we mean funding liquidity and not trading liquidity. The US term is that banks are required to be “collateralised”, that is deposits must be backed with short-dated high quality assets. Generally banking liquidity books invest in T-Bills, clearing banks Certificates of Deposit, short-dated government bonds and local authority bonds.

undistributed coupon payments at the redemption yield level at which the bond was issued. Essentially this guarantees the investor a fixed interest rate on her coupons for the life of the bond, so in effect the redemption yield becomes a true redemption yield, unlike that quoted for a conventional bond. The maturity value of such bonds are calculated using standard present value techniques, as illustrated in Example 24.1.

EXAMPLE 24.1

- A 6.00% 20-year municipal multiplier bond is issued at par. What is the value of the bond on maturity?

The bond is sold at 100 so that the redemption yield is 6.00%. The market has a semi-annual coupon convention, so the maturity value is given by the future value of the stream of coupon payments. This is:

$$100 \times (1.03)^{40} = \$326.21.$$

- If the issuing authority wishes to place 20-year bonds at a yield of 6.00% with a maturity value of 100, at what price must they be sold? This is a standard present-value calculation and is given by:

$$100 \times \frac{1}{(1.03)^{40}} = \$30.65568.$$

The maturity date on municipal bonds may be a conventional fixed date, or a dual date. Fixed term bonds have maturity terms of up to 40 years from the date of issue, although there may be a sinking fund that kicks in after the first say, ten years. It is also common for municipal bonds to mature over a number of years, which is known as a serial maturity. Such securities are known as *serial bonds*. Municipal bonds with serial maturities may have as many as 10 or more maturity dates. A serial maturity structure requires a proportion of the outstanding debt to be redeemed each year.

Municipal bonds may be broken down into two basic types, known as *general obligation bonds* and *revenue bonds*. The two types are not mutually exclusive and it is possible to encounter bonds that exhibit features of both types. They are reviewed next.

24.1.1 General obligation bonds

Municipal bonds that are issued by entities such as states, cities, towns and local districts are known as general obligation bonds. The unlimited tax-raising powers of the issuing body serves as the security for the issue of the bonds. This source of revenue is to all intents guaranteed, although amounts will fluctuate with the general health of the economy; however this fixity of income renders municipal bonds a lower credit risk than corporate bonds, at least in theory. A large borrower such as a state or large city will have a wide range of tax-raising sources, including income tax, corporation tax, value-added tax and property tax. Some general obligation bonds are issued without an unlimited tax-raising backing, and instead are secured with a tax base that is limited, such as a form of rates or property tax. The smaller entities such as local districts often issue bonds that are secured by this more limited tax-raising power. It is worth noting the term *double-barrelled security*, which refers to a general obligation bond that is backed with two income sources, the general tax-raising ability and other specific charges or fees, which are a source of additional for the issuing authority.

24.1.2 Revenue bonds

Revenue bonds are issued to finance a specific project or undertaking such as a bridge or road. The bonds are backed with the income that will be received from the running of the completed project itself. The terms of the issue must stipulate that revenues generated from the running of the particular project will be used to pay the interest costs associated with the bond issue. The redemption proceeds may well be financed through another issue of bonds. Revenues from the operation of the enterprise may be ring-fenced in a *revenue fund*, as a further protection for bondholders. Revenue bonds that have been issued include *airport revenue bonds*, *college and university bonds*, *hospital revenue bonds*, *public power revenue bonds*, *seaport revenue bonds*, *student loan revenue bonds*, *toll road revenue bonds* and *water revenue bonds*. The purpose and security of these bonds is evident from their names, except perhaps for the hospital revenue bond. These are issued to finance the construction of a hospital, and the security backing is the government and other funding that is provided to certain types of patient in US hospitals, which include the Medicare and Medicaid programmes as well as private insurance payments from insurance companies.

Note that if the facility or enterprise that is the purpose of a revenue bond is destroyed, say as a result of an act of nature or for any reason, the bond itself must be called by the issuer. This call provision is known as a *catastrophic call*.

24.2 The municipal bond market

The market in municipal securities is large and liquid. Paper is issued by a number of borrowers on a weekly basis. The issue process is similar to that for corporate bonds, and paper may be placed through a public offering to investors, via an investment bank or securities house, or via a private placement. A public offering will be underwritten by a bank or syndicate of banks. Regulation for the issue of a public offer differs slightly from state to state; for example some states have a requirement that all general obligation bonds be placed via a competitive auction.

Bonds are traded in the OTC market. Liquidity is variable and ranges from as liquid as Treasuries for the larger issues to a lower level for smaller issues, which may be supported only by smaller brokers and regional banks. There are a large number of issues that are quite illiquid however, usually due either to their small size or because there is an absence of paper to trade. The staggering of maturity dates also means that individual issues experience relatively thin trading. This consequently results in a wide bid-offer spread, which further discourages liquidity. Details of issue prices and offer sizes are detailed in a publication produced by Standard & Poor's known as *The Blue List*. An interesting feature of the municipal secondary market is that bonds are quoted on a yield basis, as opposed to a price basis.⁴ Generally a redemption yield or yield-to-call is used, and the "price" of a municipal bond is known as a *basis price*. The general formula for the price of a municipal bond is given at Appendix 24.1.

As municipal bonds are exempt from tax, their redemption yield is lower than equivalent-maturity Treasury bonds. The level at which this yield differs from Treasury yields is a function of the rate of income tax; for example the lowering of the marginal rate of tax as a result of the 1986 Tax Act made the tax-exempt nature of municipal bonds less attractive to investors, thus raising their yield. If the return on municipal bonds is compared to other bonds, the yield measure of one security must be adjusted to make the two values comparable. The common approach is to convert the municipal yield to an *equivalent taxable yield*, which is given by (24.1):

$$\text{Equivalent taxable yield} = \frac{\text{Tax-exempt yield}}{(1 - \text{tax rate})}. \quad (24.1)$$

To illustrate, if the marginal tax rate is 30% and a municipal bond is trading at a yield of 7.00%, the equivalent taxable yield is:

$$\text{Equivalent taxable yield} = \frac{0.07}{(1 - 0.30)} \text{ or } 10.00\%.$$

The equivalent taxable yield is not completely suitable for discounted issue municipal bonds or zero-coupon bonds. This is for these types of bonds, only the coupon interest is exempt from federal income tax. The redemption yield on these securities therefore is calculated assuming a tax level payable on the capital gain achieved by the bondholder, and this is used in the numerator of (24.1). The maturity value of the bond is not used to calculate the yield, rather the net proceeds after payment of capital gains tax is used as the redemption amount. Capital gains tax is payable at the investor's marginal rate of income tax.

The yields payable by municipal bonds are a function of the credit quality of the issuer, supply and demand, and differences between the local and general capital markets. The spread over equivalent-maturity Treasury bonds will reflect general economic conditions, so for example during a recession or market downturn spreads will widen, while they will narrow in a healthy economic climate. This is perhaps the best indication that municipal bonds trade in a similar fashion to corporate bonds. A peculiar phenomenon is that the yield on bonds issued by certain states will trade at lower levels than bonds of other states of similar credit rating. This is partly explained by tax treatment of certain state bonds compared to paper issued by other states; for example one state may exempt local holders of its own bonds from state and local income taxes, but not the bonds of other states. A state with a high level of local

⁴ First apparent to the author during a visit to the trading floor at Fidelity Capital Markets in Boston in 1996! Municipal bonds also trade on a 30/360 basis, unlike Treasury securities.

income tax will thus find that there is a higher demand for its paper compared to the paper of other issuers. The local income taxes of states are not uniform, for example the rates in New York are higher than those in Florida.

By far the greatest proportion of municipal bonds are held by retail investors. A bondholder is exposed to credit risk because municipal bonds reflect, like corporate bonds, the credit risk of the issuer. Investors are exposed to an additional risk, which is termed *tax risk*. This is the risk that the level of federal income tax is lowered, which will lower the value of the municipal bond. As is apparent from (24.1) the value of a municipal bond increases (that is, its yield decreases) as the tax rate is increased. A fall in the level of the tax rate will reduce the value of a municipal bond.

24.3 Municipal bonds credit ratings

After a number of high profile incidents when the issuers of municipal bonds found themselves in financial difficulty, thus affecting holders of their debt, the credit ratings assigned to municipal bonds assumed ever greater importance, and today formal ratings are used by investors to assess the credit risk associated with holding municipal debt. The process is essentially similar to the credit analysis of corporate bonds. Securities issued by municipal borrowers are no longer viewed as being risk-free securities, indeed far from it. For example bonds issued by the Washington Public Power Supply System were assigned the highest possible rating – Aaa and AAA – by Moody's and Standard & Poor's in the early 1980s. In 1990 around 25% of the entire issue was in default.⁵ There have been other instances of default since 1975, all of which occurred amongst issuers whose paper had been granted investment-grade ratings. Although institutional investors often employ their own in-house credit analysts, the market pays close attention to the ratings assigned by the formal credit rating agencies, the three most quoted of which are Standard & Poor's, Moody's and FitchIBCA. The ratings used by these agencies for long-dated bonds are similar to the ones they employ for corporate bonds generally (see Chapter 30). The ratings for short-dated instruments are given in the tables below including tax-exempt commercial paper, are shown in Table 24.1.

Rating	Definition
Moody's S&P	
MIG 1 SP-1	Best quality; very strong capacity to pay principal and interest. S&P ratings are given a "+" to indicate very strong safety characteristics
MIG 2 SP-2	High quality
MIG 3	Favourable quality.
MIG 4 SP-3	Adequate quality; S&P rating defined as "speculative" capacity to pay principal and interest
"MIG" denotes Moody's Investment Grade	

Table 24.1: Municipal Note Credit Ratings. Source: Moody's, S&P.

Rating	Definition
Moody's S&P	
Prime 1 (P-1) A-1+	Superior capacity for repayment; highest degree of safety
Prime 2 A-1	Strong capacity for repayment
Prime 3 A-2	Acceptable capacity for repayment
A-3	Satisfactory degree of safety

Table 24.2: Tax-Exempt Commercial Paper Ratings. Source: Moody's, S&P.

Due to the nature of security backing for municipal bonds, undertaking credit analysis on them, although following a similar process to that for corporate bonds requires a different approach. The similarities include collating data on the issuer's debt structure, to determine the overall level of the debt burden. Additional analysis is carried out on the political leadership of the borrowing entity, including its capacity and reputation for maintaining budgetary discipline. Generally the credit rating agencies will look for a balanced budget over the last five years or

⁵ Feldstein/Fabozzi 1997.

so. Credit analysts will also consider the specific taxes and revenues that are available to the borrower, and the scope of its tax-raising power. There is a final issue to consider which is more qualitative in nature, regarding the general social and economic conditions in the state or local area as a whole. This includes the level of unemployment, incomes, population and the expected impact of these issues on the economic health of the area.

24.4 Bond insurance

Certain issuers of municipal bonds set up insurance cover for their bonds, as cover in the event of default. Under a bond insurance policy, an insurer agrees to service the debt on a bond when it is not paid by the issuer. Municipal bond insurance agreements are means by which, in return for the payment of a premium, the credit risk exposure of a bond holding may be reduced for investors. In the event of default, the principal will be paid to the investor by the insurance company. The insurance contract may be set up on purchase of the bond, and have a duration up to the maturity date. In 1995 approximately 25% of all new municipal bond issues were insured.⁶

Bonds whose issuers have a low standing in the market benefit most from bond insurance. A borrower will be able to increase the marketability of its bond issue if it obtains bond insurance. In order for the insurance contract to be viable however, the saving in interest costs to the issuer must exceed the premium payable to the insurer for arranging the insurance contract. Therefore it is only low-rated borrowers that have an incentive to arrange bond insurance. It has been observed that although insured municipal bonds trade at yield levels that are lower than they would otherwise be without the insurance, they generally exhibit yields that are higher than highly-rated bonds such as deep-discounted bonds. There are specialised firms that operate in bond insurance, the best known of which are the Financial Guaranty Insurance Company (FGIC), the Financial Security Assurance, Inc (FSA), the Municipal Bond Investors Assurance Corporation (MBIA Corp.) and Connie Lee Insurance Company.

24.5 Taxation issues

The key tax consideration for investors in municipal bonds is the rate of federal income tax. This is not the sole interest for investors however, and state and local taxes and the tax treatment of interest expenses are also significant.

We noted early in this chapter that there are differences in the tax treatment of municipal bonds, and this treatment differs from state to state. An individual state may impose income tax on coupon income, capital gains tax on any capital gain and also a personal property tax. Most states do indeed charge income tax, and while certain states may exempt coupon income earned on any municipal bond, other states exempt only the bonds that they themselves issued. In certain states coupon income on all municipal bonds is liable to income tax. The treatment of capital gains is similarly not uniform, but in most cases where coupon income is exempted from income tax, any capital gain is still taxable. The term “personal property tax” as levied in certain states is in effect another income tax.

Institutional investors often borrow funds to purchase assets that they wish to invest in. This is also common amongst bond traders and dealers, and the gain to the investor is the extra yield they receive on their assets compared to the interest cost on their borrowed funds. In the course of ordinary business, the interest payable on borrowed funds that are used to buy securities is tax deductible. This does not apply if the funds are used to purchase tax-exempt securities, a result of a specific ruling by the US Internal Revenue Service (IRS). The same rule applies to banks that invest in municipal bonds, unless the bonds are *bank qualified issues*. A bank qualified issue is defined as one that is a tax-exempt issue but not one issued for private activities, and is designated by the issuer as so qualified. There are also limitation on the amount of paper that may be issued.

24.6 Exotic municipal bonds

There are a range of municipal bonds with special characteristics, which are known as *hybrid bonds*. There have also been issues linked to derivative products. A common special feature bond is a *refunded* bond. This is a bond that originally was issued as a conventional general obligation or revenue security, but is now secured by an *escrow fund*, which holds direct federal government obligations. A municipal borrower may wish to retire an issue using an escrow fund for a number of reasons. The main one is where an original revenue bond was issued with a range of

⁶ Feldstein /Fabozzi 1997.

restrictive covenants, which the borrower now wishes to remove. Setting up an escrow fund enables the issuer to remove these covenants without breaching any legal considerations.

The municipal market was one of the first to introduce structures that combined a floating-rate note with an *inverse floating-rate note*. This combination is created from an original issue fixed-rate bond, with the total issue size being split into the FRN and the inverse floater. The rate on the FRN moves in line with a reference interest rate index, while the rate on the inverse floater moves in an opposite direction to the reference index. The net interest payment from the two newly created bonds must equal the fixed coupon rate that was paid by the original bond, which is known as the *collateral*. In order to achieve this, a cap is usually set on the FRN, while the inverse floater note will have a floor, usually zero, to prevent its interest rate moving to a negative value.

Investment banks have introduced proprietary derivative products in the municipal market. For example the inverse floating rate notes developed by Merrill Lynch are known as RITES, from Residual Interest Tax Exempt Securities, while those issued by Lehman Brothers are known as RIBS (Residual Interest Bonds).

24.7 Municipal money market instruments

Short-dated instruments in the municipal market are known as notes. There are also tax-exempt commercial paper, and variable-rate obligations which are similar to floating-rate notes. Notes in the municipal market are given special names, for example there are *revenue anticipation notes* (RANs), *tax anticipation notes* (TANs), *grant anticipation notes* (GANs) and *bond anticipation notes* (BANs). They are similar to discount instruments in the money markets, and are often issued as short-term borrowings to be redeemed after receipt of tax or other proceeds. Essentially the notes are issued to provide working capital, because the receipt of cash flows from taxation and other local government sources is irregular. The typical maturity of a note is three months, while the longest maturity is 12 months. In most cases tax-exempt notes are issued with credit backing in the form of a bank letter of credit, a bond insurance policy or a lending line at a bank.

Municipal borrowers issue commercial paper, which is similar to corporate CP and may have a maturity ranging from 1 to 270 days. It is known as tax-exempt commercial paper.

Another money market instrument is the *variable-rate demand obligation* (VRDO). This is a floating-rate security that has a long-dated maturity but has a coupon that is re-set at the very short-dated interest rate, either the overnight rate or the seven-day rate. The securities are issued with a put feature that entitles the bondholder to put the issue back to the borrower at any time, upon giving seven days' notice. The bonds may be out to the issuer at par.

Appendices

APPENDIX 24.1 The price of a municipal bond

The price of municipal bond may be expressed as a function of the term structure, assuming that there is no default risk associated with the bond. For a discounted bond, expressed in terms of an annuity A and a zero-coupon bond of equivalent maturity of n years with a discount factor for that term of Df_n , the price is given by (24.2):

$$P_M = \frac{CA + 100(1 - t)Df_n}{1 - tDf_n}. \quad (24.2)$$

For bonds redeemable at par the price is given by (24.3):

$$P_M = CA + 100Df_n. \quad (24.3)$$

Although they are exempt from income tax, discounted bonds are liable to capital gains tax on maturity, at the income tax level of t . There is no tax liability for conventional coupon bonds.

Selected bibliography and references

- Fabozzi, F., Feldtsein, S., Pollack, I. (eds.), *The Municipal Bond Handbook: Volume I*, Dow-Jones Irwin, 1983.
- Fabozzi, F., Feldtsein, S., Pollack, I. (eds.), *The Municipal Bond Handbook: Volume II*, Dow-Jones Irwin, 1983.
- Feldstein, S., Fabozzi, F., "Municipal Bonds", in Fabozzi, F., *The Handbook of Fixed Income Securities*, 5th edition, McGraw-Hill, 1997.
- Fortune, P., "The Municipal Bond Market, Parts I and II", Federal Reserve Bank of Boston, *New England Economic Review*, September–October 1991, pp. 13–36 and May–June 1992, pp. 47–64.

- Heaton H., “The Relative Yields on Taxable and Tax-Exempt Debt”, *Journal of Money Credit and Banking* 18, 1986, pp. 482–494.
- Livingstone, M., “The Pricing of Municipal Bonds”, *Journal of Financial and Quantitative Analysis* 17, June 1982, pp. 179–193.
- Zipf, R., *How Municipal Bonds Work*, New York Institute of Finance, 1995.

25

Asset-Backed Bonds I: Mortgage-backed Securities

In Chapter 5 we introduced *asset-backed bonds*, debt instruments created from a package of loan assets on which interest is payable, usually on a floating basis. The asset-backed market was developed in the United States and is a large, diverse market containing a wide variety of instruments. The characteristics of asset-backed securities (ABS) present additional features in their analysis, which are investigated in this and the next two chapters. Financial engineering techniques employed by investment banks today enable an entity to create a bond structure from any type of cash flow; the typical forms are high volume loans such as residential mortgages, car loans, and credit card loans. The loans form assets on a bank or finance house balance sheet, which are packaged together and used as backing for an issue of bonds. The interest payments on the original loans form the cash flows used to service the new bond issue.

In this chapter we consider *mortgage-backed securities*, the largest of the asset-backed bond markets. The remaining chapters deal with the other asset-backed instruments available.¹

25.1 Mortgage-backed securities

A mortgage is a loan made for the purpose of purchasing property, which in turn is used as the security for the loan itself. It is defined as a debt instrument giving conditional ownership of an asset, and secured by the asset that is being financed. The borrower provides the lender a mortgage in exchange for the right to use the property during the term of the mortgage, and agrees to make regular payments of both principal and interest. The mortgage lien is the security for the lender, and is removed when the debt is paid off. A mortgage may involve residential property or commercial property and is a long term debt, normally 25 to 30 years; however it can be drawn up for shorter periods if required by the borrower. If the borrower or *mortgagor* defaults on the interest payments, the lender or *mortgagee* has the right to take over the property and recover the debt from the proceeds of selling the property. Mortgages can be either fixed-rate or floating-rate interest. Although in the US mortgages are generally amortising loans, known as *repayment* mortgages in the UK, there are also *interest only* mortgages where the borrower only pays the interest on the loan; on maturity the original loan amount is paid off by the proceeds of a maturing investment contract taken out at the same time as the mortgage. These are known as *endowment* mortgages and are popular in the UK market, although their popularity has been waning in recent years.

A lending institution may have many hundreds of thousands of individual residential and commercial mortgages on its book. If the total loan book is pooled together and used as collateral for the issue of a bond, the resulting instrument is a *mortgage-backed security*. This process is known as *securitisation*, which is the pooling of loan assets in order to use them as collateral for a bond issue. Sometimes a *special purpose vehicle* (SPV) is set up specifically to serve as the entity representing the pooled assets. This is done for administrative reasons and also sometimes to enhance the credit rating that may be assigned to the bonds. In the UK some SPVs have a triple-A credit rating, although the majority of SPVs are below this rating, while retaining investment grade status. In the US market certain mortgage-backed securities are backed, either implicitly or explicitly, by the government, in which case they trade essentially as risk-free instruments and are not rated by the credit agencies. In the US a government agency, the Government National Mortgage Association (GNMA, known as “Ginnie Mae”) and two government-sponsored agencies, the Federal Home Loan Corporation and the Federal National Mortgage Association (“Freddie Mac” and “Fannie Mae” respectively), purchase mortgages for the purpose of pooling them and holding them in their portfolios; they may then be securitised. Bonds that are not issued by government agencies are rated in the

¹ Many texts place mortgage-backed securities in a separate category, distinct from asset-backed bonds (for example, see the highly recommended Fabozzi, F., *Handbook of Structured Finance Products*, 1998). Market practitioners also tend to make this distinction. Generally, the market is viewed as being composed of mortgage-backed securities and asset-backed securities (which encompass all other asset types).

same way as other corporate bonds. On the other hand non-government agencies sometimes obtain mortgage insurance for their issue, in order to boost its credit quality. When this happens the credit rating of the mortgage insurer becomes an important factor in the credit standing of the bond issue.

25.1.1 Growth of the market

The mortgage-backed market in the US is the largest in the world and witnessed phenomenal growth in the early 1990s. One estimate put the size of the total market at around \$1.8 trillion at the end of 1995, with around \$400 billion issued in that year alone.² The same study suggested the following advantages of mortgage-backed bonds:

- although many mortgage bonds represent comparatively high quality assets and are collateralised instruments, the yields on them are usually higher than corporate bonds of the same credit quality. This is because of the complexity of the instruments and the uncertain nature of the mortgage cash flows. In the mid-1990s mortgage-backed bonds traded at yields of around 100–200 basis points above Treasury bonds;
- the wide range of products offers investors a choice of maturities, cash flows and security to suit individual requirements;
- agency mortgage-backed bonds are implicitly backed by the government and therefore represent a better credit risk than triple-A rated corporate bonds; the credit ratings for non-agency bonds is often triple-A or double-A rated;
- the size of the market means that it is very liquid, with agency mortgage-backed bonds having the same liquidity as Treasury bonds;
- the monthly coupon frequency of mortgage-backed bonds make them an attractive instrument for investors who require frequent income payments; this feature is not available for most other bond market instruments.

In the UK the asset-backed market has also witnessed rapid growth, and many issues are triple-A rated because issuers create a *special purpose vehicle* that is responsible for the issue. Various forms of credit insurance are also used. Unlike the US market, most bonds are floating-rate instruments, reflecting the variable-rate nature of the majority of mortgages in the UK.

The growth in selected markets in the US is shown in Table 25.1 (approximate nominal value in \$ billion).

	1986	1987	1988	1989	1990	1991	1992	1993	1994	1995	1996
GNMA	45	100	89	48	49	52	64	76	120	108	65
FHLMC	44	100.5	82	52	65	55	98	170	130	105	55
FNMA	25	65	55	52	75	80	115	175	240	130	120
Private	4.4	8	9	11	13	20	40	90	95	45	31

Table 25.1: Issue of mortgage pass-through securities 1986–1996.

Source: Federal Reserve Board Bulletin.

25.1.2 Mortgages

In the US market, the terms of a conventional mortgage, known as a *level-payment fixed-rate mortgage*, will state the interest rate payable on the loan, the term of the loan and the frequency of payment. Most mortgages specify monthly payment of interest. These are in fact the characteristics of a level-payment mortgage, which has a fixed interest rate and fixed term to maturity. This means that the monthly interest payments are fixed, hence the term “level-pay”.

The singular feature of a mortgage is that, even if it charges interest at a fixed rate, its cash flows are not known with absolute certainty. This is because the borrower can elect to repay any or all of the principal before the final maturity date. This is a characteristic of all mortgages, and although some lending institutions impose a penalty on borrowers who retire the loan early, this is a risk for the lender, known as *prepayment risk*. The uncertainty of the

² Hayre, L., Mohebbi, C., Zimmermann, T., “Mortgage Pass-Through Securities”, in Fabozzi, F., (ed.) *The Handbook of Fixed Income Securities*, 5th edition, McGraw-Hill, 1997.

cash flow patterns is similar to that of a callable bond, and as we shall see later this feature means that we may value mortgage-backed bonds using a pricing model similar to that employed for callable bonds.

The monthly interest payment on a conventional fixed-rate mortgage is given by (25.3), which is derived from the conventional present value analysis used for an annuity. Essentially the primary relationship is:

$$M_{m0} = I \left(\frac{1 - (1/(1+r)^n)}{r} \right) \quad (25.1)$$

from which we can derive:

$$I = \frac{M_{m0}}{\frac{1 - (1/(1+r)^n)}{r}}. \quad (25.2)$$

This is simplified to:

$$I = M_{m0} \frac{r(1+r)^n}{(1+r)^n - 1} \quad (25.3)$$

where

- M_{m0} is the original mortgage balance (the cash amount of loan)
- I is the monthly cash mortgage payment
- r is the simple monthly interest rate, given by (annual interest rate/12)
- n is the term of the mortgage in months.

The monthly repayment includes both the interest servicing and a repayment of part of the principal. In Example 25.1 after the 264th interest payment, the balance will be zero and the mortgage will have been paid off. Since a portion of the original balance is paid off every month, the interest payment reduces by a small amount each month, that is, the proportion of the monthly payment dedicated to repaying the principal steadily increases. The remaining mortgage balance for any particular month during the term of the mortgage may be calculated using (25.4):

$$M_{mt} = M_{m0} \frac{(1+r)^n - (1+r)^t}{(1+r)^n - 1} \quad (25.4)$$

where M_{mt} is the mortgage cash balance after t months and n remains the original maturity of the mortgage in months.

The level of interest payment and principal re-payment in any one month during the mortgage term can be calculated using the equations below. If we wish to calculate the value of the principal repayment in a particular month during the mortgage term, we may use (25.5):

$$p_t = M_{m0} \frac{r(1+r)^{t-1}}{(1+r)^n - 1} \quad (25.5)$$

where p_t is the scheduled principal repayment amount for month t , while the level of interest payment in any month is given by (25.6):

$$i_t = M_{m0} \frac{r((1+r)^n - (1+r)^{t-1})}{(1+r)^n - 1} \quad (25.6)$$

where i_t is the interest payment only in month t .

EXAMPLE 25.1 Mortgage contract calculations

A mortgage borrower enters into a conventional mortgage contract, in which he borrows £72,200 for 22 years at a rate of 7.99%. What is the monthly mortgage payment?

This gives us n equal to 264 and r equal to $(0.0799/12)$ or 0.0066583. Inserting the above terms into (25.3) we have:

$$I = 72,200 \left(\frac{0.0066583(1.0066583)^{264}}{(1.0066583)^{264} - 1} \right)$$

or I equal to £581.60

The mortgage balance after ten years is given below, where t is 120:

$$M_{m120} = 72,200 \left(\frac{(1.0066583)^{264} - (1.0066583)^{120}}{(1.0066583)^{264} - 1} \right)$$

or a remaining balance of £53,756.93.

In the same month the scheduled principal repayment amount is:

$$p_{120} = 72,200 \frac{0.0066583(1.0066583)^{120-1}}{(1.0066583)^{264} - 1}$$

or £222.19.

The interest only payable in month 120 is shown below:

$$i_{120} = 72,200 \frac{0.0066583((1.0066583)^{264} - (1.0066583)^{120-1})}{(1.0066583)^{264} - 1}$$

and is equal to £359.41. The combined mortgage payment is £581.60, as calculated before.

Some mortgage contracts incorporate a *servicing fee*. This is payable to the mortgage provider to cover the administrative costs associated with collecting interest payments, sending regular statements and other information to borrowers, chasing overdue payments, maintaining the records and processing systems and other activities. Mortgage providers also incur costs when re-possessing properties after mortgagors have fallen into default. Mortgages may be serviced by the original lender or another third-party institution that has acquired the right to service it, in return for collecting the fee. When a servicing charge is payable by a borrower, the monthly mortgage payment is comprised of the interest costs, the principal repayment and the servicing fee. The fee incorporated into the monthly payment is usually stated as a percentage, say 0.25%. This is added to the mortgage rate.

Another type of mortgage in the US market is the *adjustable-rate mortgage* or ARM, which is a loan in which the interest rate payable is set in line with an external reference rate. The re-sets are at periodic intervals depending on the terms of the loan, and can be on a monthly, six-monthly or annual basis, or even longer. The interest rate is usually fixed at a spread over the reference rate. The reference rate that is used can be a market-determined rate such as the prime rate, or a calculated rate based on the funding costs for US savings and loan institutions or *thrifts*. The cost of funds for thrifts is calculated using the monthly average funding cost on the thrifts' activities, and there are "thrift indexes" that are used to indicate to the cost of funding. The two most common indices are the Eleventh Federal Home Loan Bank Board District Cost of Funds Index (COFI) and the National Cost of Funds Index. Generally borrowers prefer to fix the rate they pay on their loans to reduce uncertainty, and this makes fixed-rate mortgages more popular than variable rate mortgages. A common incentive used to entice borrowers away from fixed-rate mortgages is to offer a below-market interest rate on an ARM mortgage, usually for an introductory period. This comfort period may be from two to five years or even longer.

Mortgages in the UK are predominantly *variable rate mortgages*, in which the interest rate is moves in line with the clearing bank base rate. It is rare to observe fixed-rate mortgages in the UK market, although short-term fixed-rate mortgages are more common (the rate reverts to a variable basis at the termination of the fixed-rate period).

A *balloon mortgage* entitles a borrower to long-term funding, but under its terms, at a specified future date the interest rate payable is re-negotiated. This effectively transforms a long-dated loan in to a short-term borrowing.

The balloon payment is the original amount of the loan, minus the amount that is amortised. In a balloon mortgage therefore the actual maturity of the bonds is below that of the stated maturity.

A *graduated payment mortgage* (GPM) is aimed at lower-earning borrowers, as the mortgage payments for a fixed initial period, say the first five years, are set at lower the level applicable for a level-paying mortgage with an identical interest rate. The later mortgage payments are higher as a result. Hence a GPM mortgage will have a fixed term and a mortgage rate, but the offer letter will also contain details on the number of years over which the monthly mortgage payments will increase and the point at which level payments will take over. There will also be information on the annual increase in the mortgage payments. As the initial payments in a GPM are below the market rate, there will be little or no repayment of principal at this time. This means that the outstanding balance may actually increase during the early stages, a process known as *negative amortisation*. The higher payments in the remainder of the mortgage term are designed to pay off the entire balance in maturity. The opposite to the GPM is the *growing equity mortgage* or GEM. This mortgage charges fixed-rate interest but the payments increase over time; this means that a greater proportion of the principal is paid off over time, so that the mortgage itself is repaid in a shorter time than the level-pay mortgage.

In the UK market it is more common to encounter hybrid mortgages, which charge a combination of fixed-rate and variable-rate interest. For example the rate may be fixed for the first five years, after which it will vary with changes in the lender's base rate. Such a mortgage is known as *fixed/adjustable hybrid mortgage*.

25.1.3 Mortgage risk

Although mortgage contracts are typically long-term loan contracts, running usually for 20 to 30 years or even longer, there is no limitation on the amount of the principal that may be repaid at any one time. In the US market there is no penalty for repaying the mortgage ahead of its term, known as a mortgage prepayment. In the UK some lenders impose a penalty if a mortgage is prepaid early, although this is more common for contracts that have been offered at special terms, such as a discounted loan rate for the start of the mortgage's life. The penalty is often set as extra interest, for example six months' worth of mortgage payments at the time when the contract is paid off. As a borrower is free to prepay a mortgage at a time of their choosing, the lender is not certain of the cash flows that will be paid after the contract is taken out. This is known as *prepayment risk*.

A borrower may pay off the principal ahead of the final termination date for a number of reasons. The most common reason is when the property on which the mortgage is secured is subsequently sold by the borrower; this results in the entire mortgage being paid off at once. The average life of a mortgage in the UK market is eight years, and mortgages are most frequently prepaid because the property has been sold.³ Other actions that result in the prepayment of a mortgage are when a property is repossessed after the borrower has fallen into default, if there is a change in interest rates making it attractive to refinance the mortgage (usually with another lender), or if the property is destroyed due to accident or natural disaster.

An investor acquiring a pool of mortgages from a lender will be concerned at the level of prepayment risk, which is usually measured by projecting the level of expected future payments using a financial model. Although it would not be possible to evaluate meaningfully the potential of an individual mortgage to be paid off early, it is tenable to conduct such analysis for a large number of loans pooled together. A similar activity is performed by actuaries when they assess the future liability of an insurance provider who has written personal pension contracts. Essentially the level of prepayment risk for a pool of loans is lower than that of an individual mortgage. Prepayment risk has the same type of impact on a mortgage pool's performance and valuation as a call feature does on a callable bond. This is understandable because a mortgage is essentially a callable contract, with the "call" at the option of the borrower of funds.

The other significant risk of a mortgage book is the risk that the borrower will fall into arrears, or be unable to repay the loan on maturity (in the UK). This is known as *default risk*. Lenders take steps to minimise the level of default risk by assessing the credit quality of each borrower, as well as the quality of the property itself. A study has also found that the higher the deposit paid by the borrower, the lower the level of default.⁴ Therefore lenders prefer to advance funds against a borrower's *equity* that is deemed sufficient to protect against falls in the value of the

³ Source: Halifax plc.

⁴ Brown, S., *et al.*, *Analysis of Mortgage Servicing Portfolios*, Financial Strategies Group, Prudential-Bache Capital Funding, 1990.

property. In the UK the typical deposit required is 25%, although certain lenders will advance funds against smaller deposits such as 10% or 5%.

25.1.4 Securities

Mortgage-backed securities are bonds created from a pool of mortgages. They are formed from mortgages that are for residential or commercial property or a mixture of both. Bonds created from commercial mortgages are known as *commercial mortgage-backed securities*. There are a range of different securities in the market, known in the US as *mortgage pass-through securities*. There also exist two related securities known as *collateralised mortgage securities* and *stripped mortgage-backed securities*. Bonds that are created from mortgage pools that have been purchased by government agencies are known as *agency mortgage-backed securities*, and are regarded as risk-free in the same way as Treasury securities.

A mortgage-backed bond is created by an entity out of its mortgage book or a book that it has purchased from the original lender (there is very often no connection between a mortgage-backed security and the firm that made the original loans). The mortgage book will have a total nominal value comprised of the total value of all the individual loans. The loans will generate cash flows, consisting of the interest and principal payments, and any prepayments. The regular cash flows are received on the same day each month, so the pool resembles a bond instrument. Therefore bonds may be issued against the mortgage pool. Example 25.2 is a simple illustration of a type of mortgage-backed bond known as a *mortgage pass-through security* in the US market.

EXAMPLE 25.2 Mortgage pass-through security

An investor purchases a book consisting of 5000 individual mortgages, with a total repayable value of \$500,000,000. The loans are used as collateral against the issue of a new bond, and the cash flows payable on the bond are the cash flows that are received from the mortgages. The issuer sells 1000 bonds, with a face value of \$500,000. Each bond is therefore entitled to $1/1000$ or 0.02% of the cash flows received from the mortgages.

The prepayment risk associated with the original mortgages is unchanged, but any investor can now purchase a bond with a much lower value than the mortgage pool but with the same level of prepayment risk, which is lower than the risk of an individual loan. This would have been possible if an investor buying all 100 mortgages, but by buying a bond that represents the pool of mortgages, a smaller cash value is needed to achieve the same performance. The bonds will also be more liquid than the loans, and the investor will be able to realise her investment ahead of the maturity date if she wishes. For these reasons the bonds will trade at higher prices than would an individual loan. A mortgage pass-through security therefore is a way for mortgage lenders to realise additional value from their loan book, and if it is sold to another investor (who issues the bonds), the loans will be taken off the original lender's balance sheet, thus freeing up lending lines for other activities.

A *collateralised mortgage obligation* (CMO) differs from a pass-through security in that the cash flows from the mortgage pool are distributed on a prioritised basis, based on the class of security held by the investor. In Example 25.2 this might mean that three different securities are formed, with a total nominal value of \$100 million each entitled to a pro-rata amount of the interest payments but with different priorities for the repayment of principal. For instance, \$60 million of the issue might consist of a bond known as "class A" may be entitled to receipt of all the principal repayment cash flows, after which the next class of bonds is entitled to all the repayment cash flow; this bond would be "class B" bonds, of which say, \$25 million was created, and so on. If 300 class A bonds are created, they would have a nominal value of \$200,000 and each would receive 0.33% of the total cash flows received by the class A bonds. Note that all classes of bonds receive an equal share of the interest payments; it is the principal repayment cash flows received that differ. What is the main effect of this security structure? The most significant factor is that, in our illustration, the class A bonds will be repaid earlier than any other class of bond that is formed from the securitisation. It therefore has the shortest maturity. The last class of bonds will have the longest maturity. There is still a level of uncertainty associated with the maturity of each bond, but this is less than the uncertainty associated with a pass-through security.

Let us consider another type of mortgage bond, the *stripped mortgage-backed security*. As its name suggests, this is created by separating the interest and principal payments into individual distinct cash flows. This allows an issuer to create two very interesting securities, the IO-bond and the PO-bond. In a stripped mortgage-backed bond the interest and principal are divided into two classes, and two bonds are issued that are each entitled to receive one

class of cash flow only. The bond class that receives the interest payment cash flows is known as an *interest-only* or IO class, while the bond receiving the principal repayments is known as a *principal only* or PO class. The PO bond is similar to a zero-coupon bond in that it is issued at a discount to par value. The return achieved by a PO-bond holder is a function of the rapidity at which prepayments are made; if prepayments are received in a relatively short time the investor will realise a higher return. This would be akin to the buyer of a zero-coupon bond receiving the maturity payment ahead of the redemption date, and the highest possible return that a PO-bond holder could receive would occur if all the mortgages were prepaid the instant after the PO bond was bought! A low return will be achieved if all the mortgages are held until maturity, so that there are no prepayments. Stripped mortgage-backed bonds present potentially less advantage to an issuer compared to a pass-through security or a CMO, however they are liquid instruments and are often traded to hedge a conventional mortgage bond book.

The price of a PO-bond fluctuates as mortgage interest rates change. As we noted earlier in the US market the majority of mortgages are fixed-rate loans, so that if mortgage rates fall below the coupon rate on the bond, the holder will expect the volume of prepayments to increase as individuals refinance loans in order to gain from lower borrowing rates. This will result in a faster stream of payments to the PO-bond holder as cash flows are received earlier than expected. The price of the PO rises to reflect this, and also because cash flows in the mortgage will now be discounted at a lower rate. The opposite happens when mortgage rates rise and the rate of prepayment is expected to fall, which causes a PO bond to fall in price. An IO bond is essentially a stream of cash flows and has no par value. The cash flows represent interest on the mortgage principal outstanding, therefore a higher rate of prepayment leads to a fall in the IO price. This is because the cash flows cease once the principal is redeemed. The risk for the IO-bond holder is that prepayments occur so quickly that interest payments cease before the investor has recovered the amount originally paid for the IO-bond. The price of an IO is also a function of mortgage rates in the market, but exhibits more peculiar responses. If rates fall below the bond coupon, again the rate of prepayment is expected to increase. This would cause the cash flows for the IO to decline, as mortgages were paid off more quickly. This would cause the price of the IO to fall as well, even though the cash flows themselves would be discounted at a lower interest rate. If mortgage rates rise, the outlook for future cash flows will improve as the prepayment rate fall, however there is also a higher discounting rate for the cash flows themselves, so the price of an IO may move in either direction. Thus IO bonds exhibit a curious characteristic for a bond instrument, in that their price moves in the same direction as market rates. Both versions of the stripped mortgage bond are interesting instruments, and they have high volatilities during times of market rate changes. Note that PO and IO bonds could be created from the hypothetical mortgage pool described above; therefore the combined modified duration of both instruments must equal the modified duration of the original pass-through security.

The securities described so far are essentially plain vanilla mortgage-backed bonds. Currently, there are more complicated instruments trading in the market.

25.2 Cash flow patterns

We stated that the exact term of a mortgage-backed cannot be stated with accuracy at the time of issue, because of the uncertain frequency of mortgage prepayments. This uncertainty means that it is not possible to analyse the bonds using the conventional methods used for fixed coupon bonds. The most common approach used by the market is to assume a fixed prepayment rate at the time of issue and use this to project the cash flows, and hence the life span, of the bond. The choice of prepayment selected therefore is significant, although it is recognised also that prepayment rates are not stable and will fluctuate with changes in mortgage rates and the economic cycle. In this section we consider some of the approaches used in evaluating the prepayment pattern of a mortgage-backed bond.

25.2.1 Prepayment analysis

Some market analysts assume a fixed life for a mortgage pass-through bond based on the average life of a mortgage. Traditionally a “12-year prepaid life” has been used to evaluate the securities, as market data suggested that the average mortgage has paid off after the twelfth year. This is not generally favoured because it does not take into account the effect of mortgage rates and other factors. A more common approach is to use a *constant prepayment rate* (CPR). This measure is based on the expected number of mortgages in a pool that will be prepaid in a selected period, and is an annualised figure. The measure for the monthly level of prepayment is known as the *constant monthly repayment*, and measures the expected amount of the outstanding balance, minus the scheduled principal, that will be prepaid in each month. Another name for the constant monthly repayment is the *single monthly*

mortality rate or SMM. In Fabozzi (1997) the SMM is given by (25.7) and is an expected value for the percentage of the remaining mortgage balance that will be prepaid in that month.

$$SMM = 1 - (1 - CPR)^{1/12}. \quad (25.7)$$

EXAMPLE 25.3 Constant prepayment rate

The constant prepayment rate for a pool of mortgages is 2% each month. The outstanding principal balance at the start of the month is £72,200, while the scheduled principal payment is £223. This means that 2% of £71,977, or £1,439 will be prepaid in that month. To approximate the amount of principal prepayment, the constant monthly prepayment is multiplied by the outstanding balance.

In the US market the convention is to use the prepayment standard developed by the Public Securities Association (PSA), which is the domestic bond market trade association.⁵ The PSA benchmark, known as 100% PSA, assumes a steadily increasing constant prepayment rate each month until the 30th month, when a constant rate of 6% is assumed. The starting prepayment rate is 0.2%, increasing at 0.2% each month until the rate levels off at 6%.

For the 100%PSA benchmark we may set, if t is the number of months from the start of the mortgage, that if $t < 30$, the $CPR = 6\% \cdot t/30$ while if $t > 30$, then CPR is equal to 6%.

This benchmark can be altered if required to suit changing market conditions, so for example the 200% PSA has a starting prepayment rate and an increase that is double the 100% PSA model, so the initial rate is 0.4%, increasing by 0.4% each month until it reaches 12% in the 30th month, at which point the rate remains constant. The 50% PSA has a starting (and increases by a) rate of 0.1%, remaining constant after it reaches 3%.

The prepayment level of a mortgage pool will have an impact on its cash flows. As we saw in Example 25.1 if the amount of prepayment is nil, the cash flows will remain constant during the life of the mortgage. In a fixed-rate mortgage the proportion of principal and interest payment will change each month as more and more of the mortgage amortises. That is, as the principal amount falls each month, the amount of interest decreases. If we assume that a pass-through security has been issued today, so that its coupon reflects the current market level, the payment pattern will resemble the bar chart shown at Figure 25.1 below.

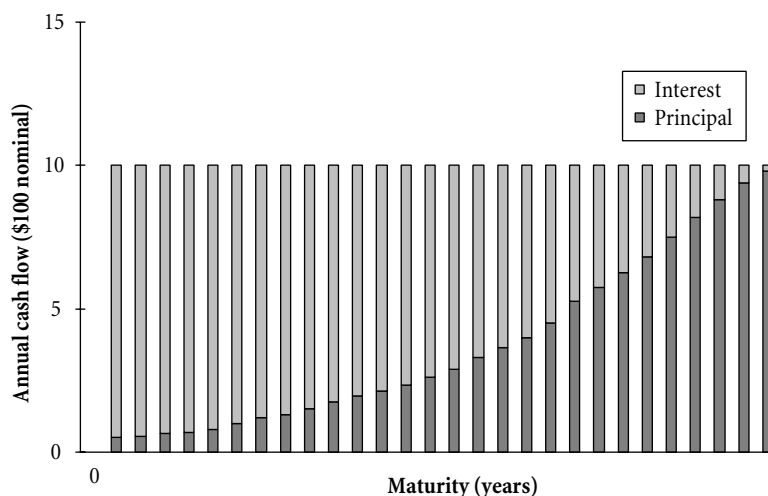


Figure 25.1: Mortgage pass-through security with 0% constant prepayment rate.

When there is an element of prepayment in a mortgage pool, for example as in the 100%PSA or 200%PSA model, the amount of principal payment will increase during the early years of the mortgages and then becomes more steady, before declining for the remainder of the term; this is because the principal balance has declined to such an extent that the scheduled principal payments become less significant. The two examples are shown at Figures 25.2 and 25.3 below.

⁵ Since renamed the Bond Market Association.

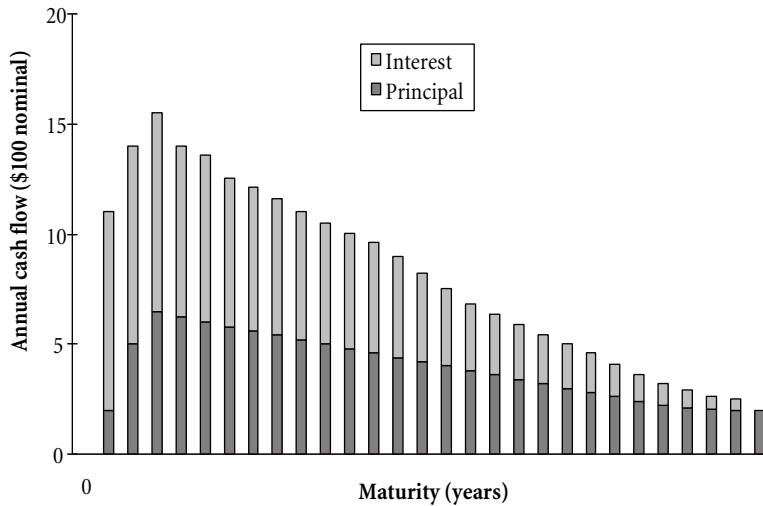


Figure 25.2: 100%PSA model.

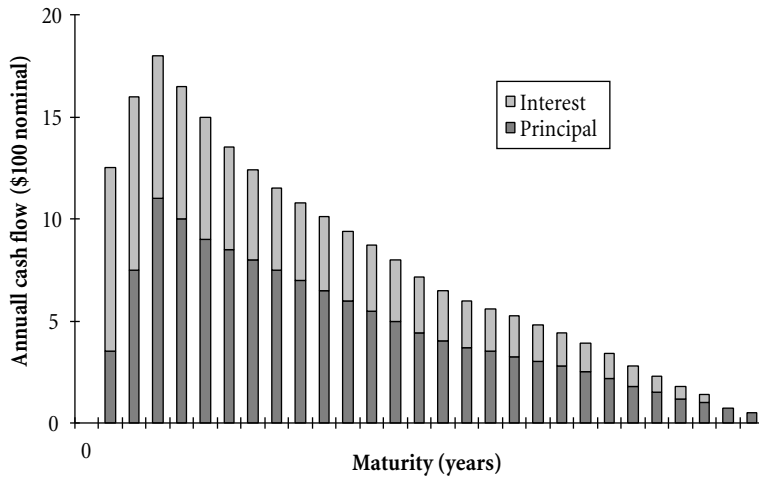


Figure 25.3: 200%PSA model.

The prepayment volatility of a mortgage-backed bond will vary according to the interest rate of the underlying mortgages. It has been observed that where the mortgages have interest rates of between 100 and 300 basis points above current mortgage rates, the prepayment volatility is the highest. At the bottom of the range, any fall in interest rates often leads to a sudden increase in refinancing of mortgages, while at the top of the range, an increase in rates will lead to a decrease in the prepayment rate.

The actual cash flow of a mortgage pass-through of course is dependent on the cash flow patterns of the mortgages in the pool. The relationships described in Example 25.1 can be used to derive further expressions to construct a cash flow schedule for a pass-through security, using a constant or adjustable assumed prepayment rate. Fabozzi (1997) describes the projected monthly mortgage payment for a level-paying fixed rate mortgage in any month as

$$\bar{I}_t = \bar{M}_{mt-1} \frac{r(1+r)^{n-t+1}}{(1+r)^{n-t+1} - 1} \quad (25.8)$$

where

$\bar{I}_t$ is the projected monthly mortgage payment for month t
 $\bar{M}_{mt-1}$ is the projected mortgage balance at the end of month t assuming that prepayments have occurred in the past.

To calculate the interest proportion of the projected monthly mortgage payment we use (25.9) where $\bar{i}_t$ is the projected monthly interest payment for month t .

$$\bar{i}_t = \bar{M}_{mt-1} \cdot i. \quad (25.9)$$

Expression (25.9) states that the projected monthly interest payment can be obtained by multiplying the mortgage balance at the end of the previous month by the monthly interest rate. In the same way the expression for calculating the projected monthly scheduled principal payment for any month is given by (25.10), where $\bar{p}_t$ is the projected scheduled principal payment for the month t .

$$\bar{p}_t = \bar{I}_t - \bar{i}_t. \quad (25.10)$$

The projected monthly principal prepayment, which is an expected rate only and not a model forecast, is given by (25.11):

$$\overline{pp}_t = SMM_t (\bar{M}_{mt-1} - \bar{p}_t) \quad (25.11)$$

where $\overline{pp}_t$ is the projected monthly principal prepayment for month t .

The above relationships enable us to calculate values for:

- the projected monthly interest payment;
- the projected monthly scheduled principal payment;
- and the projected monthly principal prepayment.

These values may be used to calculate the total cash flow in any month that a holder of a mortgage-backed bond receives which is given by (25.12) below, where cf_t is the cash flow receipt in month t .

$$cf_t = \bar{i}_t + \bar{p}_t + \overline{pp}_t. \quad (25.12)$$

The practice of using a prepayment rate is a market convention that enables analysts to evaluate mortgage-backed bonds. The original PSA prepayment rates were arbitrarily selected, based on the observation that prepayment rates tended to stabilise after the first 30 months of the life of a mortgage. A linear increase in the prepayment rate is also assumed. However this is a market convention only, adopted by the market as a standard benchmark. The levels do not reflect seasonal variations in prepayment patterns, or the different behaviour patterns of different types of mortgages.

The PSA benchmarks can be (and are) applied to default assumptions to produce a default benchmark. This is used for non-agency mortgage-backed bonds only, as agency securities are guaranteed by the one of the three government or government-sponsored agencies. Accordingly the PSA *standard default assumption* (SDA) benchmark is used to assess the potential default rate for a mortgage pool. For example the standard benchmark, 100SDA assumes that the default rate in the first month is 0.02% and increases in a linear fashion by 0.02% each month until the 30th month, at which point the default rate remains at 0.60%. In month 60 the default rate begins to fall from 0.60% to 0.03% and continues to fall linearly until month 120. From that point the default rate remains constant at 0.03%. The other benchmarks have similar patterns.

25.2.2 Prepayment models

The PSA standard benchmark reviewed in the previous section uses an assumption of prepayment rates and can be used to calculate the prepayment proceeds of a mortgage. It is not, strictly speaking, a prepayment *model* because it cannot be used to estimate actual prepayments. A prepayment model on the other hand does attempt to predict the prepayment cash flows of a mortgage pool, by modelling the statistical relationships between the various factors that have an impact on the level of prepayment. These factors are the current mortgage rate, the characteristics of the mortgages in the pool, seasonal factors and the general business cycle. Let us consider them in turn.

The prevailing mortgage interest rate is probably the most important factor in the level of prepayment. The level of the current mortgage rate and its spread above or below the original contract rate will influence the decision to refinance a mortgage; if the rate is materially below the original rate, the borrower will prepay the mortgage. As the mortgage rate at any time reflects the general bank base rate, the level of market interest rates has the greatest effect on mortgage prepayment levels. The current mortgage rate also has an effect on housing prices, since if mortgages are seen as “cheap” the general perception will be that now is the right time to purchase: this affects housing market turnover. The pattern followed by mortgage rates since the original loan also has an impact, a phenomenon known as *refinancing burnout*.

Observation of the mortgage market has suggested that housing market and mortgage activity follows a strong seasonal pattern. The strongest period of activity is during the spring and summer, while the market is at its quietest in the winter. The various factors may be used to derive an expression that can be used to calculate expected prepayment levels. For example a US investment bank uses the following model to calculate expected prepayments:⁶

Monthly prepayment rate = (Refinancing incentive) $\times$ (Season multiplier) $\times$ (Month multiplier) $\times$ (Burnout).

25.3 Evaluation and analysis of mortgage-backed bonds

25.3.1 Term to maturity

The term to maturity cannot be given for certain for a mortgage pass-through security, since the cash flows and prepayment patterns cannot be predicted. To evaluate such a bond therefore it is necessary to estimate the term for the bond, and use this measure for any analysis. The maturity measure for any bond is important, as without it it is not possible to assess over what period of time a return is being generated; also, it will not be possible to compare the asset to any other bond. The term to maturity of a bond also gives an indication of its sensitivity to changes in market interest rates. If comparisons with other securities such as government bonds are made, we cannot use the stated maturity of the mortgage-backed bond because prepayments will reduce this figure. The convention in the market is to use other estimated values, which are *average life* and the more traditional duration measure.

The *average life* of a mortgage-pass through security is the weighted-average time to return of a unit of principal payment, made up of projected scheduled principal payments and principal prepayments. It is also known as the *weighted-average life*. It is given by (25.13):

$$\text{Average life} = \frac{1}{12} \sum_{t=1}^n \frac{t (\text{Principal received at } t)}{\text{Total principal received}} \quad (25.13)$$

where n is the number of months remaining. The time from the term measured by the average life to the final scheduled principal payment is the bond's *tail*.

In Chapter 7 we saw that, to calculate duration (or Macaulay's duration) for a bond we required the weighted present values of all its cash flows. To apply this for a mortgage-backed bond therefore it is necessary to project the bond's cash flows, using an assumed prepayment rate. The projected cash flows, together with the bond price and the periodic interest rate may then be used to arrive at a duration value. The periodic interest rate is derived from the yield. This calculation for a mortgage-backed bond produces a periodic duration figure, which must be divided by 12 to arrive at a duration value in years (or by 4 in the case of a quarterly-paying bond).

EXAMPLE 25.4 Macaulay duration

- A 25-year mortgage security with a mortgage rate of 8.49% and monthly coupon is quoted at a price of \$98.50, a bond-equivalent yield of 9.127%. To calculate the Macaulay duration we require the present value of the expected cash flows using the interest rate that will make this present value, assuming a constant prepayment rate, equate the price of 98.50. Using the expression below,

$$rm = 2((1 + r)^n - 1)$$

where rm is 9.127% and $n = 5$, this is shown to be 9.018%.

⁶ Fabozzi (1997).

For the bond above this present value is 6,120.79. Therefore the mortgage security Macaulay duration is given by:

$$D_m = \frac{6,120.79}{98.50} = 62.14.$$

Therefore the bond-equivalent Macaulay duration in years is given by $D = \frac{62.14}{12} = 5.178$.

25.3.2 Calculating yield and price: static cash flow model

There are a number of ways that the yield on a mortgage-backed bond can be calculated. One of the most common methods employs the *static cash flow model*. This assumes a single prepayment rate to estimate the cash flows for the bond, and does not take into account how changes in market conditions might impact the prepayment pattern.

The conventional yield measure for a bond is the discount rate at which the sum of the present values of all the bond's expected cash flows will be equal to the price of the bond. The convention is usually to compute the yield from the *clean* price, that is excluding any accrued interest. This yield measure is known as the bond's *redemption yield* or *yield-to-maturity*. However for mortgage-backed bonds it is known as a *cash flow yield* or *mortgage yield*. The cash flow for a mortgage-backed bond is not known with certainty, due to the effect of prepayments, and so must be derived using an assumed prepayment rate. Once the projected cash flows have been calculated, it is possible to calculate the cash flow yield. The formula is given by (25.14):

$$P = \sum_{n=1}^N \frac{C(t)}{(1 + ri/1200)^{t-1}}. \quad (25.14)$$

Note however that a yield so computed will be for a bond with monthly coupon payments,⁷ so it is necessary to convert the yield to an annualised equivalent before any comparisons are made with conventional bond yields. In the US and UK markets, the bond-equivalent yield is calculated for mortgage-backed bonds and measured against the relevant government bond yield, which (in both cases) is a semi-annual yield. Although it is reasonably accurate to simply double the yield of a semi-annual coupon bond to arrive at the annualised equivalent,⁸ to obtain the bond equivalent yield for a monthly paying mortgage-backed bond we use (25.15):

$$rm = 2((1 + ri_M)^6 - 1) \quad (25.15)$$

where rm is the bond equivalent yield (we retain the designation that was used to denote yield to maturity in Chapter 4) and ri_M is the interest rate that will equate the present value of the projected monthly cash flows for the mortgage-backed bond to its current price. The equivalent semi-annual yield is given by (25.16):

$$rm_{sa} = (1 + ri_M)^6 - 1. \quad (25.16)$$

The cash flow yield calculated for a mortgage-backed bond in this way is essentially the redemption yield, using an assumption to derive the cash flows. As such the measure suffers from the same drawbacks as it does when used to measure the return of a plain vanilla bond, which are that the calculation assumes a uniform reinvestment rate for all the bond's cash flows and that the bond will be held to maturity. The same weakness will apply to the cash flow yield measure for a mortgage-backed bond. In fact the potential inaccuracy of the redemption yield measure is even greater with a mortgage-backed bond because the frequency of interest payments is higher, which makes the reinvestment risk greater. The final yield that is returned by a mortgage-backed bond will depend on the performance of the mortgages in the pool, specifically the prepayment pattern.

Given the nature of a mortgage-backed bond's cash flows, the exact yield cannot be calculated, however it is common for market practitioners to use the cash flow yield measure and compare this to the redemption yield of the equivalent government bond. The usual convention is to quote the spread over the government bond as the main

⁷ The majority of mortgage-backed bonds pay interest on a monthly basis, since individual mortgages usually do as well; certain mortgage-backed bonds pay on a quarterly basis.

⁸ See Chapter 4 for the formulae used to convert yields from one convention basis to another.

measure of value. When measuring the spread, the mortgage-backed bond is compared to the government security that has a similar duration, or a term to maturity similar to its average life.

As we noted in Chapter 4, it is possible to calculate the price of a mortgage-backed bond once its yield is known (or vice-versa). As with a plain vanilla bond, the price is the sum of the present values of all the projected cash flows. It is necessary to convert the bond-equivalent yield to a monthly yield, which is then used to calculate the present value of each cash flow. The cash flows of IO and PO bonds are dependent on the cash flows of the underlying pass-through security, which is itself dependent on the cash flows of the underlying mortgage pool. Again, to calculate the price of an IO or PO bond, a prepayment rate must be assumed. This enables us to determine the projected level of the monthly cash flows of the IO and the principal payments of the PO. The price of an IO is the present value of the projected interest payments, while the price of the PO is the present value of the projected principal payments, comprising the scheduled principal payments and the projected principal prepayments.

25.3.3 Bond price and option-adjusted spread

The concept of option-adjusted spread (OAS) and its use in the analysis and valuation of bonds with embedded options was first considered in Chapter 15, when OAS was discussed in the context of callable bonds. The behaviour of mortgage securities often resembles that of callable bonds, because effectively there is a call feature attached to them, in the shape of the prepayment option of the underlying mortgage holders. This option feature is the principal reason why it is necessary to use average life as the term to maturity for a mortgage security. It is frequently the case that the optionality of a mortgage-backed bond, and the volatility of its yield, have a negative impact on the bond holders. This is for two reasons: the actual yield realised during the holding period has a high probability of being lower than the anticipated yield, which was calculated on the basis of an assumed prepayment level, and mortgages are frequently prepaid at the time when the bondholder will suffer the most; that is, prepayments occur most often when rates have fallen, leaving the bondholder to reinvest repaid principal at a lower market interest rate.

These features combined represent the biggest risk to an investor of holding a mortgage security, and market analysts attempt to measure and quantify this risk. This is usually done using a form of OAS analysis. Under this approach the value of the mortgagor's prepayment option is calculated in terms of a basis point penalty that must be subtracted from the expected yield spread on the bond. This basis point value is calculated using a binomial model or a simulation model to generate a range of future interest rate paths, only some of which will cause a mortgagor to prepay her mortgage. The interest rate paths that would result in a prepayment are evaluated for their impact on the mortgage bond's expected yield spread over a government bond.⁹ As OAS analysis takes account of the option feature of a mortgage-backed bond, it will be less affected by a yield change than the bond's yield spread. Assuming a flat yield curve environment, the relationship between the OAS and the yield spread is given by:

$$\text{OAS} = \text{Yield spread} - \text{Cost of option feature.}$$

This relationship can be observed occasionally when yield spreads on current coupon mortgages widen during upward moves in the market. As interest rates fall, the cost of the option feature on a current coupon mortgage will rise, as the possibility of prepayment increases. Put another way, the option feature begins to approach being in-the-money. To adjust for the increased value of the option traders will price in higher spreads on the bond, which will result in the OAS remaining more or less unchanged.

25.3.4 Effective duration and convexity

The modified duration of a bond measures its price sensitivity to a change in yield; the calculation is effectively a snapshot of one point in time. It also assumes that there is no change in expected cash flows as a result of the change in market interest rates. Therefore it is an inappropriate of interest rate risk for a mortgage-backed bond, whose cash flows would be expected to change after a change in rates, due to the prepayment effect. Hence mortgage-backed bonds react differently to interest rate changes compared to conventional bonds, because when rates fall, the level of prepayments is expected to rise (and vice-versa). Therefore when interest rates fall, the duration of the bond

⁹ The yield spread from OAS analysis is based on the discounted value of the expected cash flow using the government bond-derived forward rate. The yield spread of the cash flow yield to the government bond is based on yields-to-maturity. For this reason, the two spreads are not strictly comparable. The OAS spread is added to the entire yield curve, whereas a yield spread is a spread over a single point on the government bond yield curve.

may also fall, which is opposite to the behaviour of a conventional bond. This feature is known as *negative convexity* and is similar to the effect displayed by a callable bond. The prices of both these types of security react to interest rate changes differently compared to the price of conventional bonds.

For this reason the more accurate measure of interest rate sensitivity to use is *effective duration* described by Fabozzi (1997). To recap from Chapter 9, effective duration is the approximate duration of a bond as given by (25.17):

$$D_{app} = \frac{P_- - P_+}{2P_0(\Delta rm)} \quad (25.17)$$

where

- P_0 is the initial price of the bond
- P_- is the estimated price of the bond if the yield decreases by Δrm
- P_+ is the estimated price of the bond if the yield increases by Δrm
- Δrm is the change in the yield of the bond.

The approximate duration is the effective duration of a bond when the two values P_- and P_+ are obtained from a valuation model that incorporates the effect of a change in the expected cash flows (from prepayment effects) when there is a change in interest rates. The values are obtained from a pricing model such as the static cash flow model, binomial model or simulation model. The calculation of effective duration uses higher and lower prices that are dependent on the prepayment rate that is assumed. Generally analysts will assume a higher prepayment rate when the interest rate is at the lower level of the two.

Figure 25.4 illustrates the difference between modified duration and effective duration for a range of agency mortgage pass-through securities, where the effective duration for each bond is calculated using a 20 basis point change in rates. This indicates that the modified duration measure effectively overestimates the price sensitivity of lower coupon bonds. This factor is significant when hedging a mortgage-backed bond position, because using the modified duration figure to calculate the nominal value of the hedging instrument will not prove effective for anything other than very small changes in yield.

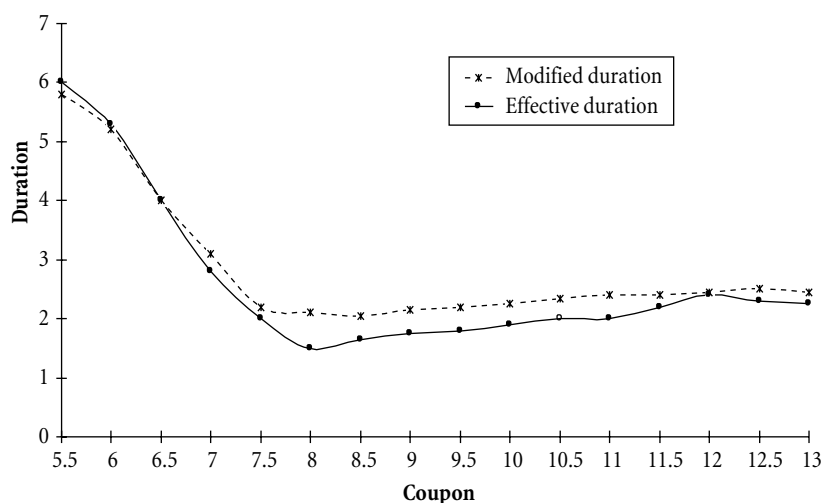


Figure 25.4: Modified duration and effective duration for agency mortgage-backed bonds.

The formula to calculate approximate convexity (or *effective convexity*) is given below as (25.18); again if the values used in the formula allow for the cash flow to change, the convexity value may be taken to be the effective convexity. The effective convexity value of a mortgage pass-through security is invariably negative.

$$CV_{app} = \frac{P_+ + P_- - 2P_0}{P_0(\Delta rm)^2}. \quad (25.18)$$

25.3.5 Simulation modelling

The earlier section alluded to the main shortcomings of the static cash flow model when used for valuation and analysis, and usually to determine the spread between the cash flow yield of the mortgage-backed bond and the redemption yield of the equivalent government bond. In short the main weaknesses of this model are that:

- neither yield measure takes sufficient account of the real government bond term structure (the spot yield curve);
- the cash flow yield ignores the cash flow effects of a change in yields.

To overcome these drawbacks certain analysts use simulation methodology to value mortgage-backed bonds. Put simply this involves generating a set of cash flows based on simulated future mortgage refinancing rates, which themselves imply simulated prepayment rates. The simulation model used is usually a Monte Carlo simulation. Although running a simulation in order to generate future prices is straightforward enough to describe, it is in fact a complex procedure, and one that requires a considerable amount of computing power. In this section we describe only the concept behind the simulation model.

To run a simulation to generate future interest-rates and hence future prices, a model requires the current government bond zero-coupon yield curve, and an assumption of interest rate volatility. The current zero-coupon curve, also known as the spot curve or the term structure of interest rates, is the starting point of the simulation, while the interest rate volatilities are used to generate a range of values for future interest rates. The average of this range of future spot rates for any maturity is equal to the current spot rate for that maturity. The simulation is run by generating scenarios of future interest rate paths. For each future month, a monthly interest rate and a mortgage rate are generated; this mortgage rate is in effect the refinancing rate for that month. The monthly mortgage rates are used to discount the projected cash flows in the scenario, and as a refinancing rate it is used to calculate the cash flow, as it represents the opportunity cost the mortgage borrower is facing at that point. Mortgage prepayments are determined by inputting the projected mortgage rate for that month and other parameters into a prepayment model. Using the projected prepayments it is possible to calculate the cash flow along an interest-rate path. Once we have the interest rate path, it is possible to determine the present value of the cash flow on that path. The correct discount rate to use to calculate the present value is the simulated spot rate for each month on the interest-rate path (there is also a spread to consider). The relationship between the simulated spot rate for month T on interest-rate path n , with the simulated future one-month rate is given by (25.19):

$$s_T(n) = ((1 + f_1(n))(1 + f_2(n)) \cdots (1 + f_T(n)))^{1/T} - 1 \quad (25.19)$$

where

$s_T(n)$ is the simulated spot rate for month T on price path n
 $f_i(n)$ is the simulated future 1-month rate for month I on path n .

Hence we may set the present value of the cash flow for month T on interest-rate path n discounted at the simulated spot rate for month T , together with an amount of spread as given by (25.20):

$$PV(C_T(n)) = \frac{C_T(n)}{(1 + s_T(n) + K)^{1/T}} \quad (25.20)$$

where

$PV(C_T(n))$ is the present value of the cash flow for month T on path n
 $C_T(n)$ is the cash flow for month T on path n
 K is the spread.

The present value of path n is then the sum of all the present values of the cash flows for each month along path n , shown as (25.21):

$$PV(\text{path}(n)) = \sum_{t=1}^T PV(C_T(n)). \quad (25.21)$$

We are now in a position to calculate a market fair value or theoretical value for the bond. If a particular interest-rate path n is actually realised, the present value of that path, determined using (25.21) is the theoretical price of the mortgage-backed bond for that path. Therefore the theoretical price of the bond is the average of all the theoretical values for all the interest rate paths, shown as (25.22). This theoretical value is then compared to the actual observed price to determine if the bond is trading cheap or dear in the market.

$$P_{theo} = \frac{PV(\text{path}(1)) + PV(\text{path}(2)) + \cdots + PV(\text{path}(N))}{N} \quad (25.22)$$

where N is the number of interest-rate paths.

25.3.6 Total return

To assess the value of a mortgage-backed bond over a given investment horizon it is necessary to measure the return generated during the holding period from the bond's cash flows. This is done using what is known as the *total return* framework. The cash flows from a mortgage-backed bond are comprised of (1) the projected cash flows of the bond (which are the projected interest payments and principal repayments and prepayments), (2) the interest earned on the reinvestment of all the payments, and (3) the projected price of the bond at the end of the holding period. The first sum can be estimated using an assumed prepayment rate during the period the bond is held, while the second cash flow requires an assumed reinvestment rate. To obtain (3) the bondholder must assume first, what the bond equivalent yield of the mortgage bond will be at the end of the holding period, and second what prepayment rate the market will assume at this point. The second rate is a function of the projected yield at the time. The total return during the time the bond is held, on a monthly basis, is then given by (25.23),

$$\left(\frac{\text{Total future cash flow amount}}{P_m} \right)^{1/n} - 1 \quad (25.23)$$

which can be converted to an annualised bond-equivalent yield using (25.15) or (25.16).

Note that the return calculated using (25.23) is based on a range of assumptions, which render it almost academic. The best approach to use is to calculate a yield for a range of different assumptions, which then give some idea of the likely yield that may be generated over the holding period, in the form of a range of yields (that is, an upper and lower limit).

EXAMPLE 25.5 Mortgage-backed bond issue

■ Bradford & Bingley Building Society £1 billion three-tranche MBS due 2031

Bradford & Bingley is a UK building society that plans to list on the London Stock Exchange (and convert to a bank) during 2000/2001. In August 2000 it issued a mortgage-backed security that was underwritten by UBS Warburg, via Aire Valley Finance (No. 2), a special purpose vehicle.

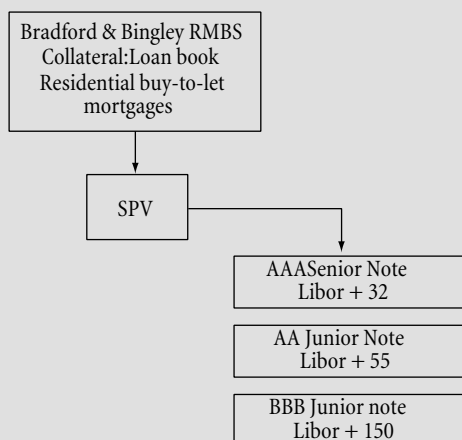


Figure 25.5

Loan portfolio

The underlying collateral consists of approximately 14,000 residential mortgages with an average loan balance of £74,000. Around two-thirds of the mortgages are on properties located in London and the south-east of England. An interesting feature of the mortgages is that they are *buy-to-let* loans; the issuer had previously undertaken a securitisation of its owner-occupier mortgage portfolio. These buy-to-let mortgages allow overpayment of the principal without penalty, consequently early prepayment is more likely with this type of collateral.

Bond structure

The issue is callable, with the structure composed of the following notes:

- £892.5 million senior note with AAA-rating, offered at a yield of three-month Libor plus 32 bps;
- £57.5 million junior note with AA-rating, offered at three-month Libor plus 55 bps;
- £50 million junior note with BBB-rating, offered at Libor plus 150 bps.

Although the issue has a legal maturity to 2031, it is callable and the senior note has a feature that raises its coupon to Libor plus 80 bps if it is not called after September 2008. This makes it likely that the bond will be called at that time.

Investor profile

The issue was placed with around 50 institutional investors, with other UK banks and large building societies and European banks being the largest buyers of the senior notes; fund managers were purchasers of the junior notes.

Selected bibliography and references

- Fabozzi, F., Ramsey, C., Ramirez, F., *Collateralized Mortgage Obligations: Structures and Analysis*, FJF Associates 1994, Chapter 3.
- Fabozzi, F., *Fixed Income Mathematics*, McGraw-Hill, 1997.
- Fabozzi, F. (ed.), *Handbook of Structured Financial Products*, FJF Associates, 1998.
- Hayre, L., Mohebbi, C., "Mortgage Pass-Through Securities", in Fabozzi, F. (ed.), *Advances and Innovations in the Bond and Mortgage Markets*, Probus Publishing, 1989, pp. 259–304.
- Pinkus, S., Chandoha, M., "The Relative Price Volatility of Mortgage Securities", in Fabozzi, F. (ed.), *Mortgage-Backed Securities: New Strategies, Applications and Research*, Probus Publishing, 1987.

Questions and exercises

1. What are the components of the cash flow arising from a mortgage?
2. What is a servicing cost?
3. Why would an analyst be unable to determine the cash flow stream arising from a pool of mortgages?
4. How is the yield-to-maturity for a mortgage-backed bond calculated?
5. Describe why a mortgage-backed security is said to incorporate a call option exercisable by the mortgagor.
6. It is common for market practitioners to evaluate mortgage securities in terms of a spread over an equivalent government bond. How is this equivalent bond identified?
7. What effect would higher interest rate volatility have on the option-adjusted spread of a mortgage security?
8. The cash flow yield of a mortgage-backed bond is 7.25%. An investor requires a yield of at least 7.00% from holding a bond instrument. Is the mortgage security an appropriate investment? What further information is required in order to allow a proper assessment of the mortgage-backed bond?
9. Mortgage-backed securities are said to share similar characteristics with callable bonds. Explain why this might be so.
10. Discuss the drawbacks of the cash flow yield as a measure of return from holding a mortgage-backed bond.

11. What is a hybrid mortgage?
12. Write down the formula for calculating the total return from holding a mortgage security. What assumptions are made in the calculation?
13. A first-time buyer in the housing market takes out a mortgage with the following terms:

Maturity	25 years
Loan size	\$250,000
Loan rate	7.75%

 - (a) The loan is a level-paying fixed rate mortgage. Calculate the monthly mortgage payment.
 - (b) In month 150, how much of the mortgage payment is made up of principal repayment and much of interest?
 - (c) Draw up a table listing the monthly payments and the amortisation of the mortgage for the first five years.
14. Why would it be necessary to forecast the level of prepayments in a mortgage pool?
15. How is a prepayment model used in the calculation of effective duration and effective convexity for a mortgage security?
16. The cash flow yield of a mortgage security is 7.55% per month. Calculate the bond equivalent yield.
17. Certain commentators have described the PSA standard prepayment benchmark as a prepayment cash flow forecasting model. Is this accurate?
18. John Paul George Investments Limited are considering placing a sizeable part of their fixed income portfolio in mortgage-pass through securities and IO bonds. The fund manager is looking for bonds with higher option-adjusted spreads than government-sponsored paper, and also believes that the value of IOs is set to increase, because prepayment levels are expected to fall later this year as interest rates rise. The fund manager believes the risks associated with holding IOs, principally the prepayment risk, are more than compensated for by their higher yield in the asset-backed market, and also feels that prepayment levels will fall later this year.
 - (a) Explain why the IO bonds trade at higher yields in the market compared to other mortgage instruments. Is this connected with their negative convexity?
 - (b) What would happen to the price of JPG Investments Ltd IO bonds if interest rates fell later in the year?
 - (c) What measure could the fund manager use to evaluate the level of prepayment risk?
19. Describe the main types of mortgage pass-through security.
20. Assuming a 200%PSA, what is the CPR for the first 30 months of a level-paying fixed rate mortgage?
21. What is *average life* for an asset-backed bond?

26

Mortgage-backed Bonds II

The previous chapter introduced mortgage-backed securities and discussed the salient features of the main type of this bond, the mortgage pass-through security. Part of the discussion reviewed the use, in general terms, of a simulation model to value a mortgage security, given that the cash flow pattern of such a bond cannot be determined with certainty. This subject is investigated further in this chapter, where we review a valuation methodology based on the binomial model first introduced in Chapter 15. However, the mathematical derivations are not included. The second part of the chapter considers the position and risk implications associated with running a portfolio of mortgage securities, and the issues concerned in tracking an index.

26.1 Basic concepts

Mortgage-backed bonds present some slight difficulty in their valuation, due to the uncertain nature of their cash flow stream, as well as the option feature that is attached to them. This option feature is the freedom of the underlying mortgage borrowers to repay their loan, known as *prepayment*, before the full term of the mortgage has run, at their option. This prepayment ability is in effect a call option on the underlying mortgage. Although an investor has no way of ascertaining when this option will be exercised, it is possible to come to an informed idea of when the most likely time of exercise will occur, based on an assessment of the current mortgage rate, the path interest rates have taken to now, the market's views on the likely direction of future interest rates, and the general economic climate at the time (and forecast). The market standard approach to pricing a mortgage security is to use the assessment of the above factors, in conjunction with another process we shall look at shortly, to arrive at a set of all the possible future interest rate paths that the bond may take, and the cash flows that will arise at each price path. The valuation must consider all the expected cash flows (both interest and principal payments) in addition to the level of prepayment and the possibility that this level might increase in the near future. The most common method used to generate the interest rate (hence price) paths and hence the valuations is to run a simulation on the likely path of future interest rates, to the point when the last scheduled principal payment for the underlying mortgages is due.

Overall the valuation process follows the three phases noted below:

- generate an “arbitrage-free” interest rate scenario, derived from the current government spot yield curve or “term structure of interest rates”, as discussed in Chapter 15;
- generate expected cash flows for the mortgage security for each interest rate path or scenario that was calculated in the previous step; unlike the illustration in Chapter 15, this requires a model that can generate, usually using a constant prepayment rate, the prepayment pattern of mortgage borrowers, in conjunction with the general business climate around at the time;
- calculate the present value of the cash flows at each price path, using the model generated interest rates as the discount rates, and sum these to arrive at a net present value, which is the price of the bond, or to determine the option-adjusted spread premium over the government bond yield curve.

The projected cash flows generated in the last step up, which are arrived at under certain assumptions, can also be used to calculate the total return generated from a holding of the bond during a specified period. This is known as the *holding period return*.

26.2 Pricing and modelling techniques

26.2.1 Valuation

In Chapter 4 the fair value of a conventional fixed income bond is given as the present value of all its expected cash flows, discounted at the current market interest rate. The discount rate that is normally used is the government or *risk-free* rate. This has been formally modelled in several instances, for example the Cox–Ingersoll–Ross model

amongst others.¹ The general approach is to use a path-dependent model, where a binomial or Monte Carlo simulation is used to generate the range of possible interest rates; the financial instrument is broken down into its cash flows which are valued using the discount rates at each point; the average of all of the present values is used to determine the theoretical value for the instrument.

The first step in pricing a security therefore requires a model of the term structure of interest rates. One approach is to use a binomial model² as described in Chapter 15. The binomial lattice is reproduced here as Figure 26.1.

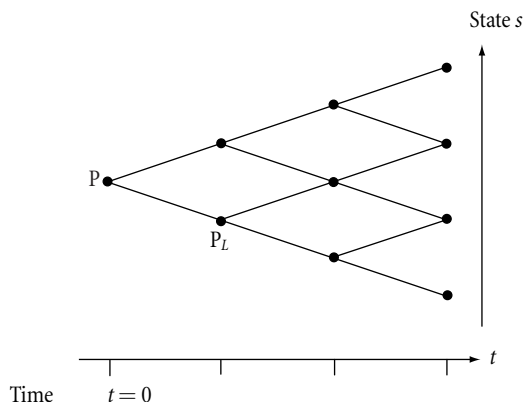


Figure 26.1: A binomial tree of path-dependent interest rates.

In a binomial tree there are discrete time periods, at each point of which the future possible short-term interest rate may move to only two possible points, either up or down. Using the tree it is possible to set all possible short-term interest rates using the base rate r at time t_0 and interest rate volatility levels σ and then modelling the possible future rate at time points 1, 2, 3, and so on to time T using the volatility level at each of these points. The discrete time points may be set at any interval, for example daily, monthly or six-monthly. The short-term interest rate at any state s of the binomial tree at any point t is then given by (26.1):

$$r_t^s = r_t^s(\sigma_t^s). \quad (26.1)$$

The base rate and volatility levels are set using historic market data. For a model such as Black–Derman–Toy,³ the parameters required are the term structure of interest rates and the term structure of interest rate volatilities.

Say that S_0 is the set of different interest rate scenarios that evolve from the initial zero state of the binomial tree. If we let r_t^s denote the short-term discount rate at time t that is associated with scenario $s \in S_0$ and set C_{jt}^s as the cash flow paid by security j at time t , it can be shown that the fair price for the security is given by (26.2):

$$P_{j0} = \frac{1}{|S_0|} \sum_{s=1}^{|S_0|} \sum_{t=1}^T \frac{C_{jt}^s}{\prod_{i=1}^t (1 + r_i^s)}. \quad (26.2)$$

Equation (26.2) is the basic price equation that has been used to develop more advanced pricing tools that we consider later in this section, principally with respect to the price of security at a future time period τ . The complexity of the equation is apparent given that we are calculating the price outcome that emanates from the number of paths in the binomial model. From the point of origin of the lattice the number of paths in the tree is 2^t , a very large number over even a short period of time. For plain vanilla bonds the process is essentially straightforward, requiring the calculation of discounted cash flows at the vertices of the binomial tree, of which there are $T^2/2$. For

¹ See Cox, J.C., Jr., Ingersoll, J.E., Ross, S.A., “A Theory of the Term Structure of Interest Rates”, *Econometrica*, March 1985, pp. 385–407.

² See Black, F., Derman, E., Toy, W., “A One-Factor Model of Interest Rates and its Application to Treasury Bond Options”, *Financial Analysts Journal*, Jan–Feb 1990, pp. 33–39.

³ *Ibid.*

bonds with embedded option features attached, including mortgage-backed bonds, because the cash flows from the bond are path-dependent, it is necessary to consider the outcome of interest rates that result from the binomial tree paths. For mortgage securities the ultimate cash flows depend not only on the current level of interest rates, but also the path that interest rates followed from the issue of the bond until the valuation date. This is because prepayment rates for mortgage bonds are affected by the path followed by interest rates subsequent to the start dates of the underlying mortgages. The prepayment rate for a mortgage-backed bond, and the cash flows, are therefore path dependent.

26.2.2 Using the option-adjusted spread

Strictly speaking the equation at (26.2) is not applicable to corporate bonds because they are not priced using the risk-free discount rates derived from the government bond coupon yield curve. Corporate bond yields reflect their credit risk and liquidity, and for bonds such as mortgage securities, the option element represented by the prepayment risk. Therefore the valuation of mortgage-backed bonds, as with callable bonds must account for the *option-adjusted spread* included in the instrument's yield.⁴ The OAS analysis estimates the adjustment factor required for the government risk-free rates that will equate the current observed market price for the bond with the theoretical fair value calculated from the expected present value of the bond's cash flows. Any difference between the observed market price and the theoretical price of a bond reflects the additional risks that the bond exposes its holders to, which are not contained in a government bond. It may also reflect uncertainties in the market about where the bond should be priced.

The OAS for a given instrument, given here as ρ_j is estimated based on the current market price P_{j0} . It is given by the non-linear equation at (26.3):

$$P_{j0} = \frac{1}{|S_0|} \sum_{s=1}^{|S_0|} \sum_{t=1}^T \frac{C_{jt}^s}{\prod_{i=1}^t (1 + \rho_j r_i^s)}. \quad (26.3)$$

Expression (26.3) cannot be solved analytically, at least not initially. This is because it sets the price of security j as a function of the option-adjusted spread ρ . However this spread premium is calculated using the market price of the security. It is not possible to price a risky bond unless we are able to quantify the risks associated with it. Therefore for non-vanilla bonds such as mortgage securities, a value for the option-adjusted spread premium is input to equation (26.3) from values observed in the market for already-existing securities that have similar characteristics to the bond we wish to price. This OAS premium is then used in (26.3) to calculate the price of a new bond, or current bonds with similar characteristics. Any difference in spread premium between bonds that are deemed to be similar is also an indication of securities that are mis-priced.

After we have obtained the current price of the bond it is possible to use the interest-rate model to calculate the bond price at some future time period which is dependent on the state of the vertices of the binomial lattice. If we assume a set of interest rate scenarios S_s originating from the state s at time period τ , the option-adjusted price of the bond $P_{j\tau}^s$ is given by (26.4) below, which is equation (26.3) modified to calculate the price at time period τ .

$$P_{j\tau}^s = \frac{1}{|S_0|} \sum_{s=1}^{|S_0|} \sum_{t=1}^T \frac{C_{jt}^s}{\prod_{i=\tau}^t (1 + \rho_j r_i^s)}. \quad (26.4)$$

Note that the price of the bond is dependent not only on the state s at the final vertex of the lattice, but also on the path of interest rates from during the period $t = 0$ to $t = \tau$ that pass through this state. The cash flows payable by a mortgage-backed bond after time τ will reflect the economic conditions experienced prior to this point. For instance if the bond has experienced a relatively low level of prepayment, a subsequent change in mortgage rates will have a higher impact on the generated cash flows. The economic situation up to point τ is therefore used when estimating the cash flows C_{jt}^s for the time period after τ .

⁴ See for example Babbel, D., Zenios, S., "Pitfalls in the Analysis of Option-Adjusted Spreads", *Financial Analysts Journal*, Jul-Aug 1992, pp. 65-69.

Therefore to price the bond at the future point τ we sample interest-rate paths from $t = 0$ that pass through state s at time $t = \tau$. If we denote the group of such paths as $S_{0,s}$ and set $P_{j\tau}, s \in S_{0,s}$ as the price of the bond at state s after applying (26.4), then the expected price of the bond at s is given by (26.5):

$$P_{j\tau}^s = \frac{1}{|S_{0,s}|} \sum_{s \in S_{0,s}} P_{j\tau}^s. \quad (26.5)$$

26.2.3 Assumptions of OAS analysis

The application of the path-dependent pricing model to mortgage-backed securities assumes a number of conditions, without which the price calculation will not have any value. The most important assumption is that prepayment rates and market interest rates are related. If the relationship between market rates and prepayment rates over time is not correlated, the price of a mortgage security calculated using OAS analysis will be inaccurate. A change in the level of prepayment rates will alter the price calculated using the OAS model. Another important assumption is that market interest rates follow what is known as a Gaussian diffusion process. The majority of interest rate models assume that market rates follow a random log-normal process and are *mean reverting*, that is the path drift centres on the mean of the rates over the time period. The log-normal distribution assumption implies that rates will centre on the implied forward rates. In a positively sloping yield curve environment, the implied forward rates indicate that short-term rates will be moving higher. This may or may not reflect the market's view of short-term rates. Using implied forward rates however forces long-dated instruments to have yields that are higher than short-dated instruments if they are to be fairly priced, and is a constituent of option pricing theory.

All interest rate models including the binomial model are sensitive to the volatility level that is assumed for interest rates. A higher volatility level will result in a greater dispersion of simulated interest-rate paths, which will increase the price of the option element of a bond. Mortgage-backed bonds have an implied call option feature, given the prepayment of the underlying mortgages, therefore a higher volatility will increase the value of the option element. This will decrease the amount of the option-adjusted spread.

Interest rate models simulate a set of randomly generated interest-rate paths, which carry an element of uncertainty as to their accuracy. The more interest-rate paths that are generated in a model, the less uncertain the simulated outcomes are deemed to be. The price calculated by an interest-rate model is therefore assumed to carry less uncertainty with the more price paths that it simulates.

26.3 Interest rate risk

The most common interest rate measure applied to vanilla bonds is duration, and the related sensitivity measure modified duration. As conventionally defined they cannot be calculated for mortgage-backed bonds, because the cash flows and the maturity date for such instruments cannot be stated with certainty. Although the average life of a mortgage security is often used as a crude measure of interest rate sensitivity, with longer term bonds deemed to carry greater interest rate risk, it is not accurate enough for most applications. Using the OAS methodology however it is possible to calculate meaningful values for both duration and convexity. In the first instance we can shift the simulated interest-rate paths either upwards or downwards by a small amount, while keeping the OAS constant, and measure the difference between the two prices. Essentially the average percentage price change can be used to calculate a bond's OAS *effective duration*, and then measuring the *effective convexity* by observing the rate of change of the effective duration. As they are based on OAS prices the effective duration measure includes the effect of the prepayment option and also may be used for hedging purposes.

More formally we may use the OAS model to estimate the sensitivity of the model-generated prices to changes in market rates. To allow for the complex dependency of the cash flows of a mortgage-backed bond to changes in the market term structure we may use a model such as a Monte Carlo simulation. Assuming that interest rates follow a *stochastic* process, we generate a range of interest rate paths based on the current term structure, and calculate the OAS premium ρ_j implied by the current market price P_{j0} given by (26.3). We then shift the term structure by some specified amount, say -10 basis points, and then regenerate the stochastic process of interest rates. The interest rate paths (r_t^{-s}) from the initial stochastic process are sampled and used to calculate the OAS price, with assumed cash flows from the bond input to the price equation (26.6):

$$P_j^- = \frac{1}{|S_0|} \sum_{s=1}^{|S_0|} \sum_{t=1}^T \frac{C_{jt}^s}{\prod_{i=1}^t (1 + \rho_j r_i^{-s})}. \quad (26.6)$$

We then shift the term structure by +10 basis points and recalculate the stochastic process of interest rates. The price under the new term structure is then calculated in the same way as before. If we denote the first price calculated as P_j^- and the second price as P_j^+ then the *option-adjusted duration* of the bond is given by (26.7):

$$D_{jOAS} = \frac{P_j^+ - P_j^-}{100}. \quad (26.7)$$

The option-adjusted convexity is given by (26.8):

$$CV_{jOAS} = \frac{P_j^+ - 2P_{j0} + P_j^-}{50^2}. \quad (26.8)$$

The OAS-adjusted duration measure is often used to judge the level of prepayment risk of a mortgage security. Prepayment risk may be defined as a measure of the exposure to unforeseen changes in the market's assumed long-term prepayment rates, above those expected to occur with movements in interest rates. Such a change can take place for a number of reasons, including changes in mortgage finance that alters prepayment forecasts or changes in the attitude of mortgagors about prepayment. For obvious reasons a change in prepayment rates can have a significant impact on the performance of a portfolio that is composed of mortgage securities. The OAS-adjusted duration measure for a mortgage security is sometimes called *prepayment duration*, its percentage price change assuming constant OAS resulting from a specified change in projected prepayment rates. Figure 26.2 shows the prepayment duration of selected mortgage-backed bonds as calculated on Bloomberg. Note that a bond such as a current-coupon pass-through security has a relatively low sensitivity to prepayment rates, whereas bonds such as interest-only (IO) or principal only (PO) bonds have a much higher prepayment sensitivity. We can also see that an IO or a high-coupon pass-through has a positive prepayment sensitivity, indicating that its price will fall if prepayment rates increase, whereas the opposite is true for a PO or discount mortgage, which have a negative prepayment sensitivity. This characteristic was described in the previous chapter, when we observed that PO bond prices will rise following an increase in prepayment rates.

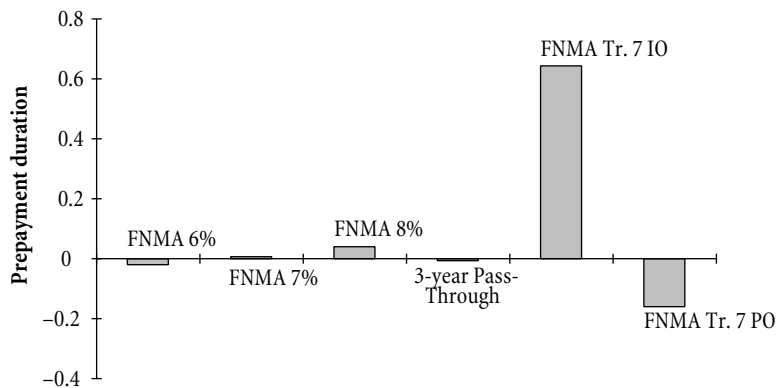


Figure 26.2: Prepayment duration of selected mortgage-backed bonds, May 1998. Source: Bloomberg.

Under certain conditions, under small changes in market rate the binomial lattice model produces errors and sometimes the wrong sign for the convexity value. For the specific situations where this occurs a simulation pricing model will produce more accurate results. A good exposition of this problem is contained in the paper by Douglas Howard (1997), listed in the selected bibliography section.

26.4 Portfolio performance

There are additional considerations in measuring the return of a portfolio of mortgage-backed securities. The market uses the valuation methods described earlier to generate scenarios of returns achieved during a specified holding period. The scenarios are usually generated using a simulation model such as Monte Carlo simulation.

The rate of return of a bond j during a holding period τ is determined by the price of the security at the end of the holding period, together with the accrued value of the cash flows that have been generated by the bond. For a mortgage-backed bond it is necessary to estimate the value of the principal, interest, and prepayments made during the holding period, and then calculate the price of the unpaid balance of the bond at the end of the holding period. This requires the simulation of different scenarios of the term structure, and the prepayment activity that is projected under each different scenario. For any given interest rate scenario s the rate of return of security j during τ is given by (26.9):

$$R_{j\tau}^s = \frac{F_{j\tau}^s + PV_{j\tau}^s}{PP_{j0}} \quad (26.9)$$

where

$F_{j\tau}^s$ is the accrued value of the cash flows generated by the security, reinvested at the short-term interest rate generated for scenario s

P_{j0} is the current market price of the bond.

$PV_{j\tau}^s$ is the value of the unpaid balance of the bond at the end of the holding period, conditional under scenario s . This is given by:

$$PV_{j\tau}^s = M_{j\tau}^s \times P_{j\tau}^s$$

where $M_{j\tau}^s$ is the unpaid balance of the mortgage security and $P_{j\tau}^s$ is the price per unit nominal value of the bond. Both values are calculated at the end of the holding period and are conditional on scenario s .

Figure 26.3 shows how the computed price for the FNMA 7% under different holding period horizons. For shorter time period horizons the price distribution is very sensitive to the number of simulations, which reflects the value of the embedded prepayment option. Under longer holding periods, and as the period approaches the bond's average life maturity, the prepayment option declines in value and the computed prices are more symmetric. As the bond approaches maturity the average price of the bond approaches par, which is similar to the "pull to par" effect on a conventional plain vanilla bond.

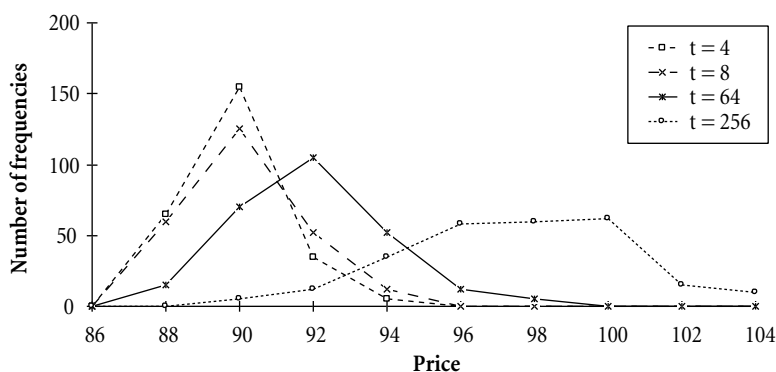


Figure 26.3: Distribution of the computed prices of a mortgage-backed bond at different discrete time points, using Monte Carlo simulation.

Different yield curve models are reviewed in Part VIII.

Selected bibliography and references

Boyce, W., Koenigsberg, M., *Effective Duration of Callable Bonds: The Salomon Brothers Term Structure-Based Option Pricing Model*, Salomon Brothers Inc., April 1987.

Douglas Howard, C., *Numerical Pitfalls of Lattice-Based Duration and Convexity Calculations*, working paper, Department of Applied Mathematics and Physics, Polytechnic University, 1997.

Herskovitz, M.D., "Option-Adjusted Spread Analysis for Mortgage-Backed Securities", in Fabozzi, F. (ed.), *The Handbook of Fixed-Income Options*, Probus Publishing, 1989, Chapter 21.

Waldman, M., Modzelewski, S., "A Framework for Evaluating Treasury-Based Adjustable Rate Mortgages", in Fabozzi, F. (ed.), *The Handbook of Mortgage-Backed Securities*, Probus Publishing, 1985.

27 Asset-backed Securities III

The market in asset-backed securities contains instruments that have widely varying payment terms and conditions, and different collateral bases. The subject matter is a large one and so in this chapter we present only a summary of the main instruments. Interested readers may wish to consult the texts listed under the selected bibliography. Here we review the characteristics of the main asset-backed instruments, which were introduced in US market but are also now issued in other markets including those in the United Kingdom and Europe. The instruments we will consider are:

- collateralised mortgage obligations, (in the US market) both agency and non-agency;
- commercial mortgage-backed securities;
- motor car-loan-backed securities;
- credit card-backed securities.

We also present some further issues connected with the analysis of these securities.

27.1 Collateralised mortgage securities

Mortgage-backed bonds were introduced in Chapter 25. In this section we review some of the newer structures of these instruments. A large number of the instruments in the US market are collateralised mortgage obligations (CMOs), the majority of which are issued by government-sponsored agencies and so offer virtual Treasury bond credit quality but at significantly higher yields. This makes the paper attractive to a range of institutional investors, as does the opportunity to tailor the characteristics of a particular issue to suit the needs of a specific investor. The CMO market in the US experienced rapid growth during the 1990s, with a high of \$324 billion issued in 1993; this figure had fallen to just under \$100 billion during 1998.¹ The growth of the market has brought with a range of new structures, for example bondholders who wished to have a lower exposure to prepayment risk have invested in *planned amortisation classes* (PACs) and *targeted amortisation classes* (TACs). The uncertain term to maturity of mortgage-backed bonds has resulted in the creation of bonds that were guaranteed not to extend beyond a stated date, which are known as *very accurately defined maturity* (VDAM) bonds. In the United Kingdom and certain overseas markets mortgage-backed bonds pay a floating-rate coupon, and the interest from foreign investors in the US domestic market led to the creation of bonds with coupons linked to the LIBOR rate. Other types of instruments in the market include interest-only (IO) and principal-only (PO) bonds, also sometimes called Strips, and inverse floating-rate bonds, which are usually created from an existing fixed-rate bond issue. Figure 27.1 illustrates the relative size of the different instruments in the US market at the beginning of 1997.

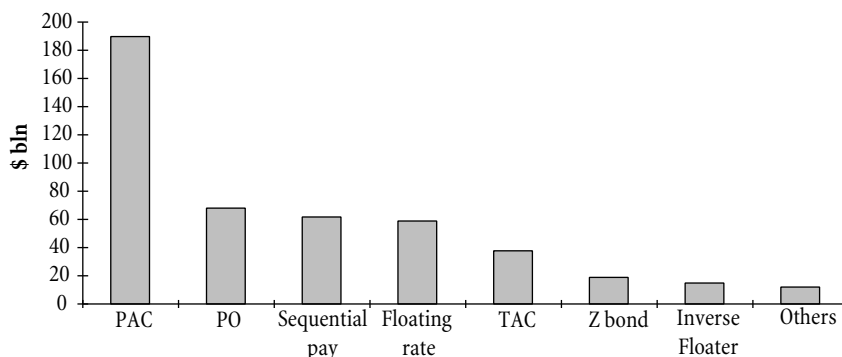


Figure 27.1: US market agency CMOs in issue 1997. Source: Chase Manhattan.

¹ The source for statistical data in this section is *Inside Mortgage Securities*, Chase Manhattan, February 1997 and *Asset-Backed Alert* (www.ABAlert.com).

The primary features of US-market CMOs are summarised as follows:

- **Credit quality.** CMOs issued by US government agencies have the same guarantee as agency pass-through securities, so may be considered risk-free. These bonds therefore do not require any form of credit insurance or credit enhancement. Whole-loan CMOs do not carry any form of government guarantee, and are rated by credit rating agencies. Most bonds carry a triple-A rating, either because of the quality of the mortgage pool or issuing vehicle or because a form of credit enhancement as been used.
- **Interest frequency.** CMOs typically pay interest on a monthly basis, which is calculated on the current outstanding nominal value of the issue.
- **Cash flow profile.** The cash flow profile of CMOs is based on an assumed prepayment rate. This rate is based on the current market expectation of future prepayment levels and expected market interest rates, and is known as the *pricing speed*.
- **Maturity.** Most CMOs are long-dated instruments, and originally virtually all issues were created from underlying mortgage collateral with a 30-year stated maturity. During the 1990s issues were created from shorter-dated collateral, including 5–7 year and 15–20 year mortgages.
- **Market convention.** CMOs trade on a yield, as opposed to a price, basis and are usually quoted as a spread over the yield of the nearest maturity Treasury security. The yields are calculated on the basis of an assumed prepayment rate. Agency CMOs are settled on a T+3 basis via an electronic book-entry system known as “Fedwire”, the clearing system run by the Federal Reserve. Whole-loan CMOs also settle on a T+3 basis, and are cleared using either physical delivery or by electronic transfer. New issues CMOs settle from one to three months after the initial offer date.

Originally mortgage-backed bonds were created from individual underlying mortgages. Agency CMOs are created from mortgages that have already been pooled and securitised, usually in the form of a pass-through security (see Chapter 25). Issuers of *whole loan* CMOs do not therefore need to create a pass-through security from a pool of individual mortgages, but structure based on cash flows from the entire pool. In the same was as for agency pass-through securities, the underlying mortgages in a whole-loan pool are generally of the same risk type, maturity and interest rate. The other difference between whole-loan CMOs and agency pass-throughs is that the latter are comprised of mortgages of up to a stated maximum size, while larger loans are contained in CMOs. There are essentially two CMO structures, those issues that re-direct the underlying pool interest payments and issues re-directing both interest and principal. The main CMO instrument types pay out both interest and principal and are described below.

Whole-loan CMO structures also differ from other mortgage-backed securities in terms of what is known as *compensating interest*. Virtually all mortgage securities pay principal and interest on a monthly basis, on a fixed coupon date. The underlying mortgages however may be paid off on any day of the month. Agency mortgage securities guarantee their bondholders an interest payment for the complete month, even if the underlying mortgage has been paid off ahead of the coupon date (and so has not attracted any interest). Whole-loan CMOs do not offer this guarantee, and so any *payment interest shortfall* will be lost, meaning that a bondholder would receive less than one month’s worth interest on the coupon date. Some issuers, but not all, will pay a compensating interest payment to bondholders to make up this shortfall

27.1.1 Planned amortisation class

The first issue of *planned amortisation classes* (PACs) was in 1986, after a period of sustained falls in the market interest rates led to a demand for less interest-rate volatile mortgage-backed structures. PAC structures are designed to reduce prepayment risk and also the volatility of the weighted average life measure, which is related to the prepayment rate. The securities have a principal payment schedule that is maintained irrespective of any change in prepayment rates. The process is similar to a corporate bond sinking fund schedule, and is based on the minimum amount of principal cash flow that is produced by the underlying mortgage pool at two different prepayment rates, known as *PAC bands*. The PAC bands are set as a low and high PSA standard, for example 50%PSA and 250%PSA. This has the effect of constraining the amount of principal repayment, so that in the early years of the issue the minimum of principal received is at the level of the lower of the two bands, while later in the bond’s life the payment

schedule is constrained by the upper PAC band. The total principal cash flow under the PAC schedule will determine the value of PACs in a structure.

A PAC schedule follows an arrangement whereby cash flow uncertainty of principal payment is directed to another class of security known as *companions*, or *support classes*. When prepayment rates are high, companion issues support the main PACs by absorbing any principal prepayments that are in excess of the PAC schedule. When the prepayment rate has fallen, the companion amortisation rates are delayed as the level of principal prepayment is not sufficient to reach the minimum level stipulated by the PAC bands. Essentially then the structure of PACs results in the companion issues carrying the prepayment uncertainty, since when prepayment rates are high the average life of the companions will be reduced as they are paid off, and when prepayment rates are low the companions will see their measure of average life increase as they remain outstanding for a longer period.

The principal cash flows of PACs and companions can be divided sequentially, similar to a sequential-pay structure, which are reviewed in the next section. PACs have a lower price volatility than other mortgage securities, the level of which is relatively stable when the prepayment rates are within the PAC bands. When prepayment rates move outside the bands, the volatility increases by a lower amount than they otherwise would, because the prepayment risk is transferred to the companions. For this reason, PAC issues trade at lower spreads to the Treasury yield curve than other issues of similar average life. The companion bonds are always priced at a higher spread than the PACs, reflecting their higher prepayment risk.

PAC bonds that have a lower prepayment risk than standard PACs are known as *Type II* and *Type III PACs*. A Type II PAC is created from an existing PAC/companion structure, and has a narrower band than the original PAC. This reduces the prepayment risk. If prepayment rates remain within the bands, Type II PACs trade as PACs. If prepayment rates move outside the narrower band, the extra cash flow is redirected to the companion. Type II PACs are second in priority to the PACs and so carry a higher yield. If prepayment rates remain high over a period of time such that all the companions are redeemed, the Type II PACs take over the function of companion, so that they then carry higher prepayment risk. A further PAC with yet narrower bands, created from the same structure, is known as a *Type III PAC*.

The upper and lower bands in a PAC may “drift” during the life of the CMO, irrespective of the level that actual prepayment rates are at. This drift arises because of the interaction between actual cash prepayments and the bands, and changes in collateral balance and the ratio between the PAC and companion nominal values. The impact of this drift differs according to where the prepayment rate is. The three possible scenarios are:

- **prepayment level lies within the current bands:** in this case the PAC will receive principal in line with the payment schedule, while any prepayments above the schedule amount will be re-directed to the companion issue. The effect of this if it continues is that both the lower and upper bands will drift upwards; this is because any prepayment that occurs is within the bands. This causes the upper band to rise, the rationale being that as prepayments have been below it, they have been received at a slower rate than expected, leaving more companion issues to receive future prepayments. The lower band rises as well, the reasoning being that prepayments have been received faster than expected (as they have been above the minimum level) so that a lower amount of collateral is available to generate future principal payments. It has been observed that upper bands tend to rise at a higher rate than the lower rate, so that prepayment levels lying within the bands causes them to widen over time;
- **prepayment level lies above the upper band:** where prepayment rates are above the upper band, the PAC will receive principal in line with the payment schedule until it is paid off. If the prepayment rates stays above the upper band, the two bands will begin to narrow, because the number of companions available to receive faster prepayments will fall. The two bands will converge completely once all the companion bonds have been redeemed, and the PAC will trade as a conventional sequential pay security after that until it is paid off;
- **prepayment level lies below the lower band:** if prepayment rates lie below the lower PAC band, the upper band will drift upwards as more companion bonds are available to receive a greater level of prepayments in the future; the lower band may also rise by a small amount. This situation is relatively rare however as PAC have the highest priority of all classes in a CMO structure until the payment schedule is back on track.

The band drift process occurs over a long time period and is sometimes not noticeable. Significant changes to the band levels only takes place if the prepayment rate is outside the band for a long period. Prepayment rates that move outside the bands over a short time period do not have any effect on the bands.

27.1.2 Sequential-pay classes

One of the requirements that CMOs were designed to meet was the demand for mortgage-backed bonds with a wider range of maturities. Most CMO structures re-direct principal payments *sequentially* to individual classes of bonds within the structure, in accordance with the stated maturity of each bond. That is, principal payments are first used to pay off the class of bond with the shortest stated maturity, until it is completely redeemed, before being re-allocated to the next maturity band class of bond. This occurs until all the bonds in the structure are retired.

Sequential pay CMOs are attractive to a wide range of investors, particularly those with shorter-term investment horizons, as they are able purchase only the class of CMO whose maturity terms meets their requirements. In addition investors with more traditional longer-dated investment horizons are protected from prepayment risk in the early years of the issue, because principal payments are used to pay off the shorter-dated bonds in the structure.

27.1.3 Targeted amortisation class

These bonds were created to cater for investors who require an element of prepayment protection but at a higher yield than would be available with a PAC. Targeted amortisation class bonds (TAC) offer a prepayment principal in line with a schedule, providing that the level of prepayment is within the stated range. If the level of principal prepayment moves outside of the range, the extra principal amounts are used to pay off TAC companion bonds, just as with PACs. The main difference between TACs and PACs is that TACs have extra prepayment risk if the level of prepayments falls below the amount required to maintain the payment schedule; this results in the average life of the TAC being extended. Essentially a TAC is a PAC but with a lower “PAC band” setting. The preference to hold TACs over PACs is a function of the prevailing interest rate environment; if current rates are low and/or are expected to fall, there is a risk that increasing prepayments will reduce the average life of the bond. In this scenario, investors may be willing to do without the protection against an increase in the average life (deeming it unlikely to be required), and take the extra yield over PACs as a result. This makes TACs attractive compared to PACs under certain conditions, and because one element of the “PAC band” is removed, TACs trade at a higher yield.

27.1.4 Z-class bonds

This type of bond is unique to the domestic US market, and has a very interesting structure. The Z-class bond (or Z-bond) is created from a CMO structure and has re-allocation of both principal and interest payments. It is essentially a coupon-bearing bond that ranks below all other classes in the CMO's structure, and pays no cash flows for part of its life. When the CMO is issued, the Z-bond has a nominal value of relatively small size, and at the start of its life it pays out cash flows on a monthly basis as determined by its coupons. However at any time that the Z-bond itself is receiving no principal payments, these cash flows are used to retire some of the principal of the other classes in the structure. This results in the nominal value of the bond being increased each month that the coupon payments are not received, so that the principal amount is higher at the end of the bond's life than at the start. This process is known as *accretion*. At the point when all classes of bond ahead of the Z-bond are retired, the Z-bond itself starts to pay out principal and interest cash flows.

The following example of a Z-bond cash flow structure is one described in Chapter 28 of *The Handbook of Fixed Income Securities*, edited by Frank Fabozzi (1997). The table is reproduced with the kind permission of McGraw-Hill.

EXAMPLE 27.1 Sample Z-bond cash flows

- This is an example of a Z-bond created from a 7½% 30-year mortgage pass-through bond. The nominal value of the bond on issue is \$118,000, but the initial coupon payments of \$737.50 (the bond pays a monthly coupon) are used as principal prepayments for other bonds in the CMO structure. These cash flows therefore accrete to the nominal value of the Z-bond. The accretion amounts increase as the principal amount increases. In month 133 though, the last sequential-pay bond in the structure is paid off, leaving the Z-class as the only left in the structure. At this point the bond has a principal value of \$270,203. From the next month the bond receives all the interest and principal payments and prepayments that are paid out by the underlying mortgage collateral.

Sequential Pay Z-Bond Structure
CMO created from 7.50% 30-year Mortgage Pass-Through Security
 Cash flows \$

Month	Starting balance	Coupon accretion	Coupon cash flow	Amortisation	Ending balance	Total cash flows
1	118,000.00	737.50	0.00	0.00	118,737.50	0.00
2	118,737.50	742.11	0.00	0.00	119,479.61	0.00
3	119,479.61	746.75	0.00	0.00	120,226.36	0.00
:	:	:	:	:	:	0.00
:	:	:	:	:	:	0.00
131	265,245.57	1,657.78	0.00	0.00	266,903.35	0.00
132	266,903.35	1,668.15	0.00	0.00	268,571.50	0.00
133	268,571.50	1,678.57	0.00	47.44	270,202.63	47.44
134	270,202.63	0.00	1,688.77	3131.55	267,071.08	4,820.31
135	267,071.08	0.00	1,669.19	3,099.51	263,971.57	4,768.70

Table 27.1: Example of Z-Bond cash flows in CMO structure. Source: Ames, C., “Collateralized Mortgage Obligation”, in Fabozzi, F., *The Handbook of Fixed Income Securities*, 5th edition, McGraw-Hill, 1997. Reproduced with permission.

In the conventional sequential pay structure, the existence of a Z-bond will increase the principal prepayments in the CMO structure. The other classes in the structure receive some of their principal prepayments from the Z-bond, which lowers their average life volatility. Creating a Z-bonds in a CMO structure reduces therefore the average life volatility of all the classes in the structure. Z-bonds are an alternative to investors who might otherwise purchase Treasury zero-coupon bonds, with a similar feature of no reinvestment risk. They also offer the added attraction of higher yields compared to those available on Treasury strips with a similar average life.

27.1.5 Interest-only and principal-only class

Stripped coupon mortgage securities were introduced in Chapter 25. They are created when the coupon cash flows payable by a pool of mortgages are split into interest-only (IO) and principal-only (PO) payments. The cash flows will be a function of the prepayment rate, since this determines the nominal value of the collateral pool. IO issues, also known as IO strips, gain whenever the prepayment rate falls, as interest payments are reduced as the principal amount falls. If there is a rise in the prepayment rate, PO bonds benefit because they are discount securities and a higher prepayment rate result in the redemption proceeds being received early. Early strip issues were created with an unequal amount of coupon and principal, resulting in a synthetic coupon rate that was different to the coupon on the underlying bond. These instruments were known as *synthetic coupon pass-throughs*. Nowadays it is more typical for all the interest to be allocated to one class, the IO issue, and all the principal to be allocated to the PO class. The most common CMO structures have a portion of their principal stripped into IO and PO bonds, but in some structures the entire issue is made up of IO and PO bonds. The amount of principal used to create stripped securities will reflect investor demand. In certain cases IO issues created from a class of CMO known as *real estate mortgage investment conduits* (REMICs) have quite esoteric terms. For example the IO classes might be issued with an amount of principal attached known as the nominal balance. The cash flows for bonds with this structure are paid through a process of amortising and prepaying the nominal balance. The balance itself is a small amount, resulting in a very high coupon, so that the IO has a multi-digit coupon and very high price (such as 1183% and 3626-12).²

Strips created from whole-loan CMOs trade differently from those issued out of agency CMOs. Agency CMOs pay a fixed coupon, whereas whole-loan CMOs pay a coupon based on a weighted average of all the individual mortgage coupons. During the life of the whole-loan issue this coupon value will alter as prepayments change the amount of principal. To preserve the coupon payments of all issues within a structure therefore, a portion of the principal and

² Ames (1997).

interest cash flows are stripped from the underlying mortgages, leaving collateral that has a more stable average life. This is another reason that IOs and POs may be created.

IO issue prices exhibit the singular tendency of moving in the same direction as interest rates under certain situations. This reflects the behaviour of mortgages and prepayment rates: when interest rates fall below the mortgage coupon rate, prepayment rates will increase. This causes the cash flow for an IO strip to fall, as the level of the underlying principal declines, which causes the price of the IO to fall as well. This is despite the fact that the issue's cash flows are now discounted at a lower rate. Figure 27.2 shows the price sensitivity of a 7% pass-through security compared to the prices of a IO and a PO that have been created from it. Note that the price of the pass-through is not particularly sensitive to a fall in the mortgage rate below the coupon rate of 7%. This illustrates the *negative convexity* property of pass-through securities. The price sensitivity of the two strip issues is very different. The PO experiences a dramatic fall in price as the mortgage rate rises above the coupon rate. The IO on the other hand experiences a rise in price in the same situation, while its price falls significantly if mortgage rates fall below the coupon rate.

Both PO and IO issues are extremely price volatile at times of moves in mortgage rates, and have much greater interest rate sensitivity than the pass-through securities from which they are created.

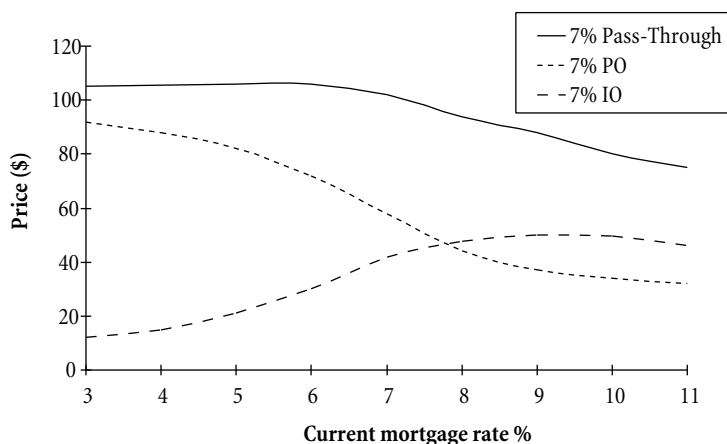


Figure 27.2: Price sensitivity of pass-through security, IO and PO to changes in mortgage rate. Price source and calculation: Bloomberg.

27.1.6 Structuring a CMO

The process of setting up a CMO structure can be quite involved, and is dependent on the level of customer demand for each of the different classes of instruments. Customer demand is a function of their individual requirements as well as the current market environment. Factors such as current mortgage rates and interest rate expectations also influence the demand for certain instruments. We can illustrate a CMO structure using a hypothetical example of an agency pass-through security. We have called this the “Agency 7% issue”. The structuring bank then creates instruments to meet particular requirements. A series of sequential-pay CMOs are created first, which is typical but most frequent under expectations of stable interest and prepayment rates. The last of this series of issues is a Z-bond, aimed at investors who believe that interest rates will rise, and hence that prepayment rates will fall. Part of the cash flows from the Z-bond issue are used to provide principal payments to support a VADM issue. These are offered to investors who wish to purchase bonds that have the greatest protection against an extension in their average life. The structuring bank then creates an IO bond by stripping some of the interest cash flows. This may be done before or after the creation of any further issues. Assume that $\frac{1}{2}\%$ of the coupon is stripped, leaving the original bond now with a coupon of $6\frac{1}{2}\%$. The different classes of instrument are shown in Figure 27.3.

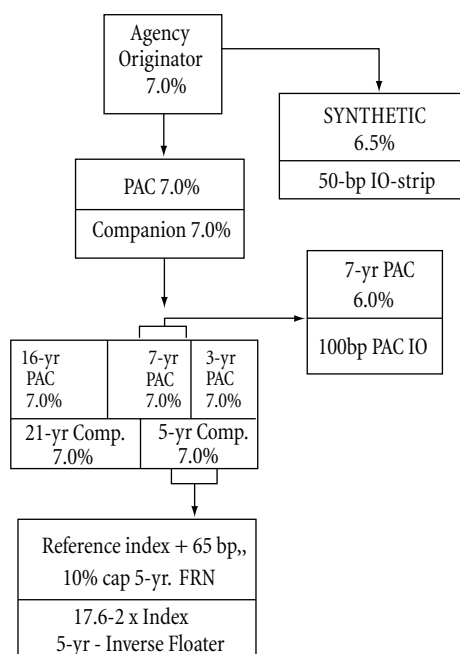


Figure 27.3: Hypothetical example of CMO structure.

In Figure 27.4 we show an alternative structure that might be set up in a different interest rate environment. If the market believes that interest rates and prepayment rates will experience high volatility levels, the structuring bank may create instead a series of PACs (or TACs) and companion bonds. A share of the collateral is used to create the PACs and their companions, with a range of maturities (measured as average lives). In addition a floating-rate bond is created to meet demand from overseas investors, which is created from the five-year companion issue. The floating-rate bond is issued in conjunction with an inverse floater bond. The final element is an instrument created to meet the requirement of an investor who wishes to purchase a bond with lower coupon together with a strip. The 7-year PAC is used for this and split into the two new issues.

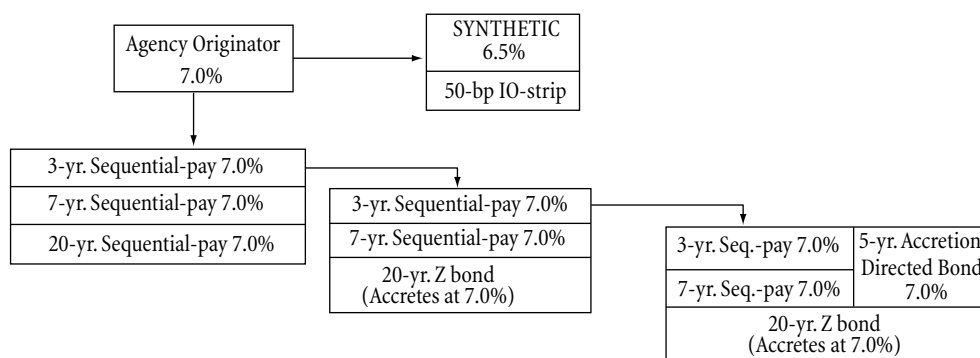


Figure 27.4: Alternative CMO structure.

The examples illustrated at Figures 27.3 and 27.4 are an indication of how flexible CMO structures can be, and they are suitable for a wide range of investors with varying requirements.

27.2 Non-agency CMO bonds

There are no significant difference in the structure and terms of non-agency CMOs compared to agency CMOs. The key feature of non-agency CMOs however, is that they are not guaranteed by government agencies, and so carry an

element of credit risk, in the same way that corporate bonds expose investors to credit risk. To attract investors therefore most non-agency CMOs incorporate an element of *credit enhancement* designed to improve the credit standing of the issue. The use of credit enhancement usually results in a triple-A rating, indeed a large majority of non-agency CMOs are triple-A rated, with very few falling below a double-A rating. All of the four main private credit rating agencies are involved in credit analysis and rating of non-agency CMOs. The rating granted to a particular issue of CMOs is dependent on a range of factors, which include:

- the term of the underlying loans;
- the size of the loans, whether *conforming* or *jumbo* (agency mortgages do not include mortgages above a certain stated size, whereas non-agency issues often are comprised of larger size loans known as *jumbo loans*);
- the interest basis for the loans, whether level-pay fixed-rate, variable or other type;
- the type of property;
- the geographical area within which the loans have been made;
- the purpose behind the loans, whether a first purchase or a refinancing.

In this section we discuss the credit enhancement facility that is used in non-agency CMOs.

27.2.1 Credit enhancements

CMOs are arranged with either an external or internal credit enhancement. An *external* credit enhancement is a guarantee made by a third-party to cover losses on the issue. Usually a set amount of the issue is guaranteed, such as 25%, rather than the entire issue. The guarantee can take the form of a letter of credit, bond insurance or *pool insurance*. A pool insurance policy would be written to insure against losses that arose as a result of default, usually for a cash amount of cover that would remain in place during the life of the pool. Certain policies are set up so that the cash coverage falls in value during the life of the bond. Pool insurance is provided by specialised agencies. Note that only defaults and foreclosures are included in the policy, which forces investors to arrange further cover if they wish to be protected any other type of loss. A CMO issue that obtains credit enhancement from an external party still has an element of credit risk, but now linked to the fortunes of the provider of insurance. That is, the issue is at risk from a deterioration in the credit quality of the provider of insurance. Investors who purchase non-agency CMOs must ensure that they are satisfied with the credit quality of the third-party guarantor, as well as the quality of the underlying mortgage pool. Note that an external credit enhancement has no impact on the cash flow structure of the CMO.

Internal credit enhancements generally have more complex arrangements and sometimes also affect the cash flow structures of the instruments themselves. The two most common types of internal credit enhancement are *reserve funds* and a senior/subordinated structure.

- **Reserve funds.** There are two types of reserve funds. A *cash reserve fund* is a deposit of cash that has been built up from payments arising from the issue of the bonds. A portion of the profits made when the bonds were initially issued are placed in a separate fund. The fund in turn places this cash in short-term bank deposits. The cash reserve fund is used in the event of default to compensate investors who have suffered capital loss. It is often set up in conjunction with another credit enhancement product, such as a letter of credit. An *excess servicing spread account* is also a separate fund, generated from excess spread after all the payments of the mortgage have been made, that is the coupon, servicing fee and other expenses. For instance if an issue has a gross weighted average coupon of 7.50%, and the service fee is 0.10% and the net weighted average coupon is 7.25%, then the excess servicing amount is 0.15%. This amount is paid into the spread account, and will grow steadily during the bond's life. The funds in the account can be used to pay off any losses arising from the bond that affect investors.
- **Senior/subordinated structure.** This is the most common type of internal credit enhancement method encountered in the market. Essentially it involves a bond ranking below the CMO that absorbs all the losses arising from default, or other cause, leaving the main issue unaffected. The subordinated bond clearly has the higher risk attached to it, so it trades at a higher yield. Most senior/subordinated arrangements also incorporate a "shifting interest structure". This arranges for prepayments to be re-directed from the subordinated class to the senior class. Hence it alters the cash flow characteristics of the senior notes, irrespective of the presence of defaults or otherwise.

27.3 Commercial mortgage-backed securities

The mortgage-backed bond market includes a sector of securities that are backed by commercial, as opposed to residential, mortgages. These are known as *commercial mortgage-backed securities* (CMBS). They trade essentially as other mortgage securities but there are detail differences in their structure, which are summarised in this section.

27.3.1 Issuing a CMBS

As with a residential mortgage security, a CMBS is created from a pool or “trust” of commercial mortgages, with the cash flows of the bond backed by the interest and principal payments of the underlying mortgages. A commercial mortgage is a loan made to finance or refinance the purchase of a commercial (business) property. There is a market in direct purchase of a commercial loan book in addition to the more structured CMBS transaction. An issue of CMBS is rated in the same way as a residential mortgage security and usually has a credit enhancement arrangement to raise its credit rating. The credit rating of a CMBS takes into account the size of the issue as well as the level of credit enhancement support.

Classes of bonds in a CMBS structure are usually arranged in a sequential-pay series, and bonds are retired in line with their rating in the structure; the highest-rated bonds are paid off first.

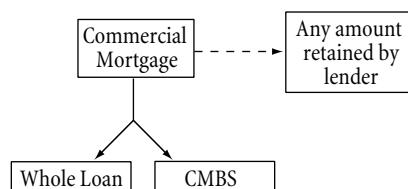


Figure 27.5: Commercial mortgage security issue.

Commercial mortgages impose a penalty on borrowers if they are redeemed early, usually in the form of an interest charge on the final principal. There is not such penalty in the US residential mortgage market, although early retirement fees are still a feature of residential loans in the UK. The early payment protection in a commercial loan can have other forms as well, such as a prepayment “lockout”, which is a contractual arrangement that prevents early retirement. This early prepayment protection is repeated in a CMBS structure, and may be in the form of call protection of the bonds themselves. There is already a form of protection in the ratings of individual issues in the structure, because the highest-rated bonds are paid off first. That is, the triple-A rated bonds will be retired ahead of the double-A rated bonds, and so on. The highest-rated bonds in a CMBS structure also have the highest protection from default of any of the underlying mortgages, which means that losses of principal arising from default will affect the lowest-rated bond first.

As well as the early retirement protection, commercial mortgages differ from residential loans in that many of them are *balloon* mortgages. A balloon loan is one on which only the interest is paid, or only a small amount of the principal is paid as well as the interest, so that all or a large part of the loan remains to be paid off on the maturity date. This makes CMBSs potentially similar to conventional vanilla bonds (which are also sometimes called “bullet” bonds) and so attractive to investors who prefer less uncertainty on term to maturity of a bond.

27.3.2 Types of CMBS structures³

In the US market there are currently five types of CMBS structures. They are:

- liquidating trusts;
- multi-property single borrower;
- multi-property conduit;
- multi-property non-conduit;
- single property single-borrower.

We briefly describe the three most common structures here.

³ The structures described in this section borrow heavily from data contained in Dunlevy (1996), Chapter 30 in Fabozzi/Jacob (1996).

- **Liquidating trusts.** This sector of the market is relatively small by value and represents bonds issued against non-performing loans, hence the other name of *non-performing CMBS*. The market is structured in a slightly different way to regular commercial mortgage securities. The features include a *fast-pay structure*, which states that all cash flows from the mortgage pool be used to redeem the most senior bond first, and *overcollateralisation*, which is when the value of bonds created is significantly lower than the value of the underlying loans. This overcollateralisation results in bonds being paid off sooner. Due to the nature of the asset backing for liquidating CMBSs, bonds are usually issued with relatively short average lives, and will receive cash flows on only a portion of the loans. A target date for paying off is set and in the event that the target is not met, the bonds usually have a provision to raise the coupon rate. This acts as an incentive for the borrower to meet the retirement target.
- **Multi-property single borrower.** The single borrower/multi-property structure is an important and large-size part of the CMBS market. The special features of these bonds include *cross-collateralisation*, which is when properties that are used as collateral for individual loans are pledged against each loan. Another feature known as *cross-default* allows the lender to call each loan in the pool if any one of them defaults. Since cross-collateralisation and cross-default links all the properties together, sufficient cash flow is available to meet the collective debt on all of the loans. This influences the grade of credit rating that is received for the issue. A *property release provision* in the structure is set up to protect the investor against the lender removing or pre-paying the stronger loans in the book. Another common protection against this risk is a clause in the structure terms that prevents the issuer from substituting one property for another.
- **Multi-borrower/conduit.** A *conduit* is a commercial lending entity that has been set up solely to generate collateral to be used in securitisation deals. The major investment banks have all established conduit arms. Conduits are responsible for originating collateral that meets requirements on loan type (whether amortising or balloon, and so on), loan term, geographic spread of the properties and the time that the loans were struck. Generally a conduit will want to have a diversified range of underlying loans, known as *pool diversification*, with a wide spread of location and size. A diversified pool reduced the default risk for the investor. After it has generated the collateral the conduit then structures the deal, on terms as similar to CMOs but with the additional features described in this section.

27.4 Motor-car-backed securities

27.4.1 Description

Bonds issued against car loan finance are known as *auto-loan-backed securities* (auto-ABS) in the US or simply as *car-loans* in the UK. The auto-loan-backed market in the US is one of the largest in the asset-backed market. It is also one of the oldest sectors, with the first deal being issued in May 1985.⁴ Auto-loans are amortising private loans in the manner of residential mortgages, but are typically of much shorter duration than mortgages and carry no prepayment risk. This makes bonds that are issued against them attractive instruments for investors who are seeking more stable, shorter-dated paper of high credit quality but yielding more than Treasury bonds. As they are more straightforward, their analysis presents fewer problems than other asset-backed securities. Up to the middle of 1998 approximately \$170 billion of bonds auto-loan bonds had been issued in the US, and the sector is a liquid and fairly transparent one.

Car loans in the US are short-term amortising deals, ranging from three to six years in maturity. The average maturity is four years. Loans are made either direct at the dealership, in which case finance is supplied by an arm of the car manufacturer, or at a commercial (retail) bank. In the UK car loans are of two types, the amortising loan from a bank or a bullet loan usually taken from a finance house, known as leasing. Both types of loans are used as collateral in a car-loan bond issue. The US car loan market is dominated by the “Big Three” car manufacturers General Motors, Ford Motor Corporation and Chrysler Corporation, who issued over 36% of all auto-ABS in 1998. The large commercial banks are also big issuers of auto-ABS bonds, and include Capital One, NationsBank and Chase Manhattan. Independent finance companies make up the third category of issuers, companies such as Western Financial and Union Acceptance Corporation. The independent houses are much more reliant on the auto-

⁴ Statistics in this section are sourced from Zimmerman (1996) in Bhattacharya/Fabozzi (1996), Chapter 31 and *Asset-Backed News*.

ABS market and can fund their book at lower rates directly as a result, since they create AAA securities out of securitised car loans that pay a lower yield than their individual credit standings would allow. To achieve a high credit rating most auto-ABS issues are structured with a form of credit enhancement. The most common form of enhancement is a senior/subordinated structure, similar to mortgage-backed bonds, although letters of credit and over-collateralisation are also used.

The underlying assets in an auto-ABS issue are bank or finance house car loans, which may be purchased or originated by a special purpose vehicle called a *special purpose corporation* (SPC). Interest payments from the original borrowers are passed to investors via the SPC. There are two main types of auto-ABS bonds, pass-through securities and *pay-through* securities. The majority of issues are pass-throughs.

The prepayment risk on auto-ABS bonds is low. Car loans have a very low response to changes in market interest rates, and loans are rarely repaid ahead of the maturity date. Auto-ABS bonds therefore do not exhibit the negative convexity of mortgage-backed bonds. There are non-interest-rate reasons why prepayment might occur however, such as customers “trading up” to another car, or theft or destruction of the vehicle.

27.4.2 Yield spreads

The structure of auto-ABS bonds is relatively straightforward compared to other asset-backed securities, and possibly simpler than credit card-backed securities. They are amortising bonds (in the US) but because they have little prepayment risk they do not exhibit negative convexity. They are also shorter-dated securities compared to other ABS issues; most auto pass-throughs have average lives of 1.8 to 2.2 years, which is considerably lower than mortgage bonds. The yield spread over Treasury securities for auto-ABS issues is tighter than mortgage securities, but slightly over that for credit card securities. During 1996 they typically yielded around 10–25 basis points over credit card securities, and on average 30–40 basis points below manufactured housing mortgage securities. The market places a premium on auto-ABS bonds issued by the independent finance companies, which trade at around 5–10 basis points above issues made by the Big Three manufacturers and the commercial banks.

Tranche Issues	Date	Amount (\$m)	Average life (years)	Spread (bps)
Premier 1996-1	21-Mar-96	1,250	2.0	34
Olympic 1996-A	07-Mar-96	600	2.0	40
Western Finance	21-Mar-96	484	2.0	42
Pass-Throughs				
Chase 1996-A	14-Feb-96	1,474	1.6	41
Banc One 1996-A	15-Feb-96	537	1.6	42
Fifth Third 1996-A	19-Mar-96	408	1.7	43
UAC 1996-A	09-Feb-96	203	1.9	49

Table 27.2: Auto-ABS issues during 1996. Reprinted from Zimmerman, T., “Auto-Loan-Backed Securities”, in Fabozzi, F., *The Handbook of Fixed Income Securities*, 5th edition, McGraw-Hill, 1997. Used with permission.

This is illustrated in Table 27.2. The Olympic 1996-A and Western Finance 1996-A were issued by independent finance houses, as was the UAC 1996-A. Note that each of these traded at a yield that was 6–8 basis points higher than the others, which were issued by commercial banks, although their relatively small issue will impact in their liquidity, for which there is a small yield premium.

Auto-ABS bonds have a wider range of payment windows than comparable securities and this makes their relative value more closely related to the shape of the yield curve. Securities such as credit card ABS bonds provide a higher return in an environment with a steeply positive short-date yield curve, due to their “roll down” effect, so that in a steepening yield curve scenario auto-ABS bonds will be outperformed by credit card securities. This reflects that in a stable yield curve environment auto-ABS bonds will achieve a higher total return, compared to their spread, than credit card securities.

27.5 Credit card asset-backed securities

Another large sector in the asset-backed market is the credit card market. It experienced steady growth during the 1990s, with over \$369 billion nominal outstanding in June 1998.⁵ The rise in credit card securitisation reflects the increasing use of credit cards by consumers, such that it is now a significant national economic indicator. The public are now able to pay for a large number of goods and services using credit cards, and as a payment medium they are increasingly used for regular, “day-to-day” purchases as well as larger size one-off transactions. The first example of a credit card securitisation was in 1987 in the US domestic market, as banks sought to move credit card assets off their balance sheets, thus freeing up lending lines, and diversify their funding sources. The first banks to securitise their credit card books were Capital One, MBNA and Advanta, among others. They were able to benefit from funding at triple-A rates, due to the credit enhancement used on the bond structures and consequently low charges on their capital. The largest issuers of credit card-backed bonds in the US market are shown at Table 27.3 below.

Securitising banks	\$ billion
Citibank	41.5
MBNA America	22.3
Discover	13.6
First USA	12.1
First Chicago	11.9
Household	10.2
Capital One	7.1
Chase Manhattan	6.0
Advanta	5.1

Table 27.3: Largest issuers of credit card-backed bonds, US domestic market, June 1998. Source: Chase Manhattan.

Credit card ABS bonds, along with auto-ABS issues, remain the most accessible instruments for investors who are interested in having an exposure to the asset-backed market. As they have maturities that are shorter than the average-lives of mortgage-backed securities, as well as cash flow patterns that are more akin to conventional bonds, they are attractive investments to institutions such as banks and money market funds. In the US, and increasingly in the UK and Europe, the combination of credit card, auto-loan and mortgage-backed securities means that there are highly-rated asset-backed instruments, paying reasonable spreads over government securities, available along the entire yield curve.

27.5.1 Issuing structures

In the US market there are two different structures under which credit card debt is securitised. The *stand-alone trust* is a pool of credit card accounts that have been sold to a trust, and then used as collateral for a single security. As with CMOs there are usually several classes of bonds within a single issue. If a subsequent class of bonds is issued, the issuer must designate a new pool of *receivables* and sell these card accounts to a separate trust. This structure was used at the start of the market, but has been all but replaced by the *master trust*, which is a structure that allows the securitising bank to issue successive securities from the same trust, which are all backed by the same pool of credit card loan collateral. Under this arrangement the issuer retains greater flexibility for issuing securities, and without the cost associated with setting up a new trust each time a new issue is required.

EXAMPLE 27.2 Credit Card ABS master trust structure

- An issuing bank transfers the cash flows represented by 100,000 credit card accounts, with a nominal value of \$200 million, to a separate trust. It then issues different classes of securities, with different coupons, maturities and nominal values, all backed by the same collateral. At a later date the issuer requires further funding it transfers the receivables from more card accounts to the same trust and issues more securities. The

⁵ The source of the statistical data in this section is *Asset-Backed Alert*.

different tranches of receivables, transferred at two different times, are not differentiated from one another and are not segregated; that is, the combined receivables in the trust back all the bond issues made against the trust.

As a master trust will comprise receivables transferred to it over time, it will contain cash flows that have different payment terms and interest rates (or *annual payment rate* [APR], used in quoting credit card interest rates), reflecting the terms and conditions of cards issued at different times. For instance, a bank may issue cards under a special low interest rate as part of a special promotion designed to attract new customers. New cardholders will have accounts with different terms and cash flows to existing card holders. Investors in credit card ABS must be aware therefore that the composition of accounts in a master trust pool are liable to alter over time, sometimes significantly, as existing cardholder accounts are closed or fall dormant, and as new accounts are added, with or without different terms and conditions.

Issuers of credit card ABS in the US market are required to retain ownership, or a shared ownership, in the trust. This is so that the issuer can ensure that fluctuations in the balance of the receivables due to seasonal factors, as well as factors such as returned goods and credit card fraud. It is also an incentive for the issuer to maintain the credit quality of the pool. The participation level is set at the minimum level of the receivables balance in the trust, which is 7% in the US market. This minimum level is set at 10% in the UK market and for sterling credit card ABS. If the level of its participation falls below the minimum required, the issuer must add further credit card accounts to the trust.

The issuing structure for credit card ABS is similar irrespective of the type of trust that is used. Generally there are three types of cash flow periods, which are *revolving*, *amortisation* (or otherwise *accumulation*) and *early amortisation*. Each period generates a different cash flow. This structure essentially replicates a conventional plain vanilla bond, with regular coupon payments and a single redemption amount on maturity, sometimes called the *bullet* payment. The main difference however is that credit card ABS pay monthly interest, instead of the traditional semi-annual interest basis. Credit card bonds also differ from other asset-backed securities because they do not have an amortising payment structure. Such an arrangement would not be appropriate for credit card bonds, since the average life of a receivable is shorter, ranging from between 5 months to 10 months. Under the amortising structure, which is used for mortgage-backed bonds and some auto-ABS bonds, payments of principal and interest are passed directly to investors each month. This would lead to an uncertain and volatile payment structure for credit card bonds. Therefore a revolving structure is used instead, with all cash flows being split into interest payments and principal payments. The monthly interest payment is used to cover the coupon on the bond, and cash flow that remain after this liability has been discharged, known as *excess spread*, is passed to the issuer or seller. The collection of principal differs according to the payment period, which is reviewed below.

- **Revolving period.** The revolving period in the structure is not fixed, and has ranged from 2 to 11 years. During the period bondholders receive only interest payments, when monthly payments of principal are used to purchase new receivables in accounts held in the trust; if there are no new receivables, the cash flows are used to purchase a share of the issuer's participation. If there are insufficient receivables available, this effects an early amortisation, as the issuer's participation would have fallen to below the minimum level. This is an incentive to the issuer to maintain a participation above the minimum.
- **Amortisation/accumulation.** This period starts after the revolving period has ceased. An amortisation or *controlled amortisation*, principal payments are no longer used for reinvestment in more receivables, but instead are used to pay off bondholders in a series of amortisation payments. The size of these payments depends on how long the amortisation period runs for; the usual length is one year, so that investors receive 12 equal amortisation periods over the year. Excess principal payments received during this period are used to purchase new receivables, similar to the process followed in the revolving period. Accumulation is similar to controlled amortisation, but for investors the results of the process are akin to a conventional bond. The monthly principal payments are deposited in a trust account or *principal funding account* over the 12 month period, and then paid out to bondholders as one lump sum on the maturity date. During this time, interest payments are made each month on the balance of the total invested amount. Accumulation is therefore attractive to investors who wish the redemption payment to be a single bullet payment, and during the process they will not notice any change as the bond moves from the revolving period to the accumulation period.

- **Early amortisation.** An early amortisation is not a planned event, and is triggered if there is decline in asset quality, or the issuer is found to be in financial trouble. The bonds enter into an early amortisation so that investors start to receive their principal immediately, and not be subject to delays in repayment that might otherwise occur. The terms under which credit card ABS deals are issued specify a range of events that, if they occur, will cause the issue to enter early amortisation. These include failure to make required interest payments, failure to transfer receivables to the trust when required, and certain events of bankruptcy and default connected with the issuer. Other triggering events include if the excess spread fall to zero, and if the issuer's participation level falls below the minimum level. The occurrence of early amortisation is a rare event.

27.5.2 Credit enhancement

Bonds issued against credit card receivables are unsecured in that the receivables provide no collateral if cardholders default. Credit card bonds require therefore some form of credit enhancement if they are to receive investment-grade credit ratings, and the majority of them are highly rated once the enhancement has been set up. In this section we briefly describe the main types of credit enhancement that are arranged for credit card issues; many structures have two or more of these enhancements.

- **Excess spread.** The excess spread payment is the amount of cash flow over and above the amount required to discharge the bond's interest obligations. There is usually a significant excess spread due to the interest rate charged on credit cards being substantially higher than money market and bond market interest rates. For example, the average credit card APR in the UK market during 1998 was over 19%, and sometimes as high as 23%. This means that once the coupon on the bond has been issued and service and other charges paid, there is usually a substantial excess spread, often as high as 5%. The excess spread is used in one or more ways, including to pay the fees of credit enhancement institutions, being deposited in a trust reserve account or being released to the issuer.
- **Senior/subordinated notes.** Issues are often split into different classes of bonds, to suit investor taste. A common credit enhancement is the senior/subordinated structure, with bonds split into higher and lower rated issues. The subordinated notes absorb the losses in the structure ahead of the senior notes, if any other credit enhancement features have not proved sufficient. On maturity or early amortisation the senior notes will be repaid ahead of the subordinated notes. The latter trade at a higher yield as a result.
- **Letter of credit.** As with other asset-backed markets, a bank letter of credit is a common form of credit enhancement used in the credit card market. This is a guarantee from a bank to provide payment in the event that the trust is unable to meet the commitments of the bond issue. The bank guarantee is up to an amount stipulated in the letter of credit. A fee is payable to the bank for providing the letter. This type of enhancement has proved less popular in recent years, after some issuers were themselves downgraded, leading to a downgrading of assets that they had provided letters of credit against.
- **Cash collateral account.** This is a segregated bank account for the benefit of the trust. It is set up and funded at the time of the issue, and funds in it may be used to cover occasions when there is insufficient payments received to cover required interest and principal payments. The funds in the account are borrowed from a third-party bank, and the terms of the issue usually provide for repayment of the cash collateral account only after all the bonds in the issue have been repaid. The excess spread may also be used to fund or top-up the account.
- **Collateral invested amount.** The collateral invested amount, also known as *collateral interest* or *enhancement invested amount* is a share of ownership in the trust and works in the same way as the cash collateral account. It is usually placed with a third-party bank, and is also funded by part of the excess spread.

27.5.3 Credit analysis

Credit card ABS are rated by the four major credit ratings agencies, in the same way as other asset-backed bonds. The variables in the credit card market mean that there are a larger number of scenarios that the agencies must consider. For example the quality of a pool of credit card receivables is a function of (among other issues):

- the type of credit cards in the pool, whether standard, low-rate, affinity cards, store cards, etc.;
- the credit scoring model used by the card issuer;

- the interest rates charged on the cards, whether fixed or floating, and the frequency with which rates are changed or re-set;
- amount of dormant cards in the pool;
- behaviour of cardholders in the pool;
- yield on the portfolio;
- monthly payment rate, which will fluctuate as the number of accounts changes, or as cardholders pay a smaller amount of their balance each month or pay off balances entirely;
- coupon on the bonds: a floating coupon on the bond is more at risk if it is backed with card receivables that are fixed rate, and vice-versa.

The credit rating a bond issue receives will also consider what possibility there exists that it will be forced into early amortisation. This is separate to the issue of credit enhancement, and unaffected by it. The rating of a credit card issue indicates the likelihood that the issue will be able to redeem in full all the bonds in the structure under any possible scenario. A triple-A rating indicates that investors should receive all their principal.

27.6 Static spread analysis of asset-backed bonds

It is apparent that the traditional gross redemption yield or yield-to-maturity method for measuring the return available from a particular instrument, and its relative value compared to the government yield curve, is not appropriate in the case of asset-backed bonds. In order to calculate a meaningful redemption yield measure, the cash flows for a particular instrument must be known with a degree of certainty. This is not the case for many asset-backed securities. The inaccuracy of the redemption yield measure also must take into account the fact that, even for conventional bonds, it ignores the reinvestment and dispersion of coupon payments, discounting them all at the same rate. This is, to all intents and purposes, glossed over for conventional bonds because the calculation treats all bonds the same, creating in effect a level playing field.

Asset-backed bonds have subtle differences in cash flow profiles and prepayment rates, making it more difficult to analyse them all in the same way. The exception to this is most credit card-ABS issues, which have a single bullet payment on maturity and (usually, but not always) a fixed coupon payment every month. It would be in order to compare the yield on such a bond with a conventional corporate bond. Other asset-backed issues such as mortgage and car loan securities do not have such a stable cash flow, mainly due to the amortising nature of the underlying loans. The simplest way to analyse these securities is to price them as a spread to the yield of the government bond whose maturity is closest to the weighted average life of the bonds being analysed. This is still an approximate measure and can be inaccurate under certain circumstances, and the convention now is to use the *static spread* measure.

27.6.1 Static spread analysis

Static spread analysis assumes that a particular bond itself represents a portfolio of individual securities, so that each cash flow is viewed as a zero-coupon bond. Under the static spread method each of these individual cash flows is discounted using a rate made up of the relevant government zero-coupon rate plus a spread. The spot rate used is the one whose duration matches that of the specific cash flow.⁶ The spread at which the sum of the discounted cash flows equalled their nominal price is known as the static spread, *zero-volatility spread*, or *Z-spread*.

This is illustrated with the following example, which uses three hypothetical securities. The first bond (Bond A) has a maturity of one year (12 months), while the remaining bonds B and C have average maturities of 12 months. Bond A has a single bullet redemption payment in exactly 12 months' time, exemplified by a conventional corporate bond or certain credit card-backed bonds. Bond B repays its principal over a 12-month period beginning six months from now, similar to a credit card-backed bond that has a controlled amortisation process. Bond C has an amortising principal repayment pattern, so that the amount of repayment declines gradually starting in the next month and finishing 24 months from now. This bond resembles an auto-ABS bond and certain mortgage pass-

⁶ In the US, UK and certain European markets there is a market in zero-coupon government bonds, known as strips. Therefore it is straightforward to find the appropriate spot rate to use in discounting the asset-backed bond cash flows, although banks frequently use an implied spot-rate curve because of liquidity effects of the observed zero-coupon curve. In the absence of an actual strip market, the implied spot rate calculated from the benchmark government bond yield curve is used.

through securities. The principal payment cash flows are shown as Figures 27.6 (i), (ii) and (iii). Assume that all bonds are trading at a yield to maturity of 6.33%. The one-year government bond is trading at a yield of 5.98%, therefore the yield spread on the three bonds is 35 basis points.⁷ This spread is also called the *nominal spread*.

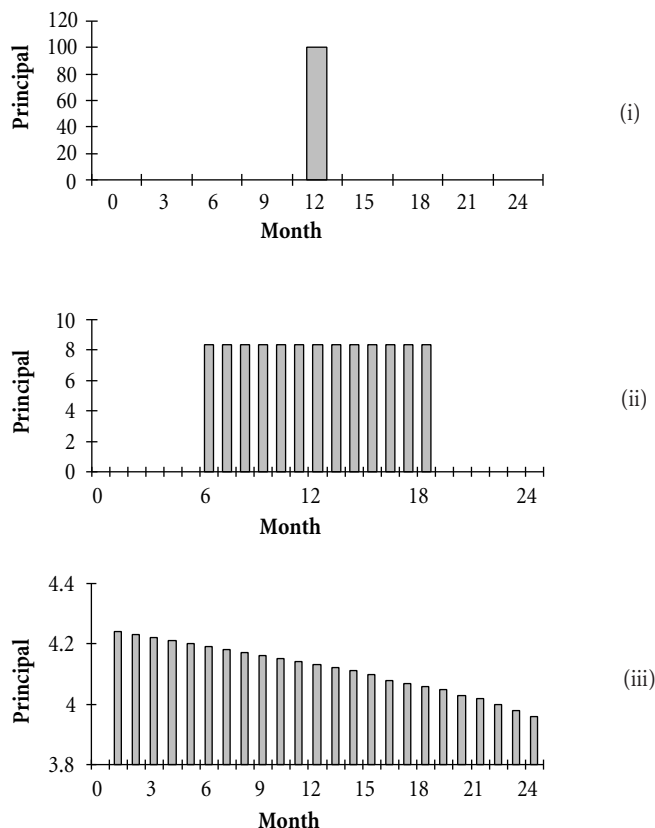


Figure 27.6: Cash flow profiles for bullet, controlled amortisation and amortised principal bonds (i), (ii), (iii).

We know that the redemption yield calculation discounts each cash flow at the internal rate of return, here 6.33%, which has been based on the average maturity of 12 months for receipt of the principal. In a positively sloping short-term yield curve environment, cash flows received early on would be discounted at a rate that was above the actual rate for that term, while cash flows received later would be discounted at a rate that was below the actual rate. This is an accepted drawback of the redemption yield method, and is accepted on the belief that the two effects will cancel each other out.

The static spread gives a more accurate picture yield. Assume that the short-term zero-coupon yield curve is as follows, shown in Figure 27.7:

⁷ The yields are based on market conditions for sterling asset-backed bonds in October 1999, against the gilt yield curve at that time.

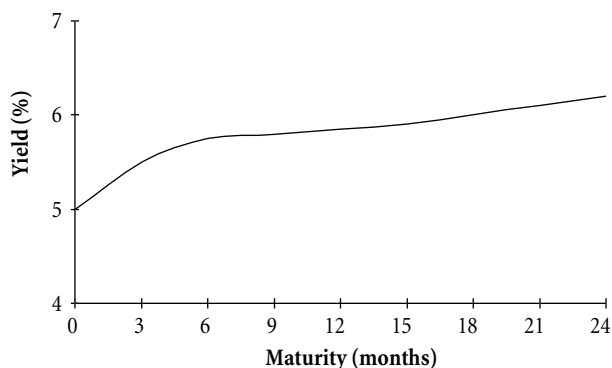


Figure 27.7: Short-term yield curve.

obtained by observing the yield of government discount instruments in the market. If we use the relevant yields from this curve to discount each of the principal cash flows for bonds A, B and C, the static spreads that are obtained, in the case of bonds B and C, differ from the nominal spread. This is because using this method more accurately captures the value of each cash flow over the period that they are received. The static spreads are shown in Table 27.4.

	Bond A	Bond B	Bond C
Nominal spread	35	35	35
Static spread	35	30	28

Table 27.4: Static and nominal spreads of hypothetical bonds A, B and C.

What is behind the difference in spreads for the two bonds with non-bullet principal repayment? Essentially these bonds, when based on an average maturity yield level plus spread, are over-valued. This is because cash flows received after the average maturity date are more sensitive to the discount rate which, using the 12-month yield, is below the actual rate in a positive yield curve environment. To allow for the lower present value of the individual cash flows to equate the higher price of the cash flows obtained using the single average maturity yield, the spread used to discount each cash flow must be lower. This is the static spread. Bond C has the greatest difference between the nominal spread and the static spread, because the cash flows are dispersed over a longer period, making each cash flow more sensitive to the discount rate used to obtain its present value. There is no difference in the two spreads for bond A, because the cash flow has been discounted at the correct rate.

In a positively sloped yield curve environment, the static spread on an amortising security will be lower than the nominal yield spread, while it will be higher than the nominal yield spread in a negative yield curve environment. The extent of the difference between the static spread and the nominal spread is a function of the slope of the yield curve and the dispersion of the bond's cash flows.

27.7 Conclusion

The asset-based markets represent a large and diverse group of securities which are suited to a varied group of investors. We have only touched upon them in these three chapters. Often they are the only way for institutional investors to pick up yield while retaining assets with high credit ratings. They are popular with issuers because they represent a cost-effective means of removing assets off the balance sheet, thus freeing up lending lines, and enabling them to have access to lower cost funding. Depending on the nature of the underlying asset backing, there are instruments available that cover the entire term of the yield curve. They are also available paying fixed- or floating-rate coupon. Although the market was developed in the United States, there are also liquid markets in the United Kingdom and other European markets. In the UK it is common for mortgage-backed bonds to have a floating coupon, reflecting the interest basis of UK mortgages, although there have been structures paying fixed coupon rates.

Yield analysis of asset-backed securities must take into account their uncertain cash flows. We have demonstrated that the traditional redemption yield measure, calculated using the security's average life maturity, is inappropriate, because it does not allow for the dispersed nature of the bond's cash. A mortgage or auto-loan backed bond has a principal repayment structure that disperses the payments over a length of time, so the more accurate yield measure to use is static spread. As well as accounting for dispersion, this approach also overcomes the traditional drawback of yield-to-maturity as it discounts each cash flow at the correct rate for that cash flow's maturity.

Selected bibliography and references

- Bear Stearns, *Asset-Backed Securities Special Report*, 5 December 1994.
 Bhattacharya, A., Fabozzi, F. (eds.), *Asset-Backed Securities*, FJF Associates, 1996.
 Fabozzi, F., Jacob, D., *The Handbook of Commercial Mortgage-Backed Securities*, FJF Associates, 1996.
 Fabozzi, F., Ramsey, C., Ramirez, F., *Collateralized Mortgage Obligations*, FJF Associates, 1994.
 Fabozzi, F., *Handbook of Structured Financial Products*, FJF Associates, 1998.
 Oppenheimer & Co., "Qualitatively Assessing Prepayments of Current- and Discount-Coupon PassThroughs", *Mortgage Research*, Oppenheimer & Co., Inc., 20 October 1994.

Questions and exercises

1. A mortgage pass-through security is used to create two separate instruments, the Class 1 and Class 2 issues. The Class 1 bond comprises 80% of the principal and 5% of the interest, while the Class 2 issue is made up of 20% of the principal and 95% of the interest. What is the name given to this type of mortgage-backed bond?
2. Why can the cash flow stream in a pool of mortgages not be determined with certainty?
3. What is the price/yield relationship for an IO security?
4. An investor in mortgage-backed bonds is concerned that interest rates will rise shortly, and wishes to reduce the prepayment risk level of his portfolio. Which class of issue in a CMO structure is best suited to him?
5. What is another name for a companion bond?
6. Consider a PAC issue for which the prepayment rates remain within the PAC bands for the first five years after its issue. What is likely to have happened to the upper and lower bands at the end of this period?
7. A CMO structure is made up of several PAC and companion bonds. After a period of time all the companion issues are retired, so that the structure becomes identical to a sequential-pay CMO. Explain what has happened and why.
8. To what extent does the creation of a CMO eliminate the prepayment risk associated with a conventional mortgage-backed security? Name the class of issues within a CMO that offer the greatest protection from prepayment risk for an investor.
9. Describe the main features of a sequential-pay CMO issue.
10. What is a *balloon* payment?
11. An asset-backed bond is calculated as having a redemption yield of 7.80%, based on an average life of 10.9 years. Explain what is meant by the average life of the bond. Why is this yield measure considered inappropriate for most asset-backed securities? What is a more accurate measure of the return of an amortising asset-backed security?
12. Detail the process by which cash flows that are received from the underlying mortgage pool by a CMO structure are re-directed so as to reduce prepayment risk for the bonds that make up the issue.
13. How do PAC bands work in reducing the level of prepayment risk faced by the holder of a PAC issue?
14. Ali Moheydin is a junior fund manager of an investment fund that holds asset-backed securities. He is considering proposing they invest up to \$500 million in PACs, IOs and POs that have been created from pass-

through securities with a coupon of 6% or 7%. He agrees that should hold only agency securities to minimise credit risk. Ali believes that these are attractive instruments because they have higher option-adjusted spreads than conventional pass-through securities; he also thinks that, as interest rates are expected to rise later in the year, IOs should outperform. He thinks the higher yield on these instruments is adequate compensation for the negative convexity of these bonds. Advise Ali on the following:

- (a) the yield levels that IOs would trade at if the market expects prepayment rates to fall, relative to the other securities being considered;
 - (b) is his view on negative convexity reasonable?
 - (c) what will happen to prepayment rates, and hence the value of the IOs, if Ali's view on future market rates turns out to be correct?
15. Stafford Bonnet Securities Limited are marketing various classes of mortgage-backed securities to their clients. A client suggests via e-mail that, while the yield on a mortgage bond is attractive compared to a government bond, whilst retaining triple-A quality, the uncertainty of the maturity, and the prepayment risk make it unsuitable for him to hold. Their senior analyst, Mr Win fires off a reply explaining how certain mortgage securities, including CMO structures, are able to alter the cash flow from mortgages so as to shift the prepayment risk across various classes of bondholders. Suggest a possible wording for the email.
 16. Explain the sequential-pay structure of a CMO issue.
 17. A stockbroker suggests to a client that a certain mortgage pass-through is yielding 7.50% at the moment and, as the client is looking at securities with a yield of 7.00% or above, it may be a suitable investment. What further information might the client request before deciding whether to purchase the security?
 18. Why are credit enhancements used as part of the arrangements for asset-backed securities? Describe the structure behind these four commonly-used enhancements:
 - (a) third-party guarantees
 - (b) letter of credit
 - (c) over-collateralisation
 - (d) reserve funds or cash collateral
 19. An asset-backed security is assigned an AAA-rating from a rating agency, and its only credit enhancements are third-party guarantees and a letter of credit from a bank. The bank is subsequently downgraded to AA grade. What will happen to the credit rating of the asset-backed issue?
 20. What is static spread?
 21. Rank the following securities in order of sensitivity to movements to market interest rates, the most sensitive first:
 - (a) auto-loan ABS
 - (b) mortgage pass-throughs
 - (c) CMOs
 - (d) credit-card ABS
 22. Explain why most credit card ABS bonds are regarded as essentially similar to conventional corporate bonds.

28 Collateralised Debt Obligations

Collateralised debt obligations (CDOs) are a form of securitised debt. The market in such bonds emerged in the US in the late 1980s, first as a form of repackaged high-yield bonds. The market experienced sharp growth in the second half of the 1990s, due to a combination of investor demand for higher yields allied to credit protection, and the varying requirements of originators, such as balance sheet management and lower-cost funding. The term “CDO” is a generic one, used to cover what are known as *collateralised bond obligations* (CBOs) and *collateralised loan obligations* (CLOs). Put simply a CBO is an issue of rated securities backed or “collateralised” by a pool of debt securities. A CLO on the other hand is an issue of paper that has been secured by a pool of bank loans. As the market has grown the distinction between the two types of structure has become blurred somewhat. Practitioners have taken to defining different issues in terms of the issuer’s motivation, the type of asset backing and the type of market into which the paper is sold, for example the commercial paper market. Different structures are now more often categorised as being *balance sheet* transactions, usually a securitisation of assets in order to reduce regulatory capital requirements and provide the originator with an alternative source of funding, or *arbitrage* transactions, in which the originator sets up a managed investment vehicle in order to benefit from a funding gap that exists between assets and liabilities. Balance sheet transactions are issuer driven, whereas arbitrage transactions are typically investor driven.

Over \$107 billion of CDOs was issued during 1999.¹ This included over \$51 billion of CBOs backed by high-yield bonds and just under \$45 billion of CLOs. During 1999 the combined CBO/CLO market was thought to account for approximately 18–20% of the total US asset-backed bond market.² In this chapter we provide an introduction to the most common CDO structures, as well as an overview of the analysis of these instruments. We also discuss the use of credit derivatives in the CDO market.

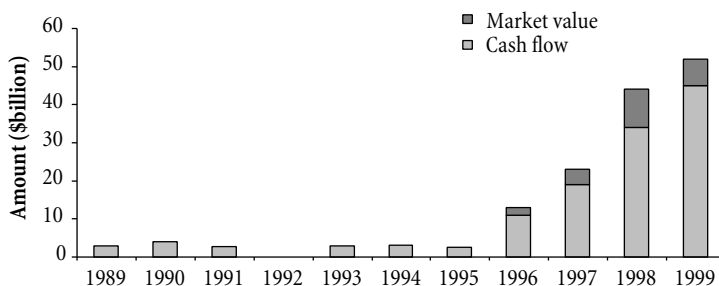


Figure 28.1: CDO issuance 1989–1999. Source: Moodys, Fitch, Bloomberg.

28.1 An overview of CDOs

28.1.1 Introduction

Collateralised debt obligation or CDO is the generic term for two distinct products, so-called balance sheet transactions and arbitrage transactions. The common thread between these structures is that they are both backed by some form of commercial or corporate debt or loan receivable. The primary differences between the two types are the type of collateral backing the newly-created securities in the CDO structure, and the motivations behind the transaction. The growth of the market has been in response to two key requirements: the desire of investors for higher-yield investments in higher-risk markets, managed by portfolio managers skilled at extracting value out of poorly performing or distressed debt, and the need for banks to extract greater value out of assets on their balance sheet, almost invariably because they are generating a below-market rate of return. By securitising bond or loan

¹ Source: *Risk*, 2000. This refers to CDO deals that had been rated by at least one of Moody’s, S&P and Fitch rating agencies.

² *Ibid.*

portfolios, banks can lower their capital charge by removing them off their balance sheet and funding them at a lower rate. The market has its origins in investor-driven arbitrage transactions, with bank balance sheet transactions a natural progression after banks applied securitisation techniques to their own asset base. Figure 28.2 is a summary of the key differences between balance sheet arbitrage and CDOs.

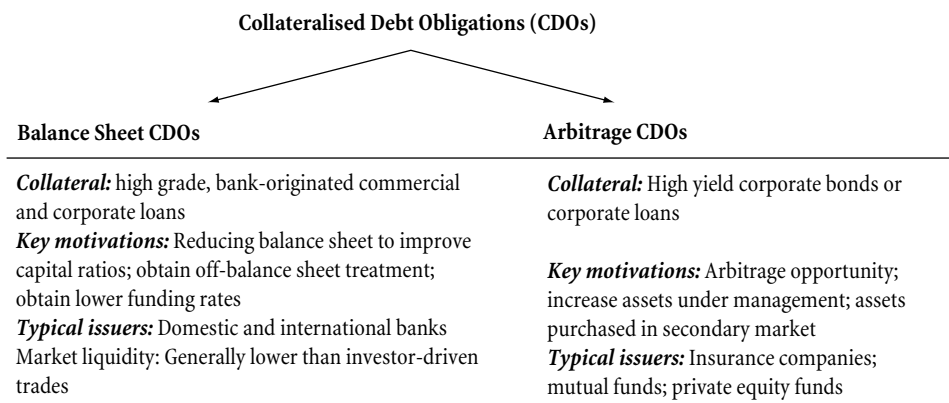


Figure 28.2: Collateralised debt obligations.

Balance sheet CDOs are structured securities that are usually backed with bank-originated, investment grade commercial and corporate loans. Since this form of collateral is almost invariably loans, and very rarely bonds, these transactions are usually referred to as collateralised loan obligations or CLOs. Why would a bank wish to securitise part of its loan portfolio? In short, in order to improve its capital adequacy position. Securitising a bank's loans reduces the size of its balance sheet, thereby improving its capital ratio and lowering its capital charge. The first domestic balance sheet CLO in the US market was the NationsBank Commercial Loan Master Trust, series 1997-1 and 1997-2, issued in September 1997, which employed what is known as a Master Trust structure to target investors who had previously purchased asset-backed securities.³

As balance sheet CLOs are originated mainly by commercial banks, the underlying collateral is usually part of their own commercial loan portfolios, and can be fixed term, revolving, secured and unsecured, syndicated and other loans. Although we note from Figure 28.6 that most CLOs have been issued by banks that are domiciled in the main developed economies, the geographical nature of the underlying collateral often have little connection with the home country of the originating bank. Most bank CLOs are floating-rate loans with average lives of five years or less. They are targeted mainly at bank sector Libor-based investors, and are structured with an amortising payoff schedule.

Arbitrage CDOs are backed with high-yield corporate bonds or loans. As the collateral can take either forms, arbitrage CDOs can be either CLOs or collateralised bond obligations (CBOs). Market practitioners often refer to all arbitrage deals as CDOs for simplicity, irrespective of the collateral backing them. The key motivation behind arbitrage CDOs is, unsurprisingly, the opportunity for arbitrage, or the difference between investment grade funding rates and high-yield investment rates. In an arbitrage CDO, the income generated by the high-yield assets should exceed the cost of funding, as long as no credit event or market event takes place.

Although CDOs are not a recent innovation, the market only experienced high growth rates from 1995 onwards, and certain investors are still prone to regard it as an "emerging" asset class. However in terms of volume in the US market, CDOs are comparable to credit card and automobile loan asset-backed securities, as illustrated in Figure 28.3 below.

³ The Master Trust structure is a generic set up that allows originators to issue subsequent asset-backed deals under the same legal arrangement, thus enabling such issues to be made quicker than they otherwise might be. Investors also welcome such a structure, as they indicate a commitment to liquidity by implying further issues into the market.

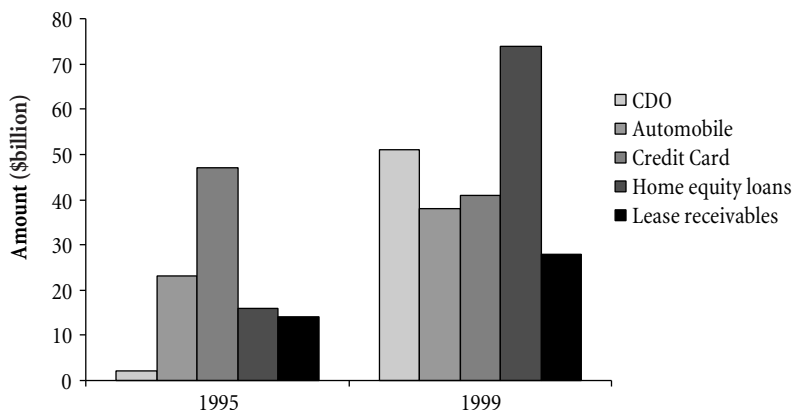


Figure 28.3: CDO supply versus other ABS products in US market, 1995 and 1999.

Source: Bloomberg, Bank of America.

28.1.2 Security structure

Arbitrage and balance sheet CDOs generally have similar structures. In essence a special purpose vehicle (SPV) purchases loans or bonds directly from the originator or from the secondary market. SPVs have been set up in a number of ways, these include a *special purpose corporation*, a limited partnership or a limited liability corporation. An SPV will be bankruptcy-remote, that is unconnected to any other entities that support it or are involved with it. The parties involved in a transaction, apart from the investors, are usually a portfolio manager, a bond trustee appointed to look after the interests of the investors, a credit enhancer and a back-up servicer. Some structures involve a swap arrangement where this is required to alter cash flows or set up a hedge, so in such cases a swap counterparty is also involved. A basic structure is shown in Figure 28.4, which is applicable to both CBOs and CLOs.

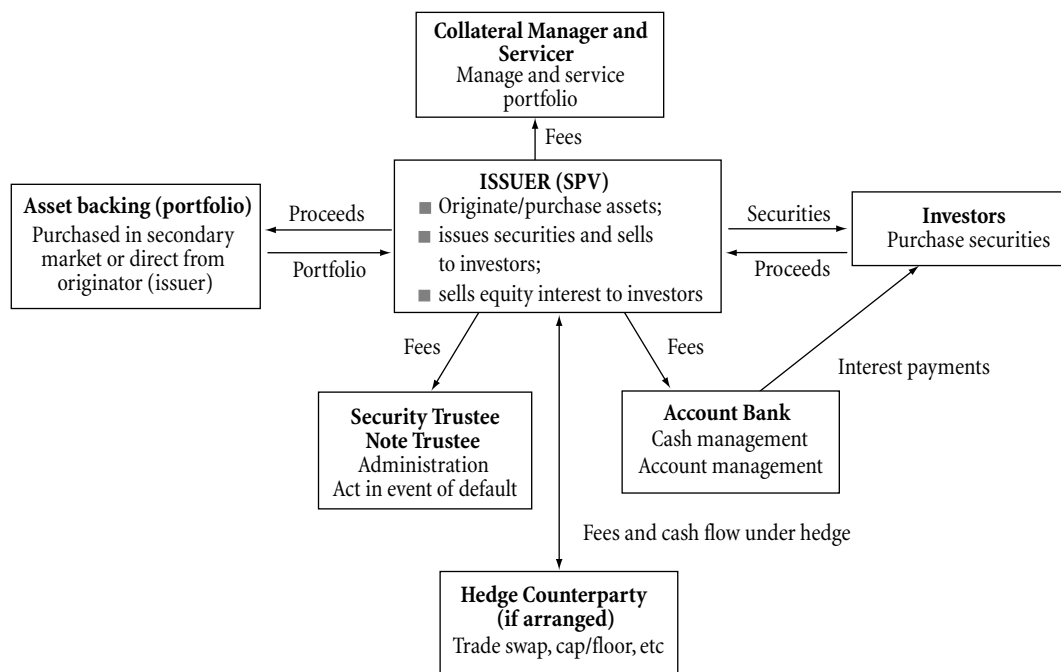
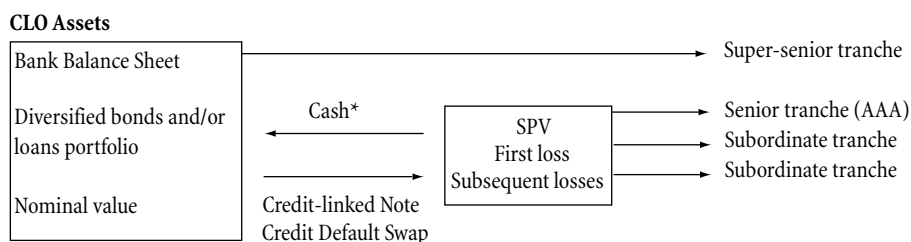


Figure 28.4: Basic CDO structure.

Invariably the SPV will issue a number of classes of debt, with credit enhancements included in the structure such that different tranches of security can be issued, each with differing levels of credit quality. Other forms of protection may also be included, including a cash reserve.

A further development has been the issue of *synthetic* CDOs. Synthetic CLO structures use credit derivatives that allows the originating bank to transfer the risk of the loan portfolio to the market. In a synthetic structure there is no actual transfer of the underlying reference assets; instead the economic effect of a traditional CDO is synthesised by passing to the end investor(s) an identical economic risk associated with the underlying assets, the same that would have been transferred had the SPV actually purchased the assets. This effect is achieved by the provision by a counterparty of a *credit default swap*, or by the issue of *credit-linked notes* by the originating bank, or by a combination of these approaches. In a credit-linked CLO the loan portfolio remains on the sponsoring bank's balance sheet, and investors in the securities are exposed to the credit risk of the bank itself, in addition to the market risk of the collateralised portfolio. Therefore, the credit rating of the CLO can be no higher than that of the originating bank. We will look at these structures in more detail later in the chapter. Figure 28.5 shows a simplified synthetic CDO structure.



* Cash raised by the SPV goes to the originating bank in "linked" transactions or is invested in Treasury securities in "delinked" transactions.

Figure 28.5: Synthetic CDO structure.

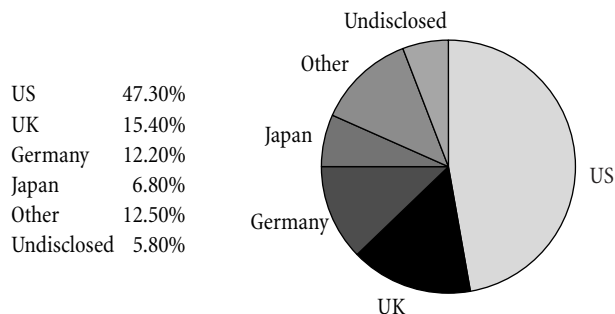


Figure 28.6: Outstanding bank CLOs by country of originating bank in 1999.

Source: Fitch, Bank of America.

28.1.3 Comparisons with other asset-backed securities

The CDO asset class has similarities in its fundamental structure with other securities in the ABS market. Like other asset-backed securities, a CDO is a debt obligation issued by a special purpose vehicle, secured by a form of receivable. In this case though, the collateral concerned is high yield loans or bonds, rather than say, mortgage or credit card receivables. Again similar to other ABS, CDO securities typically consist of different credit tranches within a single structure, and the credit ratings range from AAA to B or unrated. The rating of each CDO class is determined by the amount credit enhancement in the structure, the ongoing performance of the collateral, and the priority of interest in the cash flows generated by the pool of assets. The credit enhancement in a structure is among items scrutinised by investors, who will determine the cash flow *waterfalls* for the interest and principal, the prepayment conditions, and the methods of allocation for default and recovery. Note that the term "waterfall" is

used in the context of asset-backed securitisations that are structured with more than one tranche, to refer to the allocation of principal and interest to each tranche in a series. If there is excess cash and this can be shared with other series, the cash flows are allocated back through the waterfall, running over the successive tranches in the order of priority determined at issue.

A significant difference between CDOs and other ABS is the relationship to the servicer. In a traditional ABS the servicing function is usually performed by the same entity that sources and underwrites the original loans. These roles are different in a CDO transaction; for instance there is no servicer that can collect on non-performing loans. Instead the portfolio manager for the issuer must actively manage the portfolio. This might include sourcing higher quality credits, selling positions before they deteriorate and purchasing investments that are expected to appreciate. In essence portfolio managers assume the responsibility of a servicer. Therefore investors in CDOs must focus their analysis on the portfolio manager as well as on the credit quality of the collateral pool. CDO structures also differ from other ABS in that they frequently hold non-investment grade collateral in the pool, which is not a common occurrence in traditional ABS structures. Finally CDO transactions are (or rather, have been to date) private and not public securities.

28.1.4 CDO asset types

The arbitrage CDO market can be broken down into two main asset types, *cash flow* and *market value* CDOs. Balance sheet CDOs are all cash flow CDOs.

Cash flow CDOs share more similarities to traditional ABS than market value transactions. Collateral is usually a self-amortising pool of high-yield bonds and loans, expected to make principal and interest payments on a regular basis. Most cash flow CDO structures allow for a reinvestment period, and while this is common in other types of ABS, the period length tends to be longer in cash flow CDOs, typically with a minimum of four years. The cash flow structure relies upon the collateral's ability to generate sufficient cash to pay principal and interest on the rated classes of securities. This is similar to an automobile ABS, in which the auto-backed securities rely upon the cash flows from the fixed pool of automobile loans to make principal and interest payments on the liabilities. Trading of the CDO collateral is usually limited, for instance in the event of a change in credit situation, and so the value of the portfolio is based on the par amount of the collateral securities.

Market value CDOs, which were first introduced in 1995, resemble hedge funds more than traditional ABS. The main difference between a cash flow CDO and a market value CDO is that the portfolio manager has the ability to freely trade the collateral. This means investors focus on expected appreciation in the portfolio, and the portfolio itself may be quite different in say, three months' time compared to its composition today. This leads to the analogy with the hedge fund. Investors in market value CDOs are as concerned with the management and credit skills of the portfolio manager as they are with the credit quality of the collateral pool. Market value CDOs rely upon the portfolio manager's ability to generate total returns and liquidate the collateral in timely fashion, if necessary, in order to meet the cash flow obligations (principal and interest) of the CDO structure.

Different portfolio objectives result in distinct investment characteristics. Cash flow CDO assets consist mainly of rated, high-yield debt or loans that are *current* in their principal and interest payments, that is they are not in default. In a market value CDO the asset composition is more diversified. The collateral pool might consist of say, a 75:25 percentage split between assets to support liability payments and investments to produce increased equity returns. In this case, the first 75% of assets of a market value CDO assets will resemble those of a conventional cash flow CDO, with say 25% invested in high-yield bonds and 50% in high-yield loans. These assets should be sufficient to support payments on 100% of the liabilities. The remaining 25% of the portfolio might be invested in "special situations" such as distressed debt, foreign bank loans, hybrid capital instruments and other investments. The higher yielding investments are required to produce the higher yields that are marketed to equity investors in market value CDOs.

We have described in general terms the asset side of a CDO. The liability side of a CDO structure is similar to other ABS structures, and encompasses several investment grade and non-investment grade classes with an accompanying equity tranche that serves as the first loss position. In say, a mortgage-backed transaction the equity class is not usually offered but instead held by the issuer. Typically in the US market rated CDO liabilities have a 10–12 year legal final maturity. The four main rating agencies⁴ all actively rate cash flow CDOs, although commonly transactions carry ratings from only one or two of the agencies.

⁴ That is, Standard& Poor's, Moody's, Fitch IBCA and Duff & Phelps. [Since this was written, the last two have merged to become Fitch IBCA, Duff & Phelps].

Liabilities for market value CDOs differ in some ways from cash flow CDOs. In most cases senior bank facilities provide more than half of the capital structure, with a 6–7 year final maturity. When a market value transaction is issued, cash generated by the issuance is usually not fully invested at the start. There is a *ramp-up* period to allow the portfolio manager time to make investment decisions and effect collateral purchases. Ramp-up periods result in a risk that cash flows on the portfolio's assets will not be sufficient to cover liability obligations at the start. Rating agencies consider this ramp-up risk when evaluating the transaction's credit enhancement. Ramp-up periods are in fact common to both cash flow and market value CDOs, but the period is longer with the latter transactions, resulting in more significant risk.

We noted that although CDOs were created at almost the same time as the first ABS issues, with the first structure appearing in 1988, it was only in the latter half of the 1990s that the product evolved sufficiently and in enough volume to be regarded as a distinct investment instrument. The US market has witnessed the most innovative structures, but interesting developments have also taken place in the UK and Germany. Figure 28.7 summarises the evolution in the CDO product in the US market from its first appearance to present arrangements. In particular collateral types backing the securities have grown considerably, with increasing sophistication in structure and cash flow mechanics. In 1999 CDOs covered a wide spectrum of credit risk and investment returns, from a diverse pool of high-yielding assets. Investors analyse CDOs as investment instruments in their own right and also with regard to the relative value offered by them vis-à-vis other ABS products.

Early CDO balance sheet

<i>Assets</i>	<i>Liabilities</i>
US domestic high yield bonds	Fixed-rate private securities
	Equity

Present-day CDO balance sheet

<i>Assets</i>	<i>Liabilities</i>
US domestic high yield bonds	AAA to BBB fixed rate securities
US domestic high yield loans	AAA to BBB floating rate securities
Emerging market debt	BB Mezzanine securities
Special situation/distressed debt	Contingent interest securities
Foreign bank loan	Credit-linked notes
	Equity

Figure 28.7: CDO product evolution.

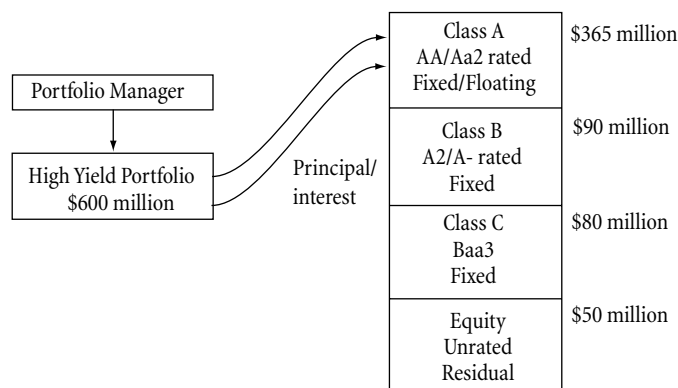


Figure 28.8: Hypothetical cash flow CBO structure.

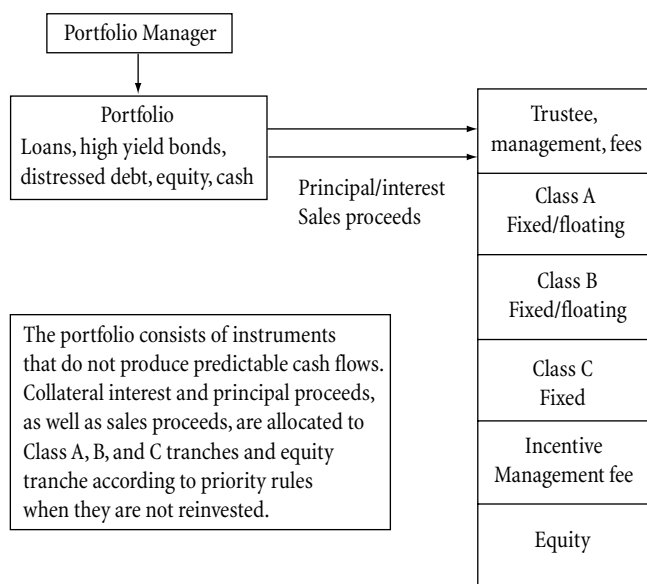


Figure 28.9: Hypothetical market value CBO structure.

28.2 Relative value analysis

Investors have a number of motivations when considering the CDO market both in their domestic market and abroad. These include:

- the opportunity to gain exposure to a high-yield market on a diversified basis, without committing significant resources;
- the ability to choose from a number of portfolio managers that manage the CDO;
- CDOs acting as an initial entry point into the high yield market;
- with respect to lower-rated (BBB and below) tranches, achieving leveraged returns while gaining benefit from a diversified portfolio;
- the appeal of a wide investor base, with ratings ranging from AAA to B and maturities from four years to as long as 20 years;
- wide variety of collateral.

CDOs offer investors a variety of risk/return profiles, as well as market volatilities, and their appeal has widened as broader macroeconomic developments in the global capital markets have resulted in lower yields on more traditional investments.

Investors analysing CDO instruments will focus on particular aspects of the market. For instance those with a low appetite for risk will concentrate in the higher rated classes of cash flow transactions. Investors that are satisfied with greater volatility of earnings but still wish to hold AA- or AAA-rated instruments may consider market value deals. The “arbitrage” that exists in the transaction may be a result of:

- industry diversification;
- differences between investment grade and high-yield spreads;
- the difference between implied default rates in the high yield market and expected default rates;
- the liquidity premium embedded in high yield investments;
- the LIBOR rate versus the Treasury spread.

The CDO asset class cannot be compared in a straightforward fashion to other ABS classes, which makes relative value analysis difficult. Although a CDO is a structured finance product, it does not have sufficient common

characteristics with other such products. The structure and cash flow of a CDO are perhaps most similar to a commercial mortgage-backed security; the collateral backing the two types share comparable characteristics. Commercial mortgage pools and high yield bonds and loans both have fewer obligors and larger balances than other ABS collateral, and each credit is rated. On the other hand CDOs often pay floating-rate interest and are private securities,⁵ whereas commercial MBS (in the US market) pay fixed rate and are often public securities.

Historically during 1998 and 1999 CDO spreads offered greater opportunity for spread tightening than traditional ABS, as CDO yields were observed to take a longer time to recover from the bond bear market that existed in the second half of 1998. This is illustrated in Figure 28.10, with a comparison to commercial mortgage-backed securities. Although these had returned to pre-bear market levels, CDO spreads remained at the higher levels, that is they required a longer recovery period. From this picture at the start of the second quarter in 1999, CDP spreads had potentially a significant scope for tightening before reaching historical low spreads. This was in fact confirmed in the second half of the year, when spreads came in to lower levels. In assessing relative value therefore an investor will assess the difference in spreads between CDOs and other ABS asset classes.

We have presented only an overview here; interested readers may wish to consult the references at the end of this chapter, principally Fabozzi (1998; Chapter 18) and the study by ING Barings.

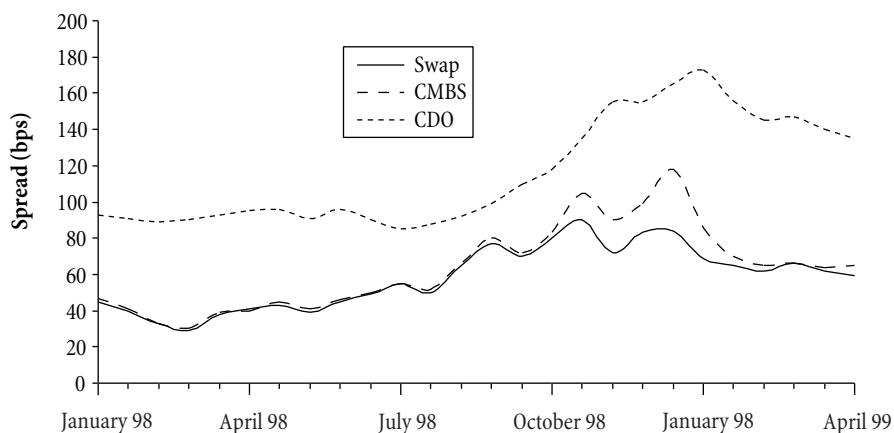


Figure 28.10: Historical spreads January 1998–March 1999.

Source: Bloomberg, Bank of America.

28.3 Credit derivatives⁶

Credit derivatives include a range of instruments designed to transfer credit risk without requiring the sale or purchase of bonds or loans. They originated in the early 1990s in the US market, among banks that wished to manage the credit risk in their loan portfolios while preserving customer relationships. Credit derivatives are reviewed in Part XI of this book. Here we consider their use in the CDO market.

Credit derivatives allow banks to provide the following enhanced services to their clients:

- tailored exposure to credit risk;
- the ability to take short positions in credits of underlying securities without having to take a position in the security itself;
- providing investors with access to the bank loan market, generally on a leveraged basis;

⁵ In the US market, they are also filed under Rule 144A, as opposed to public securities which must be registered with the Securities and Exchange Commission. Rule 144A securities may only be sold to investors classified as professional investors under specified criteria. Rule 144A provides an exemption from the registration requirements of the Securities Act (1933) for resale of privately placed securities to qualified institutional buyers. Such buyers are deemed to be established and experienced institutions, and so the SEC does not regulate or approve disclosure requirements.

⁶ Credit derivatives and their use in the debt markets are investigated in more detail in Part XI.

- the ability to extract and hedge specific sections of credit risk, for example defaulted par, defaulted coupon and credit rating migration.

The use of credit derivatives has added to the liquidity in the CDO market, by providing a more accurate asset-liability in CBO/CLO structures, increasing diversification in CBO collateral portfolios and repackaging illiquid CBO bonds to tailor risk to specific investor preferences.

The main credit derivative instruments are credit default swaps, total return swaps and credit-linked notes.

28.3.1 Application of credit derivatives

Synthetic CDOs originated for balance sheet purposes are intended to reduce regulatory capital requirements. Examples include *Glacier* in September 1997 and *Bistro* in December 1997, issued by (the then) Swiss Bank Corporation and JP Morgan respectively. These are believed to be the first structures that included credit derivatives. As we noted earlier, in a synthetic CDO a financial institution sets up an SPV to finance asset purchases. However as it is a synthetic structure, total return swaps (TRS) are used to pass through underlying security returns to a credit-linked note. These notes are in turn placed in an SPV that sells a range of liabilities. The use of TRSs in such structures allows for the maintenance of client confidentiality as well as a reduction in credit exposure. Compared to traditional structures, synthetic CDOs require a higher return, a downside for issuers, and this is achieved via the leverage offered by TRSs.

Another application of credit derivatives is in subordinated debt and default protection. The subordinated or mezzanine debt in a cash flow CBO is debt protected by an equity layer and an excess spread. One method by which liquidity of the mezzanine debt can be increased is to extract the mezzanine cash flows and place them inside a credit derivative. These repackaged cash flows can in turn be sold as an investment grade security to investors.

Here we illustrate the application of credit derivatives in CDO deals with the following hypothetical examples.

EXAMPLE 28.1

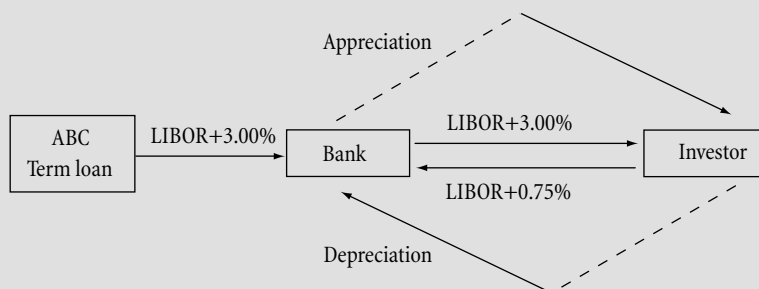


Figure 28.11: Total return swap: single reference asset.

1(i) Summary terms

<i>Reference asset</i>	ABC Bank loan
<i>Notional amount</i>	\$30 million
<i>Upfront collateral</i>	\$3 million
<i>Collateral return</i>	6.00%
<i>Term</i>	One year
<i>Initial price</i>	100
<i>Investor (buyer) pays</i>	LIBOR + 0.75% plus price depreciation
<i>Investor receives</i>	Interest + fees + price appreciation

1(ii) Remarks

This structure preserves the capital structure for the originating bank and requires minimal administration. However it allows the bank access to off-balance sheet financing.

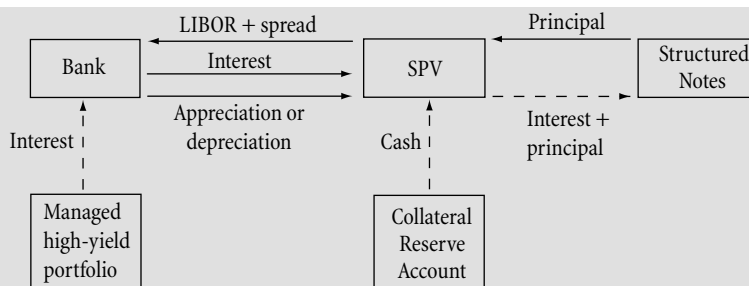


Figure 28.12: Synthetic CBO using total return swaps: TRS financing.

2(i) Summary terms

<i>Reference asset</i>	High yield loan portfolio
<i>Notional amount</i>	\$500 million
<i>Collateral return</i>	LIBOR + 3%
<i>Term</i>	12 years
<i>Initial price</i>	100
<i>Investor (buyer) pays</i>	Price of note
<i>Investor receives</i>	Interest + par

2(ii) Remarks

This structure enables investors to have access to the bank loan market, and provides protection against market downside. It generates high return for investors with lower transaction costs, and for originators is an efficient use of capital.

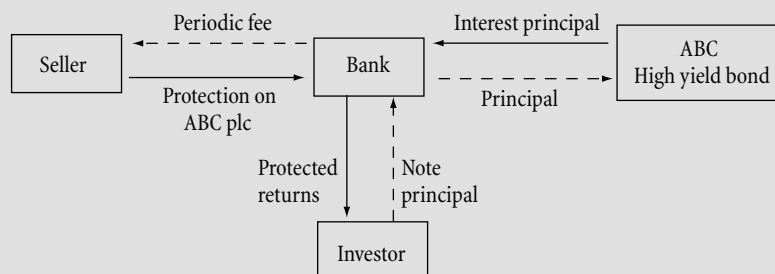


Figure 28.13: Credit-linked note: single reference asset.

3(i) Summary terms

<i>Reference asset</i>	High yield bond
<i>Notional amount</i>	\$50 million
<i>Collateral return</i>	9.00%
<i>Term</i>	Two years
<i>Initial price</i>	100
<i>Investor (buyer) pays</i>	Price of note
<i>Investor receives</i>	Interest + par

3(ii) Remarks

A simple structure with minimal administration. Another form of off-balance sheet financing.

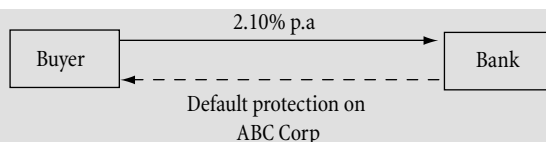


Figure 28.14: Credit default swap: purchase swap.

4(i) Summary terms

<i>Reference asset</i>	ABC Corporation bond (8.5% December 2005)
<i>Notional amount</i>	\$10 million
<i>Term</i>	Five years
<i>Buyer pays</i>	2.10% per annum
<i>Buyer receives</i>	Notional amount $\times$ (100% — market price) if credit event occurs

4(ii) Remarks

This is a simple credit default protection for the holder of the corporate bond, with the asset remaining on the balance sheet

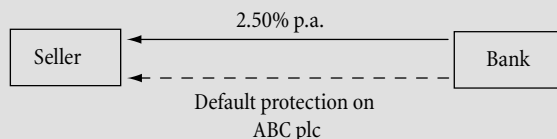


Figure 28.15: Credit default swap: sell swap.

5(i) Summary terms

<i>Reference asset</i>	ABC plc senior secured revolving loan (due 2005)
<i>Notional amount</i>	\$30 million
<i>Term</i>	Two years
<i>Investor receives</i>	2.50% per annum
<i>Investor pays</i>	Notional amount $\times$ (100% — market price) if credit event occurs

5(ii) Remarks

This is an unfounded investment that is off-balance sheet. It requires minimal administration and can be flexible in terms of maturity.

Selected bibliography

- Braddock, J., *Derivatives demystified: using structured finance products*, John Wiley and Sons, 1997.
 Fabozzi, F., *Handbook of Structured Financial Products*, FJF Associates, 1998.
 ING Barings, *International Asset Securitisation*, 1999.
 Mattoo, M., *Structured Derivatives*, Pitman, 1996.
 Norton Rose, *Collateralised Debt Obligations*, 2000.
 Peng, S., Dattatreya, R., *The Structured Note Market*, Pitman, 1994.

29

High-yield Bonds

The high-yield corporate bond market developed in the United States during the 1980s, and was a symbol of market growth in that decade. The subsequent collapse of the “junk bond” market in 1990 signalled a downturn in the market, but in recent years it has been re-developed into a liquid and relatively stable market, regarded as a viable source of capital for non-rated companies and companies that are rated below investment grade. It is now referred to as the *high-yield* bond market and an equivalent market has also developed in the United Kingdom and Europe. Economic performance through the second half of the 1990s, which has seen low inflation and steady growth in the US, and relatively low inflation and convergence of yields in Europe, as a result of introduction of the euro currency, has depressed bond yields to historically low levels across in developed countries. Investors who require higher yielding debt assets must look to “emerging” markets or, increasingly, the high-yield market. In this chapter we provide a brief review of the US high-yield market, including the different types of securities that have been issued and the returns that were generated through the last decade.

29.1 Growth of the market

The US domestic high-yield market stood at approximately \$200 billion nominal outstanding in 1990, and had grown to over \$335 billion at the end of 1998.¹ The largest amount issued in one year was in 1993, when just under \$70 billion was issued. Prior to 1990 around half of all issuance in the high-yield market was used to fund acquisitions and “leveraged buy-outs” (LBOs), but by 1995 over half of new issues was used for the purposes of refinancing existing debt and as growth capital. The proportion used for LBOs had fallen to 4% in that year. The overall credit quality of the market improved steadily during the second half of the 1990s, which was explained by analysts as reflecting the performance and health of the US economy as a whole. Figure 29.1 shows the change of composition of the credit ratings in the market as a whole. The proportion of issuers rated at BB-grade had increased from 17% in 1987 to 48% in 1997.

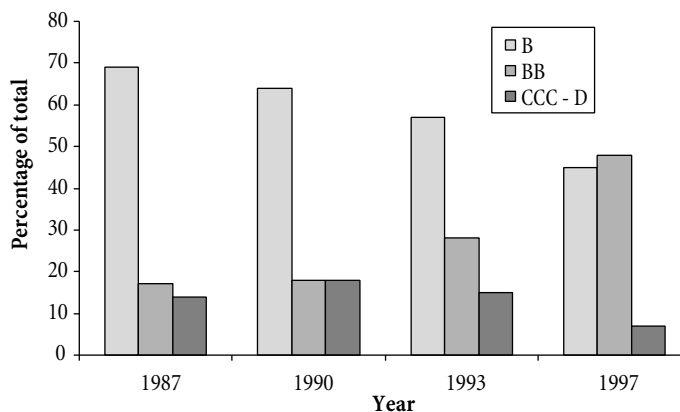


Figure 29.1: US high-yield bond market credit ratings, 1987–1997.

Source: Standard & Poor's.

Bonds have been issued by representatives of a number of industry sectors. The highest growth has been observed in the media, telecommunications and cable sectors, which represented over 15% of total debt outstanding in 1997. Outside the industrial sectors, high-yield bonds have been issued by energy, utility and finance companies, as well as by Canadian companies, whose bonds are officially designated as Yankee bonds.

¹ The statistical data in this chapter is sourced from Bloomberg and Reuters, except where indicated.

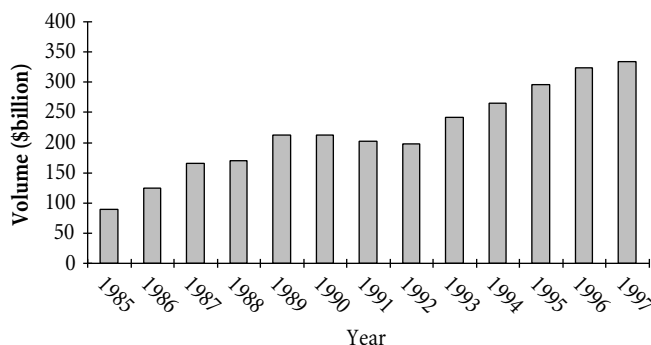


Figure 29.2: Growth of US high-yield bond market.

Source: Lehman Brothers.

29.2 High-yield securities

The high-yield market contains a range of securities, which have developed as investor interest in the market has grown. The majority of issues have been conventional plain vanilla issues however, although the market has also seen deferred-pay securities, which recognises that issuing companies will in all probability encounter cash-flow difficulties in their early years, as well as increasing-rate bonds. We present here a summary of the types of instruments traded in the market.

29.2.1 Conventional securities

The majority of bond issues are plain vanilla securities, with a fixed coupon; this type of security comprised over 90% of all domestic high-yield debt in 1997. The maturity of these bonds is between 7–12 years, although most bonds incorporate a call provision that is effective after the first three or five years. A call feature is important to issuers in the high-yield market, as it gives them an opportunity to refinance their debt at a lower interest rate when, as is hoped, their credit rating has improved.

29.2.2 Split-coupon securities

A *split-coupon* security is one that pays no coupon for the first few years of its life, and then transforms into a fixed-coupon bond for the remaining years of its life. They therefore begin life as zero-coupon bonds, typically for the first five years, although the no-interest period has been as low as two years and as high as seven years, before turning into conventional bonds. The bonds are issued at a discount, using the stated coupon as the discount rate. A split-coupon bond allows its issuer to build up its financial health before it has to service its debt. The most frequent issuers of split-coupon bonds have been operating in high-growth industry such as technology companies, or companies in the process of re-structuring their operations.

Another approach adopted by new companies has been to “over-fund” their required proceeds, and use the overfunding to pay the interest charges on the debt for the first few years of the bond’s life. This mirrors the effect achieved using a split-capital issue. For example a funding requirement of \$60 million is met through the issue of \$100 million of a 9% coupon bond; the extra capital raised is used to pay interest on the debt for the first four years.² An overfunded bond is designed to appeal to investors who have an appetite for high-yield debt but do not wish to purchase instruments that do not pay any interest.

29.2.3 Payment-in-kind securities

A *payment-in-kind* (PIK) bond is one that allows the issuer the option of paying coupon interest either in cash or similar securities. The option is usually set only for a short, limited period of time, say three or five years. If the issuer chooses to pay in the form of in-kind securities, the principal amount of the debt will increase, raising the liability of the issuer after the in-kind option falls away. When the in-kind option is in effect, PIK bonds trade

² There is “spare” \$4 million left over at the end of the four years, but this is invested at the time of the issue in the same way as the required \$60 million.

differently to other high-yield bonds, because no interest is payable this period. Hence the bonds trade without accrued interest, and the coupon payment is reflected in the accreted price of the security. This is similar to the method for quoting cumulative preference shares whose preferred dividend has not been paid in one year.

29.2.4 Step-up coupon securities

These types of bonds are common in conventional corporate markets, issued by companies who wish to make their paper more attractive to investors, or to a wider group of investors. A step-up bond is similar to the split-coupon bond, and is issued for similar reasons, except during the initial period a coupon is payable, rising to a higher coupon at a specified date. The higher coupon is set at the time of issue, and remains in force until the bond is redeemed. The initial period is usually for around five years, but can be for a shorter period.

29.2.5 Exchangeable variable-rate notes

Exchangeable variable-rate notes (EVRNs) are subordinated, medium-term instruments that pay a floating, quarterly coupon. The interest is actually fixed for a short term, known as the “teaser” period, but after this initial period the rate changes to a floating one, linked to a reference rate such as the prime rate or the 90-day Treasury bill rate. In most cases the issuer has the option to exchange the notes for fixed-coupon notes with conventional features such as a fixed maturity date or call facility. The maximum maturity period of the bonds is usually five years.

29.2.6 Bond and stock units

Certain companies in high-growth and/or high-risk business sectors have issued combinations of bonds and equity. This is designed to appeal to investors who require an element of equity ownership in order to benefit from upside growth, to compensate them for the high risk incurred by debt holders. The resulting debt issue therefore has an element of equity attached, in the form of straight shares or warrants. Investors usually strip the equity or the warrants from the bonds on issue and may realise them immediately if they wish. Bonds issued with equity warrants are also known as *usable bonds*.

29.2.7 Springing issues

Start-up companies operating in highly competitive, volatile environment sometimes issue securities that are specified to change one or more of their characteristics in response to a specific event occurring. For example the market has seen a springing warrant issue, which are exercisable only if another party attempted to acquire the issuer. Another version of a springing security was one that was originally issued as a subordinated bond, but was converted to senior debt once an old outstanding debenture had been discharged. The analysis of a springing security must take into account the probability that the security will change its terms and conditions and what impact such an event will have on the security's price.

29.2.8 Extendible/re-set securities

An extendible or reset security allows the issuer to reset the coupon or extend the life of the bond on pre-specified dates. The general rule, stated on issue, is that the coupon must be reset to a level that results in it trading at a certain yield, and which is determined by a third-party bank (usually the issuing investment bank). The alternative is for the bond to be issued with a cap or floor, or both. The ability to reset the coupon can be a valuable asset if market rates have fallen since the time of issue, while it is also attractive to investors in a rising interest rate environment and as a form of protection against a decline in the issuer's credit quality. This will add to the overall debt burden of the issuer however, so may have negative effects.

Extendible bonds are issued with a final redemption date already stated. The issuer may choose to have the bonds redeemed at one of a range of dates ahead of the final maturity date, and these dates are usually also associated with a *put* feature. If the issuer decides to extend the redemption date, a certain portion of the debt may have to be retired, or may be put back if investors so wish. This provision sometimes applies if the coupon is reset as well.

Only a small number of such issues were still trading in the US market in 1997.

29.2.9 High-yield bank loans

The development of a market in tradeable bank loans has served to break down many of the distinctions between high-yield loans and bonds. There has been a steady increase in secondary loan trading during the 1990s, indeed after the “junk” bond market declines in 1990, high-yield bank loans comprised virtually all of the debt issued to

non-investment grade companies in that year. Over \$40 billion of high-yield bank loans were issued in 1997. Another type of bank loan that trades in the secondary market is the “B loan”, which is a long-dated loan that is serviced with interest payments only (that is, no repayment of principal) in the first few years after issue.

The main differences between high-yield bank loans and high-yield bonds are as follows:

- bank loans are typically floating-rate liabilities, while high-yield debt is almost invariably fixed-rate;
- bank loans are callable at any time;
- there is an element of security associated with most bank loans.

These differences ensure that the two markets remain distinct, if part of the overall “high-yield” market.

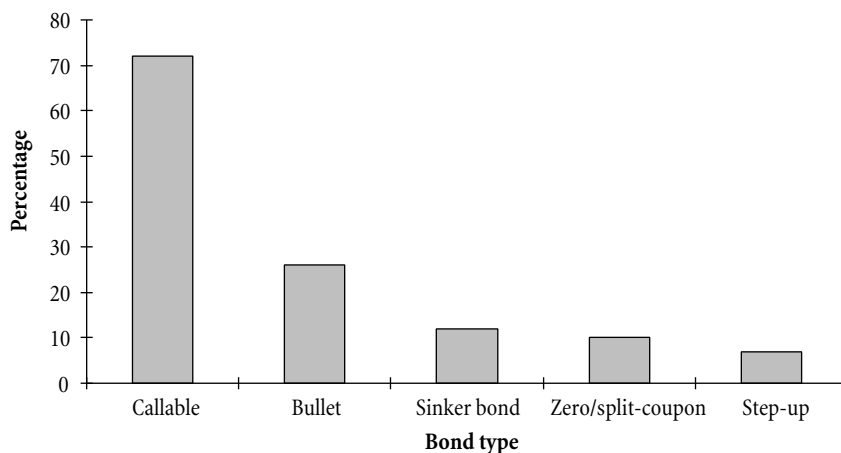


Figure 29.3: Types of high-yield securities, US domestic market, December 1997.

Source: Lehman Brothers.

Notwithstanding that the large majority of high-yield bonds are plain vanilla securities, there has been a proliferation of the types of instrument traded in the high-yield market. Bonds have been issued that offer investors a share in the firm’s profits, as well as a fixed coupon. Other issues have been backed by commodities.

A common method of issue of high-yield debt in the US is via the SEC rule known as Rule 144A. Under this rule, securities sold initially through a private placement which are then offered for re-sale to institutional investors, is not subject to review by the SEC. The borrower must register the securities within a short period of time, usually one to three months after issuance. By placing debt under this rule, the issue may be got away fairly quickly and at minimal cost. The type of bond issued is not relevant: any of the securities described here may be issued pursuant to Rule 144A.

29.3 High-yield bond performance

The performance of high-yield bonds in the US domestic market mirrors economic performance in the US overall. The yield spread of high-yield debt over investment grade corporate bonds is more of a function of the general outlook of the economy than of the health of the issuing companies themselves, although the latter is obviously important. Thus high-yield debt outperformed corporate bonds and other securities such as Treasury bonds and mortgage-backed securities in 1991 and 1992 as the US economy recovered from a recession, after having generated negative returns in 1990. During the period 1992–1995 the US economy was marked by a favourable combination of steady economic growth and low inflation and spreads moved to historical lows, before widening out considerably after the 1997 and 1998 bond market downturns, brought on (first) by South-east Asian currency collapse and debt worries and then by the Russian bond technical default. This triggered a “flight to quality” that affected both emerging market and high-yield debt alike.

Figure 29.4 illustrates how the yield spread on high-yield bonds of three different ratings compared during the late 1980s and 1990s.

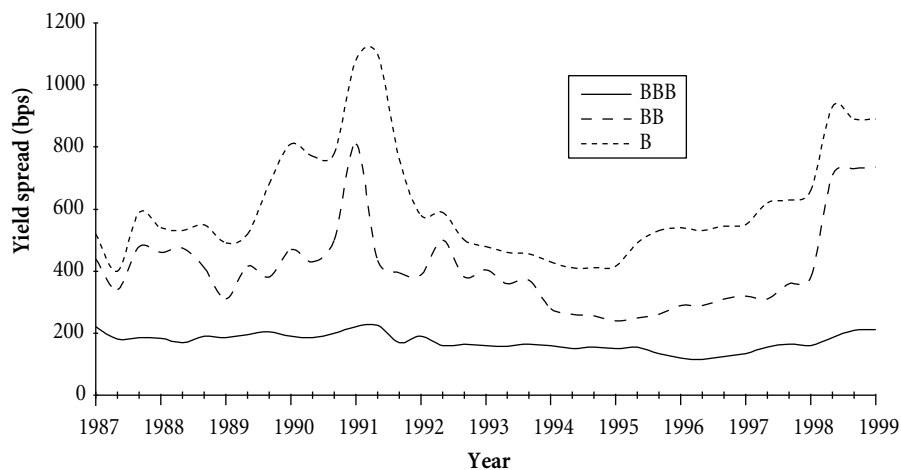


Figure 29.4: US high-yield bond spreads.

Source: Standard & Poor's, Bloomberg.

Another key indicator of high-yield bond performance is the level of default. The level of defaults have not unexpectedly, been at the historically highest levels during times of recession. The default rate in 1990–1991 reflected the recession as well as the hangover from the large-scale highly-leveraged buy-outs of the late 1980s. The average default rate during the 1980s and 1990s has been approximately 3% to 3.5%. Figure 29.5 illustrates the actual default rates in the two decades.

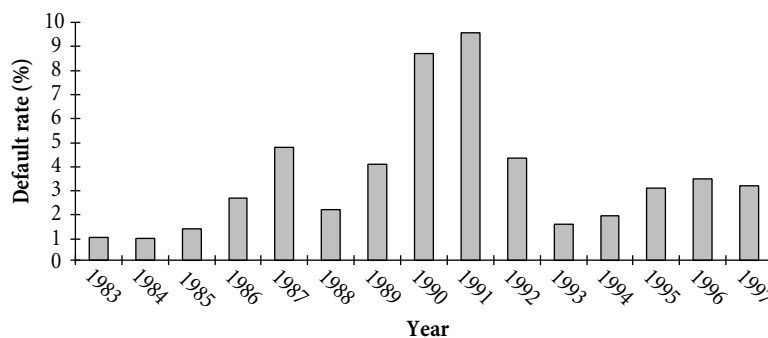


Figure 29.5: US high-yield market historical default rates, as percentage of outstanding debt.

Source: Standard & Poor's, Lehman Brothers.

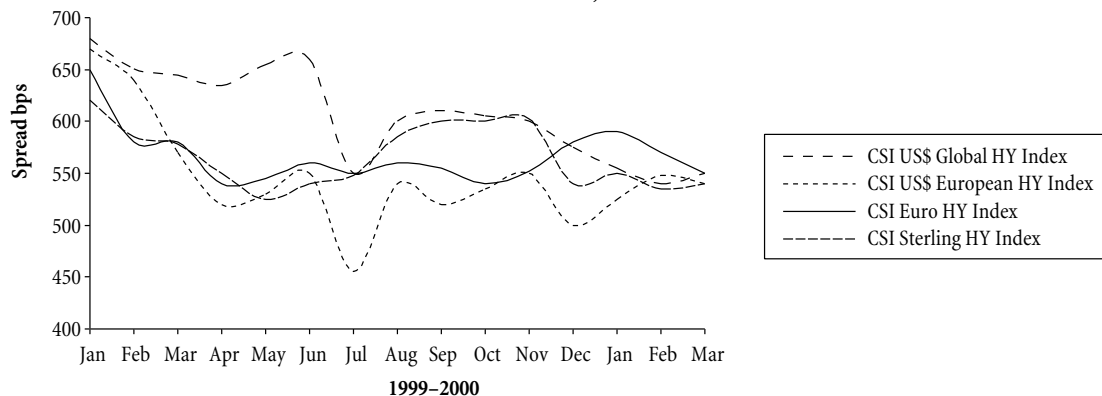


Figure 29.6: Average spreads in the European high-yield market, 1999–2000.

Source: Chase Manhattan.

Issuer	Coupon (%)	Maturity	Nominal (\$m)	Rating	Bid price	Spread (bp)
Adelphia Comms	Zero	Aug 2008	605	B1/BB-	42.000	464
Advanced Micro Devices	11.000	Aug 2003	400	B2/B	99.000	463
Advantica Rest Group	11.250	Jan 2008	550	B3/B	68.000	1,264
AK Stl	9.125	Dec 2006	546	Ba2/BB	98.250	279
Allied Waste North America	7.875	Jan 2009	875	Ba3/BB-	85.500	380
Amazon.Com	10.558	May 2008	530	Caa1/B	60.000	662
Budget Group	9.125	Apr 2006	400	B1/BB-	90.000	473
Caremark Rx	7.375	Oct 2006	450	B3/B	84.000	413
Chancellor Media Corp of LA	8.125	Dec 2007	500	B1/B	99.250	160
Charter Communication Holdings	8.625	Apr 2009	1,500	B2/B+	91.500	346
Crown Paper	11.000	Sep 2005	250	Caa1/CCC+	40.000	3,069
CSC Holdings	7.625	Jul 2018	500	Ba1/BB+	92.050	205
DR Horton	8.000	Feb 2009	385	Ba1/BB	87.000	364
Emmis Commun	8.125	Mar 2009	300	B2/B-	93.750	254
Exide	10.000	Apr 2005	300	B1/B+	95.250	449
Fairchild	10.750	Apr 2005	225	B3/B-	73.000	997
Flening Co	10.625	Dec 2001	300	B1/B+	100.000	397
Formica	10.875	Mar 2009	215	B3/B-	88.000	657
Fox Family Worldwide	10.250	Nov 2007	605	B1/B	64.000	640
Friendly Ice Cream	10.500	Dec 2007	200	B2/B	80.000	827
Gaylord Container	9.375	Jun 2007	200	Caa1/B-	90.500	464
Globalstar	11.375	Feb 2004	499	Caa1/B	59.000	2,283
HMH Pptys	7.875	Aug 2008	1,200	Ba2/BB	87.500	348
Intermedia Comms	11.250	Jul 2007	648	B2/B	78.750	403
Level 3 Comms	9.125	May 2008	1,999	B3/B	91.500	406
Lyondell Comms	10.875	May 2009	500	B2/B+	95.500	507
Mandalay Resort Group	9.250	Dec 2005	275	Ba2/BB+	99.000	275
Nextel Communications	9.963	Feb 2008	1,627	B1/B	70.000	432
Ocean Energy	8.875	Jul 2007	200	Ba3/BB-	98.750	244
Playtex Products	9.000	Dec 2003	360	B2/B	99.500	242
Pride International	9.375	May 2007	325	Ba3/BB	97.500	319
RBF Finance	11.000	Mar 2006	400	Ba3/BB-	105.750	301
REV Holdings	Zero	Mar 2001	770	Caa3/CCC+	23.000	19,344
Riverwood International	10.875	Apr 2008	400	Caa1/CCC+	95.000	520
Sterling Chemical	11.750	Aug 2006	275	Caa3/B	84.000	908
Tenet Healthcare	8.000	Jan 2005	900	Ba1/BB+	96.000	227
Time Warner Telecom	9.750	Jul 2008	400	B2/B	100.000	304
Trump Atlantic City Association	11.250	May 2006	1,200	B2/B-	70.000	1,316
WCI STL	10.000	Dec-04	300	B2/B+	99.500	337
Westpoint Stevens	7.875	Jun-05	525	Ba3/BB	88.000	417

Table 29.1: "Focus 40" US high-yield bond index, February 2000.

Source: IFR, Thompson Financial Bank Watch.

Issuer	Coupon (%)	Maturity	Currency	Nominal (mn)	Rating	Bid price	Spread (bp)
Atlantic Telecom	12.875	Feb 2010	EUR	200	B3/B-	113.00	510
Colt	7.625	Dec 2009	EUR	307	B1/B	96.00	290
Huntsman ICI	10.250	Jun 2009	EUR	200	B2/B+	105.00	375
IPC Magazines	9.625	Mar 2008	GBP	120	B3/B-	75.00	915
Jazztel	13.250	Dec 2009	EUR	400	Caa1/CCC+	105.00	680
Kappa	10.625	Jul 2009	EUR	370	B2/B	105.50	420
KPNQwest	7.125	Jun 2009	EUR	340	Ba1/BB	95.75	235
Luxfer	10.125	Mar 2009	GBP	160	B2/B	94.50	540
NTL	9.875	Nov 2009	EUR	350	B3/B-	100.50	435
NTL	9.500	Apr 2008	GBP	125	B3/B-	95.50	435
Telewest	9.875	Jan 2010	GBP	180	B1/B+	99.00	440
UPC	10.875	Aug 2009	EUR	300	B2/B-	98.00	570
UPC	13.375	Nov 2009	EUR	191	B2/B-	55.50	780
Versatel	11.875	Jul 2009	EUR	120	Caa1	107.00	510
William Hill	10.625	Apr 2008	GBP	150	B3/B-	101.00	440

Table 29.2: European high-yield bond pricing, February 2000.

Source: IFR, Euroweek.

Selected bibliography and references

Barnhill, T. (ed.), *High-yield bonds: Market structure, valuation and portfolio strategies*, McGraw-Hill, 1999.

Hickman, W., *Corporate Bond Quality and Investor Experience*, Princeton University Press and National Bureau of Economic Research, 1958.

Howe, J.T., *Junk Bonds*, Probus Publishing, 1987.

“Corporate Bond Defaults and Default Rates 1970–1992”, *Moody’s Special Report*, Moody’s Investors Service, Inc., January 1993.

Rubin, *Junk Bonds*, Prentice-Hall, 1990.

Yago, G., *Junk Bonds: How high-yield securities restructured corporate America*, OUP, 1990.

30 Corporate Bonds and Credit Analysis

The risks associated with holding a fixed interest debt instrument are closely connected with the ability of the issuer to maintain the regular coupon payments as well as redeem the debt on maturity. Essentially the *credit risk* is the main risk of holding a bond. Only the highest quality government debt, and a small number of supra-national issues, may be considered to be entirely free of credit risk. Therefore at any time the yield on a bond reflects investors' views on the ability of the issuer to meet its liabilities as set out in the bond's terms and conditions. A delay in paying a cash liability as it becomes due is known as technical default and is a cause for extreme concern for investors; failure to pay will result in the matter being placed in the hands of the legal court as investors seek to recover their funds. To judge the ability of an issue to meet its obligations for a particular debt issue, for the entire life of the issue, requires judgemental analysis of the issuer's financial strength and business prospects. There are a number of factors that must be considered, and larger banks, fund managers and corporates carry out their own *credit analysis* of individual borrowers' bond issues. The market also makes a considerable use of formal *credit ratings* that are assigned to individual bond issues by a formal credit rating agency. In the international markets arguably the two most influential ratings agencies are Standard & Poor's Corporation (S&P) and Moody's Investors Service, Inc (Moody's), based in the US. In the US domestic market Duff & Phelps Credit Rating Co. (D&P) and Fitch Investors Service, Inc (Fitch) also have a high profile, as do IBCA and Dun & Bradstreet in the UK. The four US agencies all have similar ratings.¹

The specific factors that are considered by a ratings agency, and the methodology used in conducting the analysis, differ slightly amongst the individual ratings agencies. Although in many cases the ratings assigned to a particular issue by different agencies are the same, they occasionally differ and in these instances investors usually seek to determine what aspect of an issuer is given more weight in an analysis by which individual agency. Note that a credit rating is not a recommendation to buy (or equally, sell) a particular bond, nor is it a comment on market expectations. Credit analysis does take into account general market and economic conditions; the overall purpose of credit analysis is to consider the financial health of the issuer and its ability to meet the obligations of the specific issue being rated. Credit ratings play a large part in the decision-making of investors, and also have a significant impact on the interest rates payable by borrowers.

In this chapter we review credit ratings and their function, and then go on to consider the main factors involved in corporate bond credit analysis.

30.1 Credit ratings

A credit rating is a formal opinion given by a rating agency, of the *credit risk* for investors in a particular issue of debt securities. Ratings are given to public issues of debt securities by any type of entity, including governments, banks and corporates. They are also given to short-term debt such as commercial paper as well as bonds and medium-term notes.

30.1.1 Purpose of credit ratings

Investors in securities accept the risk that the issuer will default on coupon payments or fail to repay the principal in full on the maturity date. Generally credit risk is greater for securities with a long maturity, as there is a longer period for the issuer potentially to default. For example if company issues ten-year bonds, investors cannot be certain that the company will still exist in ten years' time. It may have failed and gone into liquidation some time before that. That said, there is also risk attached to short-dated debt securities, indeed there have been instances of default by issuers of commercial paper, which is a very short-term instrument.

The prospectus or offer document for an issue provides investors with some information about the issuer so that some credit analysis can be performed on the issuer before the bonds are placed. The information in the offer documents enables investors themselves to perform their own credit analysis by studying this information before deciding whether or not to invest. Credit assessments take up time however and also require the specialist skills of

¹ Fitch and IBCA subsequently merged to become FitchIBCA.

credit analysts. Large institutional investors do in fact employ such specialists to carry out credit analysis, however often it is too costly and time-consuming to assess every issuer in every debt market. Therefore investors commonly employ two other methods when making a decision on the credit risk of debt securities:

- name recognition;
- formal credit ratings.

Name recognition is when the investor relies on the good name and reputation of the issuer and accepts that the issuer is of such good financial standing, or sufficient financial standing, that a default on interest and principal payments is highly unlikely. An investor may feel this way about say, Microsoft or British Petroleum plc. However the experience of Barings in 1995 suggested to many investors that it may not be wise to rely on name recognition alone in today's marketplace. The tradition and reputation behind the Barings name allowed the bank to borrow at Libor and occasionally at sub-Libor interest rates in the money markets, which put it on a par with the highest-quality clearing banks in terms of credit rating. However name recognition needs to be augmented by other methods to reduce the risk against unforeseen events, as happened with Barings. Credit ratings are a formal assessment, for a given issue of debt securities, of the likelihood that the interest and principal will be paid in full and on schedule. They are increasingly used to make investment decisions about corporate or lesser-developed government debt.

30.1.2 Formal credit ratings

Credit ratings are provided by the specialist agencies. The major credit rating agencies are Standard & Poor's, Fitch and Moody's, based in the United States, and the UK-based IBCA. There are other agencies both in the US and other countries. On receipt of a formal request, the credit rating agencies will carry out a rating exercise on a specific issue of debt capital. The request for a rating comes from the organisation planning the issue of bonds. Although ratings are provided for the benefit of investors, the issuer must bear the cost. However it is in the issuer's interest to request a rating as it raises the profile of the bonds, and investors may refuse to buy paper that is not accompanied with a recognised rating. Although the rating exercise involves a credit analysis of the issuer, the rating is applied to a specific debt issue. This means that in theory the credit rating is applied not to an organisation itself, but to specific debt securities that the organisation has issued or is planning to issue. In practice it is common for the market to refer to the creditworthiness of organisations themselves in terms of the rating of their debt. A highly-rated company such as Commerzbank is therefore referred to as a "triple-A rated" company, although it is the bank's debt issues that are rated as triple-A.

The rating for an issue is kept constantly under review and if the credit quality of the issuer declines or improves, the rating will be changed accordingly. An agency may announce in advance that it is reviewing a particular credit rating, and may go further and state that the review is a precursor to a possible downgrade or upgrade. This announcement is referred to as putting the issue under *credit watch*. The outcome of a credit watch is in most cases likely to be a rating downgrade, however the review may re-affirm the current rating or possibly upgrade it. During the credit watch phase the agency will advise investors to use the current rating with caution. When an agency announces that an issue is under credit watch, the price of the bonds will fall in the market as investors look to sell out of their holdings. This upward movement in yield will be more pronounced if an actual downgrade results. For example in October 1992 the government of Canada was placed under credit watch and subsequently lost its AAA credit rating; as a result there was an immediate and sharp sell-off in Canadian government Eurobonds, before the rating agencies had announced the actual results of their credit review.

Credit ratings vary between agencies. Separate categories are used by each agency for short-term debt (with original maturity of 12 months or less) and long-term debt of over one year original maturity. It is also usual to distinguish between higher "investment grade" ratings where the credit risk is low and lower quality "speculative grade" ratings, where the credit risk is greater. High-yield bonds are speculative-grade bonds and are generally rated no higher than double-B, although some issuers have been upgraded to triple-B in recent years and a triple-B rating is still occasionally awarded to a high-yield bond. A summary of long-term ratings is shown at Table 30.1.

Duff & Phelps	FitchIBCA	Moody’s	Standard & Poors	Summary Description
Investment Grade – High credit quality				
AAA	AAA	Aaa	AAA	Gilt edged, prime, lowest risk, risk-free
AA+	AA+	Aa1	AA+	High-grade, high credit quality
AA	AA	Aa2	AA	
AA-	AA-	Aa3	AA-	
A+	A+	A1	A+	Upper-medium grade
A	A	A2	A	
A-	A-	A3	A-	
BBB+	BBB+	Baa1	BBB+	Lower-medium grade
BBB	BBB	Baa2	BBB	
BBB-	BBB-	Baa3	BBB-	
Speculative – Lower credit quality				
BB+	BB+	Ba1	BB+	Low grade; speculative
BB	BB	Ba2	BB	
BB-	BB-	Ba3	BB-	
B	B+	B1	B+	Highly speculative
	B	B2	B	
	B-	B3	B-	
Highly speculative, substantial risk or in default				
CCC	CCC	Caa	CCC+	Considerable risk, in poor standing
DD	CC	Ca	CCC-	May already be in default, very speculative Extremely speculative Income bonds – no interest being paid
	C	C	CC	
			C	
				CI
	DDD			Default
	DD			
	D		D	

Table 30.1: Summary of credit rating agency bond ratings.² Source: Rating agencies.

30.2 Credit analysis

When ratings agencies were first set up the primary focus of credit analysis was on the default risk of the bond, or the probability that the investor would not receive the interest payments and the principal repayment as they fell due. Although this is still important, credit analysts these days also considers the overall economic conditions as well as the chance that an issuer will have its rating changed during the life of the bond. There are differences in approach depending on which industry or market sector the issuing company is part of.

In this section we review the main issues of concern to a credit analyst when rating bond issues. Analysts usually adopt a “top-down” approach, or a “big picture” approach, and concentrate on the macro-issues first before looking at the issuer specific points in detail. The process therefore involves reviewing the issuer’s industry, before looking at its financial and balance sheet strength, and finally the legal provisions concerning the bond issue. There are also detail differences in analysis depending on which industry the issuer is in.

² Following subsequent mergers, Fitch Inc. incorporates Fitch, IBCA and Duff & Phelps.

30.2.1 The issuer industry

In the first instance the credit analysis process of a specific issue will review the issuer's industry. This is in order to place the subsequent company analysis in context. For example, a company that has recorded growth rates of 10% each year may appear to be quality performer, but not if its industry has been experiencing average growth rates of 30%. Generally the industry analysis will review the following issues:

- **Economic cycle.** The business cycle of the industry and its correlation with the overall business cycle are key indicators. That is, how closely does the industry follow the rate of growth of its country's GNP? Certain industries such as the electricity and food retail sectors are more resistant to recession than others. Other sectors are closely tied to changes in population and birth patterns, such as residential homes, while the financial services industry is influenced by the overall health of the economy as well as by the level of interest rates. As well as the correlation with macro-factors, credit analysts review traditional financial indicators in context, for example the issuing company's *earnings per share* (EPS) against the growth rate of its industry.
- **Growth prospects.** This review is of the issuer industry's general prospects. A company operating within what is considered a high-growth industry is generally deemed to have better credit quality expectations than one operating in a low-growth environment. A scenario of anticipated growth in the industry has implications for the issuing company, for example the extent to which the company will be able cope with capacity demands and the financing of excess capacity. A fast-growth industry also attracts new entrants, which will lead to over-supply, intensified competition and reduced margins. A slow-growth industry has implications for diversification, so that a company deemed to have plans for diversifying when operating in stagnant markets will be marked up.
- **Competition.** A review of the intensity of competitive forces within an industry, and the extent of pricing and over- or under-capacity, is an essential ingredient of credit analysis. Competition is now regarded as a global phenomenon and well-rated companies are judged able to compete successfully on a global basis while concentrating on the highest-growth regions. Competition within a particular industry is related to that industry's structure and has implications for pricing flexibility. The type of market, for example, monopoly, oligopoly, and so on, also influences pricing policy and relative margins. Another issue arises if there is obvious over-capacity in an industry; this has been exemplified in the past in the airline industry and (in some countries) financial services. When over-capacity often leads to intense price competition and price wars. This is frequently damaging for the industry as a whole, as all companies suffer losses and financial deterioration in the attempt to maintain or grow market share.
- **Supply sources.** The availability of suppliers in an industry has influences for a company's financial well-being. Monopoly sources of supply are considered a restrictive element and have negative implications. A vertically-integrated company that is able to supply its own raw materials is less susceptible to economic conditions that might affect suppliers or leave it hostage to price rises. A company that is not self-sufficient in its factors of production but is nevertheless in strong enough a position to pass on its costs is in a good position.
- **Research and development.** A broad assessment of the growth prospects of a company must also include a review of its research and development (R&D) position. In certain industries such as telecommunications, media and information technology, a heavy investment in R&D is essential simply in order to maintain market share. In a high-technology field it is common for products to obsolesce very quickly, therefore it is essential to maintain high R&D spend. In the short-term however a company with a low level of research expenditure may actually post above-average (relative to the industry) profits because it is operating at higher margins. This is not considered a healthy strategy for the long term though.

Evaluating the R&D input of a company is not necessarily a straightforward issue of comparing ratios however, as it is also important to assess correctly the direction of technology. That is, a successful company needs not only to invest a sufficient amount in R&D, it must also be correct in its assessment of the direction the industry is heading, technology-wise. A heavy investment in developing Betamax videos for example, would not have assisted a company in the early 1980s.

- **Level of regulation.** The degree of regulation in an industry, its direction and its effect on the profitability of a company are relevant in a credit analysis. A highly regulated industry such as power generation, production of medicines or (in certain countries) telecommunications can have a restrictive influence on company profits. On

the other hand if the government has announced a policy of de-regulating an industry, this is considered a positive development for companies in that industry.

- **Labour relations.** An industry with a highly unionised labour force or generally tense labour relations is viewed unfavourably compared to one with stable labour relations. Credit analysts will consider historic patterns of say, strikes and production days lost to industrial action. The status of labour relations is also more influential in a highly labour-intensive industry than one that is more automated for example.
- **Political climate.** The investment industry adopts an increasingly global outlook and the emergence of sizeable tradeable debt markets in for example, “emerging” countries means that ratings agencies frequently must analyse the general political and economic climate in which an industry is operating. Failure to foresee certain political developments can have far-reaching effects for investors, as recently occurred in Indonesia when that country experienced a change of government; foreign investors lost funds as several local banks went bankrupt.

30.2.2 Financial analysis

The traditional approach to credit analysis concentrated heavily on financial analysis. The more modern approach involves a review of the industry the company is operating in first, discussed above, before considering financial considerations. Generally the financial analysis of the issuer is conducted in three phases, namely:

- the ratio analysis for the bonds;
- analysing the company’s return on capital;
- non-financial factors such as management expertise and extent of overseas operations.

Ratio analysis

There are a number of investor ratios that can be calculated. In themselves ratios do not present very much insight, although there are various norms that can be applied. Generally ratio analysis is compared to the levels prevalent in the industry, as well as historical values, in an effort to place the analysis in context and compare the company with those in its peer group. The ratios that can be considered are:

- pre-tax interest cover, the level of cover for interest charges in current pre-tax income;
- fixed interest charge level;
- *leverage*, which is commonly defined as the ratio of long-term debt as a percentage of the total capitalisation;
- level of leverage compared to industry average;
- nature of debt, whether fixed- or floating-rate, short- or long-term;
- cash flow, which is the ratio of cash flow as a percentage of total debt. Cash flow itself is usually defined as net income from continuing operations, plus depreciation and taxes, while debt is taken to be long-term debt;
- net assets, as a percentage of total debt. The liquidity of the assets – meaning the ease with which they can be turned into cash – is taken into account when assessing the net asset ratio.

The ratings agencies maintain benchmarks that are used to assign ratings, and these are monitored and if necessary modified to allow for changes in the economic climate. For example, Standard & Poor’s guidelines for pre-tax interest cover, leverage level and cash flow in 1997 are shown in Table 30.2. A pre-tax cover of above 9.00 for example, is consistent with a double-A rating.

Credit rating	Pre-tax interest cover	Leverage	Cash flow
AAA	17.99	13.2	97.5
AA	9.74	19.7	68.5
A	5.35	33.2	43.8
BBB	2.91	44.8	29.9

Table 30.2: S&P ratio benchmarks, 1997. Source: S&P.

Other ratios that are considered include:

- intangibles, that is the portion of intangibles relative to the asset side of a balance sheet;
- unfunded pension liabilities; generally a fully-funded pension is not seen as necessary, however an unfunded liability that is over 10% of net assets would be viewed as a negative point;
- age and condition of plant;
- working capital.

Return on equity

There are range of performance measures used in the market that are connected with return on equity (generally the analysis concentrates on return on capital, or more recently return on risk adjusted capital or RAROC). In analysing measures of return, analysts seek to determine trends in historical performance and comparisons with peer group companies. Different companies also emphasise different target returns in their objectives, usually an expression of their corporate philosophy, so it is common for companies in the same industry to have different return ratios. The range of ratios used by the credit ratings agencies is shown in below. Note that “EBIT” is “earnings before interest and tax”.

$$\text{Return on net assets} = \frac{\text{Profit}}{\text{Net assets}} \times 100$$

$$\text{Return on sales} = \frac{\text{Profit}}{\text{Sales turnover}} \times 100$$

$$\text{Return on equity} = (\text{Return on net assets} \times \text{Gearing}) \times 100$$

$$\text{Pre-tax interest cover} = \frac{\text{Pre-tax income from continuing operations}}{\text{Gross interest}}$$

$$\text{EBIT interest cover} = \frac{\text{Pre-tax income from continuing operations} + \text{interest expense}}{\text{Gross interest}}$$

$$\text{Long-term debt as \% of capitalisation} = \frac{\text{Long-term debt}}{\text{Long-term debt} + \text{equity}} \times 100$$

$$\text{Funds flow as \% of debt} = \frac{\text{Funds from operations}}{\text{Total debt}} \times 100$$

$$\text{Free cash flow as \% of debt} = \frac{\text{Free cash flow}}{\text{Total debt}} \times 100$$

The agencies make available data that may be consulted by the public, for example Standard & Poor’s has a facility known as “CreditStats”, which was introduced in 1989. It contains the main financial ratios for a large number of companies, organised by their industry sectors.

Non-financial factors

The non-financial element of a company credit analysis has assumed a more important role in recent years, especially with regard to companies in exotic or emerging markets. Credit analysts review the non-financial factors relevant to the specific company after they have completed the financial and ratio analysis. These include the strength and competence of senior management, and the degree of exposure to overseas markets. The depth of overseas exposure is not always apparent from documents such as the annual report, and analysts sometimes need to conduct further research to determine this. Companies with considerable overseas exposure, such as petroleum companies, also need to be reviewed with respect to the political situation in their operating locations. A bank such as Standard Chartered for example, has significant exposure to more exotic currencies in Asian, middle-eastern and African countries, and so is more at risk from additional market movements than a bank with almost exclusively

domestic operations. The global, integrated nature of the bond markets also means that the foreign-exchange exposure of a company must be evaluated and assessed for risk.

The quality of management is a subjective, qualitative factor that can be reviewed in a number of ways. A personal familiarity with senior directors, acquired over a period of time, may help in the assessment. A broad breadth of experience, diversity of age, and strong internal competition for those aspiring to very senior roles, is considered positive. A company that had been founded by one individual, and in which there were no clear plans of “succession”, might be marked down.

30.3 Industry-specific analysis

Specific industries will be subject to review that is more relevant to the particular nature of the operations of the companies within them. In this section we briefly consider two separate industries, power generation, water and certain other public service companies (or utilities) and financial companies.

30.3.1 Utility companies

The industry for power generation, water supply and until recently telecommunications has a tradition of being highly regulated. Until the mid-1980s, utility companies were public sector companies, and the first privatisation of such a company was for British Telecom in 1984. In certain European countries utility companies are still nationalised companies, and their debt trades virtually as government debt. Credit analysis for utility companies therefore emphasises non-financial factors such as the depth of regulation and the direction in which regulation is heading, for example towards an easing or tightening. Even in a privatised industry for example, new government regulation maybe targeted only at the utility sector; for example, the Labour government in the UK imposed a “windfall tax” on several privatised utility companies shortly after being elected in May 1997.

Another consideration concerns government direction on how the companies may operate, such restrictions on where a power generation company may purchase coal from. In some countries such as Germany, coal must be bought from the country’s own domestic coal industry only, which imposes costs on the generating company that it would escape if it were free to purchase coal from other, lower-cost producers.

The financial analysis of a utility company essentially follows the pattern we described earlier.

30.3.2 Financial sector companies

The financial sector encompasses a large and diverse group of companies. They conduct an intermediary function in that they are a conduit for funds between borrowers and lenders of capital. At its simplest, financial service companies such as banks may earn profit by taking the spread between funds lent and borrowed. They also play an important role in managing the risk exposure for industrial companies, utilising option structures. In analysing a financial sector company the credit analyst will consider the type of customer base served by the company, for example how much of a bank’s lending is to the wholesale sector, how much is retail and so on. The financial strength and prospects of its customer base are important elements of a bank’s credit rating.

Financial analysis of banks and securities houses is concerned (in addition to the factors discussed above) with the asset quality of the institution, for example the extent of diversification of bank’s lending book. Diversification can be across customer base as well as geographically. A loan book that is heavily concentrated in one sector is considered to be a negative factor in the overall credit assessment of the bank. A credit analyst will be concerned with the level of loans compared with levels in peer companies and the risk involved with this type of lending. For example the expected frequency of bad loans from direct unsecured retail customer loans is higher than for retail customer loans secured by a second mortgage on a property. The higher lending rate charged for the former is designed to compensate for this higher lending risk. There are a range of financial ratios that can be used to assess a bank’s asset quality. These include:

- loss reserves/net charge-off level;
- net losses/average level of receivables;
- non-performing loans/average level of receivables.

However unlike the more “concrete” financial ratios given earlier, there is a higher subjective element with these ratios as banks themselves will designate which loans are non-performing and those loans against which have been assigned charges. Nevertheless these ratios are useful indicators and may be used to identify trends across the sector

as well. The loss reserves/net charge-off ratio is perhaps the most useful as it indicates the level of “cushion” that a bank has; a falling ratio suggests that the bank may not be adding sufficient reserves to cover for future charge-offs. This trend, if continued, may then result in a future increase in the reserves and therefore a decrease in earnings levels as the expense of the reserves increase.

The leverage ratio is particularly important for financial sector companies as the industry and business itself are highly leveraged. Banks and securities companies are therefore permitted a significantly higher leverage level than other companies. For example in a diversified banking group with a high level of asset quality, a leverage ratio of 5:1 or even higher is considered satisfactory by ratings agencies.

Another important measure for financial companies is *liquidity*. Due to the nature of the industry and the capital structure of banks, liquidity or more accurately the lack of liquidity is the primary reason behind banking failures. A bank that is unable to raise funds sufficiently quickly to meet demand will most probably fail, and certainly so if external support is not provided. An inability to raise funds may arise due to internal factors, such as a deterioration in earnings or a very poorly performing loan book, connected perhaps with a downgrade in credit rating, or from external factors such as a major structural fault in the money markets. For credit analysis purposes the traditional liquidity measures are:

- cash;
- cash equivalents;
- level of receivables under one year/level of short-term liabilities.

A higher ratio indicates a greater safety cushion. A further consideration is the extent of lines of credit from other banks in the market.

Other measures of strength for financial companies are *asset coverage*, the bank’s earnings record including *earnings per share* (profit attributable to shareholders/number of shares in issue) and finally, the size of the institution. There is an element of thought which states that a very large institution, measured by asset size, cannot go bankrupt. This type of thinking can lead to complacency however and did not prevent several large Japanese banks from getting into financial difficulty in the 1990s.³

30.4 The art of credit analysis

As bond markets become ever larger and integrated across a global market, the demand for paper is increasing and with it the demand for high quality credit research. There are now large numbers of companies for whom investors will have virtually no recognition at all, leading to a greater reliance on formal credit ratings. Also the rapid change in economic conditions and the effect of the business cycle frequently result in a company’s credit outlook changing rapidly. Investors look to credit ratings as their main source of indicator of a borrower’s health. The process by which a bond issue is rated is not purely quantitative however and analysts frequently apply their own qualitative criteria, to take account of changing environments and other, political and macro-economic factors.

Selected bibliography and references

- Cohen, J., Zinbarg, E., Zeikel, A., *Investment Analysis and Portfolio Management*, Richard D Irwin, 1977.
 S&P, *Credit Week*, 8 November 1993, Standard & Poor’s.
 S&P, *Credit Week*, 15 October 1999, Standard & Poor’s.
 Mills, R., *Strategic Value Analysis*, Mars Business Associates, 1994.
 Wilson, R., Fabozzi., *Corporate Bonds: Structures and Analysis*, FJF Associates, 1996.

Questions and exercises

1. Describe what type of non-financial factors a credit analyst should consider when rating a bond issue.
2. What variables are considered in the analysis of a specific industry.
3. Consider the two following companies, ABC plc and XYZ plc. Using their last set of consolidated report and accounts, a credit analyst determines the following:

³ In fact the Japanese government gave an implicit guarantee for the largest 20 “city” banks at one stage, shortly after the collapse of Yamaichi Securities in 1998.

	ABC plc	XYZ plc
Pre-tax interest coverage ratio	5.39	4.17
Leverage	2.66	2.31
Cash flow/spending ratio	1.39	2.11
Cash flow/capital ratio	0.85	1.24
Return on equity	15.45%	19.81%

- (a) describe what these ratios mean and how they are used in credit analysis.
- (b) does the analyst have sufficient information to make any assessment of the companies?
- (c) is it possible to tell which company is in better financial health from the above ratios? If the ratios for both were given to the analyst over the last five years, would that help his assessment?

Part IV The Money Markets

Although they are part of the global debt markets, the money markets are a separate market in their own right. Money market securities are defined as debt instruments with an original maturity of less than one year. It is common to find that the maturity profile of banks' money market desks runs out to two years, so that in practice bonds with less than two years to maturity trade as money market instruments.

In Part IV of the book we review the main money market securities, both cash and off-balance sheet. We have expanded the usual coverage however, to include a review of bank asset and liability management, or ALM, which is the essential art of banking. This subject has been developed considerably beyond bank short-term liquidity management, and we consider the main points here. This part of the book will therefore be useful those who are involved in bank and security house money market trading, banking liquidity or those involved with bank asset & liability committees.

In a separate chapter we review a substantial topic in its own right, the bond repo markets; this covers the mechanics of repo and its uses as part of the bond and money markets. In the second half of the chapter there is a detail exposition of a subject that is often misunderstood, namely basis trading and the implied repo rate. Additional chapters deal with banking capital regulatory issues and the main money market derivative instruments, short-dated interest-rate futures and forward rate agreements (FRAs).

31

The Money Markets

In terms of trading volumes the *money markets* are the largest and most active market in the world. Money market securities are securities with maturities of up to twelve months, so they are short term debt obligations. Money market debt is an important part of the global financial markets, and facilitates the smooth running of the banking industry as well as providing working capital for industrial and commercial corporate institutions. The diversity of the money market is such that it provides market users with a wide range of opportunities and funding possibilities, and the market is characterised by the range of products that can be traded within it. Money market instruments allow issuers, who are financial organisations as well as corporates, to raise funds for short term periods at relatively low interest rates. These issuers include sovereign governments, who issue Treasury bills, corporates issuing commercial paper and banks issuing bills and certificates of deposit. At the same time investors are attracted to the market because the instruments are highly liquid and carry relatively low credit risk. The Treasury bill market in any country is that country's lowest-risk instrument, and consequently carries the lowest yield of any debt instrument. Indeed the first market that develops in any country is usually the Treasury bill market. Investors in the money market include banks, local authorities, corporations, money market investment funds and individuals; the money market is essentially a wholesale one and the denominations of individual instruments are relatively large.

Although the money market has traditionally been defined as the market for instruments maturing in one year or less, frequently the money market desks of banks trade instruments with maturities of up to two years, both cash and off-balance sheet.¹ In addition to the cash instruments that go to make up the market, the money markets also consist of a wide range of over-the-counter off-balance sheet derivative instruments. These instruments are used mainly to establish future borrowing and lending rates, and to hedge or change existing interest rate exposure. This activity is carried out by both banks, central banks and corporates. The main derivatives are short-term interest rate futures, forward rate agreements, and short-dated interest rate swaps.

In this chapter we review the cash instruments traded in the money market. In further chapters we review banking asset and liability management and capital arrangements, and the market in repurchase agreements. Finally we consider the market in money market derivative instruments including interest-rate futures and forward-rate agreements.

31.1 Introduction

The cash instruments traded in the money market include the following:

- Treasury bill;
- Time deposit;
- Certificate of Deposit;
- Commercial Paper;
- Bankers Acceptance;
- Bill of exchange.

A Treasury bill is used by sovereign governments to raise short-term funds, while certificates of deposit (CDs) are used by banks to raise finance. The other instruments are used by corporates and occasionally banks. Each instrument represents an obligation on the borrower to repay the amount borrowed on the maturity date together with interest if this applies. The instruments above fall into one of two main classes of money market securities: those quoted on a *yield* basis and those quoted on a *discount* basis. These two terms are discussed below. A *repurchase agreement* or “repo” is also a money market instrument and is considered in a separate chapter.

The calculation of interest in the money markets often differs from the calculation of accrued interest in the corresponding bond market. Generally the day-count convention in the money market is the exact number of days

¹ The author has personal experience in market making on a desk that combined cash and derivative instruments of up to two years maturity as well as government bonds of up to three years maturity.

that the instrument is held over the number of days in the year. In the sterling market the year base is 365 days, so the interest calculation for sterling money market instruments is given by (31.1):

$$i = \frac{n}{365}. \quad (31.1)$$

Money markets that calculate interest based on a 365-day year are listed at Appendix 31.1. The majority of currencies including the US dollar and the euro calculate interest on a 360-day base. The process by which an interest rate quoted on one basis is converted to one quoted on the other basis is shown in Appendix 31.1.

Settlement of money market instruments can be for value today (generally only when traded in before mid-day), tomorrow or two days forward, known as *spot*.

UK INTEREST RATES						
LONDON MONEY RATES						
Jul 22	Over-night	7 days notice	One month	Three months	Six months	One year
Interbank Sterling	6 - 4½	5½ - 4½	5½ - 5	5½ - 5½	5½ - 5½	5½ - 5½
Sterling CDs	-	-	4½ - 4½	5 - 4½	5½ - 5½	5½ - 5½
Treasury Bills	-	-	4½ - 4½	4½ - 4½	-	-
Bank Bills	-	-	4½ - 4½	4½ - 4½	-	-
Local authority deps.	4½ - 4½	4½ - 4½	5½ - 5	5½ - 5	5½ - 5½	5½ - 5½
Discount Market deps	4½ - 4½	5½ - 4½	-	-	-	-
UK clearing bank base lending rate 5 per cent from Jun 10, 1999						
	Up to 1 month	1-3 months	3-6 months	6-9 months	9-12 months	
Certs of Tax dep. (£100,000)	¾	2	2	2	2	
Certs of Tax dep. under £100,000 is 4pc. Deposits withdrawn for cash 2pc. Ave. tender rate of discount on Jul 9. 4.7759pc. ECGD fixed rate Sttg. Export Finance. Make up day Jun 30, 1999. Agreed rate for period Jul 26, 1999 to Aug 24, 1999, Scheme III 6.48pc. Reference rate for period May 29, 1999 to Jun 30, 1999, Schemes IV & V 5.230pc. Finance House Base Rate 5.5pc for Jul 1999.						
■ THREE MONTH STERLING FUTURES (LIFFE) £500,000 points of 100%						
	Open	Sett price	Change	High	Low	Est. vol
Oct		94.700	-			0
Dec	94.500	94.460	-0.050	94.530	94.450	11152
Mar	94.400	94.360	-0.050	94.430	94.310	39206
Jun	94.100	94.050	-0.050	94.130	94.000	15217
Sep	93.760	93.750	-0.030	93.830	93.710	3846
Also traded on APT. All Open interest figs. are for previous day.						
■ SHORT STERLING OPTIONS (LIFFE) £500,000 points of 100%						
Strike Price	Sep	Dec	Mar	Sep	Dec	Mar
94750	0.100	0.110	0.150	0.050	0.400	0.540
94875	0.045			0.120		
95000	0.020	0.050	0.080	0.220	0.590	0.720
95125	0.010			0.335		
95250	0.005	0.020	0.040	0.455	0.810	0.930
95375	0			0.575		
Est. vol. total, Calls 10566 Puts 3773. Previous day's open int., Calls 552486 Puts 436499						

Figure 31.1: London sterling money market rates. Extract from *Financial Times*, 23 July 1999. © *Financial Times*, 23/7/99. Reproduced with permission.

31.2 Securities quoted on a yield basis

Two of the instruments in the list above are yield-based instruments.

31.2.1 Money market deposits

These are fixed-interest term deposits of up to one year with banks and securities houses. They are also known as *time deposits* or *clean deposits*. They are not negotiable so cannot be liquidated before maturity. The interest rate on the deposit is fixed for the term and related to the London Interbank Offer Rate (LIBOR) of the same term. Interest and capital are paid on maturity.

LIBOR

The term LIBOR or “Libor” comes from London Interbank Offered Rate and is the interest rate at which one London bank offers funds to another London bank of acceptable credit quality in the form of a cash deposit. The rate is “fixed” by the British Bankers Association at 11am every business day morning (in practice the fix is usually about 20 minutes later) by taking the average of the rates supplied by member banks. The term LIBID is the bank’s “bid” rate, that is the rate at which it pays for funds in the London market. The quote spread for a selected maturity is therefore the difference between LIBOR and LIBID. The convention in London is to quote the two rates as LIBOR-LIBID, thus matching the yield convention for other instruments. In some other markets the quote convention is reversed. EURIBOR is the interbank rate offered for euros as reported by the European Central Bank. Other money centres also have their rates fixed, for example STIBOR is the Stockholm banking rate, while pre-euro the Portuguese escudo rate fixing out of Lisbon was LISBOR.

The effective rate on a money market deposit is the annual equivalent interest rate for an instrument with a maturity of less than one year.

EXAMPLE 31.1

- A sum of £250,000 is deposited for 270 days, at the end of which the total proceeds are £261,000. What are the simple and effective rates of return on a 365-day basis?

$$\begin{aligned}\text{Simple rate of return} &= \left(\frac{\text{Total proceeds}}{\text{Initial investment}} - 1 \right) \times \frac{M}{n} \\ &= \left(\frac{261,000}{250,000} - 1 \right) \times \frac{365}{270} = 5.9481\%.\end{aligned}$$

$$\begin{aligned}\text{Effective rate of return} &= \left(\frac{\text{Total proceeds}}{\text{Initial investment}} \right)^{\frac{M}{n}} - 1 \\ &= \left(\frac{261,000}{250,000} \right)^{\frac{365}{270}} - 1 = 5.9938\%.\end{aligned}$$

31.2.2 Certificates of Deposit

Certificates of Deposit (CDs) are receipts from banks for deposits that have been placed with them. They were first introduced in the sterling market in 1958. The deposits themselves carry a fixed rate of interest related to LIBOR and have a fixed term to maturity, so cannot be withdrawn before maturity. However the certificates themselves can be traded in a secondary market, that is, they are negotiable.² CDs are therefore very similar to negotiable money market deposits, although the yields are about 0.15% below the equivalent deposit rates because of the added benefit of liquidity. Most CDs issued are of between one and three months’ maturity, although they do trade in maturities of one to five years. Interest is paid on maturity except for CDs lasting longer than one year, where interest is paid annually or occasionally, semi-annually.

Banks, merchant banks and building societies issue CDs to raise funds to finance their business activities. A CD will have a stated interest rate and fixed maturity date and can be issued in any denomination. On issue a CD is sold for face value, so the settlement proceeds of a CD on issue are always equal to its nominal value. The interest is paid, together with the face amount, on maturity. The interest rate is sometimes called the *coupon*, but unless the CD is held to maturity this will not equal the yield, which is of course the current rate available in the market and varies over time. In the United States CDs are available in smaller denomination amounts to retail investors.³ The largest group of CD investors however are banks themselves, money market funds, corporates and local authority treasurers.

Unlike coupons on bonds, which are paid in rounded amounts, CD coupon is calculated to the exact day.

² A small number of CDs are non-negotiable.

³ This was first introduced by Merrill Lynch in 1982.

CD yields

The coupon quoted on a CD is a function of the credit quality of the issuing bank, and its expected liquidity level in the market, and of course the maturity of the CD, as this will be considered relative to the money market yield curve. As CDs are issued by banks as part of their short-term funding and liquidity requirement, issue volumes are driven by the demand for bank loans and the availability of alternative sources of funds for bank customers. The credit quality of the issuing bank is the primary consideration however; in the sterling market the lowest yield is paid by “clearer” CDs, which are CDs issued by the clearing banks such as RBS NatWest plc, HSBC and Barclays plc. In the US market “prime” CDs, issued by highly-rated domestic banks, trade at a lower yield than non-prime CDs. In both markets CDs issued by foreign banks such as French or Japanese banks will trade at higher yields.

Euro-CDs, which are CDs issued in a different currency to the home currency, also trade at higher yields, in the US because of reserve and deposit insurance restrictions.

If the current market price of the CD including accrued interest is P and the current quoted yield is r , the yield can be calculated given the price, using (31.2):

$$r = \left(\frac{M}{P} \times \left(1 + C \left(\frac{N_{im}}{B} \right) \right) - 1 \right) \times \left(\frac{B}{N_{sm}} \right). \quad (31.2)$$

The price can be calculated given the yield using (31.3):

$$\begin{aligned} P &= M \times \left(1 + C \left(\frac{N_{im}}{B} \right) \right) / \left(1 + r \left(\frac{N_{sm}}{B} \right) \right) \\ &= F / \left(1 + r \left(\frac{N_{sm}}{B} \right) \right) \end{aligned} \quad (31.3)$$

where

C	is the quoted coupon on the CD
M	is the face value of the CD
B	is the year day-basis (365 or 360)
F	is the maturity value of the CD
N_{im}	is the number of days between issue and maturity
N_{sm}	is the number of days between settlement and maturity
N_{is}	is the number of days between issue and settlement.

After issue a CD can be traded in the secondary market. The secondary market in CDs in the UK is very liquid, and CDs will trade at the rate prevalent at the time, which will invariably be different from the coupon rate on the CD at issue. When a CD is traded in the secondary market, the settlement proceeds will need to take into account interest that has accrued on the paper and the different rate at which the CD has now been dealt. The formula for calculating the settlement figure is given at (31.4) which applies to the sterling market and its 365-day count basis.

$$\text{Proceeds} = \frac{M \times \text{Tenor} \times C \times 100 + 36500}{\text{Days remaining} \times r \times 100 + 36500}. \quad (31.4)$$

The *tenor* of a CD is the life of the CD in days, while *days remaining* is the number of days left to maturity from the time of trade.

The return on holding a CD is given by (31.5):

$$R = \left(\frac{(1 + \text{purchase yield} \times (\text{days from purchase to maturity}/B))}{1 + \text{sale yield} \times (\text{days from sale to maturity}/B)} - 1 \right) \times \frac{B}{\text{days held}}. \quad (31.5)$$

EXAMPLE 31.2

- A three-month CD is issued on 6 September 1999 and matures on 6 December 1999 (maturity of 91 days). It has a face value of £20,000,000 and a coupon of 5.45%. What are the total maturity proceeds?

$$\text{Proceeds} = 20 \text{ million} \times \left(1 + 0.0545 \times \frac{91}{365}\right) = \text{£}20,271,753.42.$$

- What is the secondary market proceeds on 11 October if the yield for short 60-day paper is 5.60%?

$$P = \frac{20.271 \text{m}}{\left(1 + 0.056 \times \frac{56}{365}\right)} = \text{£}20,099,066.64.$$

- On 18 November the yield on short three-week paper is 5.215%. What rate of return is earned from holding the CD for the 38 days from 11 October to 18 November?

$$R = \left(\frac{1 + 0.0560 \times \frac{56}{365}}{1 + 0.05215 \times \frac{38}{365}} - 1 \right) \times \frac{365}{38} = 9.6355\%.$$

CDs issued with more than one coupon

CDs issued for a maturity of greater than one year pay interest on an annual basis. The longest-dated CDs are issued with a maturity of five years. The price of a CD paying more than one coupon therefore depends on all the intervening coupons before maturity, valued at the current yield. Consider a CD that has four more coupons remaining to be paid, the last of which will be paid on maturity together with the face value of the CD. The value of this last coupon will be:

$$M \times C \times \frac{n_{3-4}}{B} \quad (31.6)$$

where n_{3-4} is the number of days between the third and fourth (last) coupon dates and C is the coupon rate on the CD. The maturity proceeds of the CD are therefore:

$$M \times \left(1 + C \times \frac{n_{3-4}}{B}\right). \quad (31.7)$$

The present value of this amount to the date of the second coupon payment is therefore its discounted value using the current yield r is given by (31.8):

$$P_2 = \frac{M \times \left(1 + C \times \frac{n_{3-4}}{B}\right)}{1 + r \times \frac{n_{3-4}}{B}}. \quad (31.8)$$

This value is then added to the actual cash flow received on the same date, that is, the second coupon date, which is given by:

$$M \times C \times \frac{n_{2-3}}{B}. \quad (31.9)$$

The total of these two amounts is given by (31.10):

$$P_1 = M \times \left(\frac{\left(1 + C \times \frac{n_{3-4}}{B}\right)}{\left(1 + r \times \frac{n_{3-4}}{B}\right)} + C \times \frac{n_{2-3}}{B} \right). \quad (31.10)$$

This amount is then discounted again to obtain the present value on the first coupon date at the current yield r , and added to the total cash flow. This process is repeated so that the final total amount can be discounted to the purchase date, at the current yield.

In general for a CD with N coupon payments remaining the price is given by (31.11):

$$P = M \times \left(\frac{1}{A_N} + \left(\frac{C}{B} \times \sum_{k=1}^N \left(\frac{n_{k-1;k}}{A_k} \right) \right) \right) \quad (31.11)$$

where

$$A_k = \left(1 + r \times \frac{n_{p1}}{B}\right) \left(1 + r \times \frac{n_{1-2}}{B}\right) \left(1 + r \times \frac{n_{2-3}}{B}\right) \cdots \left(1 + r \times \frac{n_{k-1;k}}{B}\right)$$

$n_{k-1;k}$ is the number of days between the $(k-1)$ th coupon and the k th coupon

n_{p1} is the number of days between purchase date and the first coupon.

An example of the way CDs and time deposits are quoted on screen is shown at Figure 31.2, which shows one of the rates screens displayed by Garban ICAP, money brokers in London, on a Telerate screen. Essentially the same screen is displayed on Reuters. The screen has been reproduced with permission from Garban ICAP and Dow-Jones Telerate. The screen displays sterling interbank and CD bid and offer rates for maturities up to one year as at 8 December 1999, together with FRA rates and repo rates (in fact, there were no repo quotes at that time). The maturity marked “O/N” is the over-night rate, which at that time was 5.0625–4.9375. Rates for cash instruments in the sterling market, except for repo, are quoted in 32nds, so a spread of one thirty-second is 0.03125. The maturity marked “T/N” is “tom-next”, or “tomorrow-to-the-next”, which is the over-night rate for deposits commencing tomorrow. As we noted earlier, the liquidity of CDs means that they trade at a lower yield to deposits, and this can be seen from Figure 31.2. The bid-offer convention in sterling is that the lending rate – the rate at which funds are lent – placed on the left. This is the same for both CDs and deposits, so a six-month CD rate would be purchased at 6.125%, which means that funds are lent at 6.125%, while a six-month time deposit is lent at 6.15625%.

Dow Jones Markets : martin_g Telerate 4734 Wed Dec 08 09:00:48 1999

08/12	8:54 GMT	[GARBAN INTERCAPITAL-EUROPE]				12/08 02:12	4734
	FRA		GBP CDS DEPO	GBP INTERBANK DEP		GBP REPO (GC)	
1X4	6.020-990	O/N	-	5 1/16-4 15/16	-	-	O/N
2X5	6.110-080	T/N	-	5 3/8 -5 1/4	-	-	T/N
3X6	6.230-200	1WK	-	5 1/4 -5 1/8	-	-	1WK
4X7	6.330-300	1MO	5 25/32-5 23/32	5 7/8 -5 13/16	-	-	2WK
5X8	6.420-390	2MO	5 15/16-5 7/8	5 31/32-5 29/32	-	-	3WK
6X9	6.510-480	3MO	6-5 15/16	6-5 15/16	-	-	1MO
9X12	6.760-730	4MO	6 1/32-5 31/32	6 1/32-5 31/32	-	-	2MO
		5MO	6 1/16-	6 1/16-	6	-	3MO
1X7	6.240-210	6MO	6 1/8 -6 1/16	6 5/32-6 3/32	-	-	4MO
2X8	6.330-300	7MO	6 5/32-6 3/32	6 7/32-6 5/32	-	-	5MO
3X9	6.420-390	8MO	6 7/32-6 5/32	6 9/32-6 7/32	-	-	6MO
4X10	6.520-490	9MO	6 9/32-6 7/32	6 5/16-6 1/4	-	-	9MO
5X11	6.610-580	10M	6 11/32-6 9/32	6 3/8 -6 5/16	-	-	1YR
6X12	6.700-670	11M	6 13/32-6 11/32	6 7/16-6 3/8	-	-	
		12M	6 15/32-6 13/32	6 1/2 -6 7/16	-	-	
[FRA 695-2040, EUROSTG 695-2030, GBP REPOS 695-2255]							

Figure 31.2: Garban ICAP brokers sterling money markets screen, 8 December 1999.

©Garban ICAP and ©Dow-Jones Telerate. Reproduced with permission.

31.3 Securities quoted on a discount basis

The remaining money market instruments are all quoted on a *discount* basis, and so are known as “discount” instruments. This means that they are issued on a discount to face value, and are redeemed on maturity at face value. Hence Treasury bills, bills of exchange, bankers acceptances and commercial paper are examples of money market securities that are quoted on a discount basis, that is, they are sold on the basis of a discount to par. The difference between the price paid at the time of purchase and the redemption value (par) is the interest earned by the holder of the paper. Explicit interest is not paid on discount instruments, rather interest is reflected implicitly in the difference between the discounted issue price and the par value received at maturity. Note that in some markets CP is quoted on a yield basis, but not in the UK or in the US where they are discount instruments.

31.3.1 Treasury bills

Treasury bills or T-bills are short-term government “IOUs” of short duration, often three-month maturity. For example if a bill is issued on 10 January it will mature on 10 April. Bills of one-month and six-month maturity are also issued, but only rarely in the UK market. On maturity the holder of a T-Bill receives the par value of the bill by presenting it to the Central Bank. In the UK most such bills are denominated in sterling but issues are also made in euros. In a capital market, T-Bill yields are regarded as the *risk free* yield, as they represent the yield from short-term government debt. In emerging markets they are often the most liquid instruments available for investors.

A sterling T-bill with £10 million face value issued for 91 days will be redeemed on maturity at £10 million. If the three-month yield at the time of issue is 5.25%, the price of the bill at issue is:

$$P = \frac{10\text{m}}{(1 + 0.0525 \times \frac{91}{365})} = £9,870,800.69.$$

In the UK and US markets the interest rate on discount instruments is quoted as a *discount rate* rather than a yield. This is the amount of discount expressed as an annualised percentage of the face value, and not as a percentage of the original amount paid. By definition the discount rate is always lower than the corresponding yield. If the discount rate on a bill is d , then the amount of discount is given by (31.12):

$$d_{\text{value}} = M \times d \times \frac{n}{B}. \quad (31.12)$$

The price P paid for the bill is the face value minus the discount amount, given by (31.13):

$$P = 100 \times \left(1 - \frac{d \cdot (N_{sm}/365)}{100}\right). \quad (31.13)$$

If we know the yield on the bill then we can calculate its price at issue by using the simple present value formula, as shown at (31.14):

$$P = M/(1 + r(N_{sm}/365)). \quad (31.14)$$

The discount rate d for T-Bills is calculated using (31.15):

$$d = (1 - P) \times \frac{B}{n}. \quad (31.15)$$

The relationship between discount rate and true yield is given by (31.16):

$$\begin{aligned} d &= \frac{r}{\left(1 + r \times \frac{n}{B}\right)} \\ r &= \frac{d}{1 - d \times \frac{n}{B}}. \end{aligned} \quad (31.16)$$

EXAMPLE 31.3

- A 91-day £100 Treasury bill is issued with a yield of 4.75%. What is its issue price?

$$\begin{aligned} P &= £100 / \left(1 + 0.0475 \left(\frac{91}{365}\right)\right) \\ &= £98.80. \end{aligned}$$

- A UK T-bill with a remaining maturity of 39 days is quoted at a discount of 4.95% What is the equivalent yield?

$$\begin{aligned} r &= \frac{0.0495}{1 - 0.0495 \times \frac{39}{365}} \\ &= 4.976\%. \end{aligned}$$

If a T-Bill is traded in the secondary market, the settlement proceeds from the trade are calculated using (31.17):

$$\text{Proceeds} = M - \left(\frac{M \times \text{days remaining} \times d}{B \times 100} \right). \quad (31.17)$$

Bond equivalent yield

In certain markets including the UK and US markets the yields on government bonds that have a maturity of less than one year are compared to the yields of treasury bills; however the comparison can be made the yield on a bill must be converted to a “bond equivalent” yield. Therefore the bond equivalent yield of a US Treasury bill is the coupon of a theoretical Treasury bond trading at par that has an identical maturity date. If the bill has 182 days or less until maturity, the calculation required is the conventional conversion from discount rate to yield, with the exception that it is quoted on a 365-day basis (in the UK market, the quote basis is essentially the same unless it is a leap year. So the conversion element in (31.18) is not necessary). The calculation in the US market is given by (31.18):

$$rm = \frac{d}{\left(1 - d \times \frac{\text{days}}{360}\right)} \times \frac{365}{360} \quad (31.18)$$

where rm is the bond-equivalent yield that is being calculated.

If there are more than 182 days remaining to maturity on the bill though, the calculation must take into account the fact that an equivalent bond would pay a coupon during the period as well as on maturity. To convert the yield to a bond equivalent one (31.19) is used instead.

$$rm = \frac{-\frac{n}{365} + \left(\left(\frac{n}{365} \right)^2 + 2 \times \left(\frac{n}{365} - \frac{1}{2} \right) \times \left(\frac{1}{1 - d \times n/360} - 1 \right) \right)^{\frac{1}{2}}}{\left(\frac{n}{360} - \frac{1}{2} \right)}. \quad (31.19)$$

If 29 February falls in the 12-month period that begins on the purchase date, 365 is replaced by 366.

Note that if there is a bill and a bond that mature on the same day in a period under 182 days, the bond-equivalent yield will not be precisely the same as the yield quoted for the bond in its final coupon period, although it is a very close approximation. This is because the bond is quoted on actual/actual basis, so its yield is actually made up of $2 \times$ the actual number of days in the interest period.

US Treasury bills

The Treasury bill market in the United States is one of the most liquid and transparent debt markets in the world. Consequently the bid-offer spread on them is very narrow. The Treasury issues bills at a weekly auction each Monday, made up of 91-day and 182-day bills. Every fourth week the Treasury also issues 52-week bills as well. As a result there are large numbers of Treasury bills outstanding at any one time. The interest earned on Treasury bills is not liable to state and local income taxes.

Federal funds

Commercial banks in the US are required to keep reserves on deposit at the Federal Reserve. Banks with reserves in excess of required reserves can lend these funds to other banks, and these interbank loans are called *federal funds* or *fed funds* and are usually overnight loans. Through the fed funds market, commercial banks with excess funds are able to lend to banks that are short of reserves, thus facilitating liquidity. The transactions are very large denominations, and are lent at the *fed funds rate*, which is a very volatile interest rate because it fluctuates with market shortages.

Prime rate

The *prime interest rate* in the US is often said to represent the rate at which commercial banks lend to their most creditworthy customers. In practice many loans are made at rates below the prime rate, so the prime rate is not the best rate at which highly rated firms may borrow. Nevertheless the prime rate is a benchmark indicator of the level of US money market rates, and is often used as a reference rate for floating-rate instruments. As the market for bank loans is highly competitive, all commercial banks quote a single prime rate, and the rate for all banks changes simultaneously.

31.3.2 Bankers acceptances

A bankers acceptance is a written promise issued by a borrower to a bank to repay borrowed funds. The lending bank lends funds and in return accepts the bankers acceptance. The acceptance is negotiable and can be sold in the secondary market. The investor who buys the acceptance can collect the loan on the day that repayment is due. If the borrower defaults, the investor has legal recourse to the bank that made the first acceptance. Bankers acceptances are also known as *bills of exchange*, *bank bills*, *trade bills* or *commercial bills*.

Essentially bankers acceptances are instruments created to facilitate commercial trade transactions. The instrument is called a *bankers acceptance* because a bank accepts the ultimate responsibility to repay the loan to its holder. The use of bankers acceptances to finance commercial transactions is known as *acceptance financing*. The transactions in which acceptances are created for include import and export of goods, the storage and shipping of goods between two overseas countries, where neither the importer nor the exporter is based in the home country,⁴ and the storage and shipping of goods between two entities based at home. Acceptances are discount instruments and are purchased by banks, local authorities and money market investment funds. The creation of a bankers acceptance is illustrated at Example 31.4.

The rate that a bank charges a customer for issuing a bankers acceptance is a function of the rate at which the bank thinks it will be able to sell it in the secondary market. A commission is added to this rate. For ineligible bankers acceptances (see below) the issuing bank will add an amount to offset the cost of the additional reserve requirements.

EXAMPLE 31.4 Creation of hypothetical bankers acceptance in sterling market

■ The following fictitious institutions are involved in this process:

- ▶ PCTools plc, a firm in London that sells information technology equipment including personal computers, laptops and so on
- ▶ Rony Ltd, a manufacturer of personal computers based in Japan
- ▶ ABC Bank plc, a clearing bank based in London
- ▶ Samurai Bank, a bank based in Japan
- ▶ XYZ Bank plc, another bank based in London
- ▶ Thistle Investors plc, a money market fund based in Edinburgh

PCTools and Rony Ltd are to enter into a deal in which PCTools will purchase (that is, import) a consignment of personal computers (PCs) with a transaction value of £1 million; however Rony Ltd is concerned about the ability of PCTools to make payment on the PCs when they are delivered. To get around this uncertainty both parties decided to fund the transaction using acceptance financing. The terms of the transaction are that payment must be made by PCTools within 60 days after the PCs have been shipped to the United Kingdom. In determining whether it is willing to accept the £1 million, it must calculate the present value of the amount because it will not be receiving this sum until 60 days after shipment. Therefore it agrees to the following terms:

- ▶ PCTools arranges with its bankers, ABC Bank plc to issue a letter of credit (LOC, also known as a *time draft*). The LOC states that ABC Bank plc will guarantee the payment of £1 million that PCTools must make to Rony 60 days after shipment. The LOC is sent by ABC Bank plc to Rony's bankers, who are Samurai Bank. On receipt of the LOC, Samurai Bank notifies Rony, who will then transport the PCs. After the PCs are shipped, Rony presents the shipping documents to Samurai Bank and receives the present value of £1 million. This completes the transaction for Rony Ltd.
- ▶ Samurai Bank presents the LOC and the shipping documents to ABC Bank plc. The latter will stamp the LOC as "accepted", thus creating a bankers acceptance. This means that ABC Bank plc agrees to pay the holder of the bankers acceptance the sum of £1 million on the maturity date. PCTools will receive the shipping documents so that it can then take delivery of the PCs once it signs a note or financing arrangement with ABC Bank plc.

⁴ A bankers acceptance created to finance such a transaction is known as a *third-party acceptance*.

At this point the holder of the bankers acceptance is Samurai Bank. It has two choices open to it: it may retain the bankers acceptance as an investment in its loan portfolio, or it may request that ABC Bank plc makes a payment of the present value of £1 million. Let us assume that Samurai elects to request payment of the present value of £1 million. At this point, the holder of the bankers acceptance is ABC Bank plc. It also has two choices that it may make: it may retain the bankers acceptance as an investment, or it may sell it to an investor. Again, we assume that it chooses the latter, and one of its clients, Thistle Investors, is interested in a high-quality instrument with the same maturity as the bankers acceptance. ABC Bank plc sells the acceptance to Thistle Investments at the present value of £1 million, calculated using the appropriate discount rate for paper of that maturity and credit quality. Alternatively, it may have sold the acceptance to another bank, such as XYZ Bank plc, that also runs a book in acceptances. In either case, on maturity of the bankers acceptance, its holder presents it to ABC Bank plc and receives the maturity value of £1 million, which the bank in turn recovers from PCTools plc.

The holder of a bankers acceptance is exposed to credit risk on two fronts: the risk that the original borrower is unable to pay the face value of the acceptance, and the risk that the accepting bank will not be able to redeem the paper. For this reason the rate paid on a bankers acceptance will be on average 10–25 basis points higher than the equivalent maturity treasury bill. Someone trading in acceptances though will need to know the identity and credit risk of the original borrower as well as the accepting bank.

Eligible bankers acceptance

An accepting bank that chooses to retain a bankers acceptance in its portfolio may be able to use it as collateral for a loan obtained from the central bank during open market operations, for example the Bank of England in the UK and the Federal Reserve in the US. Not all acceptances are eligible to be used as collateral in this way, as they must meet certain criteria set by the central bank. The main requirement for eligibility is that the acceptance must be within a certain maturity band (a maximum of six months in the US and three months in the UK), and that it must have been created to finance a self-liquidating commercial transaction. In the US eligibility is also important because the Federal Reserve imposes a reserve requirement on funds raised via bankers acceptances that are ineligible. Bankers acceptances sold by an accepting bank are potential liabilities of the bank, but reserve imposes a limit on the amount of eligible bankers acceptances that a bank may issue. Bills eligible for deposit at a central bank enjoy a finer rate than ineligible bills, and also act a benchmark for prices in the secondary market.

31.3.3 Commercial paper

Commercial paper (CP) is a short-term money market funding instrument issued by corporates. In the UK and US it is a discount instrument. CP was reviewed in Chapter 22

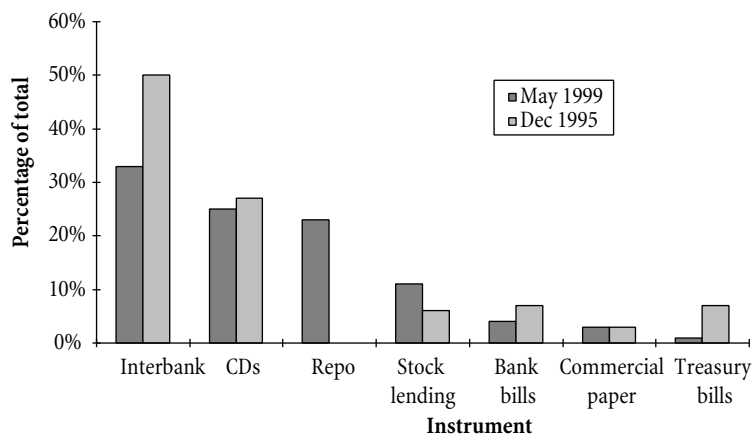


Figure 31.3: Composition of sterling money markets: May 1999.

Source: Bank of England.

31.4 Foreign exchange

The market in foreign exchange is an excellent example of a liquid, transparent and immediate global financial market. Rates in the foreign exchange (FX) markets move at an extremely rapid pace and in fact, it is a very different discipline to bond trading or money markets trading. There is a considerable literature on the FX markets, as it of course a separate subject in its own right. However some banks organise their forward desk as part of the money markets and not the foreign exchange desk, necessitating its inclusion in this book. For this reason we present an overview summary of FX in this chapter, both and forward. Interested readers may wish to follow up some of the references listed in the bibliography.

The quotation for currencies generally follows the ISO convention, which is also used by the SWIFT and Reuters dealing systems, and is the three-letter code used to identify a currency, such as USD for US dollar and GBP for sterling. The rate convention is to quote everything in terms of one unit of the US dollar, so that the dollar and Swiss franc rate is quoted as USD/CHF, and is the number of Swiss francs to one US dollar. The exception is for sterling, which is quoted as GBP/USD and is the number of US dollars to the pound. The rate for euros has been quoted both ways round, for example EUR/USD although some banks, for example NatWest Bank in the UK quotes euros to the pound, that is GBP/EUR.

The complete list of currency codes is given at Appendix 31.2.

POUND SPOT FORWARD AGAINST THE POUND											
Jul 22		Closing mid-point	Change on day	Bid/offer spread	Day's Mid high low	One month Rate %PA	Three months Rate %PA	One year Rate %PA	Bank of Eng. Index		
Europe											
Austria*	(Sch)	20.7608	+0.1367	519 - 697	20.7939 20.6215	20.7174 2.5	20.6354 2.4	20.2777 2.3	102.1		
Belgium*	(Bfr)	60.8627	+0.4008	366 - 887	60.9600 60.4540	60.7355 2.5	60.4951 2.4	59.4464 2.3	101.4		
Denmark	(DKr)	11.2251	+0.0710	214 - 288	11.2438 11.1504	11.2081 1.8	11.1772 1.7	11.037 1.7	104.2		
Finland*	(Fm)	8.9706	+0.0590	668 - 744	8.9850 8.9100	8.952 2.5	8.9164 2.4	8.7619 2.3	80.0		
France*	(FFr)	9.8968	+0.0652	925 - 010	9.9126 9.8304	9.8761 2.5	9.8369 2.4	9.6665 2.3	104.6		
Germany*	(DM)	2.9509	+0.0195	496 - 521	2.9559 2.9305	2.9447 2.5	2.933 2.4	2.8822 2.3	102.1		
Greece	(Dr)	490.517	+3.8590	147 - 887	491.224 486.870	492.321 -4.4	495.831 -4.3	505.308 -3.0	61.5		
Ireland*	(Ir)	1.1882	+0.0078	877 - 887	1.1901 1.1802	1.1857 2.5	1.181 2.4	1.1605 2.3	91.7		
Italy*	(L)	2921.34	+19.2400	009 - 259	2926.01 2901.74	2915.24 2.5	2903.7 2.4	2853.36 2.3	74.6		
Luxembourg*	(Lfr)	60.8627	+0.4008	366 - 887	60.9600 60.4540	60.7355 2.5	60.4951 2.4	59.4464 2.3	101.4		
Netherlands*	(Fl)	3.3249	+0.0219	234 - 263	3.3302 3.3025	3.3179 2.5	3.3048 2.4	3.2475 2.3	100.6		
Norway	(Nkr)	12.4693	+0.0803	630 - 756	12.4969 12.3688	12.4846 -1.5	12.5092 -1.3	12.5409 -0.6	94.7		
Portugal*	(Es)	302.476	+1.9920	347 - 606	302.954 300.446	301.844 2.5	300.649 2.4	295.437 2.3	91.1		
Spain*	(Pta)	251.034	+1.6530	927 - 142	251.430 249.360	250.509 2.5	249.518 2.4	245.192 2.3	76.0		
Sweden	(SKr)	13.2535	+0.1051	430 - 639	13.2731 13.1530	13.2308 2.1	13.1907 1.9	13.0373 1.6	82.5		
Switzerland	(Sfr)	2.4247	+0.0179	234 - 259	2.4280 2.4107	2.4165 4.1	2.4008 3.9	2.3363 3.6	105.5		
UK	(£)	-	-	-	-	-	-	-	102.9		
Euro	(€)	1.5088	+0.0100	081 - 094	1.5109 1.4987	1.5056 2.5	1.4996 2.4	1.4737 2.3	86.71		
SDR†	-	1.170470	-	-	-	-	-	-	-		
Americas											
Argentina	(Peso)	1.5827	+0.0090	824 - 830	1.5873 1.5743	-	-	-	-		
Brazil	(RS)	2.8825	+0.0322	803 - 846	2.8879 2.8593	-	-	-	-		
Canada	(C\$)	2.3783	+0.0157	771 - 795	2.3850 2.3624	2.3775 0.4	2.3767 0.3	2.375 0.1	78.5		
Mexico	(New Peso)	14.8207	+0.1205	100 - 314	14.8524 14.7101	15.0348 -17.3	15.4933 -18.2	17.6608 -19.2	-		
USA	(S)	1.5829	+0.0090	826 - 832	1.5875 1.5745	1.583 -0.1	1.5837 -0.2	1.5863 -0.2	109.0		
Pacific/Middle East/Africa											
Australia	(A\$)	2.4447	+0.0102	423 - 470	2.4531 2.4298	2.4442 0.3	2.4435 0.2	2.4386 0.2	83.7		
Hong Kong	(HK\$)	12.2851	+0.0708	819 - 882	12.3198 12.2202	12.298 -1.3	12.3269 -1.4	12.5173 -1.9	-		
India	(Rs)	68.4502	+0.4184	316 - 687	68.5440 68.1050	68.6891 -4.2	69.1656 -4.2	71.8554 -5.0	-		
Indonesia	(Rupiah)	10716.24	+163.23	046 - 201	10746.00 10577.30	10788.32 -8.1	10967.1 -9.4	11552.62 -7.8	-		
Israel	(Shk)	6.4823	+0.0285	755 - 891	6.4895 6.4854	-	-	-	-		
Japan	(Y)	185.880	-0.4150	781 - 979	187.290 185.650	185.085 5.1	183.54 5.0	176.35 5.1	134.3		
Malaysia	(M\$)	6.0150	+0.0342	150 - 150	6.0323 5.9839	-	-	-	-		
New Zealand	(NZ\$)	3.0008	+0.0046	973 - 042	3.0097 2.9866	2.9995 0.5	2.9975 0.4	2.9912 0.3	91.5		
Philippines	(Peso)	60.8625	+0.4247	718 - 532	60.9683 60.6600	61.0937 -4.6	61.5867 -4.8	64.2076 -5.5	-		
Saudi Arabia	(SR)	5.9368	+0.0341	354 - 381	5.9538 5.9053	5.9392 -0.5	5.9472 -0.7	5.9814 -0.8	-		
Singapore	(S\$)	2.6878	+0.0181	865 - 891	2.6948 2.6749	2.6829 2.2	2.6707 2.5	2.6237 2.4	-		
South Africa	(R)	9.6616	+0.0482	526 - 705	9.6910 9.6019	9.7253 -7.9	9.8344 -7.2	10.2308 -5.9	-		
South Korea	(Won)	1912.14	+22.6700	020 - 409	1916.11 1895.18	-	-	-	-		
Taiwan	(Ts)	51.1079	+0.2945	942 - 215	51.2524 50.8406	51.1752 -1.6	51.4108 -2.4	51.9252 -1.6	-		
Thailand	(Bt)	59.1926	+0.6907	418 - 433	59.2930 58.4790	59.4268 -4.7	59.6024 -2.8	60.0038 -1.4	-		

† Rates for Jul 21. Bid/offer spreads in the Pound Spot table show only the last three decimal places. Sterling index calculated by the Bank of England. Base average 1990 = 100. Index rebased 1/2/95. * EMU member. The exchange rates printed in this table are also available on the internet at <http://www.FT.com>.

Figure 31.4: Spot and forward exchange rates against pounds sterling, July 1999. Reproduced from *The Financial Times* 23 July 1999. ©Financial Times, 23/7/99. Used with permission.

EXCHANGE CROSS RATES

	Jul 22	Bfr	Dkr	Ffr	DM	IE	L	FI	Nkr	Es	Pta	SKr	Sfr	£	C\$	\$	Y	€
Belgium* (Bfr)	100	18.44	16.26	4.848	1.952	4800	5.463	20.49	497.0	412.5	21.78	3.984	1.643	3.908	2.601	305.4	2.479	
Denmark (DKr)	54.22	10	8.817	2.629	1.059	2602	2.962	11.11	269.5	223.6	11.81	2.160	0.891	2.119	1.410	165.6	1.344	
France* (FFr)	61.50	11.34	10	2.982	1.201	2952	3.360	12.60	305.6	253.7	13.39	2.450	1.010	2.403	1.600	187.8	1.525	
Germany* (DM)	20.63	3.804	3.354	1	0.403	990.0	1.127	4.226	102.5	85.07	4.491	0.822	0.339	0.806	0.536	62.99	0.511	
Ireland* (IE)	51.22	9.447	8.329	2.483	1	2459	2.798	10.49	254.6	211.3	11.15	2.041	0.842	2.001	1.332	156.4	1.270	
Italy* (L)	2.083	0.384	0.339	0.101	0.041	100	0.114	0.427	10.35	8.593	0.454	0.083	0.034	0.081	0.054	6.363	0.052	
Netherlands* (Fl)	18.31	3.376	2.977	0.888	0.357	878.6	1	3.750	90.97	75.50	3.986	0.729	0.301	0.715	0.476	55.91	0.454	
Norway (Nkr)	48.81	9.002	7.937	2.366	0.953	2343	2.666	10	242.6	201.3	10.63	1.945	0.802	1.907	1.269	149.1	1.210	
Portugal* (Pta)	20.12	3.711	3.272	0.976	0.393	965.8	1.099	4.122	100	82.99	4.382	0.802	0.331	0.786	0.523	61.45	0.499	
Spain* (Pta)	24.24	4.472	3.942	1.175	0.473	1164	1.324	4.967	120.5	100	5.280	0.966	0.398	0.947	0.631	74.05	0.601	
Sweden (SKr)	45.92	8.470	7.467	2.226	0.897	2204	2.509	9.408	228.2	189.4	10	1.829	0.755	1.794	1.194	140.3	1.138	
Switzerland (Sfr)	25.10	4.630	4.082	1.217	0.490	1205	1.371	5.143	124.7	103.5	5.466	1	0.412	0.981	0.653	76.66	0.622	
UK (£)	60.86	11.23	9.897	2.951	1.188	2921	1.398	5.243	302.5	251.0	13.25	2.425	1	2.378	1.583	185.9	1.509	
Canada (C\$)	25.59	4.720	4.161	1.241	0.500	1228	1.398	5.243	127.2	105.6	5.573	1.020	0.420	1	0.666	78.16	0.634	
USA (\$)	38.45	7.092	6.252	1.864	0.751	1845	2.108	6.788	181.1	158.6	8.373	1.532	0.632	1.503	1	117.4	0.953	
Japan (Y)	32.74	6.039	5.324	1.588	0.639	1572	1.789	6.708	162.7	135.1	7.130	1.304	0.538	1.279	0.852	100	0.812	
Euro (€)	40.34	7.440	6.560	1.956	0.788	1936	2.204	8.265	200.5	166.4	8.785	1.607	0.663	1.576	1.049	123.2	1	

Danish Kroner, French Franc, Norwegian Kroner, and Swedish Kronor per 100; Belgian Franc, Yen, Escudo, Lira and Peseta per 100. * EMU member.

Figure 31.5: Foreign exchange cross rates, July 1999. Reproduced from *The Financial Times* 23 July 1999.

©Financial Times, 23/7/99. Used with permission.

31.4.1 Spot exchange rates

A *spot* FX trade is an outright purchase or sale of one currency against another currency, with delivery two working days after the trade date. Non-working days do not count, so a trade on a Friday is settled on the following Tuesday. There are some exceptions to this, for example trades of US dollar against Canadian dollar are settled the next working day; note that in some currencies, generally in the Middle East, markets are closed on Friday but open on Saturday. A settlement date that falls on a public holiday in the country of one of the two currencies is delayed for settlement by that day. An FX transaction is possible between any two currencies, however to reduce the number of quotes that need to be made the market generally quotes only against the US dollar or occasionally sterling or euro, so that the exchange rate between two non-dollar currencies is calculated from the rate for each currency against the dollar. The resulting exchange rate is known as the *cross-rate*. Cross-rates themselves are also traded between banks in addition to dollar-based rates. This is usually because the relationship between two rates is closer than that of either against the dollar, for example the Swiss franc moves more closely in line with the euro than against the dollar, so in practice one observes that the dollar/Swiss franc rate is more a function of the euro/franc rate.

The spot FX quote is a two-way bid-offer price, just as in the bond and money markets, and indicates the rate at which a bank is prepared to buy the base currency against the variable currency; this is the “bid” for the variable currency, so is the lower rate. The other side of the quote is the rate at which the bank is prepared to sell the base currency against the variable currency. For example a quote of 1.6245–1.6255 for GBP/USD means that the bank is prepared to buy sterling for \$1.6245, and to sell sterling for \$1.6255. The convention in the FX market is uniform across countries, unlike the money markets. Although the most common money market convention for bid-offer quotes is for example, 5½%–5¼%, meaning that the “bid” for paper – the rate at which the bank will lend funds, say in the CD market – is the higher rate and always on the left, this convention is reversed in certain countries. In the FX markets the convention is always the same one just described.

The difference between the two sides in a quote is the bank’s dealing spread. Rates are quoted to 1/100 of a cent, known as a *pip*. In the quote above, the spread is 10 pips, however this amount is a function of the size of the quote number, so that the rate for USD/JPY at say, 110.10–110.20, indicates a spread of 0.10 yen. Generally only the pips in the two rates are quoted, so that for example the quote above would be simply “45–55”. The “big figure” is not quoted.

EXAMPLE 31.5 Exchange cross-rates

- Consider the following two spot rates:

EUR/USD	1.0566–1.0571
AUD/USD	0.7034–0.7039

The EUR/USD dealer buys euros and sells dollars at 1.0566 (the left-hand side), while the AUD/USD dealer sells Australian dollars and buys US dollars at 0.7039 (the right-hand side). To calculate the rate at which the

bank buys euros and sells Australian dollars, we need $1.0566/0.7039 = 1.4997$ is the rate at which the bank buys euros and sells Australian dollars. In the same way the rate at which the bank sells euros and buys Australian dollars is given by $1.0571/0.7034$ or 1.5028 . So the spot EUR/AUD rate is $1.4997 - 1.5028$.

The derivation of cross-rates can be depicted in the following way. If we assume two exchange rates XXX/YYY and XXX/ZZZ, the cross-rates are:

$$YYY/ZZZ = XXX/ZZZ \div XXX/YYY$$

$$ZZZ/YYY = XXX/YYY \div XXX/ZZZ$$

Given two exchange rates YYY/XXX and XXX/ZZZ, the cross-rates are:

$$YYY/ZZZ = YYY/XXX \times XXX/ZZZ$$

$$ZZZ/YYY = 1 \div (YYY/XXX \times XXX/ZZZ)$$

31.4.2 Forward exchange rates

Forward outright

The spot exchange rate is the rate for immediate delivery (notwithstanding that actual delivery is two days forward). A *forward contract* or simply *forward* is an outright purchase or sale of one currency in exchange for another currency for settlement on a specified date at some point in the future. The exchange rate is quoted in the same way as the spot rate, with the bank buying the base currency on the bid side and selling it on the offered side. In some emerging markets no liquid forward market exists so forwards are settled in cash against the spot rate on the maturity date. These *non-deliverable forwards* are considered at the end of this section.

Although some commentators have stated that the forward rate may be seen as the market's view of where the spot rate will be on the maturity date of the forward transaction, this is incorrect. A forward rate is calculated on the current interest rates of the two currencies involved, and the principle of no-arbitrage pricing ensures that there is no profit to be gained from simultaneous (and opposite) dealing in spot and forward. Consider the following strategy:

- borrow US dollars for six months starting from the spot value date;
- sell dollars and buy sterling for value spot;
- deposit the long sterling position for six months from the spot value date;
- sell forward today the sterling principal and interest which mature in six months' time into dollars.

The market will adjust the forward price so that the two initial transactions if carried out simultaneously will generate a zero profit/loss. The forward rates quoted in the trade will be calculated on the six months' deposit rates for dollars and sterling; in general the calculation of a forward rate is given as (31.20):

$$\text{Fwd} = \text{Spot} \times \frac{(1 + \text{variable currency deposit rate} \times (\text{days}/B))}{(1 + \text{base currency deposit rate} \times (\text{days}/B))}. \quad (31.20)$$

The year day-count base will be either 365 or 360 according to the day-base for each currency.

EXAMPLE 31.6 Forward rate

90-day GBP deposit rate: 5.75%

90-day USD deposit rate: 6.15%

Spot GBP/USD rate: 1.6315 (mid-rate)

The forward rate is given by:

$$1.6315 \times \frac{(1 + 0.0575 \times \frac{90}{365})}{(1 + 0.0615 \times \frac{90}{360})} = 1.6296.$$

Therefore to deal forward the GBP/USD mid-rate is 1.6296, so in effect £1 buys \$1.6296 in three months' time as opposed to \$1.6315 today. Under different circumstances sterling may be worth more in the future than at the spot date.

Forward swaps

The calculation given above illustrates how a forward rate is calculated and quoted in theory. In practice as spot rates change rapidly, often many times even in one minute, it would be tedious to keep re-calculating the forward rate so often. Therefore banks quote a forward spread over the spot rate, which can then be added or subtracted to the spot rate as it changes. This spread is known as the *swap points*. An approximate value for the number of swap points is given by (31.21):

$$\text{Forward swap} \approx \text{spot} \times \text{deposit rate differential} \times \frac{\text{days}}{B}. \quad (31.21)$$

The approximation is not accurate enough for forwards maturing more than 30 days from now, in which case another equation must be used. This is given as (31.22). It is also possible to calculate an approximate deposit rate differential from the swap points by rearranging (31.21):

$$\text{Forward swap} = \text{spot} \times \frac{(\text{variable currency depo rate} \times (\text{days}/B) - \text{base currency depo rate} \times (\text{days}/B))}{(1 + \text{base currency depo rate} \times (\text{days}/B))} \quad (31.22)$$

EXAMPLE 31.7 Forward swap points

Spot EUR/USD: 1.0566–1.0571
 Forward swap: 0.0125–0.0130
 Forward outright: 1.0691–1.0701

The forward outright is the spot price + the swap points, so in this case,

$$\begin{aligned} 1.0691 &= 1.0566 + 0.0125 \\ 1.0701 &= 1.0571 + 0.0130. \end{aligned}$$

Spot EUR/USD rate: 0.9501
 31-day EUR rate: 3.15%
 31-day USD rate: 5.95%

$$\text{Forward swap} = 0.9501 \times \frac{0.0595 \times \frac{31}{360} - 0.0315 \times \frac{31}{360}}{1 + 0.0315 \times \frac{31}{360}} = 0.0024, \text{ or } +24 \text{ swap points.}$$

The swap points are quoted as two-way prices in the same way as spot rates. In practice a middle spot price is used and then the forward swap spread around the spot quote. The difference between the interest rates of the two currencies will determine the magnitude of the swap points and whether they are added or subtracted from the spot rate. When the swap points are positive and the forwards trader applies a bid-offer spread to quote a two-way price, the left-hand side of the quote is smaller than the right-hand side as usual. When the swap points are negative, the trader must quote a “more negative” number on the left and a “more positive” number on the right-hand side. The “minus” sign is not shown however, so that the left-hand side may appear to be the larger number. Basically when the swap price appears larger on the right, it means that it is negative and must be subtracted from the spot rate and not added.

Forwards traders are in fact interest rate traders rather than foreign exchange traders, which explains why in many books the forwards trader sits on the money market and not the FX desk; although they will be left positions that arise from customer orders, in general they will manage their book based on their view of short-term deposit rates in the currencies they are trading. A forward trader expecting the interest rate differential to move in favour of the base currency, for example, a rise in base currency rates or a fall in the variable currency rate, will “buy and sell” the base currency. Trading in forwards enables the trader to transact a simultaneous borrowing in the base currency against a deposit in the variable currency. The relationship between interest rates and forward swaps means that banks can take advantage of different opportunities in different markets. Assume that a bank requires funding in one currency but is able to borrow in another currency at a relatively cheaper rate. It may wish to borrow in the second currency and use a forward contract to convert the borrowing to the first currency. It will do this if the all-in cost of borrowing is less than the cost of borrowing directly in the first currency.

Forward cross-rates

A forward cross-rate is calculated in the same way as spot cross-rates. The formulas given for spot cross-rates can be adapted to forward rates.

Forward-forwards

A *forward-forward* swap is a deal between two forward dates rather than from the spot date to a forward date; this is the same terminology and meaning as in the bond markets, where a forward or a forward-forward rate is the zero-coupon interest rate between two points both beginning in the future. In the foreign exchange market, an example would be a contract to sell sterling three months forward and buy it back in six months' time. Here, the swap is for the three-month period between the three-month date and the six-month date. The reason a bank or corporate might do this is to hedge a forward exposure or because of a particular view it has on forward rates, in effect deposit interest rates.

EXAMPLE 31.8 Forward-forward contract

GBP/USD spot rate: 1.6315–20
 3-month swap: 45–41
 6-month swap: 135–125

If a bank wished to sell GBP three month forward and buy them back six months forward, this is identical to undertaking one swap to buy GBP spot and sell GBP three months forward, and another to sell GBP spot and buy it six months forward. Swaps are always quoted as the quoting bank buying the base currency forward on the bid side, and selling the base currency forward on the offered side; the counterparty bank can “buy and sell” GBP “spot against three months” at a swap price of –45, with settlement rates of spot and (spot – 0.0045). It can “sell and buy” GBP “spot against six months” at the swap price of –125 with settlement rates of spot and (spot – 0.0125). It can therefore do both simultaneously, which implies a difference between the two forward prices of $(-125) - (-45) = -90$ points. Alternatively, the bank can “buy and sell” GBP “three months against six months” at a swap price of $(-135) - (-41)$ or –94 points. The two-way price is therefore 94–90 (we ignore the negative signs).

Long-dated forward contracts

The formula for calculating a forward rate was given earlier (see 31.20). This formula applies to any period that is under one year, hence the adjustment of the deposit rate by the fraction of the day-count. However if a forward contract is traded for a period greater than one year, the formula must be adjusted to account for the fact that deposit rates are compounded if they are in effect for more than one year. To calculate a long-dated forward rate, in theory (31.23) should be used. In practice the formula may not give an answer to the required accuracy, because it does not consider reinvestment risk. To get around this it is necessary to use spot (zero-coupon) rates in the formula. However the market in long-dated forward contracts is not as liquid as the sub-1-year market, so banks may not be as keen to quote a price.

$$\text{Long-dated forward} = \text{spot} \times \frac{(1 + \text{variable currency deposit rate})^N}{(1 + \text{base currency deposit rate})^N} \quad (31.23)$$

where N is the contract's maturity in years.

31.4.3 Non-deliverable forward

A market in *non-deliverable forwards* was established first in Latin American currencies and then in certain Asian currencies, as organisations interested in investing and trading with counterparties in countries in these areas were constrained by local exchange control regulations and by the absence of a forward foreign exchange market in the currencies of the countries concerned. A non-deliverable forward (NDF) is conceptually similar to an outright forward foreign exchange transaction. In essence a (notional) principal amount, forward foreign exchange rate and forward delivery date are all agreed at the time of the trade. In the case of an NDF however, there is no physical exchange of principal amount; the deal is agreed on the basis that net settlement will be made in US dollars, or another fully convertible currency, to reflect any differential between the agreed forward rate and the actual exchange

rate on the agree forward date. An NDF therefore is a cash-settled outright forward and is more similar to a *contract for difference* than a conventional forward.

A fixing methodology is agreed when the NDF is contracted. It specifies how a fixing spot rate is to be determined on the fixing date, which is normally one or two business days before settlement. Generally the fixing spot rate is based on a reference page on Reuters or Telerate. Settlement is made in the major currency, paid to or by the counterparty and reflects the differential between the agreed NDF forward rate and the fixing rate. Using an NDF a corporate can hedge any forward exposure in the currency even though no forward market is available. As NDFs are cash settled instruments with no exchange of principal amount involved, settlement risk is reduced and the credit risk is therefore lower than for a conventional forward contract.

In Asia NDF markets developed through 1995 and 1996 in several currencies including the Korean won, Taiwan dollar, Philippine peso and Indian rupee.

Once a NDF currency becomes fully convertible then the need for a non-deliverable market falls away, as a deliverable offshore forward market then develops. This has happened with certain Latin American currencies such as the Argentina peso. As the economies of the other emerging markets develop, investors may focus on other currencies such as those of South Asia and south-east Asia.

EXAMPLE 31.9 Non-deliverable forward

- A fund manager has invested \$2 million in the Taiwanese stock market for one year. She expects the stock market to rise but is concerned about potential Taiwan dollar (TWD) depreciation. She wishes to hedge her foreign exchange exposure using an NDF. A USD/TWD non-deliverable forward rate of 35.00 is agreed between the fund manager and her bank for a notional principal of \$2 million.

There are three possible outcomes in one year's time; the TWD may have reached the forward rate, depreciated further or appreciated relative to the forward rate. These scenarios are shown below.

	Outcome A	Outcome B	Outcome C
TWD	Depreciated	–	Appreciated
Fixing spot rate	35.3	35	34.7
Equivalent rate	US\$ 1,983,002.83	US\$ 2 million	US\$ 2,017,291.07
Settlement	Bank pays customer: US\$ 16,997.17	No net payment	Customer pays bank: US\$ 17,291.07

Figure 31.6: Example of non-deliverable forward.

Whatever the outcome, the fund manager has achieved the objective of her Taiwan dollar exposure at 35.00. In outcome A, the hedge has worked in the fund manager's favour and when she physically sells the Taiwan dollars in the spot market, she will be compensated for the higher rate by the proceeds of the NDF, received from the bank. In outcome C the fund manager can achieve a better rate in the spot market, but this negated by the payment required on the NDF.

THE CENTRAL MONEYMARKETS OFFICE

The Central Moneymarkets Office (CMO) is the clearing system for the London money markets. It supports the settlement of sterling- and euro-denominated money markets instruments. The CMO was originally developed by the Bank of England but merged with CREST, the London market equity settlement system; hence CMO is now operated by CRESTCo on behalf of the Bank. The CMO mechanism provides secure facilities for what is termed the *immobilisation* and transfer of bills, CDs and commercial paper. Settling through CMO means that the clearing system is *dematerialised*, that is by computer book-entry transfer, obviating the need for physical settlement.

CMO settlement

When settled through CMO, money market instruments are immobilised within the CMO Depository operated

by the Bank of England on behalf of CRESTCo. The instrument is lodged into CMO by the Issuing and Paying Agent (IPA), which acts as agent for the issuer of the paper. Lodgement usually is into the account of the primary dealer, against payment sent via CMO. The primary dealer may retain the instruments in its CMO account, or they may be passed on to other CMO members.

CMO settlement occurs on the same day of issue, between 08.30 and 17.00 hours. This same-day settlement is termed *London Good Delivery*. The service supports the transfer of instruments against sterling or euro payment; paper may be used as collateral against loans or in repo within the CMO mechanism, which assists in maintaining market liquidity. Once paper is lodged in CMO it is of course dematerialised, but it may be withdrawn into physical form before it matures. However this is rare. Paper may also be transferred to Euro-clear, which maintains an account at CMO. When an instrument matures, CMO will automatically transfer it to the IPA, in return for the redemption proceeds.

Only CMO members may lodge instruments into the CMO facility. As at May 2000 there were 31 members of the service, which included the main commercial banks such as RBS NatWest, Lloyds TSB, Barclays, Chase Manhattan and Citibank, and descendants of the discount houses including Gerrard & King and Cater Allen International.

CRESTCo is currently engaged in full integration of CMO and the gilts settlement system CGO with the CREST service, which is planned for 2002. Once complete this will result in dematerialisation of money market instruments, gilts and London market equities.

Appendices

APPENDIX 31.1 Currencies using money market year base of 365 days

- Sterling
- Hong Kong dollar
- Malaysian ringgit
- Singapore dollar
- South African rand
- Taiwan dollar
- Thai baht

In addition the domestic markets, but not the international markets, of the following currencies also use a 365-day base:

- Australian dollar
- Canadian dollar
- Japanese yen
- New Zealand dollar

To convert an interest rate i quoted on a 365-day basis to one quoted on a 360-day basis (i^*) use the expressions given at (31.24):

$$\begin{aligned} i &= i^* \times \frac{365}{360} \\ i^* &= i \times \frac{360}{365}. \end{aligned} \tag{31.24}$$

APPENDIX 31.2 Country SWIFT/ISO Currency Codes

Country	Currency	Code	Country	Currency	Code
Abu Dhabi	UAE dirham	AED	Kuwait	dinar	KWD
Albania	lek	ALL	Lebanon	Lebanese pound	LBP
Algeria	dinar	DZD	Libya	dinar	LYD
Angola	kwanza	AON	Liechtenstein	Swiss franc	CHF
Argentina	peso	ARS	Luxembourg	euro	EUR
Australia	Australian dollar	AUD		Luxembourg franc	LUF
Austria	schilling	ATS	Macao	pataca	MOP
	euro	EUR	Macedonia	denar	MKD
Bahamas	Bahama dollar	BSD	Madeira	euro	EUR
Bahrain	dinar	BHD		Portuguese escudo	PTE
Bangladesh	taka	BDT	Malagasy Republic	franc	MGF
Belarus	rouble	BYR	Malawi	kwacha	MWK
Belgium	Belgian franc	BEF	Malaysia	ringgitt	MYR
	euro	EUR	Maldives	rufiyaa	MVR
Bermuda	Bermuda dollar	BMD	Mali	CFA franc	XOF
Bolivia	boliviano	BOB	Malta	lira	MTL
Brazil	real	BRL	Martinique	euro	EUR
Brunei	Brunei dollar	BND		French franc	FRF
Bulgaria	lev	BGL	Mauritania	ouguiya	MRO
Burkina Faso	CFA franc	XOF	Mauritius	rupee	MUR
Burundi	Burundi franc	BIF	Mexico	peso nuevo	MXN
Cambodia	riel	KHR	Morocco	dirham	MAD
Cameroon	CFA franc	XAF	Namibia	rand	NAD
Canada	Canadian dollar	CAD	Nepal	rupee	NPR
Canary Islands	euro	EUR	Netherlands	euro	EUR
	Spanish peseta	ESP		guilder	NLG
Central African Republic	CFA franc	XAF	New Zealand	NZ dollar	NZD
Chad	CFA franc	XAF	Nigeria	naira	NGN
Chile	peso	CLP	Norway	krone	NOK
China	renmimbi yuan	CNY	Oman	riyal	OMR
Colombia	peso	COP	Pakistan	rupee	PKR
Costa Rica	colon	CRC	Peru	solnuevo	PEN
Croatia	kuna	HRK	Philippines	peso	PHP
Cuba	peso	CUP	Poland	zloty	PLN
Cyprus	Cyprus pound	CYP	Portugal	euro	EUR
Czech Republic	koruna	CZK		escudo	PTE
Denmark	krone	DKK	Qatar	riyal	QAR
Dubai	UAE dirham	AED	Romania	leu	ROL
Ecuador	sucre	ECS	Russia	rouble	RUB
Egypt	Egyptian pound	EGP	Rwanda	franc	RWF
Equatorial Guinea	CFA franc	XAF	Saudi Arabia	riyal	SAR
Estonia	kroon	EEK	Senegal	CFA franc	XOF
Ethiopia	birr	ETB	Seychelles	rupee	SCR
European Monetary Union	euro	EUR	Sierra Leone	leone	SLL
Finland	euro	EUR	Singapore	Singapore dollar	SGD
	markka	FIM	Slovakia	koruna	SKK

Country	Currency	Code	Country	Currency	Code
France	euro	EUR	Slovenia	tolar	SIT
	French franc	FRF	Solomon Islands	dollar	SBD
Gambia	dalasi	GMD	Somalia	shilling	SOS
Germany	euro	EUR	South Africa	rand	ZAR
	mark	DEM	Spain	euro	EUR
Ghana	cedi	GHC		peseta	ESP
Gibraltar	Gibraltar pound	GIP	Sri Lanka	rupee	LKR
Great Britain	pound	GBP	St Lucia	E Caribbean dollar	XCD
Greece	drachma	GRD	Sudan	dinar	SDD
Greenland	Danish krone	DKK	Sudan	Sudanese pound	SDP
Grenada	E Caribbean dollar	XCD	Surinam	Surinam guilder	SRG
Guatemala	quetzal	GTQ	Swaziland	lilangeni	SZL
Guinea Republic	Guinean franc	GNF	Sweden	krona	SEK
Haiti	gourde	HTG	Switzerland	Swiss franc	CHF
Honduras	lempira	HNL	Syria	Syrian pound	SYF
Hong Kong	Hong Kong dollar	HKD	Taiwan	dollar	TWD
Hungary	forint	HUF	Tanzania	shilling	TZS
Iceland	krona	ISK	Thailand	baht	THB
India	rupee	INR	Togo	CFA franc	XOF
Indonesia	rupiah	IDR	Trinidad & Tobago	TT dollar	TTD
Iran	rial	IRR	Tunisia	dinar	TND
Iraq	dinar	IQD	Turkey	lira	TRL
Irish Republic	euro	EUR	Turkmenistan	manat	TMM
	punt	IEP	Uganda	shilling	UGX
Israel	shekel	ILS	Ukraine	hryvna	UAH
Italy	euro	EUR	United Arab Emirates	dirham	AED
	lira	ITL	Uruguay	peso	UYF
Ivory Coast	CFA franc	XOF	USA	US dollar	USD
Jamaica	Jamaican dollar	JMD	Venezuela	bolivar	VEB
Japan	yen	JPY	Vietnam	dong	VND
Jordan	dinar	JOD	Virgin Islands (USA)	US dollar	USD
Kenya	shilling	KES	Yemen	rial	YER
Korea (North)	won	KPW	Yugoslavia	dinar	YUM
Korea (South)	won	KRW	Zambia	kwacha	ZMK
			Zimbabwe	Zimbabwe dollar	ZWD

Selected bibliography and references

- Bishop, P., Dixon, D., *Foreign Exchange Handbook*, McGraw-Hill, 1992.
- Cook, T., Rowe, T. (eds.), *Instruments of the Money Market*, 6th edition, Federal Reserve Bank of Richmond, 1986.
- Grabbe, J., *International Financial Markets*, Elsevier, 1986.
- Sarver, E., *The Eurocurrency Market Handbook*, New York Institute of Finance, 1990.
- Stigum, M., *The Money Market*, 4th edition, Irwin, 1995.
- Stigum, M., Mann, J., *Money Market Calculations*, Dow-Jones Books, 1981.
- Walmsley, J., *The Foreign Exchange and Money Markets Guide*, Wiley, 1992.

32

Banking Regulatory Capital Requirements

Commercial banks and securities houses are subject to a range of regulations and controls, a primary one of which is concerned with the level of capital that a bank holds, and that this level is sufficient to provide a cushion for the activities that the bank enters into. Typically an institution is subject to regulatory requirements of its domestic regulator, but may also be subject to cross-border requirements such as the European Union's Capital Adequacy Directive.¹ A capital requirements scheme proposed by a committee of central banks acting under the auspices of the Bank for International Settlements (BIS) in 1988 has been adopted universally by banks around the world. These are known as the BIS regulatory requirements or the Basle capital ratios, from the town in Switzerland where the BIS is based.² Under the Basle requirements all cash and off-balance sheet instruments in a bank's portfolio are assigned a risk weighting, based on their perceived credit risk, that determines the minimum level of capital that must be set against them. The Basle rules came into effect in 1992. The BIS is currently inviting comment on proposals for a new system of capital adequacy to replace the current rules. Further proposals are expected during 2000, with the BIS hoping to implement the agreed requirements during 2002.

In this chapter we summarise the essential elements of the current requirements, and then discuss reaction to the BIS proposals for updating them.

32.1 Regulatory issues

The need for adequate regulation of the banking industry is widely recognised, and a string of banking failures in the 1990s have possibly emphasised this. The most spectacular instances of failure, for example the collapse of Barings Bank in 1995, were found to have resulted from dishonest action and lack of internal management control, while other large-scale trading losses such as those at Kidder Peabody and Daiwa Securities allegedly resulted from individual traders mis-marking their books, to cover up losses. The closure of Yamaichi Securities was an instance of large-scale bad loans no longer being tenable, although other issues were involved as well. Although such cases raise the importance of a strong system and culture of risk management and internal controls in commercial and investment banking, they do not in themselves necessarily strengthen the need for excessive capital levels. By the nature of their activities, bank trading and lending desks are risk takers, and the reward culture in many banks provides strong incentives for perhaps excessive risk taking. However the regulators are more concerned with *systemic risk*, the risk that, as a result of the failure of one bank, the whole banking system is put in danger, due to knock-on effects. This did not arise in 1995 in the United Kingdom when Barings collapsed, because the bank was not a large enough part of the monetary system. However the integrated nature of the global financial industry means that banks are closely entwined, and the failure of one bank generates a risk of failure for all those banks that have lend funds to the failed bank. So systemic risk is a major challenge for regulators. While a bank will be concerned with risk management of its own operations, regulators are concerned with the risk to the whole financial system. The inter-relationships between banks means that they have exposures to one another, while the profit motive encourages risk-taking. The systemic risk inherent in the banking system means that it is important to have sufficiently adequate financial regulation, of which the capital requirements rules are one example.

¹ In the United Kingdom banking regulation is now the responsibility of the Financial Services Authority, which took over responsibility for this area from the Bank of England in 1998. In the United States banking supervision is conducted by the Federal Reserve; it is common for the central bank to be a country's domestic banking regulator.

² Bank for International Settlements, Basle Committee on Banking Regulations and Supervisory Practice, *International Convergence of Capital Measurement and Capital Standards*, July 1988. At the time of writing, the BIS is soliciting comment on its Basle II draft proposals, which are required by summer 2001. The final version of Basle II is due to become effective in 2004.

32.2 Capital adequacy requirements

32.2.1 The Basle rules

The BIS rules set a minimum ratio of capital to assets of 8% of the value of the assets. Assets are defined in terms of their risk, and it is the *weighted risk assets* that are multiplied by the 8% figure. Each asset is assigned a risk weighting, which is 0% for risk-free assets such as certain country government bonds, up to 100% for the highest-risk assets such as certain corporate loans. So while a loan in the interbank market would be assigned a 20% weighting, a loan of exactly the same size to a corporate would receive the highest weighting of 100%.

Formally, the BIS requirements are set in terms of the type of capital that is being set aside against assets. International regulation (and UK practice) defines the following types of capital for a bank:

- **Tier 1:** perpetual capital, capable of absorbing loss through the non-payment of a dividend. This is shareholders' equity and also non-cumulative preference shares;
- **Upper Tier 2:** this is also perpetual capital, subordinated in repayment to other creditors; this may include for example undated bonds such as building society PIBS, and other irredeemable subordinated debt;
- **Lower Tier 2:** this is capital that is subordinated in repayment to other creditors, such as long-dated subordinated bonds.

The level of capital requirement is given by (32.1):

$$\begin{aligned} \frac{\text{Tier 1 capital}}{\text{Risk-adjusted exposure}} &> 4\% \\ \frac{\text{Tier 1} + \text{Tier 2 capital}}{\text{Risk-adjusted exposure}} &> 8\%. \end{aligned} \tag{32.1}$$

The ratios at (32.1) therefore set minimum levels. A bank's *risk-adjusted exposure* is the cash risk-adjusted exposure together with the total risk-adjusted off-balance sheet exposure. For cash products the risk-adjusted exposure is given by:

$$\text{nominal value} \times \text{risk weighting}$$

calculated for each instrument. The risk weights are given in Table 32.1.

Asset	Risk weighting (%)
Treasury bill	0.0
Interbank loan	20.0
Residential mortgage loan	50.0
Corporate loan	100.0

Table 32.1: BIS requirements risk weighting.

Example 32.1 illustrates a simple capital adequacy calculation for a hypothetical bank. To illustrate, consider a bank with a loan book made up of the following assets:

- £100m gilts;
- £315m corporate loans;
- £600m residential mortgages.

The risk-adjusted exposure of the bank's portfolio is $(0.0 \times 100) + (1.0 \times 315) + (0.5 \times 600)$ or £615 million. Therefore the bank would require a minimum tier 1 capital level of £24.6 million (that is, $4\% \times 615$ million). If the capital available to support the loan book comprised both tier 1 and tier 2 capital, the minimum amount required would be higher, at £49.2 million.

There is of course a cost associated with maintaining capital levels, which is one of the main reasons for the growth in the use of derivative (off-balance sheet) instruments, as well as the rise in securitisation. Derivative instruments attract a lower capital charge than cash instruments, because the principal in a derivative instrument does not change hands and so is not at risk, while the process of securitisation removes assets from a bank's balance sheet, thereby reducing its capital requirements.

The capital rules for off-balance sheet instruments are slightly more involved. For certain instruments such as FRAs and swaps with a maturity of less than one year have no capital requirement at all, while longer-dated interest-rate swaps and currency swaps are assigned a risk weighting of between 0.08% and 0.20% of the nominal value. This is a significantly lower level than for cash instruments. For example, a £50 million 10-year interest-rate swap conducted between two banking counterparties would attract a capital charge of only £40,000, compared to the £800,000 capital an interbank loan of this value would require; a corporate loan of this value would require a higher capital level still, of £4 million.

The capital calculation for derivatives have detail differences between them, depending the instrument that is being traded. For example for interest-rate swaps the exposure includes an “add-on factor” to what is termed the instrument’s “current exposure”. This add-on factor is a percentage of the nominal value, and is shown in Table 32.2.

Maturity	Plain vanilla	Floating/Floating swaps	Currency swaps
Up to 1 year	0.0	0.0	1.0
Over 1 year	0.5	0.0	5.0

Table 32.2: Add-on risk adjustment for interest-rate swaps, percentage of nominal value.

EXAMPLE 32.1 Simple illustration of calculation of capital adequacy

ABC Bank plc Balance Sheet

Assets	Weighting (%)	Value (£m)	Capital risk weighting (£m)
Treasury Bills	0	250	0
Cash	0	30	0
Interbank loans	20	790	158
Mortgage book	50	652	326
Commercial loan book	100	814	814
TOTAL		<u><u>2536</u></u>	1298
Capital charge (8%)			103.84
Liabilities			
Shareholders funds	100		
Reserves	356	456	
Long-term debt	500		
Deposits	1580	2080	
		<u><u>2536</u></u>	

Table 32.3: Example of capital adequacy calculation.

The assets of ABC Bank plc are £2.536 billion, which are balanced by shareholders funds and long-term borrowings, as well as the deposit base of the bank. The Basle risk weighting assigns the various types of assets a certain risk weighting, and using the rules we calculate a capital at risk value of £1.298 billion. The capital required is 8% of this sum, or just over £103 million. The Basle rule states that at least 50% of this amount must be sourced from Tier 1 capital. We see from the table that the level of Tier 1 capital is well above the sum required. The combination of Tier 1 and Tier 2 capital is also well above the minimum required.

32.2.2 Issues in applying the Basle rules

The BIS rules do not allow for credit quality of a counterparty beyond the rigid framework described above. For example in calculating the capital requirement on a commercial loan book, there is no differentiation between a double-A rated counterparty and a double-B rated one. Another example is that certain country governments, those which are members of the OECD,³ have a zero risk-weighting, so that lending to say the South Korean government attracts a lower charge than lending to a triple-A rated corporate or a company such as Microsoft. This is an anomaly that the BIS is hoping to correct with its proposed changes to the rules.

Banks use their own internal models to allocate capital according to the risk taken by each transactions department, so that finance department may analyse not only the performance of each desk in terms of return on capital employed, but also the return on the basis of the risk that has been taken on.

The extent of *market risk* in a bank's portfolio is also of greater concern to regulators now than when the original rules were introduced. The BIS has published proposals that require banks to set aside capital to cover for market risk as well as credit risk. Similar proposals were contained in the EU's CAD II guidelines, which became operative from the beginning of 1999. The main BIS proposals include:

- a calculation of the capital requirement from the level of *delta*, *gamma* and *vega* risk that the bank's book is exposed to;⁴
- using a bank's own internal market risk model to estimate the level of capital requirements, using a VaR methodology and calculated for a holding period of 10 days and with a confidence level of 99%.

32.3 Proposed changes to Basle rules

The perceived shortcomings of the 1988 Basle capital accord attracted much comment from academics and practitioners alike, almost as soon as they were adopted. The main criticism was, as we noted above, that the requirements made no allowance for the credit risk ratings of different corporate borrowers, and was too rigid in its application of the risk weightings. That these were valid issues was recognised when, on 3 June 1999 the BIS published proposals to update the capital requirements rules. The new guidelines are designed "to promote safety and soundness in the financial system, to provide a more comprehensive approach for addressing risks, and to enhance competitive equality". The proposals also are intended to apply to all banks worldwide, and not simply those that active across international borders. The greatest changes is to the four risk weight buckets of the current regime. The revised ruling would redistribute the capital required for different types of lending and also add an additional category for very low-rated assets. For sovereign lending there is a smooth scale from 0% to 8%, while the scale is more staggered for corporates. An unusual feature is that low-rated companies attract a higher charge than non-rated borrowers. For lending to other banks there are two options; in the first, the sovereign risk of the home country of the bank is used, and the bank placed in the next lower category. In the second option, the credit rating of a bank itself is used. Whatever option is selected, the main effect will be that the capital charge for interbank lending will increase significantly, to virtually double the current level. The new proposed risk weights are summarised in Table 32.4, as percentages of the standard 8% ratio.

Asset	Credit rating					
	AAA to AA	A+ to A-	BBB+ to BBB-	BB+ to B-	Below B-	Unrated
Sovereign	0	20	50	100	150	100
Banks – option 1 *	0	20	50	100	150	100
Banks – option 2 ~	20	50 ^	50 ^	100 ^	150	50 ^
Corporates	20	100	10	100	150	100

* Based on the risk weighting of the sovereign in which the bank is incorporated.

~ Based on the assessment of the individual bank.

^ Claims on banks of a short original maturity, for example lower than six months, would receive a weighting that was one level higher.

Table 32.4: Basle capital requirement proposals, percentage weightings. Source: BIS.

³ Organisation for Economic Cooperation and Development, loosely a grouping of developed countries in North America, Western Europe and Asia.

⁴ These risk measures are reviewed in the chapter on options. The Value-at-Risk methodology is reviewed in Chapter 37.

The Basle committee wishes to expand capital requirements to cover other eras of risk, such as market risk and operational risk. It is recognised that a bank's capital should reflect the level of risk of its own portfolio, but this may best be estimated by a bank's own internal model rather than any standard ruling provided by a body such as the BIS. In any event the proposed rule changes have attracted considerable comment and the final form of the rules that are eventually adopted may bear little resemblance to the proposals listed above. There is a growing consensus among practitioners that perhaps the markets themselves should carry more of the supervisory burden rather than regulators, for example narrowing the scope of deposit insurance,⁵ or by requiring banks to issue specific kinds of uninsured debt, similar to the PIBS issued by UK building societies. Holders of such subordinated debt are more concerned with the financial health of a bank, because their investment is not guaranteed, and at the same time they are not interested in high-risk strategies because their return is the same every year irrespective of the profit performance of the bank, that is the fixed coupon of their subordinated bond. Therefore the yield on this subordinated debt is in effect the market's assessment of the risk exposure of the bank. An academic at Columbia University⁶ has suggested that regulators should place a cap on this yield, which would force the bank to cap the level of its risk exposure, but this level would have been evaluated by the market, and not the regulatory authority.

Bank rules in disarray

Reproduced from *The Economist*, 27 November 1999.

©Pearson Group. Used with permission.

The Basle committee's attempts to update its rules on bank capital are running into serious trouble. At first glance, it all seems to be going smoothly enough. On November 22nd the European Commission issued proposals to update the regulations governing how much capital European financial institutions must set aside as a cushion against the risks they are taking. These are closely based on the recommendations of the Basle Committee, which is trying to modernise its regulations. But the implication, that revisions to the Basle Accord, first proposed in June, are going well, is wide of the mark. Basle is proving mighty hard to fix.

That seems odd. Almost everybody agrees that the rules need to be changed so that the capital that banks have to set aside is more closely linked to the risks they are running. The question is: how? Big banks wanted to be able to use their own sophisticated (in their own eyes, at least) in-house risk-management models. These reflect the benefits of their diversified portfolios and of other ways of mitigating risks, such as the use of credit derivatives. But Basle folk, while accepting that this was good stuff, responded that these models were insufficiently tested over a long enough period to be used now – though they held out the prospect that they might be in the (distant) future.

Instead, the committee offered two other approaches. The first is a modification of the original accord. At present, there are four broad categories, or “risk buckets”, into which a loan is placed. But curiously, these bear little relation to actual riskiness. For example, since South Korea is a member of the OECD, lending to one of its banks carries a lower capital charge than lending to, say, General Electric.

More generally, if the capital required to support a loan takes no account of a loan's riskiness, then banks will – and do – take advantage of the anomaly. That is, they will shed high-quality assets that they think should not require as much capital as the Basle rules demand. Instead, they will add assets that, in their opinion, should demand more capital than the rules say. In other words, banks have tended to plump for lower-quality assets. This was a big reason why regulators were so keen to change the rules. Unfortunately, the way in which they have tried to do it has been roundly criticised. So roundly, indeed, that one committee member believes the traditional “standardised approach” is dead. Why so? After all, the committee wants to increase the number of buckets and to make them bear some relationship to the risks that banks actually run in their lending business.

The trouble lies in the way it proposes to do this: by using credit-rating agencies. This was bound to raise hackles, since two American firms, Moody's and Standard & Poor's, dominate the business. The committee would have preferred a body that was less politically sensitive. But other candidates, such as the OECD, were not interested.

⁵ Many countries operate a deposit insurance scheme that guarantees the level of a private customer's deposits in a bank should that bank fail. In the UK for example, the arrangement is that if a bank or building society is declared bankrupt, individuals are entitled to compensation of 90% of their savings with that institution, up to a maximum of £18,000 per individual.

⁶ Charles Calomiris, as described in “Better than Basle”, *The Economist*, 19 June 1999.

Far from being gleeful at this official recognition, the rating agencies detest the idea. They fear that less rigorous newcomers will nab some of their business, as companies shop around for the best ratings. New rating agencies would have to be approved by the various national regulators. If experience is any guide, they will be sympathetic to domestic rating firms – although regulators counter that markets will penalise companies that opt for generous raters. Nor, for their part, are banks keen on the reform. They think they are better-informed about their borrowers than are the rating agencies. In any case, outside America, few companies carry ratings. So for the time being they are not of much use.

Worse, the way in which capital charges have been affixed to different credit ratings is perverse. To avoid giving American banks an inbuilt advantage (since so many more companies are rated there), the compromise was to have lending to unrated companies carry the same capital charge as all but the most highly rated bonds, those with AAA or AA ratings. Equally odd – since the point of the exercise was to link capital to riskiness – loans to AA borrowers will require 2% capital, and loans to A companies 8%. And yet the default rates for both types of borrower are almost the same. Even more peculiar, a loan to an A-rated company would require the same capital as lending to a high-risk firm rated as “junk”.

Faced with such hostility to the use of external ratings, regulators are now concentrating on allowing banks to use their second option – banks’ internal ratings – instead. This, quips one regulator, is to swap the tractor for the Maserati. Originally, the committee thought that this option would apply only to about 20 of the world’s biggest and most sophisticated banks. Now, they say, far more will be eligible: perhaps 100 or so, reckons one committee member. That is likely to be an underestimate, since the European Commission is keen that the system be widely adopted in Europe; and no big bank is eager to admit that, for want of expertise, it has to use the standardised approach.

But using internal ratings is fraught with problems. Not least, ideas on how to do it are woefully undeveloped, because some countries (Germany in particular) have been so reluctant to use them. They have come round, but there are plenty of gaps to fill in. How will regulators compare different banks’ internal ratings? How much capital to attach to a given bucket? Will this mean that supervisors become too involved in running the bank? Some big banks in America already give regulators direct access to their balance sheets. And how much capital should firms that use internal ratings put aside, compared with those that use the standard approach?

This last question is particularly tricky. Regulators think that banks using internal ratings should be required to set aside less capital against their credit risks, because that would encourage sound risk management. Yet they do not want less capital overall in the banking system. So they want to take with one hand what they give with the other – by, for example, charging extra capital for non-credit risks, such as “operational” ones. Nobody can agree on what these are, or on how capital should be set against them. (But, as it happens, regulators are convinced that big banks have more of them.)

And if – a big if – the committee sorts out all these problems, will banks be safer? Certainly, an incentive to manage risk well is a good thing; banks would not then be in such a rush to dump decent assets. On the other hand, risk-based capital could have one big drawback: banks may be even more likely to lend more when times are good (and problems few), and to reduce lending when things sour – thus causing themselves and others even more pain. It would be worse than ironic if the outcome of the effort to fix Basle’s faults were to raise the risk of bank failures during a cyclical downturn.

Selected bibliography and references

Basle Committee on Banking Supervision, *An Internal Model-Based Approach to Market Risk Capital Requirements*, Bank for International Settlements, 1995.

Bhattacharya, S., Thakor, Contemporary banking theory, *Journal of Financial Intermediation*, 3–1, October 1993, pp. 2–50.

Dimson, E., Marsh, P., “Capital requirements for securities firms”, *Journal of Finance* 50, 1995, pp. 821–851.

Moir, C., “Basle or bust?”, *Securities and Investment Review*, September 1999, pp. 6–7.

Price Waterhouse, *Bank Capital Adequacy and Capital Convergence*, PriceWaterhouseCoopers, 1991.

Wall, L., Pringle, J., McNulty, J., “Capital requirements for interest-rate and foreign exchange hedges”, Federal Reserve Bank of Atlanta, *Economic Review* 75, May–June 1990, pp. 14–29.

33

Asset and Liability Management

As part of our discussion on the money markets, we will consider in this chapter a major part of banking activity, and one that is closely related to the main business of banking, which is the subject of *asset and liability management*. The art of asset and liability management (ALM) is essentially one of risk management and capital management, and although the day-to-day activities are run at the desk level, overall direction is given at the highest level of a banking institution. The risk exposure in a banking environment is multi-dimensional, for example they encompass interest-rate risk, liquidity risk, credit risk and operational risk. Interest-rate risk is one type of market risk. Risks associated with moves in interest rates and levels of liquidity¹ are those that result in adverse fluctuations in earnings levels due to changes in market rates and bank funding costs. By definition, banks' earnings levels are highly sensitive to moves in interest rates and the cost of funds in the wholesale market. Asset and liability management covers the set of techniques used to manage interest rate and liquidity risks; it also deals with the structure of the bank's balance sheet, which is heavily influenced by funding and regulatory constraints and profitability targets.

In practically all cases the funding or liquidity desk of a bank is separate to the bond trading arm and other capital markets areas. However in some institutions such as securities houses, the Treasury arm may be responsible for both funding and certain sectors of the debt markets, such as short-dated bonds and money markets. The funding desk performs a vital function not just for the bank, whose traditional role of lending and deposit-taking is underpinned by the funding desk, but also for other areas, who will also keep a close eye on their cost of capital and the effect this has on their overall profit levels.²

In this chapter we review the concept of balance sheet management, the role of the ALM desk, liquidity risk and maturity gap risk. We also review a basic gap report. The increasing use of *securitisation* and the responsibility of the ALM desk in enhancing the return on assets on the balance sheet is also introduced. For readers who are interested in developing their knowledge further, we list a selection of articles and publications in the bibliography.

33.1 Introduction

For newcomers to the subject, an excellent introduction to the primary activity of banking is contained in an article in *The Economist* entitled "The business of banking".³ Those who are complete beginners may wish to refer to this article. In this section we provide an overview of the main business of banking before considering the subject of ALM.

One of the major areas of decision-making in a bank involves the maturity of assets and liabilities. Typically longer-term interest rates are higher than shorter-term rates, that is it is common for the yield curve in the short-term (say 0–3 year range) to be positively sloping. To take advantage of this banks usually raise a large proportion of their funds from the short-dated end of the yield curve and lend out these funds for longer maturities at higher rates. The spread between the borrowing and lending rates is in principle the bank's profit. The obvious risk from such a strategy is that the level of short-term rates rises during the term of the loan, so that when the loan is refinanced the bank makes a lower profit or a net loss. Managing this risk exposure is the key function of an ALM desk. As well as managing the interest rate risk itself, banks also match assets with liabilities – thus locking in a profit – and diversify their loan book, to reduce exposure to one sector of the economy.

Another risk factor is liquidity. From a banking and Treasury point of view the term *liquidity* means funding liquidity, or the "nearness" of money. The most liquid asset is cash money. Banks bear several inter-related liquidity risks, including the risk of being unable to pay depositors on demand, an inability to raise funds in the market at reasonable rates and an insufficient level of funds available with which to make loans. Banks keep only a small portion of their assets in the form of cash, because this earns no return for them. In fact once they have met the

¹ In this chapter the term *liquidity* is used to refer to funding liquidity.

² The author was lucky enough to trade on a desk that combined trading in short-dated bonds, money markets, futures, FRAs and swaps as well as interbank deposits, repo and stock lending. The Treasury arm of some banking institutions also combines various functions onto one desk.

³ *The Economist*, 30 October 1999.

minimum cash level requirement, which is something set down by international regulation (and which was reviewed in the previous chapter), they will hold assets in the form of other instruments. Therefore the ability to meet deposit withdrawals depends on a bank's ability to raise funds in the market. The market and the public's perception of a bank's financial position heavily influences liquidity. If this view is very negative, the bank may be unable to raise funds and consequently be unable to meet withdrawals or loan demand. Thus liquidity management is running a bank in a way that maintains confidence in its financial position. The assets of the banks that are held in near-cash instruments, such as Treasury bills and clearing bank CDs, must be managed with liquidity considerations in mind. The asset book on which these instruments are held is sometimes called the *liquidity book*.

33.1.1 The balance sheet

ALM and transactions required in managing the bank's traditional activity may first be viewed in the context of the balance sheet. A banking balance sheet essentially is a grouping of the following activities:

- treasury and banking transactions;
- collection of deposits and disbursing loans;
- financial assets;
- long-dated assets, and capital (equity and long-term debt).

A simplified balance sheet is shown in Table 33.1.

Assets	Liabilities
Cash	Short-term debt
Loans	Deposits
Financial assets	Financial assets
Fixed assets	Long-term debt
	Equity capital
Off-balance sheet	Off-balance sheet
(contingencies received)	(contingencies paid)

Table 33.1: Banking balance sheet.

The Financial Accounting Standards Board has defined assets as “probable future economic benefits obtained or controlled” by the bank that have arisen as a result of transactions entered into by the bank. Liabilities are defined as “probable future sacrifices of economic benefits arising from present obligations” of the bank to transfer assets to other bodies as a result of transactions it has entered into. Assets are further sub-divided into *current assets* which are cash or can be converted into cash within one year, and *long-term assets* which are expected to provide benefits over periods longer than one year. A similar classification is applied to *current liabilities* and *long-term liabilities*.

The relative shares of each constituent in a bank balance sheet will depend on the type of activity carried out by the bank. Commercial banks have a higher share of deposit-taking and loan activity, which are held in the *banking book*. Integrated banking groups combining commercial activity and investment activity, and investment banks, will have a greater proportion of market transactions in the capital markets, such as bond trading, equity trading, foreign-exchange and derivatives market making. These activities will be placed in the *trading book*. Risk management in a bank is concerned (among other things) with the funding and hedging of the balance sheet. In terms of the activities undertaken, there is therefore an obvious distinction between each of the four types of transaction listed above.

33.1.2 The banking book

Traditionally ALM has been concerned with the banking book. The conventional techniques of ALM were developed for application to a bank's banking book that is, the lending and deposit-taking transactions. The core banking activity will generate either an excess of funds, when the receipt of deposits outweighs the volume of lending the bank has undertaken, or a shortage of funds, when the reverse occurs. This mis-match is balanced via financial transactions in the wholesale market. The banking book generates both interest-rate and liquidity risks, which are

then monitored and managed by the ALM desk. Interest-rate risk is the risk that the bank suffers losses due to adverse movements in market interest rates. Liquidity risk is the risk that the bank cannot generate sufficient funds when required; the most extreme version of this is when there is a “run” on the bank, and the bank cannot raise the funds required when depositors withdraw their cash.

Note that the asset side of the banking book, which is the loan portfolio, also generates credit risk.

The ALM desk will be concerned with risk management that focuses on the quantitative management of the liquidity and interest-rate risks inherent in a banking book. The major areas of ALM include:

- **measurement and monitoring of liquidity and interest-rate risk.** This includes setting up targets for earnings and volume of transactions, and setting up and monitoring interest-rate risk limits;
- **funding and control of any constraints on the balance sheet.** This includes liquidity constraints, debt policy and *capital adequacy* ratio and solvency;
- **hedging of liquidity and interest-rate risk.**

33.2 The ALM desk

The ALM desk or unit is a specialised business unit that fulfils a range of functions. Its precise remit is a function of the type of the activities of the financial institution that it is a part of. Let us consider the main types of activities that are carried out.

If an ALM unit has a profit target of zero, it will act as a cost centre with a responsibility to minimise operating costs. This would be consistent with a strategy that emphasises commercial banking as the core business of the firm, and where ALM policy is concerned purely with hedging interest-rate and liquidity risk.

The next level is where the ALM unit is responsible for minimising the cost of funding. That would allow the unit to maintain an element of exposure to interest-rate risk, depending on the view that was held as to the future level of interest rates. As we noted above, the core banking activity generates either an excess or shortage of funds. To hedge away all of the excess or shortage, while removing interest-rate exposure, has an opportunity cost associated with it since it eliminates any potential gain that might arise from movements in market rates. Of course, without a complete hedge, there is an exposure to interest-rate risk. The ALM desk is responsible for monitoring and managing this risk, and of course is credited with any cost savings in the cost of funds that arise from the exposure. The saving may be measured as the difference between the funding costs of a full hedging policy and the actual policy that the ALM desk adopts. Under this policy, interest-rate risk limits are set which the ALM desk ensures the bank's operations do not breach.

The final stage of development is to turn the ALM unit into a profit centre, with responsibility for optimising the funding policy within specified limits. The limits may be set as *gap* limits, *value-at-risk* limits or by another measure, such as level of earnings volatility. Under this scenario the ALM desk is responsible for managing all financial risk.

The final development of the ALM function has resulted in it taking on a more active role. The previous paragraphs described the three stages of development that ALM has undergone, although all three versions are part of the “traditional” approach. Practitioners are now beginning to think of ALM as extending beyond the risk management field, and being responsible for adding value to the net worth of the bank, through proactive positioning of the book and hence, the balance sheet. That is, in addition to the traditional function of managing liquidity risk and interest-rate risk, ALM should be concerned additionally with managing the regulatory capital of the bank and with actively positioning the balance sheet to maximise profit. The latest developments mean that there are now financial institutions that run a much more sophisticated ALM operation than that associated with a traditional banking book.

Let us review the traditional and developed elements of an ALM function.

33.2.1 Traditional ALM

Generally a bank's ALM function has in the past been concerned with managing the risk associated with the banking book. This does not mean that this function is now obsolete, rather that additional functions have now been added to the ALM role. There are a large number of financial institutions that adopt the traditional approach, indeed the nature of their operations would not lend themselves to anything more. We can summarise the role of the traditional ALM desk as follows:

- **Interest rate risk management.** This is the interest-rate risk arising from the operation of the banking book. It includes net interest income sensitivity analysis, typified by maturity gap and duration gap analysis, and the sensitivity of the book to parallel changes in the yield curve. The ALM desk will monitor the exposure and position the book in accordance with the limits as well as its market view. Smaller banks, or subsidiaries of banks that are based overseas, often run no interest-rate risk, that is there is no short gap in their book. Otherwise the ALM desk is responsible for hedging the interest-rate risk or positioning the book in accordance with its view.
- **Liquidity and funding management.** There are regulatory requirements that dictate the proportion of banking assets that must be held as short-term instruments. The liquidity book in a bank is responsible for running the portfolio of short-term instruments. The exact make-up of the book is however the responsibility of the ALM desk, and will be a function of the desk's view of market interest rates, as well as its opinion on the relative value of one asset over another. For example it may decide to move some assets into short-dated government bonds, above what it normally holds, at the expense of high-quality CDs, or vice-versa.
- **Reporting on hedging of risks.** The ALM fulfils a senior management information function by reporting on a regular basis on the extent of the bank's risk exposure. This may be in the form of a weekly hardcopy report, or via some other medium.
- **Setting up risk limits.** The ALM unit will set limits, implement them and enforce them, although it is common for an independent "middle office" risk function to monitor compliance with limits.
- **Capital requirement reporting.** This function involves the compilation of reports on capital usage and position limits as percentage of capital allowed, and reporting to regulatory authorities.

All financial institutions will carry out the activities described above.

Gap analysis

Maturity gap analysis measures the cash difference or *gap* between the absolute values of the assets and liabilities that are sensitive to movements in interest rates. Therefore the analysis measures the relative interest-rate sensitivities of the assets and liabilities, and thus determines the risk profile of the bank with respect to changes in rates. The *gap ratio* is given as (33.1):

$$\text{Gap ratio} = \frac{\text{Interest — rate sensitive assets}}{\text{Interest — rate sensitive liabilities}} \quad (33.1)$$

and measures whether there are more interest-rate sensitive assets than liabilities. A gap ratio higher than one for example, indicates that a rise in interest rates will increase the net present value of the book, thus raising the return on assets at a rate higher than the rise in the cost of funding. This also results in a higher income spread.

A gap ration lower than one indicates a rising funding cost. *Duration gap* analysis measures the impact on the net worth of the bank due to changes in interest rates by focusing on changes in market value of either assets or liabilities. This is because duration measures the percentage change in the market value of a single security for a one percent change in the underlying yield of the security (strictly speaking, this is *modified duration* but the term for the original "duration" is now almost universally used to refer to modified duration). The duration gap is defined as (33.2):

$$\text{Duration gap} = \text{Duration of assets} - w(\text{Duration of liabilities}) \quad (33.2)$$

where w is the percentage of assets funded by liabilities. Hence the duration gap measures the effects of the change in the net worth of the bank. A higher duration gap indicates a higher interest rate exposure. As duration only measures the effects of a linear change in interest rate, that is a parallel shift yield curve change, banks with portfolios that include a significant amount of instruments with elements of optionality, such as callable bonds, asset-backed securities and convertibles, also use the *convexity* measure of risk exposure to adjust for the inaccuracies that arise in duration over large yield changes.

33.2.2 Developments in ALM

A greater number of financial institutions are enhancing their risk management function by adding to the responsibilities of the ALM function. These have included enhancing the role of the head of Treasury and the *asset and liability committee* (ALCO), using other risk exposure measures such as option-adjusted spread and value-at-risk (VaR), and integrating the traditional interest-rate risk management with credit risk and operational risk. The increasing use of credit derivatives has facilitated this integrated approach to risk management.

The additional roles of the ALM desk may include:

- using the VaR tool to assess risk exposure;
- integrating market risk and credit risk;
- using new *risk-adjusted* measures of return;
- optimising portfolio return;
- proactively managing the balance sheet; this includes giving direction on securitisation of assets (removing them from the balance sheet), hedging credit exposure using credit derivatives and actively enhancing returns from the liquidity book, such as entering into stock lending and repo.

An expanded ALM function will by definition expand the role of the Treasury function and the ALCO. This may see the Treasury function becoming active “portfolio managers” of the bank’s book. The ALCO, traditionally composed of risk managers from across the bank as well as the senior member of the ALM desk or liquidity desk, is responsible for assisting the head of Treasury and the Finance Director in the risk management process. In order to fulfil the new enhanced function the Treasurer will require a more strategic approach to his or her function, as many of the decisions with running the bank’s entire portfolio will be closely connected with the overall direction that the bank wishes to take. These are Board-level decisions.

33.3 Liquidity and interest-rate risk

33.3.1 The liquidity gap

Liquidity risk arises because a bank’s portfolio will consist of assets and liabilities with different sizes and maturities. When assets are greater than resources from operations, a funding gap will exist which needs to be sourced in the wholesale market. When the opposite occurs, the excess resources must be invested in the market. The differences between the assets and liabilities is called the *liquidity gap*. For example if a bank has long-term commitments that have arisen from its dealings and its resources are exceeded by these commitments, and have a shorter maturity, there is both an immediate and a future deficit. The liquidity risk for the bank is that, at any time, there are not enough resources, or funds available in the market, to balance the assets.

Liquidity management has several objectives; possibly the most important is to ensure that deficits can be funded under all foreseen circumstances, and without incurring prohibitive costs. In addition there are regulatory requirements that force a bank to operate certain limits, and state that short-term assets be in excess of short-run liabilities, in order to provide a safety net of highly liquid assets. Liquidity management is also concerned with funding deficits and investing surpluses, with managing and growing the balance sheet, and with ensuring that the bank operates within regulatory and in-house limits. In this section we review the main issues concerned with liquidity and interest-rate risk.

The liquidity gap is the difference, at all future dates, between assets and liabilities of the banking portfolio. Gaps generate liquidity risk. When liabilities exceed assets, there is an excess of funds. An excess does not of course generate liquidity risk, but it does generate interest-rate risk, because the present value of the book is sensitive to changes in market rates. When assets exceed liabilities, there is a funding deficit and the bank has long-term commitments that are not currently funded by existing operations. The liquidity risk is that the bank requires funds at a future date to match the assets. The bank is able to remove any liquidity risk by locking in maturities, but of course there is a cost involved as it will be dealing at longer maturities.⁴

⁴ This assumes a conventional upward-sloping yield curve.

33.3.2 Gap risk and limits

Liquidity gaps are measured by taking the difference between outstanding balances of assets and liabilities over time. At any point a positive gap between assets and liabilities is equivalent to a deficit, and this measured as a cash amount. The *marginal gap* is the difference between the changes of assets and liabilities over a given period. A positive marginal gap means that the variation of value of assets exceeds the variation of value of liabilities. As new assets and liabilities are added over time, as part of the ordinary course of business, the gap profile changes.

The gap profile is tabulated or charted (or both) during and at the end of each day as a primary measure of risk. For illustration, a tabulated gap report is shown at Figure 33.1 and is an actual example from a UK banking institution. It shows the assets and liabilities grouped into maturity *buckets* and the net position for each bucket. It is a snapshot today of the exposure, and hence funding requirement of the bank for future maturity periods.

	Time periods											
	Total		0–6 months		6–12 months		1–3 years		3–7 years		7+ years	
Assets	40,533	6.17%	28,636	6.08%	3,801	6.12%	4,563	6.75%	2,879	6.58%	654	4.47%
Liabilities	40,533	4.31%	30,733	4.04%	3,234	4.61%	3,005	6.29%	2,048	6.54%	1,513	2.21%
Net Cumulative												
Positions	0	1.86%	(2,097)		567		1,558		831		(859)	
Margin on total assets:				2.58%								
Average margin on total assets:				2.53%								

Figure 33.1: Example Gap profile.

Figure 33.1 is very much a summary figure, because the maturity gaps are very wide. For risk management purposes the buckets would be much narrower, for instance the period between zero and 12 months might be split into 12 different maturity buckets. An example of a more detailed gap report is shown at Figure 33.3, which is from another UK banking institution. Note that the overall net position is zero, because this is a balance sheet and therefore, not surprisingly, it balances. However along the maturity buckets or grid points there are net positions which are the gaps that need to be managed.

Limits on a banking book can be set in terms of gap limits. For example a bank may set a six-month gap limit of £10 million. The net position of assets and maturities expiring in six months' time could then not exceed £10 million. An example of a gap limit report is shown at Figure 33.2, with the actual net gap positions shown against the gap limits for each maturity. Again this is an actual limit report from a UK banking institution.

Time Periods	0–1	1–3	3–6	6–12	1–2	2–3	3–4	4–5	5–6	6–7	7–8	8–9	9–10	10+ years
	0–6 months				1–3 years			3–7 years			7–10 years			
Current Gaps														
Individual	0	0	0	710	–520	771	417	484	104	7	4	2	2	–117
Cumulative	–1,864				251			1,011			9			
Limits														
Individual (+/–)				+/-1250	–2000	+/-1000	+1000–200	+1000–200	+250–100	+200–75	+/-50	+/-25	+/-25	–125
Cumulative	+500 to –2500				+750 to –1000			–2,000			+100			
Excess														
	0				0			0			0			

Figure 33.2: Gap limit report.

	Total (£m)	Up to 1 month	1–3 months	3–6 months	6 months to 1 year	1–2 years	2–3 years	3–4 years	4–5 years	5–6 years	6–7 years	7–8 years	8–9 years	9–10 years	10+ years
ASSETS															
Cash & Interbank Loans	2,156.82	1,484.73	219.36	448.90	3.84	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Certificates of Deposit purchased	1,271.49	58.77	132.99	210.26	776.50	92.96	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Floating Rate Notes purchased	936.03	245.62	586.60	12.68	26.13	45.48	0.00	0.00	19.52	0.00	0.00	0.00	0.00	0.00	0.00
Bank Bills	314.35	104.09	178.36	31.90	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Other Loans	13.00	0.00	1.00	0.00	0.00	7.00	0.00	1.00	0.00	0.00	2.00	2.00	0.00	0.00	0.00
Debt Securities/Gilts	859.45	0.00	25.98	7.58	60.05	439.06	199.48	26.81	100.50	0.00	0.00	0.00	0.00	0.00	0.00
Fixed rate Mortgages	4,180.89	97.72	177.37	143.13	964.98	1,452.91	181.86	661.36	450.42	22.78	4.30	3.65	3.10	2.63	14.67
Variable & Capped Rate Mortgages	14,850.49	14,850.49	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Commercial Loans	271.77	96.62	96.22	56.52	0.86	2.16	1.12	3.64	8.85	1.06	0.16	0.17	0.16	4.23	0.00
Unsecured Lending and Leasing	3,720.13	272.13	1,105.20	360.03	507.69	694.86	400.84	195.19	79.98	25.45	14.06	10.03	10.44	10.82	33.42
Other Assets	665.53	357.72	0.00	18.77	5.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	284.03
TOTAL CASH ASSETS	29,239.95	17,567.91	2,523.06	1,289.77	2,345.05	2,734.43	783.31	888.00	659.26	49.28	20.53	15.85	13.71	17.68	332.12
Swaps	9,993.28	3,707.34	1,462.32	1,735.59	1,060.61	344.00	146.50	537.60	649.00	70.00	5.32	200.00	75.00	0.00	0.00
Forward Rate Agreements	425.00	0.00	50.00	0.00	220.00	5.00	150.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Futures	875.00	0.00	300.00	0.00	175.00	400.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
TOTAL	40,533.24	21,275.24	4,335.38	3,025.36	3,800.66	3,483.43	1,079.81	1,425.60	1,308.26	119.28	25.84	215.85	88.71	17.68	332.12
LIABILITIES (£m)															
Bank Deposits	3,993.45	2,553.85	850.45	233.03	329.06	21.07	1.00	0.00	5.00	0.00	0.00	0.00	0.00	0.00	0.00
Certificates of Deposit issued	1,431.42	375.96	506.76	154.70	309.50	60.00	20.00	3.50	1.00	0.00	0.00	0.00	0.00	0.00	0.00
Commercial Paper – CP & Euro	508.46	271.82	128.42	108.21	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Subordinated Debt	275.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	200.00	75.00	0.00	0.00
Eurobonds + Other	2,582.24	768.75	1,231.29	121.94	53.86	9.77	13.16	150.43	150.53	0.00	7.51	0.00	0.00	0.00	75.00
Customer Deposits	17,267.55	15,493.65	953.60	311.70	340.50	129.10	6.60	24.90	0.00	7.50	0.00	0.00	0.00	0.00	0.00
Other Liabilities (incl capital/reserves)	3,181.83	1,336.83	0.00	0.00	741.72	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1,103.28
TOTAL CASH LIABILITIES	29,239.96	20,800.86	3,670.52	929.58	1,774.64	219.93	40.76	178.83	156.53	7.50	7.51	200.00	75.00	0.00	1,178.28
Swaps	9,993.28	1,754.70	1,657.59	1,399.75	1,254.24	1,887.97	281.44	905.06	770.52	15.76	6.48	7.27	8.13	13.06	31.30
FRAs	425.00	0.00	150.00	70.00	55.00	150.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Futures	875.00	0.00	0.00	300.00	150.00	425.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
TOTAL	40,533.24	22,555.56	5,478.11	2,699.33	3,233.89	2,682.90	322.20	1,083.90	927.05	23.26	13.99	207.27	83.13	13.06	1,209.58
Net Positions	0.00	-1,351.09	-1,234.54	265.58	583.48	929.10	803.46	341.70	404.88	104.28	11.85	8.58	5.57	4.62	-877.45

Figure 33.3: Detailed gap profile.

The maturity gap can be charted to provide an illustration of net exposure, and an example is shown at Figure 33.4 below, from yet another UK banking institution. In some firms' reports both the assets and the liabilities are shown for each maturity point, but in our example only the net position is shown. This net position is the gap exposure for that maturity point. A second example, used by the overseas subsidiary of a middle eastern commercial bank, which has no funding lines in the interbank market and so does not run short positions, is shown at Figure 33.5, while the gap report for a UK high-street bank is shown at Figure 33.6. Note the large short gap under the maturity labelled "non-int"; this stands for *non-interest bearing liabilities* and represents the balance of current accounts (cheque or "checking" accounts) which are funds that attract no interest and are in theory very short-dated (because they are demand deposits, so may be called at instant notice).

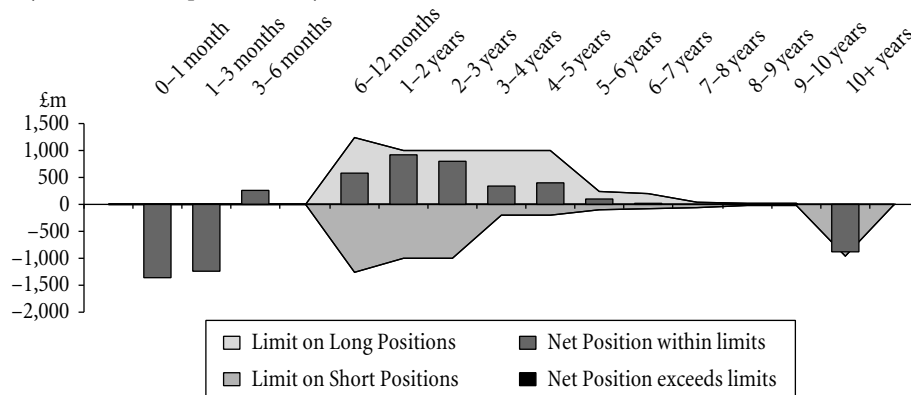


Figure 33.4: Gap maturity profile in graphical form.

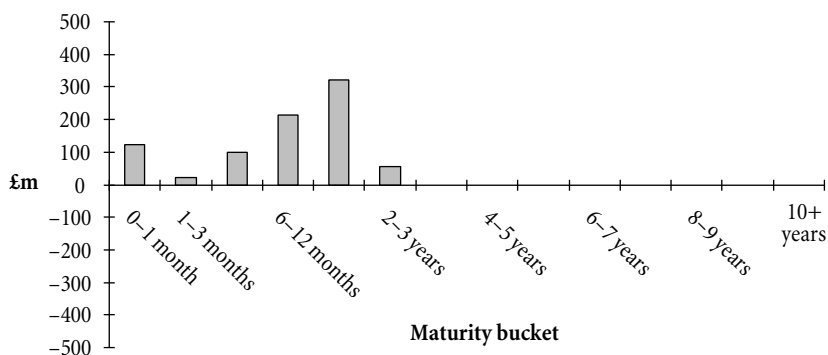


Figure 33.5: Gap maturity profile, bank with no short funding allowed.

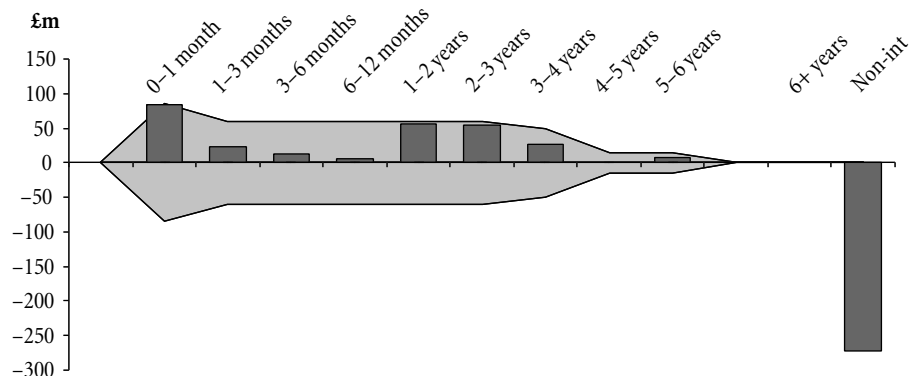


Figure 33.6: Gap maturity profile, UK high-street bank.

Gaps represent cumulative funding required at all dates. The cumulative funding is not necessarily identical to the new funding required at each period, because the debt issued in previous periods is not necessarily amortised at subsequent periods. The new funding between for example months 3 and 4 is not the accumulated deficit between months 2 and 4 because the debt contracted at month 3 is not necessarily amortised at month 4. Marginal gaps may be identified as the new funding required or the new excess funds of the period that should be invested in the market. Note that all the reports are snapshots at a fixed point in time and the picture is of course a continuously moving one. In practice the liquidity position of a bank cannot be characterised by one gap at any given date, and the entire gap profile must be used to gauge the extent of the book's profile.

The liquidity book may decide to match its assets with its liabilities. This is known as *cash matching* and occurs when the time profiles of both assets and liabilities are identical. By following such a course the bank can lock in the spread between its funding rate and the rate at which lends cash, and run a guaranteed profit. Under cash matching the liquidity gaps will be zero. Matching the profile of both legs of the book is done at the overall level; that is, cash matching does not mean that deposits should always match loans. This would be difficult as both result from customer demand, although an individual purchase of say, a CD can be matched with an identical loan. Nevertheless, the bank can elect to match assets and liabilities once the net position is known, and keep the book matched at all times. However it is highly unusual for a bank to adopt a cash matching strategy.

33.3.3 Liquidity management

The continuous process of raising new funds or investing surplus funds is known as liquidity management. If we consider that a gap today is funded, thus balancing assets and liabilities and squaring-off the book, the next day a new deficit or surplus is generated which also has to be funded. The liquidity management decision must cover the amount required to bridge the gap that exists the following day as well as position the book across future dates in line with the bank's view on interest rates. Usually in order to define the maturity structure of debt a target profile of resources is defined. This may be done in several ways. If the objective of ALM is to replicate the asset profile with resources, the new funding should contribute to bringing the resources profile closer to that of the assets, that is, more of a matched book looking forward. This is the lowest-risk option. Another target profile may be imposed on the bank by liquidity constraints. This may arise if for example the bank has a limit on borrowing lines in the market so that it could not raise a certain amount each week or month. For instance, if the maximum that could be raised in one week by a bank is £10 million, the maximum period liquidity gap is constrained by that limit. The ALM desk will manage the book in line with the target profile that has been adopted, which requires it to try to reach the required profile over a given time horizon.

Managing the banking book's liquidity is a dynamic process, as loans and deposits are known at any given point, but new business will be taking place continuously and the profile of the book looking forward must be continuously re-balanced to keep it within the target profile. There are several factors that influence this dynamic process, the most important of which are reviewed below.

Demand deposits

Deposits placed on demand at the bank, such as current accounts (known in the US as "checking accounts") have no stated maturity and are available on demand at the bank. Technically they are referred to as "non-interest bearing liabilities" because the bank pays no or very low rates of interest on them, so they are effectively free funds. The balance of these funds can increase or decrease throughout the day without any warning, although in practice the balance is quite stable. There are a number of ways that a bank can choose to deal with these balances, which are:

- to group all outstanding balances into one maturity bucket at a future date that is the preferred time horizon of the bank, or a date beyond this. This would then exclude them from the gap profile. Although this is considered unrealistic because it excludes the current account balances from the gap profile, it is nevertheless a fairly common approach;
- to rely on an assumed rate of amortisation for the balances, say 5% or 10% each year;
- to divide deposits into stable and unstable balances, of which the core deposits are set as a permanent balance. The amount of the core balance is set by the bank based on a study of the total balance volatility pattern over time. The excess over the core balance is then viewed as very short-term debt. This method is reasonably close to reality as it is based on historical observations;

- to make projections based on observable variables that are correlated with the outstanding balances of deposits. For instance such variables could be based on the level of economic growth plus an error factor based on the short-term fluctuations in the growth pattern.

Pre-set contingencies

A bank will have committed lines of credit, the utilisation of which depends on customer demand. Contingencies generate outflows of funds that are by definition uncertain, as they are contingent upon some event, for example the willingness of the borrower to use a committed line of credit. The usual way for a bank to deal with these unforeseen fluctuations is to use statistical data based on past observation to project a future level of activity.

Prepayment options of existing assets

Where the maturity schedule is stated in the terms of a loan, it may still be subject to uncertainty because of prepayment options. This is similar to the prepayment risk associated with a mortgage-backed bond. An element of prepayment risk renders the actual maturity profile of a loan book to be uncertain; banks often calculate an “effective maturity schedule” based on prepayment statistics instead of the theoretical schedule. There are also a range of prepayment models that may be used, the simplest of which use constant prepayment ratios to assess the average life of the portfolio. The more sophisticated models incorporate more parameters, such as one that bases the prepayment rate on the interest rate differential between the loan rate and the current market rate, or the time elapsed since the loan was taken out.

Interest cash flows

Assets and liabilities generate interest cash inflows and outflows, as well as the amortisation of principal. The interest payments must be included in the gap profile as well.

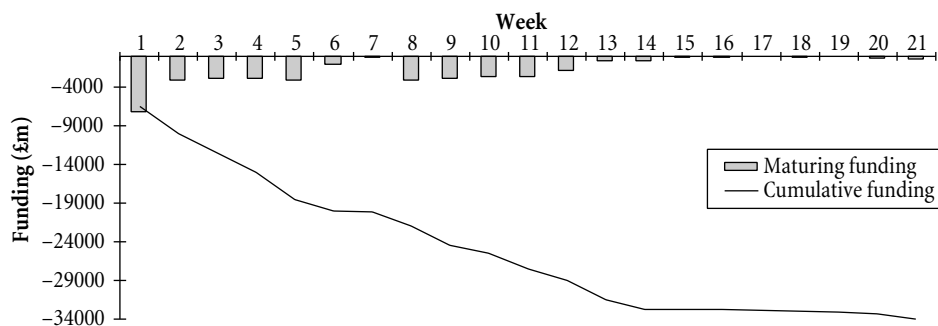


Figure 33.7: Liquidity analysis – example of UK bank profile of maturity of funding.

33.3.4 Interest-rate gap

The interest-rate gap is the standard measure of the exposure of the banking book to interest-rate risk. The interest-rate gap for a given period is defined as the difference between fixed-rate assets and fixed-rate liabilities. It can also be calculated as the difference between interest-rate sensitive assets and interest-rate liabilities. Both differences are identical in value when total assets are equal to total liabilities, but will differ when the balance sheet is not balanced. This only occurs intra-day, when for example, a short position has not been funded yet. The general market practice is to calculate interest-rate gap as the difference between assets and liabilities. The gap is defined in terms of the maturity period that has been specified for it.

The convention for calculating gaps is important for interpretation. The “fixed-rate” gap is the opposite of the “variable-rate” gap when assets and liabilities are equal. They differ when assets and liabilities do not match and there are many reference rates. When there is a deficit, the “fixed-rate gap” is consistent with the assumption that the gap will be funded through liabilities for which the rate is unknown. This funding is then a variable-rate liability and is the bank’s risk, unless the rate has been locked-in beforehand. The same assumption applies when the banks runs a cash surplus position, and the interest rate for any period in the future is unknown. The gap position at a given time bucket is sensitive to the interest rate that applies to that period.

The gap is calculated for each discrete time bucket, so there is a net exposure for say, 0–1 month, 1–3 months and so on. Loans and deposits do not, except at the time of being undertaken, have precise maturities like that, so they are “mapped” to a time bucket in terms of their relative weighting. For example, a £100 million deposit that matures in 20 days’ time will have most of its balance mapped to the three-week time bucket, but a smaller amount will also be allocated to the two-week bucket. Interest-rate risk is measured as the change in present value of the deposit, at each grid point, given a 1 basis point change in the interest rate. So a £10 million one-month CD that was bought at 6.50% will have its present value move upwards if on the next day the one-month rate moves down by a basis point.

The net change in present value for a 1 basis point move is the key measure of interest-rate risk for a banking book and this is what is usually referred to as a “gap report”, although strictly speaking it is not. The correct term for such a report is a “PVBP” or “DV01” report, which are acronyms for “present value of a basis point” and “dollar value of an 01 [1 basis point]” respectively. The calculation of interest-rate sensitivity assumes a *parallel shift* in the yield curve, that is that every maturity point along the term structure moves by the same amount (here one basis point) and in the same direction. An example of a PVBP report is given at Figure 33.8, split by different currency books, but with all values converted to sterling.

	1 day	1 week	1 month	2 months	3 months	6 months	12 months	2 years
GBP	8,395	6,431	9,927	8,856	(20,897)	(115,303)	(11,500)	(237,658)
USD	1,796	(903)	10,502	12,941	16,784	17,308	(13,998)	(18,768)
Euro	1,026	1,450	5,105	2,877	(24,433)	(24,864)	(17,980)	(9,675)
Total	11,217	6,978	25,534	24,674	(28,546)	(122,859)	(43,478)	(266,101)

	3 years	4 years	5 years	7 years	10 years	15 years	20 years	30 years
GBP	(349,876)	(349,654)	5,398	(5,015)	(25,334)	(1,765)	(31,243)	(50,980)
USD	(66,543)	(9,876)	(1,966)	237	2,320	(5,676)	(1,121)	0
Euro	(11,208)	(3,076)	1,365	1,122	3,354	(545)	(440)	(52)
Total	(427,627)	(362,606)	4,797	(3,656)	(19,660)	(7,986)	(32,804)	(51,032)

GBP total: (1,160,218); USD total: (56,963); Euro total: (75,974); Grand total: (1,293,155)
All figures £

Figure 33.8: Banking book PVBP Grid report.

The basic concept in the gap report is the net present value (NPV) of the banking book, which was introduced in Chapter 2. The PVBP report measures the difference between the market values of assets and liabilities in the banking book. To calculate NPV we require a discount rate, and it represents a *mark-to-market* of the book. The rates used are always the zero-coupon rates derived from the government bond yield curve, although some adjustment should be made to this to allow for individual instruments.

Gaps may be calculated as differences between outstanding balances at one given date, or as differences of variations of those balances over a time period. A gap number calculated from variations is known as a *margin gap*. The cumulative margin gaps over a period of time, plus the initial difference in assets and liabilities at the beginning of the period are identical to the gaps between assets and liabilities at the end of the period.

The interest-rate gap differs from the liquidity gap in a number of detail ways, which include:

- whereas for liquidity gap all assets and liabilities must be accounted for, only those that have a fixed rate are used for the interest-rate gap;
- the interest-rate gap cannot be calculated unless a period has been defined because of the fixed-rate/variable-rate distinction. The interest-rate gap is dependent on a maturity period and an original date.

The primary purpose in compiling the gap report is in order to determine the sensitivity of the interest margin to changes in interest rates. As we noted earlier the measurement of the gap is always “behind the curve” as it is an historical snapshot; the actual gap is a dynamic value as the banking book continually undertakes day-to-day business.

33.3.5 Portfolio modified duration gap

From Chapter 7 we know that modified duration measures the change in market price of a financial instrument that results from a given change in market interest rates. The duration gap of a net portfolio value is a measure of the interest-rate sensitivity of a portfolio of financial instruments and is the difference between the weighted-average duration of assets and liabilities, adjusted for the net duration of any off-balance sheet instruments. Hence it measures the percentage change in the net portfolio value that is expected to occur if interest rates change by 1%.

The net portfolio value, given by net present value (*NPV*) of the book, is the market value of assets *A* minus the market value of the liabilities *L*, plus or minus the market value *OBS* of off-balance sheet instruments, shown by (33.3):

$$NPV = A - L \pm OBS. \quad (33.3)$$

To calculate the duration gap of the net present value, we obtain the modified duration of each instrument in the portfolio and weight this by the ratio of its market value to the net value of the portfolio. This is done for assets, liabilities and off-balance sheet instruments. The modified duration of the portfolio is given by (33.4):

$$MD_{NPV} = MD_A - MD_L \pm MD_{OBS}. \quad (33.4)$$

The modified duration of the NPV may be used to estimate the expected change in the market value of the portfolio for a given change in interest rates, shown by (33.5):

$$\Delta NPV = NPV \times -MD_{NPV} \times \Delta r. \quad (33.5)$$

It is often problematic to obtain an accurate value for the market value of every instrument in a banking book. In practice book values often are used to calculate the duration gap when market values are not available. This may result in inaccurate results when actual market values differ from book values by a material amount.

The other points to note about duration gap analysis are:

- the analysis uses modified duration to calculate the change in NPV and therefore provides an accurate estimate of price sensitivity of instruments for only small changes in interest rates. For a change in rates of more than say, 50 basis points the sensitivity measure given by modified duration will be significantly in error;
- the duration gap analysis, like the maturity gap model, assumes that interest rates change in a parallel shift, which is clearly unrealistic;

As with the maturity gap analysis, the duration gap is favoured in ALM application because it is easily understood and summarises a banking book's interest-rate exposure in one convenient number.

33.4 Critique of the traditional approach

Traditionally the main approach of ALM concentrated on interest sensitivity and net present value sensitivity of a bank's loan/deposit book. The usual interest sensitivity report is the maturity gap report, which we reviewed briefly earlier. The maturity gap report is not perfect however, and can be said to have the following drawbacks:

- the re-pricing intervals chose for gap analysis are ultimately arbitrary, and there may be significant mismatches within a re-pricing interval. For instance a common re-pricing interval chosen is the one-year gap and the 1–3 year gap; there are (albeit extreme) circumstances when mismatches would go undetected by the model. Consider a banking book that is composed solely of liabilities that re-price in one month's time, and an equal cash value of assets that re-price in 11 months' time. The one-year gap of the book (assuming no other positions) would be zero, implying no risk to net interest income. In fact under our scenario the net interest income is significantly at risk from a rise in interest rates;
- maturity gap models assume that interest rates change by a uniform magnitude and direction. For any given change in the general level of interest rates however, it is more realistic for different maturity interest-rates to change by different amounts, what is known as a non-parallel shift;
- maturity gap models assume that principal cash flows do not change when interest rates change. Therefore it is not possible effectively to incorporate the impact of options embedded in certain financial instruments. Instruments such as mortgage-backed bonds and convertibles do not fall accurately into a gap analysis, as only their first-order risk exposure is captured.

Notwithstanding these drawbacks, the gap model is widely used as it is easily understood in the commercial banking and mortgage industry, and its application does not require a knowledge of sophisticated financial modelling techniques.

33.5 Securitisation

It is common for ALM units in banks to take responsibility for a more proactive balance sheet management role, and *securitisation* is a good example of this. Securitisation is a process undertaken by banks to both realise additional value from assets held on the balance sheet as well as to remove them from the balance sheet entirely, thus freeing up lending lines. Essentially it involves selling assets on the balance sheet to third-party investors. In principle the process is straightforward, as assets that are sold generate cash flows in the future, which provide the return to investors who have purchased the securitised assets. To control the risk exposure for investors, the uncertainty associated with certain asset cash flows is controlled or re-engineered, and there are a range of ways that this may be done.

For balance sheet management one of the principal benefits of securitisation is to save or reduce capital charges through the sale of assets. The other added benefit of course is that the process generates additional return for the issuing bank; therefore securitisation is not only a method by which capital charges may be saved, but an instrument in its own right that enables a bank to increase its return on capital.

33.5.1 The securitisation process

An introduction to asset-backed instruments was given in Chapters 25–28. In this section we consider the implications of securitisation from the point of view of asset and liability management.

The basic principle of securitisation is to sell assets to investors, usually through a medium known as a *special purpose vehicle* or some other intermediate structure, and to provide the investors with a fixed or floating rate return on the assets they have purchased; the cash flows from the original assets are used to provide this return. It is rare, though not totally unknown, for the investors to buy the assets directly, instead a class of securities is created to represent the assets and the investors purchase these securities. The most common type of assets that are securitised include mortgages, car loans, and credit card loans. However in theory virtually any asset that generates a cash flow that can be predicted or modelled may be securitised.⁵ The vehicle used is constructed so that securities issued against the asset base have a risk-return profile that is attractive to the investors that are being targeted.

To benefit from diversification asset types are usually pooled, and this pool then generates a range of interest payments, principal repayments and principal prepayments. The precise nature of the cash flows is uncertain because of the uncertainty of payment and prepayment patterns, and also because of the occurrence of loan defaults and delays in payment. However the pooling of a large number of loans means that cash flow fluctuation can be ironed out to a large extent, sufficient to issues notes against. The cash flows generated by the pool of assets are re-routed to investors through a dedicated structure, and a credit rating for the issue is usually requested from one or more of the private credit agencies. Most asset-backed securities carry investment-grade credit ratings, often triple-A or double-A, mainly because of various credit insurance facilities that are set up to guarantee the bonds. The securitisation structure disassociates the quality of the original cash flows from the quality of the flows accruing to investors. In many cases the original borrowers are not aware that the process has occurred and notice no difference in the way their loan is handled. The credit rating on the securitisation issue has no bearing on the rating of the selling bank and often will be different.

33.5.2 Benefits of securitisation

Securitisating assets produces a double benefit for the issuing bank. Those assets which are sold to investors generate a saving in the cost of required capital for the bank, as they are no longer on the balance sheet, so the bank's capital requirement is reduced. Secondly if the credit rating of the issued securities is higher than that of the originating bank, there is a potential gain in the funding costs of the bank. For example, if the securities issued are triple-A rated, a double-A rated bank will have lower funding costs for those securities. The bank benefits from paying a

⁵ For example, the rock musician David Bowie has issued bonds whose coupons were backed by expected royalty payments from sales of his back catalogue. Other musicians have since issued similar securities. Now, who said rock stars don't have any cool?!

lower rate on the borrowed funds than if it had borrowed those funds directly in the market. This has led to strong growth in for example, the specialised “credit card” banks in the United States, where banks such as Capital One, First USA and MBNA Bank have benefited from triple-A rated funding levels and low capital charges. It is doubtful if such banks could have grown as rapidly as they did without securitisation. Although there is a cost associated with securitising assets, which include the direct issue transaction costs and the cost of running the payment structure, these are outweighed by the benefits obtained from the process.

The major benefit of securitisation is reduced funding costs. Several factors influence such costs. These include:

- the lower cost of funds due to the enhanced credit rating of the issued bonds. The extent of this gain is a function of current spreads in the market and the current rating of the originating bank, and will fluctuate in line with market conditions;
- the saving in capital charges obtained from reducing the size of assets on the balance sheet. This decreases the minimum earnings required to ensure an adequate return for shareholders, in effect improving return on capital at a stroke.

The costs of the process include:

- those associated with setting up the issuing structure, and subsequently the payment mechanism that channels cash flows to investors. These costs are a function of the structure and risk of the original assets; the higher the risk of the original assets, the higher the cost of insuring the cash flows for investors;
- the legal costs of origination, plus operating costs and servicing costs.

However the reduction in funding cost obtained as a result of securitisation should significantly outweigh the cost of the process itself. In order to determine whether a securitisation is feasible, as well as the impact on the return on capital, the originating bank will conduct a cost and benefit analysis prior to embarking on the process. This is frequently the responsibility of the ALM unit.

EXAMPLE 33.1 Securitisation transaction

- We illustrate the impact of securitising the balance sheet with an hypothetical example from ABC Bank plc.

The bank has a mortgage book of £100 million, and the regulatory weight for this asset is 50%. The capital requirement is therefore £4 million (that is, $8\% \times 0.5\% \times £100 \text{ million}$). The capital is comprised of equity, estimated to cost 25% and subordinated debt, which has a cost of 10.2%. The cost of straight debt is 10%. The ALM desk reviews a securitisation of 10% of the asset book, or £10 million. The loan book has a fixed duration of 20 years, but its effective duration is estimated at seven years, due to re-financings and early repayment. The net return from the loan book is 10.2%.

The ALM desk decides on a securitised structure that is made up of two classes of security, subordinated notes and senior notes. The subordinated notes will be granted a single-A rating due to their higher risk, while the senior notes are rated triple-A. Given such ratings the required rate of return for the subordinated notes is 10.61%, and that of the senior notes is 9.80%. The senior notes have a lower cost than the current balance sheet debt, which has a cost of 10%. To obtain a single-A rating, the subordinated notes need to represent at least 10% of the securitised amount. The costs associated with the transaction are the initial cost of issue and the yearly servicing cost, estimated at 0.20% of the securitised amount. The summary information is given at Table 33.2.

ABC Bank plc	
Current funding	
Cost of equity	25%
Cost of subordinated debt	10.20%
Cost of debt	10%
Mortgage book	
Net yield	10.20%
Duration	7 years
Balance outstanding	100 million

Proposed structure	
Securitised amount	10 million
Senior securities:	
Cost	9.80%
Weighting	90%
Maturity	10 years
Subordinated notes:	
Cost	10.61%
Weighting	10%
Maturity	10 years
Servicing costs	0.20%

Table 33.2: ABC Bank plc mortgage loan book and securitisation proposal.

A bank's cost of funding is the average cost of all the funds it employed. The funding structure in our example is capital 4%, divided into 2% equity at 25%, 2% subordinated debt at 10.20% and 96% debt at 10%. The weighted funding cost F therefore is:

$$\begin{aligned}
 F_{\text{balance sheet}} &= 96\% \times 10\% + ((8\% \times 50\%) \times (25\% \times 50\%) + (10.20\% \times 50\%)) \\
 &= 10.30\%.
 \end{aligned}$$

This average rate is consistent with the 25% before-tax return on equity given at the start. If the assets do not generate this return, the received return will change accordingly, since it is the end result of the bank's profitability. As currently the assets generate only 10.20%, they are currently performing below shareholder expectations. The return actually obtained by shareholders is such that the average cost of funds is identical to the 10.20% return on assets. We may calculate this return to be:

$$\text{Asset return} = 10.20\% = (96\% \times 10\%) + 8\% \times 50\% \times (roe \times 50\% + 10.20\% \times 50\%).$$

Solving this relationship we obtain a return on equity of 19.80%, which is lower than shareholder expectations. In theory the bank would find it impossible to raise new equity in the market because its performance would not compensate shareholders for the risk they are incurring by holding the bank's paper. Therefore any asset that is originated by the bank would have to be securitised, which would also be expected to raise the shareholder return.

The ALM desk proceeds with the securitisation, issuing £9 million of the senior securities and £1 million of the subordinated notes. The bonds are placed by an investment bank with institutional investors. The outstanding balance of the loan book decreases from £100 million to £90 million. The weighted assets are therefore £45 million. Therefore the capital requirement for the loan book is now £3.6 million, a reduction from the original capital requirement of £400,000, which can be used for expansion in another area, a possible route for which is given in Table 33.3.

Outstanding balances	Value (£m)	Capital required (£m)
Initial loan book	100	4
Securitised amount	10	0.4
Senior securities	9	Sold
Subordinated notes	1	Sold
New loan book	90	3.6
Total asset	90	
Total weighted assets	45	3.6

Table 33.3: Impact of securitisation on balance sheet.

The benefit of the securitisation is the reduction in the cost of funding. The funding cost as a result of securitisation is the weighted cost of the senior notes and the subordinated notes, together with the annual servicing cost. The cost of the senior securities is 9.80%, while the subordinated notes have a cost of 10.61% (for simplicity

here we ignore any differences in the duration and amortisation profiles of the two bonds). This is calculated as:

$$(90\% \times 9.80\%) + (10\% \times 10.61\%) + 0.20\% = 10.08\%.$$

This overall cost is lower than the target funding cost obtained through the direct from the balance sheet, which was 10.30%. This is the quantified benefit of the securitisation process. Note that the funding cost obtained through securitisation is lower than the yield on the loan book. Therefore the original loan can be sold to the structure issuing the securities for a gain.

EXAMPLE 33.2 Position management

■ Starting the day with a flat position, a money market interbank desk transacts the following deals:

1. £100 million borrowing from 16/9/99 to 7/10/99 (3 weeks) at 6.375%
2. £60 million borrowing from 16/9/99 to 16/10/99 (1 month) at 6.25%
3. £110 million loan from 16/9/99 to 18/10/99 (32 days) at 6.45%

The desk reviews its cash position and the implications for refunding and interest rate risk, bearing in mind the following:

- There is an internal overnight roll-over limit of £40 million (net);
- The bank's economist feels more pessimistic about a rise in interest rates than most others in the market, and has recently given an internal seminar on the dangers of inflation in the UK as a result of recent increases in the level of average earnings;
- Today there are some important figures being released including inflation (RPI) data. If today's RPI figures exceed market expectations, the dealer expects a tightening of monetary policy by *at least* 0.50% almost immediately.
- A broker's estimate of daily market liquidity for the next few weeks is one of low shortage, with little central bank intervention required, and hence low volatilities and rates in the overnight rate.
- Brokers' screens indicate the following term repo rates:

O/N	6.350% – 6.300%
1 week	6.390% – 6.340%
2 week	6.400% – 6.350%
1 month	6.410% – 6.375%
2 month	6.500% – 6.450%
3 month	6.670% – 6.620%

- The indication for a 1v2 FRA is:

1v2 FRA	6.680% – 6.630%

- The quote for an 11 day forward borrowing in 3 weeks' time (the "21v32 rate") is 6.50% bid
- The book's exposure looks like this:

16 Sept	7 Oct	16 Oct	18 Oct
long £50m	short £50m	short £110m	flat

What courses of action are open to the desk, bearing in mind that the book needs to be squared off such that the position is flat each night?

Possible solutions

Investing early surplus

From a cash management point of view, the desk has a £50 million surplus from 16/9 up to 7/10. This needs to be invested. It may be able to negotiate a 6.31% loan with the market for overnight, or 6.35% term desposit for 1 week to 6.38% for 1 month.

The overnight roll is the most flexible but offers a worse rate, and if the desk expects the overnight rate to remain both low and stable (due to forecasts of low market shortages), it may not opt for this course of action.

However it may make sense from an interest rate risk point of view. If the desk agrees with the bank's economist, it should be able to benefit from rolling at higher rates soon – possibly in the next three weeks. Therefore it may not want to lock in a term rate now, and the overnight roll would match this view. However, it exposes them to lower rates, if their view is wrong, which will limit the extent of the positive funding spread. The market itself appears neutral about rate changes in the next month, but appears to factor in a rise thereafter.

The forward “gap”

Looking forward, the book is currently on course to exceed the £40 million overnight position limit on 7/10, when the refunding requirement is £50 million. The situation gets worse on 16/10 (for two days) when the refunding requirement is £110 million. The desk needs to fix a term deal before those dates to carry it over until 18/10 when the funding position reverts to zero. A borrowing from 7/10 to 18/10 of £50 million will reduce the roll-over requirement to within limit.

However given that interest rates will rise, should the desk wait until the 7th to deal in the cash? Not if it has a firm view. They may end up paying as much as 6.91% or higher for the funding (after the 0.50% rate rise). So it would be better to transact now a forward starting repo to cover the period, thus locking in the benefits obtainable from today's yield curve. The market rate for a 21x32 day repo is quoted at 6.50%. This reflects the market's consensus that rates may rise in about a month's time. However, the desk's own expectation is of a larger rise, hence its own logic suggests trading in the forward loan. This strategy will pay dividends if their view is right, as it limits the extent of funding loss.

An alternative means of protecting the interest rate risk alone is to BUY a 1v2 month Forward Rate Agreement (FRA) for 6.68%. This does not exactly match the gap, but should act as an effective hedge. If there is a rate rise, the book gains from the FRA profit. Note that the cash position still needs to be squared off. Should the desk deal before or after the inflation announcement? That is of course down to its, but most dealers like, if at all possible, to sit tight ahead of releases of key economic data.

Appendices

APPENDIX 33.1 NPV and value-at-risk

The NPV of a banking book is an appropriate target of interest-rate policy because it captures all future cash flows and is equal to the discounted value of future margins when the discount rate is the cost of all debt. The sensitivity of the NPV is derived from the duration of the assets and liabilities. Therefore we may write the change in NPV as (33.6):

$$\frac{\Delta NPV}{\Delta r} = \left(\frac{1}{(1+r)} \right) (-D_A MV_A + D_L MV_L) \quad (33.6)$$

where D_A is the duration of assets and MV_A is the market value of assets. (33.6) is applicable when only one interest-rate is used for reference. The sensitivity with respect to the interest rate r is known. It is then possible to derive the value-at-risk (VaR) from these simple relationship above. With one interest rate we are interested in the maximum variation of the NPV that results from a change in the reference interest rate. The volatility of the NPV can be derived from its sensitivity and from the interest-rate volatility. If we set S_r as the sensitivity of the NPV with respect to the interest rate r , the volatility of the NPV is given by (33.7):

$$\sigma(NPV) = S_r \times \sigma(r). \quad (33.7)$$

Once the volatility is known, the maximum change at a given confidence level is obtained as a multiple of the volatility. The multiple is based on assumptions with respect to the shape of the distribution of interest rates. Under a curve of the normal distribution, a multiple of 1.96 provides the maximum expected change at a 2.5% two-tailed confidence level, so that we are able to say that the VaR of the book is as given by (33.8):

$$VaR = 1.96 \times S_r \times \sigma(r). \quad (33.8)$$

Where there is more than one interest rate, the variation of the NPV can be approximated as a linear combination of the variations due to a change of each interest rate. This is written as (33.9):

$$NPV = S_r \times \Delta r + S_s \times \Delta s + S_t \times \Delta t + \dots \quad (33.9)$$

where r , s and t are the different interest rates. Since all interest rate changes are uncertain, the volatility of the NPV is the volatility of a sum of random variables. Deriving the volatility of this sum requires assumptions on correlations between interest rates.

This problem is identical to the general problem of measuring the market risk of a portfolio when bearing in mind that its change in market value arises as a result of changes generated by the random variations of market parameters. The main concern is to calculate the volatility of the mark-to-market value of the portfolio, expressed as the sum of the random changes of the mark-to-market values of the various individual transactions. These random changes can be interdependent, in the same way that the underlying market parameters are. The volatility of the value of the portfolio depends upon the sensitivities of individual transactions, upon the volatilities of the individual market parameters and also upon their interdependency, if any exists. The methodology that calculates this volatility is known as *delta-VaR*. This is based on the delta sensitivity of the portfolio to changes in market interest rates.

Selected bibliography and references

- Asarnow, E., "Best practices in loan portfolio management", *Journal of Lending and Credit Risk Management*, March 1996, pp. 14–24.
- Avery, R.B., Berger, A.N., "Loan commitments and bank risk exposure", *Journal of Banking and Finance* 15, 1994, pp. 173–192.
- Benston, G., Henwick, G., Humphrey, D., "Operating Costs in Commercial Banking", Federal Reserve Bank of Atlanta, *Economic Review*, November 1982, pp. 6–21.
- Berger, A., Udell, G., "Collateral, loan quality and bank risk", *Journal of Monetary Economics* 25, 1990, pp. 21–42.
- Bitner, J., *Successful Bank Asset-Liability Management*, Wiley, 1992.
- Brewer, E., "Bank gap management and the use of financial futures", *Economic Perspectives*, Federal Reserve Bank of Chicago, March/April 1985, pp. 12–22.
- Calstrom, C., Samolyk, K., "Examining the microfoundations of market incentives for asset-backed lending", *Economic Review*, Federal Reserve Bank of Cleveland, 29, Quarter 1 1993, pp. 27–38.
- Cumming, C., "The economics of securitisation", *Quarterly Review*, Federal Reserve Bank of New York, 12, Autumn 1987, pp. 11–23.
- Deshmukh, S., Greenbaum, S., Kamatas, G., "Interest rate uncertainty and the financial intermediary's choice of exposure", *Journal of Finance* 38, 1990, pp. 141–147.
- Greenbaum, S., Thakor, A., *Contemporary Financial Intermediation*, Dryden Press, 1995.
- Gup, B., Brooks, R., *Interest Rate Risk Management*, Bankers Publishing Company and Probus Publishing Company, 1993.
- Howe, D., *A Guide to Managing Interest-Rate Risk*, New York Institute of Finance, 1992.
- Johnson, H., *Bank Asset/Liability Management*, Probus Publishing, 1994.
- Kim, D., Santomero, A., "Risk in banking and capital regulation", *Journal of Finance*, 43, 1988, pp. 1218–1233.
- Koch, T., *Bank Management*, Dryden Press, 1988.
- Sinkey, J., *Commercial Bank Financial Management*, 4th edition, Macmillan, 1992.
- Stevenson, B., Fadil, M., "Modern Portfolio Theory: can it work for commercial loans?", *Commercial Lending Review*, 10(2), Spring 1995, pp. 4–12.
- Toevs, A., Haney, W., "Measuring and managing interest rate risk: a guide to asset/liability models used in banks and thrifts", in Platt, R. (ed.), *Controlling Interest Rate Risk, New Techniques and Applications for Money Management*, Wiley, 1986.
- Wilson, J.S.G. (ed.), *Managing Banks' Assets and Liabilities*, Euromoney Publications, 1988.

34

The Repo Markets

One of the largest segments of the money markets worldwide is the market in repurchase agreements or *repo*. A most efficient mechanism by which to finance bond positions, repo transactions enable market makers to take long and short positions in a flexible manner, buying and selling according to customer demand on a relatively small capital base. Repo is also a flexible and relatively safe investment opportunity for investors such as money market funds and corporate and local authority treasurers. The ability to execute repo is particularly important to overseas firms who might not have access to a domestic deposit base; where no repo market exists, funding is in the form of unsecured lines of credit from the banking system, which is restrictive for some market participants. An open market in repo, and its close cousin securities lending, is often cited as a key ingredient of a liquid bond market. Repo is therefore a very important instrument.

In the United States repo is a well-established alternative money market instrument. By providing ready access to secured borrowing, and by enhancing liquidity in the securities markets, repo facilitates portfolio financing and the ability to run a short position in any bond. Banks can also use repo to extend credit to securities houses, who provide collateral in the form of government bonds and other high quality bonds. In this chapter we review the main uses of repo and its structure; there follows a comprehensive review of basis trading strategy and the implied repo rate.

34.1 Development of the repo market

A market in *repo* – from “sale and repurchase” agreement – was introduced by the US Federal Reserve in 1918, as the main tool of the Fed’s open market operations. Repo was used both to drain liquidity (in the form of surplus cash) from the banking system, and to add liquidity as required. The US government bond market and the repo market in US Treasuries are the largest in the world; daily volume in US repo is estimated at well over \$1,000 billion. Repo has developed in other markets around the world, and exists in both developed and emerging economy capital markets. The instrument now covers corporate bonds and Eurobonds as well as equity and money market instruments.

The growth in repo across markets globally can be attributed to the following:

- growth in non-bank funding
- ease of transaction
- expansion in public sector debt levels
- increased volatility in interest rates
- arbitrage opportunities against other money market instruments.

The instrument is attractive to market participants due to its flexibility when compared to other money market instruments. Bondholders can pick up extra yield on their portfolios, there is potential for reduction of borrowing costs allied to reduced credit risk due to collateralisation and a facility to borrow bonds to cover short positions.

Country	Daily turnover (€m)
Belgium	25
France	40
Germany	60
Great Britain	25.6
Italy	50
Spain	20
Other European	35
USA	420

Table 34.1: Repo markets, estimated daily turnover, 1998.
Source: National depositories, Euroclear, Clearstream.

In Europe repo markets have grown substantially in recent years, although the size and importance of the market varies considerably across different countries. The volumes in selected countries are shown in Table 34.1.

34.2 Introduction to repo

A repo agreement is a transaction in which one party sells securities to another, and at the same time and as part of the same transaction commits to repurchase identical securities on a specified date at a specified price. The seller delivers securities and receives cash from the buyer. The cash is supplied at a predetermined rate – the *repo rate* – that remains constant during the term of the trade. On maturity the original seller receives back collateral of equivalent type and quality, and returns the cash plus repo interest. One party to the repo requires either the cash or the securities and provides *collateral* to the other party, as well as some form of compensation for the temporary use of the desired asset. Although legal title to the securities is transferred, the seller/lender retains both the economic benefits and the market risk of owning them.

There are two main repo types in operation in different markets, to begin with we shall consider the operation of a *classic* repo, the type prevalent in the UK and US bond markets.

34.2.1 Classic Repo

There are two parties to a repo trade, let us say Bank A (the “seller” of securities) and Bank B (the “buyer” of securities). On the trade date the two banks enter into an agreement whereby on a set date, the *settlement* date Bank A will sell to Bank B a nominal amount of securities in exchange for cash. The price received for the securities will be linked to the settlement price of the stock on the trade date. The agreement also demands that on the termination date Bank B will sell identical stock back to Bank A at the previously agreed price, and consequently Bank B will have its cash returned with interest at the agreed repo rate.

In essence a repo agreement is a secured loan (or *collateralised* loan) in which the repo rate reflects the interest charged. The mechanism is illustrated at Figure 34.1.

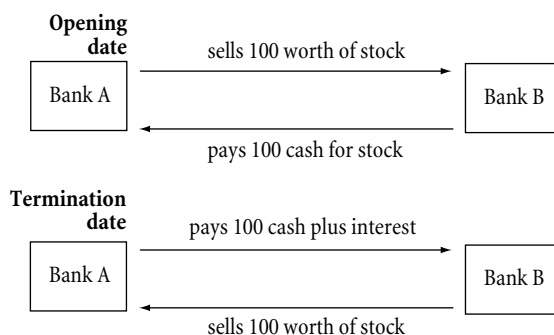


Figure 34.1: Basic repo structure.

A *reverse repo* is the mirror image of a repo, that is, purchasing the bond and then selling it back on termination. Of course, every repo is a reverse repo, depending from which party’s point of view one is looking at the transaction.

34.2.2 Repo example

To illustrate the basic principle, consider the following. This illustrates a *specific* repo, that is one in which the collateral supplied is specified as a particular stock, as opposed to a *general collateral* (GC) trade in which a basket of collateral can be supplied, of any particular issue, as long as it is of the required type and credit quality.¹ In our example, on 6 September 1997, Bank A agrees to sell £1 million nominal of a UK gilt, the 8% Treasury 2000, which is trading at a *dirty* price of 104.3. The agreement will begin on 7 September, the value date. The term of the trade is 30 days, so the termination date is 7 October 1997 and the agreed repo rate for the (effectively collateralised) loan is set at 6.75%. On 7 September Bank B receives £1m nominal 8% Treasury 2000 stock, which has a settlement value of £1,043,000 (clean price plus accrued interest).

¹ Stock that is regarded as general collateral for selected countries is listed in Appendix 34.1

On 7 October Bank A receives back the gilt and returns the original cash amount of £1,043,000 along with a repo interest payment of £5786.50. This is shown in Figure 34.2 below.

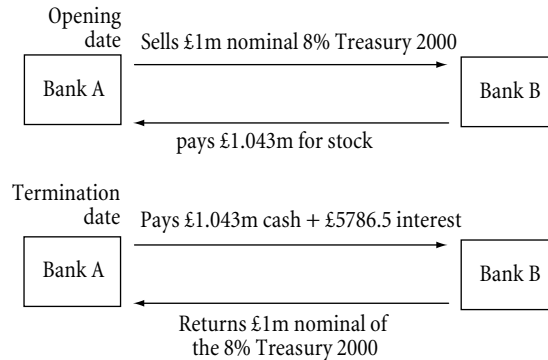


Figure 34.2: Gilt repo example.

The repo interest in the example is based on a 30 day repo rate of 6.75% and a 365 day count basis, which is the convention in the sterling markets. If the repo had been conducted for a US dollar bond for example, the interest would have been calculated on the basis of 360-day year. Repo rates are agreed at the time of the trade and are quoted, like all interest rates, on an annualised basis. The repo interest under the agreement equals the cash loaned multiplied by the repo rate, multiplied by the term of the loan as a proportion of the year:

$$1,043,000 \times \frac{6.75}{100} \times \frac{30}{365} = £5786.50.$$

The settlement price (dirty price) is used because it is the market value of the bonds on the particular trade date and hence indicates the cash value of the gilts. The object is to minimise credit exposure by equating value of the cash and the collateral.

34.2.3 Sell/buy back and stock lending

In addition to classic repo there also exists *sell/buy back* and *securities lending*. A sell/buy back is defined as an outright sale of a bond on the value date, and an outright repurchase of that bond for value on a *forward* date. In the diagram above, the cash flows therefore become a sale of the bond at a *spot* price, followed by repurchase of the bond at the *forward* price. The forward price calculated includes the interest on the repo, and is therefore a different price to the spot price. Securities lending is defined as a temporary transfer of securities in exchange for collateral. It is not a repo in the normal sense; there is no sale or repurchase of the securities. The use of the desired asset is reflected in a fixed fee payable by the party temporarily taking the desired asset. Both these mechanisms will be reviewed again shortly.

34.3 Uses and economic functions of repo

The repo mechanism allows for compensation for use of a desired asset. If cash is the desired asset, the compensation for its use is simply the repo rate of interest paid on it. If bonds are the desired asset, the buyer (borrower) compensates the seller (lender) by accepting a repo interest rate that is below the market level.

34.3.1 Funding bond positions

In the normal course of business a bond trader or market-maker will need to finance her long and short positions. The diagram at Figure 34.3 illustrates the basic principle for a bond trader running a bond market making book.

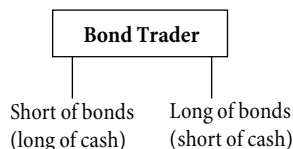


Figure 34.3: Financing bond positions.

To finance a long position the bond trader can borrow money unsecured in the interbank market, assuming that he has a credit line in this market. However a collateralised loan will invariably be offered to him at a lower rate, and counterparties are more likely to have a credit line for the bond trader if the loan is secured.

34.3.2 Repo as a financing transaction

Cash rich money market investors finance bond traders, by lending out cash in a repo. They receive *general collateral* (GC) in return for their cash, which is any bond of the required credit quality. Legally this is a sale and repurchase of bonds; economically it is a secured loan of cash. The cash investor receives the repo rate of interest for making the loan.

The advantages of a repo transaction for the cash investor are:

- it is a secured investment;
- the returns are competitive with bank deposits and occasionally higher;
- this is a diversification from bank risk, that is, an extra form of investment outside a regular bank deposit.

A bond trader will enter into a *reverse repo* when he requires a specific issue to deliver into a short sale. In this case the trader is effectively borrowing bonds and putting up cash as collateral. The bond trader receives the repo rate on his cash.

The position is shown in Figure 34.4 below.

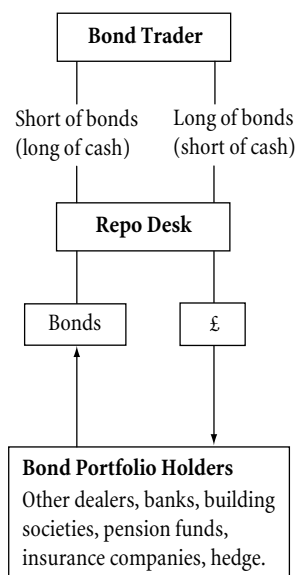


Figure 34.4: Financing a short position.

In this transaction the bond lender's compensation is the difference between the repo rate paid on the dealer's cash and the market rate at which he can reinvest the cash. If the bond is particularly sought after, that is it is *special*, the repo rate may be significantly below the GC rate. Special status in a bond will push the repo rate downwards. Zero rates and even negative rates are possible when dealing in specials. The repo rate will reflect supply and demand in the market. In a financing transaction, the dealer is paying the repo rate on the investor's cash. The GC rate tends to trade below the LIBOR rate, and also below the LIBID rate, reflecting its status as a secured loan. In a positioning transaction, the dealer receives the repo rate on his cash. If the bond being borrowed (for this is, in effect what is happening) is special, the repo rate receivable will be lower to reflect the demand for the bond.

Active players in repo and interbank markets can enhance yield by lending bonds at the GC rate and then reinvesting the cash at a higher rate. This would of course introduce an element of credit risk. A market counterparty could also borrow bonds in the stock lending market, on-lend these bonds via repo and invest the cash proceeds in say, CDs. Where the collateral is government bonds, the institution will usually be receiving a higher rate on the CD

than the repo rate it is paying in the repo. The use of repo for arbitrage and basis trading will be considered in a later section.

34.4 Repo mechanics

We noted that there are essentially two main types of repo, the *classic repo* and *sell/buy back*. Markets also engage in stock lending, which is economically similar to a repo trade but without the associated trade monitoring and interest rate exposure aspects of repo or sell/buy back.

In a *classic repo*, as we have already seen, one party sells bonds to another while simultaneously agreeing to repurchase them on a future date at a specified price. The sale and repurchase prices are the same, although settlement values will differ because on termination of the repo the interest is added on. If a coupon is paid it will be handed over to the seller on the value date. This reflects the fact that although legal title to the collateral passes to the buyer in a repo, economic costs and benefits of the collateral remain with the seller.

A classic repo transaction is subject to a legal contract signed in advance by both parties. A standard document will suffice – it is not necessary to sign a legal agreement prior to each transaction.

EXAMPLE 34.1 Classic repo

On 21 June 1998, a corporate wishes to invest DEM 50 million against German government bonds for 7 days. The collateral is the 5½% bunds due in April 2003. The repo rate is agreed at 5.60% The bund price is 101.2305 clean, which together with 1.0542 accrued interest (69 days) gives a dirty price of 102.2847.

The borrower of cash will need to determine the face value of bunds required at current market price that will equate to DEM 50 million. This is shown below.

$$\frac{102.2847}{100.0000} = \frac{50,000,000}{X}$$

The nominal value of bunds required (X) is 48,883,000. This is rounded to the nearest 1,000 because bunds traded in denominations of 1,000 (with the introduction of the euro, euroland bonds trade down to EUR 0.01).

The trade details are summarised below.

Nominal	DEM 48,883,000 of Bund 5½% 2003
Clean start price	101.2305
Accrued	1.0542
Dirty start price	102.2847
Settlement money	DEM 50,000,000
Dirty end price	102.2847
Repo interest	DEM 54,444 ($50,000,000 \times 5.60\% \times 7/360$)
Termination money	DEM 50,054,444

Note that the sale and repurchase prices are the same

A *sell and buy-back* on the other hand refers to a spot sale and forward repurchase of bonds transacted simultaneously. The repo rate is not explicit, but is implied in the forward price. If initial margin is required it is given to the provider of cash (the buyer). Any coupon payments during the term are paid to the seller, however this is done through incorporation into the forward price, so the seller will not receive it immediately. Generally sell/buy backs are not subject to a legal agreement, so in effect the seller has no legal right to any coupon, and there is no provision for *variation margining*. This makes the sell/buy back a higher risk transaction when compared to classic repo, more so in volatile markets. It is more common to encounter sell/buy-backs in emerging markets, or being conducted between counterparties whose settlement systems are not able to handle classic repo, necessitating the input of both legs of a repo trade as a sale and a buy-back.²

² The author has experience of a classic repo being treated as a sell/buy-back due to the bank's settlement systems not being able to handle classic repo. The trade ticket on that occasion was one complete side of A4 paper!

EXAMPLE 34.2 Sell/buy-back transaction

- Consider the same terms as Example 34.1 above, but in this case as a sell/buy back transaction. We require the forward bond price, and this is calculated by converting the termination money.

$$\frac{\text{DEM } 50,054,444}{\text{DEM } 48,883,000} \times 100 = 102.396424.$$

The accrued interest *at the time of termination* is subtracted from this price to obtain a forward clean price, as shown below.

$$102.396424 - 1.1611 \text{ (76 days)} = 101.235324.$$

The trade details are summarised below.

Nominal	DEM 48,883,000 of Bund 5½% 2003
Clean start price	101.2305
Accrued	1.0542
Dirty start price	102.2847
Settlement money	DEM 50,000,000
Clean end price	101.235324
Accrued	1.1611
Dirty end price	102.396424
Termination money	DEM 50,054,444 (includes rep interest of DEM 54,444)

Note that the sale and repurchase prices are now different.

34.4.1 Stock lending

Institutional investors such as pension funds and insurance companies often prefer to enhance the income from their fixed interest portfolios by lending their bonds, for a fee, rather than get involved in repo. A stock loan is a contract committing one party to lend, and the other to borrow, agreed securities for an agreed period. The borrower of stock is required to provide collateral to the lender in the form of cash, other securities or a letter of credit. The origins and history of the stock lending market are different from that of the repo market. The range of counterparties is also different, although of course a large number of counterparties are involved in both markets. Most stock loans are on an “open” basis, although term loans also occur. Initial margin is given to the lender of the securities.

EXAMPLE 34.3 Stock loan transaction

- A dealer needs to borrow DEM 50 million nominal of a specific issue, the 5½% bund due April 2003, from 21 June to 28 June. A pension fund has agreed to lend the stock against collateral, and requires a margin of 102%. The agreed rebate is 5.10%. The cash flows for this stock loan are shown below.

Bonds borrowed	DEM 50 million nominal of 5½% 2003
Clean price	101.2305
Accrued	1.0542
Dirty price	102.2847
Market value	$\text{DEM } 50,000,000 \times 102.2847 = 51,423,350$
Settlement money	$\text{DEM } 52,165,197 \text{ (} 102\% \times 51,423,350 \text{)}$
Term	7 days
Rebate rate	5.10%
Rebate interest	$\text{DEM } 51,730 \text{ (} 52,165,197 \times 5.10\% \times 7/360 \text{)}$
Termination money	$\text{DEM } 52,216,927 \text{ (} \text{DEM } 52,165,197 + 51,730 \text{)}$

Note that initial margin is provided to the lender of the bonds, and that the rebate interest rate is lower than the GC repo rate used in Example 34.1.

34.4.2 Initial margin

In both classic repo and sell/buy back, any initial margin is given to the supplier of cash in the transaction. This remains the case in the case of specific issue transactions. For initial margin the market value of the bond collateral is reduced (or given a “haircut”) by the percentage of the initial margin and the nominal value determined from this reduced amount.

There are two methods for calculating the margin; for a 2% margin this could be one of the following:

- the dirty price of the bonds $\times 0.98$;
- the dirty price of the bonds $/ 1.02$

The repo page on the Bloomberg® system, which is “RRRA”, uses the second method for its calculations.³ The size of margin required in any particular transaction is a function of the following:

- the credit quality of the counterparty supplying the collateral;
- the term of the repo;
- the duration (price volatility) of the collateral;
- the existence or absence of a legal agreement.

However in the final analysis margin is required to guard against market risk, the risk that the value of collateral will drop during the course of the repo. Therefore the margin call must reflect the risks prevalent in the market at the time; extremely volatile market conditions may call for large increases in initial margin.

34.4.3 Variation margin

The standard market documentation that exists for the three structures covered so far (in addition to Tri-Party repo, covered in a later section) include clauses that allow parties to a transaction to call for variation margin during the term of a repo. This can be in the form of extra collateral, if the value of the collateral has dropped in relation to the asset exchanged, or a return of collateral, if the value has risen. Both parties have an interest in making and meeting margin calls, although there is no obligation. The level at which variation margin is triggered is often agreed beforehand in the legal agreement put in place between individual counterparties.

34.5 Other repo structures

34.5.1 Tri-party repo

This is a mechanism that allows dealers maximum control over their inventory, incurs minimal settlement cost to the investor, but gives the investor independent confirmation that their cash is fully collateralised. Under a tri-party agreement, the dealer delivers collateral to an independent third-party custodian, such as Euroclear or Clearstream, who will place it into a segregated tri-party account. The dealer maintains control over which precise securities are in this account (multiple substitutions are permitted) but the custodian undertakes to confirm each day to the investor that their cash remains fully collateralised by securities of suitable quality. A tri-party agreement needs to be in place with all three parties before trading can commence. This arrangement reduces the administrative burden for the end-investor, but is not in theory as secure as a conventional delivery-versus-payment structure. Consequently the yield on the investor’s cash (assuming collateral of identical credit quality) should be slightly higher. The structure is shown in Figure 34.5 below.

The tri-party agent is an agent to both parties in the repo transaction. It provides a collateral management service overseeing the exchange of securities and cash, and managing collateral during the life of the repo. It also carries out daily marking-to-market, and substitution of collateral as required. The responsibilities of the agent can include:

- the preparation of documentation;
- the setting up of the repo account;
- monitoring of cash against purchased securities, both at inception and at maturity;
- initial and ongoing testing of concentration limits;

³ Trust me, the methods give you different answers!

- the safekeeping of securities handed over as collateral;
- managing the substitution of securities, where this is required;
- monitoring the market value of the securities against the cash lent out in the repo;
- issuing margin calls to the borrower of cash.

The tri-party agent will issue close-of-business reports to both parties. The contents of the report can include some or all of the following:

- tri-party repo cash and securities valuation;
- corporate actions;
- pre-advice of expected income;
- exchange rates;
- collateral substitution.

The extent of the duties performed by the tri-party agent is dependent of the sophistication of an individual party's operation. Smaller market participants who do not wish to invest in sophisticated infrastructure may outsource all repo-related functions to the tri-party agent.

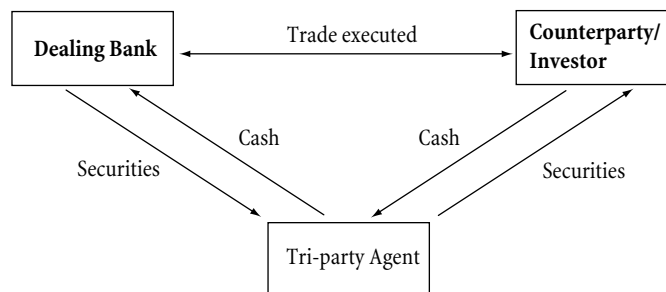


Figure 34.5: Tri-party repo structure.

34.5.2 Hold-in-Custody repo

This is a sector of the general collateral market, and is more common in the United States than elsewhere. Consider a case of a cash-rich institution investing in GC as an alternative to deposits or commercial paper. When it comes to the rate of return on their cash, the rules of risk and reward apply. The better the quality of collateral, the lower the yield the institution can expect. Similarly the mechanics of settlement may also affect the repo rate. The most secure procedure is to take physical possession of the collateral. However if the dealer needs one or more substitutions during the term of the trade, the settlement costs involved may make the trade unworkable for one or both parties. Therefore the dealer may offer to hold the securities in his own custody against the investor's cash. This is known as *hold in custody* (HIC) repo. The advantage of this trade is that, since securities do not physically move, no settlement charges are incurred. However this is a risky trade for the investor because they only have the dealer's word that their cash is indeed fully collateralised in the event of default. Thus this type of trade is sometime referred to as a "Trust Me" repo. In the US market there have been cases where securities houses that went into bankruptcy and defaulted on loans, were found to have pledged the same collateral for multiple HIC repo trades. Investors dealing in HIC repo must ensure:

- they only invest with dealers of good credit quality, since an HIC repo may be perceived as an unsecured transaction;
- the investor receives a higher yield on their cash in order to compensate them for the higher credit risk involved.

A *safekeeping repo* is a form of repo whereby the collateral from the repo seller is not delivered to the cash lender but held in "safe keeping" by the seller. This has advantages in that there is no administration and cost associated with the movement of stock. The risk is that the cash lender must entrust the safekeeping of collateral to the counterparty, and has no means of confirming that the security is indeed segregated, and only being used for one transaction.

34.5.3 Borrow/Loan vs Cash

This is similar in almost all respects to a classic repo/reverse repo. A legal agreement between the two parties is necessary, and trades generally settle DVP. The key difference is that under a repo agreement legal title over the collateral changes hands. Under a securities lending agreement this is not necessarily the case. The UK standard securities lending agreement does involve transfer of title, but it is possible to construct a securities lending agreement where legal title does not move. This can be an advantage for customers who may have accounting or tax problems in doing a repo trade. Such institutions will opt to transact a *loan versus cash*. The UK standard lending agreement also covers items such as dividends and voting rights, and is therefore the preferred transaction structure in the equity repo market.

34.5.4 Bonds borrowed/collateral pledged

In this instance the institution lending the bonds does not want or need to receive cash against them, as it is already cash-rich and would only have to re-invest any further cash generated. As such this transaction only occurs with *special collateral*. The dealer borrows the special bonds and pledges securities of similar quality and value (general collateral). The dealer builds in a fee payable to the lending institution as an incentive to do the trade.

EXAMPLE 34.4

- ABC Bank plc wishes to borrow DKK 300 million of the Danish government bond 8% 2001. ABC owns the Danish government bond 7% 2007. ABC is prepared to pay a customer a 40 basis point fee in order to borrow the 8% 2001 for one month.

The market price of the 8% 2001 (including accrued interest) is 112.70. The total value of DKK 300 million nominal is therefore DKK 338,100,000. The market price of the 7% 2007 (including accrued interest) is 102.55.

In order to fully collateralise the customer ABC needs to pledge $338,100,000 / 1.0255$ which is 329,692,832.76, which rounded to the nearest DKK 1 million becomes DKK 330 million nominal of the 7% 2007.

In a bonds borrowed/collateral pledged trade, both securities are delivered free of payment and ABC Bank plc would pay the customer a 40bp borrowing fee upon termination. In our example the fee payable would be:

$$338,100,000 \times 31/360 \times 0.4/100 = \text{DKK } 112,700.$$

34.5.5 Borrow vs Letter of Credit

In this case the institution lending securities does not require cash, but takes a third-party bank letter of credit as collateral. However since banks typically charge 25–50 basis points for this facility, transactions of this kind are relatively rare.

34.5.6 Cross-currency repo

All of the examples discussed so far have used cash and securities denominated in the same currency, for example Bunds trading versus euro cash, and so on. In fact there is no requirement to limit oneself to single-currency transactions. It is possible to trade say, UK Gilts versus US dollar cash (or any other currency), or pledge Spanish government bonds against borrowing Japanese government bonds. A cross-currency repo is essentially a plain vanilla transaction, except that:

- there may be significant daylight credit exposure on the transaction if securities cannot settle versus payment;
- the transaction must be covered by appropriate legal documentation;
- fluctuating foreign exchange rates mean that it is likely that the transaction will need to be marked-to-market frequently in order to ensure that cash or securities remain fully collateralised.

34.6 Pricing and margin

The cash proceeds in a repo are typically no more than the market value of the collateral. This minimises credit exposure by equating the value of the cash to that of the collateral. The market value of the collateral is calculated at its *dirty* price, not *clean* price, that is including accrued interest. This is referred to as *accrual pricing*.

To calculate the accrued interest on the (bond) collateral we need to know the day-count basis for the particular bond.

EXAMPLE 34.5

- DEM 10 million nominal of the 7% bund 2006 is quoted at 99.15. There have been 100 days since the last coupon (at the time of this example, bunds paid on a 30/360 basis).

The full value is therefore:

$$(10,000,000 \times 99.15/100) + (10,000,000 \times 7\% \times 100/360) = \text{DEM } 10,109,444.$$

The start proceeds of a repo can be less than the market value of the collateral by an agreed amount or percentage. This is known as the *initial margin* or *haircut*. The initial margin protects the buyer against:

- a sudden fall in the market value of the collateral;
- illiquidity of collateral;
- other sources of volatility of value (for example, approaching maturity);
- counterparty risk.

The margin level of repo varies from 0%-2% for collateral such as UK gilts to 5% for cross-currency and equity repo to 10%-35% for emerging market debt repo.

To illustrate the calculation of initial margin, we will use a simple example. An initial margin of say, 2% can be either 2% of the value of the collateral or 2% of the value of the cash. The first example would be:

$$\begin{aligned} \text{collateral value} \times (1.00 - 0.02) &= 98\%; \text{ or} \\ \text{nominal value} \times \text{dirty price} \times (1.00 - 0.02) &= 98\%. \end{aligned}$$

The second case is calculated as follows:

$$\begin{aligned} &\frac{\text{collateral value}}{(1.00 + 0.02)}; \text{ or} \\ &\text{nominal} \times \frac{\text{dirty price}}{(1.00 + 0.02)}. \end{aligned}$$

The PSA/ISMA Global Repurchase Master Agreement defines a “margin ratio” as

$$\frac{\text{collateral value}}{\text{cash}} = 102\%.$$

The market value of the collateral is maintained through the use of *variation margin*. So if the market value of the collateral falls, the buyer calls for extra cash or collateral. If the market value of the collateral rises, the seller calls for extra cash or collateral. In order to reduce the administrative burden, margin calls can be limited to changes in the market value of the collateral in excess of an agreed amount or percentage, which is called a *margin maintenance limit*. An example of variation margin being applied during the term of a trade is given at Example 34.6.

EXAMPLE 34.6 Variation margin for repo in UK gilt 8% 2000

- The diagram shows a 60-day repo where a margin of 2% is taken. The repo rate is 5.625%. The start of the trade is 5 January 2000. The repo start is shown as Figure 34.6.

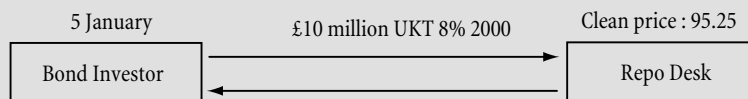
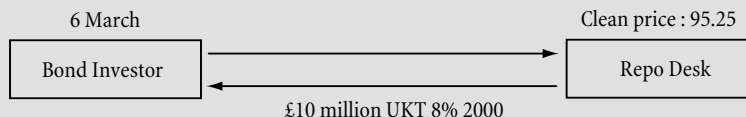


Figure 34.6

Principal	9,525,000.00
Accrued (29 days)	39,617.49
Total Consideration	£9,564,617.42

The consideration is divided by 1.02, the amount of margin, to give £9,377,075.97. This is rounded to £9,377,000.

**Figure 34.7**

Original loan	9,377,000.00
Repo interest @ 5.625%	86,705.14
Total maturity proceeds	9,463,705.14

Assume that half-way through the trade there has been a catastrophic fall in the bond market and the 8% 2000 gilt is trading down at 91.00. Following this drop in market price the securities are now worth:

Principal	9,100,000
Accrued (59 days)	129,315.07
Total	£9,229,315.07

However, the repo desk has lent £9,377,000 against this security. If it wishes it can call margin from the counterparty in the form of eligible securities or cash.

To restore the original margin of 2% the repo desk would call for an adjustment calculated as follows:

$$((\text{original consideration} + \text{repo interest accrued on consideration}) \times (1 + \text{initial margin})) \\ - (\text{new all-in price} \times \text{nominal amount}).$$

This therefore becomes:

$$((9,377,000 + 43,352.57) \times 1.02) - (0.9229315 \times 10,000,000) = £379,444.62$$

34.7 Risks in dealing repo

As with all capital market transactions there are risks attached to trading in repo. These risks can be grouped into the following areas:

- credit risk/counterparty risk;
- legal risk;
- collateral risk/issuer risk;
- market risk;
- daylight exposure;
- systems and controls (operational risk).

In certain cases the risk exposure by a market counterparty will be a function of more than one of the above, hence we refer to these risks as being interdependent.

34.7.1 Financial market risks

Generally credit risk, issuer risk, legal risk and market risk can be thought of as functions of counterparty risk, as opposed to operational risk which is generally an internal issue (unless a firm is affected by operational weakness in a counterparty).

Credit risk is the risk that the counterparty defaults on the transaction, due to financial difficulty or going out of business entirely. Although this is a serious issue, it is more so for unsecured creditors of the company, including those who have dealt in (unsecured) interbank transactions with the company. A market participant that has entered into a repo transaction may feel that he is protected by the collateral he is holding. Banks and other market participants guard against credit risk by internally rating all their counterparties, and assigning trading limits to each of them. The trading limit may be higher for repo compared to interbank trading, or set at the same level. In some cases counterparties deemed unsuitable for an interbank limit are not assigned secured lending limits either; an example of this occurred after the collapse of the Barings merchant bank in 1995. Many UK building societies

withdrew or drastically reduced limits for other similar merchant banks in the unsecured market. This continued to be the case when the gilt repo market was introduced the following year, despite the quality of collateral involved in this market.

The different credit ratings of market participants are reflected in the rates paid by each of them in the debt market. This is illustrated diagrammatically in Figure 34.8 below.

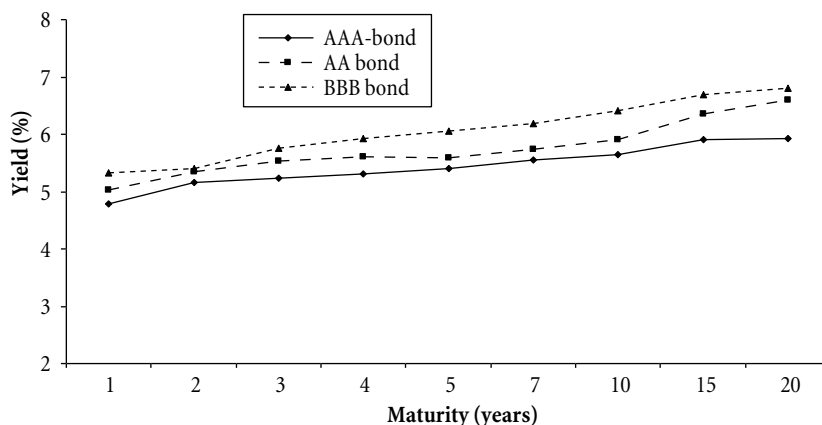


Figure 34.8: The credit structure of interest rates.

Although the rating agencies specify all paper at the level of Baa3/BBB- and above as being of “investment grade”, in practice often only banks and corporates rated at Baa1/BBB+ or higher are deemed suitable counterparties for unsecured transactions in the money markets.

- **Issuer risk** refers also to collateral risk, the risk that the quality of collateral held will suffer because the fortunes of the issuer decline. For this reason many repo market participants will only accept AAA-rated paper such as government bonds as collateral. In the case of equity repo usually only blue-chip shares are accepted, for example the shares of FTSE-100 or S&P 500 companies. The market for lower quality collateral is less liquid, and the equivalent “GC” rate will be higher than that for government collateral, reflecting the higher risk involved. In the UK the equity repo rate typically trades at around 35–50 basis points over the government rate.
- **Market risk** is the risk of a change in the value of an asset due to moves in market levels/prices. Repo market participants are exposed to movements in interest rates during the course of each transaction. They are also exposed to changes in the value of collateral. One reason for the continuing popularity of the stock lending market is because lenders such as fund managers and insurance companies prefer to lend stock in return for a fixed fee rather than engage in repo, which requires a dealing desk and interest rate management and exposes the lender to market risk. Changes in the value of collateral are the main reason for engaging in *margining* in repo transactions.
- **Legal risk** refers to the risk incurred in translating legal agreements, or the difficulty in enforcing such agreements in the event of default.

34.7.2 Dealing with risk

The issue of credit risk is still relevant in a secured transaction such as repo. There may be an inclination to attach a lower level of importance compared to an unsecured transaction, but this not correct. A holding of collateral may still create problems in the event of default, as the lender of cash may not have legal title to the collateral, or a sufficient amount of collateral. When a counterparty is in default, the process of liquidation or administration can be a lengthy one; it also may not be possible to enforce ownership through legal title, as other creditors may be “ahead of the queue”. This is an example of *insolvency risk*, as the liquidator may be able to “cherry pick” the assets of the company, which may include the assets held by the repo counterparty.

Ultimately the most effective means of dealing with the various risks is to “know your counterparty”. Some market participants go further, and only engage in repo with firms with whom they would also deal in the unsecured market.

There has been also some progress in reducing the legal risk exposure in repo trading. These include:

- a formal binding legal agreement, of which the PSA/ISMA agreement is the best example. In the UK gilt repo market for example, parties are required to sign this document before being able to commence dealing. A participant has to have the signed agreement in place with each counterparty it deals with;
- the International Stock Lenders Association OSLA agreement;
- standard domestic agreements (such as the *pension livree* in France).

Most markets have provision for a set-off mechanism in the event of default, known as *netting*. This means that:

- all outstanding loans/repos are recalled or repurchased;
- each party’s obligations to the other are valued and converted into a monetary amount;
- the resulting cash sums are set off and only the net balance is payable by the party owing the greater amount.

The common response to exposure to market risk is to adopt the practice of *margining*. In order for margin calls to take place, market participants have to engage in *marking-to-market* of all their positions. A bond would be marked at its current market price at the close of business. Where the value has fallen by a pre-determined amount, the lender of cash will ask for margin. This is not a water-tight arrangement in the event of default. The cash lender may still find herself short of a sufficient of collateral, due to the following:

- there may have been adverse market movements between the last margin call and payment;
- there may be an element of *concentration risk*, associated with illiquid issues, where the lender is holding a high proportion of bonds from the same issuer, which then become illiquid and difficult to realise.

A solution to this potential problem is to engage in daily margin calls.

34.8 Legal issues

34.8.1 PSA/ISMA Agreement

The Public Securities Association (PSA; recently renamed the Bond Market Association) is a US-based body that originally developed the market standard documentation for repo in the US domestic market, introduced in February 1986. It developed in conjunction with the International Securities Market Association (ISMA) the Global Master Repurchase Agreement. This is the market standard repo document used as the legal basis for repo in non-US dollar markets, introduced in November 1992. It was updated three years later to include UK gilts, buy-sell transactions and relevant agency annexes.

The agreement covers transactions between parties including repo, buy/sell back and agency trades, and has adapted for securities paying net, as well as for equities.

The key features of the agreement are that:

- repo trades are structured as outright sales and repurchases;
- full ownership is conferred of securities transferred;
- there is an obligation to return “equivalent” securities;
- there is provision for initial and variation margin;
- coupon is paid over to the repo seller at the time of payment;
- legal title to collateral is confirmed in the event of default.

The main advantages of the agreement are (i) its allowance for close-out and netting are capital efficient for CAD purposes, (ii) specifying action in the event of default, (iii) its rules on margining.

34.8.2 Gilt Repo Legal Agreement

The Gilt Repo Legal Agreement is an amended version of the revised (November 1995) PSA/ISMA agreement for the UK gilt repo market. The PSA/ISMA agreement was extended by supplemental terms and conditions for gilt repo forming Part 2 to Annex I of the PSA/ISMA agreement and modified by a side letter in connection with the upgrade to the Central Gilts Office service in November 1997. Participants in the gilt repo market are strongly recommended to adopt the Gilt Repo Legal Agreement for gilt repo transactions, as set out in the Gilt Repo Code of Best Practice. The Code was issued by the Bank of England. Use of the legal agreement is subject to legal confirmation of its effectiveness, if the specific circumstances in which it is to be used are not straightforward. The agreement is recommended as the umbrella documentation for all types of repo, including buy/sell back.

The agreement provides for the following:

- the absolute transfer of title to securities;
- daily marking-to-market;
- appropriate initial margin and for maintenance of margin whenever the mark-to-market reveals a material change of value;
- clear events of default and the consequential rights and obligations of the counterparties;
- in the event of default, full set-off of claims between counterparties;
- clarification of rights of parties regarding substitution of collateral and the treatment of coupon payments;
- terms subject to English law.

These are essentially the provisions as contained in the PSA/ISMA agreement.

34.9 Accounting, Tax and capital issues

34.9.1 Accounting

The accounting treatment of repos reflects the commercial substance of the transaction, which is as a *secured loan*. For tax purposes the transfer of securities at the start of the trade does not count as a disposal, which tallies with the accounting treatment. However tax treatment is different for the income in a repo. As a collateralised financing transaction, repos are on-balance sheet transactions. In a repo transaction for the seller, bonds given as collateral remain on the balance sheet of the seller. The corresponding liability is the repo cash. Coupon continues to accrue to the seller. The opposite is the case for the buyer. As an accounting entry a repo appears as a secured loan and not an actual sell transaction.

With regard to the profit & loss account, the repo interest (repo return) is treated as the payment of interest and is taken as a charge on an accruals basis, that is, it is entered in the books at the time of the transaction.

34.9.2 Taxation

The tax treatment of repo differs in each jurisdiction. In the UK the return on the cash leg of a repo is treated as interest and is taxed as income. Coupon payments during the term of a repo is treated for tax purposes as being to the benefit of the counterparty, the taxable date of which is taken to be the dividend date. When trading in overseas markets where the tax treatment is uncertain, institutions must investigate the principal tax issues, both from the point of view of repo seller and buyer.

- **Repo seller:** For the seller, the principal issue is whether the “sale” of securities will trigger a taxable event and/or result in transfer taxes. The type of institution engaged in the trade may affect the resulting tax treatment, as it may be taxable or tax-exempt, as will the firm’s country of residence, as there may be a form of double taxation treaty. For a cross-border trade the accounting treatment in both countries must be taken into consideration, and whether the repo is treated as a sale or not. The basic accounting treatment is that a repo is not a sale, however this is not the case in all countries.
- **Repo buyer:** For the buyer, the principal issue to ascertain is whether the “purchase” of securities will result in transfer taxes.

A taxation issue may arise whenever a coupon payment is made during the term of a repo. If a repo runs over a record date (coupon date) a “manufactured dividend” will arise. In the UK this is treated as income accruing to the beneficial owner of the securities, and therefore not the income of the cash lender. For this reason the dividend is

deductible for the cash lender. There is an additional issue if the coupon is actually paid net of any withholding taxes or gross. The coupon on government bonds is usually paid gross, the major exception is Japanese government bonds which pay coupon net. The UK only recently introduced gross payment of coupon (previously foreign-domiciled investors had to register to receive gross coupons). However holders are still liable to tax which is paid in the normal course of tax assessment.

34.9.3 Capital treatment

The Bank for International Settlements (BIS) originally introduced a standard for capital adequacy in July 1988. This was known as the Basle Capital Accord or Capital Adequacy Directive (CAD) and specified a relatively uniform system for defining exactly how much capital a bank required. The rules came into effect at the end of 1992. They set a minimum ratio of capital to *weighted risk assets* of 8%. Each asset on the bank's balance sheet is assigned a weighting, which can be from 0% for assets considered riskless, to 100% for the most risky assets. For example, most interbank deposits are given a 20% weighting, while most bank lending receives the full 100% weighting. A £100 million corporate loan would therefore consume £8 million of the bank's capital (that is, the bank would have to set aside £8m against the loan), while a £100 million interbank deposit would require a £1.6 million allocation of bank capital. The risk weighting applied varies with the type of counterparty; broadly speaking the weightings are:

0%	Cash, Zone A Central Governments/Banks
20%	EIB, Zone B Credit Institutions
50%	Fully secured loans
100%	Zone A or Zone B non-bank sectors (for example, corporates).

An institution's capital charge calculation is therefore:

$$\text{principal value} \times \text{risk weighting} \times \text{capital charge [8\%]}.$$

The sum of the exposures is taken. Firms may use netting or portfolio modelling to reduce the total principal value.

The capital requirements for off-balance sheet instruments are lower because for these instruments the *principal* is rarely at risk. Interest-rate derivatives such as Forward Rate Agreements (FRAs) of less than one year's maturity have no capital requirement at all, while a long term currency swap requires capital of between 0.08% and 0.2% of the nominal principal.

The BIS makes a distinction between *banking book* transactions as carried out by retail and commercial banks (primarily deposits and lending) and *trading book* transactions as carried out by investment banks and securities houses. Capital treatment sometimes differs between banking and trading books. A repo transaction attracts a charge on the *trading book*. The formula for calculating the capital allocation is:

$$CA = \max(((C_{mv} - S_{mv}) \times 8\% \times RW), 0) \quad (34.1)$$

where

C_{mv}	is the value of cash proceeds
S_{mv}	is the market value of securities
RW	the counterparty risk weighting (as percentage)

EXAMPLE 34.7

- The CAD charge for a repo transaction with the following terms:

Clean price of collateral	100
Accrued interest	0
Cash proceeds on £50 mn nominal	£50,000,000
Counterparty	OECD bank
Counterparty risk weighting	20%

$$\begin{aligned} CA &= (((50,000,000 - 50,000,000) \times 8\% \times 20\%), 0) \\ &= 0. \end{aligned}$$

The CAD charge for a loan/deposit transaction of the same size is as follows:

Unsecured loan	£50,000,000
Counterparty	OECD bank
Counterparty risk weighting	20%

$$\begin{aligned} \text{CA} &= \max((50,000,000 \times 8\% \times 20\%), 0) \\ &= £800,000. \end{aligned}$$

Repo conducted under legal documentation, such as the PSA/ISMA agreement, are given favourable treatment for CAD purposes compared with undocumented repo. Buy/sell backs attract the full charge based on the counterparty risk weighting. Documented repos attract a capital charge at the counterparty risk weighting based on the mark-to-market value of all positions between the parties.

34.10 Market participants

The structure of the repo market in any one country will determine the exact nature of the participants involved. In a developed market interested parties will include investors and cash-rich institutions, those seeking to finance asset positions and their intermediaries. Some firms will cross over these broad boundaries and engage all aspects of repo trading. The main market parties are listed below.

- **Financial institutions:** retail and commercial banks, building societies, securities houses and investment banks
- **Investors:** fund managers, insurance companies and pension funds, investment funds, hedge funds, local authorities and corporate treasuries
- **Intermediaries:** Inter-dealer brokers and money brokers

Central banks are major players in repo markets and use repo as part of daily liquidity operations and as a tool of monetary policy.

34.11 The United Kingdom gilt repo market

34.11.1 Introduction and background

Trading in UK gilt repo market began on 2 January 1996. Prior to this stock borrowing and lending in the gilt market was available only to gilt-edged Market Makers (GEMMs), dealing through approved intermediaries, the Stock Exchange Money Brokers (SEMBs). The introduction of Gilt Repo allowed all market participants to borrow and lend gilts. The market reforms also liberalised gilt stock lending by removing the restrictions on who could borrow and lend stock, thus ensuring a “level playing field” between the two types of transaction. The gilt-edged stock lending agreement (GESLA) was also updated to ensure that it dovetailed with the new gilt repo legal agreement; the revised GESLA was issued in December 1995 and repo and stock lending are inter-linked aspects of the new, open market. In the run up to the start of repo trading market practitioners and regulators drew up recommended market practices, set out in the Gilt Repo Code of Best Practice. The associated legal agreement is the PSA/ISMA Global Master Repurchase Agreement, with an addendum to cover special features of gilts such as the use of Delivery by Value (DBV) within the Central Gilts Office settlement mechanism.

The market grew to about £50 billion of repos and stock lending outstanding in the first two months, further growth took it to nearly £95 billion by February 1997, of which £70 billion was in repos. This figure fell to about £75 billion by November 1998, compared with £100 billion for sterling certificates of deposit (CDs). Data collected on turnover in the market suggests that average daily turnover in gilt repo was around £16 billion through 1999.

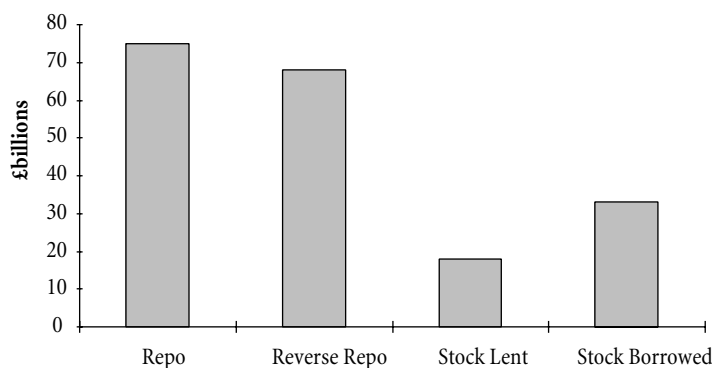


Figure 34.9: Gilt repo market volumes November 1999. Source: BoE.

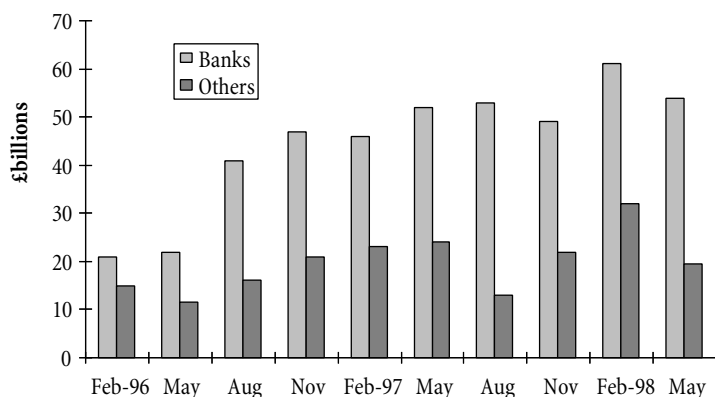


Figure 34.10: Gilt repo market growth, 1996–1998. Source: BoE.

34.11.2 Gilt repo and other sterling money markets

Gilt repo has developed alongside growth in the existing unsecured money markets. There has been a visible shift in short term money market trading patterns from unsecured to secured money. According to the Bank of England (BoE) market participants estimate that gilt repo now accounts for about half of all overnight transactions in the sterling money markets. The repo general collateral (GC) rate tends to trade below the interbank rate, on average about 10–15 basis points below, reflecting its status as government credit. The gap is less obvious at very short maturities, due to the lower value of such credit over the short term and also reflecting the higher demand for short term funding through repo by securities houses that may not have access to unsecured money.

The Certificate of Deposit (CD) market has grown substantially, partly because the growth of the gilt repo and stock lending market has contributed to demand for CDs for use as collateral in stock loans. One effect of gilt repo on the money market is a possible association with a reduction in the volatility of overnight unsecured rates. Fluctuations in the overnight unsecured market have been reduced since the start of an open repo market, although the evidence is not conclusive. This may be due to repo providing an alternative funding method for market participants, which may have reduced pressure on the unsecured market in overnight funds. It may also have enhanced the ability of financial intermediaries to distribute liquidity.

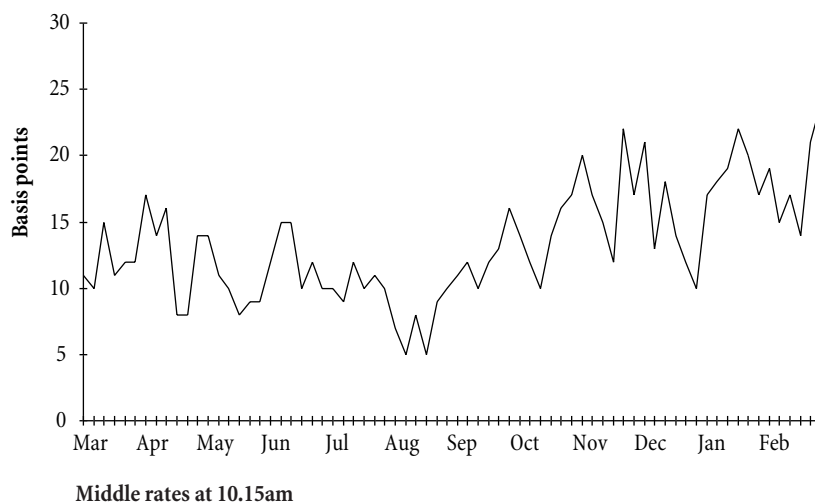


Figure 34.11: Three-month sterling interbank rate minus three-month Gilt Repo GC rate 1997/98. Source: Bank of England, Bloomberg, Reuters.

34.12 Market structure

34.12.1 Repo and stock lending

The UK market structure comprises both gilt repo and gilt stock lending. Some institutions will trade in one activity although of course many firms will engage in both. Although there are institutions which undertake only one type of activity, there are many institutions trading actively in both areas. For example, an institution that is short of a particular gilt may cover its short position (which could result from either an outright sale or a repo) in either the gilt repo or the stock lending market. Certain institutions prefer to use repo because they feel that the value of a *special* stock is more rapidly and more accurately reflected in the repo than the stock lending market. Some firms have preferred to remain in stock lending because their existing systems and control procedures can accommodate stock lending more readily than repo. For example, a firm may have no cash reinvestment facility or experience of managing interest rate risk. Such a firm will prefer to receive collateral against a stock loan for a fee, rather than interest bearing cash in a repo. They may also feel that their business does not need or cannot justify the costs of setting up a repo trading facility. In addition the stock lending has benefited from securities houses and banks who trade in both it and repo; for example, borrowing a stock in the lending market, repoing this and then investing the cash in say, the CD market. Other firms have embraced repo due, for instance to the perception that value from tight stock is more readily obtained in the repo market than in the lending market.

34.12.2 Market participants

Virtually from the start of the market some firms have provided what is in effect a market making function in gilt repo. Typical of these are the former stock exchange money brokers (SEMBs) and banks that run large matched books. According to the Bank of England during 1999 there were approximately 20 firms, mostly banks and securities houses, which quoted two-way repo rates on request, for GC, specifics and specials, up to three months. Longer maturities are also readily quoted. Examples of market making firms include former SEMBs such as Gerrard & National, Lazards, Cater Allen (part of the Abbey National group) and Rowe & Pitman (part of the UBS group), and banks such as Greenwich NatWest (part of the NatWest group), Deutsche Bank and Barclays Capital. Some firms will quote only to established customers, such as ABN Amro Securities Limited. Many of the market making firms quote indicative repo rates on screen services such as Reuters and Bloomberg.

A number of sterling broking houses are active in gilt repo. Counterparties still require signed legal documentation to be in place with each other, along with credit lines, before trading can take place, which is not the case in the interbank broking market. A gilt repo agreement is not required with the broker, although of course firms will have counterparty agreements in place with them. Typical of the firms providing broking services are Garban ICAP,

Tullet & Tokyo, RP Martin's and King & Shaxson Bond Brokers Limited. Brokers tend to specialise in different aspects of the gilt market. For example some concentrate on GC repo, and others on *specials* and *specifics*; some on very short maturity transactions and others on longer term trades. Brokerage is usually 1 basis point of the total nominal amount of the bond transferred for GC, and 2 basis points for specific and special repo. Brokerage is paid by both sides to a gilt repo.⁴

The range of participants has grown as the market has expanded. The overall client base now includes banks, building societies, overseas banks and securities houses, hedge funds, fund managers (such as Standard Life, Scottish Amicable, and so on), insurance companies and overseas central banks. Certain corporates have also begun to undertake gilt repo transactions. The slow start in the use of tri-party repo in the UK market has probably constrained certain corporates and smaller financial institutions from entering the market. Tri-party repo would be attractive to such institutions because of the lower administrative burden of having an external custodian. The largest users of gilt repo will remain banks and building societies, who are required to hold gilts as part of their BoE liquidity requirements.

34.13 Trading patterns

34.13.1 The influence of the yield curve

Generally repo trading has been found to be more active when the yield curve is positively sloped, with overnight GC trading at lower rates than 1–2 weeks up to 1 month. This allows the repo trader to enjoy positive funding by borrowing cash overnight on repo while lending funds in the 1 week or 1 month. The trader is of course exposed to unexpected upward movements in overnight rates while covering his positions.

The short-term money market curve acts occasionally acts independently of the long-dated cash gilt curve, especially with regard to long gilt yields, and may move independently of its movements.

EXAMPLE 34.8

- A good illustration of the low correlation in movement between the short term money market yield curve and the gilt yield curve took place in summer 1997. Following the granting of interest rate setting responsibility to the Bank of England, the gilt yield curve changed from a positive to a negative (inverted) curve; the Treasury 8% 2021 was yielding 6.54% in September 1997 and by December was yielding 6.37%. (The yield as at September 1998 was 5.15%). Some firms' view was that the short term curve would behave in the same manner. One market participant bid for 1 year GC at 7.18% at that time (September 1997), but by December 1997 this rate was 7.58%! An expensive trade...

In this case the gilt yield curve had behaved as the trader had expected but the short-term money market curve had not. Note that to "bid" is to bid for stock, that is, lend the cash.

34.13.2 Hedging through repo

Hedging positions in other markets is one of the main motives for some participant's involvement in gilt repo. This is evident in the sterling bond market, where underwriters have benefited from the ability to hedge the interest rate risk on their (long) underwriting positions, by taking an offsetting short position in the gilt they are using to price their bond. This improves the quality of their interest rate hedge. The underwriter uses (reverse) repo to cover the short position in the gilt. Previously underwriters that weren't GEMMs would have used less exact hedges such as the long gilt future. The introduction of open repo has therefore benefited the sterling bond market.

Gilt repo activity is concentrated at the very short end of the yield curve, with around 90% of trading at overnight to one week maturity. This is longer than stock lending, which generally undertaken overnight or on call. As liquidity has improved the volume, if not the proportion of longer maturity trades has increased. Trades of up to three months' maturity are common, and three month repo rates are routinely quoted with a spread of around 5 basis points. Trades of up to 6 months are also not unusual.

⁴ As a friend of the author's in the gilt repo market once said, if you see your broker cruising down Bishopsgate in a Rolls-Royce, it's time to negotiate a discount on brokerage...

34.13.3 Specials trading

The emergence of *specials* trading is a natural part of a repo market. One purpose in introducing a repo market was to allow the demand to borrow and lend stocks to be cleared by the price mechanism. Hence it is natural that when stocks are in demand, for example because firms wish to cover underwriting positions, the premium on obtaining them rises. Figure 34.12 shows the extent of specials premium for three gilt stocks at the start of 1997.

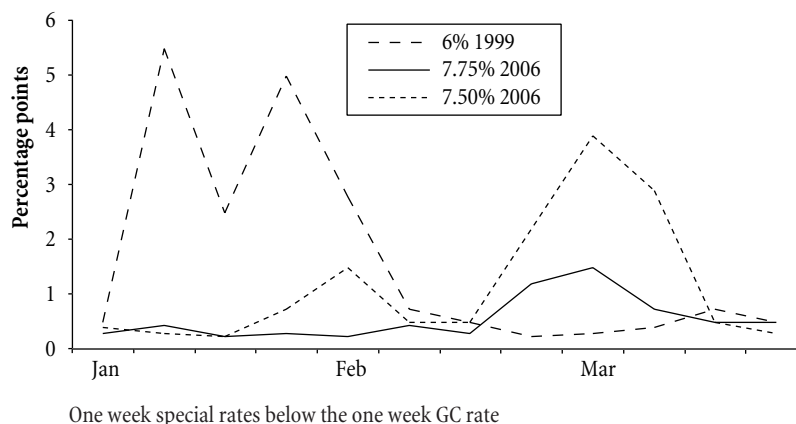


Figure 34.12: Gilt special rates in early 1997.

Source: Bank of England.

EXAMPLE 34.9

- On rare occasions the Bank of England will intervene to relieve excess demand for a specific stock:

In November 1996 there was an increase in demand for 7.75% 2006, mainly because investors switched out of a similar maturity stock as it approached its ex-dividend period. Market makers sold the stock to meet this demand and then simultaneously covered their shorts in the repo market. This sudden demand coupled with the fact that a large amount of this issue was held by investors that did not lend it out led to the special rate trading at close to 0%. The Bank issued £100 million of the stock to help relieve demand.

For delivery into the March 1998 LIFFE long gilt futures contract, the BoE made supplies of the cheapest to deliver stock (Treasury 9% 2008) available if required, albeit at a rate of 0%!

The Bank of England has conducted a study into the relationship between cash prices and repo rates for stocks that have traded special (BoE, *Quarterly Bulletin*, February 1998). This showed a positive correlation between changes in a stock trading expensive to the yield curve and changes in the degree to which it trades special. Theory would predict this: traders maintain short positions for paper which has high associated funding costs only if the anticipated fall in the price of the paper is large enough to give a profit. (One implication of this is that longer duration stocks should be less expensive for a given specials premium, because their prices are more sensitive to changes in yields and therefore a given rise in yields will give a trader running a short position a higher profit to offset any increase in the cost of the repo.)

Both types of cause and effect can be explained:

- the stock may be perceived as expensive, for example after an *auction* announcement. This creates a greater demand for short positions, and hence greater demand for the paper in repo (to cover short positions).
- at other times stock might go tight in the repo market. It would then tend to be bid higher in the *cash* market as traders closed out existing shorts, which were now too expensive to run; another reason would be that traders and *investors* would try to buy the stock outright since it would now be cheap to finance by repoing out.

The Bank of England report has suggested that the link between dearness in the cash market and specialness in the repo market flows both ways: in some cases changes in dearness have preceded changes in specialness and in other cases the sequence has been the other way round. In both cases the stock remains expensive until existing holders take profits by selling their stock or making it available for repo or lending.

34.14 Open market operations

The Bank of England (BoE) introduced gilt repo into its open market operations in April 1997. The Bank aims to meet the banking system's liquidity needs each day via its open market operations. Almost invariably the market's position is one of a shortage of liquidity, which the Bank generally relieves via open market operations conducted at a fixed official interest rate. The Bank's repo operation in this case is actually a reverse repo. The Bank will reverse in gilts and eligible Bills. The reason central banks choose repo as the money market instrument to relieve shortages is because it provides a combination of security (government debt as collateral) and liquidity to trade in large size.

The average daily shortage of liquidity in the sterling market in September 1999 was £700 million.⁵ Table 34.2 shows what the average daily shortage has been since 1996.

	Average daily shortage, £m
1996	900
1997	1200
1998	1400
Q1 1999	1700
Q2 1999	1200
July 1999	1200
August 1999	1000
September 1999	700

Table 34.2: Average daily shortage, sterling market.

Source: BoE.

Figure 34.13 shows how the Bank's daily refinancing was provided during the third quarter of 1999; about 70% was by repo of gilts and eligible bills. The stock of eligible bank bills (bills that may be sold to the BoE as part of its daily operations) was roughly unchanged during 1999, at £21 billion. Therefore the introduction of gilt repo to the daily operation has been a significant element in the market being able to deal with the larger daily shortages.

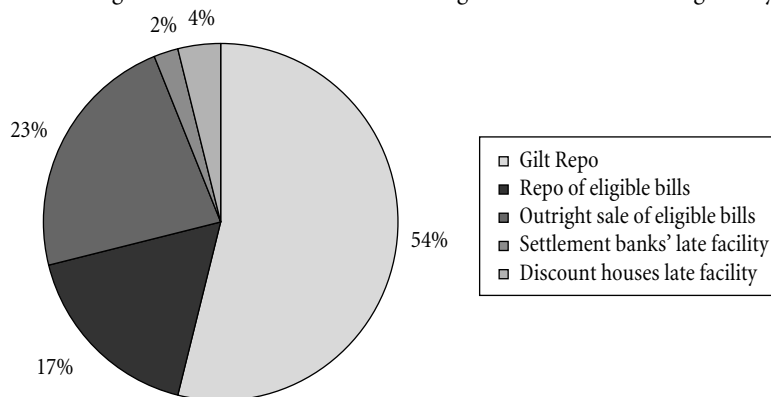


Figure 34.13: Open market operations, instruments used. Source: BoE.

The BoE deals with a wide range of financial institutions active in the gilt repo and/or bill markets, all of whom must satisfy a number of financial criteria, designed to ensure that operations function efficiently and that the

⁵ The BoE announced its estimate of the shortage, which is based on early morning discussion with clearing banks, large building societies, discount houses and certain other banks and securities houses, at 9.45am each day. If there is a shortage larger than £1 billion, an "early round" is held at which the Bank offers to buy bills or undertake repo operation, which releases funds into the market. Otherwise the Bank will hold a round at mid-day and at 2.30pm. Very rarely, (in the author's experience, on no more than three or four occasions in over five years!) there is a surplus in the market, at which point the Bank sells Treasury bills, thus soaking up liquidity.

liquidity supplied is available to all market participants. Counterparty banks, building societies and securities houses must satisfy the BoE requirements that they:

- have the technical capability to respond quickly and efficiently to the Bank's operations;
- maintain an active presence in the gilt repo and/or bill markets, thus contributing to the distribution of liquidity;
- participate regularly in the Bank's operations;
- provide the Bank with useful information on market conditions and movements.

There is no formal underwriting commitment, but the BoE monitors compliance with its functional requirements and reserves the right to cease dealing with any counterparty at its own discretion.

34.15 Gilts settlement and the CREST service

CREST is the London market computerised settlement system, initially implemented to settle exchange-traded equities but since, following the merger of CREST with the Bank of England's Central Gilts Office (CGO) in June 1999, now also the gilts settlement system as well. It is run by CRESTCo, which is an independent body. The original CGO service was established in 1986 by the BoE and the London Stock Exchange to facilitate the settlement of specified securities, essentially gilts and certain sterling bonds such as Bulldogs for which the BoE acts as registrar, and was upgraded by the BoE in 1997. In particular the service was upgraded to enhance gilt repo trading activity, which commenced in January 1996, and to cater for the introduction of the gilt Strips facility in December 1997. It also provides a vehicle for the development of real-time Delivery versus Payment (DVP) through links to the Real Time Gross Settlement System (RTGS) for wholesale payments, which was introduced in mid-1996.

The basic concept of the gilts settlement facility at CREST remains the same, that is the provision of secure settlement for gilt-edged securities through an efficient and reliable system of electronic book entry transfers in real time against an assured payment. For gilts CREST is a real-time, communication-based system. Settlement on the specified business day (T+1 for normal gilt trades) is dependent on the matching by CREST of correctly input and authenticated instructions by both of the parties and the successful completion of pre-settlement checks on the parties' stock account balances and credit headroom.

The service upgrade to CGO during 1995 provided additional features including:

- greater control by settlement banks over the credit risks run on their customers (by means of a debit-capped payment mechanism);
- the movement of stock free of payment;
- matching of instructions between counterparties;
- a flexible membership structure (allowing the names of "sponsored" as well as "direct" members to appear on the register;
- multiple account designations.

Banks and securities houses must have an account at CREST in order to settle via the system; due to the high charges many banks opt for an agent settlement bank to handle their transactions. It is also possible to settle gilts through Euroclear and Cedel, both of which have accounts at CREST.

34.15.1 CREST reference prices

After a repo trade has been agreed, the back offices of both parties will often use the CREST reference price as the basis for settlement proceeds and other calculations.

The CREST system uses data supplied by GEMMs for the calculation of CREST reference prices. The reference prices for conventional CREST stocks are based on the clean mid-market closing (normally 4.15pm) prices supplied by members of the GEMM Association. These mid-prices are then adjusted to include accrued interest and quoted to five decimal places, expressed in £100 nominal of stock. Reference prices are updated daily. Gilt strips trade on a yield basis. The reference price for strips is calculated from gross redemption yields using an "actual/actual" formula, that is, compound interest for all strips divided by the actual number of days in the coupon period.

Delivery-by-Value (DBV) is a mechanism whereby a CREST member may borrow money from or lend money to another CREST member against overnight gilt collateral. The CREST system automatically selects and delivers securities to a specified aggregate value on the basis of the previous night's CREST reference prices; equivalent securities

are returned the following business day. The DBV functionality allows the giver and taker of collateral to specify the classes of security to be included within the DBV. The options are: all classes of security held within CREST, including strips and bulldogs; coupon bearing gilts and bulldogs; coupon bearing gilts and strips; only coupon bearing gilts.

DBV repo is a repo transaction in which the delivery of the securities is by the DBV mechanism in CGO; a series of DBV repos may be constructed to form an “open” or “term” DBV repo. The DBV functionality allows repo interest to be automatically calculated and paid.

34.16 Gilt repo Code of Best Practice

The Gilt Repo Code of Best Practice sets out standards of best practice for gilt repo. It was introduced by the BoE in November 1995 ahead of the commencement of gilt repo trading in January 1996. The Code is set out in various sections, which we summarise below.

Preliminary issues

Market participants should ensure that they have adequate systems and controls for the business they intend to undertake. This includes internal controls, credit risk control systems, written procedures and systems for accounting and taxation.

Market professionals

Before dealing with a client for the first time, market professionals should either confirm that the client is already aware of the Code or draw it to the client’s attention.

Legal agreement

Gilt repo transactions should be subject to a legal agreement between the two parties concerned. A market standard is the Gilt Repo Legal Agreement, and participants to gilt repo are strongly recommended to adopt this. This agreement is based on the PSA/ISMA Global Master Repurchase Agreement (section 8.1).

Margin

Participants in gilt repo should negotiate suitable initial margin reflecting both their assessment of their counterparty’s creditworthiness and the market risks (for example, duration of collateral) involved in the transaction. Participants should also monitor their net exposure to all counterparties on a daily basis.

Custody

Clients need to ensure that stock loan and repo transactions are identified accordingly to their custodian.

Default and close-out

Once the decision to default has been taken it is important that the process be carried out carefully. This includes the non-defaulting party doing everything in its power to ensure that default market values used in the close-out calculations are fair.

34.17 Trading approach

The repo desk will have a central role on the trading floor, supporting the fixed interest sales desk, hedging new issues, and working with the swaps and OTC options desks. In some banks and securities houses it will be placed within the Treasury or money markets areas, whereas other firms will organise the repo desk as part of the bond operation. It is also not unusual to see equity repo carried out in a different area from bond repo.

34.17.1 Yield curve environment

When the yield curve is positively sloped, the conventional approach is to fund at the short end and lend at the long end of the curve. In essence therefore a bank would borrow say one-week funds while simultaneously lending out at two-week or one-month. This is known as *funding short* and is illustrated with Figure 34.14.

A bank can effect the economic equivalent of borrowing at the short end of the yield curve and lending at the longer end through repo transactions, in our example a 1-week repo and a 6-month reverse repo. The bank then continuously rolls over its funding at one week intervals for the six month period. This is also known as *creating a tail*; here the “tail” is the gap between one week and six months, the interest rate “gap” that the bank is exposed to.

During the course of the trade, as the reverse repo has locked in a loan for six months, the bank is exposed to interest rate risk should the slope or shape of the yield curve change. In this case if short rates rise, the bank may see its profit margin shrink or turn into a funding loss.

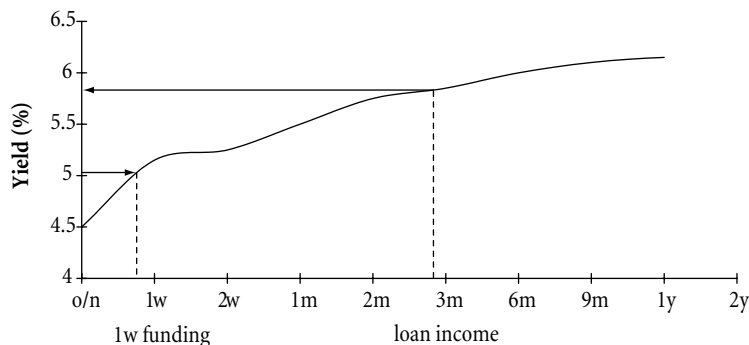


Figure 34.14: Positive yield curve – funding short.

In the case of an inverted yield curve, a bank will (all else being equal) lend at the short end of the curve and borrow at the longer end. This is known as *funding long* and is shown in Figure 34.15.

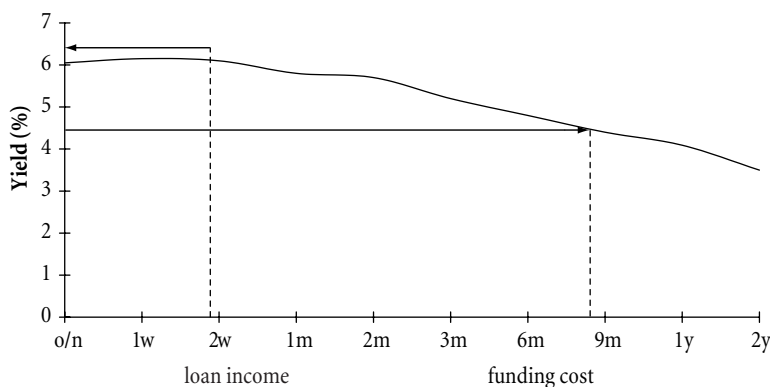


Figure 34.15: Funding long.

The example in Figure 34.15 shows a short cash position of two-week maturity against a long cash position of six-month maturity. The interest rate *gap* of 22 weeks is the book's interest rate exposure.

34.17.2 Yield curve arbitrage

This is a first principles-type of *relative value* trading common on fixed interest desks. If a trader believes that the shape of the yield curve is going to change, thus altering the *yield spread* between two bonds of differing maturities, she can position the book to benefit from such a move. A yield spread arbitrage trade is not market directional, that is, it is not necessarily dependent on the direction that market moves in, but rather the move in the shape of the yield curve. As long as the trade is *duration weighted* there is no first-order risk involved, although there is second-order risk in that if the shape of the yield curve changes in the opposite direction the trade will suffer a loss.

Consider the yield spread between 2-year and 5-year bonds; the trader believes that this spread will widen in the near future. The trade therefore looks like this:

- buy £x million of the 2-year bond;
- sell £y million of the 5-year bond, and borrow in the repo market.

The nominal amount of the 5-year bond will be a ratio of the respective *basis point values* multiplied by the amount of the 2-year bond. The trader will arrange the repo transaction simultaneously (or instruct the repo desk to do so). The funding for both bonds forms an important part of the rationale for the trade. As repo rates can be fixed

for the anticipated term of the trade, the trader will know the net funding cost – irrespective of any change in market levels or yield spreads – and this cost is the break-even cost for the trade. A disciplined trader will have a time horizon for the trade, and the trade will be reviewed if the desired spread increase has not occurred by the expected time. In the case of the repo however, the trader may wish to fix this at a shorter interval than the initial time horizon, and roll over as necessary.

Figure 34.16 illustrates the yield curve considerations involved.

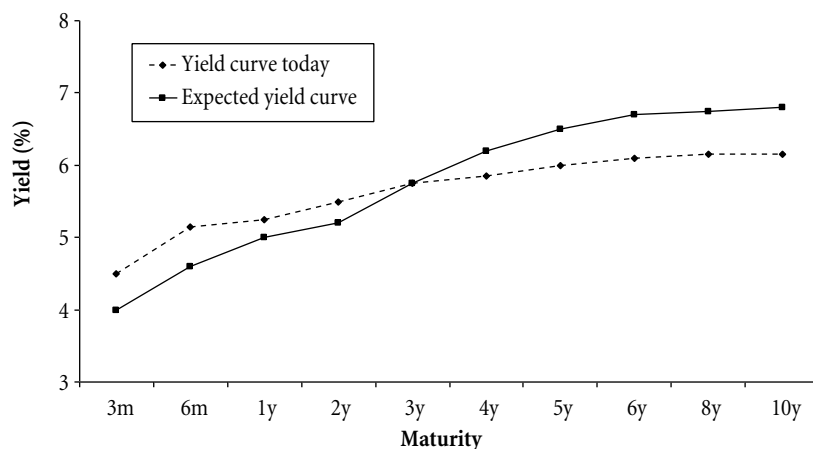


Figure 34.16: Bond spread trade.

The increase in the two-year versus five-year spread is the profit made from the trade, minus the net funding. The trader must check with the repo desk what the funding cost will be over the expected time horizon of the trade, and whether either of the two stocks are trading special in the repo market. The net funding cost is the trader's break-even for the trade strategy. An example of a spreadsheet used to calculate the breakeven funding level is shown in Example 34.10.

EXAMPLE 34.10 Spread trade funding calculator

The spreadsheet shown at Figure 34.17, below, calculates the net funding cost associated with running a spread position of two bonds, and was written by Dr Didier Joannas for ABN Amro Hoare Govett Sterling Bonds Limited. It comprises an Excel front-end and Visual Basic program. The bonds shown in the spreadsheet are gilts. The user selects which bonds to enter into the spreadsheet; the first half shows benchmark bonds, which represent the short position of a trade, and the bond that is taken on as the long position is entered into a cell in the lower half. The specific repo rate that applies to each bond is also entered, as is the term of the trade. The calculator then works out the net funding, which if it is a loss is the traders break-even cost for the trade. Any combination of bonds can be selected. The spreadsheet also calculates each bond's basis point value (bpv) and bpv ratio, because of course this is used to determine the nominal value of the bond positions. A spread trade is first-order risk neutral because both the long and short positions are duration-weighted. Note that a trade made up of a long position in the 8.50% 2005, against a short position in the 8% 2021, makes a 1.29 basis point gain in funding over a 30 day period, so that if the yield curve stays unchanged during that time, which admittedly is unlikely, the trade would still make a net gain for the book, composed of the funding profit. The funding of bond trades is a vital ingredient in devising trade strategy.

Yield Spread										18Feb00 21Feb00		Settlement							
UK gilt										Yield	bpv					Basis points			
																funding		TERM	
SHORT								3	5	10	20	in pence	Repo Rate	23Feb00	2	Days			
2	103.24	103.75	7.00	07-Jun-02	5.234	2.08	2	40	1	-1	-50	0.09	5	103.740	0.04	98			
3	107.50	107	8.00	10-Jun-03	5.638	2.86	3	-39		-42	-91	-0.18	5.9	106.991	-0.06	101			
5	114.66	116.0625	8.50	07-Dec-05	5.244	4.59		5		-2	-51	-0.17	5.5	116.051	-0.04	98			
10	104.30	104	5.75	07-Dec-09	5.221	7.39				10	-49	-0.18	5.5	104.000	-0.02	98			
20	141.82	143.5625	8.00	07-Jun-21	4.730	11.86						-0.72	6	143.566	-0.06	98			
30	120.60	120.03125	6.00	07-Dec-28	4.720	17.92						0.11	4.5	120.028	0.01	98			
														funding forward					
LONG						Cpn	Maturity	Yield	bpv	Benchmark	Spread	bpv ratio	gain	spread	Repo Rate	23Feb00	2	Days	
UK	114.66	116.0625	8.5	07-Dec-05	5.24389915	5.41	20			51.39	0.45631285	0.14	51.54	0.44	4.5	116.045	0.08	98	
UK	141.82	143.5625	8	07-Jun-21	4.72997011	17.23	30			1.04	0.96180726	-0.04	0.99	-0.6490288	5.5	143.562	-0.04	98	
UK	103.24	103.75	7	07-Jun-02	5.2336894	2.19	10			1.28	0.2958155	-0.06	1.22	-0.1901567	5.5	103.743	-0.09	98	

The trade described in the previous section is an example of relative value trading. There are many variations on this, including trades spanning different currencies and markets.

EXAMPLE 34.11 Cross-market spread trade

- The spread between 10-year UK gilts and 10-year German bunds has narrowed from a high of 160 basis points six months ago to a level of 91 basis points today. A trader feels that this spread will widen out again over the next three weeks. She therefore sells the gilt and buys the bund in anticipation of this move. Both trades are funded/covered in the respective repo markets. This trade also requires the trader to have a view on the sterling/deutschmark exchange rate, as any profit from the trade could be reduced or eliminated by adverse movements in the exchange rate. The graph in Figure 34.18 illustrates the starting point for the trade.

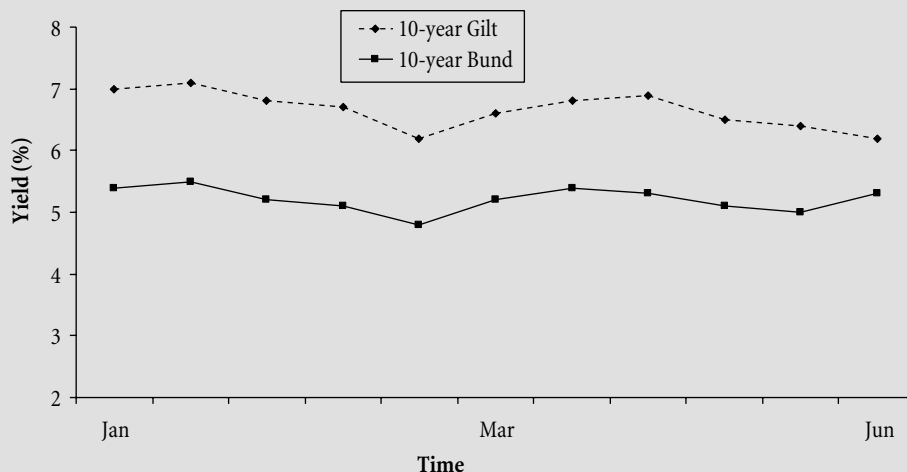


Figure 34.18: Cross-market spread trade.

34.17.3 Specials trading

The existence of a repo market allows the demand to borrow and lend stocks to be cleared by the price mechanism (and more efficiently than in traditional stock lending). It is to be expected that when specific stocks are in demand, for one of a number of reasons, the premium on obtaining them rises. Factors contributing to demand for *specials* include the following:

- government bond auctions – the bond to be issued is shorted by market makers in anticipation of new supply of stock and due to client demand;
- outright short selling, whether deliberate position taking on the trader's view, or market makers selling stock on client demand;
- hedging, including bond underwriters who will short the benchmark government bond that the corporate bond is priced against;
- derivatives trading such as basis ("cash-and-carry") trading creating demand for a specific stock.

Natural holders of government bonds can benefit from issues *going special*, which is when the demand for specific stocks is such that the rate for borrowing them is reduced. The lower repo rate reflects the premium for borrowing the stock. Note that the party borrowing the special stock is lending cash; it is the rate payable on the cash that he has lent that is depressed. The holder of a stock that has gone special can obtain cheap funding for the issue itself, by lending it out. Alternatively the holder can lend the stock and obtain cash in exchange in a repo, for which the rate payable is lower than the interbank rate. These funds can then be lent out as either secured funding (in a repo) or as unsecured funding, enabling the specials holder to lock in a profit. For example, a repo dealer holds an issue trading at 5.5% in the 1-week. The equivalent GC rate is 7%. By lending the stock out the dealer can lock in the

profit by on-lending 1-week cash at 7%, or at a higher rate in the interbank market. This is illustrated in Figure 34.19.

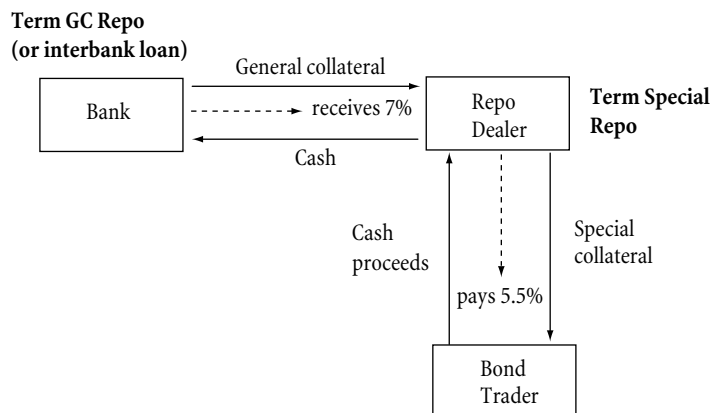


Figure 34.19: Specials trading.

There is a positive correlation between changes in a stock trading expensive to the yield curve and changes in the degree to which it trades special. Theory would predict this, since traders will maintain short positions for bonds with high funding (repo) costs only if the anticipated fall in the price of the bond is large enough to cover this funding premium. When stock is perceived as being expensive, for example after an auction announcement, this creates a demand for short positions and hence greater demand for the paper in repo. At other times the stock may go tight in the repo market, following which it will tend to be bid higher in the *cash* market as traders closed out existing shorts (which had become expensive to finance). At the same time traders and investors may attempt to buy the stock outright since it will now be cheap to finance in repo. The link between dearthness in the cash market and special status in the repo market flows both ways.

34.17.4 Credit intermediation

The government bond market will trade at a lower rate than other money market instruments, reflecting its status as the best credit. This allows the spreads between markets of different credits to be exploited. The following are examples of credit intermediation trades:

- a repo dealer lends general collateral currently trading at Libor-25, and uses the cash to buy CDs trading at Libor-12.5;
- a repo dealer borrows specific collateral in the stock lending market, paying a fee, and on-lends the stock in the repo market at the GC rate; the cash is then lent in the interbank market at a higher rate;
- a repo dealer trades repo in the GC market, and using this cash reverses in emerging market collateral at a spread say, 400 basis points higher.

34.17.5 Matched Book trading

The growth of repo markets has led to repo match book trading desks. Essentially this is market-making in repo; dealers make two-way trading prices in various securities, irrespective of their underlying positions. In fact the term 'matched book' is a misnomer, most matched books are deliberately mismatched as part of a view on the short term yield curve. Traders put on positions to take advantage of (i) short term interest rate movements and (ii) anticipated supply and demand in the underlying stock. Many of the trading ideas and strategies described in this book are example of match book trading.

Matched book trading can involve the following types of trades:

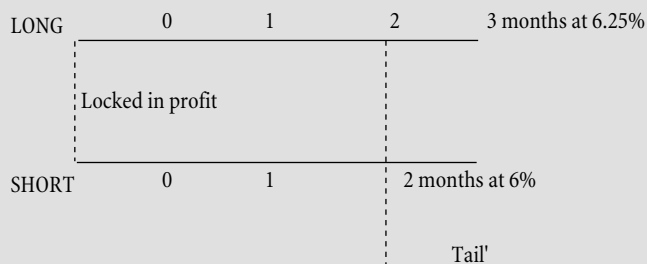
- taking a view on interest rates; for example the dealer bids for 1-month GC and offers 3-month GC, expecting the yield curve to invert;
- taking a view on specials; for example the trader borrows stock in the stock lending market for use in repo once (as she expects) is goes *special*;

- credit intermediation; for example a dealer reverses in Brady bonds from a Latin American bank, at a rate of Libor +200 and offers this stock to a US money market investor at a rate of Libor +20.

Principals and principal intermediaries with large volumes of repos and reverse repos, such as the market makers mentioned above, are said to be running ‘matched books’. An undertaking to provide two-way prices is made to provide customers with a continuous financing service for long and short positions and also as part of proprietary trading. Traders will mismatch positions in order to take advantage of a combination of two factors, which are short term interest rate movements and anticipated supply/demand in the underlying bond.

EXAMPLE 34.12

- ABC Bank reverses in gilts/other collateral for 3 months at 6.25%
- ABC Bank repos out gilts/other collateral for 2 months at 6.00%.



ABC Bank has therefore lent cash for 3 months at 6.25% and simultaneously borrowed cash for 2 months at 6%. The mismatch exposure in 2–3 month period is the tail. In this case the break-even on the tail of the trade is 6.75%. ABC has locked in a 25bp profit for 2 months, therefore it would have to lose 50 bp or more in the last month in order to lose money on the trade. This might happen if interest rates were to rise significantly, forcing the Bank to fund the exposure at a higher rate.

34.17.6 Hedging tools

For dealers who are not looking to trade around term mismatch or other spreads, there is more than one way to hedge the repo trade. The best hedge for any trade is an exact off-setting trade. This is not always possible, nor indeed always desirable as it may reduce profit. However a similar off-setting trade will limit *basis risk*. The residual risk will be that between say GC and special or interest rate gap risk. A forward term interest rate gap exposure can be hedged using interest rate futures. In the sterling market typically the instrument will be the 90-day short sterling future traded on LIFFE. A strip of futures can be used to hedge the term gap. The trader buys futures contracts to the value of the exposure and for the term of the gap. Any change in cash rates should be hedged by offsetting move in futures prices.

Forward rate agreements (FRAs) are similar in concept to interest rate futures and are also off-balance sheet instruments. Under a FRA a buyer agrees notionally to borrow and a seller to lend a specified notional amount at a fixed rate for a specified period, the contract to commence on an agreed date in the future. On this date (the “fixing date”) the actual rate is taken and, according to its position versus the original trade rate, the borrower or lender will receive a an interest payment on the notional sum equal to the difference between the trade rate and the actual rate. The sum paid over is present-valued as it is transferred at the start of the notional loan period, whereas in a cash market trade interest would be handed over at the end of the loan period. As FRAs are off-balance sheet contracts no actual borrowing or lending of cash takes place, hence the use of the term “notional”. In hedging an interest rate gap in the cash period, the trader will buy a FRA contract that equates to the term gap for a nominal amount equal to his exposure in the cash market. Should rates moves against him in the cash market, the gain on the FRA should (in theory) compensate for the loss in the cash trade.

An *interest rate swap* is an off-balance sheet agreement between two parties to make periodic interest payments to the other. Payments are on a predetermined set of dates in the future, based on a notional principal amount; one party is the *fixed rate payer*, the rate agreed at the start of the swap, and the other party is the *floating rate payer*, the floating rate being determined during the life of the swap by reference to a specific market rate or index. There is no exchange of principal, only of the interest payments on this principal amount. Note that our description is for a plain

vanilla swap contract, it is common to have variations on this theme, for instance *floating – floating* swaps where both payments are floating rate, as well as *cross-currency* swaps where there is an exchange of an equal amount of different currencies at the start and end-dates for the swap.

An interest rate swap can be used to hedge the fixed rate risk arising from the purchase of a bond during a repo arbitrage or spread trade. The terms of the swap should match the payment dates and maturity date of the bond. The idea is to match the cash flows from the bond with equal and opposite payments in the swap contract, which will hedge the bond position. For example, if a trader has purchased a bond, he will be receiving fixed-rate coupon payments on the nominal value of the bond. To hedge this position the trader buys a swap contract for the same nominal value in which he will be paying the same fixed rate payment; the net cash flow is a receipt of floating interest rate payments. A bond issuer, on the other hand, may issue bonds of a particular type because of the investor demand for such paper, but prefer to have the interest exposure on his debt in some other form. So for example a UK company issues fixed rate bonds denominated in say, Australian dollars, swaps the proceeds into sterling and pays floating rate interest on the sterling amount. As part of the swap he will be receiving fixed rate Australian dollars which neutralises the exposure arising from the bond issue. At the termination of the swap (which must coincide with the maturity of the bond) the original currency amounts are exchanged back, enabling the issuer to redeem the holders of the bond in Australian dollars.

34.18 Electronic repo trading

Various forms of electronic trading exist in repo markets, ranging from screen-based repo rates to fully automated systems. In this section we describe one system in operation in the UK gilt repo market.

34.18.1 King & Shaxson Bond Brokers Limited

King & Shaxson Bond Brokers Limited (KSBB) is part of Gerrard Group plc. The firm is active in sterling and other markets as cash bond brokers as well as repo brokers. In September 1998 KSBB introduced “GiltKING”, an automated electronic trading system for gilt repo. GiltKING offers the speed and freedom of dealer input whilst retaining the benefits of voice broking, the traditional method employed by brokers in the bond and money markets. Dealers connected to the system can view real-time GC and specific rates. If they wish to trade on a price they activate a “hit” or “lift” on the screen at the touch of a mouse button. The system will prompt the dealer to confirm the trade, after which the trade goes through.

Overnight									
⁷ 62 ₁₇	⁷ 61 ₁₂	⁷ 60 ₅₀	GC	⁷ 60 ₁₂₁	⁷ 55 ₅₀	⁷ 52 ₂₀			
⁷ 60 ₅₀	⁷ 59 ₅₀	⁷ 57 ₅₀	DBV	⁷ 56 ₁₅	⁷ 50 ₉₇	⁷ 48 ₇₅			
		⁷ 52 ₄₅	6 99	⁷ 50 ₅₀					
			8 00	⁷ 45 ₇₆					
	⁷ 51 ₂₅	⁷ 49 _{38A}	7 01	⁷ 46 ₂₅	⁷ 42 ₁₈				
			7 02	⁷ 47 ₆₀					
	⁷ 44 ₁₀	⁷ 41 ₁₅	8 03						
		⁷ 43 ₁₉	6H 03	⁷ 38 ₇₆					
	⁷ 45 ₂₅	⁷ 41 ₂₃	6T 04						
		⁷ 35 ₃₀	8H 05	⁷ 29 ₁₆₀	⁷ 27 ₈₀	⁷ 25 ₂₅			
⁷ 23 ₅₀		⁷ 21 ₂₀	7H 06	⁷ 11 ₃₀					
		⁷ 10 ₇₅	7Q 07						
8H 05									
		⁷ 35 ₃₀	ON	⁷ 29 ₁₆₀	⁷ 27 ₈₀	⁷ 25 ₂₅			
		⁷ 37 ₄₅	TN						
		⁷ 55 ₂₀	1W	⁷ 45 ₁₅					
		⁷ 45 ₂₀	1M						
			2M						
⁷ 42 ₂₅		⁷ 37 ₂₀	3M	⁷ 35 ₂₀					
			6M						
			9M						
			1Y						

Figure 34.20: GiltKING dealing screen. ©KSBB 1998. Used with permission.

The markets available on GiltKING are currently limited to gilt repo, cash gilts and sterling swaps; other markets and currencies are planned. The trading screen displays real-time live prices from the market, input by repo dealers. Prices are displayed anonymously and may be dealt on directly through the screen. A single command button enables the dealer to *refer* all his prices in an instant. The right-hand side of the screen is a dialogue box where dealers input and maintain prices, with additional sections showing trade confirmations and trades history. There is also an “on call” help facility that puts the dealer in touch with the KSBB broking team. Voice broking is still available from KSBB, and the system is initially aimed as a supplement to voice broking.

Figure 34.20 shows an extract from the trading screen on GiltKING, which has been reproduced with permission. The screen displays the overnight rates for gilt GC and specials and the sizes at which the dealer inputting the price is prepared to deal. The bottom half shows term rates for a specific stock, the 8½% Treasury 2005, abbreviated to “8H 05”. Other pages will show the rates for term rates up to 1 year. The page is in colour so that on the trader’s screen the firm’s own prices are in green. Prices are firm (unless indicated as referred) and can be dealt on straight away on screen. Figure 34.21 shows a sample trade history box taken from the right-hand side of the screen. This shows time of trade, stock dealt, term of trade, whether buy or sell, size of bargain and rate dealt.

11:26:36	7H 06	ON	B	30	7.11
11:15:24	8 03	1W	B	15	7.27
10:49:14	8 00	ON	B	76	7.45
09:49:42	6 99	ON	S	56	7.18
09:12:00	DBV	ON	B	50	7.55

Figure 34.21: GiltKING trade history dialogue box.
©KSBB 1998. Used with permission.

34.19 Repo netting

The introduction of netting across repo markets is one of the most significant developments in the repo markets in recent years. The advantages offered by netting should help to make the market even more liquid and accessible than it is already. Although it has been established in the US for some years now, it is still attaining critical mass in Europe, but analysts believe that this is only a matter of time. In this section we summarise the main points concerning netting.

When a bank enters into a number of reverse and reverse repo transactions with the same counterparty, it records a series of assets (the loans, from reverse repos) and liabilities (borrowing of cash, from repo) on its balance sheet. *Netting* is the ability to offset these assets and liabilities in the balance sheet, but across all the counterparties of the bank, via a central clearing system. A pre-requisite for netting is that legal agreements must be in place between all counterparties, and that they should as far as possible try to eliminate any translation risk associated with one of the a liquidation or bankruptcy. There are different legal definitions of netting, which are:

- **payment or settlement netting**, which addresses the settlement risk arising when two or more same currency payment or same securities delivery obligations are due between two parties on the same maturity date and from the same clearing location. This form of netting consolidates the relevant payment or delivery obligations to leave only the net amount payable or deliverable;
- **close-out netting**, which covers two concepts, (i) the right to terminate transactions early and (ii) the right to offset the accelerated obligations, whether they were due and payable or not. In most jurisdictions this is an exception to insolvency laws and has to be implemented by a special law. Generally close-out netting has to be provided for in a contract, unlike payment netting which applies by virtue of law;
- **novation netting**, which is a form of netting principally associated with the involvement of central clearing counterparties. In clearing a repo, a central clearing counterparty (CCC) becomes the counterparty to, and responsible for, the corresponding trade obligations arising from the original bilateral trade. It interposes itself and becomes the buyer to the seller, and the seller to the buyer. By using a CCC, a member of the clearing service is able to replace its existing bank-to-bank bilateral repo relationships (on the settlement side, and regarding lending limits) with multilateral netting. The original bilateral repo trade is still subject to the master repo legal agreement, but at the point of registration or novation the transaction becomes subject to CCC legal documentation.

34.19.1 Netting through central clearing service

In netting through a central clearing facility, all repo transactions between counterparties are assigned or *novated* to the CCC. This then becomes one of the parties to all the transactions. Netting takes place on transactions between two counterparties, and the CCC is always one of the parties to a transaction. The concept is exactly the same as that used in futures exchanges. There are a number of advantages to netting from a bank's point of view; these include:

- **balance sheet reduction:** a party repoing a bond retains the securities on its balance sheet as an asset and includes the obligation to repay the cash advanced by the buyer as a liability. The party reversing the bond will have opposite accounting entries. Repo netting reduces balance sheet usage if the dealer undertakes repo and reverse repo transactions with the same counterparty;
- **freeing up credit lines:** when a CCC registers a repo, it becomes the buyer to the selling bank and the seller to the buying bank. The two banks no longer have any exposure to each other, so the CCC frees up their bilateral credit lines for additional business;
- **centralised margin calculations:** the CCC will mark-to-market all repo transactions each day, and calculate and collect (or pay) all the margin payments. This eliminates the need for banks to bilaterally calculate and transfer margin payments, which should reduce the administrative burden in the bank's operations department;
- **anonymous trading:** although a CCC is not in itself a trading facility, certain CCCs have facilitated the introduction of automated trading systems, which becomes in effect an inter-dealer broker system, and allows for anonymous trading between banks; a centralised clearing system provides the anonymity that is required in an on-line, real-time dealing system
- **netted settlement deliveries:** each day a CCC will net all trades settling that day that are due in a security to or from a counterparty. This reduces the amount of deliveries taking place in depositories, for example if Bank A has sold bonds to Bank B and the same bonds on to Bank C, the process of netting eliminates the need for Bank B to make any deliveries. This should help to reduce the number of failed deliveries in the market;
- **cross-product netting:** one CCC, the "RepoClear" system run by London Clearing House, is planning to provide offsets between a member bank's repos, swaps and financial futures calculating margins; this frees up collateral that may then be used for other purposes, or cash which would then be available to finance other positions;
- **reduction in credit risk:** the amount of bilateral credit lines required in a cleared environment is lower in a netting system;
- **potential brokerage savings:** automated trading of repo transactions may possibly be charged at a lower rate than the current "voice broking" in most markets; however in the US the volume of repo transactions doubled, as they were brokered over the screen and cleared by the central clearing house.

34.19.2 The RepoClear system

A repo netting system is well established in the US market, provided by the Government Securities Clearing Corporation (GSCC). In Europe the first cross-border netting system was introduced by the London Clearing House (LCH), which introduced *RepoClear* during 1999. LCH is the clearing house for the LIFFE futures exchange. Their repo netting system went "live" in August 1999, initially for German government bonds and repo. The LCH has stated an intention to extend the service to other bond and repo markets over the next few years, including Belgian, French, Italian and UK government repo, and euro interest-rate swaps.

The members of RepoClear are banks, investment banks and securities houses. A potential member firm requires approval from LCH as a repo dealer; the main criteria for membership are that the firm must:

- be a dealer in the wholesale market, for example a primary dealer in one or more European government bonds;
- have an investment-grade credit rating, that is triple-B or better;
- be a member of LCH, or have a clearing arrangement with a member of LCH, with a minimum of £250 million cash margin deposited at LCH.

The RepoClear system has trade feeds with external systems, such as the ISMA "TRAX" trade matching system. All members have just one counterparty for repo trades conducted with other members, which is the LCH. RepoClear calculates the net amount of each bond that a member is due to deliver as a result of its trades with member firms, and these are delivered to LCH. If a member of RepoClear defaults, the procedure is that LCH will carry

out that firm's obligations to other members, including covering losses if necessary. This counterparty risk is managed at LCH by through the collection of margin from members each day. The margin comprises initial margin, which may be cash, government securities or letters of credit, variation margin, which requires a marking-to-market of all positions at the end of business each day, and delivery margin, to cover the risk of exposure when conducting next day settlement. The system is very similar to that operated by LCH for exchange-traded futures contracts.

As at October 1999 there were 11 users of the system, including Chase Manhattan Bank, Barclays Capital, CSFB, Morgan Stanley, Goldman Sachs and Warburg Dillon Read.

34.20 The implied repo rate and basis trading

Basis trading, also known as *cash and carry* trading refers to the activity of simultaneously trading cash bonds and the related bond futures contract. An open repo market is essential for the smooth operation of basis trading. Most futures exchanges offer at least one bond futures contract. Major exchanges such as CBOT offer contracts along the entire yield curve; others such as LIFFE provide a market in contracts on bonds denominated in a range of major currencies. The theoretical price of a bond futures contract is calculated from the cost of constructing the hedge for a trader who was short of the futures contract; the hedge is a long position in the underlying bond, and holding this until the futures contract matures, at which point the bond is delivered against it. There is a cost associated with holding the bond, which is the net difference actual repo rate for the term of the trade (the bond is purchased using borrowed funds in the repo market), and the coupon income on the bond itself. In the market, the actual price of the futures contract should match the theoretical price, otherwise there would be an arbitrage opportunity which could be exploited. In practice therefore, we may use the actual current futures price in the market and the current price of the underlying bond, and calculate the repo rate that would ensure a zero net profit from a strategy of selling the future and buying the bond. This interest rate is what is known as the *implied repo rate* and it is one of two ways in which to analyse bond futures and bond prices. The implied repo rate is in effect the break-even rate at which a short futures position can be hedged. It is known as an "implied" rate because the long cash bond position used in the hedge would be financed in the repo market; therefore the break-even rate is the repo borrowing rate that is implied by the current price of the bond future.

In this chapter we review the fundamentals of *basis* trading.

34.20.1 Futures contract definition

Bond futures, like commodity futures are based on the delivery of a tangible asset – a particular bond – at some date in the future. As an example the LIFFE Long Gilt Contract, one of the most active financial futures contracts, is defined in Table 34.3 below.

Unit of trading	UK Gilt bond having a face value of £100,000 and a coupon of 7% and a notional maturity of 10 years (changed from contract value of £50,000 from the September 1998 contract)
Deliverable grades	UK gilts with a maturity ranging from 8¾ to 13 years from the first day of the delivery month (changed from 10–15 years from the December 1998 contract)
Delivery months	March, June, September, December
Delivery date	Any business day during the delivery month
Last trading day	11:00 hours two business days before last business day of delivery month
Quotation	Percent of par expressed as points and hundredths of a point, eg., 114.56 (changed from points and 1/32nds of a point, as in 114-17 meaning 114 17/32, or 114.53125, from the June 1998 contract)
Minimum price movement	0.01 of 1 per cent (one tick)
Tick value	£10
Trading hours	08:00–18:00 hours
	All trading conducted electronically on LIFFE CONNECT™ platform

Table 34.3: LIFFE Long Gilt future contract specifications. Source: LIFFE.

The definition of the gilt contract detailed in Table 34.3 above calls for the delivery of a UK Gilt with an effective maturity of between 8¾ to 13 years and a 7% coupon. Of course there would be problems if the definition of

deliverable bonds were restricted solely to those with a coupon of exactly 7%. At times there may be no bonds having this precise coupon. Where there was one or more such bonds, the size of the futures market in relation to the size of the bond issue would expose the market to price manipulation, as banks sought to corner the market in the underlying bond. To avoid this, futures exchanges design contracts in such a way as to prevent manipulation of the market. In the case of the Long Gilt and most similar contracts this is achieved by allowing the delivery of *any* bond with a sufficient maturity. Of course the holder of a long position in futures would prefer to receive a high-coupon bond with significant accrued interest, while those short of the future would favour delivering a cheaper low-coupon bond shortly after the coupon date. This conflict of interest is resolved by adjusting the *invoice amount*, the amount paid in exchange for the bond on delivery, to account for coupon rate and timing of the bond actually delivered.

Equation (34.2) defines this invoice amount.

$$Inv_{amt} = P_{fut} \times CF + AI \quad (34.2)$$

where

Inv_{amt}	is the invoice amount
P_{fut}	is the futures price
CF	is the conversion factor
AI	is the accrued interest.

Every bond deliverable under a particular futures contract, said to be in the *delivery basket* will have its own *conversion factor* or *price factor*, which is intended to compensate for the coupon and timing differences of deliverable bonds. The exchange publishes tables of conversion factors in advance of a contract starting to trade, and these remain fixed for the life of the contract. The conversion factor will be smaller than 1.0 for deliverable bonds that have a coupon lower than the notional coupon specified in the futures contract terms, (in the case of the long gilt future, that would be less than 7%), and greater than 1.0 otherwise.

A general model for the calculation of the conversion factor is given in Appendix 34.3, equation (34.13).

Conversion factor

- The conversion factor gives the price of an individual cash bond such that its yield to maturity on the delivery day of the futures contract is equal to the notional coupon of the contract. The product of the conversion factor and the futures price is the forward price available in the futures market for that cash bond (plus the cost of funding, referred to as the gross basis). See Appendices 34.2 and 34.3

As an example Table 34.4 lists the sets of conversion factors calculated by LIFFE for gilts deliverable into the December 1998 long gilt futures contract, as listed on page “DLV” of a Bloomberg® terminal on 30 October 1998. The price of the futures contract at the time was 114.55, and the actual cash market repo rate to the expiry date of the future (30 December 1998) was 7.26%.

Futures price 114.55

	Price	Source	Yield	Conversion factor	Gross Basis	Implied repo %	Actual repo %	Net Basis
UKT 9 10/13/08	130.7188	BGN	5.035	1.1407155	0.05	6.64	7.26	0.131
UKT 7 1/4 12/07/07	116.3750	BGN	4.988	1.0165266	-0.068	6.51	7.26	0.144
UKT 8 09/25/09	125.4375	BGN	4.950	1.0750106	2.295	-4.86	7.26	2.474
UKT 9 07/12/11	136.1563	BGN	5.095	1.1655465	2.643	-5.34	7.26	2.831
UKT 6 1/4 11/25/10	110.7500	BGN	5.049	0.9400748	3.064	-11.38	7.26	3.362
UKT 5 3/4 12/07/09	106.6250	BGN	4.966	0.9051249	2.943	-11.61	7.26	3.273

Table 34.4: December 1998 LIFFE Long Gilt future deliverable bonds.

34.20.2 Conversion factors

A particular bond that remains in the basket over a length of time will have different conversion factors for successive contracts. For example the 9% Treasury maturing on 13 October 2008 had conversion factors of 1.1454317, 1.1429955 and 1.1407155 for the LIFFE long gilt contracts maturing in June, September and December 1998 respectively.

The yield obtainable on bonds with differing coupons but identical maturities can be equalised by adjusting the price for each. This principle is used to calculate the conversion factors for different bonds. The conversion factor for each bond is the price per £1 (or per \$1, €1, etc) such that every bond would provide an investor with the same yield if purchased. The yield selected for the calculations is the same as the coupon rate in the definition of the futures contract, 7% in the case of the long gilt contract traded on the LIFFE.

Other things being equal, bonds with a higher coupon will have larger conversion factors than those with lower coupons. For bonds with the same coupon, maturity has an influence, though this is slightly less obvious. For bonds with coupons below the notional rate defined in the contract description, the conversion factor is smaller for bonds with a longer maturity. The opposite is true for bonds carrying coupons in excess of the notional coupon rate, for which the conversion factor will be larger the longer the maturity. This effect arises from the mathematics of fixed-interest securities. Bonds with coupon below current market yields will trade at a “discount” (that is, below par). This discount is larger the longer the maturity, because it is a disadvantage to hold a bond paying a coupon lower than current market rates, and this disadvantage is greater the longer the period to the bond maturing. Conversely bonds with coupons above current market yields trade at a premium (above par), which will be greater the longer the maturity.

Most futures exchanges calculate conversion factors effective either on the exact delivery date, where a single date is defined, or (as at LIFFE) on the first day of the delivery month if delivery can take place at any time during the delivery month.

34.20.3 The Cheapest-To-Deliver Bond

Using *conversion factors* provides an effective system for making all deliverable bonds perfect substitutes for one another. The system is not perfect however. Conversion factors are calculated to equalise returns at a single uniform yield, the notional coupon rate specified in the contract specification. In practice of course bonds trade at different yields, so that we do not observe a flat yield curve. Hence despite the use of conversion factors, bonds will not be precisely “equal” at the time of delivery. Some bonds will be relatively more expensive, some cheaper; one particular bond will be the *cheapest-to-deliver* bond. The cheapest-to-deliver (CTD) bond is an important concept in the pricing of bond futures contracts. The CTD bond will change according to the coupon and yield of bonds in the delivery basket, and it is a function of the carrying cost of each of the bonds. If we consider the potential arbitrage trade described at the beginning, where a short futures position is hedge with the underlying bond, the profit on delivery at maturity would be as given by (34.3):

$$\text{Profit} = \text{Nom}_{\text{fut}} \times \frac{Pd_{\text{bond}}}{CF} \times (IRR - r_{\text{repo}}) \times \frac{n}{B} \quad (34.3)$$

where Pd_{bond} is the dirty price of the bond. The CTD bond is the bond that maximises this profit.

Although it has been observed that, as market rates decline, bonds with lower modified duration sometimes become the cheapest-to-deliver (because the price of a bond that has a lower modified duration will fall be less than the price of a bond with a higher duration), it is incorrect to suggest, as certain texts do, that the CTD is characterised in terms of duration. This is a misconception that is frequently quoted, not only in textbooks but also by practitioners in the market. However it has been shown⁶ that this assumption is not always the case.

34.20.4 Determining the CTD bond

To determine which bond is the CTD, consider the following trading strategy executed during a delivery month:

- buy £100,000 nominal of a deliverable bond;
- sell one futures contract;
- immediately initiate the delivery process.

⁶ For example, by Benninga (1997).

The amount paid for the bond will be the market price plus accrued interest:

$$P_{con} = P_{bond} + AI \quad (34.4)$$

where

P_{con} is the total consideration paid for the bond
 P_{bond} is the market price for the bond
 AI is the accrued interest.

The invoice amount Inv_{amt} received when delivering the bond against the short futures position has already been defined in equation (34.4). The resultant profit from the trading strategy is then:

$$\begin{aligned} P_{profit} &= Inv_{amt} - P_{con} \\ &= (P_{fut} \times CF + AI) - (P_{bond} + AI) \\ &= (P_{fut} \times CF - P_{bond}). \end{aligned} \quad (34.5)$$

The bond for which this expression is maximised will be the CTD bond during the delivery month. A more complex formula can be derived to determine the CTD bond prior to the delivery month, taking into account *carrying costs*.

In an environment where the actual price is not in line with the theoretical price, in principle an arbitrage opportunity is available. If the actual repo rate for the term from the trade date to the expiry date was lower than the implied repo rate (IRR), it would be possible to buy the bond and fund it in the repo market, and short the futures contract, and then deliver the bond into the futures contract on expiry. This is known as a *cash-and-carry* arbitrage and is illustrated in Figure 34.22.

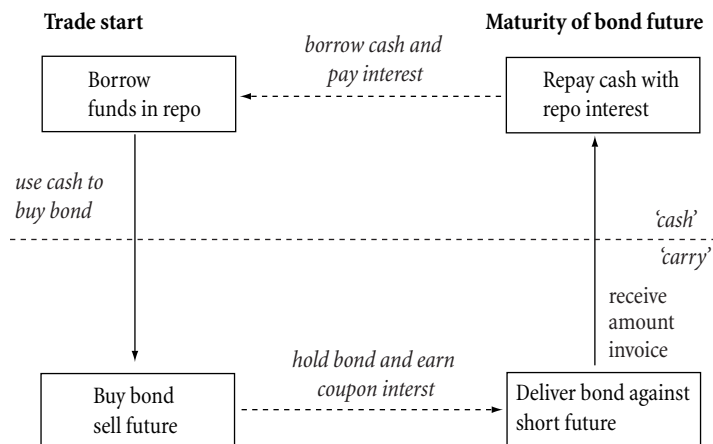


Figure 34.22: Cash-and-carry (basis trade) arbitrage.

If the actual repo rate is higher than the implied repo rate, in theory one can undertake a *reverse cash-and-carry*, in which the underlying bond is sold short and covered in the repo market, and the futures contract is bought. On expiry the trader will take delivery of a bond against the future. This strategy carries more uncertainty however because of the delivery options available to the party that is short the future; the short chooses which bond to deliver and also on which day to deliver. If the CTD bond on the delivery day has changed, the trader will be delivered a different bond to the one they have shorted, which means they will have to sell the delivered bond and purchase, in the open market, the original bond they shorted, in order to close the repo trade. This will result in large losses. In theory an exchange-traded futures contract is deliverable on any day of the expiry month, so for example the June 2000 gilts contract can be delivered into any day from 1 June 2000 to 28 June 2000. In practice however, there are only two days when a bond should be delivered into a futures contract, either the very first day or the very last day. The exact day of delivery is a function of the running yield on the CTD bond and the repo rate in the market.

34.20.5 The gross basis

The other term for cash-and-carry trading is *basis trading*, which is analysed in terms of the *gross basis*. Basis trading arises from the difference between the current clean price of a bond and the clean price at which the bond is bought through the purchase of a futures contract. The difference between these two prices is known as the gross basis.

The formula for calculating the gross basis is therefore:

$$\text{Gross basis} = P_{\text{bond}} - (P_{\text{fut}} \times C_{\text{f}_{\text{bond}}}). \quad (34.6)$$

The gross basis may also be expressed in terms of the implied repo rate, which is given as (34.7):

$$\text{Basis} = C \times \frac{n}{B} - P_{\text{d}_{\text{bond}}} \times \text{IRR} \times \frac{n}{B} \quad (34.7)$$

where n is the number of days to delivery.

The gross basis can be explained essentially as the difference between the running yield on the bond and the current repo (money market) rate. However a residual factor is due to the delivery option implicit in the design of the futures contract and to the daily marking-to-market of the contract, both of which are more difficult to quantify. This residual amount is known as the *net basis*. The bond with the lowest net basis will be the cheapest-to-deliver bond. Net basis is the gross basis adjusted for net carry, that is,

$$\text{net basis} = \text{gross basis} - \text{net cost of carry}$$

where the net cost of carry is

$$\text{net carry cost} = \text{coupon income} - \text{financing cost}.$$

Therefore net carry is the actual coupon income and re-investment less borrowing expense, which is at the security's actual repo (money market) rate. The net basis is thus the true "economic basis". A positive value represents a *loss* or net cost to the long cash/short futures position, and the net basis is the expected *profit* for the short cash/long futures position (where actual repo is the reverse repo rate). The opposite is true for negative net basis values. Net basis is also known as *basis over carry* or *basis adjusted for carry*. The net basis is given by (34.8):

$$\text{net basis} = (P_d \times (1 + r \times \text{Days}/36500)) - ((P_{\text{fut}} \times CF) + AI_{\text{del}}) \quad (34.8)$$

where

- r is the cash market repo rate
- Days is the number of days from settlement date to the futures delivery date
- AI_{del} is the additional accrued interest on the bond from settlement date to the futures date.

The expression above is for a futures contract for a bond that trades with a 365-day base market convention. In the euro market for example, the 360-day base would be used.

We can illustrate the basis calculation from actual market data. Consider the following market details, all relating to one instantaneous point in time:

Settlement date	16 March 2000
Futures delivery date	30 June 2000
Days to delivery	106
Bond price (UKT 9% 2011)	131.4610
Accrued interest	1.5780822
Accrued to delivery	4.1917808
Futures price (M0 LIFFE Long gilt)	112.98
Conversion factor	1.1525705
Cash market repo rate	6.24%

We can calculate the gross and net basis that would apply in a hypothetical cash-and-carry trade, where there is a simultaneous purchase of the bond and sale of the futures contract as shown below; the *implied repo rate* is explained in the next section.

bond purchase — outflow of funds: $131.461 + 1.5781 = 133.0390822$

futures sale — inflow of funds: $(112.98 \times 1.1525705) + 4.192 = 134.409196$.

The gross basis is $131.4610 - (112.98 \times 1.1525705)$ or 1.24358491.

The net basis is $(133.0391 \times (1 + 6.24 \times 106/36500)) - ((112.98 \times 1.15257) + 4.192)$ or 1.0405721.

Traders take a view on the size of the net basis and trade according to this view.

34.20.6 The implied repo rate

We have stated that exchange traded bond futures such as LIFFE's long gilt contract require physical delivery of a real bond. For the gilt contract this will be any eligible gilt with a maturity of 8¼ to 13 years. However only the bond that is the cheapest-to-deliver, from the point of the view of the short futures holder, will be delivered.

The CTD gilt can be delivered on any business day of the delivery month, but in practice only two days are ever used. If the current yield on the CTD gilt exceeds the money market interest rate, the bond will be delivered on the last business day of the month, because the short earns more by holding on to the bond than by delivering it and investing the proceeds in the money market; otherwise the bond will be delivered on the first business day of the delivery month.

Until very recently a gilt that was trading *special ex-dividend* on the proposed delivery day was not eligible for delivery. However from August 1998 the provision for special ex-dividend trading was removed from gilts, so this consideration no longer applies. Other gilts that are not eligible are index-linked, partly paid or convertible bonds.

Invoice Amount

When the bond is delivered, the long pays the short an invoice amount:

$$Inv_{amt} = (\text{Settlement price}/100 \times CF \times \text{Nominal value of gilt}) + AI. \quad (34.9)$$

The settlement price (or *exchange delivery settlement price*, EDSP) is the trading price per £100 nominal for the futures contract on the last day of trading, and is confirmed by the Exchange. The invoice amount includes accrued interest because the futures contract is traded at a *clean* price and does not include accrued interest. Gilts traded at a clean price but have accrued interest added on (called *dirty* price).

We have stated that the conversion factor (or price factor) determines the appropriate price of the bond that is delivered. It is calculated as being the price per £1 nominal at which the bond delivered has a yield to maturity of 7%.

Calculating the implied repo rate

Another way of looking at the concept of the cheapest-to-deliver bond is in terms of the *implied repo rate*. The CTD bond is the bond that gives the highest implied repo rate (IRR) to the short from a cash-and-carry trade, that is a strategy of buying the bond (with borrowed funds) in the cash market and selling it into the futures market.

The IRR is given by (34.10) which sets the net profit of a cash-and-carry trade as zero.

$$IRR = \frac{(P_{fut} \times CF + AI_{delivery}) - (Pd_{bond}) + \sum_i^N C_i}{n(Pd_{bond}) - \sum_i^N (C_i \cdot n_{i,delivery})} \quad (34.10)$$

where $AI_{delivery}$ is the accrued interest of the bond on delivery, n is the number of days the bond is repoe'd, that is, the number of days of the trade, and $n_{i,delivery}$ is the number of days from receipt of the i th coupon payment on the bond and the delivery date. As we notes earlier, the IRR is the breakeven rate for a cash-and-carry strategy, and if the trade could be effected at an actual repo rate that was lower than the IRR, the strategy would result in a profit.

To illustrate we can calculate the IRR for the 9% Treasury 2008, which in October 1998 was trading at 129.0834. The December 1998 long gilt futures contract was trading at 114.50.

The date is 1 October. The money market rate on this date is 7.25%. As the current (or *running*) yield on the 9% 2008, at 6.972% is lower than the money market rate, it will be delivered at the beginning of December (that is, in 61 days from now). To identify the CTD bond we would need to calculate the IRR for all eligible bonds. We will use the conversion factors given in Table 34.4, which were calculated and given out by LIFFE before the futures contract began trading.

Consider the 9% 2008. The cash outflow in a cash-and-carry trade is as follows:

Dirty price of bond	129.0834
+ interest cost (1 October – 1 December)	$129.0834 \times (0.0725 (61/365))$
	130.6474

The bond (whose price includes 171 days interest on 1 October) has to be financed at the money market rate of 7.25% for the 61 days between 1 October and 1 December, when the bond (if it happens to be the CTD) is delivered into the futures market.

The cash inflow per £100 nominal as a result of this trade is:

Implied clean price of bond (1 December)	
(futures price (1 October) $\times$ conversion factor	114.50×1.1407155
+ accrued interest (1 October – 1 December)	$\frac{£9 \times (61/365)}{132.11603}$
	132.11603

The implied price of the bond on 1 December equals the futures price on 1 October multiplied by the conversion factor for the bond. Because the futures price is quoted clean, accrued interest has to be added to obtain the implied dirty price on 1 December.

This cash-and-carry trade which operates for 61 days from 1 October to 1 December generates a rate of return or *implied repo rate* of:

$$\text{Implied repo rate} = \left(\frac{132.11603 - 130.6474}{130.6474} \right) \cdot \frac{365}{61} \times 100 = 6.726\%.$$

The rate implied by a cash-and-carry strategy is known as a repo rate because it is equivalent to a *repurchase* agreement with the futures market. In effect the short lends money to the futures market: the short agrees to buy a bond with a simultaneous provision to sell it back to the market at a pre-determined price and to receive a rate of interest on his money, the repo rate.

The IRR for all deliverable bonds can be calculated in this way. The only modification required is if a bond goes ex-dividend between trade date and delivery date, in which case the interest accrued is negative during the ex-dividend period. Nowadays it is more usual to calculate IRRs and net basis figures from an off-the-shelf spreadsheet program or an external source such as Bloomberg.

In the previous section we calculated the gross basis and net basis for the 9% 2011 gilt, part of the delivery basket for the June 2000 LIFFE long gilt contract. The implied repo rate of this bond at that time is calculated as follows:

$$\text{Implied repo rate} = 100\% \times (365/106) \times (134.4092/133.0391 - 1) \text{ or } 3.54617\%.$$

34.20.7 A model of the cheapest-to-deliver bond

Beninnga (1997) has suggested a CTD model in a non-flat yield curve environment that may be taken to be a general model for basis trading. His study analysed the character of the CTD bond under four different scenarios, as part of a test of the following; that when the term structure is flat, the CTD bond is the one with:

- the highest duration if the market interest rate is higher than the notional coupon;
- the lowest duration if the market interest rate is lower than the notional coupon.

Benninga suggests that under certain scenarios, notably when the market yield is higher than the notional coupon and there are no deliverable bonds with a coupon lower than the notional coupon, and when the market yield is higher than the notional coupon and there are no deliverable bonds with a coupon higher than the notional, the duration rule does not always apply. The conclusions of his analysis are that:

- the CTD bond invariably has either the highest coupon of the deliverable bonds, where the market yield is lower than the notional coupon, otherwise it has the lowest coupon of the deliverable bonds. The analysis assumes that the bonds possess positive convexity, but the results are not dependent on the shape of the yield curve;

- when market rates are lower than the notional coupon, the maturity of the CTD is the shortest of all deliverable bonds; again, if the market lies above the notional coupon, the CTD bond will have the longest maturity if it also has a coupon greater than the notional coupon. If the coupon of the CTD bond is lower than the notional, Benningna concludes that the CTD will have neither the longest or the shortest maturity in the delivery basket.

The basic model for value of the CTD is given in Appendix 34.3.

EXAMPLE 34.13 Calculating the gross and net basis

- This example calculation relates to the gilt contract before the structural changes in the gilt market were introduced, therefore the notional coupon is 10% and pricing is in 32nds (“ticks”). The “special ex” rule also applied.

June long gilt future	109-21/32
CTD bond (8½% 2007)	106-11/32
Conversion factor (8½% 2007)	0.9674064
Repo rate	6.36%

The clean price at which a bond is bought through use of a futures contract is:

futures price $\times$ conversion factor.

Therefore the cost of buying the 8½% 2007 through the futures contract is:

$$109-21/32 \times 0.9674064 = 106.08216.$$

The market price is 106-11/32, therefore the gross basis is:

$$106-11/32 - 106.08216 = 0.26159.$$

Due to the special ex rule in this case, the last day for delivery of 8½% 2007 into the futures contract is 12 June. Assume the bond is purchased in the market on 24 April, for settlement on 25 April. The total price paid including accrued interest will be 108.64923. To finance that using repo for 48 days until 12 June will cost £0.9087243. The holder of the gilt will however earn 48 days’ accrued interest of £1.1178082. Therefore buying the bond direct gives the owner an income advantage of £0.2090839.

The difference between the gross basis and this income advantage is £0.216159 – £0.2090839, that is £0.0525. It therefore represents the gain by buying the gilt using the futures contract rather than buying directly in the market.

Of course the long gilt contract gives the futures seller the right to deliver any of the gilts in the delivery basket and on any day of the delivery month. If the CTD is bought through a futures contract the buyer may find that because of market movements, a different gilt is delivered. The futures short in effect holds an option which decreases the value of the futures contract to the long.

For this reason the *net* basis is usually positive. The futures contract is also marked-to-market which means that the gain or loss on the contract is spread over the life of the contract, in contrast to a forward contract. This effect is small but will again lead to the net basis differing from zero.

For theoretical background to basis trading and the debate on net basis versus implied repo rates see Appendix 34.2.

34.20.8 The delivery mechanism

Basis trading is conducted on an exchange, so that the delivery mechanism is effected through the exchange’s clearing house. They will calculate the final exchange delivery settlement price, based on the expiry price of the contract, and invoice parties who have run long positions to expiry. The short future party delivers the underlying bond to the clearing house. When physical delivery occurs, the short future has the option of when to initiate delivery, and also which bond to deliver (as long as it is in the delivery basket). Most exchanges have a rule that those running short futures must declare their intention to deliver at least two days before the delivery date. This is known as the *notification date*. On the day after notification, the exchange will assign a long future to receive delivery, usually on a pro rata basis, although certain exchanges assign delivery at random. On the delivery day the

short future delivers the bond, and the long future makes payment for value the same day. Both transaction are with the clearing house.

The options available to the short future provide an element of advantage for this party, which includes:

- **timing option:** as we noted, most exchanges give the short the option to deliver on any day of the delivery month, so that if the money market rate is below the coupon rate on the bond, delivery will take place on the very last eligible day;
- **the wildcard option:** on the CBOT exchange, on the last trading day the short future, if the bond price falls after 2pm, can issue a notice of intention to deliver and proceed to buy the CTD bond in preparation for delivery. If the bond price rises, the short future can keep the position open and wait until the next day, to see if the tactic is worth repeating;
- **the short** has, having declared his delivery intention two days before delivery, the freedom to choose another bond to deliver the following day, and may do this if another cheaper bond becomes available;
- **the end-of-month option:** where exchanges allow delivery to take place on any day during the expiry month, trading in the contract itself usually stops a few days beforehand. The prices of deliverable bonds will fluctuate after the futures settlement price has been determined, ahead of the delivery date.

These features provide an element of advantage for the short future, who is able to act on favourable price movements and delay acting on unfavourable price movements. For this reason, it is common for futures contracts to trade one or two hundredths of a point below the theoretical price; this difference is the market value of the seller's options.

34.20.9 Practical implications of basis trading

Generally all bonds on all exchanges will produce a negative result for the strategy of buying the bond and simultaneously selling the futures, and initiating the delivery process. Bid-offer price spreads will also erode an theoretical advantage. This implies that the opposite strategy, buying the futures and selling the bond, known as *reverse cash-and-carry*, would lead to a profit. In theory the trader can earn the repo rate on the short sale proceeds; such a trade is indicated when the implied repo rate is lower than the actual repo rate. However the party who is short futures always initiates the delivery process and chooses, among other things, which bond to deliver. A trader tempted to execute a reverse cash-and-carry by shorting the 6¼% Treasury 2010 in a bid to secure 3.064% riskless profit (see Table 34.4) should note that the party that is short futures is most unlikely to deliver that particular bond. A reverse cash-and-carry trade in the CTD bond may also come unstuck if changing market circumstances result in the bond losing its status as the CTD bond.

The delivery date of the bonds is at the option of the short future, and will depend on the yield of the bond; if the yield is higher than the money market (repo) rate, the bond will be delivered at the end of the month, while if the yield is lower than the money market rate, delivery will be on the first day of the delivery month. Although delivery is at the short future's option and can be at any time during the delivery month, in practice the short will always deliver on one of the two dates noted above, for if the yield is higher than the money market rate the bond holder will earn the higher return until the last possible moment, whereas if the yield on the bond is lower than the money market rate delivery will take place as soon as possible, as there is no gain for the cash long to hold on to the position.

In running the arbitrage position a trader is also exposed to *basis risk*, the risk that the price of the two instruments, cash and futures, do not move "tick-for-tick" or exactly in line. The trader must mark-to-market the position every day, and as prices move out of line, which is common over time, there is the risk that a loss on one instrument will not be matched by the gain on the other.

34.21 Repo market structures

A natural progression within fixed interest markets is the development of ever more sophisticated structures. This occurs for a number of reasons. As liquidity increases and yield spreads decline, banks need to look at newer structures for the same relative value. Investors also may wish to increase exposure to more risky markets in the search for yield; this may take the form of high quality credit counterparties issuing paper in "exotic" currencies or linked to emerging market indices. The imagination of bank financial engineers is the only constraint to the development of sophisticated structures. Customers can have instruments customised to meet their exact investment parameters

and risk appetite. When engineering structured notes therefore it is important to identify clients who may wish to invest in more sophisticated structures, and also to identify their specific requirements.

Structured repo instruments have developed mainly in the US market where repo is widely accepted as a retail investment and money placement instrument. Following the introduction of new repo types it is also possible now to transact them in other liquid markets. In addition to the structures mentioned here cross-currency repo, with each leg in a different currency, is frequently traded in many markets across the world.

34.21.1 Callable repo

In this arrangement the lender of cash in a term fixed rate repo has the right to terminate the repo early, or call back a portion of the cash. In effect the lender of cash an interest rate option, which is of benefit if rates rise during the life of the repo. Should rates rise the lender will call back the cash and reinvest at the higher rate. For this reason a callable repo will trade at lower fixed rates than a conventional repo.

34.21.2 Whole loan repo

This structure has developed in the US market as a response to investor demand for higher yields in a falling interest rate environment. Whole loan repo trades at higher rate than conventional repo because a lower quality collateral is used in the transaction. There are generally two types: mortgage whole loans and consumer whole loans. Both are unsecuritised loans or interest receivables. The loans can also be credit card payments and other types of consumer loan. The repo uses the PSA repo agreement. Owners of whole loan repos are exposed to credit risk but also, due to the nature of the collateral, prepayment risk. This is the risk that the loan package is paid off earlier than the maturity date, which is often the case with consumer loans. For these reasons the yield on whole loan repo is higher than conventional repo, trading at around 20–30 basis points over Libor.

34.21.3 Total return swap

This structure is also known as a “total rate of return swap” and is economically identical to a repo. The main difference is that the transaction is governed by the International Swap Dealers Association (ISDA) Swap agreement as opposed to a repo agreement. This may change the way the trade is reflected on the bank balance sheet, and take it off-balance sheet. This is one of the main motivations for entering into this type of trade. The transaction works as follows:

1. the institution sells the security at its market price;
2. the institution then executes a swap transaction for a fixed term, exchanging the total return on the security for an agreed rate on the relevant cash amount;
3. on maturity of the swap the institution repurchases the security at the market price.

In theory each leg of the transaction can be executed separately with different counterparties; in practice the trade is bundled together and so is economically identical to a repo.

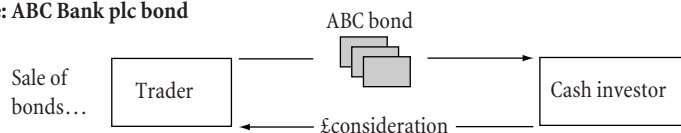
One of the main motivating factors behind the rise of the TRS was the desire to (temporarily) remove bonds off the balance sheet. This may be ahead of a ratings analysis, or the annual external audit. If there are a large number of lower-grade assets on the book, rather than sell them, a bank may choose to enter into a TRS trade. The TRS as originally transacted in the market had the following features:

- the bond trader will receive the “total return” on the bonds, which means that if bond rises in value, the trader pays the difference in value to the counterparty; while if the bonds fall in value, the trader will receive the difference from the counterparty;
- as part of the swap, the trader pays Libor plus a spread on the cash proceeds during the term of the transaction;
- the cash investor counterparty has full title and can sell securities in the open market at termination;
- the trader has no legal obligation to repurchase the bonds, and indeed the counterparty is free to sell the bonds into the open market.

The TRS trade will take the bond assets off the trader’s balance sheet, which may be desired if a year-end is approaching, for (say) credit rating analysis.

Total return swaps are common in equity repo, as a fixed term trade. They are often used as a form of hedge, as well as for financing the underlying position. In a hedge transaction a bank will pay Libor on the funds that are received from the initial sale. On termination of the trade it will receive the difference in market value if the price of the bond has dropped. This is “selling the swap”, the opposite is buying the swap.

Start of trade: ABC Bank plc bond



Termination: rise in price

Settlement (unwind) of
OTC swap agreement

Bonds bought back

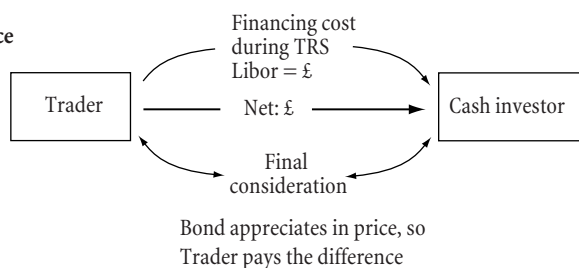


Figure 34.23: Total Return Swap trade.

EXAMPLE 34.14 Financing an investment position with a cross-currency repo

- A fund manager holding a US government bond wishes to fund the position in pounds sterling. A repo trader offers a 7.375% rate in sterling for a seven day term with a “haircut” (margin) of 2%.

Collateral	US Treasury 6 $\frac{1}{8}$ % 2001
GBP Repo rate	4.95%
Term	7 days (5 January – 12 January 2001)

The collateral has a clean price of 99-19 and a haircut of 2% is applied to the loan proceeds.

Accrued	0.0841346 ($6.125 \times 5/182 \times 0.5$)
Dirty price	99.6778746
Gross settlement price	USD 99,677,884.62
Net wired settlement amount	USD 97,723,416.29
Net settlement amount (GBP)	£59,953,016.13

This uses a GBP/USD exchange rate of 1.63

Repo interest	£56,339.41

On 12 January 2000 the repo trader returns the collateral to the fund manager in exchange for a payment of £60,009,355 which is the original principal plus repo interest.

The repo market has allowed the fund manager to borrow in sterling, which we can safely assume (as it is backed with high quality collateral) will be at a lower rate than the unsecured rate. The fund manager could have borrowed in dollars or another currency for the same loan cost benefits. The repo trader is “overcollateralised” by the difference between the value of the bonds and the loan proceeds, that is 2%. A rise in dollar yields or a fall in the value of the dollar will leave the trader undercollateralised, in which case further collateral (“margin call”) will be sought from the fund manager.

34.22 Central bank repo and overseas markets

34.22.1 Central bank operations

Central banks are an important customer base for repo business. In addition many central banks use repo as a tool of monetary policy to control liquidity in the domestic money market. A central bank “repo” operation is actually a reverse repo, as it buys in eligible securities, typically domestic government debt, against lending out cash to a list of eligible counterparties. The net effect is a short term injection of cash which alleviates shortages in the money market. Central banks prefer to use repo as a money market instrument because of the security afforded by the high quality collateral, and because of the liquid market. There is usually no direct impact on the bond market. The duration of transactions will vary but is usually two-week or four-weeks. Trades can be at a fixed rate determined by the central bank or (less common) at a variable rate resulting from auction amongst eligible participants. In addition to the objective of controlling market liquidity central banks operations are also undertaken to send a signal to the market on the intended direction of short term interest rates.

The European Central Bank actively uses repo in all aspects of its money market operations. This includes long term adjustments that involve outright bond purchases, interim financing using repo and “fine tuning” using two- to ten-day repo. The Banque de France has a five- to ten-day lending facility at its ceiling rate, and uses repo on a once-weekly basis dealt at its floor (base) rate. Repo trades are usually overnight or two-day maturity. Only primary dealers in repo can take part – the domestic market in France is unusual in that primary dealers are registered not only in cash government bond trading (which is conventional) but also in the repo market. As in France, primary dealers in US Treasury are the only counterparties in the US market. The Federal Reserve conducts a daily repo operation, dealing in overnight, term and open repo.

34.22.2 Overview of overseas markets

United States

The US market is the oldest and largest repo market in the world. The Federal Reserve initiated repo trading over eighty years ago. The size of the domestic market was quoted as \$1 trillion over ten years ago, the estimate today is perhaps 2–3 times this figure. The core of the market remains dealing in Treasury bills, notes and bonds as well as government agency paper. The US market is invariably the source of new developments in repo, including asset-backed bond repo, whole loan and high-yield debt. The key players are the primary dealers, money market mutual funds and local authorities (municipal authorities). Trades are often carried out through inter-dealer brokers. As the market has developed market participants have been able to deal on a more automated basis, with prices displayed on screens (“GovPx” screen). Most business is conducted between 0800–1000 hours; the Federal Reserve conducts its repo operation at 1130 hours. The Fed “wire” mechanism allows same-day domestic settlement, similar to the settlement arrangement in the UK market. As might be expected the majority of trades are in the overnight to three month area of the yield curve, although liquidity is maintained out to two year term trades.

Germany

The German repo market trades in high volumes both domestically and offshore out of London; it is a mature and well-established market. Until recently the German market was underdeveloped, prior to 1997 the daily volume of German domestic repo was less than DEM 15 billion. It had grown to approximately DEM 63 billion in 1998. The deregulation by the Bundesbank of the minimum reserve requirement for repo transactions contributed to the growth of the market. Repo is concentrated on classic repo transactions, but sell/buy-backs and securities lending are also common. The repo market is split into two main segments:

- the market in government bonds or “bunds”, which is very liquid, and which due to the European benchmark character of the bund market, is the leading repo instrument on the continent. One of the major differences compared to other markets is that a large number of transactions are cross-border ones, often out of London;
- the market for mortgage bonds known as *Pfandbrief*, dominated by domestic participants.

France

The French repo market is the second largest in the world. Repo has been actively encouraged since passage of the *pension livree* legislation in 1993. This is the legal documentation for the market; the PSA/ISMA agreement is not

generally accepted in the Paris market. The average outstanding volume in mid-1998 was Ffr 1,500 billion. An unusual aspect of the French market is the popularity of TMP repos, which are repo trades with interest rates linked to the TMP moving average overnight index (in effect, a variable rate repo). The Banque de France is an active player in the market and deals with SPVTs, the repo market primary dealers. There is little or no specials activity in the market. With the introduction of the euro at the beginning of 1999, the repo market in government bonds has declined slightly, thought to be a result of a reduction in the volume of European yield curve arbitrage trading.

Italy

The Italian market dates from 1970 and is the oldest repo market in Europe. It is also the largest European market. Due to tax and legal problems with classic repo, most trades are sell/buy-backs. Until recently Italian government bonds paid coupons net of tax, as such this created different opportunities for different users of the repo market. Non-resident institutions were able to reclaim withholding tax via a domestic custodian, while resident institutions accessed the repo market to generate tax credits which were then used as an offset against income from other sources. From January 1997 bonds paid coupon gross to non-residents. Repo rates are quoted on both a net and gross basis due to taxable coupon issue, depending on domicile of the investor. Due to the requirement for a domestic custodian all trades are settled onshore. As in the US the domestic market has a strong retail involvement with interest from savings institutions and fund managers. There is a liquid government bond market which trades mainly on the MTS screen-based system, and this has resulted in a liquid repo market. There is a strong segmentation between the domestic market, which operates as sell/buy-back (without the use of legal agreements such as PSA-ISMA) and a London market which trades mainly in classic repo.

Spain

The repo market in Spain is another market with strong domestic participation and retail interest; the instrument is regarded as a savings product. Trades are generally sell/buy-backs and there is little or no special activity. The market is characterised by a split between a liquid government market, with average daily trading volumes around PTA 8 billion, and a less liquid corporate fixed income market. The main reason for the low level of liquidity in the domestic corporate market is because there are only a limited number of issues, of small size, and a high proportion of which are not rated. The corporate market account for less than 10% of average daily volume in repo.

Other European markets

Active government bond repo trading exist in the Netherlands, Belgium, Denmark, Sweden, Austria, Portugal and Ireland. In all these markets there is a domestic market interacting with a cross-border market based in London.

Appendices

APPENDIX 34.1 List of general collateral as agreed by ISMA, 1999

Australia Government guaranteed bonds and federal state-guaranteed bonds and bills	Austria Government guaranteed bonds and bills	Belgium Philippe bonds, Government guaranteed bonds and bills
Canada Government guaranteed bonds and bills	France OATs, BTANs (fixed coupon securities only), BTF	Germany German Unity Fund, Bund, Bobl, Tobl, Treuhand bonds, Schatz, World Bank global
Italy CCT, CTO, BTP, BOT, CTZ	Japan Government "clean" bonds only	Netherlands Dutch State loans, and bills
Spain Obligaciones, Letras, Bonos,	Sweden, Denmark, Norway Government guaranteed bonds and bills	United Kingdom Treasury bills, Gilts (fixed coupon, floating coupon, index-linked, undated, strips), Local authority bonds

APPENDIX 34.2 Basis trading and the cheapest-to-deliver bond

There are two competing definitions for the cheapest-to-deliver issue which usually but not always identify the same issue as “cheapest” namely (i) the issue with the highest implied repo rate, known as the IRP method and (ii) the issue with the lowest net basis (basis method). Most academic literature uses the first definition, whereas market practitioners often argue that the net basis method should be used since it measures the actual profit & loss (p&l) for a “real world” trade. We define net basis below.

- **Net basis** is the gross basis adjusted for net carry. Net carry is actual coupon income and re-investment less borrowing expense, which is at the security’s actual repo (money market) rate. Net basis is the true “economic basis”. A positive value represents a *loss* or net cost to the long cash/short futures position. Net basis is the expected *profit* for the short cash/long futures position (where actual repo is the reverse repo rate). The opposite is true for negative net basis values. For example, if gross basis for a US treasury bond is 5 ticks (32nds of a point) and net basis is 2 ticks, the p&l for a long cash/short futures trade is a *loss* of 2 ticks.

It is up to the individual to decide on which method to use as the basis for analysis. For example Bloomberg® terminals use the IRP method. It is accepted that the IRP method is appropriate to the cash-and-carry investor seeking maximum return per pound invested. The main area of disagreement regards those cases where an arbitrageur finances (repos) the cash side of the trade and the net basis measures his resulting profit or loss. In a Bloomberg analysis this net basis is presented as percentage points of par (the same units as price), although some practitioners express it as p&l per million bonds. It is primarily because the net basis is per par amount rather than per pound invested that the two methods occasionally identify different “cheapest” issues. Note that in practice net basis will always be a loss, otherwise traders would arbitrage an infinite amount of any issue with a profitable net basis. Therefore the basis method identifies the issue which has the *smallest loss* per million as the cheapest issue.

The only reason a trader is willing to accept this guaranteed loss is that he doesn’t intend to follow through exactly with this trade to maturity. Being long of the basis, that is short futures, essentially gives the trader numerous delivery and trading options; the cost of these is the net basis that the trader pays. In effect the trader is buying options for the cost of the net basis. The number of options he buys is indicated by the *conversion factor* since that is the hedge factor for the cheapest issue. Therefore the cost per option is the net basis divided by the conversion factor. When ranked by net basis per contract (that is, divided by the conversion factor), the cheapest by this method invariably agrees with the IRP method.

The formula for the calculation of the implied repo rate for a bond with no or one interim coupon is shown as equation (34.11), which states:

$$\begin{aligned} \text{cost to purchase and finance bond} &= \text{coupon and reinvested earnings} + \text{amount received from contract delivery} \\ (P + AI)(1 + IRR \times D_1/365) &= C(1 + IRRD_2/365) + (DP + AI_{del}). \end{aligned}$$

Rearranging for the implied repo rate we obtain:

$$IRR = \frac{(P_{fut} \times CF) + AI_{del} - (P + AI) + C}{(P + AI)(D_1/365) - C(D_2/365)} \quad (34.11)$$

where

IRR	is the implied repo rate (as a decimal)
P	is the bond purchase clean price
P_{fut}	is the futures price
AI	is the accrued interest at the time of purchase
AI_{del}	is the accrued interest on the bond on futures delivery day
D_1	is the days from purchase settlement date to futures delivery
D_2	is the days from interim coupon receipt to futures delivery
C	is the actual interim coupon received (is zero if no coupon received).

The general formula for the conversion factor is given at Appendix 34.3.

APPENDIX 34.3 A general model of the CTD bond

The price today (or at time 0) of a bond is generally given by (34.12):

$$P = \int_0^T Ce^{-rt} dt + 100e^{-rT} \quad (34.12)$$

where C is the bond cash flow and T is the bond maturity date.

The discount factor at time t for one unit of cash at time $s \geq t$ when the time t spot interest-rate is r is given by e^{-rt} . The value of the conversion factor for a bond with maturity T and coupon C delivered at time F , the expiry date of the futures contract, is given by (34.13):

$$\begin{aligned} CF &= \int_0^{T-F} Ce^{-cs} ds + e^{-c(T-F)} \\ &= \frac{Ce^{-cs}}{-c} \Big|_0^{T-F} + e^{-c(T-F)} = \frac{C(1 - e^{-c(T-F)})}{-c} + e^{-c(T-F)} \end{aligned} \quad (34.13)$$

where c is the notional coupon of the futures contract.

APPENDIX 34.4 Cheapest-to-deliver bonds for Sep97 and Dec97 Long Gilt Futures contract, with graphical analysis of CTD bond

8H 07 7Q 07 9 08 8 09 6Q 10 9 11 9 12	SEP 30 Sep 97				DEC 31 Dec 97				
	GROSS	IRR	NET		GROSS	IRR	NET		
	BASIS	ACT/365	BASIS	hunds.	Early del	BASIS	ACT/365	BASIS	Early del.
	2.77700	7.16%	1.60040		0.400	7.23%	0.102		
	2.84400	7.75%	3.43610		1.665	7.91%	0.969		
	1.29790	7.74%	3.21210		1.512	7.89%	0.681		
	2.70543	7.84%	2.96291		2.705	7.99%	1.332		
	5.82163	8.61%	3.68984		3.094	8.89%	1.638		
	3.52430	7.25%	2.48452		0.944	7.38%	0.266		
	Repo to delivery	6.300	special		Repo	6.50			
	68 days								

Table 34.5: CTD bonds for September 97 and December 97 LIFFE long gilt future. ©Didier Joannas.

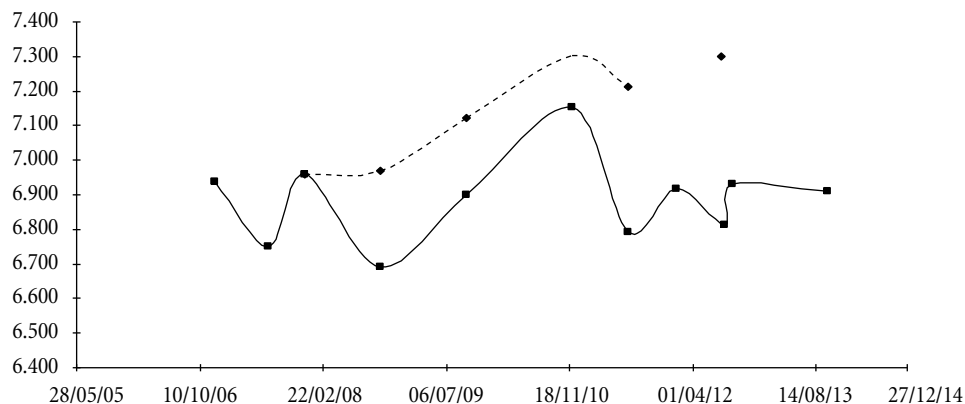


Figure 34.24: Cheapest-to-deliver graph, September 97 and December 97 gilt future.

Selected bibliography and references

Benninga, S., *Financial Modeling*, MIT Press, 1997.

Blake, D., *Financial Market Analysis*, McGraw-Hill, 1991.

Choudhry, M., *An Introduction to Repo Markets*, SI (Services) Publishing, 1999.

Choudhry, M., *Trading the basis: theory and practice*, unpublished paper, Henley Management College, 1998.

Klemkosky, R., Lasser, D., "An Efficiency Analysis of the T-Bond Futures Market", *Journal of Futures Markets* 5, 1985, pp. 607–620.

Resnick, B., Hennigar, E., "The Relationship between Futures and Cash Prices for US Treasury Bonds", *Review of Research in Futures Markets* 3, 1984, pp. 88–104.

Steiner, R., *Mastering Repo Markets*, FT Pitman, 1998.

Stigum, M., *The Money Market*, Irwin, 1990.

Questions and exercises

1. A repo desk is reviewing money market rates by comparing the T-bill market with the repo market. Assume no bid-offer spreads.

Maturity (days)	T-Bill discount	Repo rate
30	5.12%	5.18%
60	5.36%	5.39%

You are *expecting* the 30-day fixed repo rate to rise over the next month (30 days) by 0.50% to a rate of 5.68%.

Discuss the merits of the following strategies:

- (a) buying the long-dated Bill and funding it to maturity with a matching repo;
- (b) shorting the short-dated Bill and covering with a matching repo;
- (c) selling the long-dated Bill and covering with a shorter reverse repo; roll over funding;
- (d) buying the long-dated Bill for a 28-day forward date, against a forward-starting repo transaction (in effect, investing at the 30-day forward-forward rate, funded against the 30-day fwd-fwd repo rate).

We require the T-Bill equivalent yields, T-Bill prices and Bill and repo 30v58 fwd-fwd rates. Assume a money market with a 360-day base. Calculate the cash flows for each part of the trades to determine the real or potential profit from each strategy.

US market T-Bill price formula: Price from discount rate:

$$P = \frac{M}{1 + r\%(n/360)} \qquad P = 100 \times \left(1 - \frac{d(\frac{n}{360})}{100}\right)$$

Formula for conversion of Bill discount to yield:

$$r = \frac{d\%}{1 - (d\% \cdot n/360)}$$

Forward rate formula from yields:

$$rf = \left(\frac{1 + r_{long}\%(n_{long}/360)}{1 + Sr\%(n_{short}/360)} - 1 \right) \cdot \left(\frac{360}{n_{long} - n_{short}} \right)$$

2. The gilt future is trading at 114.55. Which of the following gilts is the cheapest-to-deliver?

Bond	Price	Conversion factor
9% 2008	130.7188	1.1407155
7% 2007	116.375	1.0165266
8% 2009	125.4375	1.0750106
9% 2011	136.1536	1.1655465

3. A bond desk puts on an arbitrage trade consisting of a long cash and short futures position. What risks does this trade expose the desk to?
4. Assume that the cheapest-to-deliver bond for a futures contract has a coupon of 6% and has precisely nine years to maturity. Its price is 103.5625. If its conversion factor is 0.90123564, what is the current price of the futures contract?
5. Assess the following market information and determine if there is an arbitrage opportunity available from undertaking a basis trade:

Bond coupon:	8.875%
Maturity:	December 2003
Price	102.71
Accrued interest:	3.599
Futures price:	85.31
Conversion factor:	1.20305768
Repo rate:	6.803%
Days to delivery:	23
Contract size	100,000
Accrued interest on delivery:	4.182
6. The terms of the LIFFE long gilt contract state that delivery may take place on any day during the delivery month. On 30 May 1999 the yield on the cheapest-to-deliver gilt is 5.716% while the repo rate is 6.24%. On what day will a short future deliver the bond if:
 - (a) he already owns the cash bond
 - (b) if he does not yet own the bond? Explain your answer.

Will there be any change in the current cheapest-to-deliver bond if there is a parallel shift in the yield curve of 50 basis point?
7. The first day of trading of a new futures contract is about to commence. What is the fair price of the contract?
8. Consider the following market data, with price for UK gilts and the LIFFE long gilt contract. Gilts pay semi-annual coupon on an act/act basis.

UKT 5.75% 7 Dec 2009	£102.7328	Conversion factor 0.9142255
UKT 6.25% 25 Nov 2010	107.8777	Conversion factor 0.9449312
UKT 9% 6 Aug 2012	134.4551	Conversion factor 1.1619558
Futures price	112.98	
Settlement date	16 March 2000	
Futures expiry	30 June 2000	
Actual repo rate	6.24%	

Calculate the gross basis, the net basis and the implied repo rate for each bond. Which bond is the cheapest-to-deliver? Relative to the futures contract, what is the difference in price between the cheapest-to-deliver bond and the most expensive-to-deliver bond? What does a negative net basis indicate?
9. We wish to determine by how much the yield of a deliverable bond would have to change in order for it to become the cheapest-to-deliver bond. How could we do this?
10. A junior trader feels that there are some arbitrage opportunities available in the basis, which is net positive for the cheapest-to-deliver bond, if she puts on a strategy of long futures versus short in the cheapest-to-deliver. What factors may contribute to preventing her from realising a profit equal to the current value of the net basis?
11. A long bond futures contract matures in 56 days, and its current price is 107.55. The price of the cheapest-to-deliver is 129.875, and it has a coupon of 9% and accrued interest of 79 days (act/act). What is the implied repo rate?

35 Money Markets Derivatives

The market in short-term interest-rate derivatives is a large and liquid one, and the instruments involved are used for a variety of purposes. In this chapter we review the two main contracts used in money markets trading, the short-term *interest rate future* and the *forward rate agreement*. In Chapter 6 we introduced the concept of the forward rate. Money market derivatives are priced on the basis of the forward rate, and are flexible instruments for hedging against or speculating on forward interest rates. The FRA and the exchange-traded interest-rate future both date from around the same time, and although initially developed to hedge forward interest-rate exposure, they now have a variety of uses. In this chapter the instruments are introduced and analysed, and there is a review of their main uses. We also briefly consider the concept of *convexity bias* in swap and futures pricing.

35.1 Forward rate agreements

A *forward rate agreement* (FRA) is an OTC derivative instrument that trades as part of the money markets. It is essentially a forward-starting loan, but with no exchange of principal, so that only the difference in interest rates is traded. Trading in FRAs began in the early 1980s and the market now is large and liquid; turnover in London exceeds \$5 billion each day. So an FRA is a forward dated loan, dealt at a fixed rate, but with no exchange of principal – only the interest applicable on the notional amount between the rate dealt at and the actual rate prevailing at the time of settlement changes hands. That is, FRAs are *off-balance sheet* (OBS) instruments. By trading today at an interest rate that is effective at some point in the future, FRAs enable banks and corporates to hedge interest rate exposure. They are also used to speculate on the level of future interest rates.

35.1.1 Definition of an FRA

An FRA is an agreement to borrow or lend a *notional* cash sum for a period of time lasting up to twelve months, starting at any point over the next twelve months, at an agreed rate of interest (the FRA rate). The “buyer” of a FRA is borrowing a notional sum of money while the “seller” is lending this cash sum. Note how this differs from all other money market instruments. In the cash market, the party buying a CD or Bill, or bidding for stock in the repo market, is the lender of funds. In the FRA market, to “buy” is to “borrow”. Of course, we use the term “notional” because with an FRA no borrowing or lending of cash actually takes place, as it is an OBS product. The notional sum is simply the amount on which interest payment is calculated.

So when a FRA is traded, the buyer is borrowing (and the seller is lending) a specified notional sum at a fixed rate of interest for a specified period, the “loan” to commence at an agreed date in the future. The *buyer* is the notional borrower, and so if there is a rise in interest rates between the date that the FRA is traded and the date that the FRA comes into effect, she will be protected. If there is a fall in interest rates, the buyer must pay the difference between the rate at which the FRA was traded and the actual rate, as a percentage of the notional sum. The buyer may be using the FRA to hedge an actual exposure, that is an actual borrowing of money, or simply speculating on a rise in interest rates. The counterparty to the transaction, the *seller* of the FRA, is the notional lender of funds, and has fixed the rate for lending funds. If there is a fall in interest rates the seller will gain, and if there is a rise in rates the seller will pay. Again, the seller may have an actual loan of cash to hedge or be a speculator.

In FRA trading only the payment that arises as a result of the difference in interest rates changes hands. There is no exchange of cash at the time of the trade. The cash payment that does arise is the difference in interest rates between that at which the FRA was traded and the actual rate prevailing when the FRA matures, as a percentage of the notional amount. FRAs are traded by both banks and corporates and between banks. The FRA market is very liquid in all major currencies and rates are readily quoted on screens by both banks and brokers. Dealing is over the telephone or over a dealing system such as Reuters.

The terminology quoting FRAs refers to the borrowing time period and the time at which the FRA comes into effect (or matures). Hence if a buyer of a FRA wished to hedge against a rise in rates to cover a three-month loan starting in three months’ time, she would transact a “three-against-six month” FRA, or more usually a 3x6 or 3-vs-6 FRA. This is referred to in the market as a “threes-sixes” FRA, and means a three-month loan beginning in three

months' time. So a "ones-fours" FRA (1v4) is a three-month loan in one month's time, and a "three-nines" FRA (3v9) is six-month money in three month's time.

Note that when one buys an FRA one is "borrowing" funds. This differs from cash products such as CD or repo, as well as interest-rate futures, where "buying" is lending funds.

EXAMPLE 35.1

A company knows that it will need to borrow £1 million in three months' time for a twelve-month period. It can borrow funds today at Libor + 50 basis points. Libor rates today are at 5% but the company's treasurer expects rates to go up to about 6% over the next few weeks. So the company will be forced to borrow at higher rates unless some sort of hedge is transacted to protect the borrowing requirement. The treasurer decides to buy a 3v15 ("threes-fifteens") FRA to cover the twelve-month period beginning three months from now. A bank quotes 5½% for the FRA which the company buys for a notional £1 million. Three months from now rates have indeed gone up to 6%, so the treasurer must borrow funds at 6½% (the Libor rate plus spread), however she will receive a settlement amount which will be the difference between the rate at which the FRA was bought and today's twelve-month Libor rate (6%) as a percentage of £1 million, which will compensate for some of the increased borrowing costs.

35.2 FRA mechanics

In virtually every market FRAs trade under a set of terms and conventions that are identical. The British Bankers Association (BBA) has compiled standard legal documentation to cover FRA trading. The following standard terms are used in the market.

- **Notional sum:** The amount for which the FRA is traded.
- **Trade date:** The date on which the FRA is dealt.
- **Settlement date:** The date on which the notional loan or deposit of funds becomes effective, that is, is said to begin. This date is used, in conjunction with the notional sum, for calculation purposes only as no actual loan or deposit takes place.
- **Fixing date:** This is the date on which the *reference rate* is determined, that is, the rate to which the FRA dealing rate is compared.
- **Maturity date:** The date on which the notional loan or deposit expires.
- **Contract period:** The time between the settlement date and maturity date.
- **FRA rate:** The interest rate at which the FRA is traded.
- **Reference rate:** This is the rate used as part of the calculation of the settlement amount, usually the Libor rate on the fixing date for the contract period in question.
- **Settlement sum:** The amount calculated as the difference between the FRA rate and the reference rate as a percentage of the notional sum, paid by one party to the other on the settlement date.

These terms are illustrated in Figure 35.1.

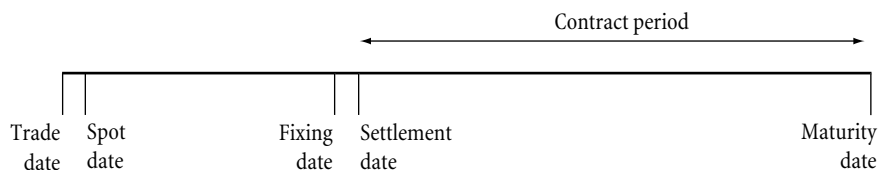


Figure 35.1: Key dates in a FRA trade.

The spot date is usually two business days after the trade date, however it can by agreement be sooner or later than this. The settlement date will be the time period after the spot date referred to by the FRA terms, for example a 1x4 FRA will have a settlement date one calendar month after the spot date. The fixing date is usually two business days before the settlement date. The settlement sum is paid on the settlement date, and as it refers to an amount over a period of time that is paid up front, at the start of the contract period, the calculated sum is discounted

present value. This is because a normal payment of interest on a loan/deposit is paid at the end of the time period to which it relates; because an FRA makes this payment at the *start* of the relevant period, the settlement amount is a discounted present value sum.

With most FRA trades the reference rate is the LIBOR fixing on the fixing date.

The settlement sum is calculated after the fixing date, for payment on the settlement date. We may illustrate this with an hypothetical example. Consider a case where a corporate has bought £1 million notional of a 1v4 FRA, and dealt at 5.75%, and that the market rate is 6.50% on the fixing date. The contract period is 90 days. In the cash market the extra interest charge that the corporate would pay is a simple interest calculation, and is:

$$\frac{6.50 - 5.75}{100} \times 1,000,000 \times \frac{91}{365} = \text{£}1869.86.$$

This extra interest that the corporate is facing would be payable with the interest payment for the loan, which (as it is a money market loan) is when the loan matures. Under a FRA then, the settlement sum payable should, if it was paid on the same day as the cash market interest charge, be exactly equal to this. This would make it a perfect hedge. As we noted above though, FRA settlement value is paid at the start of the contract period, that is, the beginning of the underlying loan and not the end. Therefore the settlement sum has to be adjusted to account for this, and the amount of the adjustment is the value of the interest that would be earned if the unadjusted cash value was invested for the contract period in the money market. The settlement value is given by (35.1):

$$\text{Settlement} = \frac{(r_{\text{ref}} - r_{\text{FRA}}) \times M \times \frac{n}{B}}{1 + \left(r_{\text{ref}} \times \frac{n}{B}\right)} \quad (35.1)$$

where

r_{ref}	is the reference interest fixing rate
r_{FRA}	is the FRA rate or <i>contract rate</i>
M	is the notional value
n	is the number of days in the contract period
B	is the day-count base (360 or 365).

The expression at (35.1) simply calculates the extra interest payable in the cash market, resulting from the difference between the two interest rates, and then discounts the amount because it is payable at the start of the period and not, as would happen in the cash market, at the end of the period.

In our hypothetical illustration, as the fixing rate is higher than the dealt rate, the corporate buyer of the FRA receives the settlement sum from the seller. This then compensates the corporate for the higher borrowing costs that he would have to pay in the cash market. If the fixing rate had been lower than 5.75%, the buyer would pay the difference to the seller, because the cash market rates will mean that he is subject to a lower interest rate in the cash market. What the FRA has done is hedge the interest rate, so that whatever happens in the market, it will pay 5.75% on its borrowing.

A market maker in FRAs is trading short-term interest rates. The settlement sum is the value of the FRA. The concept is exactly as with trading short-term interest-rate futures; a trader who buys a FRA is running a long position, so that if on the fixing date $r_{\text{ref}} > r_{\text{FRA}}$, the settlement sum is positive and the trader realises a profit. What has happened is that the trader, by buying the FRA, “borrowed” money at an interest rate, which subsequently rose. This is a gain, exactly like a *short* position in an interest-rate future, where if the price goes down (that is, interest rates go up), the trader realises a gain. Equally a “short” position in a FRA, put on by selling a FRA, realises a gain if on the fixing date $r_{\text{ref}} < r_{\text{FRA}}$.

35.2.1 FRA pricing

As their name implies, FRAs are forward rate instruments and are priced using the forward rate principles we established in Chapter 6. Consider an investor who has two alternatives, either a six-month investment at 5% or a one-year investment at 6%. If the investor wishes to invest for six months and then roll-over the investment for a further six months, what rate is required for the roll-over period such that the final return equals the 6% available from the one-year investment? If we view a FRA rate as the break-even forward rate between the two periods, we

simply solve for this forward rate and that is our approximate FRA rate. This rate is sometimes referred to as the interest rate “gap” in the money markets (not to be confused with an interbank desk’s *gap risk*, the interest rate exposure arising from the net maturity position of its assets and liabilities).

We can use the standard forward-rate breakeven formula to solve for the required FRA rate; we established this relationship earlier when discussing the calculation of forward rates that are arbitrage-free. The relationship given at (35.2) below connects simple (bullet) interest rates for periods of time up to one year, where no compounding of interest is required. As FRAs are money market instruments we are not required to calculate rates for periods in excess of one year¹, where compounding would need to be built into the equation. The expression is given by (35.2):

$$(1 + r_2 t_2) = (1 + r_1 t_1)(1 + r_f t_f) \quad (35.2)$$

where

- r_2 is the cash market interest rate for the long period
- r_1 is the cash market interest rate for the short period
- r_f is the forward rate for the gap period
- t_2 is the time period from today to the end of the long period
- t_1 is the time period from today to the end of the short period
- t_f is the forward gap time period, or the contract period for the FRA.

This is illustrated diagrammatically in Figure 35.2.

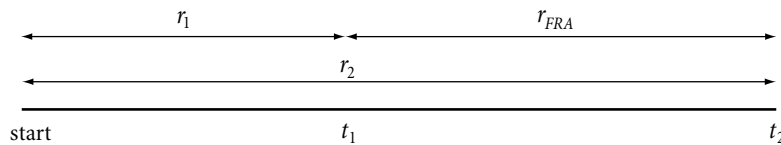


Figure 35.2: Rates used in FRA pricing.

The time period t_1 is the time from the dealing date to the FRA settlement date, while t_2 is the time from the dealing date to the FRA maturity date. The time period for the FRA (contract period) is t_2 minus t_1 . We can replace the symbol “ t ” for time period with “ n ” for the actual number of days in the time periods themselves. If we do this and then rearrange the equation to solve for r_{fra} the FRA rate, we obtain (35.3):

$$r_{FRA} = \frac{r_2 n_2 - r_1 n_1}{n_{fra} \left(1 + r_1 \frac{n_1}{365} \right)} \quad (35.3)$$

where

- n_1 is the number of days from the dealing date or spot date to the settlement date
- n_2 is the number of days from dealing date or spot date to the maturity date
- r_1 is the spot rate to the settlement date
- r_2 is the spot rate from the spot date to the maturity date
- n_{fra} is the number of days in the FRA contract period
- r_{FRA} is the FRA rate.

If the formula is applied to say the US money markets, the 365 in the equation is replaced by 360, the day count base for that market.

In practice FRAs are priced off the exchange-traded short-term interest rate future for that currency, so that sterling FRAs are priced off LIFFE short sterling futures. Traders normally use a spreadsheet pricing model that has futures prices directly fed into it. FRA positions are also usually hedged with other FRAs or short-term interest rate futures.

¹ Although it is of course possible to trade FRAs with contract periods greater than one year, for which a different pricing formula must be used.

35.2.2 FRA prices in practice

The dealing rates for FRAs are possibly the most liquid and transparent of any non-exchange traded derivative instrument. This is because they are calculated directly from exchange-traded interest-rate contracts. The key consideration for FRA market makers however is how the rates behave in relation to other market interest rates. The forward rate calculated from two period spot rates must, as we have seen, be set such that it is arbitrage-free. If for example the six-month spot rate was 8.00% and the nine-month spot rate was 9.00%, the 6v9 FRA would have an approximate rate of 11%. What would be the effect of a change in one or both of the spot rates? The same arbitrage-free principle must apply. If there is an increase in the short-rate period, the FRA rate must decrease, to make the total return unchanged. The extent of the change in the FRA rate is a function of the ratio of the contract period to the long period. If the rate for the long period increases, the FRA rate will increase, by an amount related to the ratio between the total period to the contract period. The FRA rate for any term is generally a function of the three-month LIBOR rate generally, the rate traded under an interest-rate future. A general rise in this rate will see a rise in FRA rates.

The general relationship for FRA and money market rates can be shown to be given by (35.4), which is obtained from a partial differentiation of r_{FRA} with respect to r_1 and r_2 , the rates in the pricing equation at (35.3):

$$\frac{\partial r_{FRA}}{\partial r_1} \approx -\frac{n_1}{n_{FRA}} \quad \frac{\partial r_{FRA}}{\partial r_2} \approx \frac{n_2}{n_{FRA}} \quad \frac{\partial r_{FRA}}{\partial r_{ALL}} \approx 1 \quad (35.4)$$

EXAMPLE 35.2 Pricing FRAs from futures

The following are interest-rate futures prices for short-sterling contracts:

Jun 00	94.70 (implied three-month interest rate: 5.30%)
Sep 00	94.65 (implied three-month interest rate: 5.35%)
Dec 99	94.60 (implied three-month interest rate: 5.40%)

A trader is asked for the offer side of a 3v6 FRA in £5 million and also advise on the hedge by futures. If we assume there are no bid-offer spreads, what should the rate on the FRA be?

The FRA dates are given below.

Contract period: 20 June to 20 September (92 days)
Settlement date: 18 June

As the expiry date for the June futures contract is 18 June, the FRA rate will be the implied June futures rate of 5.30%. The settlement amount is:

$$\frac{5,000,000 \times (0.0530 - Libor) \times \frac{92}{365}}{1 + Libor \times \frac{92}{365}}.$$

The profit or loss on the futures contract, which is not discounted is calculated as:

$$\text{No. of contracts} \times 500,000 \times (0.0530 - Libor) \times \frac{90}{365}.$$

In a hedge the FRA buyer requires these two values to be equal, so we have:

$$\text{No. of contracts} = 10 \times \frac{\frac{92}{90}}{(1 + Libor \times \frac{92}{365})}.$$

The future Libor rate is of course not known at this point, but if we estimate it as 5.30%, we obtain:

$$\text{No. of contracts} = 10 \times \frac{\frac{92}{90}}{(1 + 0.0530 \times \frac{92}{365})} = 10.08746.$$

The hedge here is 10 contracts.

EXAMPLE 35.3

- Using the same prices as in Example 35.2, what is the hedge and price for a sale of a 3v6 FRA against a 6v12 FRA?

The 3v6 FRA is priced at 5.30% as before. A 6v9 FRA, which is the 91 days from 20 September to 20 December, is also priced at the Sep futures price of 5.35%, and hedged with 10 futures contracts. This is only completely accurate if the futures contract delivery date is the same as the settlement date of the FRA, but it is close enough for our purposes. The 3v9 FRA is equivalent to a *strip* combining the 3v6 FRA and a 6v9 FRA, due to the principle of arbitrage-free pricing, so the 3v9 FRA price is calculated as:

$$\left(\left(1 + 0.0530 \times \frac{92}{365} \right) \times \left(1 + 0.0535 \times \frac{91}{365} \right) - 1 \right) \times \frac{365}{182} = 5.38987\%.$$

The price of the 6v12 FRA may be obtained from the price of a 6v9 FRA and a 9v12 FRA (90 days from 20 December to 20 March), because a strip of these two FRAs is the same as the 6v12 FRA. The price is calculated below.

$$\left(\left(1 + 0.0535 \times \frac{91}{365} \right) \times \left(1 + 0.0540 \times \frac{90}{365} \right) - 1 \right) \times \frac{365}{183} = 5.39815\%.$$

EXAMPLE 35.4

- Again using the prices in Example 35.2, our trader now wishes to sell a 3v8 FRA (153 days from 20 June to 20 November) and a 6v11 FRA (153 days from 20 September to 20 February). How are these priced?

The 3v8 FRA is the implied rate for five-month money in three months' time. We do not have the spot or futures rates that allow us to calculate this rate exactly, so we must interpolate using the rates we do have. This then becomes:

$$\begin{aligned} & 3v6 + (3v9 - 3v6) \times \frac{\text{days in 3v8} - \text{days in 3v6}}{\text{days in 3v9} - \text{days in 3v6}} \\ &= 5.35\% + \left((5.39 - 5.35) \times \frac{153 - 92}{182 - 92} \right) \\ &= 5.3771\%. \end{aligned}$$

EXAMPLE 35.5 Valuation of an existing FRA

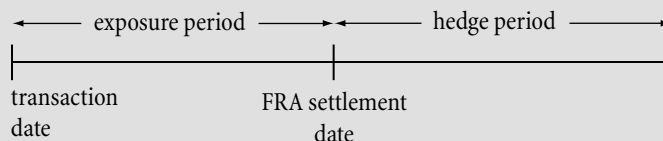
In order to value an FRA, it must be decomposed into its constituent parts, which are equivalent to a loan and deposit. Both these parts are then present valued, and the value of the FRA is simply the net present value of both legs. For example, a 6v9 FRA is equivalent to a six-month asset and a nine-month liability. If we assume that the six-month rate is 5% and the nine-month rate is 6%, on a notional principal of £1 million, the value of the FRA is given by:

Value	Term (yr)	Rate (%)	PV
£1 million	0.4932	5.00	£976,223
−£1 million	0.7397	6.00	−£957,814

The value of the FRA is therefore £18,409.

EXAMPLE 35.6 Hedging an FRA position

A FRA market maker sells an EUR 100 million 3v6 FRA, that is, an agreement to make a notional deposit (without exchange of principal) for three months in three months' time, at a rate of 7.52%. He is exposed to the risk that interest rates will have risen by the FRA settlement date in three months' time.



Date	14 December
3v6 FRA rate	7.52%
March futures price	92.50%
Current spot rate	6.85%

Action

The dealer first needs to calculate a precise hedge ratio. This is a three stage process:

- (1.) Calculate the nominal value of a basis point move in LIBOR on the FRA settlement payment;

$$BPV = FRA_{nom} \times 0.01\% \times \frac{n}{360}.$$

$$\text{Therefore: } \text{ECU } 100,000,000 \times 0.01\% \times 90/360 = \text{EUR } 2,500$$

- (2.) Find the present value of (1.) by discounting it back to the transaction date using the FRA and spot rates;

Present value of a basis point move =

$$\frac{\text{nominal value of basis point}}{\left(1 + \text{spot rate} \times \frac{\text{days in hedge period}}{360}\right) \times \left(1 + \text{FRA rate} \times \frac{\text{days in hedge period}}{360}\right)}.$$

Therefore

$$\frac{\text{EUR } 2,500}{\left(1 + 6.85\% \times \frac{90}{360}\right) \times \left(1 + 7.52\% \times \frac{90}{360}\right)}.$$

- (3.) Determine the correct hedge ratio by dividing (2.) by the futures tick value.

$$\text{Hedge ratio} = \frac{2412}{25} = 96.48.$$

The appropriate number of contracts for the hedge of an EUR 100,000,000 3v6 FRA would therefore be 96 or 97, as the fraction is under one-half, 96 is correct. To hedge the risk of an increase in interest rates, the trader sells 96 ECU three months futures contracts at 92.50. Any increase in rates during the hedge period should be offset by a gain realised on the futures contracts through daily variation margin receipts.

Outcome

Date	15 March
Three month LIBOR	7.625%
March EDSP	92.38

The hedge is lifted upon expiry of the March futures contracts. Three month LIBOR on the FRA settlement date has risen to 7.625% so the trader incurs a loss of ECU25,759 on his FRA position (i.e., ECU26,250 discounted back over the three month FRA period at current LIBOR rate), calculated as follows:

$$\frac{(\text{LIBOR} - \text{FRA rate}) \times (\text{days in FRA period}/360) \times \text{Contract Nominal Amount}}{1 + \text{LIBOR rate} \times (\text{days in FRA period}/360)}.$$

Therefore:

$$\frac{26,250^*}{1 + 7.625\% \times \frac{90}{360}} = \text{EUR } 25,759$$

$$^* \text{ i.e., } 0.105\% \times \frac{90}{360} \times \text{EUR } 100,000,000$$

Futures P/L: 12 ticks $(92.50 - 92.38) \times \text{EUR } 25 \times 96 \text{ contracts} = \text{ECU}28,800$.

Conclusion

The EUR25,759 loss on the FRA position is more than offset by the ECU28,800 profit on the futures position when the hedge is lifted.

If the dealer has sold 100 contracts his futures profit would have been EUR30,000, and, accordingly, a less accurate hedge. The excess profit in the hedge position can mostly be attributed to the arbitrage profit realised by the market-maker (i.e., the market-maker has sold the FRA for 7.52% and in effect bought it back in the futures market by selling futures at 92.50 or 7.50% for a 2 tick profit.)

35.3 Long-dated FRAs

There is a liquid market in FRAs that extend over periods longer than one year. For such instruments the calculation of the FRA rate is different because the pricing formula has to take into account the compounding effect that applies in the cash market. The correct approach is to calculate the zero-coupon discount factors for the relevant terms, and assume an equivalent cash loan over the contract period, and use these in an adjusted pricing formula. We illustrate this with an hypothetical example to calculate the price of a 15v18 FRA.

A sterling money market cash rate desk is given the following interest rates:

12 months	6.75%
15 months	6.875%
18 months	6.9375%

The 12-month discount factor is: $1/(1 + 0.0675) = 0.93677$.

An equivalent cash trade for the FRA would be 15-month deposit and a simultaneous 12-month loan. The cash flows from such transactions would be:

$$\text{Loan: } (-100) + (6.875 \times 0.93677) = -93.55971$$

$$\text{Deposit: } 100 + (6.875 \times 91/365) = 101.71404.$$

Therefore the 15-month discount factor is $93.55971/101.71404$ or 0.919831. The cash flows from a deposit of 18 months and loan of 12 months are:

$$\text{Loan: } (-100) + (6.9375 \times 0.93677) = -93.50116$$

$$\text{Deposit: } 100 + (6.9375 \times 183/365) = 103.47825.$$

The 18-month discount factor may then be calculated, and is $93.55971/103.47825$ or 0.9041486. Therefore the 15v18 FRA rate is given by:

$$r_{\text{FRA}} = \left(\frac{1/0.9041486}{1/0.919831} - 1 \right) \times \frac{365}{92} = 6.88140\%.$$

35.4 Forward contracts

A forward contract is an OTC instrument with terms set for delivery of an underlying asset at some point in the future. That is, a forward contract fixes the price and the conditions now for an asset that will be delivered in the future. As each contract is tailor-made to suit user requirements, a forward contract is not as liquid as an exchange-traded futures contract with standardised terms.

The theoretical textbook price of a forward contract is the spot price of the underlying asset plus the funding cost associated with holding the asset until forward expiry date, when the asset is delivered. More formally we may write the price of a forward contract (written on an underlying asset that pays no dividends, such as a zero-coupon bond), as (34.5):

$$P_{fwd} = P_{spot}e^{rn} \quad (35.5)$$

where

- P_{spot} is the price of the underlying asset of the forward contract
- r is the continuously compounded risk-free interest rate for a period of maturity n
- n is the term to maturity of the forward contract in days.

The rule of no-arbitrage pricing states that (35.5) must be true. If $P_{fwd} < P_{und}e^{rn}$ then a trader could buy the cheaper instrument, the forward contract, and simultaneously sell the underlying asset. The proceeds from the short sale could be invested at r for n days; on expiry the short position in the asset is closed out at the forward price P_{fwd} and the trader will have generated a profit of $P_{und}e^{rn} - P_{fwd}$. In the opposite scenario, where $P_{fwd} > P_{und}e^{rn}$, a trader could put on a long position in the underlying asset, funded at the risk-free interest rate r for n days, and simultaneously sell the forward contract. On expiry the asset is sold under the terms of the forward contract at the forward price and the proceeds from the sale used to close out the funding initially taken on to buy the asset. Again a profit would be generated, which would be equal to the difference between the two prices.

The relationship described here is used by the market to assume that forward rates implied by the price of short-term interest-rate futures contracts are equal to forward rates given by a same-maturity forward contract. Although this assumption holds good for futures contracts with a maturity of up to three or four years, it breaks down for longer-dated futures and forwards. This is examined in Appendices 35.1 and 35.2 and in Chapter 40, in the section on convexity bias.

35.5 Short-term interest rate futures

35.5.1 Description

A *futures* contract is a transaction that fixes the price today for a commodity that will be delivered at some point in the future. Financial futures fix the price for interest rates, bonds, equities and so on, but trade in the same manner as commodity futures. Contracts for futures are standardised and traded on recognised exchanges. In London the main futures exchange is LIFFE, although other futures are also traded on for example, the International Petroleum Exchange and the London Metal Exchange. The money markets trade short-term interest rate futures, which fix the rate of interest on a notional fixed term deposit of money (usually for 90 days or three months) for a specified period in the future. The sum is notional because no actual sum of money is deposited when buying or selling futures; the instrument is off-balance sheet. Buying such a contract is equivalent to making a notional deposit, while selling a contract is equivalent to borrowing a notional sum.

The three-month interest-rate future is the most widely used instrument used for hedging interest-rate risk.

The LIFFE exchange in London trades short-term interest rate futures for major currencies including sterling, euros, yen and Swiss franc. Table 35.1 summarises the terms for the short sterling contract as traded on LIFFE.

Name	90-day sterling Libor interest rate future
Contract size	£500,000
Delivery months	March, June, September, December
Delivery date	First business day after the last trading day
Last trading day	Third Wednesday of delivery month
Price	100 minus interest rate
Tick size	0.005
Tick value	£6.25
Trading hours	LIFFE CONNECT™ 0805 - 1800 hours

Table 35.1: Description of LIFFE short sterling future contract. Source: LIFFE.

The original futures contracts related to physical commodities, which is why we speak of *delivery* when referring to the expiry of financial futures contracts. Exchange-traded futures such as those on LIFFE are set to expire every quarter during the year. The short sterling contract is a deposit of cash, so as its price refers to the rate of interest on this deposit, the price of the contract is set as $P = 100 - r$ where P is the price of the contract and r is the rate of interest at the time of expiry implied by the futures contract. This means that if the price of the contract rises, the rate of interest implied goes down, and vice versa. For example the price of the June 1999 short sterling future (written as Jun99 or M99, from the futures identity letters of H, M, U and Z for contracts expiring in March, June, September and December respectively) at the start of trading on 13 March 1999 was 94.880, which implied a three-month Libor rate of 5.12% on expiry of the contract in June. If a trader bought 20 contracts at this price and then sold them just before the close of trading that day, when the price had risen to 94.96, an implied rate of 5.04%, she would have made 16 ticks profit or £2000. That is, a 16 tick upward price movement in a long position of 20 contracts is equal to £2000. This is calculated as follows:

$$\begin{aligned}\text{Profit} &= \text{ticks gained} \times \text{tick value} \times \text{number of contracts} \\ \text{Loss} &= \text{ticks lost} \times \text{tick value} \times \text{number of contracts}.\end{aligned}$$

The tick value for the short sterling contract is straightforward to calculate, since we know that the contract size is £500,000, there is a minimum price movement (tick movement) of 0.005% and the contract has a three month “maturity”.

$$\text{Tick value} = 0.005\% \times £500,000 \times \frac{3}{12} = £6.25.$$

The profit made by the trader in our example is logical because if we buy short sterling futures we are depositing (notional) funds; if the price of the futures rises, it means the interest rate has fallen. We profit because we have “deposited” funds at a higher rate beforehand. If we expected sterling interest rates to rise, we would sell short sterling futures, which is equivalent to borrowing funds and locking in the loan rate at a lower level.

Note how the concept of buying and selling interest rate futures differs from FRAs: if we buy an FRA we are borrowing notional funds, whereas if we buy a futures contract we are depositing notional funds. If a position in an interest rate futures contract is held to expiry, cash settlement will take place on the delivery day for that contract.

Short-term interest rate contracts in other currencies are similar to the short sterling contract and trade on exchanges such as Deutsche Terminbourse in Frankfurt and MATIF in Paris.

35.5.2 Pricing interest rate futures

The price of a three-month interest rate futures contract is the implied interest rate for that currency’s three-month rate at the time of expiry of the contract. Therefore there is always a close relationship and correlation between futures prices, FRA rates (which are derived from futures prices) and cash market rates. On the day of expiry the price of the future will be equal to the Libor rate as fixed that day. This is known as the Exchange Delivery Settlement Price (EDSP) and is used in the calculation of the delivery amount. During the life of the contract its price will be less closely related to the actual three-month Libor rate *today*, but closely related to the *forward rate* for the time of expiry.

Equation (35.1) was our basic forward rate formula for money market maturity forward rates, which we adapted to use as our FRA price equation. If we incorporate some extra terminology to cover the dealing dates involved it can also be used as our futures price formula. Let us say that:

T_0	is the trade date
T_M	is the contract expiry date
T_{CASH}	is the value date for cash market deposits traded on T_0
T_1	is the value date for cash market deposits traded on T_M
T_2	is the maturity date for a three-month cash market deposit traded on T_M

We can then use equation (35.2) as our futures price formula to obtain P_{fut} , the futures price for a contract up to the expiry date.

$$P_{fut} = 100 - \frac{r_2 n_2 - r_1 n_1}{n_f \left(1 + r_1 \frac{n_1}{365} \right)} \quad (35.6)$$

where

P_{fut}	is the futures price
r_1	is the cash market interest rate to T_1
r_2	is the cash market interest rate to T_2
n_1	is the number of days from T_{CASH} to T_1
n_2	is the number of days from T_{CASH} to T_2
n_f	is the number of days from T_1 to T_2 .

The formula uses a 365 day count convention which is applies in the sterling money markets; where a market uses a 360-day base this must be used in the equation instead.

In practice the price of a contract at any one time will be close to the theoretical price that would be established by (35.6) above. Discrepancies will arise for supply and demand reasons in the market, as well as because Libor rates are often quoted only to the nearest sixteenth or 0.0625. The price between FRAs and futures are correlated very closely, in fact banks will often price FRAs using futures, and use futures to hedge their FRA books. When hedging a FRA book with futures, the hedge is quite close to being exact, because of the two prices track each other almost tick for tick.² However the tick value of a futures contract is fixed, and uses (as we saw above) a 3/12 basis, while FRA settlement values use a 360 or 365 day base. The FRA trader will be aware of this when putting on her hedge.

In our discussion of forward rates in Chapter 6 we emphasised that they were the markets view on future rates using all information available today. Of course a futures price today is very unlikely to be in line with the actual three-month interest rate that is prevailing at the time of the contract's expiry. This explains why prices for futures and actual cash rates will differ on any particular day. Up until expiry the futures price is the implied forward rate; of course there is always a discrepancy between this forward rate and the cash market rate *today*. The gap between the cash price and the futures price is known as the *basis*. This is defined as:

$$\text{Basis} = \text{Cash price} - \text{Futures price}.$$

At any point during the life of a futures contract prior to final settlement – at which point futures and cash rates converge – there is usually a difference between current cash market rates and the rates implied by the futures price. This is the difference we've just explained; in fact the difference between the price implied by the current three-month interbank deposit and the futures price is known as *simple basis*, but it is what most market participants refer to as the basis. Simple basis consists of two separate components, *theoretical basis* and *value basis*. Theoretical basis is the difference between the price implied by the current three-month interbank deposit rate and that implied by the theoretical fair futures price based on cash market forward rates, given by (35.6) above. This basis may be either positive or negative depending on the shape of the yield curve; this is illustrated in Example 35.7.

² That is, the basis risk is minimised.

EXAMPLE 35.7 Theoretical basis

Let us examine the relationship between the shape of the yield curve and the basis. Assume that today is 14 March.

1. Negative yield curve; negative theoretical basis

Three-month LIBOR	6.50%		
Six-month LIBOR	6.375%		
Nine-month LIBOR	6.25%		
One-year LIBOR	6.1875%		
Cash price (100 – LIBOR)	Contract	Fair futures price	Theoretical basis
93.5	Jun	93.85	–0.35
	Sep	94.19	–0.69
	Dec	94.27	–0.77

2. Flat yield curve; negative theoretical basis

Three-month LIBOR	6.50%		
Six-month LIBOR	6.50%		
Nine-month LIBOR	6.50%		
One-year LIBOR	6.50%		
Cash price (100 – LIBOR)	Contract	Fair futures price	Theoretical basis
93.50	Jun	93.60	–0.10
	Sep	93.70	–0.20
	Dec	93.80	–0.30

3. Positive yield curve; positive theoretical basis

Three-month LIBOR	6.50%		
Six-month LIBOR	6.75%		
Nine-month LIBOR	6.9375%		
One-year LIBOR	7.125%		
Cash price (100 – LIBOR)	Contract	Fair futures price	Theoretical basis
93.50	Jun	93.11	0.39
	Sep	92.93	0.57
	Dec	92.69	0.81

The above is a very simple example and the steepness and shape of the yield curve will have an impact on the assumptions shown

The value basis is the difference between the theoretical fair futures price and the actual futures price. It is a measure of how under- or over-valued the futures contract is relative to its fair value. Value basis reflects the fact that a futures contract does not always trade at its mathematically calculated theoretical price, due to the impact of market sentiment and demand and supply. The theoretical and value can and do move independently of one another and in response to different influences. Both however converge to zero on the last trading day when final cash settlement of the futures contract is made.

Futures contracts do not in practice provide a precise tool for locking into cash market rates today for a transaction that takes place in the future, although this is what they are in theory designed to do. Futures do allow a bank to lock in a rate for a transaction to take place in the future, and this rate is the *forward rate*. The basis is the difference between today's cash market rate and the forward rate on a particular date in the future. As a futures contract approaches expiry, its price and the rate in the cash market will converge (the process is given the name *convergence*). As we noted earlier this is given by the EDSP and the two prices (rates) will be exactly in line at the exact moment of expiry.

EXAMPLE 35.8 The Eurodollar futures contract

The Eurodollar futures contract is traded on the Chicago Mercantile Exchange. The underlying asset is a deposit of US dollars in a bank outside the United States, and the contract is on the rate on dollar 90-day Libor. The Eurodollar future is cash settled on the second business day before the third Wednesday of the delivery month (London business day). The final settlement price is used to set the price of the contract, given by:

$$10000(100 - 0.25r)$$

where r is the quoted Eurodollar rate at the time. This rate is the actual 90-day Eurodollar deposit rate.

The longest-dated Eurodollar contract has an expiry date of 10 years. The market assumes that futures prices and forward prices are equal; Appendix 35.1 shows that this is indeed the case under conditions where the risk-free interest rate is constant and the same for all maturities. In practice it also holds for short-dated futures contracts, but is inaccurate for longer-dated futures contracts. Therefore using futures contracts with a maturity greater than five years to calculate zero-coupon rates or implied forward rates will produce errors in results, which need to be taken into account if the derived rates are used to price other instruments such as swaps.

35.5.3 Hedging using interest-rate futures

Banks use interest rate futures to hedge interest rate risk exposure in cash and OBS instruments. Bond trading desks also often use futures to hedge positions in bonds of up to two or three years' maturity, as contracts are traded up to three years' maturity. The liquidity of such "far month" contracts is considerably lower than for near month contracts and the "front month" contract (the current contract, for the next maturity month). When hedging a bond with a maturity of say two years' maturity, the trader will put on a *strip* of futures contracts that matches as near as possible the expiry date of the bond.

The purpose of a hedge is to protect the value of a current or anticipated cash market or OBS position from adverse changes in interest rates. The hedger will try to offset the effect of the change in interest rate on the value of their cash position with the change in value of their hedging instrument. If the hedge is an exact one the loss on the main position should be compensated by a profit on the hedge position. If the trader is expecting a fall in interest rates and wishes to protect against such a fall they will buy futures, known as a long hedge, and will sell futures (a short hedge) if wishing to protect against a rise in rates.

Bond traders also use three-month interest rate-contracts to hedge positions in short-dated bonds; for instance, a market maker running a short-dated bond book would find it more appropriate to hedge his book using short-dated futures rather than the longer-dated bond futures contract. When this happens it is important to accurately calculate the correct number of contracts to use for the hedge. To construct a bond hedge it will be necessary to use a strip of contracts, thus ensuring that the maturity date of the bond is covered by the longest-dated futures contract. The hedge is calculated by finding the sensitivity of each cash flow to changes in each of the relevant forward rates. Each cash flow is considered individually and the hedge values are then aggregated and rounded to the nearest whole number of contracts.

Figure 35.3 is a reproduction of page TED on a Bloomberg terminal, which calculates the strip hedge for short-dated bonds.³ The example shown is for short-sterling contract hedge for a position in the UK 8% 2000 gilt, for settlement on 21 May 1999. The screen shows the number of each contract that must be bought (or sold) to hedge the position, which in the example is a holding of £10 million of the bond. The "stub" requirement is met using the near-month contract. A total of 122 contracts are required.

³ This screen was introduced in 1995, the author recalls being in message communication with Bloomberg in Princeton discussing the screen before it went live!

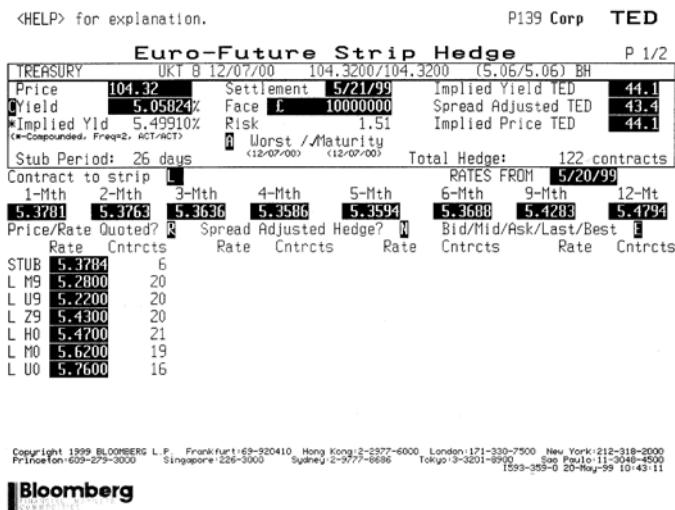


Figure 35.3: Bloomberg screen TED page.
 ©Bloomberg LP. Reproduced with permission.

The following examples illustrate hedging with short-term interest-rate contracts.

EXAMPLE 35.9 Hedging a forward three-month lending requirement

On 1 June a corporate treasurer is expecting a cash inflow of £10 million in three months' time (1 December), which they will then invest for three months. The treasurer expects that interest rates will fall over the next few weeks and wishes to protect themselves against such a fall. This can be done using short sterling futures. Market rates on 1 June are as follows:

3-mo Libor 6½%
 Sep futures price 93.220

The treasurer buys 20 September short sterling futures at 93.220, this number being exactly equivalent to a sum of £10 million. This allows them to lock in a forward *lending* rate of 6.78%, if we assume there is no bid-offer quote spread.

Expected lending rate = rate implied by futures price
 = $100 - 93.220$
 = 6.78%.

On 1 September market rates are as follows:

3-mo Libor 6¼%
 Sep futures price 93.705

The treasurer unwinds the hedge at this price.

Futures p/l = + 97 ticks (93.705 - 93.22), or 0.485%
 Effective lending rate = 3-mo Libor + futures profit
 = 6.25% + 0.485%
 = 6.735%.

The treasurer was quite close to achieving their target lending rate of 6.78% and the hedge has helped to protect against the drop in Libor rates from 6½% to 6¼%, due to the profit from the futures transaction.

In the real world the cash market bid-offer spread will impact the amount of profit/loss from the hedge transaction. Futures generally trade and settle near the offered side of the market rate (Libor) whereas lending, certainly by corporates, will be nearer the Libid rate.

EXAMPLE 35.10 Hedging a forward six-month borrowing requirement⁴

A treasury dealer has a six-month borrowing requirement for DEM30 million in three months' time, on 16 September. She expects interest rates to rise by at least ½% before that date and would like to lock in a future borrowing rate. The scenario is detailed below.

Date:	16 June
Three-month LIBOR	6.0625%
Six-month LIBOR	6.25
Sep futures contract	93.66
Dec futures contract	93.39

In order to hedge a six month DEM30 million exposure the dealer needs to use a total of 60 futures contracts, as each has a nominal value of DEM1 million, and corresponds to a three-month notional deposit period. The dealer decides to sell 30 September futures contracts and 30 December futures contracts, which is referred to as a *strip* hedge. The expected forward borrowing rate that can be achieved by this strategy, where the expected borrowing rate is rf , is calculated as follows:

$$1 + rf \times \frac{\text{days in period}}{360} = \left(1 + \text{Sep implied rate} \times \frac{\text{Sep days period}}{360} \right) \times \left(1 + \text{Dec implied rate} \times \frac{\text{Dec days period}}{360} \right).$$

Therefore we have:

$$1 + rf \times \frac{180}{360} = \left(1 + 0.0634 \times \frac{90}{360} \right) \times \left(1 + 0.0661 \times \frac{90}{360} \right) \Rightarrow rf = 6.53\%.$$

The rate rf is sometimes referred to as the “strip rate”.

The hedge is unwound upon expiry of the September futures contract. Assume the following rates now prevail:

Three-month LIBOR	6.4375%
Six-month LIBOR	6.8125
Sep futures contract	93.56
Dec futures contract	92.93

The futures profit-and-loss is:

September contract:	+10 ticks
December contract:	+46 ticks

This represents a 56 tick or 0.56% profit in three-month interest-rate terms, or 0.28% in six-month interest-rate terms. The effective borrowing rate is the six-month LIBOR rate minus the futures profit, or:

$$6.8125\% - 0.28\% \text{ or } 6.5325\%.$$

In this case the hedge has proved effective because the dealer has realised a borrowing rate of 6.5325%, which is close to the target strip rate of 6.53%.

The dealer is still exposed to the basis risk when the December contracts are bought back from the market at the expiry date of the September contract. If for example, the future was bought back at 92.73, the effective borrowing rate would be only 6.4325%, and the dealer would benefit. Of course the other possibility is that the futures contract could be trading 20 ticks more expensive, which would give a borrowing rate of 6.6325%, which is 10 basis points above the target rate. If this happened, the dealer may elect to borrow in the cash market for three months, and maintain the December futures position until the December contract expiry date, and roll over the borrowing at that time. The profit (or loss) on the December futures position will compensate for any change in three-month rates at that time.

⁴ This example pre-dates the introduction of the euro.

35.5.4 Refining the hedge ratio

A futures hedge ratio is calculated by dividing the amount to be hedged by the nominal value of the relevant futures contract and then adjusting for the duration of the hedge. When dealing with large exposures and/or a long hedge period, inaccuracy will result unless the hedge ratio is refined to compensate for the timing mismatch between the cash flows from the futures hedge and the underlying exposure. Any change in interest rates has an immediate effect on the hedge in the form of daily variation margin, but only affects the underlying cash position on maturity, that is, when the interest payment is due on the loan or deposit. In other words, hedging gains and losses in the futures position are realised over the hedge period while cash market gains and losses are deferred. Futures gains may be reinvested, and futures losses need to be financed.

The basic hedge ratio is usually refined to counteract this timing mismatch, this process is sometimes called “tailing”.

EXAMPLE 35.11 Refining the hedge ratio

A dealer is hedging a three-month SFr100 million borrowing commitment commencing in two months' time and wishes to determine an accurate hedge ratio.

Two-month LIBOR	4.75%
Five-month LIBOR	4.875%
Implied 2v5 fwd–fwd rate	4.92%

The first part of the process to refine the hedge ratio involves measuring the sensitivity of the underlying position to a change in interest rates, that is the cost of a basis point move in LIBOR on the interest payment or receipt on maturity. We therefore calculate the basis point value as follows:

$$\text{Principal} \times 0.01\% \times \frac{\text{term (days)}}{360}. \text{ In this case the PVBP is Sfr2,500.}$$

For every basis point increase (decrease) in three-month LIBOR, the dealer's interest-rate expense at the maturity of the loan will increase (decrease) by SFr2,500. Correspondingly a SFr2,500 gain (loss) will be realised over the hedge period on the futures position. The present value of the futures gain (loss) is therefore greater than the present value of the loss (gain) on the loan. In other words, a hedge position consisting of short 100 lots is an over-hedge.

To calculate a more precise hedge ratio, the dealer needs to discount the nominal basis point value of the interest payments back from the maturity date of the loan to the start date of the loan. The discounting rate used in this calculation is the forward-forward rate over the loan period, that is, three months, implied by the current LIBOR market rates (given here as the 2v5 fwd–fwd rate). The discounting period may vary depending on assumptions about the timing of cash flows. The formula for calculating the present value at the start date of the loan of a basis point move is calculated as:

$$\frac{\text{Nominal value of a basis point}}{1 + \text{fwd–fwd rate} \times \frac{\text{loan period (days)}}{360}}.$$

For the dealer then the calculation is:

$$\frac{2500}{1 + 4.92\% \times \frac{90}{360}} = \text{SFr2,469.6.}$$

To obtain the hedge ration from this figure, the dealer would divide the PVBP value by the tick value of the futures contract, which for the LIFFE Euroswiss contract is SFr25.

$$\frac{\text{SFr2,470}}{\text{SFr25}} = 98.80.$$

Therefore the correct number of contracts needed to hedge the SFR100 million exposure would be 99, rather than 100.

Appendices

APPENDIX 35.1 The forward interest rate and futures-implied forward rate

The markets assume that the forward rate implied by the price of a futures contract is the same as the futures price itself for a contract with the same expiry date. This assumption is the basis on which futures contracts are used to price swaps and other forward rate instruments such as FRAs. In the following appendix (Appendix 35.2) we summarise a strategy first described by Cox, Ingersoll and Ross (1981) to show that under certain assumptions, namely that when the risk-free interest rate is constant and identical for all maturities (that is, in a flat term structure environment), this assumption holds true. However in practice, because the assumptions are not realistic under actual market conditions, this relationship does not hold for longer-dated futures contracts and forward rates. In the first place, term structures are rarely flat or constant. The main reason however is because of the way futures contracts are settled, compared to forward contracts. Market participants who deal in exchange-traded futures must deposit daily margin with the exchange clearing house, reflecting their profit and loss on futures trading. Therefore a profit on a futures position will be received immediately, and in a positive-sloping yield curve environment this profit will be invested at a higher-than-average rate of interest. In the same way a loss on futures trading would have to be funded straight away, and the funding cost would be at a higher-than-average rate of interest. However the profit on a forward contract is not realised until the maturity of the contract, and so a position in a forward is not affected by daily profit or loss cash flows. Therefore, a long-dated futures contract will have more value to an investor than a long-dated forward contract, because of the opportunity to invest mark-to-market gains made during the life of the futures contract.

When the price of the underlying asset represented by a futures contract is positively correlated with interest rates, the price of futures contracts will be higher than the price of the same-maturity forward contract. When the price of the underlying asset is negatively correlated with interest rates, which is the case with three-month interest-rate futures like short sterling, forward prices are higher than futures prices. That is, the forward interest rate is lower than the interest rate implied by the futures contract price. This difference is not pronounced for short-dated contracts, and so is ignored by the market. There are also other factors that will cause a difference in forward and futures prices, the most significant of these being transaction costs and liquidity: it is generally cheaper to trade exchange-traded futures and they tend to more liquid instruments. However for long-dated instruments, the difference in treatment between forwards and futures means that their rates will not be the same, and this difference needs to be taken into account when pricing long-dated forward instruments. This is discussed in Chapter 40, in the section on convexity bias.

APPENDIX 35.2 Arbitrage proof of the futures price being equal to the forward price

Under certain assumptions it can be shown that the price of same-maturity futures and forward contracts are equal. The primary assumption is that interest rates are constant. The strategy used to prove this was first described by Cox, Ingersoll and Ross (1981).

Consider a futures contract with maturity of T days and with a price of P_t at the end of day t . Set r as the constant risk-free interest rate of interest per day. Assume a trading strategy that consists of:

- establishing a long position in the futures of e^r at the start of day 0;
- adding to the long position to make a total of e^{2r} at the end of day 1;
- adding to the long position to make a total of e^{3r} at the end of day 2;
- increasing the size of the position daily by the amount shown.

At the start of day t the long position is e^{rt} . The profit or loss from the position is given by (35.7):

$$\text{profit or loss} = (P_t - P_{t-1})e^{rt}. \quad (35.7)$$

If this amount is compounded on a daily basis using r , the final value on the expiry of the contract is given by:

$$(P_t - P_{t-1})e^{rt}e^{r(T-t)} = (P_t - P_{t-1})e^{rT}$$

so that the value of the position on the expiry of the contract at the end of day T is given by (35.8):

$$FV = \sum_{t=1}^T (P_t - P_{t-1}) e^{rT}. \quad (35.8)$$

The expression at (35.8) may also be written as (35.9):

$$\begin{aligned} FV &= ((P_T - P_{T-1}) + (P_{T-1} - P_{T-2}) + \cdots + (P_1 - P_0)) e^{rT} \\ &= (P_T - P_0) e^{rT}. \end{aligned} \quad (35.9)$$

In theory the price of a futures contract on expiry must equal the price of the underlying asset on that day. If we set the price of the underlying asset on expiry as $P_{T-\text{underlying}}$, as P_T is equal to the final price of the contract on expiry, the final value of the trading strategy may be written as (35.10):

$$FV = (P_{T-\text{underlying}} - P_0) e^{rT}. \quad (35.10)$$

Investing P_0 in a risk-free bond and using the same strategy as that described above will therefore return:

$$P_0 e^{rT} + (P_{T-\text{underlying}} - P_0) e^{rT}$$

or an amount equal to $P_{T-\text{underlying}} e^{rT}$ at the expiry of the contract at the close of day T . Therefore this states that an amount P_0 may be invested to return a final amount of $P_{T-\text{underlying}} e^{rT}$ at the end of day T .

Assume that the forward contract price at the end of day 0 is $P_{0-\text{forward}}$. By investing this amount in a risk-free bond, and simultaneously establishing a long forward position of e^{rT} forward contracts, we are guaranteed an amount $P_{T-\text{underlying}} e^{rT}$ at the end of day T . We therefore have two investment strategies that both return a value of $P_{T-\text{underlying}} e^{rT}$ at the end of the same time period; one strategy requires an investment of P_0 while the other requires an investment of $P_{0-\text{forward}}$. Under the rule of no-arbitrage pricing, the price of both contracts must be equal, that is,

$$P_0 = P_{0-\text{fwd}}.$$

That is, the price of the futures contract and the price of the forward contract at the end of day 0 are equal.

Selected bibliography and references

- Chicago Board of Trade, *Interest Rate Futures for Institutional Investors*, CBOT, 1987.
- Cox, J., Ingersoll, J., Ross, S., "The Relationship between Forward Prices and Futures Prices", *Journal of Financial Economics* 9, December 1981, pp. 321–346.
- Figlewski, F., *Hedging with Financial Futures for Institutional Investors*, Probus Publishing, 1986.
- French, K., "A Comparison of Futures and Forwards Prices", *Journal of Financial Economics* 12, November 1983, pp. 311–342.
- Hull, J., *Options, Futures and Other Derivatives*, 4th edition, Prentice-Hall, 1999.
- Jarrow, R., Oldfield, G., "Forward Contracts and Futures Contracts", *Journal of Financial Economics* 9, December 1981, pp. 373–382.
- Veit, W., Reiff, W., "Commercial Banks and Interest Rate Futures: A Hedging Survey", *Journal of Futures Markets* 3, 1983, pp. 283–293.

Questions and exercises

- The following are sterling cash market rates:

1 month	5.75%
3 months	5.875%
6 months	6.125%

 Assuming a month has 30 days, what is the theoretical price of a 1x6 FRA?
- During the quiet period between Christmas and New Year, a junior trader on the money market desk, Brett, is in charge of the book. He borrows £15 million at 6.625% for six months and deposits the same sum for three

months at 6.475%. He is considering how to hedge the interest rate gap, and notes the following FRA prices on screen:

3v6: 6.14 – 6.11
6v9: 6.24 – 6.21

- (a) How should he hedge the gap? Detail the FRA he should transact in, the rate and amount.
(b) On the fixing date for the FRAs in the previous question, the three-month cash rate is 5.89%–5.81%. what is the settlement amount for the FRA? Does young Brett make a profit or loss on this set of trades?
3. A news screen shows the following prices for a short-term interest rate contract. What is the theoretical price of the 3v12 FRA? Assume that the March futures contract settlement date is precisely three months from the spot settlement date.

March 96.49
June 96.62
September 96.74

4. Consider two futures contracts with prices P_1 and P_2 with maturity dates of n_1 and n_2 , where $n_1 < n_2$. Prove that the following relationship holds true:

$$P_2 \leq (P_1 + P_{und})e^{rn}$$

where r is the constant risk-free rate of interest for the period n (which is $n_2 - n_1$) and P_{und} is the spot price of the underlying asset.

5. Brett the junior money markets trader is currently running a £20 million forward interest-rate gap, in the form of funding requirement, in the six-month period from today (in June) to December. Using the money market quotes below, advise Brett on the best and cheapest way to hedge his forward gap, bearing in mind he will at some point have to cover the funding requirement by borrowing in the market. Assume that all dates match and that each month has 30 days.

Interbank: 3 month 5.25% – 5.20%
6 month 5.35% – 5.30%
9 month 5.5625% – 5.50%
FRAs: 3v6 5.57% – 5.54%
6v9 5.62% – 5.59%
3v9 5.70% – 5.67%
Futures: Jun 94.30
Sep 94.25

Part V Risk Management

During the 1990s the effects of a number of high-profile trading losses, perhaps best exemplified by the collapse of Barings Bank in 1995, alerted bank senior managers of the importance of having robust and effective procedures and controls to assist them in the management of market risk exposure. Risk management is now closely related to the activity of investment banking, particularly market risk management (ironically often the risk management department is one of the fastest growing divisions of an investment bank) and therefore a section on this subject is not out of place in a book on bond markets and bond trading. In Chapter 36 we introduce the subject of market risk management. In Chapter 37 we review Value-at-Risk, one of the main market risk measurement tools.

36 Risk Management

In this chapter we will consider aspects of the risks to which participants in the capital markets are exposed, and the risk management function to which banks and securities houses now devote a significant part of their resources. The profile of the risk management function and risk measurement tools such as Value-at-Risk has been raised during the 1990s, following bank collapses such as that of Barings and trading losses suffered by banks such as Kidder Peabody, Daiwa and Sumitomo. It was widely alleged that one of the driving forces behind the merger of the old UBS with Swiss Bank (in reality a takeover of UBS by Swiss Bank; the merged entity was named UBS) was the discovery of a multi-million loss on UBS's currency options trading book, which senior management had been unaware of right up until its discovery. In any case shareholders of banks now demand greater comfort that senior executives are aware of the trading risks that their bank is exposed to, and that robust procedures exist to deal with these risks. For this reason it is now common for all staff, front-, middle- and back- office, to be familiar with the risk management function in a bank.

In this chapter we present a brief overview of the risk management function for financial market practitioners; the following chapter goes on to review the Value-at-Risk tool, which has become the main market (and credit) risk measurement methodology used in the market. This is of course an introduction to the subject and interested readers are directed to the bibliography at the end of this and the next chapter.

36.1 Introduction

For participants in the financial markets risk is essentially a measure of the volatility of asset returns, although it has a broader definition as being any type of uncertainty as to future outcomes. The types of risk that a bank or securities house is exposed to as part of its operations in the bond and capital markets are broadly characterised as follows:

- **Market risk.** Risk arising from movements in prices in financial markets. Examples include foreign exchange (FX) risk, interest rate risk and basis risk.
- **Credit risk.** This refers to the risk that an issuer of debt will default. *Counterparty risk* refers to the risk that a counterparty from whom one has dealt with will cease trading, making recovery of funds owed difficult. Examples include sovereign risk, marginal risk and *force majeure* risk.
- **Liquidity risk.** The risk that a bank has insufficient funding to meet commitments as they arise. For a securities house, it is the risk that the market for its assets becomes too thin to enable fair and efficient trading to take place.
- **Operational risk.** Risk of loss associated with non-financial matters such as fraud, system failure, accidents and ethics.

We can look at some of these risk types in some more detail.

- **Market risk.** This risk reflects uncertainty as to an asset's price when it is sold. Market risk is the risk arising from movements in financial market prices; for bondholders it is the risk arising from movement in interest rates, and this is specifically referred to as *interest rate risk*.
- **Currency risk.** This arises from exposure to movements in FX rates. Currency risk is often sub-divided into *transaction* risk, where currency fluctuations affect the proceeds from day-to-day transactions, and *translation* risk, which affects the value of assets and liabilities on a balance sheet.
- **Other market risks.** There are residual market risks which fall in this category. Among these are *volatility* risk, which affects option traders, and *basis* risk, which has a wider impact. Basis risk arises whenever one kind of risk exposure is hedged with an instrument that behaves in a similar, but not necessarily identical manner. One example would be a company using 3-month interest rate futures to hedge its commercial paper programme. Although eurocurrency rates, to which futures prices respond, are well correlated with commercial paper rates,

they do not invariably move in lock step. If CP rates moved up by 20 basis points but futures prices dropped by only 15 basis points, the 5 basis point gap would be the basis risk in this case.

- **Liquidity risk.** This is the potential risk arising when an entity cannot meet payments when they fall due. It may involve borrowing at an excessive rate of interest, facing penalty payments under contractual terms, or selling assets at below market prices (*forced sale risk*). It also refers to an inability to trade or obtain a price when desired, due to lack of supply or demand or a shortage of market-makers.
- **Concentration risk.** Any organisation with too great a proportion of its assets invested in one type of instrument, or in one specific geographical or industrial sector, is exposed to concentration risk. Banks will seek to limit this type of risk exposure by diversifying across investment types and geographical and country boundaries.
- **Reinvestment risk.** If an asset makes any payments before the investor's horizon, whether it matures or not, the cash flows will have to be reinvested until the horizon date. Since the reinvestment rate is unknown when the asset is purchased, the final cash flow is uncertain.
- **Sovereign risk.** This is a type of credit risk specific to a government bond. There is minimal risk of default by an industrialised country. A developing country may default on its obligation (or declare a debt "moratorium") if debt payments relative to domestic product reach unsustainable levels.
- **Pre-payment risk.** This is specific to mortgage-backed and certain asset-backed bonds. For example mortgage lenders allow the homeowner to repay outstanding debt before the stated maturity. If interest rates fall prepayment will occur, which forces reinvestment at rates lower than the initial yield.
- **Model risk.** Some of the latest financial instruments such as exotic options are heavily dependent on complex mathematical models for pricing and hedging. If the model is incorrectly specified, is based on questionable assumptions or does not accurately reflect the true behaviour of the market, a bank relying on it could suffer extensive losses.

36.2 Risk management

The risk management function grew steadily in size and importance within commercial and investment banks during the 1990s. The development of the risk management function and risk management departments was not instituted from a desire to eliminate the possibility of all unexpected losses, should such an outcome indeed be feasible; rather from a wish to control the frequency, extent and size of trading losses in such a way as to provide the minimum surprise to senior management and shareholders.

Risk exists in all competitive business although the balance between financial risks of the types described above and general and management risk varies with the type of business that is being engaged in. The key objective of the risk management function within a financial institution is to allow for a clear understanding of the risks and exposures the firm is engaged in, such that any monetary loss is deemed acceptable by the firm. The acceptability of any loss should be on the basis that such (occasional) loss is to be expected as a result of the firm being engaged in a particular business activity. If the bank's risk management function is effective, there will be no over-reaction to any unexpected losses, which may increase eventual costs to many times the original loss amount.

While there is no one agreed organisation structure for the risk management function, the following may be taken as being reflective of the typical bank set-up:

- an independent, "middle office" department responsible for compiling and explicitly stating the bank's approach to risk, and defining trading limits and the areas of the market that the firm may have exposure to;
- the head of the risk function reporting to an independent senior manager, who is a member of the Executive Board;
- monitoring the separation of duties between front, middle and back office, often in conjunction with an internal audit function;
- reporting to senior management, including the firm's overall exposure and adherence of the front office to the firm's overall risk strategy;
- communication of risks and risk strategy to shareholders;

- where leading edge systems are in use, employment of the risk management function to generate competitive advantage in the market as well as control.

The risk management function is more likely to deliver effective results when there are clear lines of responsibility and accountability. “Dotted lines” of responsibility or a lack of clear accountability hamper effective decision-making in a crisis. It is also important that the risk management department interacts closely with other areas of the front and back office. In addition to the above, the following are often accepted as ingredients of a market best-practice risk management framework in an institution that is engaged in investment banking and trading activity:

- daily overview of risk exposure profile and profit & loss reports (p&l);
- *Value-at-Risk* as a common measure of risk exposure, in addition to other measures including “jump risk” to allow for market corrections;
- independent daily monitoring of risk utilisation by a middle-office risk management function;
- independent production of daily p&l, and independent review of front-office closing prices on a daily basis;
- independent validation of market pricing, and pricing and VaR models.

The risk manager will be concerned with the extent of market risk that the bank is exposed to, but also with communicating this to senior management and liaising with the front office to keep in touch with latest developments. Risk management then is:

- an understanding of how the value of the banking and trading books changes in line with movements in underlying factors;
- a quantitative measurement of how liquid hedge instruments move in line with underlying factors;
- defining “levels of comfort” with regard to risk exposure, using statistical analysis to quantify comfort levels, based on market data;
- constructing and/or appraising the hedging arrangements designed to bring a portfolio within comfort levels if at any time it has exceeded it;
- adding value in enhancing return on capital by dint of “sensible” risk taking;
- communicating management information on levels of return with respect to the level of risk taken on.

It is recognised that the traditional risk measures used in the past, for example the duration-based measures, suffered from drawbacks as they capture linear risk only. The emphasis now is more on second-order risk measures, that is the convexity exposure on a portfolio, or *gamma* in an option book. It is the area of options trading that has observed the most development in risk management techniques, as options – which, unlike all other financial instruments, do not have a linear payout profile – are not well served by the traditional risk measures. The new approach to risk management, developed in response to high profile banking failures, focuses on:

- measurement of *gamma* and *vega* risks;
- more emphasis on likely future impact on the banking portfolio, and use of scenario planning to identify major risks;
- use of risk assessment to guide trading decisions;
- a new emphasis on correlation risk, the impact of different instruments and the correlation of their returns, and the benefits available from diversification.

A key measurement tool is *value-at-risk*, introduced in its first form by JP Morgan in 1994. Their *RiskMetrics* methodology was made available to any interested party over the Internet. In the next chapter we review the VaR risk tool.

The risk management function is now recognised as an important function in commercial and investment banking. A set of formal processes and procedures is now recognised as being essential to minimise the possibility of loss due to movements in the market, counterparty credit issues and internal issues. The art of risk management

encompasses more than a measurement tool such as value-at-risk; internal controls to ensure the separation of duties, and adequate provision for the provision of effective management information are also vital.

36.3 Non-VaR measure of risk

Portfolio managers employ a number of non-VaR measures of risk. One of these is the *Van ratio*. The Van ratio expresses the probability of an investment suffering a loss for a defined period, usually one year. For example a Van ratio of 20% indicates that there is a 1 in 5 chance of a loss during every four-quarter rolling window. The ratio first uses the following fraction to calculate this probability:

$$\frac{\text{compound annual return for the measurement period}}{\text{average four-quarter standard deviation for the measurement period}}$$

The probability of a loss is then calculated using standard normal curve probability tables (for more information on the normal distribution refer to the technical appendix).

The Van ratio provides an intuitive measure of *absolute* risk, the concept of the probability of a loss. To this end its calculation has assumed a normal distribution of returns. The assumption of normality of returns is important in the concept of VaR as calculated by most of the models and methodologies in use in financial institutions.

EXAMPLE 36.1 Financial market failures

A German industrial company suffered large losses because it was unable to meet its margin calls on oil futures contracts whose prices had gone against it. A mixture of slow management appraisal of the situation and lack of adequate cash budgeting led to the problem being exacerbated.

The investment banking arm of a UK clearing bank used a pricing model to value interest-rate options that was inadequate. When the problem was discovered the option book was found to be overvalued by over £100 million. This was an example of *model risk*, and banks now make strenuous efforts to validate their pricing models and obtain external third-party verification for them.

A US Treasury bond dealer allegedly used a procedure of forward booking of pricing to falsely claim profits on a book when there were none. A procedure that insisted that someone other than the trader input prices for the book, together with independent verification of these prices, would have prevented this.

A floor trader working with a UK merchant bank suffers large losses on an option position, but conceals them by mis-marking the prices and booking failed trades to an account that only he is aware of; he then generates false profits so that his request for cash to cover margin calls is accepted by the head office. He increases his exposure in the hope that the market will move in his favour, but the extra losses he incurs cause the bank to collapse. A more effective management control procedure may have prevented this, together with an effective risk measurement system that captured the bank's risk exposure. However even this may not have worked, as the trader was allowed to book his own trades and operate a separate settlement account. This case emphasises the need for adequate separation of duties.

Selected bibliography and references

- Beaver, W., Parker, G. (eds.), *Risk Management: Problems and Solutions*, McGraw-Hill, 1995.
- Bernstein, P., *Against the Gods: the Remarkable Story of Risk*, Wiley, 1996.
- Bierwag, G., "The art of risk management", *Journal of Portfolio Management*, Spring 1987, pp. 27–36.
- Rawnsley, J., *Total Risk: Nick Leeson and the Fall of Barings Bank*, Harper, 1995.
- Uyemura, D., Van Deventer, D., *Financial Risk Management in Banking*, Bankers Publishing Company and Probus Publishing, 1993.

37

Bank Risk Exposure and Value-at-Risk

In this chapter we review the main market risk measurement tool used in banking, known as *Value-at-Risk*. The review looks at the three main methodologies used to calculate VaR, as well as some of the key assumptions used in the calculations, including those on the normal distribution of returns, volatility levels and correlations. We also consider the range of trading market risks that a bank is exposed to.

37.1 Value-at-Risk

The introduction of Value-at-Risk (VaR) as an accepted methodology for quantifying market risk, and its adoption by bank regulators is part of the evolution of risk management. The application of VaR has been extended from its initial use in securities houses to commercial banks and corporates, following its introduction in October 1994 when JP Morgan launched RiskMetrics™ free over the Internet.

37.1.1 Definition

Value-at-Risk is a measure of the worst expected loss that a firm may suffer over a period of time that has been specified by the user, under normal market conditions and a specified level of confidence. This measure may be obtained in a number of ways, using a statistical model or by computer simulation.

VaR is a measure of market risk. It is the maximum loss which can occur with X% confidence over a holding period of n days.

VaR is the expected loss of a portfolio over a specified time period for a set level of probability. For example if a daily VaR is stated as £100,000 to a 95% level of confidence, this means that during the day there is only a 5% chance that the loss the next day will be *greater* than £100,000. VaR measures the potential loss in market value of a portfolio using estimated volatility and correlation. The “correlation” referred to is the correlation that exists between the market prices of different instruments in a bank’s portfolio. VaR is calculated within a given confidence interval, typically 95% or 99%; it seeks to measure the possible losses from a position or portfolio under “normal” circumstances. The definition of normality is critical and is essentially a statistical concept that varies by firm and by risk management system. Put simply however, the most commonly used VaR models assume that the prices of assets in the financial markets follow a normal distribution. To implement VaR, all of a firm’s positions data must be gathered into one centralised database. Once this is complete the overall risk has to be calculated by aggregating the risks from individual instruments across the entire portfolio. The potential move in each instrument (that is, each risk factor) has to be inferred from past daily price movements over a given observation period. For regulatory purposes this period is at least one year. Hence the data on which VaR estimates are based should capture all relevant daily market moves over the previous year.

The main assumption underpinning VaR – and which in turn may be seen as its major weakness – is that the distribution of future price and rate changes will follow past variations. Therefore the potential portfolio loss calculations for VaR are worked out using distributions from historic price data in the observation period.

For a discussion of the normal distribution, refer to Appendix 37.1.

VaR is a measure of the volatility of a firm’s banking or trading book. A portfolio containing assets that have a high level of volatility has a higher risk than one containing assets with a lower level of volatility. The VaR measure seeks to quantify in a single measure the potential losses that may be suffered by a portfolio.

VaR is therefore a measure of a bank’s risk exposure; it a tool for measuring market risk exposure. There is no one VaR number for a single portfolio, because different methodologies used for calculating VaR produce different results. The VaR number captures only those risks that can be measured in quantitative terms; it does not capture risk exposures such as operational risk, liquidity risk, regulatory risk or sovereign risk. It is important to be aware of what precisely VaR attempts to capture and what it clearly makes no attempt to capture. Also, VaR is not “risk management”. A risk management department may choose to use a VaR measurement system in an effort to

quantify a bank's risk exposure; however the application itself is merely a tool. Implementing such a tool in no way compensates for inadequate procedures and rules in the management of a trading book.

37.1.2 Assumption of normality

A distribution is described as *normal* if there is a high probability that any observation from the population sample will have a value that is close to the mean, and a low probability of having a value that is far from the mean. The normal distribution curve is used by many VaR models, which assume that asset returns follow a normal pattern. A VaR model uses the normal curve to estimate the losses that an institution may suffer over a given time period. Normal distribution tables show the probability of a particular observation moving a certain distance from the mean. Tables are contained in Appendix 37.2. If we look along the table we see that at -1.645 standard deviations, the probability is 5%; this means that there is a 5% probability that an observation will be at least 1.645 standard deviations below the mean. This level is used in many VaR models.

37.1.3 Calculation methods

There are three different methods for calculating VaR. They are:

- the variance/covariance (or *correlation* or *parametric* method);
- historical simulation;
- Monte Carlo simulation.

Variance-covariance method

This method assumes the returns on risk factors are normally distributed, the correlations between risk factors are constant and the delta (or price sensitivity to changes in a risk factor) of each portfolio constituent is constant. Using the correlation method, the volatility of each risk factor is extracted from the historical observation period. Historical data on investment returns is therefore required. The potential effect of each component of the portfolio on the overall portfolio value is then worked out from the component's delta (with respect to a particular risk factor) and that risk factor's volatility.

There are different methods of calculating the relevant risk factor volatilities and correlations. Two alternatives are:

- *simple historic volatility*: this is the most straightforward method but the effects of a large one-off market move can significantly distort volatilities over the required forecasting period. For example if using 30-day historic volatility, a market shock will stay in the volatility figure for 30 days until it drops out of the sample range and correspondingly causes a sharp drop in (historic) volatility 30 days *after* the event. This is because each past observation is equally weighted in the volatility calculation;
- *to weight past observations unequally*: this is done to give more weight to recent observations so that large jumps in volatility are not caused by events that occurred some time ago. One method is to use exponentially-weighted moving averages.

Historical simulation method

The historic simulation method for calculating VaR is the simplest and avoids some of the pitfalls of the correlation method. Specifically the three main assumptions behind correlation (normally distributed returns, constant correlations, constant deltas) are not needed in this case. For historical simulation the model calculates potential losses using actual historical returns in the risk factors and so captures the non-normal distribution of risk factor returns. This means rare events and crashes can be included in the results. As the risk factor returns used for revaluing the portfolio are actual past movements, the correlations in the calculation are also actual past correlations. They capture the dynamic nature of correlation as well as scenarios when the usual correlation relationships break down.

Monte Carlo simulation method

The third method, Monte Carlo simulation is more flexible than the previous two. As with historical simulation, Monte Carlo simulation allows the risk manager to use actual historical distributions for risk factor returns rather than having to assume normal returns. A large number of randomly generated simulations are run forward in time using volatility and correlation estimates chosen by the risk manager. Each simulation will be different but in total the simulations will aggregate to the chosen statistical parameters (that is, historical distributions and volatility and

correlation estimates). This method is more realistic than the previous two models and therefore is more likely to estimate VaR more accurately. However its implementation requires powerful computers and there is also a trade-off in that the time required to perform calculations is longer.

The level of confidence in the VaR estimation process is selected by the number of standard deviations of variance applied to the probability distribution. A standard deviation selection of 1.645 provides a 95% confidence level (in a one-tailed test) that the potential estimated price movement will not be more than a given amount based on the correlation of market factors to the position's price sensitivity.

37.2 Explaining Value-at-Risk

37.2.1 Correlation

Measures of correlation between variables are important to fund managers who are interested in reducing their risk exposure through diversifying their portfolio. Correlation is a measure of the degree to which a value of one variable is related to the value of another. The correlation coefficient is a single number that compares the strengths and directions of the movements in two instruments values. The sign of the coefficient determines the relative directions that the instruments move in, while its value determines the strength of the relative movements. The value of the coefficient ranges from -1 to $+1$, depending on the nature of the relationship. So if, for example, the value of the correlation is 0.5 , this means that one instrument moves in the same direction by half of the amount that the other instrument moves. A value of zero means that the instruments are uncorrelated, and their movements are independent of each other.

Correlation is a key element of many VaR models, including parametric models. It is particularly important in the measurement of the variance (hence volatility) of a portfolio. If we take the simplest example, a portfolio containing just two assets, (37.1) below gives the volatility of the portfolio based on the volatility of each instrument in the portfolio (x and y) and their correlation with one another.

$$V_{port} = \sqrt{x^2 + y^2 + 2xy \cdot \rho(xy)} \quad (37.1)$$

where

x	is the volatility of asset x
y	is the volatility of asset y
ρ	is the correlation between assets x and y .

The correlation coefficient between two assets uses the covariance between the assets in its calculation. The standard formula for covariance is shown at (37.2):

$$\text{Cov} = \frac{\sum_{i=1}^n (x_i - \bar{x})(y_i - \bar{y})}{(n - 1)} \quad (37.2)$$

where the sum of the distance of each value x and y from the mean is divided by the number of observations minus one. The covariance calculation enables us to calculate the correlation coefficient, shown as (37.3):

$$\rho = \text{Cov} \frac{(1,2)}{\sigma_1 \times \sigma_2} \quad (37.3)$$

where σ is the standard deviation of each asset.

Equation 37.1 may be modified to cover more than two instruments. In practice correlations are usually estimated on the basis of past historical observations. This is an important consideration in the construction and analysis of a portfolio, as the associated risks will depend to an extent on the correlation between its constituents.

It should be apparent that from a portfolio perspective a positive correlation increases risk. If the returns on two or more instruments in a portfolio are positively correlated, strong movements in either direction are likely to occur at the same time. The overall distribution of returns will be wider and flatter, as there will be higher joint probabilities associated with extreme values (both gains and losses). A negative correlation indicates that the assets are likely to move in opposite directions, thus reducing risk.

It has been argued that in extreme situations, such as market crashes or large-scale market corrections, correlations cease to have any relevance, because all assets will be moving in the same direction. However under most market scenarios using correlations to reduce the risk of a portfolio is considered satisfactory practice, and the VaR number for diversified portfolio will be lower than that for an undiversified portfolio.

37.2.2 Simple VaR calculation

To calculate the VaR for a single asset, we would calculate the standard deviation of its returns, using either its historical volatility or *implied volatility*. If a 95% confidence level is required, meaning we wish to have 5% of the observations in the left-hand tail of the normal distribution, this means that the observations in that area are 1.645 standard deviations away from the mean. This can be checked from the normal tables at Appendix 37.2. Consider the following statistical data for a government bond, calculated using one year's historical observations.

Nominal:	£10 million
Price:	£100
Average return:	7.35%
Standard deviation:	1.99%

The VaR at the 95% confidence level is 1.645×0.0199 or 0.032736. The portfolio has a market value of £10 million, so the VaR of the portfolio is $0.032736 \times 10,000,000$ or £327,360. So this figure is the maximum loss the portfolio may sustain over one year for 95% of the time.

We may extend this analysis to a two-stock portfolio. In a two-asset portfolio, we stated at (37.1) that there is a relationship that enables us to calculate the volatility of a two-asset portfolio; this expression is used to calculate the VaR, and is shown at (37.4):

$$Var_{port} = \sqrt{w_1^2\sigma_1^2 + w_2^2\sigma_2^2 + 2w_1w_2\sigma_1\sigma_2\rho_{1,2}} \quad (37.4)$$

where

w_1	is the weighting of the first asset
w_2	is the weighting of the second asset
σ_1	is the standard deviation or <i>volatility</i> of the first asset
σ_2	is the standard deviation or volatility of the second asset
$\rho_{1,2}$	is the correlation coefficient between the two assets.

In a two-asset portfolio the undiversified VaR is the weighted average of the individual standard deviations; the diversified VaR, which takes into account the correlation between the assets, is the square root of the variance of the portfolio. In practice banks will calculate both diversified and undiversified VaR. The diversified VaR measure is used to set trading limits, while the larger undiversified VaR measure is used to gauge an idea of the bank's risk exposure in the event of a significant correction or market crash. This is because in a crash situation, liquidity dries up as market participants all attempt to sell off their assets. This means that the correlation relationship between assets cease to have any impact on a book, as all assets move in the same direction. Under this scenario then, it is more logical to use an undiversified VaR measure.

Although the description given here is very simple, nevertheless it explains what is the essence of the VaR measure; VaR is essentially the calculation of the standard deviation of a portfolio, which is the used as an indicator of the volatility of that portfolio. A portfolio exhibiting high volatility will have a high VaR number. An observer may then conclude that the portfolio has a high probability of making losses. Risk managers and traders may use the VaR measure to help them to allocate capital to more efficient sectors of the bank, as return on capital can now be measured in terms of return on risk capital. Regulators may use the VaR number as a guide to the capital adequacy levels that they feel the bank requires.

37.3 Variance-covariance Value-at-Risk

37.3.1 Calculation of variance-covariance VaR

In the previous section we showed how VaR could be calculated for a two-stock portfolio. Here we illustrate how this is done using matrices.

Consider the following hypothetical portfolio, invested in two assets, as shown in Table 37.1. The standard deviation of each asset has been calculated on historical observation of asset returns. Note that *returns* are returns of asset prices, rather than the prices themselves; they are calculated from the actual prices by taking the ratio of closing prices. The returns are then calculated as the logarithm of the price relatives. The mean and standard deviation of the returns are then calculated using the standard statistical formulae listed in Appendix 37.1. This would then give the standard deviation of daily price relatives, which is converted to an annual figure by multiplying it by the square root of the number of days in a year, usually taken to be 250.

Assets	Bond 1	Bond 2
Standard deviation	11.83%	17.65%
Portfolio weighting	60%	40%
Correlation coefficient		0.647
Portfolio value		£10,000,000
Variance		0.016506998
Standard deviation		12.848%
95% c.i. standard deviations		1.644853
Value-at-Risk		0.211349136
Value-at-Risk £		£2,113,491

Table 37.1: Two-asset portfolio VaR.

The standard equation (shown as (37.4)) is used to calculate the variance of the portfolio, using the individual asset standard deviations and the asset weightings; the VaR of the book is the square root of the variance. Multiplying this figure by the current value of the portfolio gives us the portfolio VaR, which is £2,113,491.

The RiskMetrics VaR methodology uses matrices to obtain the same results that we have shown here. This is because once a portfolio starts to contain many assets, the method we described above becomes unwieldy. Matrices allow us to calculate VaR for a portfolio containing many hundreds of assets, which would require assessment of the volatility of each asset and correlations of each asset to all the others in the portfolio. We can demonstrate how the parametric methodology uses variance and correlation matrices to calculate the variance, and hence standard deviation, of a portfolio. The matrices are shown at Figure 37.1. Note that the multiplication of matrices carries with it some unique rules; these are summarised in Appendix 37.3. Readers who are unfamiliar with matrices should refer to this appendix.

As shown at Figure 37.1, using the same two-asset portfolio described, we can set a 2×2 matrix with the individual standard deviations inside; this is labelled the “variance” matrix. The standard deviations are placed on the horizontal axis of the matrix, and a zero entered in the other cells. The second matrix is the correlation matrix, and the correlation of the two assets is placed in cells corresponding to the other asset; that is why a “1” is placed in the other cells, as an asset is said to have a correlation of 1 with itself. The two matrices are then multiplied to produce another matrix, labelled “VC” in Figure 37.1.¹ The VC matrix is then multiplied with the V matrix to obtain the variance-covariance matrix or VCV matrix. This shows the variance of each asset; for Bond 1 this is 0.01399, which is expected as that is the square of its standard deviation, which we were given at the start. The matrix also tells us that Bond 1 has a covariance of 0.0135 with Bond 2. We then set up a matrix of the portfolio weighting of the two assets, and this is multiplied by the VCV matrix. This produces a 1×2 matrix, which we need to change to a single number, so this is multiplied by the W matrix, reset as a 2×1 matrix, which produces the portfolio variance. This is 0.016507. The standard deviation is the square root of the variance, and is 0.1284795 or 12.848%, which is what we obtained before. In our illustration it is important to note the order in which the matrices were multiplied, as this will obviously affect the result. The volatility matrix contains the standard deviations along the diagonal, and zeros are entered in all the other cells. So if the portfolio we were calculating has 50 assets in it, we would require a 50×50 matrix and enter the standard deviations for each asset along the diagonal line. All the other cells would have

¹ Microsoft Excel has a function for multiplying matrices which may be used for any type of matrix. The function is “=MMULT()” typed in all the cells of the product matrix.

a zero in them. Similarly for the weighting matrix, this is always one row, and all the weights are entered along the row. To take the example just given the result would be a 1×50 weighting matrix.

	Variance matrix	Correlation matrix	
		Bond 1 Bond 2	VC matrix
Bond 1	11.83% 0	1 0.647	0.1183 0.0765401
Bond 2	0 17.65%	0.647 1	0.1141955 0.1765
	VC matrix	Variance matrix	VCV matrix
	0.1183 0.0765401	11.83% 0	0.01399489 0.013509328
	0.1141955 0.1765	0 17.65%	0.013509328 0.03115225
	Weighting matrix	VCV matrix	WVCV
	60% 40%	0.01399489 0.013509328	0.013800665 0.020566497
		0.013509328 0.03115225	
	WVCV	W	WVCVW
	0.013800665 0.020566497	60%	0.016506998
		40%	
		Standard deviation	0.128479561

Figure 37.1: Matrix variance-covariance calculation for two-asset portfolio.

The matrix method for calculating the standard deviation is more effective than the first method we described, because it can be used for a portfolio containing a large number of assets. In fact this is exactly the methodology used by RiskMetrics, and the computer model used for the calculation will be set up with matrices containing the data for hundreds, if not thousands, of different assets.

The variance-covariance method captures the diversification benefits of a multi-product portfolio because the correlation coefficient matrix used in the calculation. For instance if the two bonds in our hypothetical portfolio had a negative correlation the VaR number produced would be lower. It was also the first methodology introduced, by JP Morgan in 1994. To apply it, a bank would require data on volatility and correlation for the assets in its portfolio. This data is actually available from the RiskMetrics website (and other sources), so a bank does not necessarily need its own data. It may wish to use its own datasets however, should it have them, to tailor the application to its own use. The advantages of the variance-covariance methodology are that:

- it is simple to apply, and fairly straightforward to explain;
- datasets for its use are immediately available.

The drawbacks of the variance-covariance are that it assumes stable correlations and measures only linear risk; it also places excessive reliance on the normal distribution, and returns in the market are widely believed to have “fatter tails” than a true to normal distribution. This phenomenon is known as *leptokurtosis*, that is, the non-normal distribution of outcomes. Another disadvantage is that the process requires *mapping*. To construct a weighting portfolio for the RiskMetrics tool, cash flows from financial instruments are mapped into precise maturity points, known as grid points. We will review this later in the chapter, however in most cases assets do not fit into neat grid points, and complex instruments cannot be broken down accurately into cash flows. The mapping process makes assumptions that frequently do not hold in practice.

Nevertheless the variance-covariance method is still popular in the market, and is frequently the first VaR method installed at a bank.

37.3.2 Mapping

The cornerstone of variance-covariance methodologies such as RiskMetrics is the requirement for data on volatilities and correlations for assets in the portfolio. The RiskMetrics dataset does not contain volatilities for every maturity possible, as that would require a value for every period from 1 day to over 10,950 days (30 years) and longer, and correlations between each of these days. This would result in an excessive amount of calculation. Rather, volatilities are available for set maturity periods, and these are shown in Figure 37.2.

RiskMetrics grid points

1 month	3 months	6 month	1 year	2 years	3 years
4 years	5 years	7 years	9 years	10 years	15 years
20 years	30 years				

Figure 37.2: RiskMetrics grid points.

If a bond is maturing in six years' time, its redemption cash flow will not match the data in the RiskMetrics dataset, so it must be mapped to two periods, in this case being split to the five year and seven year grid point. This is done in proportions so that the original value of the bond is maintained once it has been mapped. More importantly, when a cash flow is mapped, it must split in a manner that preserves the volatility characteristic of the original cash flow. Therefore, when mapping cash flows, if one cash flow is apportioned to two grid points, the share of the two new cash flows must equal the present value of the original cash flows, and the combined volatility of the two new assets must be equal to that of the original asset. A simple demonstration is given at Example 37.1.

EXAMPLE 37.1 Cash flow mapping

A bond trading book holds £1 million nominal of a gilt strip that is due to mature in precisely six years' time. To correctly capture the volatility of this position in the bank's RiskMetrics VaR estimate, the cash flow represented by this bond must be mapped to the grid points for five years and seven years, the closest maturity buckets that the RiskMetrics dataset holds volatility and correlation data for. The present value of the strip is calculated using the six-year zero-coupon rate, which RiskMetrics obtains by interpolating between the five-year rate and the seven-year rate. The details are shown in Table 37.2.

Gilt strip nominal (£)	1,000,000
Maturity (years)	6
5-year zero-coupon rate	5.35%
7-year zero-coupon rate	5.50%
5-year volatility	24.50%
7-year volatility	28.95%
Correlation coefficient	0.979

Table 37.2: Bond position to be mapped to grid points.

Note that the correlation between the two interest rates is very close to 1; this is expected because five-year interest rates generally move very closely in line with seven-year rates.

We wish to assign the single cash flow to the five-year and seven-year grid points (also referred to as *vertices*). The present value of the bond, using the six-year interpolated yield, is £728,347. This is shown in Table 37.3, which also uses an interpolated volatility to calculate the volatility of the six-year cash flow. However we wish to calculate a portfolio volatility based on the apportionment of the cash flow to the five-year and seven-year grid points. To do this, we need to use a weighting to allocate the cash flow between the two vertices. In the hypothetical situation used here, this presents no problem because six years falls precisely between five years and seven years. Therefore the weightings are 0.5 for year five and 0.5 for year seven. If the cash flow had fallen in less obvious a maturity point, we would have to calculate the weightings using the formula for portfolio variance. Using these weightings, we calculate the variance for the new "portfolio", containing the two new cash flows, and then the standard deviation for the portfolio. This gives us a VaR for the strip of £265,853.

Interpolated yield	0.05425
Interpolated volatility	0.26725
Present value	728,347.0103
Weighting 5-year grid point	0.5
Weighting 7-year grid point	0.5
Variance of portfolio	0.070677824

Standard deviation	0.265853012
VaR £	265,853

Table 37.3: Cash flow mapping and portfolio variance.

37.3.3 Confidence intervals

Many models estimate VaR at a given confidence interval, under normal market conditions. This assumes that market returns generally follow a random pattern but one that approximates over time to a normal distribution. The level of confidence at which the VaR is calculated will depend on the nature of the trading book's activity and what the VaR number is being used for. The Market Risk amendment to the Basle Capital Accord stipulates a 99% confidence interval and a 10-day holding period if the VaR measure is to be used to calculate the regulatory capital requirement. However certain banks prefer to use other confidence levels and holding periods; the decision on which level to use is a function of asset-types in the portfolio, quality of market data available and the accuracy of the model itself, which will have been tested over time by the bank.

For example, a bank may view a 99% confidence interval as providing no useful information, as it implies that there should only be two or three breaches of the VaR measure over the course of one year; that would leave no opportunity to test the accuracy of the model until a relatively long period of time had elapsed, in the meantime the bank would be unaware if the model was generating inaccurate numbers. A 95% confidence level implies the VaR level being exceeded around one day each month, if a year is assumed to contain 250 days.² If a VaR calculation is made using 95% confidence, and a 99% confidence level is required for say, regulatory purposes, we need to adjust the measure to take account of the change in standard deviations required. For example, a 99% confidence interval corresponds to 2.32 standard deviations, while a 95% level is equivalent to 1.645 standard deviations. Thus to convert from 95% confidence to 99% confidence, the VaR figure is divided by 1.645 and multiplied by 2.32.

In the same way there may be occasions when a firm will wish to calculate VaR over a different holding period to that recommended by the Basle Committee. The holding period of a portfolio's VaR calculation should represent the period of time required to unwind the portfolio, that is, sell off the assets on the book. A 10-day holding period is recommended but would be unnecessary for a highly liquid portfolio, for example one holding government bonds.

To adjust the VaR number to fit it to a new holding period we simply scale it upwards or downward by the square root of the time period required. For example a VaR calculation measured for a 10-day holding period will be $\sqrt{10}$ times large than the corresponding 1-day measure.

37.3.4 Capturing volatility

Volatility is a key parameter in the calculation of value-at-risk. This makes it important to have as accurate a measure of volatility as possible, which is not necessarily straightforward because often volatility levels are not readily observable in the market. Practitioners sometimes use *implied volatility* which has been calculated from the price of exchange-traded options; for risk measurement purposes it is more common for a statistical model to be used, both to estimate volatility and correlations. This in itself leads to further potential inaccuracies, because many VaR models make assumptions about the distribution of asset returns, which may not be a true reflection of real world situations. Nevertheless it is often necessary to make these assumptions to enable the calculations to be carried out. The key assumption used in risk models are (i) normal distribution of returns and (ii) independent and identically distributed data.

Observation of the market suggests that volatility does not in fact behave as the assumptions imply. First, in many markets volatility levels are fairly stable for long periods and then jump sharply, often taking the market by surprise, stay at the higher levels for a short time, and then settle back down into a pattern resembling the original pattern. The period of clustering of high volatility levels, followed by a fairly swift descent to lower levels, implies that the volatility level for one day is influenced by the level of the previous day. This indicates that the behaviour of volatility does not act according to the assumption of no *autocorrelation*. Such behaviour makes it difficult to assume a normal distribution for volatility, as well as one of independence.

² For the 99% confidence level, $250 \times 1\% = 2.5$ days in one year, while 95% confidence is $250 \times 5\%$ or 12.5 days.

In this section we review the different statistical models that are used to model volatility; the most commonly used techniques are also the most straightforward. Value-at-Risk methodologies that employ some form of simulation use the most complex volatility models, such as the stochastic volatility models.

The *equally-weighted moving average* bases its estimate of volatility on historic data. Consider a sample of N historical observations of daily returns $\{r_1, r_2, \dots, r_N\}$. An estimate of the volatility of the daily returns in the daily weighted model is given by (37.5):

$$\sigma_t = \sqrt{\frac{1}{N-1} \sum_{n=1}^N (r_{t-n} - \bar{r})^2} \quad (37.5)$$

where

σ_t is the volatility estimate for time t at time $(t-1)$
 r_{t-n} is the daily return at time $t-n$
 $\bar{r}$ is the mean daily return of the sample.

In practice risk analysts usually estimate volatility using a moving window of a fixed number of observations, instead of continually increasing the sample size over time. This method ignores both the timing and ordering of the observations, and also gives equal importance or weight to each of the observations, hence its name. The user selects the size of the moving window of observations. A wide window produces a more stable volatility estimate, but this stability is obtained at the expense of accuracy. Another drawback of this method is that it creates what is known as a *ghost feature*, which is when an extreme volatility observation moves outside of the moving window. When this happens, it creates a jump in volatility that is not reflecting actual market characteristics.

The equally-weighted moving average is acceptable in an environment where the distribution of daily returns is relatively stationary, but only rarely will this be observed in practice. Its advantages are that it is simple to implement and explain, but its limitations relating to the ghost feature, its equal weighting of past and recent events, and its failure to handle autocorrelation mean that it is an unsuitable method for sophisticated trading institutions to use.

The *exponentially-weighted moving average* moves one step further than the equally-weighted method because it captures the dynamic features of volatility. In calculating the volatility estimate, more recent events are assigned a heavier weighting in the volatility estimate. This approach also recognises potential autocorrelation effects and diminishes the ghost feature that exists in the equal weighting method. The exponentially-weighted volatility estimate is given by (37.6):

$$\sigma_t = \sqrt{\left(\sum_{n=1}^N \lambda^{n-1} (r_{t-n} - \bar{r})^2 \right) / \sum_{n=1}^N \lambda^{n-1}} \quad (37.6)$$

where λ is the decay factor or “smoothing constant”. The larger this decay factor, the more weight is placed on past observations to estimate volatility. Conversely a smaller decay factor will place more weight on current observations. A decay factor of 1.00 will result in moving average volatility estimates with equal weighting on all observations. For very large N , (37.6) may be approximated as (37.7):

$$\sigma_t = \sqrt{(1 - \lambda) \sum_{n=1}^N \lambda^{n-1} (r_{t-n} - \bar{r})^2}. \quad (37.7)$$

In its recursive form the variance estimate of the exponentially-weighted moving average may therefore be written as (37.8):

$$\sigma_t^2 = \lambda \sigma_{t-1}^2 + (1 - \lambda) r_{t-1}^2. \quad (37.8)$$

The RiskMetrics methodology uses the exponentially-weighted moving average model, with a decay factor of 0.94 for a daily time period and 0.97 for a monthly time period, for all variables in its dataset. The VaR calculator on a Bloomberg system allows the user to specify a decay factor, from a range of 1.00 to 0.93. Note that the Basle Committee capital adequacy rule update stipulates that historical observations and associated decay factor should be for a minimum period of six months.

More sophisticated volatility models can be divided into two classes, so-called GARCH models (for *generalised autoregressive conditional heteroscedasticity*) and stochastic volatility models. Both types consider the sequence of volatilities, and the fact that empirical studies³ on volatility time series indicate that volatility may be serially correlated. To capture serial volatility, an ARCH model⁴ seeks to express conditional variance of returns as a distributed lag of past squared daily asset returns, given by (37.9):

$$\sigma_t^2 = \alpha + \sum_{i=1}^q \beta_i r_{t-i}^2 \quad (37.9)$$

where α and β_i are the lag coefficients. The conditional variance will be positive if the lag coefficients are constrained to be non-negative. An alternative method of modelling persistent movements in volatility is the GARCH model for conditional variance,⁵ given by (37.10):

$$\sigma_t^2 = \alpha + \sum_{i=1}^p \beta_i \sigma_{t-i}^2 + \sum_{i=1}^q \gamma_i r_{t-i}^2. \quad (37.10)$$

The ARCH models do not require the estimation of a large number of lag coefficients. The p term indicates the persistence of the first parameter, the variance, while the q term indicates the persistence of the returns, in the second half of the equation. The number of lag coefficients p and q will depend on the nature of time series data and the in-sample and out-of-sample performance of the model for different choices of p and q . In most financial applications p and q are set equal to 1, which in most circumstances is sufficient to model the dynamics of volatility. From this then we may set the GARCH(1,1) model for the estimation of daily (37.11):

$$\begin{aligned} r_t &= \mu + \sigma_t \varepsilon_t \\ \sigma_t^2 &= \alpha + \beta \sigma_{t-1}^2 + \gamma r_{t-1}^2 \\ \alpha, \beta, \gamma &> 0 \\ \varepsilon &\sim N(0,1). \end{aligned} \quad (37.11)$$

Most applications set μ to zero when working with recent data. GARCH models react faster to sudden changes in market volatility than do other models, and have been observed to go up faster with the sudden movements and also recover faster than the traditional techniques. The unconditional variance corresponding to the GARCH(1,1) model is given by (37.12):

$$\sigma^2 = \frac{\alpha}{1 - \beta - \gamma}. \quad (37.12)$$

To ensure the unconditional variance is positive, we also need to further impose the constraint that $\beta + \gamma < 1$.

Implementing a GARCH model requires an empirical analysis of data. The use of GARCH modelling will allow a risk manager to capture sudden extreme movements in volatility more accurately than is possible with the traditional methods, and they also reflect the clustering effect of volatility around the higher levels after there has been a significant move. The parameters can also be adjusted to fit structural changes in the market. Their disadvantages are that they are difficult to implement and consume excessive amounts of computing power. They are also less easier to explain than the traditional models.

In attempting to capture market volatility, a bank is faced with the observation that actual market volatility behaviour often contradicts any statistical model. The VaR calculation is an approximation of forces that do not always fit the assumptions and statistical models that are required. Volatility is unstable and occasional shocks in volatility levels have spill-over or persistence effects that hamper attempts to measure the subsequent volatility level. When large market movements occur VaR-based trading limits are frequently exceeded because of non-normal extreme events that are observed in the market but not included in normalised data.

³ For example, see Engle (1982).

⁴ *Ibid.*

⁵ See Bollerslev (1986).

37.4 Historical VaR methodology

The historical approach to value-at-risk is a relatively simple calculation, and it is also easy to implement and explain. To implement it, a bank requires a database record of its past profit/loss figures for the total portfolio; the required confidence interval is then applied to this record, to obtain a cut-off of the worst-case scenario. For example, to calculate the VaR at a 95% confidence level, the 5th percentile is value for the historical data is taken, and this is the VaR number. For a 99% confidence level measure, the 1% percentile is taken. The advantage of the historical method is that it uses the actual market data that a bank has recorded (unlike RiskMetrics for example, for which the volatility and correlations are not actual values, but estimated values calculated from average figures over a period of time, usually the last five years), and so produces a reasonably accurate figure. Its main weakness is that as it is reliant on actual historical data built up over a period of time; generally at least one year's data is required to make the calculation meaningful. Therefore it is not suitable for portfolios whose asset weightings frequently change, as another set of data would be necessary before a VaR number could be calculated.

In order to overcome this drawback banks use a method known as *historical simulation*. This calculates VaR for the current portfolio weighting, using the historical data for the securities in the current portfolio. To calculate historical simulation VaR for our hypothetical portfolio considered earlier, comprising 60% of bond 1 and 40% of bond 2, we require the closing prices for both assets over the specified previous period (usually three or five years); we then calculate the value of the portfolio for each day in the period assuming constant weightings.

37.5 Simulation methodology

The most complex calculations use computer simulations to estimate value-at-risk. The most common is the Monte Carlo method. To calculate VaR using a Monte Carlo approach, a computer simulation is run in order to generate a number of random scenarios, which are then used to estimate the portfolio VaR. The method is probably the most realistic, if we accept that market returns follow a similar “random walk” pattern. However Monte Carlo simulation is best suited to trading books containing large option portfolios, whose price behaviour is not captured very well with the RiskMetrics methodology. The main disadvantage of the simulation methodology is that it is time-consuming and uses a substantial amount of computer resources.

A Monte Carlo simulation generates simulated future prices, and it may be used to value an option as well as for VaR applications. When used for valuation, a range of possible asset prices are generated and these are used to assess what intrinsic value the option will have at those asset prices. The present value of the option is then calculated from these possible intrinsic values. Generating simulated prices, although designed to mimic a “random walk”, cannot be completely random because asset prices, although not a pure normal distribution, are not completely random either. The simulation model is usually set to generate very few extreme prices. Strictly speaking, it is asset price *returns* that follow a normal distribution, or rather a *lognormal* distribution. Monte Carlo simulation may also be used to simulate other scenarios, for example the effect on option “greeks” for a given change in volatility, or any other parameters. The scenario concept may be applied to calculating VaR as well. For example, if 50 000 simulations of an option price are generated, the 95th lowest value in the simulation will be the VaR at the 95% confidence level. The correlation between assets is accounted for by altering the random selection programme to reflect relationships.

EXAMPLE 37.2 Portfolio volatility using variance-covariance and simulation methods

A simple two-asset portfolio is composed of the following instruments:

	Gilt strip	FTSE100 stock
Number of units	£100 million	5 million
Market value	£54.39 million	£54 million
Daily volatility	£0.18 million	£0.24 million

The correlation between the two assets is 20%. Using (37.4) we calculate the portfolio VaR as follows:

$$\begin{aligned}
 Vol &= \sqrt{\sigma_{bond}^2 + \sigma_{stock}^2 + 2\sigma_{bond}\sigma_{stock}\rho_{bond,stock}} \\
 Vol &= \sqrt{0.18^2 + 0.24^2 + (2 \cdot 0.18 \cdot 0.24 \cdot 0.2)} \\
 &= 0.327.
 \end{aligned}$$

We have ignored the weighting element for each asset because the market values are roughly equal. The calculation gives a portfolio volatility of £0.327 million. For a 95% confidence level VaR measure, which corresponds to 1.645 standard deviations (in a one-tailed test) we multiply the portfolio volatility by 1.645, which gives us a portfolio value-at-risk of £0.538 million.

In a Monte Carlo simulation we also calculate the correlation and volatilities of the portfolio. These values are used as parameters in a random number simulation to throw out changes in the underlying portfolio value. These values are used to re-price the portfolio, and this value will be either a gain or loss on the actual mark-to-market value. This process is repeated for each random number that is generated. In Table 37.4 we show the results for 15 simulations of our two-asset portfolio. From the results we read off the loss level that corresponds to the required confidence interval.

Simulation	Market value: bond	Market value: stock	Portfolio value	Profit/Loss
1	54.35	54.9	109.25	0.86
2	54.64	54.02	108.66	0.27
3	54.4	53.86	108.26	-0.13
4	54.25	54.15	108.4	0.01
5	54.4	54.17	108.57	0.18
6	54.4	54.03	108.43	0.04
7	54.31	53.84	108.15	-0.24
8	54.3	53.96	108.26	-0.13
9	54.46	54.11	108.57	0.18
10	54.32	53.92	108.24	-0.15
11	54.31	53.97	108.28	-0.11
12	54.47	54.08	108.55	0.16
13	54.38	54.03	108.41	0.02
14	54.71	53.89	108.6	0.21
15	54.29	54.05	108.34	-0.05

Table 37.4: Monte Carlo simulation results.

As the number of trials is increased, the results from a Monte Carlo simulation approach those of the variance-covariance measure. This is shown in Figure 37.3.

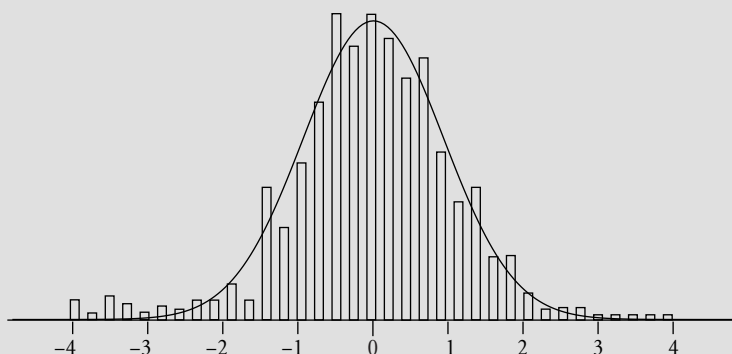


Figure 37.3: The normal approximation of returns.

37.6 Value-at-risk for fixed interest instruments

Perhaps the most straightforward instruments that one can apply VaR to are foreign exchange and interest-rate instruments such as money market products, bonds, forward-rate agreements and swaps. In this section we review the calculation of VaR for a simple portfolio of bonds.

37.6.1 Bond portfolio

Table 37.5 details the bonds that are in the portfolio; for simplicity we assume that all the bonds pay an annual coupon and have full years left to maturity. In order to calculate the value-at-risk we first need to value the bond portfolio itself. The bonds are valued by breaking them down into their constituent cash flows; the present value of each cash flow is then calculated, using the appropriate zero-coupon interest rate. Note from Figure 37.4 that the term structure is inverted. Table 37.6 shows the present values for each of the cash flows. The total portfolio value is also shown.

	Bond 1	Bond 2	Bond 3
Nominal value	10,000,000	3,800,000	9,700,000
Coupon	5%	7.25%	6%
Maturity	5	7	2

Table 37.5: Simple three-bond portfolio.

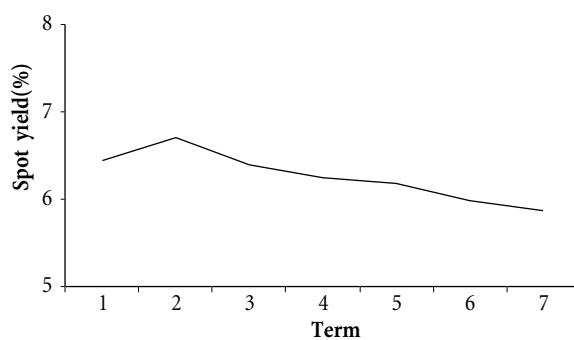


Figure 37.4: Term structure used in the valuation.

We then use the volatility for each period rate to calculate the VaR; data on interest-rate volatility is available for example, from the RiskMetrics website for all major currencies. The volatility levels for our hypothetical currency are relatively low in this example. The VaR for each maturity period is then obtained by multiplying the total present value of the cash flows for that period by its volatility level. This is shown in Table 37.7. By adding together all the individual values, we obtain an undiversified VaR for the portfolio. The total VaR is £1.77 million, for a portfolio with a market value of £23.1 million.

Period	Cash flows: Bond 1	Bond 2	Bond 2	Zero-coupon rates	Discount factor	Present value		
1	500,000	275,500	582,000	6.45	0.939408173	469,704	258,807	546,736
2	500,000	275,500	10,282,000	6.7	0.878357191	439,179	241,987	9,031,269
3	500,000	275,500		6.4	0.830185447	415,093	228,716	
4	500,000	275,500		6.25	0.784664935	392,332	216,175	
5	10,500,000	275,500		6.18	0.740945722	7,779,930	204,131	
6		275,500		5.98	0.705759136	0	194,437	
7		4,075,500		5.87	0.670794678	0	2,733,824	
Totals						9,496,238	4,078,077	9,578,004
Portfolio value						23,152,319		

Table 37.6: Bond portfolio valuation.

Period	Cash flows	Present value	Volatility	Value-at-Risk
1	1,357,500	1,275,246.60	0.0687	87,609.44
2	11,057,500	9,712,434.64	0.0695	675,014.21
3	775,500	643,808.81	0.07128	45,890.69
4	775,500	608,507.66	0.0705	42,899.79
5	10,775,500	7,984,060.63	0.08501	678,724.99
6	275,500	194,436.64	0.08345	16,225.74
7	4,075,500	2,733,823.71	0.08129	222,232.53
Undiversified VaR				1,768,597.392

Table 37.7: Bond portfolio undiversified VaR.

The figure just calculated is the undiversified VaR for the bond portfolio. To obtain the diversified VaR for the book, we require the correlation coefficient of each interest rate with the other interest rates (the correlation will be very close to unity, although the shorter-dated rates will cluster closer in line with each other than they will be with long-dated rates). We may then use the standard variance-covariance approach, using a matrix of the undiversified VaR values and a matrix with the correlation values. However the diversification benefit of a portfolio of bonds will be small, mainly because their volatilities will be closely correlated.

37.6.2 Forward-rate agreements

The VaR calculation for a forward-rate agreement (FRA) follows the same principles reviewed in the previous section. A FRA is a notional loan or deposit for a period starting at some point in the future; in effect it is used to fix a borrowing or lending rate. The derivation of a FRA rate is based on the principle of what it would cost for a bank that traded one to hedge it; this is known as the “break-even” rate. So a bank that has bought 3v6 FRA (referred to as a “threes-sixes FRA”) has effectively borrowed funds for three months and placed the funds on deposit for six months. Therefore a FRA is best viewed as a combination of an asset and a liability, and that is how one is valued. So a long position in a 3v6 FRA is valued as the present value of a three-month cash flow asset and the present value of a six-month cash flow liability, using the three-month and six-month deposit rates. The net present value is taken of course, because one cash flow is an asset and the other a liability.

Consider a 3v6 FRA that has been dealt at 5.797%, the three-month forward-forward rate. The value of its constituent (notional) cash flows is shown in Table 37.8. The three-month and six-month rates are cash rates in the market, while the interest-rate volatilities have been obtained from RiskMetrics. The details are summarised in Table 37.8.

Cash flow	Term (days)	Cash rate	Interest rate volatilities	Present value	Undiversified VaR
10,000,000	91	5.38%	0.14%	+9,867,765	13,815
−10,144,536	182	5.63%	0.21%	−9,867,765	20,722

Table 37.8: Undiversified VaR for 3v6 FRA.

The undiversified VaR is the sum of the individual VaR values, and is £34,537. It has little value in the case of a FRA however, and would overstate the true VaR, because a FRA is made up of a notional asset and liability, so a fall in the value of one would see a rise in the value of the other. Unless a practitioner was expecting three-month rates to go in an opposite direction to six-month rates, there is an element of diversification benefit. There is a high correlation between the two rates, so the more logical approach is to calculate a diversified VaR measure.

For an instrument such as a FRA, the fact that the two rates used in calculating the FRA rate are closely positively correlated will mean that the diversification effect will be to reduce the VaR estimate, because the FRA is composed notionally of an asset and a liability. From the values in Table 37.8 therefore, the six-month VaR is actually a negative value (if the bank had sold the FRA, the three-month VaR would have the negative value). To calculate the diversified VaR then requires the correlation between the two interest rates, which may be obtained

from the RiskMetrics dataset. This is observed to be 0.87. This value is entered into a 2×2 correlation matrix and used to calculate the diversified VaR in the normal way. The procedure is:

- transpose the weighting VaR matrix, to turn it into a 2×1 matrix;
- multiply this by the correlation matrix;
- multiply the result by the original 1×2 weighting matrix;
- this gives us the variance; the VaR is the square root of this value.

The result is an diversified VaR of £11,051. For readers who wish to follow the matrix procedures, it is given in Appendix 37.4.

37.6.3 Interest-rate swaps

To calculate a variance-covariance VaR for an interest-rate swap, we use the process described earlier for a FRA. There are more cash flows that go to make up the undiversified VaR, because a swap is essentially a strip of FRAs. In a plain vanilla interest-rate swap, one party pays fixed rate basis on an annual or semi-annual basis, and receives floating-rate interest, while the other party pays floating-rate interest payments and receives fixed-rate interest. Interest payments are calculated on a notional sum, which does not change hands, and only interest payments are exchanged. In practice, it is the net difference between the two payments that is transferred.

The fixed rate on an interest-rate swap is the break-even rate that equates the present value of the fixed-rate payments to the present value of the floating-rate payments; as the floating-rate payments are linked to a reference rate such as LIBOR, we do not know what they will be, but we use the forward rate applicable to each future floating payment date to calculate what it would be if we were to fix it today. The forward rate is calculated from zero-coupon rates today. A “long” position in a swap is to pay fixed and receive floating, and is conceptually the same as being short in a fixed-coupon bond and being long in a floating-rate bond; in effect the long is “borrowing” money, so a rise in the fixed rate will result in a rise in the value of the swap. A “short” position is receiving fixed and paying floating, so a rise in interest rates results in a fall in the value of the swap. This is conceptually similar to a long position in a fixed-rate bond and a short position in a floating-rate bond.

Describing an interest-rate swap in conceptual terms of fixed- and floating-rate bonds gives some idea as to how it is treated for value-at-risk purposes. The coupon on a floating-rate bond is reset periodically in line with the stated reference rate, usually LIBOR. Therefore the duration of a floating-rate bond is very low, and conceptually the bond may be viewed as being the equivalent of a bank deposit, which receives interest payable at a variable rate. For market risk purposes,⁶ the risk exposure of a bank deposit is nil, because its present value is not affected by changes in market interest rates. Similarly, the risk exposure of a floating-rate bond is very low and to all intents and purposes its VaR may be regarded as zero. This leaves only the fixed-rate leg of a swap to measure for VaR purposes.

Pay date	Swap rate	Principal (£)	Coupon (£)	Coupon present value (£)	Volatility	Undiversified VaR
07-Jun-00	6.73%	10,000,000	337,421	327,564	0.05%	164
07-Dec-00	6.73%	10,000,000	337,421	315,452	0.05%	158
07-Jun-01	6.73%	10,000,000	335,578	303,251	0.10%	303
07-Dec-01	6.73%	10,000,000	337,421	294,898	0.11%	324
07-Jun-02	6.73%	10,000,000	335,578	283,143	0.20%	566
09-Dec-02	6.73%	10,000,000	341,109	277,783	0.35%	972
09-Jun-03	6.73%	10,000,000	335,578	264,360	0.33%	872
08-Dec-03	6.73%	10,000,000	335,578	256,043	0.45%	1,152
07-Jun-04	6.73%	10,000,000	335,578	248,155	0.57%	1,414
07-Dec-04	6.73%	10,000,000	337,421	242,161	1.90%	4,601
Total						10,528

Table 37.9: Fixed-rate leg of five-year interest rate swap and undiversified VaR.

⁶ We emphasise for *market* risk purposes; the credit risk exposure for a floating-rate bond position is a function of the credit quality of the issuer.

Table 37.9 shows the fixed-rate leg of a five-year interest rate swap (this swap is discussed in greater detail in Chapter 40). To calculate the undiversified VaR we use the volatility rate for each term interest rate; this may be obtained from RiskMetrics. Note that the RiskMetrics dataset supports only liquid currencies; for example, data on volatility and correlation is not available for certain emerging market economies. Below we show the VaR for each payment; the sum of all the payments constitutes the undiversified VaR. We then require the correlation matrix for the interest rates, and this is used to calculate the diversified VaR. The weighting matrix contains the individual term VaR values, which must be transposed before being multiplied by the correlation matrix.

Using the volatilities and correlations supplied by RiskMetrics the diversified VaR is shown to be £10 325. This is very close to the undiversified VaR of £10 528. This is not unexpected because the different interest rates are very closely correlated. The matrices are shown in Appendix 37.4.

Using VaR to measure market risk exposure for interest rate products enables a risk manager to capture non-parallel shifts in the yield curve, which is an advantage over the traditional duration measure and interest-rate gap measure. Therefore estimating a book's VaR measure is useful not only for the trader and risk manager, but also for senior management, who by using VaR will have a more accurate idea of the risk market exposure of the bank. Value-at-Risk methodology captures pivotal shifts in the yield curve by using the correlations between different maturity interest rates; this reflects the fact that short-term interest rates and long-term interest rates are not perfectly positively correlated.

37.7 Derivative products and Value-at-Risk

The variance-covariance methodology for calculating value-at-risk is considered adequate for trading books that contain mostly products that have a linear payoff profile. This covers money market interest-rate instruments; however the price/yield relationship for bonds exhibits a curved relationship, which gives rise to the convexity property. A trading book with convex instruments will have an added convexity risk exposure, and while most VaR methodologies are able to capture convexity risks adequately, an adjustment to the basic calculation has to be made. Such an adjusted measure is known as the *delta-gamma VaR*. Option products however, have a non-linear payoff profile, and it is more difficult to capture risks associated with option trading books using the variance-covariance approach. In this section we review the delta-gamma approach and its application to bonds and options. The specific risk measures used by option traders are considered in the next section.

37.7.1 Bond convexity

The duration and convexity of fixed income instruments were reviewed in Chapters 7–10. To recap, the *modified duration* of a bond is an indicator of its sensitivity to interest rates; it measures the change in price of the bond for a 1% change in yield. The higher the level of modified duration in a bond, the more sensitive it is to changes in market interest rates. However the relationship between price and yield in a bond is not a straight-line one; for changes in yield much above 50 basis points, the result given by the modified duration measure becomes increasingly inaccurate. Thus the measure given by a bond's modified duration is only an approximation. The extent of the approximation of modified duration is given by *convexity*, which might be said to be a measure of the error made in using modified duration. Bonds with greater convexity perform differently under the same conditions compared to low convex bonds. For option products, the *delta* of an option is a measure of how much its price moves with respect to changes in the price of the underlying asset. Therefore delta for an option is a similar risk measure to modified duration for bonds. Due to the way it is calculated, the delta figure is also an approximation, and the delta of an option changes as the price of the underlying changes. To measure the change in an option delta with respect to changes in the price of the underlying, traders calculate the *gamma* of the option.

The convex relationship between bond price and yield illustrates that the change in prices for a given change in interest rates is not constant, and nor is it identical, for all but very small amounts, for both upward and downward changes in yield. This feature makes it a difficult property to capture in VaR calculations. Generally the slope of the convex curve flattens out as the yield increases, while it steepens as yields decrease. As this property cannot be captured, and differs among individual bonds, risk managers often overcome the problem with modelling it by simply assuming that the relationship is constant, and using modified duration as the usual risk measure. Under this assumption, we would calculate the VaR for a bond portfolio as described in Example 37.3.

EXAMPLE 37.3 Simplified VaR for a bond portfolio

A portfolio consists of several conventional bonds. The portfolio modified duration is 6.794, and it has a market value of £368 million. Market yields are expected to rise during the year, and the worst-case scenario, to 95% confidence, is a rise in yields of 75 basis points. As the interest-rate scenario is already given to the required degree of confidence, the VaR of the portfolio is given by: $368 \text{ million} \times -6.794 \times 0.75\%$ or £18 751 440.

The drawback in assuming constant changes in the price/yield relationship is that it is very inaccurate for large changes in yield. One of the main objectives of risk management is to provide accurate management information on the risk exposure of the bank, particularly under volatile conditions such as a market correction. As these are exactly the type of situation where there are large-scale moves in interest rates, the information contained in a bank's risk reports would be of questionable value.

To overcome this problem, it is necessary to make a convexity adjustment to the duration-based VaR measure. Convexity is the second derivative of duration risk measure, and is derived using a Taylor expansion (this is discussed in greater detail in a separate chapter). The convexity measure is used to provide a more accurate measure of modified duration, and is given by:

$$\frac{1}{2} \times CV \times (\Delta r)^2. \quad (37.13)$$

The convexity adjustment is added to the modified duration value. This is demonstrated in Example 37.4.

EXAMPLE 37.4 Applying the convexity adjustment for a bond

The analysis is conducted on a hypothetical bond, the 6% 2020, which pays an annual coupon on an actual/actual basis. Its redemption date is 24 January 2021, so the bond has precisely 20 years to maturity. The details are listed in Table 37.10.

Bond	6% 2020
Yield	6.15%
Price	98.30027
Duration	12.08839
Modified duration	11.38803
Convexity	184.21158

Table 37.10: Modified duration and convexity of 6% 2020 bond.

The bond has a modified duration of 11.38803%, a high value and not unexpected as the bond is very long-dated. The modified duration value is used to estimate the price of the bond resulting from a 1% upward move in yields; we see from Table 37.11 that already, using only modified duration, the new price is over-estimated by approximately one point. The convexity adjustment, using (37.13), is 0.92106%. However the over-estimation is pronounced for a large change, as shown by the 3% upward move in yields. Using the convexity adjustment, which is 8.28952%, the estimated price is considerably closer to the actual price. Note that we are estimating prices for a rise in yields; this means that the bond price will fall, so the modified duration measure is given a negative value. When making the convexity adjustment, we add the value for convexity to the (negative) modified duration value, thus reducing it. For a downward move in yield, modified duration is a positive value, as prices will rise. Adding the convexity adjustment will then increase the value for modified duration.

Yield	7.15%	9.15%	5.00%
Price	87.95764	71.54983	112.46221
MD estimate	86.91224	64.13618	111.39650
MD + CV estimate	87.8333	72.42570	113.46889

Table 37.11: Price estimate using convexity adjustment.

Convexity is a valuable property in a bond and fund managers frequently look to invest in high convexity bonds; there is usually a convexity premium in a bond, so that it will trade at a lower yield to a bond of similar duration but lower convexity. The attraction of convexity is apparent if the price/yield profile for two bonds is plotted; the bond with higher convexity will outperform the other bond no matter what happens to interest rates. That is, it will rise in price by more for a given fall in yields, and it will drop in price by less for a given rise in yields. Equally therefore, to be short convexity, as a result of running a short position in a bond, can significantly add to a trader's risk. This is because, being short, even a quite small fall in interest rates can substantially increase the risk exposure of the book. This property is even more pronounced with option products.

37.7.2 Option gamma

The gamma measurement for an option is conceptually similar to convexity for a bond. Convexity is a measure of the error made in using modified duration, that is, the curvature of the price/yield relationship. Gamma is the second derivative of an option's delta, so in effect measures the same thing as convexity. As with convexity, it is important for a trader to be aware of the gamma exposure of his book, as at a high gamma level, even very small changes in the price of the underlying asset may lead to substantial mark-to-market losses. A trader who writes options, whether put or call options, is effectively short gamma.

The gamma effect on an option book cannot be captured accurately by most VaR models. This is because the relationship between gamma and the price of the underlying is non-linear. To approximate the VaR measure for an option book, a *delta-gamma* calculation is made, and although it is still not completely accurate, it is a better estimate than the conventional delta-normal approach. However, although intuitively delta-gamma is similar to the convexity adjustment for a bond portfolio, it is not as good an approximation as the convexity measure. This is because behaviour of an option is more unpredictable than that of a bond. A bond instrument may be broken down into a series of zero coupon bonds, so that volatility and other data maybe adjusted for convexity with relative ease. This is not as easy for options, and becomes particularly acute as an option approaches maturity. For example, an at-the-money option will experience extreme movement in its gamma as it approaches maturity, in a way that unpredictable. It is difficult to capture this effect in a VaR model. Nevertheless, the delta-gamma measure is recognised as a close approximation of option book risk, short of using simulation-type VaR models.

The gamma effect has an impact on the distribution of returns from an option book. This transforms the distribution from normal to one with slightly skewed tails, as illustrated by Figure 37.5.

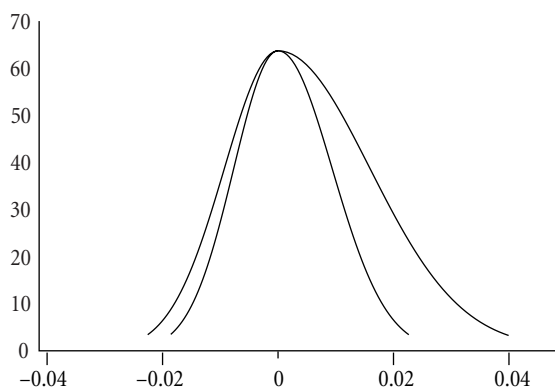


Figure 37.5: Delta + gamma effect.

To illustrate the gamma adjustment, consider a position in a bond instrument and a put option on foreign exchange. The details are set out in Table 37.12. The interest-rate and FX volatility and correlation data may be obtained from RiskMetrics. Using these, we calculate the undiversified VaR in the normal manner, multiplying the market value of the instrument by the volatility value to obtain VaR. For the option, we also multiply the value and the volatility by the delta (that is, $1,507,000 \times 0.54 \times 6.10\%$). The delta adjustment is required because the price of the option does not “tick-for-tick” with the underlying, but by 0.54 for each unit change in the underlying. The undiversified VaR is £73,753.

Bond nominal	2,000,000
Maturity (years)	2
Market value	1,507,000
Volatility	1.60%
Undiversified VaR	24,112
Nominal value FX option	1,507,000
Delta	0.54
Gamma	3.9
FX volatility	6.10%
Undiversified VaR	49,641
Correlation coefficient	−0.31

Table 37.12: Hypothetical portfolio and undiversified VaR.

To calculate the undiversified VaR we require the portfolio variance, which would normally be done in the conventional way using matrices; here there are only two assets so we may use the standard variance equation (37.4). The square root of this is the VaR, which is calculated as:

$$\begin{aligned} Var_{port} &= \sqrt{24,112^2 + 49,641^2 + (2 \cdot -0.31 \cdot 24,112 \cdot 49,641)} \\ &= 41,498. \end{aligned}$$

Although the diversified VaR is more realistic a measure, it will not take into account the gamma effect of the option. Previously we allowed for the delta of the option, which was used to modify the volatility level, which changed from 6.10% to 3.294%. The gamma adjustment is made by using equation (37.13), which in this case gives a gamma adjustment of 0.7256%. The delta-gamma approximation for the volatility is therefore 2.568%. Multiplying this by the weighting (the option value) we have a new diversified VaR for the option of 38 700. If we use the same portfolio variance equation we obtain a delta-gamma adjusted diversified VaR of £27,488.

The delta-gamma adjustment is only an approximation of an option book's gamma risk exposure, and it is not as close as a convexity adjustment. This is due mainly to the unpredictable behaviour of gamma as an option approaches maturity, more so if it is at-the-money.

37.8 Stress testing

Risk measurement models and their associated assumptions are not without limitation. It is important to understand what will happen should some of the model's underlying assumptions break down. Stress testing is a process whereby a series of scenario analyses or simulations are carried out to investigate the effect of extreme market conditions on the VaR estimates calculated by a model. It is also an analysis of the effect of violating any of the basic assumptions behind a risk model. If carried out efficiently stress testing will provide a clearer information on the potential exposures at risk due to significant market corrections, which is why the Basel Committee recommends that it be carried out.

37.8.1 Simulating stress

There is no standard way to do stress testing. It is a means of experimenting with the limits of a model; it is also a means to measure the residual risk which is not effectively captured by the formal risk model, thus complementing the VaR framework. If a bank uses a confidence interval of 99% when calculating its VaR, the losses on its trading portfolio due to market movements should not exceed the VaR number on more than one day in 100. For a 95% confidence level the corresponding frequency is one day in 20 or roughly one trading day each month. The question to ask is "what are the expected losses on those days?" Also what can an institution do to protect itself against these losses? Assuming that returns are normally distributed provides a workable daily approximation for estimating risk but when market moves are more extreme these assumptions no longer add value. The 1% of market moves that are not used for VaR calculations contain events such as the October 1987 crash, the bond market collapse of February 1994 and the Mexican peso crisis at the end of 1994. In these cases market moves were much larger than any VaR

model could account for; in fact the October 1987 crash was a 20 standard deviation move. Under these circumstances correlations between markets also increase well above levels normally assumed in models.

An approach used by risk managers is to simulate extreme market moves over a range of different scenarios. One method is to use Monte Carlo simulation. This allows dealers to push the risk factors to greater limits; for example a 99% confidence interval captures events up to 2.33 standard deviations from the mean asset return level. A risk manager can calculate the effect on the trading portfolio of a 10 standard deviation move. Similarly risk managers may want to change the correlation assumptions under which they normally work. For instance if markets all move down together, something that happened in Asian markets from the end of 1997 and emerging markets generally from July 1998 after the Russian bond technical default, losses will be greater than if some markets are offset by other negatively correlated markets.

Only by pushing the bounds of the range of market moves that are covered in the stress testing process can financial institutions have an improved chance of identifying where losses might occur, and therefore a better chance of managing their risk effectively.

37.8.2 Stress testing in practice

For effective stress testing, a bank has to consider non-standard situations. The Basel policy group has recommended certain minimum standards in respect of specified market movements; the parameters chosen are considered large moves to overnight marks, including:

- parallel yield curve shifts of 100 basis points up and down;
- steepening and flattening of the yield curve (2-year to 10-year) by 25 basis points;
- increase and decrease in 3-month yield volatilities by 20%;
- increase and decrease in equity index values by 10%;
- increase and decrease in swap spread by 20 basis point.

These scenarios represent a starting point for a framework for routine stress testing.

Banks agree that stress testing must be used to supplement VaR models. The main problem appears to be difficulty in designing appropriate tests. The main issues are:

- difficulty in “anticipating the unanticipated”;
- adopting a systematic approach, with stress testing carried out by looking at past extremes and analysing the effect on the VaR number under these circumstances;
- selecting 10 scenarios based on past extreme events and generating portfolio VaRs based on re-runs of these scenarios.

The latest practice is to adapt stress tests to suit the particular operations of a bank itself. On the basis that one of the main purposes of stress testing is to provide senior management with accurate information of the extent of a bank's potential risk exposure, more valuable data will be gained if the stress test is particularly relevant to the bank. For example, an institution such as Standard Chartered Bank, which has a relatively high level of exposure to exotic currencies, may design stress tests that take into account extreme movements in say, regional Asian currencies. A mortgage book holding option positions only to hedge its cash book, say one of the former UK building societies that subsequently converted to banks, may have no need for excessive stress testing on perhaps, the effect of extreme moves in derivatives liquidity levels.

37.8.3 Issues in stress testing

It is to be expected that extreme market moves will not be captured in VaR measurements. The calculations will always assume that the probability of events such as the Mexican peso devaluation are extremely low when analysing historical or expected movements of the currency. Stress tests need to be designed to model for such occurrences. *Back testing* a firm's qualitative and quantitative risk management approach for actual extreme events often reveals the need to adjust reserves, increase the VaR factor, adopt additional limits and controls and expand risk calculations. With back testing a firm will take say, its daily VaR number, which we will assume is computed to 95% degree of confidence; the estimate will be compared to the actual trading losses suffered by the book over a 20-day period, and if there is a significant discrepancy the firm will need to go back to its model and make adjustments to

parameters. Frequent and regular back testing of the VaR model's output with actual trading losses is an important part of stress testing. To conduct back-testing efficiently, a firm would need to be able to strip out its intra-day profit-and-loss figures, so it could compare the actual change in P/L to what was forecast by the VaR model.

The procedure for stress testing in banks usually involves:

- creation of hypothetical extreme scenarios;
- computation of corresponding hypothetical P&Ls.

One method is to imagine *global* scenarios. If one hypothesis is that the euro appreciates sharply against the dollar,⁷ the scenario needs to consider any related areas, such as the effect if any on the Swiss franc and Norwegian krone rate, or the effect on the yen and interest rates. Another method is to generate many *local* scenarios and so consider a few risk factors at a time. For example given an FX option portfolio a bank might compute the hypothetical P&L for each currency pair under a variety of exchange rate and implied volatility scenarios. There is then the issue of amalgamating the results: one way would be to add the worst case results for each of the sub-portfolios, but this ignores any portfolio effect and cross-hedging. This may result in an over-estimate that is of little use in practice.

Nevertheless stress testing is one method to account for the effect of extreme events that occur more frequently than would be expected were asset returns to follow a true normal distribution. For example five-standard-deviation moves in a market in one day have been observed to occur twice every 10 years or so, which is considerably more frequent than given by a normal distribution. Testing for the effects of such a move gives a bank an idea of its exposure under these conditions.

37.9 Value-at-Risk methodology for credit risk

Credit risk emerged as a significant risk management issue in the 1990s. As returns and interest spreads in developed markets have been reducing over time, in increasingly competitive markets banks and securities houses are taking on more forms of credit risk, in a bid to boost returns. This has led to both retail and investment banks being exposed to higher levels of *credit risk*. There are two main types of credit risk:

- credit spread risk;
- credit default risk.

Credit spread is the excess premium required by the market for taking on a certain assumed credit exposure. Credit spread risk is the risk of financial loss resulting from changes in the level of credit spreads used in the marking-to-market of a product. It is exhibited by a portfolio for which the credit spread is traded and marked. Changes in observed credit spreads affect the value of the portfolio. Credit default risk is the risk that an issuer of debt (*obligor*) is unable to meet its financial obligations. Where an obligor defaults a firm generally incurs a loss equal to the amount owed by the obligor less any recovery amount which the firm recovers as a result of foreclosure, liquidation or restructuring of the defaulted obligor. By definition all portfolios of exposures, except those of developed country government bonds, exhibit an element of credit default risk.

Therefore, there are two types of *credit event* that a credit VaR model can model; the event that an obligor goes bankrupt and defaults completely, and the event that the obligor suffers a credit downgrade from a credit rating agency.

37.9.1 Distribution of credit events

The crucial difference between modelling market risk and credit risk is that the distribution of credit default returns does not follow a normal distribution; moreover, it is not symmetrical either. Market risk, although generally believed to have “fatter tails” than a true distribution, at least follows a symmetrical pattern, which essentially implies that returns from portfolio have an equal chance of being positive or negative. The returns from credit events are skewed, which implies that in most cases the returns have a greater possibility of being positive. The distribution is shown at Figure 37.6. Losses due to negative returns, though less frequently occurring, tend to be relatively large. The main impact on risk management is that variance-covariance VaR methodologies will not capture accurately risk arising from credit exposure. And for the same reasons as with market risk, simulation methods will be able estimate the best credit risk approximations.

⁷ Unlikely, granted, if one were to listen to most FX dealers!

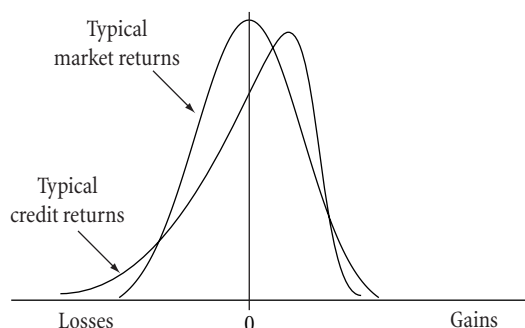


Figure 37.6: Distribution of credit returns.

37.9.2 Probability of default

Credit rating agencies such as Moody's and S&P publish probabilities that bonds will suffer a credit event. These probabilities are assigned to rating class of bonds, for example the one-year probabilities for bonds with triple-B and triple-C ratings are shown in Table 37.13.

Probability (%)	BBB bond	CCC bond
AAA	0.03	0.22
AA	0.32	0.01
A	5.94	0.23
BBB	87.00	1.29
BB	4.40	2.39
B	1.20	11.25
CCC	0.13	68.00
Default	0.98	16.61

Table 37.13: One-year probability of credit event, BBB and CCC bonds. Source: Moody's.

Not surprisingly, for both bonds the highest probability event over one year is that they stay at the same rating. A holder of both bonds will suffer a loss due to a credit event if either bond is downgraded, so the risk of loss is 6.71% for the triple-B bond and 16.61% for the triple-C bond. The combined probability of making a loss on both bonds then is 1.1145%; in a normal distribution the probability of loss for both bonds would be 50% for each. The distribution for bonds of varying credit quality is shown in Figure 37.7, which is reproduced from the CreditMetrics technical document.

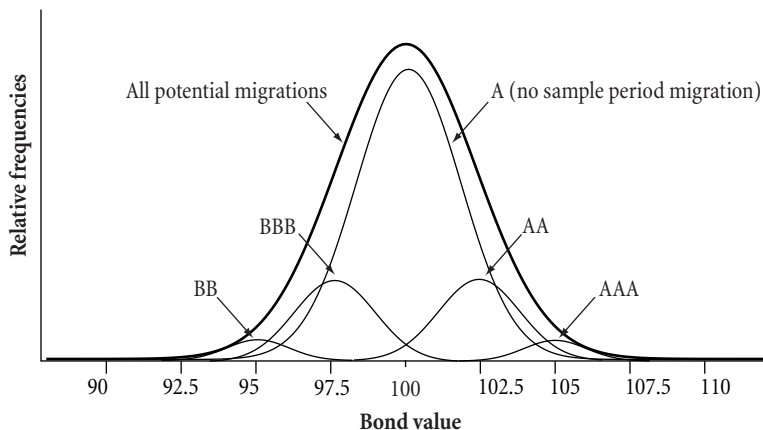


Figure 37.7: Distribution of credit returns by rating. Source: JPMorgan.

To make use of the probabilities of default, they must be adjusted to standard deviations. This requires an assumption of normality for bond prices, so that the normal distribution may then be applied to see over how many standard deviations distance away a change in credit rating is.

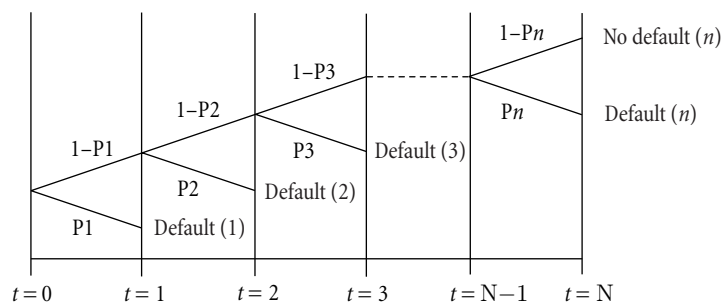


Figure 37.8: Probability of default. Source: J. P. Morgan.

Figure 37.8, reproduced from the CreditMetrics technical document, shows how the binomial tree method for calculating loss due to credit events may be applied, and also illustrates why the assumption of a normal distribution for credit returns is unsuitable.

37.9.3 Modelling VaR for credit risk

After its initial introduction as a measurement tool for market risk in 1994, practitioners began to apply VaR methodology in the estimation of credit risk exposure. CreditMetrics™ applies the same methodology that is used in the JPMorgan RiskMetrics™ VaR model. CreditMetrics™ calculates probabilities of loss on a portfolio due both to default of any issuer, or due to any change in credit rating of an issuer. Among other institutions, the investment bank CSFB introduced an in-house credit risk VaR model that calculates the probability of loss due solely to instances of default of any issuer; their system is known as CreditRisk+. The main credit risk VaR methodologies take a *portfolio* approach to credit risk analysis. This means that:

- credit risks to each obligor across the portfolio are re-stated on an equivalent basis and aggregated in order to be treated consistently, regardless of the underlying asset class;
- correlations of credit quality moves across obligors are taken into account.

This allows portfolio effects – the benefits of diversification and risks of concentration – to be quantified. The *portfolio* risk of an exposure is determined by four factors:

- size of the exposure;
- maturity of the exposure;
- probability of default of the obligor;
- systematic or concentration risk of the obligor.

All of these elements need to be accounted for when attempting to quantify credit risk exposure.

Credit VaR, like market risk VaR considers (credit) risk in a mark-to-market framework. That is it views credit risk exposure to arise because of changes in portfolio value that result from credit events, which are changes in obligor credit quality that include defaults, credit rating upgrades and rating downgrades. Nevertheless credit risk is different in nature from market risk. Typically market return distributions are assumed to be relatively symmetrical and approximated by normal distributions, for the purposes of VaR calculations. (In fact the occurrence of extreme market movements, such as stock market crashes, is more frequent than would be predicted by pure normal distributions. If we were to model the frequency of actual market returns, our resulting distribution would exhibit fatter tails than the conventional normal curve, a phenomenon referred to as *leptokurtosis*.) In credit portfolios value changes will be relatively small as a result of minor credit rating upgrades or downgrades, but can be substantial upon actual default of a firm. This remote probability of large losses produces skewed distributions with heavy

downside tails that differ from the more normally distributed returns assumed for market VaR models. We illustrate the different curves in Figure 37.9.

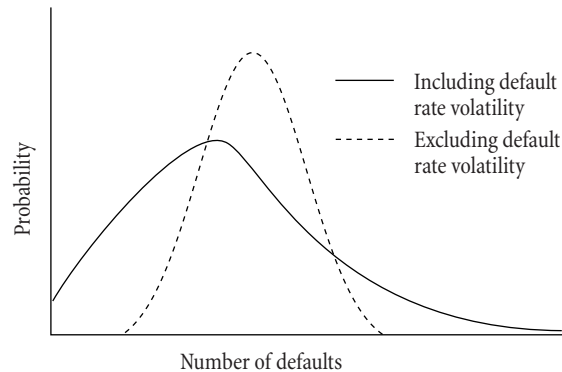


Figure 37.9: Distribution of market and credit returns.

This difference in risk profiles does not prevent us from assessing risk on a comparable basis. Analytical method market VaR models consider a time horizon and estimate value-at-risk across a distribution of estimated market outcomes. Credit VaR models similarly look to a horizon and construct a distribution of values given different estimated credit outcomes.

When modelling credit risk the two main measures of risk are:

- distribution of loss: obtaining distributions of loss that may arise from the current portfolio. This considers the question of what the expected loss is for a given confidence level;
- identifying extreme or catastrophic outcomes; this is addressed through the use of scenario analysis and concentration limits.

To simplify modelling no assumptions are made about the causes of default. Mathematical techniques used in the insurance industry are used to model the event of an obligor default.

37.9.4 Time horizon

The choice of time horizon will not be shorter than the time frame over which “risk-mitigating” actions can be taken, that is, the time to run down a book or off-load the exposure. In practice this can be a fairly time-consuming and costly process. The time horizon therefore is essentially a function of the type of instrument that is traded on the book; illiquid or low credit-quality bonds will be more difficult to unwind than higher rated paper. CSFB (who introduced the CreditRisk+ model) suggest two alternatives:

- a constant time horizon such as one year;
- a hold-to-maturity time horizon.

Modelling credit risk requires certain data inputs; for example CreditRisk+ uses the following:

- credit exposures;
- obligor default rates;
- obligor default rate volatilities;
- recovery rates.

These data requirements present some difficulties. There is a lack of comprehensive default and correlation data and assumptions need to be made at certain times, which will affect the usefulness of any final calculation. For more liquid bond issuers there is obviously more data available. In addition rating agencies such as Moody’s have published data on for example, the default probabilities of bonds of each category.

The annual probability of default of each obligor can be determined by its credit rating and then mapping between default rates and credit ratings. A default rate can then be assigned to each obligor. Default rate volatilities can be observed from historic volatilities of such rates, and are available from credit rating agencies.

37.9.5 Applications of credit VaR

One of the objectives of a risk management system is to direct and prioritise actions, with a view to minimising the level of loss or expected loss. If we are looking at firm's credit exposure, when considering risk-mitigating actions there are various features of risk worth targeting, including obligors having:

- the largest absolute exposure;
- the largest percentage level of risk (volatility);
- the largest absolute amount of risk.

In theory a credit-VaR methodology helps to identify these areas and allow the risk manager to prioritise risk-mitigating action. This is clearly relevant in a bond dealing environment, for example in times of market volatility or economic recession, when banks will seek to limit the extent of their loan book. Bond desks will seek to limit the extent of their exposure to obligors.

Another application that applies in a bond dealing environment is in the area of exposure limits. Within bank dealing desks credit risk limits are often based on intuitive, but arbitrary, exposure amounts. It can be argued that this is not a logical approach because resulting decisions are not risk-driven. Risk statistics used as the basis of VaR methodology can be applied to credit limit setting, in conjunction with the standard qualitative analysis that is normally used. For this reason the limit setting departments of banks may wish to make use of a credit VaR model to assist them with their limit setting.

37.9.6 Assessment of VaR tool

Although the methodology behind value-at-risk is based on well established statistical techniques, it is a more complex exercise to apply VaR in practice. Applying VaR to the whole firm can bring with it problems that hinder the calculation, including unstable market data, issues in synchronising trading book positions across the bank and across global trading books, and the issues presented by the differing characteristics of different instruments. As one might expect, a VaR calculation can be undertaken more easily (and is likely to be proved inaccurate on fewer occasions) for a foreign exchange trading book than an exotic option trading book, due to the different behaviour of the prices of those two instruments in practice. Often banks will use a correlation method VaR model for some of their trading books and a Monte Carlo simulation approach for books holding exotic option instruments.

It is important to remember that VaR is a tool that attempts to quantify the size of a firm's risk exposure to the market. It can be viewed as a management information tool, useful for managing the business. It is conceptually straightforward to grasp because it encompasses the *market* risk of a firm into one single number, however it is based on a statistical model of that firm's risks, and it does not capture – nor does it attempt to capture – all the risks that the firm is faced with. In the real world, the statistical assumptions used in VaR calculation will sometimes not apply, for example in times of extreme market movements, such as market crashes or periods of high volatility, as experienced in 1997 and 1998 with the volatility in Asian currency markets. For example, VaR makes no allowance for *liquidity* risk. In times of market correction and/or high market volatility, the inability of a bank to trade out of its positions (possible because all of the other market participants have the same positions, and wish to trade out of them at the same time) will result in higher losses than normal, losses that a VaR model is unlikely to have catered for. In such a case, it would have been a combination of market and liquidity risk that the bank was exposed to and which resulted in trading losses. In addition it has been argued that the normal distribution underestimates the risks of large market movements (as experienced in market crashes) and therefore is not an accurate representation of real market conditions. Banks may need to allow for this when calculating their VaR estimate, for example by building in a compensating factor into their model.

Our discussion needs to be borne in mind at senior management level, so that it is clearly understood what the VaR figure means to a bank. It is important not to be over-reliant on VaR as the only measure of a firm's risk exposure, but rather as a tool forming part of an integrated and independent risk management function operating within the firm.

Appendices

APPENDIX 37.1(i) Statistical concepts and the normal distribution

One of the most accessible introductions to the normal and log-normal distributions is given in Julian Walmsley's excellent book, *The Foreign Exchange and Money Markets Guide*. For the benefit of readers, and with the kind permission of John Wiley & Sons, we reproduce pages 492–498 of Walmsley's book here as Appendix 37.1(i).

Mean and standard deviation

Two important concepts are the mean and standard deviation of a series of numbers. The mean of a sample of numbers is useful information in its own right, but we are often interested in the variability of the underlying data, which is why we use the standard deviation. Dividing a total of the population by the number of observations n (for technical reasons this is $n - 1$) gives us the variance of a set of numbers, however the variance itself is not very meaningful. The standard deviation is the square root of the variance and so is measured in more understandable units. Thus we can say the average (often called the mean) of a set of numbers is say, 10, and the standard deviation is 2. This gives an idea of how the variable the underlying data is.

The formal definitions are as follows:

$$\text{Mean} \quad \bar{x} = \frac{1}{n} \sum_{i=1}^n x_i$$

$$\text{Variance} \quad \sigma^2 = \frac{1}{n-1} \sum_{i=1}^n (x_i - \bar{x})^2$$

If the variance is for a population rather than a sample of the population we divide by n .

Probability distributions

A probability distribution is a model for an actual or empirical distribution. Consider an experiment in which three coins are tossed simultaneously and the number of heads which show is recorded. The number of heads, X , can take any one of the values 0, 1, 2 or 3. Thus X is called a discrete random variable. There are eight possible outcomes to the experiment: TTT, TTH, THT, HTT, THH, HTH, HHT, HHH. Assuming the coins to be fair each outcome is equally likely so in repeated trials we would expect X to take the value 0 in one out of eight, the value 1 in three out of eight, and so on. Therefore the probability distribution for the experiment, denoted by $P(X)$, is:

n	0	1	2	3
$P(X=n)$	1/8	3/8	3/8	1/8

Actually performing the experiment would produce an empirical distribution which should grow closer to the theoretical distribution as the number of tosses increases.

The cumulative distribution function gives the probability that an observation X is less than or equal to the value n ; it is usually denoted by $F(x)$. For the example above we have:

n	0	1	2	3
$F(n)$	1/8	4/8	7/8	1

It is certain that n will be less than or equal to 3; the chances are 7 out of 8 that it will be less than or equal to 2, and so on. In this example we were considering a discrete set of outcomes (0, 1, 2, 3) and hence a discrete probability distribution. It is equally possible to have a continuous probability distribution: for example the probability that the return on a portfolio lies between 5% and 10% is associated with a continuous probability distribution.

Important probability distributions

Here we discuss three important probability distributions: the binomial distribution for discrete variables and the normal and lognormal distributions for continuous variables.

The binomial distribution is a discrete probability distribution and is defined as follows. Suppose there are n independent trials of an experiment with only the same two possible outcomes at each trial, usually denoted "success" and "failure", where p is the probability success in a single trial. Then the probability, $p(x)$ of obtaining x "successes" from the n trials is given by the binomial distribution.

This is:

$$p(x) = \frac{n!}{x!(n-x)!} p^x (1-p)^{n-x}$$

where $n!$ means “ n factorial”, that is $n \times (n-1) \times (n-2) \times (n-3) \times \cdots \times (n-(n-1))$. It will be shown later that for large numbers of observations the binomial distribution can be approximated by the normal distribution.

The normal distribution

The Central Limit Theorem (often loosely described as the law of large numbers) is the reason for the central role of the normal distribution in statistical theory. Very many distributions tend towards the normal, given a sufficient number of observations.

Consider a simple example as follows. A conventional six-sided die is tossed n times and the total score T is noted. If $n = 1$, T may take the values 1, 2, 3, 4, 5, 6 with probability $1/6$ in each case. This binomial distribution, which has mean 3.5 and variance 2.916, is plotted in Figure 37.10. The normal distribution, with the same mean and variance, is also shown. Clearly the two probability distributions are quite different.

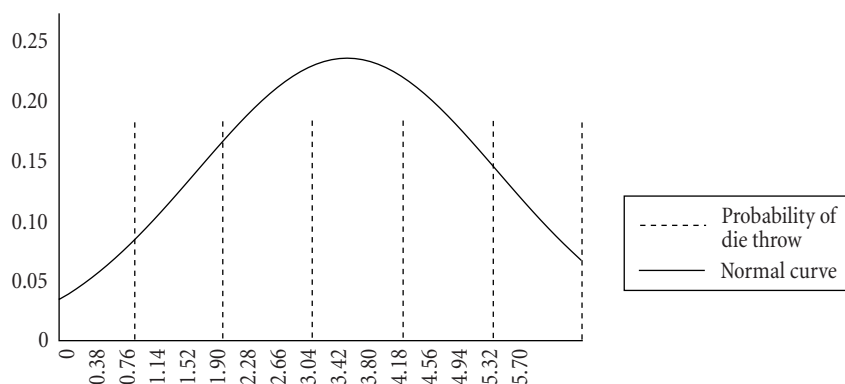


Figure 37.10

Suppose now we toss the die twice, i.e., $n = 2$. Now T may take the values 2, 3 ... 12 with probabilities $1/36$, $2/36$, $3/36$, $4/36$, $5/36$, $6/36$... $1/36$. That is, the probability of throwing a total of 2 is the probability of throwing 1 each time: which we know is $(1/6) \times (1/6) = 1/36$. But we can throw a total of three in two ways: by throwing 2 and 1, or 1 and 2. Hence the probability that $T = 3$ is $2/36$. We can throw 4 in three ways: 1 + 3, 3 + 1, or 2 + 2, hence its probability is $3/36$, and so on. This distribution, with mean 7 and variance 5.83, is plotted in Figure 37.11 together with a normal distribution having the same mean and variance. They resemble one another more closely than before.

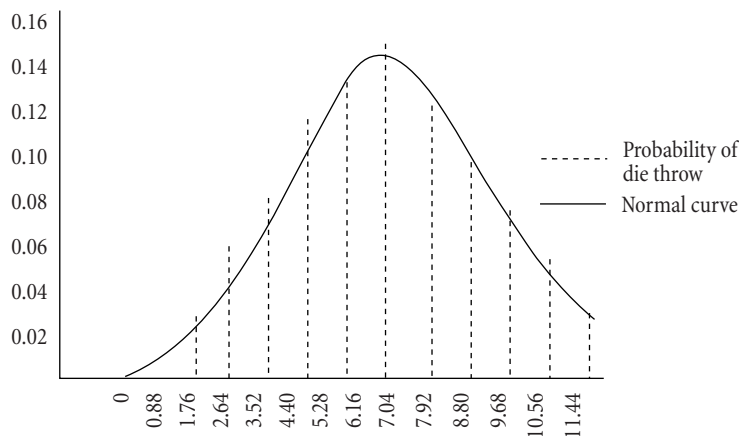


Figure 37.11

Similarly when $n = 3$ we get a probability distribution with a mean of 10.5 and variance 8.75 and in Figure 37.12 we plot the normal distribution having the same mean and variance. Here the resemblance between the two distributions is quite noticeable.

This example was chosen because the two distributions converge quickly. The same process holds good, even when taking place slowly, for many statistical distributions. Hence the great importance of the normal distribution in statistics. If we have large numbers of observations – for example, percentage changes in exchange rates, closing prices in government bonds or claims under motor insurance policies – it is often practical to assume that they are normally distributed.

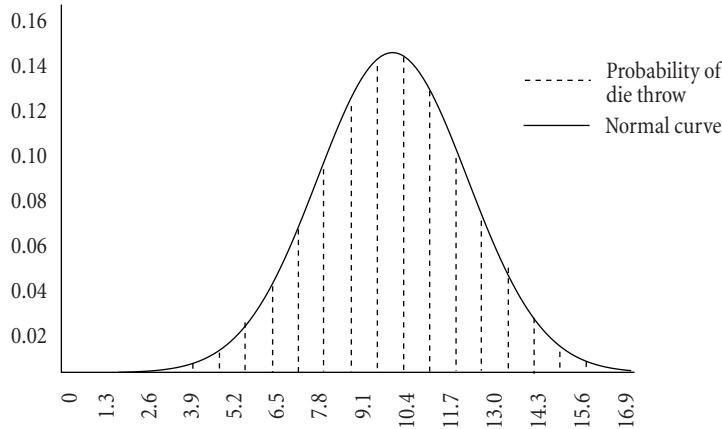


Figure 37.12

The log-normal distribution

A random variable X has a log-normal distribution with mean μ and variance σ if $Y = \ln(X)$ has the normal distribution with mean μ and variance σ . Often, as we have seen in the discussions on Value-at-Risk, it is convenient to assume that the returns from holding an asset are normally distributed. It is often convenient to define the return in logarithmic form as:

$$\ln\left(\frac{P_t}{P_{t-1}}\right)$$

where P_t is the price today and P_{t-1} is the previous price.

If this is assumed to be normally distributed then the underlying price will have a log-normal distribution. The log-normal distribution never goes to a negative value, unlike the normal distribution, and hence is intuitively more suitable for asset prices. The distribution is illustrated as Figure 37.13.

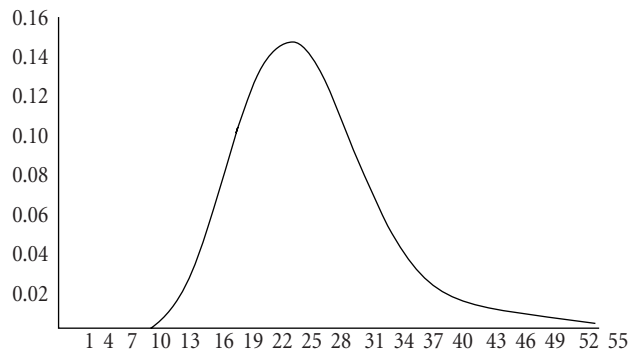


Figure 37.13

Confidence intervals

Suppose that we have an estimate, x , of the average of a given statistical population where the true mean of the population is μ . Suppose that we believe that on average x is an unbiased estimator of μ . Although this means that on average x is accurate, the specific sample that we observe will almost certainly be above or below the true level. Accordingly if we want to be reasonably confident that our inference is correct, we cannot claim that μ is precisely equal to the observed x . Instead we must construct an interval estimate or confidence interval of the following form:

$$\mu = \bar{x} \pm \text{sampling error.}$$

The crucial question is: how wide must this confidence interval level be? The answer of course will depend on how much x fluctuates. The first step is to establish how stringent our requirements are. Suppose that we decide we want to be 95% confident that our estimate is accurate. Suppose also that we believe that the elements of the population are normally distributed. In that case we would expect that the population would be distributed along the lines portrayed in Figure 37.14 below.

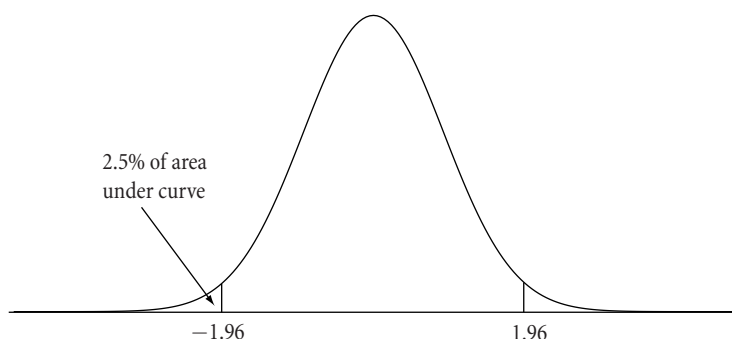


Figure 37.14

A well-known feature of the normal distribution is that 2.5% of the outcomes of a normally distributed process can be expected to fall more than 1.96 standard deviations from the mean. Clearly therefore 95% of the outcomes can be expected to fall within ± 1.96 standard deviations. That is to say there is a 95% chance that the random variable x will fall between $\mu - 1.96$ standard deviations and $\mu + 1.96$ standard deviations. This would be referred to as a “two-sided” (or “two-tailed”) confidence interval. It measures the probability of a move upwards or downwards by the random variable outside the limits we would normally expect.

A case can be made however that we should only consider a one-sided test if we are concerned with the risk of loss: a move upward into profit is of less concern (certainly to a risk manager anyway!). It is well known that 5% of the outcomes of a normally distributed process can be expected to fall more than 1.645 (rounded to 1.65) standard deviations from the mean. This would be referred to as a one-sided confidence interval.

Putting that less technically, suppose that we have a position in sterling: we are long of sterling at \$1.70. We believe that \$1.70 represents the mean of sterling’s distribution; we believe that the sterling exchange rate has an annual standard deviation of 17 cents, which is 10% volatility. In that case if we wish to have a 95% confidence interval for sterling we will set the interval at 1.65×17 cents (assuming a one-sided confidence interval). That is, an interval of 28.05 cents. In other words we are 95% confident that sterling will remain above \$1.3695 during the next year.

Alternatively if we were to set a two-sided confidence interval we would use an interval of 1.96×17 cents, an interval of 33.32 cents. In this case we would expect with 95% confidence that sterling remain in the range \$1.3668 to \$2.0332.

Clearly an annual standard deviation of 17 cents will translate into a smaller daily movement; in fact to convert from annual to daily we must divide the annual level by $\sqrt{250}$. (The denominator of 250 arises from the assumed number of 250 working days in a year. We use the square root to conform with the definition of the standard deviation and the normal distribution.) Thus the daily standard deviation is $17 / 15.811 = 1.07517$ cents.

Hence on a daily business using a one-sided confidence interval we would be 95% confident that sterling would not fall by more than $1.65 \times 1.07517 = 1.77404$ cents. That is, we would be 95% confident that sterling would remain above \$1.6822. Using a two-sided confidence interval we would be 95% confident that sterling would remain within the range $\$1.70 \pm (1.96 \times 1.07517) = \$1.6789 - \$1.7211$.

APPENDIX 37.1(ii) Assumption of normality

The RiskMetrics™ assumption of conditional multivariate normality is open to criticism that financial series tend to produce “fat tails” (leptokurtosis). That is, in reality there is a greater occurrence of non-normal returns than would be expected for a purely normal distribution. This is shown in Figure 37.15. There is evidence that fat tails are a problem for calculations. The RiskMetrics™ technical document defends its assumptions by pointing out that if volatility changes over time there is a greater likelihood of incorrectly concluding that the data is not normal when in fact it is. In fact conditional distribution models can generate data that possesses fat tails.

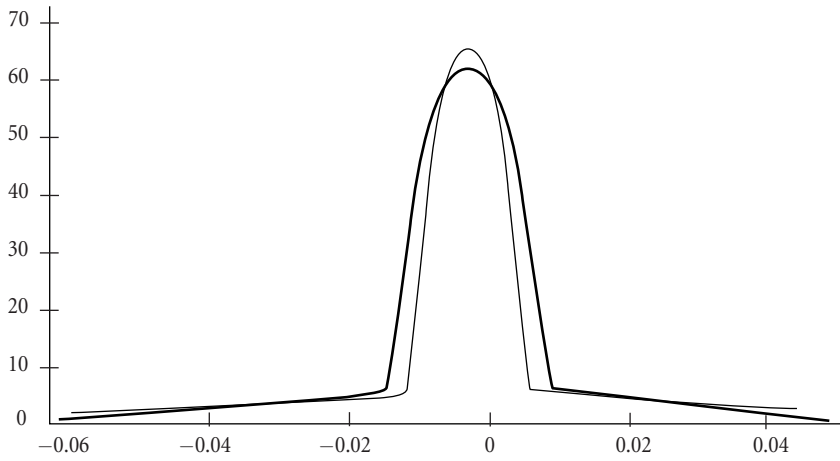


Figure 37.15: Leptokurtosis.

Higher moments of the normal distribution

The *skewness* of a price data series is measured in terms of the third moment about the mean of the distribution. If the distribution is symmetric, the skewness is zero. The measure of skewness is given by

$$\text{Skewness} = \frac{\frac{1}{n} \sum_{i=1}^n (x_i - \bar{x})^3}{\sigma^3}. \quad (37.14)$$

The *kurtosis* describes the extent of the peak of a distribution, that is how peaked it is. It is measured by the fourth moment about the mean. A normal distribution has a kurtosis of three. The kurtosis is given by

$$\text{Kurtosis} = \frac{\frac{1}{n} \sum_{i=1}^n (x_i - \bar{x})^4}{\sigma^4}. \quad (37.15)$$

Distributions with a kurtosis higher than three are commonly observed in asset market prices and are called *leptokurtic*. A leptokurtic distribution has higher peaks and fatter tails than the normal distribution. A distribution with kurtosis lower than 3 is known as *platykurtic*.

APPENDIX 37.2 The Normal distribution**Normal distribution table**

Number of standard deviations away from the mean: $Z = \frac{X - \mu}{\sigma}$

Z Value	0.0000	0.0100	0.0200	0.0300	0.0400	0.0500	0.0600	0.0700	0.0800	0.0900
0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9247	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916

Z Value	0.0000	-0.0100	-0.0200	-0.0300	-0.0400	-0.0500	-0.0600	-0.0700	-0.0800	-0.0900
0	0.5000	0.4960	0.4920	0.4880	0.4840	0.4801	0.4761	0.4721	0.4681	0.4641
-0.1	0.4602	0.4562	0.4522	0.4483	0.4443	0.4404	0.4364	0.4325	0.4286	0.4247
-0.2	0.4207	0.4168	0.4129	0.4090	0.4052	0.4013	0.3974	0.3936	0.3897	0.3859
-0.3	0.3821	0.3783	0.3745	0.3707	0.3669	0.3632	0.3594	0.3557	0.3520	0.3483
-0.4	0.3446	0.3409	0.3372	0.3336	0.3300	0.3264	0.3228	0.3192	0.3156	0.3121
-0.5	0.3085	0.3050	0.3015	0.2981	0.2946	0.2912	0.2877	0.2843	0.2810	0.2776
-0.6	0.2743	0.2709	0.2676	0.2643	0.2611	0.2578	0.2546	0.2514	0.2483	0.2451
-0.7	0.2420	0.2389	0.2358	0.2327	0.2296	0.2266	0.2236	0.2206	0.2177	0.2148
-0.8	0.2119	0.2090	0.2061	0.2033	0.2005	0.1977	0.1949	0.1922	0.1894	0.1867
-0.9	0.1841	0.1814	0.1788	0.1862	0.1736	0.1711	0.1685	0.1660	0.1635	0.1611
-1	0.1587	0.1562	0.1539	0.1515	0.1492	0.1469	0.1446	0.1423	0.1401	0.1379
-1.1	0.1357	0.1335	0.1314	0.1292	0.1271	0.1251	0.1230	0.1210	0.1190	0.1170
-1.2	0.1151	0.1131	0.1112	0.1093	0.1075	0.1056	0.1038	0.1020	0.1003	0.0985
-1.3	0.0968	0.0951	0.0934	0.0918	0.0901	0.0885	0.0869	0.0853	0.0838	0.0823
-1.4	0.0808	0.0793	0.0778	0.0764	0.0749	0.0735	0.0721	0.0708	0.0694	0.0681
-1.5	0.0668	0.0655	0.0643	0.0630	0.0618	0.0606	0.0594	0.0582	0.0571	0.0559
-1.6	0.0548	0.0537	0.0526	0.0516	0.0505	0.0495	0.0485	0.0475	0.0465	0.0455
-1.7	0.0446	0.0436	0.0427	0.0418	0.0409	0.0401	0.0392	0.0384	0.0375	0.0367
-1.8	0.0359	0.0351	0.0344	0.0336	0.0329	0.0322	0.0314	0.0307	0.0301	0.0294
-1.9	0.0287	0.0281	0.0274	0.0268	0.0262	0.0256	0.0250	0.0244	0.0239	0.0233
-2	0.0228	0.0222	0.0217	0.0212	0.0207	0.0202	0.0197	0.0192	0.0188	0.0183
-2.1	0.0179	0.0174	0.0170	0.0166	0.0162	0.0158	0.0154	0.0150	0.0146	0.0143
-2.2	0.0139	0.0136	0.0132	0.0129	0.0125	0.0122	0.0119	0.0116	0.0113	0.0110
-2.3	0.0107	0.0104	0.0102	0.0099	0.0096	0.0094	0.0091	0.0089	0.0087	0.0084

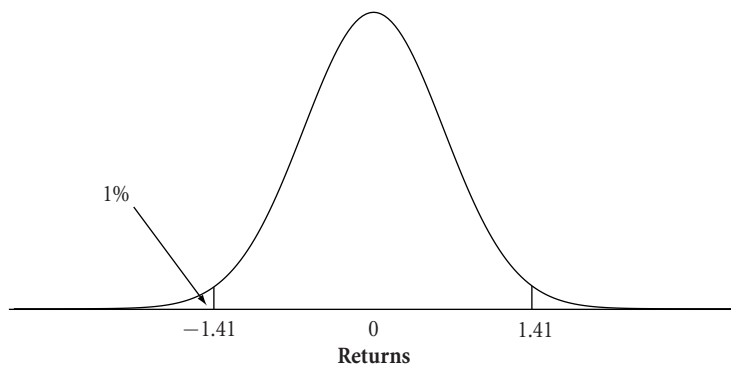


Figure 37.16

APPENDIX 37.3 Multiplication of matrices

The multiplication of matrices involves applying unique rules that are not generally observed in other branches of mathematics. In the first instance, the commutative rule of multiplication does not apply to matrices, so that it is important to multiply them in the right order. Consider the two matrices below, both 2×2 matrices.

$$\begin{array}{cc} \text{matrix 1} & \text{matrix 2} \\ \begin{pmatrix} 23 & 38 \\ 12 & 16 \end{pmatrix} & \begin{pmatrix} 91 & 15 \\ 24 & 121 \end{pmatrix} \end{array}$$

Figure 37.17: Matrices 1 and 2.

If we multiply matrix 1 by matrix 2, we obtain matrix P (for “product”), shown below. To multiply the two matrices:

- row 1 of matrix 1 is multiplied by column 1 of matrix 2, which is $(23 \times 91) + (38 \times 24) = 3005$
- row 1 of matrix 1 is then multiplied by column 2 of matrix 2, which is $(23 \times 15) + (38 \times 121) = 4943$
- row 2 of matrix 1 is then multiplied by column 1 of matrix 2, giving $(12 \times 91) + (16 \times 24) = 1476$
- finally, row 2 of matrix 1 is then multiplied by column 2 of matrix 2, giving us $(12 \times 15) + (16 \times 121) = 2116$

$$\begin{array}{l} \begin{pmatrix} 23 & 38 \\ 12 & 16 \end{pmatrix} \begin{pmatrix} 91 & 15 \\ 24 & 121 \end{pmatrix} \quad \begin{pmatrix} (23 \times 91) + (38 \times 24) & \\ & \end{pmatrix} \quad \begin{pmatrix} 3005 & \\ & \end{pmatrix} \\ \\ \begin{pmatrix} 23 & 38 \\ 12 & 16 \end{pmatrix} \begin{pmatrix} 91 & 15 \\ 24 & 121 \end{pmatrix} \quad \begin{pmatrix} 3005 & (23 \times 15) + (38 \times 121) \\ & \end{pmatrix} \quad \begin{pmatrix} 3005 & 4943 \\ & \end{pmatrix} \\ \\ \begin{pmatrix} 23 & 38 \\ 12 & 16 \end{pmatrix} \begin{pmatrix} 91 & 15 \\ 24 & 121 \end{pmatrix} \quad \begin{pmatrix} 3005 & 4943 \\ (12 \times 91) + (16 \times 24) & \end{pmatrix} \quad \begin{pmatrix} 3005 & 4943 \\ 1476 & \end{pmatrix} \\ \\ \begin{pmatrix} 23 & 38 \\ 12 & 16 \end{pmatrix} \begin{pmatrix} 91 & 15 \\ 24 & 121 \end{pmatrix} \quad \begin{pmatrix} 3005 & 4943 \\ 1476 & (12 \times 15) + (16 \times 121) \end{pmatrix} \quad \begin{pmatrix} 3005 & 4943 \\ 1476 & 2116 \end{pmatrix} \\ \hline \text{Final matrix, } P = \begin{pmatrix} 3005 & 4943 \\ 1476 & 2116 \end{pmatrix} \end{array}$$

Figure 37.18: Matrix product $P = \text{matrix 1} \times \text{matrix 2}$.

We can see now why the commutative rule does not apply to matrices; if we multiplied matrix 2 by matrix 1, we obtain a quite different matrix, shown below as matrix PP .

$$\begin{array}{l} \text{matrix 2} \times \text{matrix 1} = PP \\ \begin{pmatrix} 91 & 15 \\ 24 & 121 \end{pmatrix} \begin{pmatrix} 23 & 38 \\ 12 & 16 \end{pmatrix} = \begin{pmatrix} 2273 & 3698 \\ 2004 & 2848 \end{pmatrix} \end{array}$$

Figure 37.19: Matrix product $PP = \text{matrix 2} \times \text{matrix 1}$.

There are additional rules that apply when matrices of different shapes are multiplied. Essentially to multiply two matrices, the number of columns in the first matrix must equal the number of rows in the second matrix. If this does not apply, the matrices cannot be multiplied.

There are occasions during the calculation of VaR using variance-covariance when the multiplying rule for matrices cannot be followed. To overcome this, a *transpose* of the matrix is used instead, which may allow us to multiply it as required. The transpose of a matrix is one where its rows have been turned into columns, for example matrix T is shown below with its transpose, referred to as matrix T' . Matrix T is a 3×4 matrix, while its transpose is a 4×3 matrix.

$$\begin{array}{cc} \text{Matrix } T & \text{Matrix } T' \\ \begin{pmatrix} 1 & 2 & 3 \\ 2 & 7 & 9 \\ 2 & 5 & 6 \\ 4 & 1 & 7 \end{pmatrix} & \begin{pmatrix} 1 & 2 & 2 & 4 \\ 2 & 7 & 5 & 1 \\ 3 & 9 & 6 & 7 \end{pmatrix} \end{array}$$

Figure 37.20: Transposing matrices.

APPENDIX 37.4

■ Example of diversified VaR calculation for 3v6 FRA position

VaR weighting transposed		Correlation matrix			
13815	−20722	1	0.87	−4213.14	−8702.95
		0.87	1		
		VaR weighting			
−4213.14	−8702.95	13815	1.22E+08		
		−20722			
		VaR			
			11051.61		
			11051		

Figure 37.21: Diversified VaR for 3v6 FRA, correlation coefficient 0.87.

■ Example of diversified VaR calculation for five-year interest-rate swap

The weighting matrix W is composed of the individual VaR values for each interest rate period in the swap. This is shown as Figure 37.22.

Figure 37.22: Interest-rate swap weighting matrix.	164
	158
	303
	324
	566
	972
	872
	1152
	1414
	4601

In order to multiply this by the correlation matrix C it needs to be transposed, and this is shown as Figure 37.23(i). The correlation matrix is Figure 37.23(i); correlation data is as listed from the RiskMetrics dataset. This may be obtained from RiskMetrics direct or downloaded from <http://www.Riskmetrics.com>.

(i)

WV transpose

164	158	303	324	566	972	872	1152	1414	4601

Correlation matrix

1	0.89	0.91	0.92	0.94	0.87	0.91	0.89	0.95	0.97
0.89	1	0.97	0.96	0.95	0.97	0.89	0.91	0.98	0.93
0.91	0.97	1	0.96	0.95	0.94	0.98	0.91	0.92	0.92
0.92	0.96	0.96	1	0.95	0.94	0.95	0.97	0.98	0.97
0.94	0.95	0.95	0.95	1	0.92	0.93	0.93	0.94	0.96
0.87	0.97	0.94	0.94	0.92	1	0.97	0.98	0.95	0.94
0.91	0.89	0.98	0.95	0.93	0.97	1	0.89	0.91	0.95
0.89	0.91	0.91	0.97	0.93	0.98	0.89	1	0.95	0.97
0.95	0.98	0.92	0.98	0.94	0.95	0.91	0.95	1	0.96
0.97	0.93	0.92	0.97	0.96	0.94	0.95	0.97	0.96	1

(ii)

WVC

9882.823	9880.203	9806.312	10165.1	9990.291	10022.81	9920.518	10094.76	10082.95	10262.1

Figure 37.23 (i) and (ii)

The usual procedure is then followed, with the WVC matrix multiplied by the weightings matrix; this gives us the variance, from which we calculate the VaR to be £10,325, shown in Figure 37.24.

WCVW	106614498.1
VaR	10325.42968

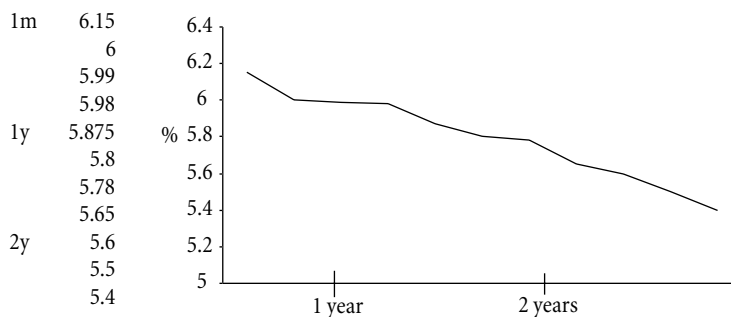
Figure 37.24

Selected bibliography and references

- Alexander, C., *Risk Management and Analysis*, John Wiley and Sons Ltd, 1996.
- Beckstrom, R., Campbell, A. (eds.), *An introduction to VAR*, CATS Software, 1995.
- Beder, T., "VAR: seductive but dangerous", *Financial Analysts Journal* 51, 1995, pp. 12–24.
- Bollerslev, T., "Generalised Autoregressive Conditional Heteroscedasticity", *Journal of Econometrics* 31, 1986, pp. 307–327.
- Chew, L., *Managing Derivatives Risks*, Wiley, 1996.
- Engle, R., "Autoregressive Conditional Heteroscedasticity with Estimates of the Variance of UK Inflation", *Econometrica* 50, 1982, pp. 987–1008.
- Jorion, P., *Value-at-Risk*, Irwin, 1997.
- JP Morgan, *RiskMetrics Technical Manual*, JP Morgan Bank, 1995.
- JP Morgan & Co., Inc., CreditMetrics® – Technical Document, 1997.
- Schwartz, R., Clifford, W., *Advanced Strategies in Financial Risk Management*, New York Institute of Finance, 1993.

Questions and exercises

1. A risk manager updates a daily volatility forecast using the RiskMetrics method of weighting observations, which uses a decay factor of 0.97. The volatility forecast for the previous day was 1%, and the actual market returns were 2%. What is the risk manager's new forecast? (*Hint: use the RiskMetrics decay formula.*)
2. A bond portfolio has a one-day VaR measure of £1 million. The market has been observed to be following an autocorrelation trend of 0.10. Calculate the two-day VaR using $\sigma_{2\text{-day}} = \sqrt{\sigma_{1\text{-day}}^2 + \sigma_{1\text{-day}}^2 + 2\sigma_{1\text{-day}}^2\rho}$.
3. What is the one-year probability of a triple-B rated bond going into default?
4. The cumulative probability of a B-rated counterparty defaulting over the next 12 months is approximately 6.00%. From your observation of the graph below, what is the expected probability of the counterparty defaulting in the first month?



5. To fulfil regulatory requirements, a risk manager converts a one-day holding period VaR measure to a 10-day holding period. How would he do this?
6. What requirements are stipulated by the Basle Committee for banks wishing to calculate VaR for their trading books?
7. What methodology is JP Morgan's RiskMetrics based on?
8. A bank calculates its overnight VaR measure to be £12.85 million, given a 95% confidence interval. What is the appropriate interpretation of this measure?
9. Estimate the approximate VaR of a \$23 million long position in a 10-year Brady bond if the 10-year volatility level is 5.78%.
10. A commodity trader has an option position in wheat with a delta of 5 000 bushels and a gamma of -200 per dollar move in price. Using the delta-gamma approximation calculate the VaR on the trader's position, assuming that the volatility level for wheat is equivalent to \$3 per bushel.
11. A bond portfolio has a one-day VaR measure, at 95% confidence, of \$1 million. How would you convert this measure to meet Basle committee VaR measure requirements? What would the equivalent VaR measure be?
12. ABC Bank plc calculates its VaR with more observations and a higher confidence level (99%, as opposed to 95%) than XYZ Bank plc. Which bank is likely to have a smaller measurement error due to sampling variation?
13. From the following observations of returns, what is the correlation between A and B?
 A: 20, 18, 16, 14, 12, 10
 B: 10, 12, 18, 14, 16, 20
14. The VaR of one instrument is 1 000, while the VaR of another instrument is 800. The combined VaR of both instruments is 1 200. What is the correlation between the instruments? (*Hint: use portfolio variance equation.*)

38 Interest-rate Risk and a Critique of Value-at-Risk

The primary risk exposure of a bond trading book is interest-rate risk. In this chapter we review interest-rate risk reporting. As a conclusion to both this and the previous two chapters, we review the value-at-risk risk measurement tool and comment on its place in the risk management policy, processes and procedures of a bank.

38.1 Interest-rate risk

38.1.1 Interest-rate risk reporting

A bank's trading book will have an interest rate exposure arising from its net position. For example an interest-rate swap desk will have exposure for each point of the term structure, out to the longest-dated swap that is holds on the book. A first order measure of risk would be to calculate the effect of a 1 basis point change in interest rates, along the entire yield curve, on the value of the net swaps position. This measures the effect of a *parallel shift* in interest rates. For large moves in interest rates, a bank's risk management department will also monitor the effect of a large parallel shift in interest rates, say 1% or 5%. This is sometimes referred to as a bank's *jump risk*, although the term is also used to refer to risk exposures in connection with something quite different, the jump diffusion model. Generally however jump risk refers to the effect on value of an upward move of 100 basis points for all interest rates, that is for all points of the term structure. Each selected point on the term structure is called an interest rate bucket or *grid point*. The jump risk figure is therefore the change in the value of the portfolio for a 1% parallel shift in the yield curve.

Derivatives desks often produce reports for trading books showing the effect on portfolio value of a 1 basis point move, along each part of the term structure of interest rates. For example such a report would show that a change of 1 basis point in 3-month rates would result in a change in value of £x (this measure is often referred to as a price variation per basis point, present value of a basis point, PVBP or as the dollar value of a 01, DV01). As banks deal in a large number of currencies their jump risk reports will amalgamate the risk exposures from all parts of the bank. Figure 38.1 shows an interest rate risk report for the London office of a US bank, with combined currency and interest-rate risk for a 100 basis point upward and downward move in interest rates.

38.1.2 The bond yield curve

Banks generally model interest-rate delta risk based on the zero-coupon curve derived from either interest-rate swap rates or government bond yields. A bond yield curve is usually constructed from bonds with set maturities of between one year and 30 years; the choice of the bond will depend on what instruments are on the trading book. When constructing a yield curve, the short-end of the curve is frequently modelled as Libor minus a fixed spread, although it is also reasonable to use the government repo rate. The yields for the standard grid-points from one month to 30 years are usually obtained from linear interpolation of government bond yields. Some banks do not calculate zero-coupon rates but instead use actual bond redemption yields; this involves interpolating from actual bond yields. This results in the curve exhibiting the following characteristics:

- using actual bond yields means that individual bonds may be replaced (for example, when their maturity falls out of their maturity bucket) without disturbing the whole curve;
- if the yield on one bond is excessively cheap or dear to the curve, because of liquidity reasons or when it falls out of the maturity bucket, only that point of the yield curve will be affected;

+100 bps change in interest rate																
Currency	Overnight	1 week	1 month	2 months	3 months	6 months	9 months	1 year	2 years	3 years	4 years	5 years	7 years	10 years	30 years	Int. Rate
Gross sensitivity	1,533.77	87,395.50	190,461.72	114,042.32	62,102.97	295,189.13	121,623.80	222,762.55	84,704.71	66,704.87	185,080.99	349,422.46	656,115.64	246,049.16	60,098.31	588,991.98
AUD	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
EUR	-0.42	-1.04	27.42	5.29	721.09	-1026.02	-536.57	-2549.55	-6047.58	-2348.8	-2115.63	-1365.93	-201.66	0.00	0.00	-15287.89
GBP	0.00	0.00	-42996.52	-1466.18	-93.53	-222.58	-322.62	-1209.38	-3573.17	-3629.9	-6958.94	-10598	-12022.96	-3158.02	0.00	-85430.49
USD	1,533.35	87,394.45	147,437.77	112,570.85	61,288.35	-293940.53	-120764.61	219,004.63	74,619.96	60,726.09	176,366.41	337,458.39	643,891.07	242,891.14	60,098.31	488,273.60
JPY	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Figures in US dollars																
-100 bps change in interest rate																
Currency	Overnight	1 week	1 month	2 months	3 months	6 months	9 months	1 year	2 years	3 years	4 years	5 years	7 years	10 years	30 years	Int. Rate
Gross sensitivity	0.42	1.05	43,426.90	1,466.95	94.36	306,126.42	125,089.77	3,646.92	9,709.11	88,332.50	319,347.26	475,880.08	99,080.68	3,260.59	0.00	104,012.30
AUD	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
EUR	0.42	1.05	-27.67	-5.3	-703	1,020.54	528.06	2,421.02	6,060.80	2,409.28	2,185.53	1,420.95	206.73	0.00	0.00	15,673.87
GBP	0.00	0.00	43,426.90	1,466.95	94.36	224.82	326.41	1,225.90	3,648.31	3,708.80	6,809.40	11,021.24	12,627.84	3,260.59	0.00	88,338.43
USD	-1557.31	-87701.77	-147859.83	-110884.73	-55021.22	304,881.06	124,235.30	-221794.1	-54572.73	82,214.42	310,352.33	463,437.89	86,246.11	-22003.7	-48866.83	-395417.33
JPY	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00

Figure 38.1: Example of interest-rate and currency sensitivity report.

- individual bonds may be phased in and out of the curve in a smooth manner. For example if a new five-year bond is issued, and the actual yield of this bond differs by a few basis points from the yield implied by the present curve, then this jump in the yield curve can be spread over several days.

The bond value-at-risk (VaR) for the markets for which bond yield curves are used may be calculated using any VaR methodology. Although the academic literature and most banking practice suggests that a bond yield curve used for valuation or risk measurement should be obtained from a zero-coupon curve, in practice it has been observed that volatilities and implied extreme moves of a curve constructed using actual bond yields (as described above) are not significantly different from zero-coupon curves. Similarly, when valuing swaps for say, audit year-end purposes, a zero-coupon curve is usually used; however the valuation is quite often not greatly dissimilar to that obtained using the actual bond yield curve.

38.2 Comparison with traditional duration-based risk measurement

Adherents to VaR-based risk measurement techniques point out a number of advantages that the methodology has over traditional duration-based measures. In the realm of fixed income trading and portfolio management, market best-practice has tended to incorporate VaR as part of an overall risk management framework. We highlight some of the advantages over duration measures below.

38.2.1 Interpreting risk exposure

In one sense, value-at-risk provides a more definite interpretation of *risk* per se, as it quantifies risk exposure as a cash amount potential loss, within a specified time horizon. (Of course the accuracy of this measure is another issue.) Within a portfolio management context therefore VaR may be used to identify concentrations of risk, for examples in industrial or geographical sectors or across different segments of the yield curve. In the same way duration measures do not account for foreign exchange risk, which can be significant for international investors' portfolios. The VaR measure covers FX risk in theory because it takes into account the volatility of foreign exchange rates and the correlation between these and interest rates.

Duration does not effectively measure portfolio risk when investments are traded according to different yield curves, because they are denominated in different currencies. Because there is no account of the correlation between different currency interest rates, an internationally-invested portfolio duration measure assumes implicitly that there is perfect correlation between all yield curves – which we know to be incorrect. This lack of perfect correlation is in theory captured by a VaR measure.

38.2.2 Interest-rate sensitivity

The well-known weakness of the duration measure is its assumption of parallel shifts in the yield curve. VaR is not dependent on this assumption, and so should be more accurate under a greater number of scenarios. A simple illustration will suffice: consider a bond with exactly five years to maturity that has a duration of 4.74 years; and a portfolio that consists of two bonds, a four-year bond of 3.61 years duration and a 30-year bond of 14.1 years duration. The shorter-dated bond makes up 85% of the portfolio. The arithmetic works out a portfolio duration of 4.74 years as well. However the VaR measure for the portfolio, at 95% confidence, represents just over half the present value of the bond, but just over 40% that of the portfolio. This illustrates the risk reduction achieved through segmentation along the yield curve, a diversification not picked up by the duration measure.

A VaR measure of market risk accounts for the interest-rate sensitivity of a fixed income instrument or portfolio, which is in effect the modified duration risk element, however unlike duration it also incorporates the level of interest-rate volatility. This is a significant advantage. For example, bond price volatility increased considerably during August-December 1998, a reflection of the Russian bond and Brazilian currency crises. The duration measure for any bond would remain roughly constant during this time, in other words it would give no indication to a bondholder that risk had increased during this time. However VaR measures would reflect the increased volatility levels that were observed.

38.3 A critique of Value-at-Risk

The value-at-risk tool is now a widely accepted risk measurement tool among banks and securities houses. As we saw in the previous chapter there are a number of methodologies available, but the complete details on the approach

used and assumptions made are available for only a few of these, including the original RiskMetrics model. The variance-covariance method, while still popular at many banks, has generally been supplanted by in-house developed historical simulation models at the larger or more sophisticated securities houses. The choice of which VaR model to adopt in a bank is a function of a number of factors, including:

- the type of activity engaged in; this ranges from commercial bank lending and market making in cash instruments to trading in exotic derivative instruments; mortgage banks do not require the same type of VaR model as an integrated banking house that makes markets in the full range of products;
- the expertise of senior management, and the extent of appreciation of risk management factors among senior managers; VaR performs a vital function as a management information tool, and it may be that a less sophisticated model, but one that is easily explained and understood, is a more appropriate choice for certain firms;
- the sophistication of the firm's information technology (IT) and testing suite. The most complex models such as historical simulation models require a large investment in computing power, so the decision to implement them must consider the IT culture of the firm;
- the efficacy of a bank's other risk management tools and processes, for example (among other things) the extent to which there is adequate separation of duties, an independent middle office, reporting direct to a board-level director such as the Finance Director, documented procedures and strict guidelines for escalation in the event of a crisis.

It is important for both risk managers and shareholders alike to be aware that VaR is a measurement of risk, but as with other statistical measurements carries with it standard error terms. Therefore a risk manager should not place over-reliance on one value. To prevent over-reliance, both bank personnel and shareholders must be aware of the technicalities behind the model in use at their institution and its limitations. When RiskMetrics was originally introduced, JP Morgan was at pains to stress that it was a risk measurement tool designed for management use, that is, a source of further management information. It was not designed to be a predictor of extreme events such as market corrections, or as a measure with which to calculate capital adequacy levels. It is a tool for internal use, and the adoption of the VaR concept by the BIS as a calculation methodology for calculating capital is a sign of its acceptance in the wider field of banking risk management. To quote from RiskMetrics technical document:

"RiskMetrics™ is a risk measurement tool designed for management use. In management it is important that you can observe and check your estimate frequently. The 95%/5% cut-off level implies that you should observe a loss greater than the estimated DEaR about every 20 business days. If you had chosen the 1% cut-off you would have to wait an average 100 days for a confirmation that the estimate reflects reality. Regulators have a different perspective: they look to define capital levels necessary to support any losses that could happen."

JP Morgan

The key issue then is to realise that any VaR number is a measurement, often based on statistical assumptions, of a bank's market (or credit) exposure, and this number should be used *as part of* management decision-making. Let us consider some of the criticisms levelled at the VaR concept.

38.3.1 Liquidity risk

A value-at-risk number does not deal with liquidity risk, and nor does it claim to. Under "normal" market conditions, a VaR tool such as RiskMetrics measures the risk exposure of a bank's trading book, based on the previous day's market-to-market. However under conditions of extreme market movement, such as crashes or corrections, as participants all attempt to close out of positions at the same time, the liquidity in a market, best measured by the extent of the bid-offer spread, dries up. A one-day or one-week VaR number becomes meaningless as a bank finds that, with bids disappearing or considerably lower than the previous day, it takes more than one day or one week to liquidate assets on the trading book. A VaR number makes no allowance for this. The inaccuracy of the VaR exposure of Long Term Capital Management, a highly leveraged and aggressive hedge fund, was only apparent when, with the market falling at alarming speed in the aftermath of the Russian bond technical default and currency problems in Brazil, the lack of liquidity led to losses many times greater than the VaR number.

The liquidity risk criticism of VaR is not entirely valid because, as we note, VaR does not attempt to measure it. There is no doubt that in the event of an extreme market event such as a crash or significant market correction, the VaR measure will underestimate risk. Therefore it is important to calculate, alongside VaR, an estimate for liquidity risk; the easiest way to do this is to monitor bid-offer spreads on a daily basis. Widening spreads are one indication

of a fall in liquidity levels. The true market risk exposure of a bank then, is its market risk measure as based on its value-at-risk, together with the extent of liquidity risk. Note that the type of assets held on a book will also indicate the potential liquidity exposure; to consider two extremes, a book of US Treasury bonds will be relatively to liquidate compared to a book of long-dated interest-rate swaps.

38.3.2 Confidence interval

Value-at-risk models calculate market risk exposure at a level of confidence that is selected by the user. An off-the-shelf model such as RiskMetrics uses a 95% confidence interval, which means that the risk number calculated will not be exceeded in 19 days out of 20. A criticism of this is that this confidence level is insufficient, because it does not cover extreme events. However, RiskMetrics is, as we quoted earlier, designed as a management tool, not a calculation of capital adequacy. Regulatory authorities are interested in a higher level of confidence because they are interested in a bank's ability to withstand extreme events. This is why the BIS sets a 99% confidence level.

The best-practice approach then is to calculate two figures, one at 95% and the other at 98% or 99%, so that a bank does not underestimate its risk exposure in the event of extreme market moves or crashes.

38.3.3 The normal distribution

A key assumption behind many VaR models is that of normal distribution of asset price returns, or strictly speaking lognormal returns. Observation of returns in practice indicates that in actual fact the distribution of asset returns exhibits “fat tails” and a narrower median range. This phenomenon, known as *leptokurtosis*, suggests that the normal distribution does not capture the full extent of extreme movements, indeed the occurrence of market crashes in recent years has been more frequent than would be predicted by a true normal distribution. The result is that a VaR model that assumes lognormal distribution of returns will underestimate the risk exposure posed by the occurrence of extreme events. The difference between the two distributions is shown at Figure 38.2

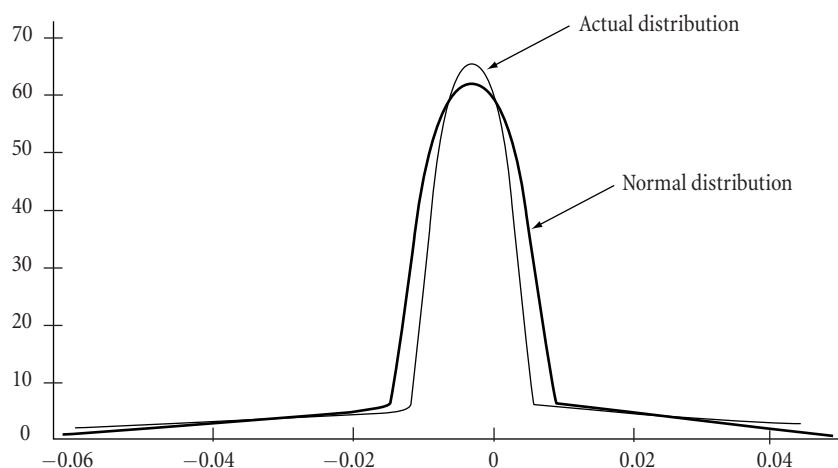


Figure 38.2: The normal distribution and actual distribution of price returns.

38.3.4 Using VaR

The key factor to remember about VaR tool is that it is a measurement tool, developed to enhance the level and quality of management information. Placing over-reliance on a calculated VaR number is not satisfactory market practice. The proper place for VaR is as part of an all-encompassing risk management process. This process should ideally follow the guidelines summarised in Chapter 36, although risk management is, despite the extensive use of quantitative techniques and mathematical models, an art rather than science, and one that calls for sound judgement and experience. While it is important to understand VaR and what it seeks to measure, it is equally vital to be aware of the limits of the methodology that one is using. Banks are in the business of taking on risk, investment banks especially so, and a system that combines prudence and professionalism with sensible procedures that separate duties and quantify risk will probably serve a bank's objectives best, rather than any total reliance on mathematical models.

Part VI Derivative Instruments

Nothing illustrates the importance of the global capital markets more than the phenomenal growth in the use of derivative instruments. In the financial markets derivatives are instruments whose price is derived from the price of another, underlying asset, hence their name. The underlying asset can be any number of products, for example a commodity such as wheat or sugar, to something more exotic such as the level of rainfall; in the debt capital markets it can be a bond, bond index, interest rate or a combination of these. Since the early 1980s derivatives, also known as *off-balance sheet* instruments have become very important in the financial markets. An understanding of the key derivative instruments is vital in order to attain an understanding of the bond markets. The main instruments are swaps, of which the most important arguably are interest-rate swaps, futures and forward contracts, and options. These instruments are traded in their own right and are also used as the building blocks of more complex instruments known as *structured products*.

In Part VI of the book we concentrate on derivative instruments. We begin with a look at swaps, including interest-rate swaps, currency swaps and other forms of swap contracts. This is followed by chapters on futures contracts and options. Due to the complex nature of option pricing and analysis, we devote eight chapters to this subject. As always a comprehensive bibliography is provided for readers who wish to investigate derivative instruments further. Readers who are beginners to the subject may wish to consider some of the key concepts in derivatives analysis, which we review below.

The key to the valuation of financial instruments that include derivatives is that there exist *state prices* for each security (Duffie 1996). This term refers to discount factors for each possible future date and “state”; given these state prices we can calculate the value of an instrument as a state-price weighted sum of its future payoff values. This thinking dates from Arrow (1953) and the concept of the *equilibrium* of asset markets. The process of calculating asset prices involves identifying their state prices. There have been a number of key breakthroughs in research that have enabled market participants to value and analyse derivative securities. The ground-breaking paper by Black and Scholes in 1973, and the work carried out contemporaneously by Merton (1973) are widely believed to be the start of the derivatives revolution. In the context of debt markets the first term structure model was presented by Vasicek (1977), followed by the influential paper by Cox, Ingersoll and Ross (1985, although the basic content actually appeared well before this publication date). Other research advances include Harrison and Kreps (1979) and Harrison and Pliska (1981) in the field of dynamic asset pricing theory, Hull and White (1991) on mean reversion within an interest rate model and Heath, Jarrow and Morton (1992) and their whole yield curve model. More references are contained in the chapters that follow.

The concept of a traded security

A market in traded securities is essential in the pricing of derivatives contracts. Such a security would be one that was traded in a liquid secondary market, by a large number of market participants. Most financial instruments can be considered to be traded securities, whereas certain commodities (such as copper) are not. If a traded security is available, a derivative contract that has been written on it can be priced on the basis of the security, whereas a (option) contract written on a non-traded security will need to be priced based on the dynamics of the price process of the underlying asset. Traded securities are used in the construction of a *risk-free hedge*, essential in the theory of option pricing.

Following on from the concept of a traded security is the concept of *risk-neutral valuation*. This underpins much of derivatives pricing and analysis. A derivative that is priced against traded securities will be valued using a differential equation that does not require parameters that reflect the market price for risk. Therefore the price of a derivative in an environment where market participants are risk-neutral will be identical to its price in a real-world environment. Hence in derivatives pricing we assume that market participants are risk-neutral. This is then used in valuation analysis. In an environment that is risk-neutral, the return from an investment in a traded security is the risk-free interest rate. We can then use the return achieved using this risk-free rate, which is known, to discount the

payoff from an option contract (discounted at the risk-free rate). This gives us the present value of the option contract.

The risk-free hedge

In order to derive the valuation formula for an option contract, a hypothetical risk-free hedge is constructed. This is also known as a *replicating portfolio*. The final payoff of the risk-free hedge is then used to construct the price formula. This is because a risk-free hedge will earn the risk-free rate of interest, and the final return can be used to construct an equation that an option contract must satisfy, in order to fulfil the principle of no-arbitrage. In a risk-free hedge, the final return is independent of the dynamics of any price process, or any *stochastic process*. Therefore, if two financial instruments exist, both dependent on one underlying variable, a risk-free hedge can be constructed of equal and opposite positions in the two instruments. The value of the replicating portfolio must be equal to the value of any derivative contract written on the underlying asset, under the rules of economic equilibrium. The solution of a equation that has been derived from the construction of a risk-free hedge is not unique, as any derivative instrument that is being priced against a risk-free hedge must satisfy the same equation. For this reason specific conditions known as *boundary conditions* are set that determine the solution for each particular security.

Let us consider now the main derivative instruments, beginning with swaps.

References

- Arrow, K., *Essays in the Theory of Risk Bearing*, North-Holland, 1970.
- Black, F., Scholes, M., "The Pricing of Options and Corporate Liabilities", *Journal of Political Economy* 81, 1973, pp. 637–654.
- Cox, J., Ingersoll, J., Ross, S., "A Theory of the Term Structure of Interest Rates", *Econometrica* 53, 1985, pp. 385–408.
- Duffie, D., *Dynamic Asset Pricing Theory*, 2nd edition, Princeton University Press, 1996.
- Harrison, M., Kreps, D., "Martingales and Arbitrage in Multi-Period Securities Markets", *Journal of Economic Theory* 29, 1979, pp. 381–408.
- Harrison, M., Pliska, S., "Martingales and Stochastic Integrals in the Theory of Continuous Trading", *Stochastic Processes and Their Applications* 11, 1981, pp. 215–260.
- Heath, D., Jarrow, R., Morton, A., "Bond Pricing and the Term Structure of Interest Rates: A New Methodology for Contingent Claims Valuation", *Econometrica* 60, 1992, pp. 77–106.
- Hull, J., White, A., "One-Factor Interest Rate Models and the Valuation of Interest-Rate Derivative Securities", *Journal of Financial and Quantitative Analysis* 28, 1993, pp. 235–254.
- Merton, R., "The Theory of Rational Option Pricing", *Bell Journal of Economics and Management Science* 4, 1973, pp. 141–183.
- Vasicek, O., "An Equilibrium Characterization of the Term Structure", *Journal of Financial Economics* 5, 1977, pp. 177–188.

39 Swaps I

39.1 Introduction

The growth in the swaps market illustrates perfectly the flexibility and application of financial engineering in the capital markets. The market is large and liquid and has a variety of applications; the notional principal outstanding of interest-rate swaps exceeded \$6,000 billion at the end of 1998. The annual turnover in swaps and forward-rate agreements (FRAs) combined has been estimated at \$37 trillion notional.¹ *Swaps* are synthetic securities involving combinations of two or more basic building blocks. Most swaps currently traded in the market involve combinations of cash market securities, for example a fixed interest rate security combined with a floating interest rate security, possibly also combined with a currency transaction. However the market has also seen swaps that involve a futures or forward component, as well as swaps that involve an option component. The first example of a swap is thought to be that set up between IBM and the World Bank in 1981, since when the swap market has grown considerably. The market in say, dollar, euro and sterling interest rate swaps is very large and very liquid. The main types of swap are interest rate swaps, asset swaps, basis swaps, fixed-rate currency swaps and currency coupon swaps. The market for swaps is organised by the International Swap Dealers Association (ISDA).

The origin of the swap market was the market in parallel and “back-to-back” loans that existed during the 1970s, when restrictions on the exchange of foreign currency limited the ability of companies and investors to finance overseas activities. So-called parallel loans could only be arranged where there were two companies, with subsidiaries in each other’s countries, each of whom required the same level of funding; as such they were very inflexible. We do not need to concern ourselves with the structure of such loans. The elimination of foreign exchange controls at the end of the 1970s in the US and UK, soon followed in other countries, allowed straight cross-border loans to take place. The first swaps were *cross-currency swaps*, which as we shall later remove the exchange risk associated with a loan made in two different currencies. These swaps were bilateral agreements between two companies directly, which required both counterparties to have essentially exactly opposite borrowing requirements. Gradually banks ceased acting as brokers between two companies with opposite requirements, charging a fee for the service, and instead became direct counterparties to companies who required both cross-currency and interest-rate swaps. If the other side to a swap could be found by the bank in the market, then the first swap could be hedged. However often there would not be another counterparty with the exact opposite requirements, so the bank would manage the swap on its own book, which would be hedged with another swap or other financial instruments. Banks thus became principals and market makers in swaps, and the activity of maintaining swaps on a bank’s own book was known as *warehousing*. The profit on a swap book was made by a combination of the bid-offer spread and managing the book’s interest-rate exposure by correctly calling the direction on market interest rates.

The evolution of interest-rate swaps followed from the same requirement of companies to adjust their interest-rate basis, be it fixed- or floating-rate, and the comparative advantage in borrowing of one company vis-à-vis another. Example 39.1 illustrates an hypothetical example of how two companies might benefit from an interest-rate swap.

EXAMPLE 39.1 Comparative advantage and interest-rate swap structure

When entering into a swap either for hedging purposes or to alter the basis of an interest rate liability, the opposite to the current cash flow profile is required. Consider a homeowner with a variable rate mortgage. The homeowner is at risk from an upward move in interest rates, which will result in her being charged higher interest payments. She wishes to protect against such a move and (in theory! Don’t try this with your building society!), as she is *paying floating*, must *receive floating* in a swap. Therefore she will pay fixed in the swap. The floating interest payments cancel each other out, and the homeowner now has a fixed rate liability. The same

¹ *Risk Professional* 1–3, May 1999.

applies in a hedging transaction: a bondholder *receiving fixed* coupons from the bond issuer (that is, the bondholder is a *lender of funds*) can hedge against a rise in interest rates that lowers the price of the bond by *paying fixed* in a swap with the same basis point value as the bond position; the bondholder receives floating interest. *Paying fixed* in a swap is conceptually the same as being a *borrower of funds*; this borrowing is the opposite of the loan of funds to the bond issuer and therefore the position is hedged.

Consider two companies borrowing costs for a 5-year loan of £50m

- **Company A:** can pay fixed at 8.75% or floating at Libor. Its desired basis is floating.
- **Company B:** can pay fixed at 10% or floating at Libor+100bp. Its desired basis is fixed.

Without a swap:

Company A borrows fixed and pays 8.75%.

Company B borrows floating and pays Libor + 100bps

Let us say that the two companies decide to enter into a swap, whereby company A borrows floating-rate interest and therefore receives fixed from company B at the 5-year swap rate of 8.90%. Company B, who has borrowed at Libor+100bp, pays fixed and receives Libor in the swap. Company A ends up paying floating-rate interest, and company B ends up paying fixed. This is shown in the diagram below.

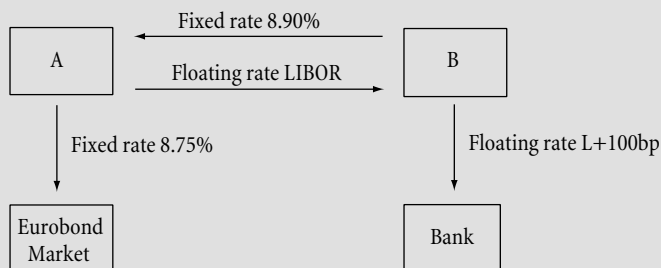


Figure 39.1: Interest-rate swap.

Result after swap:

A pays $8.75\% + \text{Libor} - 8.90\% = \text{Libor} - 15\text{bps}$

B pays $\text{Libor} + 100\text{bps} + 8.90\% - \text{Libor} = 9.90\%$

Company A saves 15 bps (pays L-15bp rather than L flat) and B saves 10 bps (pays 9.90% rather than 10%).

Both parties benefit from a *comparative advantage* of A in the fixed rate market and B in the floating rate market (spread of B over A is 125 bps in the fixed-rate market but 100 bps in the floating rate market). Initially swap banks were simply brokers, and charged a fee to both counterparties for bringing them together. In the example company A deals direct with company B, although it is more likely that an intermediary bank would have been involved. As the market developed banks would become principals and deal direct with counterparties, eliminating the need to find someone who had requirements that could be met by the other side of an existing requirement.

Swaps are now one of the most useful instruments in the debt capital markets. They are used by a wide range of institutions, including banks, mortgage banks and building societies, corporates and local authorities. The demand for them has grown as the continuing uncertainty and volatility of interest rates and exchange rates has made it ever more important to hedge their exposures. As the market has matured the instrument has gained wider acceptance, and is regarded as a “plain vanilla” product in the debt capital markets. Virtually all commercial and investment banks will quote swap prices for their customers, and as they are OTC instruments, dealt over the telephone, it is possible for banks to tailor swaps to match the precise requirements of individual customers. There is also a close relationship between the bond market and the swap market, and corporate finance teams and underwriting banks keep a close eye on the government yield curve and the swap yield curve, looking out for possibilities regarding new issue of debt.

In this chapter we review the different types of swap product, their pricing and valuation, as well as the main uses for swaps. The following chapter reviews hedging techniques, the relationship with FRAs and also related issues like the convexity bias between swaps and futures contracts.

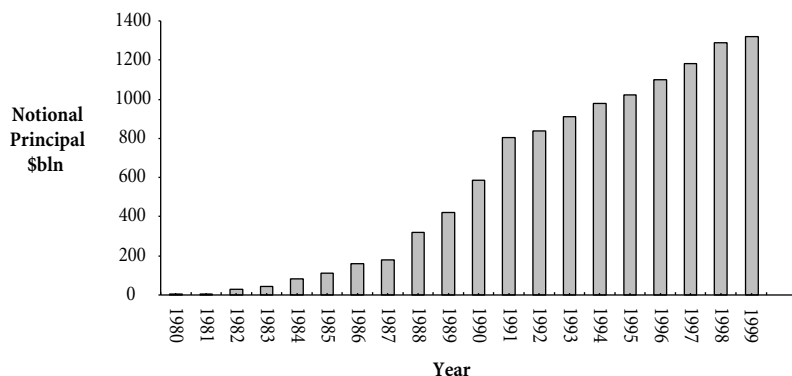


Figure 39.2: Growth of interest rate swap market. Source: ISDA.

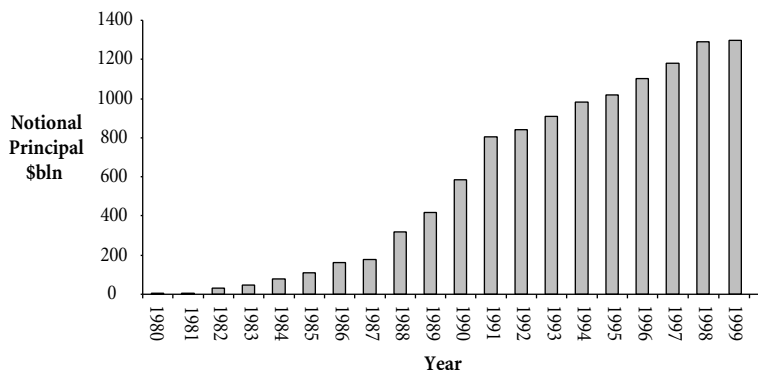


Figure 39.3: Growth of currency swap market. Source: ISDA.

INTEREST RATE SWAPS

Jul 22	Euro-£		£ Sfr		SwFr		US \$		Yen	
	Bid	Ask	Bid	Ask	Bid	Ask	Bid	Ask	Bid	Ask
1 year	3.09	3.12	5.59	5.62	1.72	1.75	5.74	5.77	0.19	0.22
2 year	3.53	3.57	5.99	6.03	2.15	2.23	6.06	6.09	0.38	0.41
3 year	3.86	3.90	6.28	6.32	2.48	2.56	6.23	6.26	0.64	0.67
4 year	4.12	4.16	6.37	6.41	2.71	2.79	6.33	6.36	0.90	0.93
5 year	4.32	4.36	6.36	6.40	2.91	2.99	6.42	6.45	1.16	1.19
6 year	4.50	4.54	6.32	6.36	3.08	3.16	6.49	6.52	1.40	1.43
7 year	4.68	4.72	6.25	6.29	3.23	3.31	6.56	6.59	1.62	1.65
8 year	4.83	4.87	6.16	6.20	3.36	3.44	6.62	6.65	1.80	1.83
9 year	4.96	5.00	6.10	6.14	3.49	3.57	6.66	6.69	1.95	1.98
10 year	5.05	5.09	6.03	6.07	3.60	3.68	6.70	6.73	2.09	2.12
12 year	5.20	5.24	5.93	5.99	3.78	3.88	6.75	6.78	2.32	2.35
15 year	5.36	5.40	5.84	5.90	3.99	4.09	6.81	6.84	2.59	2.62
20 year	5.49	5.53	5.72	5.80	4.19	4.29	6.85	6.88	2.78	2.82
25 year	5.56	5.60	5.65	5.73	4.29	4.39	6.85	6.88	2.83	2.88
30 year	5.58	5.62	5.55	5.65	4.35	4.45	6.85	6.88	2.87	2.92

Bid and ask rates as of close of London business. US \$ is quoted annual money actual/360 basis against 3 months Libor, £ and Yen quoted on a semi-annual actual/365 basis against 6 months Libor, Euro/Swiss Franc quoted on annual bond 30/360 basis against 6 month Euribor/Libor with the exception of the 1 year rate which is quoted against 3 month Euribor/Libor.

Source: InterCapital Europe Limited.

Figure 39.4: Interest rate swaps 23 July 1999.

© Financial Times. Used with permission.

39.2 Interest rate swaps

Interest-rate swaps are the most important type of swap in terms of volume of transactions. They are used to manage and hedge interest rate risk and exposure, while market makers will also take positions in swaps that reflect their view on the direction of interest rates. An interest rate swap is an agreement between two counterparties to make periodic interest payments to one another during the life of the swap, on a pre-determined set of dates, based on a *notional* principal amount. One party is the fixed-rate payer, and this rate is agreed at the time of trade of the swap; the other party is the floating-rate payer, the floating rate being determined during the life of the swap by reference to a specific market index. The principal or notional amount is never physically exchanged, hence the term “off-balance sheet”, but is used to calculate the interest payments. The fixed-rate payer receives floating-rate interest and is said to be “long” or to have “bought” the swap. The long side has conceptually purchased a floating-rate note (because it receives floating-rate interest) and issued a fixed coupon bond (because it pays out fixed interest at intervals), that is, it has in principle borrowed funds. The floating-rate payer is said to be “short” or to have “sold” the swap. The short side has conceptually purchased a coupon bond (because it receives fixed-rate interest) and issued a floating-rate note (because it pays floating-rate interest). So an interest rate swap is:

- an agreement between two parties;
- to exchange a stream of cash flows;
- calculated as a percentage of a *notional* sum.
- and calculated on different interest bases.

For example in a trade between Bank A and Bank B, Bank A may agree to pay fixed semi-annual coupons of 10% on a notional principal sum of £1 million, in return for receiving from Bank B the prevailing six-month sterling Libor rate on the same amount. The known cash flow is the fixed payment of £50,000 every six months by Bank A to Bank B.

Like other financial instruments, interest-rate swaps trade in a secondary market. The value of a swap moves in line with market interest rates, in exactly the same fashion as bonds. If a five-year interest-rate swap is transacted today at a rate of 5%, and five-year interest rates fall to 4.75% shortly thereafter, the swap will have decreased in value to the fixed-rate payer, and correspondingly increased in value to the floating-rate payer, who has now seen the level of interest payments fall. The opposite would be true if five-year rates moved to 5.25%. Why is this? Consider the fixed-rate payer in an IR swap to be a borrower of funds; if she fixes the interest rate payable on a loan for five years, and then this interest rate decreases shortly afterwards, is she better off? No, because she is now paying above the market rate for the funds borrowed. For this reason a swap contract decreases in value to the fixed-rate payer if there is a fall in rates. Equally a floating-rate payer gains if there is a fall in rates, as he can take advantage of the new rates and pay a lower level of interest; hence the value of a swap increases to the floating-rate payer if there is a fall in rates. We can summarise the profit/loss profile of a swap position, as shown in Table 39.1

	Fall in rates	Rise in rates
Fixed-rate payer	Loss	Profit
Floating-rate payer	Profit	Loss

Table 39.1

A bank swaps desk will have an overall net interest rate position arising from all the swaps it has traded that are currently on the book. This position is an interest rate exposure at all points along the term structure, out to the maturity of the longest-dated swap. At the close of business each day all the swaps on the book will be *marked-to-market* at the interest rate quote for that day, and the resulting p/l for the book will be in line with the profile shown in Table 39.1.

A swap can be viewed in two ways, either as a bundle of forward or futures contracts, or as a bundle of cash flows arising from the “sale” and “purchase” of cash market instruments. If we imagine a strip of futures contracts, maturing every three or six months out to three years, we can see how this is conceptually similar to a three-year interest-rate swap. However in the author’s view it is better to visualise a swap as being a bundle of cash flows arising from cash instruments.

Let us imagine we have only two positions on our book:

- a long position in £100 million of a three-year FRN that pays six-month Libor semi-annually, and is trading at par;
- a short position in £100 million of a three-year gilt with coupon of 6% that is also trading at par.

Being short a bond is the equivalent to being a borrower of funds. Assuming this position is kept to maturity, the resulting cash flows are shown in Table 39.2.

Cash flows resulting from long position in FRN and short position in gilt			
Period (6mo)	FRN	Gilt	Net cash flow
0	−£100m	+£100m	£0
1	$+(\text{Libor} \times 100)/2$	−3	$+(\text{Libor} \times 100)/2 - 3.0$
2	$+(\text{Libor} \times 100)/2$	−3	$+(\text{Libor} \times 100)/2 - 3.0$
3	$+(\text{Libor} \times 100)/2$	−3	$+(\text{Libor} \times 100)/2 - 3.0$
4	$+(\text{Libor} \times 100)/2$	−3	$+(\text{Libor} \times 100)/2 - 3.0$
5	$+(\text{Libor} \times 100)/2$	−3	$+(\text{Libor} \times 100)/2 - 3.0$
6	$+(\text{Libor} \times 100)/2 + 100$	−103	$+(\text{Libor} \times 100)/2 - 3.0$

The Libor rate is the six-month rate prevailing at the time of the setting, for instance the Libor rate at period 4 will be the rate actually prevailing at period 4.

Table 39.2: Three-year cash flows.

There is no net outflow or inflow at the start of these trades, as the £100 million purchase of the FRN is netted with receipt of £100 million from the sale of the gilt. The resulting cash flows over the three year period are shown in the last column of Table 39.2. This net position is exactly the same as that of a fixed-rate payer in an IR swap. As we had at the start of the trade, there is no cash inflow or outflow on maturity. For a floating-rate payer, the cash flow would mirror exactly a long position in a fixed rate bond and a short position in an FRN. Therefore the fixed-rate payer in a swap is said to be short in the bond market, or a borrower of funds; the floating-rate payer in a swap is said to be long the bond market.

39.2.1 Market terminology

Virtually all swaps are traded under the legal terms and conditions stipulated in the ISDA standard documentation. An example of standard swap documentation for two typical interest-rate swaps is given at Appendix 39.1.

The trade date for a swap is not surprisingly, the date on which the swap is transacted. The terms of the trade include the fixed interest rate, the maturity and notional amount of the swap, and the payment bases of both legs of the swap. The date from which floating interest payments are determined is the *setting date*, which may also be the trade date. Most swaps fix the floating-rate payments to Libor, although other reference rates that are used include the US Prime rate, euribor, the Treasury bill rate and the commercial paper rate. In the same way as for FRA and eurocurrency deposits, the rate is fixed two business days before the interest period begins. The second (and subsequent) setting date will be two business days before the beginning of the second (and subsequent) swap periods. The *effective date* is the date from which interest on the swap is calculated, and this is typically two business days after the trade date. In a *forward-start* swap the effective date will be at some point in the future, specified in the swap terms. The floating interest-rate for each period is fixed at the start of the period, so that the interest payment amount is known in advance by both parties (the fixed rate is known of course, throughout the swap by both parties).

Although for the purposes of explaining swap structures, both parties are said to pay interest payments (and receive them), in practice only the net difference between both payments changes hands at the end of each interest payment. This eases the administration associated with swaps and reduces the number of cash flows for each swap. The counterparty that is the net payer at the end of each period will make a payment to the counterparty. The first payment date will occur at the end of the first interest period, and subsequent payment dates will fall at the end of successive interest periods. The final payment date falls on the maturity date of the swap. The calculation of interest is given by (39.1):

$$I = M \times r \times \frac{n}{B} \quad (39.1)$$

where I is the interest amount, M is the nominal amount of the swap and B is the day-base for the swap. Dollar and euro-denominated swaps use an actual/360 day-count, similar to other money market instruments in those currencies, while sterling swaps use an actual/365 day-count basis.

The cash flows resulting from a vanilla interest rate swap are illustrated in Figure 39.5, using the normal convention where cash inflows are shown as an arrow pointing up, while cash outflows are shown as an arrow pointing down. The counterparties in a swap transaction only pay across net cash flows however, so at each interest payment date only one actual cash transfer will be made, by the net payer. This is shown as Figure 39.5 (iii).

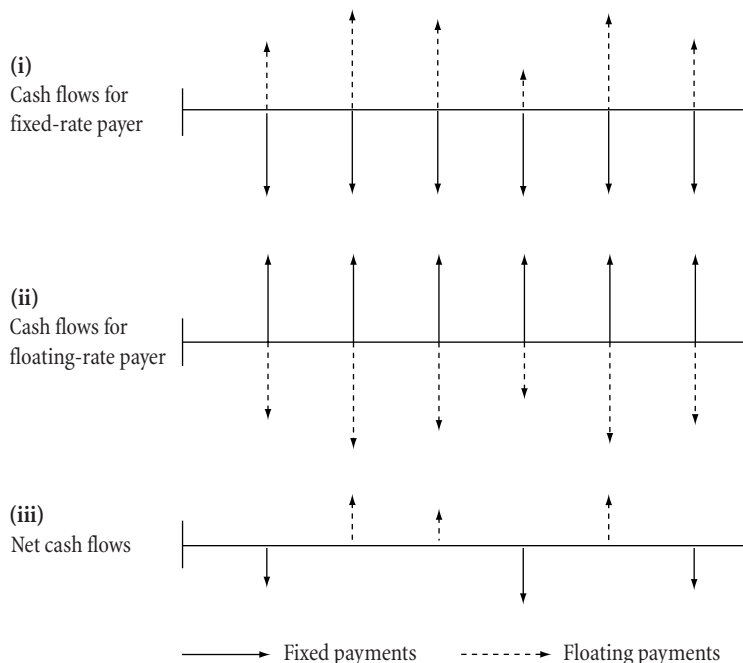


Figure 39.5: Cash flows for typical interest rate swap.

39.2.2 Example of vanilla swap

The following swap cash flows are for a “pay fixed, receive floating” interest-rate swap with the following terms:

Trade date:	3 December 1999
Effective date:	7 December 1999
Maturity date:	7 December 2004
Interpolation method:	Linear interpolation
Day-count (fixed):	Semi-annual, act/365
Day-count (floating):	Semi-annual, act/365
Nominal amount:	£10 million
Term:	Five years
Fixed rate:	6.73%

To calculate the cash flows, we construct a zero-coupon curve and value the swap with this curve (zero-coupon pricing is discussed later in this chapter). Using a conventional swap calculator, we obtain the following cash flows for the pay and receive legs of the swap, shown in Table 39.3. The summary details for the swap, valued as at 20 January 2000, are shown in Table 39.4.

Cash flow table**Pay leg**

Start date	End date	Interest period (days)	Interest rate	Nominal	Interest (£)	Discount factor	Interest present value (£)
07-Dec-99	07-Jun-00	183	6.73%	10,000,000	337,421.92	0.970787	327,564.98
07-Jun-00	07-Dec-00	183	6.73%	10,000,000	337,421.92	0.934890	315,452.27
07-Dec-00	07-Jun-01	182	6.73%	10,000,000	335,578.08	0.903668	303,251.27
07-Jun-01	07-Dec-01	183	6.73%	10,000,000	337,421.92	0.873975	294,898.27
07-Dec-01	07-Jun-02	182	6.73%	10,000,000	335,578.08	0.843748	283,143.40
07-Jun-02	09-Dec-02	185	6.73%	10,000,000	341,109.59	0.814353	277,783.70
09-Dec-02	09-Jun-03	182	6.73%	10,000,000	335,578.08	0.787777	264,360.77
09-Jun-03	08-Dec-03	182	6.73%	10,000,000	335,578.08	0.762992	256,043.24
08-Dec-03	07-Jun-04	182	6.73%	10,000,000	335,578.08	0.739486	248,155.28
07-Jun-04	07-Dec-04	183	6.73%	10,000,000	337,421.92	0.717683	242,161.99

Receive leg

07-Dec-99	07-Jun-00	183	5.87%	10,000,000	294,554.79	0.970787	285,950.11
07-Jun-00	07-Dec-00	183	7.66%	10,000,000	383,979.03	0.934890	358,978.04
07-Dec-00	07-Jun-01	182	6.93%	10,000,000	345,496.21	0.903668	312,213.97
07-Jun-01	07-Dec-01	183	6.78%	10,000,000	339,751.52	0.873975	296,934.29
07-Dec-01	07-Jun-02	182	7.18%	10,000,000	358,242.71	0.843748	302,266.64
07-Jun-02	09-Dec-02	185	7.12%	10,000,000	360,960.68	0.814353	293,949.50
09-Dec-02	09-Jun-03	182	6.77%	10,000,000	337,354.53	0.787777	265,760.22
09-Jun-03	08-Dec-03	182	6.51%	10,000,000	324,848.55	0.762992	247,856.70
08-Dec-03	07-Jun-04	182	6.37%	10,000,000	317,864.09	0.739486	235,056.03
07-Jun-04	07-Dec-04	183	6.06%	10,000,000	303,795.95	0.717683	218,029.20

Net payments:	42,867
	—46,557.11
	—9,918.13
	—2,329.60
	—22,664.63
	—19,851.09
	—1,776.45
	10,730
	17,714
	33,626

Table 39.3: Interest-rate swap cash flows.

The interest payment dates of the swap fall on 7 June and 7 December; the coupon dates of benchmark gilts also falls on these dates, so even though the swap has been traded for conventional dates, it is safe to surmise that it was put on as a hedge against a long gilt position. The fixed-rate payments are not always the same, because the actual/365 basis will calculate slightly different amounts. The net payments for the fixed-rate payer are also shown. Note also that the present value of each cash flow is also shown; the net difference of the cash, present valued, is the current value of the swap. This is shown in Table 39.4. The present values are calculated using a spot rate yield curve, which has been calculated using money market and government bond yields. The integrated discount function that is used to value the swap is shown at Table 39.5, and the graph of the function is shown at Figure 39.6. The discount function has been calculated using linear interpolation, although other methods are also used in the market.

Trade date	03-Dec-99	Present value (£)	4179.52
Effective date	07-Dec-99	Duration	2.6259
Maturity date	07-Dec-04	Modified duration	2.5395
Margin	0.00%	Convexity	9.5991
Fixed rate	6.73%		
Nominal	£10 million		

Table 39.4: Summary of swap terms.

Grid point	Discount factor	Spot rate%	Grid point	Discount factor	Spot rate%
03-Dec-1999	1.000000	5.8113	07-Dec-2012	0.462082	6.1131
07-Dec-1999	0.999383	5.7983	07-Jun-2013	0.450487	6.0803
14-Dec-2009	0.998329	5.7045	09-Dec-2013	0.439166	6.0466
21-Dec-1999	0.997260	5.7224	09-Jun-2014	0.428461	6.0130
07-Jan-2000	0.994652	5.7679	08-Dec-2014	0.418173	5.9790
07-Feb-2000	0.989882	5.8013	08-Jun-2015	0.407653	5.9553
07-Mar-2000	0.985258	5.8886	07-Dec-2015	0.398135	5.9207
07-Apr-2000	0.980537	5.8753	07-Jun-2016	0.388939	5.8862
07-May-2000	0.975813	5.9125	07-Dec-2016	0.380103	5.8512
07-Jun-2000	0.970787	5.9744	07-Jun-2017	0.371659	5.8156
07-Sep-2000	0.956849	5.9572	07-Dec-2017	0.363502	5.7795
07-Dec-2000	0.934890	6.8865	07-Jun-2018	0.355707	5.7433
07-Jun-2001	0.903668	6.9402	07-Dec-2018	0.348179	5.7065
07-Dec-2001	0.873975	6.9279	07-Jun-2019	0.340986	5.6697
07-Jun-2002	0.843748	7.0045	09-Dec-2019	0.333967	5.6319
09-Dec-2002	0.814353	7.0452	08-Jun-2020	0.325449	5.6247
09-Jun-2003	0.787777	7.0218	07-Dec-2020	0.317812	5.6073
08-Dec-2003	0.762992	6.9719	07-Jun-2021	0.310429	5.5891
07-Jun-2004	0.739486	6.9193	07-Dec-2021	0.303255	5.5705
07-Dec-2004	0.717683	6.8441	07-Jun-2022	0.296357	5.5516
07-Jun-2005	0.695827	6.8037	07-Dec-2022	0.289654	5.5323
07-Dec-2005	0.675491	6.7444	07-Jun-2023	0.283210	5.5128
07-Jun-2006	0.656228	6.6851	07-Dec-2023	0.276947	5.4928
07-Dec-2006	0.637787	6.6252	07-Jun-2024	0.270894	5.4729
07-Jun-2007	0.619586	6.5821	09-Dec-2024	0.264982	5.4523
07-Dec-2007	0.602744	6.5236	09-Jun-2025	0.259359	5.4316
09-Jun-2008	0.586535	6.4651	08-Dec-2025	0.253925	5.4105
08-Dec-2008	0.571347	6.4070	08-Jun-2026	0.248672	5.3892
08-Jun-2009	0.556883	6.3474	07-Dec-2026	0.243594	5.3675
07-Dec-2009	0.543110	6.2875	07-Jun-2027	0.238685	5.3457
07-Jun-2010	0.527778	6.2696	07-Dec-2027	0.233916	5.3234
07-Dec-2010	0.513559	6.2390	07-Jun-2028	0.229306	5.3011
07-Jun-2011	0.499967	6.2080	07-Dec-2028	0.224852	5.2786
07-Dec-2011	0.486833	6.1764	07-Jun-2029	0.220571	5.2556
07-Jun-2012	0.474211	6.1450	07-Dec-2029	0.216411	5.2323

Table 39.5: Integrated discount function for 20 January 2000.

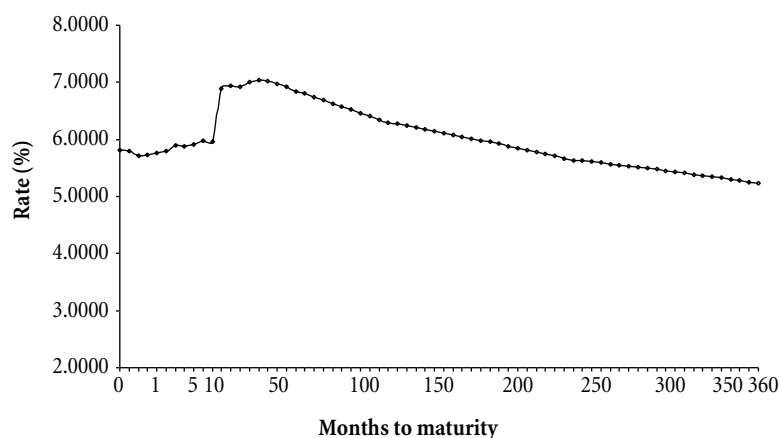


Figure 39.6: Graph of implied spot rate yield curve.

If the five-year swap rate changes, the present value of the swap will change. The swap has a fixed rate of 6.73%, therefore a fall in market interest rates will result in the swap being marked-to-market at a loss to the fixed-rate payer, and a profit to the floating-rate payer. The opposite applies if there is a rise in market rates. Table 39.6 shows the valuation of the swap where the five-year interest rate has risen to 7.00%. The present value of the swap has risen to £144,103, which is a mark-to-market profit.

Trade date	03-Dec-99	Market rate	7.00%
Effective date	07-Dec-99	Present value (£)	144,103.67
Maturity date	07-Dec-04	Duration	2.4917
Margin	0.00%	Modified duration	2.4081
Fixed rate	6.73%	Convexity	9.5089
Nominal	£10 million		

Table 39.6: Revaluation of interest-rate swap.

The swap we have described is a plain vanilla swap, which it means it has one fixed-rate and one floating-rate leg. The floating interest-rate is set just before the relevant interest period and is paid at the end of the period. Note that both legs have identical interest dates and day-count bases, and the term to maturity of the swap is exactly five years. It is of course possible to ask for a swap quote where any of these terms has been set to customer requirements; so for example both legs may be floating-rate, or the notional principal may vary during the life of the swap. Non-vanilla interest-rate swaps are very common, and banks will readily price swaps where the terms have been set to meet specific requirements. The most common variations are different interest payment dates for the fixed- and floating-rate legs, on different day-count bases, as well as terms to maturity that are not whole years.

39.2.3 Generic interest-rate swap portfolio

Table 39.7 illustrates an hypothetical swap book for a bank, with five different swaps and swap terms.

Swap	Start date	Maturity date	Principal	Fixed rate %	Current reset rate %	Fixed/floating	Present value £	Accrued interest £
5	05-Jun-99	04/06/2002	20,000,000	6.125	6.200	Pay fixed	307,573.65	1,931.51
1	20-Jan-99	17/01/2009	5,000,000	5.650	6.200	Pay floating	-240,619.85	-226.03
2	11-Oct-99	10/10/2001	10,000,000	6.000	6.500	Pay fixed	147,890.50	13,835.62
3	03-Jan-00	03/01/2004	5,000,000	5.780	6.800	Pay floating	-195,229.48	-2,375.34
4	15-Sep-99	15/09/2000	2,000,000	6.750	6.200	Pay fixed	-13,134.12	-3,827.40
Total							6,480.70	9,338.36

Table 39.7: Interest-rate swap portfolio.

Note that certain swaps are “pay fixed, receive floating” while the others are the opposite. The maturities and fixed rates are also different. The swaps have been marked-to-market at the relevant interest reset rate, which enables the current (present) value to be calculated. This shows that the swap book is currently marked at a profit of £6 480.69.

39.2.4 Swap spreads and the swap yield curve

In the market banks will quote two-way swap rates, on screens and on the telephone or via a dealing system such as Reuters. Brokers will also be active in relaying prices in the market. The convention in the market is for the swap market maker to set the floating leg at Libor and then quote the fixed rate that is payable for that maturity. So for a five-year swap a bank's swap desk might be willing to quote the following:

Floating-rate payer:	pay 6mo-Libor receive fixed rate of 5.19%
Fixed-rate payer:	pay fixed rate of 5.25% receive 6-mo Libor

In this case the bank is quoting an offer rate of 5.25%, which the fixed-rate payer will pay, in return for receiving Libor flat. The bid price quote is 5.19% which is what a floating-rate payer will receive fixed. The bid-offer spread in this case is therefore 6 basis points. The fixed-rate quotes are always at a spread above the government bond yield curve. Let us assume that the five-year gilt is yielding 4.88%; in this case then the five-year swap bid rate is 31 basis points above this yield. So the bank's swap trader could quote the swap rates as a spread above the benchmark bond yield curve, say 37-31, which is her swap spread quote. This means that the bank is happy to enter into a swap paying fixed 31 basis points above the benchmark yield and receiving Libor, and receiving fixed 37 basis points above the yield curve and paying Libor. The bank's screen on say, Bloomberg or Reuters might look something like Table 39.8, which quotes the swap rates as well as the current spread over the government bond benchmark.

1YR	4.50	4.45	+17
2YR	4.69	4.62	+25
3YR	4.88	4.80	+23
4YR	5.15	5.05	+29
5YR	5.25	5.19	+31
10YR	5.50	5.40	+35

Table 39.8: Swap quotes.

An actual interest-rate swap screen is shown at Figure 39.7, which is the sterling swaps page from Garban ICAP (a money market and bond broker) distributed via the Telerate service on 8 December 1999.

08/12 8:54 GMT		[GARBAN-INTERCAPITAL LONDON]		4922	
TEL	0171 256 9292	GBP/IEP		08/12 08:53 GMT	
2 YRS	6.800-6.760	7 01	+61.0/+57.0	1 YR/A3	6.580 6.550
3 YRS	6.910 6.870	7 02	+72.0/+67.0	DEC/DEC	6.590-6.560
4 YRS	6.850-6.800	6H 03	+87.0/+82.0	MAR/MAR	6.850 6.820
5 YRS	6.740 6.690	6T 04	+91.0 +86.0	IEP	
6 YRS	6.650-6.600	8H 05	+86.0/+81.0	1 YR	3.830-3.800
7 YRS	6.560 6.510	7H 06	+85.0 +80.0	2 YRS	4.245 4.215
8 YRS	6.470-6.420	7Q 07	+88.0/+83.0	3 YRS	4.510-4.480
9 YRS	6.380 6.330	9 08	+86.0 +81.0	4 YRS	4.710 4.680
10 YRS	6.290-6.240	5T 09	+110.0/+105.0	5 YRS	4.870-4.840
12 YRS	6.190 6.120	8 13	+122.0 +114.0	6 YRS	5.015 4.985
15 YRS	6.050-5.960	8 15	+129.0/+121.0	7 YRS	5.150-5.120
20 YRS	5.810 5.710	8 21	+127.0 +117.0	8 YRS	5.265 5.235
25 YRS	5.660-5.550	8 21	+112.0/+100.0	9 YRS	5.350-5.320
30 YRS	5.550T5.420	6 28	+120.0 +108.0	10 YRS	5.410 5.380
40 YRS	5.430-5.260	6 28	+105.0/+93.0		

Figure 39.7: Garban ICAP brokers sterling swaps page, 8 December 1999.
©Garban ICAP ©Dow-Jones Markets 1999. Reproduced with permission.

The page shows the swap bid and offer rates for swaps of maturities from two years to 40 years. The benchmark government bond for that maturity is also shown, together with the swap spread over the yield of the bond. Note that in the right-hand corner there is also a quote for a one-year swap, with quarterly payments. The other quotes are for Irish pound swaps, which of course is a euro currency as of January 1999.

The swap spread is a function of the same factors that influence the spread over government bonds for other instruments. For shorter duration swaps, say up to three years, there are other yield curves that can be used in comparison, such as the cash market curve or a curve derived from futures prices. For longer-dated swaps the spread is determined mainly by the credit spreads that prevail in the corporate bond market. Because a swap is viewed as a package of long and short positions in fixed- and floating-rate bonds, it is the credit spreads in these two markets that will determine the swap spread. This is logical; essentially it is the premium for greater credit risk involved in lending to corporates that dictates that a swap rate will be higher than the same maturity government bond yield. Technical factors will be responsible for day-to-day fluctuations in swap rates, such as the supply of corporate bonds and the level of demand for swaps, plus the cost to swap traders of hedging their swap positions.

We can conclude by saying that swap spreads over government bonds reflect the supply and demand conditions of both swaps and government bonds, as well as the market's view on the credit quality of swap counterparties. There is considerable information content in the swap yield curve, much like that in the government bond yield curve. During times of credit concerns in the market, such as the corrections in Asian and Latin American markets in summer 1998, and the possibility of default by the Russian government regarding its long-dated US dollar bonds, the swap spread will increase, more so at higher maturities. To illustrate this let us consider the sterling swap spread in 1998/98. The UK swap spread widened from the second half of 1998 onwards, a reaction to bond market volatility around the world. At such times investors embark on a "flight to quality" that results in yield spreads widening. In the swap market, the spread between two-year and ten-year swaps also increased, reflecting market concern with credit and counterparty risk. The spreads narrowed in the first quarter 1999, as credit concerns brought about by market corrections in 1998 declined. The change in swap spreads is shown in Figure 39.8.

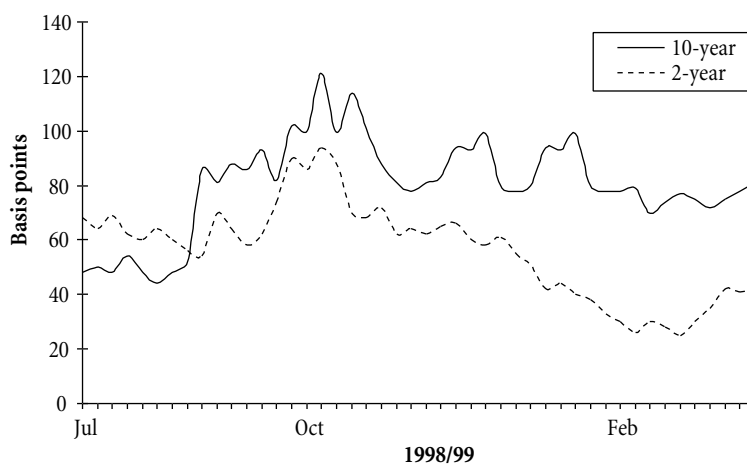


Figure 39.8: Sterling 2-year and 10-year Swap Spreads 1998/99. Source: BoE.

In an earlier chapter we discussed the interpretation of yield curves and their information content, in the context of government bond yield curves. In certain circumstances it is more relevant to use the swap curve for market analysis and as an information and interest rate predictor. This is usually the case where a liquid market in interest rate swaps exists out to long-dated maturities. A situation under which the swap curve should be examined is when it is a different shape to the government bond yield curve; usually the swap curve will mirror the shape of the government curve, but at a higher yield level due to the swap spread. In the UK market the government curve has been inverted (negative) since July 1997, and the swap curve has been slightly inverted or flat. However in the last quarter on 1998 the swap curve became slightly upward sloping, whereas the gilt curve remained inverted. This suggested that the market was predicting higher future short-term interest rates, and had priced this into swap rates, while the gilt curve remained unchanged, mainly for structural reasons and excess demand.

39.3 Relationship between interest-rate swaps and FRAs

A strip of forward-rate agreements (FRAs) is conceptually similar to an interest-rate swap. This is because one FRA fixes the interest rate for a period starting in the future, so that a strip of continuous FRAs will fix the interest rate for a continuous period. The rates implied by the FRA strip will equate to that payable on a swap, to avoid arbitrage opportunities arising. To consider this, a borrowing arranged on a rolling three-month basis at Libor, whilst simultaneously transacting a swap to receive quarterly three-month Libor against a fixed payment, is the same as borrowing on the three-month basis and fixing the interest cost with a series of rolling FRAs, starting with a 3v6 FRA, then followed by a 6v9 FRA and so on. If the swap and FRA rates were out of line, then it would be possible to put on opposing positions in both and pocket the difference as an arbitrage profit. For example if the two-year swap rate is too low, it would be possible to trade the swap to pay fixed and receive floating, simultaneously selling a strip of FRAs to offset this.

EXAMPLE 39.2 Calculating the one-year swap rate

Assume the following sterling FRA rates:

Three-month cash:	7.625%
3v6 FRA:	6.875%
6v9 FRA:	6.125%
9v12 FRA:	5.75%

If we assume the cash period is 92 days and the FRA periods are 91 days, with FRA settlement made at the end of the period rather than at the beginning, we may use the cash and FRA rates to calculate the one-year swap rate, which is given as:

$$\left(1 + 0.07625 \times \frac{92}{365}\right) \times \left(1 + 0.06875 \times \frac{91}{365}\right) \times \left(1 + 0.0615 \times \frac{91}{365}\right) \times \left(1 + 0.0575 \times \frac{91}{365}\right) = 6.761\%.$$

This means that the fixed cost of a borrowing of cash over the one-year period is 6.761%, which is the theoretical price of a one-year swap transacted today. As the swap would pay on a quarterly basis we convert the rate to a quarterly equivalent, which is

$$\left((1 + 0.0761)^{\frac{1}{4}} - 1\right) \times 4 = 6.596\%.$$

In practice a slightly different procedure is used, which we discuss in the section on pricing.

In this example we used FRAs with final maturity periods extending to one year from now. The existence of a forward rate, which is what FRA rates are, enables us to calculate a swap rate. The result is known as a *zero-coupon swap rate*, which is examined next. As FRA rates, which are calculated from interest-rate futures prices, are only available out to two or three years, we use bond yields to calculate longer-dated swaps. The convention is to calculate zero-coupon bond yields and use these to price swaps. Swap rates that have been calculated in this way are known as *par swap rates* and are the fixed-rates payable on a swap for which the floating-rate is Libor.

39.4 Generic swap valuation

Banks generally use *par swap* or *zero-coupon* swap pricing. We will look at this method in the very next section; first however we will introduce an intuitive swap valuation method.

Assume we have a vanilla interest-rate swap with a notional principal of N that pays n payments during its life, to a maturity date of T . The date of each payment is on t_j with $j = 1, \dots, n$. The present value today of a future payment is denoted by $PV(0, t)$. If the swap rate is r , the value of the fixed-leg payments is given by (39.2):

$$PV_{\text{fixed}} = N \sum_{i=1}^n PV(0, t_i) \times r \times \left(\frac{t_i - t_{i-1}}{B} \right) \quad (39.2)$$

where B is the money market day base. The term $t_i - t_{i-1}$ is simply the number of days between the i th and the $(i-1)$ th payments.

The value of the floating-leg payments at the date t_1 for an existing swap is given by

$$PV_{float} = N \times \left(rl \times \frac{t_1}{B} \right) + N - (N \times PV(t_1, t_n)) \quad (39.3)$$

where rl is the LIBOR rate that has been set for the next interest payment. We set the present value of the floating-rate payment at time 0 as follows:

$$PV(0, t_1) = \frac{1}{1 + rl(t_1)(t_1/B)}. \quad (39.4)$$

For a new swap the value of the floating payments is given by

$$PV_{float} = N \left(rl \times \frac{t_1}{B} + 1 \right) \times PV(0, t_1) - PV(0, t_n). \quad (39.5)$$

The swap valuation is then given by $PV_{fixed} - PV_{float}$. The swap rate quoted by a market making bank is that which sets $PV_{fixed} = PV_{float}$ and is known as the par or zero-coupon swap rate. We consider this next.

39.5 Zero-coupon swap pricing

So far we have discussed how vanilla swap prices are often quoted as a spread over the benchmark government bond yield in that currency, and how this swap spread is mainly a function of the credit spread required by the market over the government (risk-free) rate. This method is convenient and also logical because banks use government bonds as the main instrument when hedging their swap books. However because much bank swap trading is now conducted in non-standard, tailor-made swaps, this method can sometimes be unwieldy as each swap needs to have its spread calculated to suit its particular characteristics. Therefore banks use a standard pricing method for all swaps known as *zero-coupon* swap pricing.

In earlier chapters we referred to zero-coupon bonds and zero-coupon interest rates. A bond that has only one cash flow, its redemption payment on maturity, has no coupons and consequently offers no *reinvestment risk* to the bondholder. If a zero-coupon bond such as a gilt strip is purchased at a particular yield, and held to maturity, the return for the strip holder will be exactly the yield that applied at the time of purchase. Zero-coupon rates, or *spot rates* are therefore true interest rates for their particular term to maturity. In zero-coupon swap pricing, a bank will view all swaps, even the most complex, as a series of cash flows. The zero-coupon rates that apply now for each of the cash flows in a swap can be used to value these cash flows. Therefore to value and price a swap, each of the swap's cash flows are present valued using known spot rates; the sum of these present values is the value of the swap.

In a swap the fixed-rate payments are known in advance and so it is straightforward to present-value them. The present value of the floating rate payments is usually estimated in two stages. First the implied forward rates can be calculated using (39.6):

$$rf_i = \left(\frac{df_i}{df_{i+1}} - 1 \right) N \quad (39.6)$$

where

rf_i	is the forward rate
i	is the total number of coupons (generic bond)
df_i	is the discount factor for the relevant cash flow date
N	is the number of times per year that coupons are paid

By definition the floating-payment interest rates are not known in advance, so the swap bank will predict what these will be, using the forward rates applicable to each payment date. The forward rates are those that are currently implied from spot rates. Once the size of the floating-rate payments have been estimated, these can also be valued by using the spot rates. The total value of the fixed and floating legs is the sum of all the present values, so the value of the total swap is the net of the present values of the fixed and floating legs.

Earlier in this chapter we illustrated how cash flows in a vanilla interest-rate swap, where the party is paying floating, are conceptually the same as that from a long position in a fixed interest rate bond and a short position in

an FRN. Such a swap would also be identical to a long position in a bond that was financed by borrowings at Libor. As the resulting cash flows are identical whatever position is put on, we can say that the fixed rate on a vanilla interest-rate swap is the same as the yield (and coupon) on a bond that is trading at par (par because such a bond is priced at 100). If we wish to replicate the conditions of a swap, the initial cash flows need to be identical. At par, the cash flows from buying £100 million of the bond and financing with borrowing of £100 million at Libor is identical to that resulting from £100 million of a swap where the party is paying the floating leg). So if we wish to determine the correct rate for the fixed leg of a vanilla swap we need to calculate the correct coupon for a bond of the same maturity that we wish to issue at par. In fact the swap rate is the weighted arithmetic average of the forward rates up to the swap's maturity. For example the five-year swap rate is the weighted average of the strip of five one-year forward rates. We calculated forward rates in an earlier chapter, and using these principles we can calculate swap rates.

These procedures are discussed in detail in the remainder of this section.

39.5.1 Assumptions of zero-coupon swap pricing

While the term *zero-coupon* refers to an interest rate that applies to a discount instrument that pays no coupon and has one cash flow (at maturity), it is not necessary to have a functioning zero-coupon bond market in order to construct a zero-coupon yield curve. We noted in Chapter 6 that it is possible to derive theoretical zero-coupon rates from other cash market instruments, including coupon bonds. In practice most financial pricing models use a combination of the following instruments to construct zero-coupon yield curves:

- money market deposits;
- interest-rate futures;
- FRAs;
- government bonds.

Frequently an overlap in the maturity period of all instruments is used; FRA rates are usually calculated from interest-rate futures so it is necessary to use only of either FRA or futures rates.

Once a zero-coupon yield curve (*term structure*) is derived, this may be used to value a future cash flow maturing at any time along the term structure. This includes swaps: to price an interest-rate swap, we calculate the present value of each of the cash flows using the zero-coupon rates and then sum all the cash flows. As we noted above, while the fixed-rate payments are known in advance, the floating-rate payments must be estimated, using the forward rates implied by the zero-coupon yield curve. The net present value of the swap is the net difference between the present values of the fixed- and floating-rate legs. This has already been demonstrated in Table 39.3.

39.5.2 The discount function

Discount factors and the discount function were introduced and discussed in Part I (Chapters 3, 4 and 6). To recap, the discount factor of any period n is given by (39.7):

$$PV_n = FV_n \times df_n \quad (39.7)$$

where df_n is the discount factor for a cash flow that occurs at time n . To obtain the discount factor from zero-coupon rates we use (39.8) and (39.9) as appropriate; (39.8) is used for money market maturities,

$$df_n = \frac{1}{(1 + rs_n t_n)} \quad (39.8)$$

$$df_n = \frac{1}{(1 + rs_n)^{t_n}} \quad (39.9)$$

where t_n is the maturity of the period in question and rs_n is the relevant period zero-coupon or spot rate.

Discount factors for a set of discrete period spot rates are given in Table 39.9.² The continuous set of discount factors is of course, we remember from Chapter 6, the discount function. This is represented graphically in Figure 39.9.

Term	Spot rate	Discount factor
1 month	6.250%	0.99488893
3 months	6.500%	0.98405290
6 months	6.125%	0.97068010
1 year	6.375%	0.94007051
2 years	6.625%	0.88581315
3 years	6.750%	0.82204643
4 years	6.875%	0.76647058
5 years	7.000%	0.71298618
7 years	7.250%	0.61265911
10 years	7.500%	0.48519393

Table 39.9: Zero-coupon rates and associated discount factors.

To calculate the discount factor for every possible grid point, which will range from 1 day to 14,600 days (40 years) in the swap market, banks commonly use linear interpolation for any period between two known rates. Interpolation may be used to calculate the zero-coupon rate between two points or the discount factor between two points. However, to apply linear interpolation between two points on a yield curve is a flawed technique, because it assumes that the curve follows a straight line between the two points; from our discussion of yield curve in Chapter 6 we know this is not a realistic assumption. The discount function on the other hand, always follows an exponential curve, and from equations (39.8) and (39.9) we may set the curve in the form given as (39.10),

$$e^{-nt} \quad (39.10)$$

so we may then interpolate between two points on the function without having to make assumptions about the nature of the curve. The proof for this is given in Appendix 39.2. Therefore interpolating between two points on the discount function is known as *exponential interpolation* and is given by the expression at (39.11):

$$df_t = df_1^{(t_n(t_2-t_n)/t_1(t_2-t_1))} df_2^{(t_n(t_n-t_1)/t_2(t_2-t_1))} \quad (39.11)$$

where df_t is the discount factor for the period t in between periods 1 and 2.

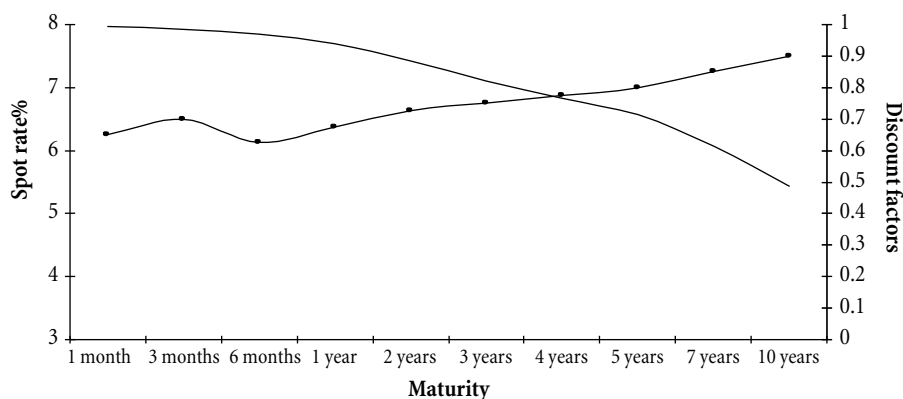


Figure 39.9: Spot yield curve and discount function.

² The discount factors are calculated using the actual/365 day-count basis.

The discount factors for periods 1 and 2 are given by df_1 and df_2 respectively.

If we wish to interpolate for a point ahead of the first known or beyond the last known discount factor, we use (39.12):

$$df_n = df_s^{t_n/t_s} \quad (39.12)$$

where

df_n	is the discount factor at the required date n
df_s	is the first or last known discount factor at date s
t_s	is the period from today to date s
t_n	is the period from today to date n .

39.5.3 Calculating the forward rate from spot rate discount factors³

Early in the chapter we suggested that one way to view a swap was as a long position in a fixed-coupon bond that was funded at Libor, or against a short position in a floating-rate bond. The cash flows from such an arrangement would be paying floating-rate and receiving fixed-rate. In the former arrangement, where a long position in a fixed-rate bond is funded with a floating-rate loan, the cash flows from the principals will cancel out, as they are equal and opposite (assuming the price of the bond on purchase was par), leaving a collection of cash flows that mirror an interest-rate swap that pays floating and receives fixed. Therefore, as the fixed-rate on an interest-rate swap is the same as the coupon (and yield) on a bond priced at par, so that in order to calculate the fixed-rate on an interest-rate swap is the same as calculating the coupon for a bond that we wish to issue at par.

The price of a bond paying semi-annual coupons is given by (39.13), which may be rearranged for the coupon rate r to provide an equation that enables us to determine the par yield, and hence the swap rate r , given by (39.14).

$$P = \frac{r_n}{2} df_1 + \frac{r_n}{2} df_2 + \cdots + \frac{r_n}{2} df_n + Mdf_n \quad (39.13)$$

where r_n is the coupon on an n -period bond with n coupons and M is the maturity payment. If we assume the bond is priced at par, we can rearrange (39.13) for r_n to obtain:

$$\begin{aligned} r_n &= \frac{1 - df_n}{(df_1/2) + (df_2/2) + \cdots + (df_n/2)} \\ &= \frac{1 - df_n}{\sum_{i=1}^n (df_i/2)}. \end{aligned} \quad (39.14)$$

For annual coupon bonds there is no denominator for the discount factor, while for bonds paying coupons on a frequency of F we replace the denominator 2 with F .⁴ The expression at (39.14) may be rearranged again, using F for the coupon frequency, to obtain an equation which may be used to calculate the n th discount factor for an n -period swap rate, given at (39.15):

$$df_n = \frac{1 - r_n \sum_{i=1}^{n-1} (df_i/F)}{1 + (r_n/F)}. \quad (39.15)$$

The expression at (39.15) is the general expression for the *bootstrapping* process that we first encountered in Chapter 6. Essentially, to calculate the n -year discount factor we use the discount factors for the years 1 to $n - 1$, and the n -year swap rate or zero-coupon rate. If we have the discount factor for any period, we may use (39.15) to determine the same period zero-coupon rate, after rearranging it, shown at (39.16):

³ The follows the approach described in Galitz (1995).

⁴ The expression also assume an actual/365 day-count basis. If any other day-count convention is used, the $1/F$ factor must be replaced by a fraction made up of the actual number of days as the numerator and the appropriate year base as the denominator.

$$rs_n = \sqrt[n]{\frac{1}{df_n}} - 1. \quad (39.16)$$

Discount factors for spot rates may also be used to calculate forward rates. We know from (39.8) that

$$df_1 = \frac{1}{1 + \frac{rs_1}{F}} \quad (39.17)$$

where the time period t_n lies within a one-year period given by $1/F$. The discount factor is then the numerator divided by one plus the zero-coupon rate. If we have a *forward rate* we may use this to calculate a second discount factor, shown by (39.18):

$$df_2 = \frac{df_1}{1 + \frac{rf_1}{F}} \quad (39.18)$$

where rf_1 is the forward rate. This is no use in itself, however we may derive from it an expression to enable us to calculate the discount factor at any point in time between the previous discount rate and the given forward rate for the period n to $n + 1$, shown at (39.19), which may then be rearranged to give us the general expression to calculate a forward rate, given at (39.20).

$$df_{n+1} = \frac{df_n}{1 + \frac{rf_n}{F}}. \quad (39.19)$$

$$rf_n = \left(\frac{df_n}{df_{n+1}} - 1 \right) F. \quad (39.20)$$

The general expression for an n -period discount rate at time n from the previous period forward rates is given by (39.21):

$$df_n = \frac{1}{1 + \frac{rf_{n-1}}{F}} \times \frac{1}{1 + \frac{rf_{n-2}}{F}} \times \cdots \times \frac{1}{1 + \frac{rf_0}{F}} \quad (39.21)$$

$$df_n = \prod_{i=0}^{n-1} \left(\frac{1}{1 + (rf_i/F)} \right).$$

From the above we may combine equations (39.14) and (39.20) to obtain the general expression for an n -period swap rate and zero-coupon rate, given by (39.22) and (39.23) respectively.

$$r_n = \frac{\sum_{i=1}^n (rf_{i-1} df_i) / F}{\sum_{i=1}^n df_i / F}. \quad (39.22)$$

$$1 + rs_n = \sqrt[n]{\prod_{i=0}^{n-1} \left(1 + \frac{rf_i}{F} \right)}. \quad (39.23)$$

The two expressions do not tell us anything new, as we have already encountered their results in Chapter 6. The swap rate, which we have denoted as r_n is shown by (39.22) to be the weighted-average of the forward rates; this is not surprising and we showed this for a one-year swap in Example 39.2. In that example, a strip of FRA rates was used to calculate the one-year swap rate, and we saw that it was the arithmetic average. This is what we would expect, as a swap rate is for a continuous period that would be covered by a strip of FRAs; therefore an average of the FRA rates would be the correct swap rate. As FRA rates are forward rates, we may be comfortable with (39.22), which states that the n -period swap rate is the average of the forward rates from rf_0 to rf_n . To be accurate we must weight the forward rates, and these are weighted by the discount factors for each period. Note that although swap

rates are derived from forward rates, interest payments under a swap are paid in the normal way at the end of an interest period, while payments for a FRA are made at the beginning of the period and must be discounted.

Equation (39.23) states that the zero-coupon rate is calculated from the geometric average of (one plus) the forward rates. The n -period forward rate is obtained using the discount factors for periods n and $n - 1$. The discount factor for the complete period is obtained by multiplying the individual discount factors together, and exactly the same result would be obtained by using the zero-coupon interest-rate for the whole period to obtain the discount factor.⁵

39.5.4 Pricing a generic interest-rate swap

The rate charged on a newly-transacted interest-rate swap is the one that gives its net present value as zero. The term *valuation* of a swap is used to denote the process of calculating the net present value of an existing swap, when marking-to-market the swap against current market interest rates. Therefore when we price a swap, we set its net present value to zero, while when we value a swap we set its fixed rate at the market rate and calculate the net present value.

To illustrate the basic principle, we price a plain vanilla interest rate swap with the terms set out below; for simplicity we assume that the annual fixed-rate payments are the same amount each year, although in practice there would be slight differences. Again for simplicity, assume we already have our zero-coupon yield curve, given in Table 39.9. We use the zero-coupon rates to calculate the discount factors, and then use the discount factors to calculate the forward rates. This is done using equation (39.20). These forward rates are then used to predict what the floating-rate payments will be at each interest period. Both fixed-rate and floating-rate payments are then present-valued at the appropriate zero-coupon rate, which enables us to calculate the net present value.

The fixed-rate for the swap is calculated using equation (39.14), to give us:

$$(1 - 0.71298618)/4.16187950 = 6.8963\%.$$

Nominal principal	£10,000,000
Fixed rate	6.8963%
Day count fixed	Actual/365
Day count floating	Actual/365
Payment frequency fixed	Annual
Payment frequency floating	Annual
Trade date	31 January 2000
Effective date	2 February 2000
Maturity date	2 February 2005
Term	Five years

Period	Zero-coupon rate %	Discount factor	Forward rate %	Fixed payment	Floating payment	PV fixed payment	PV floating payment
1	5.5	0.947867298	5.5	689,625	550,000	653,672.9858	521,327.0142
2	6	0.88999644	6.502369605	689,625	650,236.9605	613,763.7949	578,708.58
3	6.25	0.833706493	6.751770257	689,625	675,177.0257	574,944.8402	562,899.4702
4	6.5	0.777323091	7.253534951	689,625	725,353.4951	536,061.4366	563,834.0208
5	7	0.712986179	9.023584719	689,625	902,358.4719	491,693.094	643,369.1194
		4.161879501				2,870,137	2,870,137

Table 39.10: Generic interest-rate swap.

⁵ Zero-coupon and forward rates are also related in another way. If we change the zero-coupon rate rs_n and the forward rate rf_i into their continuously-compounded equivalent rates, given by $\ln(1 + rs_n)$ and $\ln(1 + rf_i)$, we may obtain an expression for the continuously-compounded zero-coupon rate as being the simple average of the continuously-compounded forward rates, given by:

$$rs'_n = \frac{1}{t_n} \sum_{i=0}^{n-1} \frac{rf'_i}{F}.$$

CELL	C	D	E	F	G	H	I	J
21			10000000					
22								
		Zero-coupon	Discount		Fixed		PV fixed	PV floating
23	Period	rate %	factor	Forward rate %	payment	Floating payment	payment	payment
24	1	5.5	0.947867298	5.5	689625	"(F24*10000000)/100	"G24/1.055	"H24/(1.055)
25	2	6	0.88999644	"((E24/E25)-1)*100	689625	"(F25*10000000)/100	"G24/(1.06)^2	"H25/(1.06)^2
26	3	6.25	0.833706493	"((E25/E26)-1)*100	689625	"(F26*10000000)/100	"G24/(1.0625)^3	"H26/(1.0625^3)
27	4	6.5	0.777323091	"((E26/E27)-1)*100	689625	"(F27*10000000)/100	"G24/(1.065)^4	"H27/(1.065)^4
28	5	7	0.712986179	"((E27/E28)-1)*100	689625	"(F28*10000000)/100	"G24/(1.07)^5	"H28/(1.07)^5
			"SUM(E24:E28)				2870137	2870137

Table 39.11: Generic interest-rate swap (Excel formulae).

For reference the Microsoft Excel® formulae are shown in Table 39.11. It is not surprising that the net present value is zero, because the zero-coupon curve is used to derive the discount factors which are then used to derive the forward rates, which are used to value the swap. As with any financial instrument, the fair value is its breakeven price or hedge cost, and in this case the bank that is pricing the five-year swap shown in Table 39.10 could hedge the swap with a series of FRAs transacted at the forward rates shown. If the bank is paying fixed and receiving floating, value of the swap to it will rise if there is a rise in market rates, and fall if there is a fall in market rates. Conversely, if the bank was receiving fixed and paying floating, the swap value to it would fall if there was a rise in rates, and vice-versa.

This method can be used to price any interest-rate swap, even exotic ones. To illustrate, consider the three-year accruing swap shown in Table 39.12, which pays fixed against floating plus 25 basis points. The fixed rate must be adjusted accordingly and is shown to be 6.475%, calculated under the same zero-coupon rate environment as the previous example.

Period	Zero-coupon rate (%)	Discount factor	Forward rate (%)	Principal	Margin	Fixed payment	Floating payment	PV fixed payment	PV floating payment
1	5.50	0.94786730	5.500000	5,00,0000	Libor + 25	323,728	287,500	306,851	272,512
2	6.00	0.88999644	6.502370	8,00,0000	Libor + 25	517,965	540,192	460,987	480,769
3	6.25	0.83370649	6.751770	10,00,0000	Libor + 25	647,456	700,180	539,788	583,745
		<u>2.67157023</u>						<u>1,337,626</u>	<u>1,337,025</u>
	swap rate	0.062245606							
		6.2246	6.4746						

Table 39.12: Accruing interest-rate swap.

To calculate the present value of the fixed-rate leg of an interest-rate swap we use (39.24), while the present value of the floating leg is given by (39.25).

$$PV_{fixed} = r_n \sum_{i=1}^n M_i \frac{d_i}{B} df_i. \quad (39.24)$$

$$PV_{floating} = \sum_{i=1}^n rf_{i-1} M_i \frac{d_i}{B} df_i. \quad (39.25)$$

where

r_n	is the swap fixed rate
M_i	is the notional principal from time $i - 1$ to time i
rf_i	is the forward rate applicable to the period $i - 1$ to i
d_i	is the number of days in the interest period (time $i - 1$ to time i)
df_i	is the discount factor at time i
B	is the year day-count, either 360 or 365.

39.5.5 Valuation using final maturity discount factor

A short-cut to valuing the floating-leg payments of an interest-rate swap involves using the discount factor for the final maturity period. This is possible because, for the purposes of valuation, an exchange of principal at the beginning and end of the swap is conceptually the same as the floating-leg interest payments. This holds because, in an exchange of principal, the interest payments earned on investing the initial principal would be uncertain, as they are floating rate, while on maturity the original principal would be returned. The net result is a floating-rate level of receipts, exactly similar to the floating-leg payments in a swap. To value the principals then, we need only the final maturity discount rate.

To illustrate, consider Table 39.10, where the present value of both legs was found to be £2,870,137. The same result is obtained if we use the five-year discount factor, as shown below.

$$PV_{\text{floating}} = (10,000,000 \times 1) - (10,000,000 \times 0.71298618) = 2,870,137.$$

The first term is the principal multiplied by the discount factor 1; this is because the present value of an amount valued immediately is unchanged (or rather, it is multiplied by the immediate payment discount factor, which is 1.0000).

Therefore we may use the principal amount of a swap if we wish to value the swap. This is of course for valuation only, as there is no actual exchange of principal in a swap.

39.5.6 Summary of pricing principles

To recap, we have stated that the first step in swap pricing is to derive the spot rates up to the maturity required. These spot rates are then used to value all the cash flows in a swap; the floating-leg cash flows are estimated using the forward rates that have been derived from the spot rates. The fixed-rate for the swap may be obtained from the discount factors for all the spot rates from the current date to the final maturity period. The technique we have described is used for both pricing and valuing the swap. The market convention is to use the term *pricing* when trying to find the correct fixed rate for a new swap such that its net present value is zero. *Valuing* is the term used to describe the process of finding the net present value of an existing swap for which the fixed rate has already been set.

EXAMPLE 39.3 Mark-to-market valuation

The following received fixed, pay floating interest-rate swap is marked-to-market on 25 February 1999.

Nominal amount:	£10 million
Effective date:	22 June 1998
Maturity:	20 June 2003
Fixed rate:	6.45%
Basis:	Annual, act/365
Floating rate:	Libor
Basis:	Annual, act/365
Previous fixing:	6.05% from 22 June 1998 to 21 June 1999

The zero-coupon rate term structure is

22 June 1999:	5.055%
22 June 2000:	5.467%
22 June 2001:	5.912%

22 June 2002: 6.348%
 22 June 2003: 6.717%

The discount factors as at the valuation date are:

22 June 1999: 0.951882
 22 June 2000: 0.899015
 22 June 2001: 0.841714
 22 June 2002: 0.781777
 22 June 2003: 0.722490

The discount factors are used to calculate the forward rates, which are used to estimate the floating rate payments. The forward rates are:

22 June 2000: 5.881%
 22 June 2001: 6.808%
 22 June 2002: 7.667%
 22 June 2003: 8.206%

The cash flows for the swap are therefore:

<i>Date</i>	<i>Net cash flows</i>	<i>Present value</i>
22 June 1999	+40,000	+38,075
22 June 2000	+56,900	+51,154
22 June 2001	−35,800	−30,133
22 June 2002	−122,700	−95,924
22 June 2003	−175,600	−126,869

The net present value of the swap, and hence its mark-to-market valuation, is −163,697.

39.5.7 Summary of IR Swap

Let us summarise the chief characteristics of interest rate swaps. A plain vanilla swap has the following characteristics:

- one leg of the swap is fixed-rate interest, while the other will be floating-rate, usually linked to a standard index such as Libor;
- the fixed rate is fixed through the entire life of the swap;
- the floating rate is set in advance of each period (quarterly, semi-annually or annually) and paid in arrears;
- both legs have the same payment frequency;
- the maturity can be standard whole years up to 30 years, or set to match customer requirements;
- the notional principal remains constant during the life of the swap.

Of course to meet customer demand banks can set up swaps that have variations on any or all of the above standard points. Some of the more common variations are discussed in the next section.

39.6 Non-vanilla interest-rate swaps

The swap market is very flexible and instruments can be tailor-made to fit the requirements of individual customers. A wide variety of swap contracts have been traded in the market. Although the most common reference rate for the floating-leg of a swap is six-month Libor, for a semi-annual paying floating leg, other reference rates that have been used include three-month Libor, the prime rate (for dollar swaps), the one-month commercial paper rate, the Treasury bill rate and the municipal bond rate (again, for dollar swaps). The term of a swap need not be fixed; swaps may be *extendable* or *putable*. In an extendable swap, one of the parties has the right but not the obligation to extend the life of the swap beyond the fixed maturity date, while in a putable swap one party has the right to terminate the swap ahead of the specified maturity date. It is also possible to transact options on swaps, known as *swaptions*. A

swaption is the right to enter into a swap agreement at some point in the future, during the life of the option. Essentially a swaption is an option to exchange a fixed-rate bond cash flow for a floating-rate bond cash flow structure. As a floating-rate bond is valued on its principal value at the start of a swap, a swaption may be viewed as the value on a fixed-rate bond, with a strike price that is equal to the face value of the floating-rate bond.

Other swaps include:

- **Constant maturity swap.** This is a swap in which the parties exchange a Libor rate for a fixed swap rate. For example, the terms of the swap might state that six-month Libor is exchanged for the five-year swap rate on a semi-annual basis for the next five years, or for the five-year government bond rate. In the US market the second type of constant maturity swap is known as a *constant maturity Treasury swap*.
- **Accreting and amortising swaps.** In a plain vanilla swap the notional principal remains unchanged during the life of the swap. However it is possible to trade a swap where the notional principal varies during its life. An accreting (or *step-up*) swap is one in which the principal starts off at one level and then increases in amount over time. The opposite, an amortising swap, is one in which the notional reduces in size over time. An accreting swap would be useful where for instance, a funding liability that is being hedged increases over time. The amortising swap might be employed by a borrower hedging a bond issue that featured sinking fund payments, where a part of the notional amount outstanding is paid off at set points during the life of the bond. If the principal fluctuates in amount, for example increasing in one year and then reducing in another, the swap is known as a *roller-coaster swap*. Accreting and amortising swaps are illustrated in Figure 39.10.

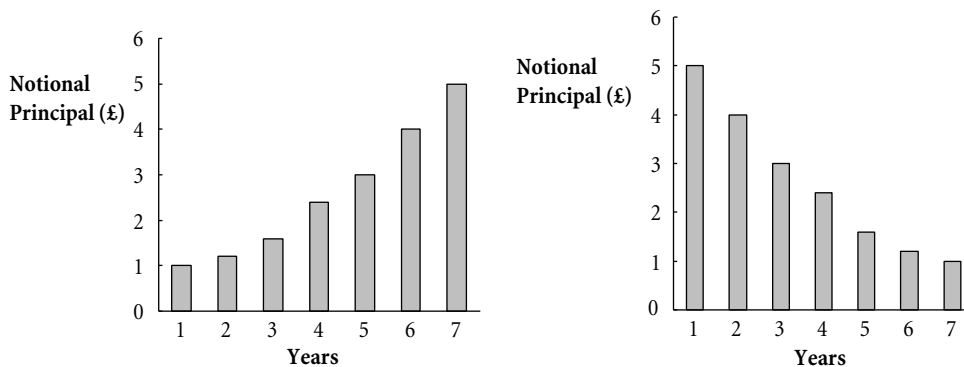


Figure 39.10: Accreting and amortising swaps.

Another application for an amortising swap is as a hedge for a loan that is itself an amortising one. Frequently this is combined with a forward-starting swap, to tie in with the cash flows payable on the loan. The pricing and valuation of an amortising swap is no different in principle to a vanilla interest-rate swap; a single swap rate is calculated using the relevant discount factors, and at this rate the net present value of the swap cash flows will equal zero at the start of the swap.

- **Zero-coupon swap.** A zero-coupon swap replaces the stream of fixed-rate payments with a single payment at the end of the swap's life, or more unusually, at the beginning. The floating-rate payments are made in the normal way. Such a swap exposes the floating-rate payer to some credit risk because it makes regular payments but does not receive any payment until the termination date of the swap.
- **Libor-in-arrears swap.** In this type of swap (also known as a *back-set swap*) the setting date is just before the end of the accrual period for the floating-rate setting and not just before the start. Such a swap would be attractive to a counterparty who had a different view on interest rates compared to the market consensus. For instance in a rising yield curve environment, forward rates will be higher than current market rates, and this will be reflected in the pricing of a swap. A Libor-in-arrears swap would be priced higher than a conventional swap. If the floating-rate payer believed that interest rates would in fact rise more slowly than forward rates (and the market) were suggesting, she may wish to enter into an arrears swap as opposed to a conventional swap.

- **Basis swap.** In a conventional swap one leg comprises fixed-rate payments and the other floating-rate payments. In a basis swap both legs are floating-rate, but linked to different money market indices. One leg is normally linked to Libor, while the other might be linked to the CD rate say, or the commercial paper rate. This type of swap would be used by a bank in the US that had made loans that paid at the prime rate, and financed its loans at Libor. A basis swap would eliminate the *basis risk* between the bank's income and expense cash flows. Other basis swaps have been traded where both legs are linked to Libor, but at different maturities; for instance one leg might be at three-month Libor and the other at six-month Libor. In such a swap the basis is different and so is the payment frequency: one leg pays out semi-annually while the other would be paying on a quarterly basis. Note that where the payment frequencies differ, there is a higher level of counterparty risk for one of the parties. For instance, if one party is paying out on a monthly basis but receiving semi-annual cash flows, it would have made five interest payments before receiving one in return.
- **Margin swap.** It is common to encounter swaps where there is a margin above or below Libor on the floating leg, as opposed to a floating leg of Libor flat. If a bank's borrowing is financed at Libor+25bps, it may wish to receive Libor+25bps in the swap so that its cash flows match exactly. The fixed rate quote for a swap must be adjusted correspondingly to allow for the margin on the floating side, so in our example if the fixed-rate quote is say, 6.00%, it would be adjusted to around 6.25%; differences in the margin quoted on the fixed leg might arise if the day-count convention or payment frequency were to differ between fixed and floating legs. Another reason why there may be a margin is if the credit quality of the counterparty demanded it, so that highly rated counterparties may pay slightly below Libor, for instance.
- **Off-market swap.** When a swap is transacted its fixed rate is quoted at the current market rate for that maturity. Where the fixed rate is different to the market rate, this is an off-market swap, and a compensating payment is made by one party to the other. An off-market rate may be used for particular hedging requirements for example, or when a bond issuer wishes to use the swap to hedge the bond as well as to cover the bond's issue costs.
- **Differential swap.** A differential swap is a basis swap but with one of the legs calculated in a different currency. Typically one leg is floating-rate, while the other is floating-rate but with the reference index rate for another currency, but denominated in the domestic currency. For example, a differential swap may have one party paying six-month sterling Libor, in sterling, on a notional principal of £10 million, and receiving euro-Libor, minus a margin, payable in sterling and on the same notional principal. Differential swaps are not very common and are the most difficult for a bank to hedge. The hedging is usually carried out using what is known as a *quanto* option.
- **Forward-start swap.** A forward-start swap is one where the *effective date* is not the usual one or two days after the trade date but a considerable time afterwards, for instance say six months after trade date. Such a swap might be entered into where one counterparty wanted to fix a hedge or cost of borrowing now, but for a point some time in the future. Typically this would be because the party considered that interest rates would rise or the cost of hedging would rise. The swap rate for a forward-starting swap is calculated in the same way as that for a vanilla swap.

SONIA SWAPS

Sterling overnight interest rate swaps (SONIA) are usually one-year swap contracts referenced to an average of the overnight sterling interest rate. This differs from a vanilla swap in the sterling market that might be indexed to quarterly or six-month Libor. SONIA is the average interest rate of all sterling interbank overnight deposits traded before 3.30pm that day. The average is weighted to reflect trading volume at each rate, and refers to trades conducted by seven members of the Wholesale Money Brokers Association. On maturity a SONIA interest-rate swap contract exchanges the net of the fixed-rate payment against the geometric average of the (floating) overnight interest rates that have been recorded during the life of the swap. SONIA swaps are used mainly by wholesale banking counterparties to hedge against overnight interest-rate liability, or to speculate on the direction of very short-dated interest rates. SONIA swaps have traded in maturities ranging from one week to two years, the interest rate horizon of most bank money market desks, although the majority of contracts are usually around one-year maturity.

When pricing a SONIA swap, the market making bank will calculate the fixed rate as a function of the average level of the overnight rate it expects during the life of the swap. The SONIA rate is closely correlated to the Bank of England's two-week repo and trades close to this rate. The spread is quite low because in the interbank market the credit risk on an overnight trade is relatively low. Due to the close relationship between the two rates, banks also use SONIA swaps to reflect their view on the direction of the Bank's repo rate.

39.7 Cancelling a swap

Where companies have entered into swap contracts to hedge interest-rate liabilities, the swap will be kept in place until expiry. However circumstances may change or a company may alter its view on interest rates, and so it may be necessary to terminate the swap. The most straightforward option is for the company to take out a second contract that negates the first. This allows the first swap to remain in place, but there may be residual cash flows unless the two swaps cancel each other out precisely. The terms for the second swap, being non-standard (and unlikely to be an exactly whole years to maturity, unless traded on the anniversary of the first), may also result in it being more expensive than a vanilla swap. As it is unlikely that the second swap will have the same rate, the two fixed legs will not net to zero. And if the second swap is not traded on an anniversary, payment dates will not match.

For these reasons the company may wish to cancel the swap entirely. To do this it will ask a swap market maker for a quotation on a cancellation fee. The bank will determine the cancellation fee by calculating the net present value of the remaining cash flows in the swap, using the relevant discount factor for each future cash flow. In practice just the fixed leg will be present valued, and then netted with Libor. The net present value of all the cash flows is the fair price for cancelling the swap. The valuation principles we established earlier will apply; that is, if the fixed rate payer is asking to cancel the swap when interest rates have fallen, he will pay the cancellation fee, and vice-versa if rates have risen.

39.8 Swaptions

39.8.1 Description

A bank or corporate may enter into an option on a swap, which is known as a *swaption*. The buyer of a swaption has the right but not the obligation to enter into an interest rate swap agreement during the life of the option. The terms of the swaption will specify whether the buyer is the fixed- or floating-rate payer; the seller of the option (the *writer*) becomes the counterparty to the swap if the option is exercised. In the market the convention is that if the buyer has the right to exercise the option as the fixed-rate payer, he has traded a *call swaption*, while if by exercising the buyer of the swaption becomes the floating-rate payer he has bought a *put swaption*. The writer of the swaption is the party to the other leg.

Swaptions are up to a point similar to forward start swaps, but the buyer has the *option* of whether or not to commence payments on the effective date. A bank may purchase a call swaption if it expects interest rates to rise, and will exercise the option if indeed rates do rise as the bank has expected. This is shown in the payout profiles in Figure 39.11.

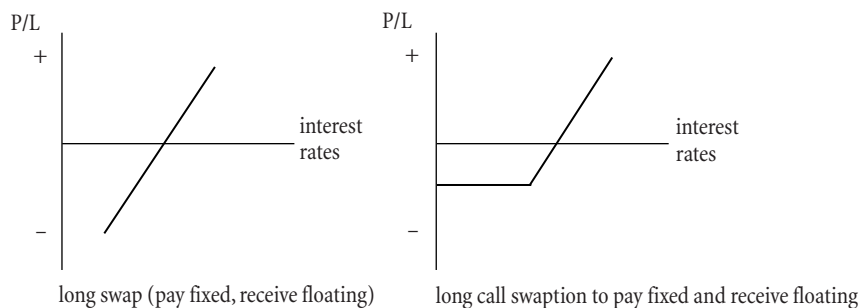


Figure 39.11: Interest-rate swap and swaption.

A company will use swaptions as part of an interest-rate hedge for a future exposure. For example, assume that a company will be entering into a five-year bank loan in three months' time. Interest on the loan is charged on a

floating-rate basis, but the company intend to swap this to a fixed-rate liability after they have entered into the loan. As an added hedge, the company may choose to purchase a swaption that gives it the right to receive Libor and pay a fixed rate, say 10%, for a five-year period beginning in three months' time. When the time comes for the company to take out a swap and exchange its interest-rate liability in three months' time (having entered into the loan), if the five-year swap rate is below 10%, the company will transact the swap in the normal way and the swaption will expire worthless. However if the five-year swap rate is above 10%, the company will instead exercise the swaption, giving it the right to enter into a five-year swap and paying a fixed rate of 10%. Essentially the company has taken out protection to ensure that it does not have to pay a fixed rate of more than 10%. Hence swaptions can be used to guarantee a maximum swap rate liability. They are similar to forward-starting swaps, but do not commit a party to enter into a swap on fixed terms. The swaption enables a company to hedge against unfavourable movements in interest rates but also to gain from favourable movements, although there is of course a cost associated with this, which is the premium paid for the swaption.

39.8.2 Valuation

Swaptions are typically priced using the Black–Scholes or Black option pricing models. These are used to value a European option on a swap, assuming that the appropriate swap rate at the expiry date of the option is lognormal. Consider a swaption with the following terms:

Strike swap rate	r_n
Maturity	n
Start date	t
Pay basis	F
Principal	M

If we imagine that the actual swap rate on the maturity of the swaption is r , the pay-off from the swaption is given by:

$$\frac{M}{F} \max(r - r_n, 0).$$

The cash flows will be received F times per year for n years, on dates $t_1, t_2, \dots, t_{Fn}$. The Black model for the price of an interest-rate option is shown at (39.26).

$$c = P(0, T)(f_0 N(d_1) - XN(d_2)) \quad (39.26)$$

where c is the price of the call option and

$P(t, T)$	is the price at time t of a zero-coupon bond maturing at time T
f	is the forward price of the underlying asset with maturity T
f_0	is the forward price at time zero
X	is the strike price of the option
σ	is the volatility of f

$$\text{and where } d_1 = \frac{\ln(f_0/X) + \sigma^2 T/2}{\sigma\sqrt{T}} \text{ and } d_2 = \frac{\ln(f_0/X) - \sigma^2 T/2}{\sigma\sqrt{T}} = d_1 - \sigma\sqrt{T}.$$

Using equation (39.26), the value of the cash flow received at time t_i is given by (39.27):

$$\frac{M}{F} P(0, t_i)(f_0 N(d_1) - r_n N(d_2)) \quad (39.27)$$

where f_0 is the forward swap rate and r_i is the continuously compounded zero-coupon interest rate for an instrument with a maturity of t_i . Using (39.27) then the total value of the swaption is given by (39.28):

$$PV = \sum_{i=1}^{Fn} \frac{M}{F} P(0, t_i)(f_0 N(d_1) - r_n N(d_2)). \quad (39.28)$$

39.9 Cross-currency swaps

So far we have discussed swap contracts where the interest payments are both in the same currency. A *cross-currency* swap is similar to an interest-rate swap, except that the currencies of the two legs are different. Like interest-rate swaps, the legs are usually fixed- and floating-rate, although again it is common to come across both fixed-rate or both floating-rate legs in a currency swap. On maturity of the swap there is an exchange of principals, and usually (but not always) there is an exchange of principals at the start of the swap. Where currencies are exchanged at the start of the swap, at the prevailing spot exchange rate for the two currencies, the exact amounts are exchanged back on maturity. During the time of the swap, the parties make interest payments in the currency that they have *received* when principals are exchanged. It may seem that exchanging the same amount on maturity gives rise to some sort of currency risk, in fact it is this feature that removes any element of currency risk from the swap transaction.

Currency swaps are widely used in association with bond issues by borrowers who seek to tap opportunities in different markets but have no requirement for that market's currency. By means of a currency swap, a company can raise funds in virtually any market and swap the proceeds into the currency that it requires. Often the underwriting bank that is responsible for the bond issue will also arrange for the currency swap to be transacted. In a currency swap therefore, the exchange of principal means that the value of the principal amounts must be accounted for, and is dependent on the prevailing spot exchange rate between the two currencies.

39.9.1 Valuation of currency swaps

The same principles we established for the pricing and valuation of interest-rate swaps may be applied to currency swaps. A generic currency swap with fixed-rate payment legs would be valued at the fair value swap rate for each currency, which would give a net present value of zero. The cash flows are illustrated in Figure 39.12. This shows that the two swap rates in a fixed-fixed currency swap would be identical to the same-maturity swap rate for each currency interest-rate swap. So the swap rates for a fixed-fixed five-year sterling/dollar currency swap would be the five-year sterling swap rate and the five-year dollar swap rate.

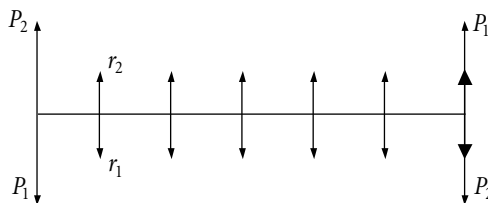


Figure 39.12: Fixed-fixed rate currency swap.

A floating-floating currency swap may be valued in the same way, and for valuation purposes the floating-leg payments are replaced with an exchange of principals, as we observed for the floating leg of an interest-rate swap. A fixed-floating currency swap is therefore valued at the fixed-rate swap rate for that currency for the fixed leg, and at Libor or the relevant reference rate for the floating leg.

EXAMPLE 39.4 Bond issue and associated cross-currency swap

We illustrate currency swaps with an example from the market. A subsidiary of a US bank that invests in projects in the United States issues paper in markets around the world, to exploit market conditions. The company's funding requirement is in US dollars, however it is active in issuing bonds in various currencies, according to where the most favourable conditions can be obtained and to meet investor demand. When an issue of debt is made in a currency other than dollars, the proceeds must be swapped into dollars for use in the USA, and interest payable on the swapped (dollar) proceeds. To facilitate this the issuer will enter into a currency swap. One of the bank's recent issues was a Swiss franc step-up bond, part of an overall Euro-MTN programme. The details of the bond are summarised below.

Issue date	March 1998
Maturity	March 2003
Size	CHF 15 million

Coupon	2.40% to 25/3/99
	2.80% to 25/3/00
	3.80% to 25/3/01
	4.80% to 25/3/02

The bond was also callable on the each anniversary from March 1999 onwards, and in fact was called by the issuer at the earliest opportunity. The issuing bank entered into a currency swap that resulted in the exchange of principals and the Swiss franc interest payments to be made by the swap counterparty; in return it paid US dollar three-month Libor during the life of the swap. At the prevailing spot rate on the effective date, CHF 15 million was exchanged for \$10.304 million; these exact same amounts would be exchanged back on the maturity of the swap. When the issue was called the swap was cancelled and the swap counterparty paid a cancellation fee. The interest payment dates on the fixed leg of the swap matched the coupon dates of the bond exactly, as shown above. The floating leg of the swap paid USD Libor on a quarterly basis, as required by the bond issuer.

The structure is shown in Figure 39.13. A currency swap structure enables a bank or corporate to borrow money in virtually any currency in which a liquid swap market exists, and swap this into a currency that is required. In our example the US bank was able to issue a bond that was attractive to investors. The swap mechanism also hedged the interest rate exposure on the Swiss franc note. The liability remaining for the issuer was quarterly floating rate interest on US dollars as part of the swap transaction.

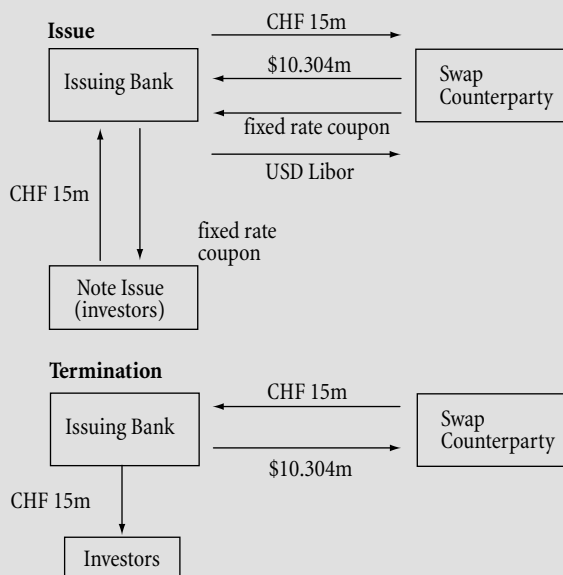


Figure 39.13: Bond issue with currency swap structure.

EXAMPLE 39.5 Cross-currency asset swap

ABC Bank plc purchases a five-year sterling Eurobond with an annual coupon of 7.0% at a price of £98.00. The investment is then transformed into an Australian dollar asset via a currency swap, at a swap rate of 7.50% for sterling against 8.0% for AUD. The exchange rate is 2.20. The net AUD yield is calculated as follows.

The cash flows from the Eurobond are:

Year 1	7.00
Year 2	7.00
Year 3	7.00
Year 4	7.00
Year 5	107.00

The net present value of these cash flows if we discount at the swap rate of 7.50% is £97.98. At the current exchange rate this is equal to AUD 215.56. The amount invested per £100 nominal was £98, which is equal to AUD 215.60. The currency swap is set up so that there is AUD cash flow to replace the coupon, and a final cash flow of AUD 215.60 on maturity, with the net present value of these cash flows, at a discount rate of 8.00%, equal to AUD 215.56. Using Microsoft Excel or a programmable calculator this is seen to be AUD 17.24 each year, with a final cash flow of AUD 232.84 on maturity. The yield on the asset is therefore the yield returned on an investment of AUD 215.60, which is returned on maturity, and an income stream of AUD 17.24, which is the swap rate of 8.00%.

39.10 Credit risk

The rate quoted for swaps in the interbank market assumes that the counterparty to the transaction has a lending line with the swap bank, so the swap rate therefore reflects the credit risk associated with interbank quality counterparty. This credit risk is reflected in the spread between the swap rate and the equivalent-maturity government bond, although the spread also reflects other considerations such as liquidity and supply and demand. The credit risk of a swap is separate from its interest-rate risk or market risk, and arises from the possibility of the counterparty to the swap defaulting on its obligations. If the present value of the swap at the time of default is net positive, then a bank is at risk at loss of this amount. While market risk can be hedged, it is more problematic to hedge credit risk. The common measures taken include limits on lending lines and diversification across counterparty sectors, as well as a form of credit value-at-risk to monitor credit exposures.

A bank therefore is at risk of loss due to counterparty default for all its swap transactions. If at the time of default the net present value of the swap is positive, this amount is potentially at risk and will probably be written off. If the value of the swap is negative at the time of default, in theory this amount is a potential gain to the bank, although in practice the counterparty's administrators will try to recover the value for their client. In this case then, there is no net gain or loss to the swap bank. The credit risk management department of a bank will therefore often assess the ongoing credit quality of counterparties with whom the swap transactions are currently positive in value.

The credit risk of a currency swap is considerably higher than that of an interest-rate swap, because a currency swap involves an exchange of principal.

Appendices

APPENDIX 39.1 Standard swap documentation: hypothetical examples

In this appendix we use hypothetical examples to illustrate standard swap documentation. The first hypothetical example is for a one-year US dollar interest-rate swap paying a fixed rate of 5.32% against one-month USD Libor. The second example is for a basis swap, with each counterparty exchanging floating-rate payments. The swap bank in both cases is ABC Bank plc, while the counterparty is a corporate called "BritCo plc".

First example

AMENDED CONFIRMATION

7003-ISDA92-1CCY-FX/FL

Date: May 19, 1999
 To: BritCo plc
 Attention: Mr AN Other
 Fax Number: 0208 134 2345
 From: ABC Bank plc
 Fax Number: 0207 311 5267
 Deal No: 0399134025

The definitions and provisions contained in the 1991 ISDA Definitions (as supplemented by the 1998 Supplement) (as published by the International Swaps and Derivatives Association, Inc.) are incorporated into this Confirmation. In the event of any inconsistency between those definitions and provisions and this Confirmation, this Confirmation will govern. References herein to a "Transaction" shall be deemed to be references to a "Swap Transaction" for the purposes of the 1991 ISDA Definitions. Each party shall deliver to the other, at the

time of its execution of this Confirmation, evidence of the specimen signature and incumbency of each party who is entering this Confirmation on the party's behalf, unless such evidence has previously been supplied in connection with this Agreement and remains true and in effect. This Confirmation supplements, forms a part of, and is subject to, the ISDA Master Agreement dated as of 11 August 1998 between you and us ("Agreement").

All provisions contained in the Agreement shall govern this Confirmation except as expressly modified below.

Terms of the particular Transaction to which this Confirmation relates are as follows:

Notional Amount:	USD 30,000,000.00
Trade Date:	14 May 1999
Effective Date:	19 May 1999
Termination Date:	9 May 2000, subject to adjustment in accordance with the Modified Following Business Day Convention

Fixed Amounts:

Fixed Rate Payer:	ABC Bank plc
Fixed Rate Payer Payment Dates:	May 19 2000, subject to adjustment in accordance with the MODIFIED FOLLOWING Business Day Convention.
Fixed Rate:	5.32000%

Fixed Rate Day:

Count Fraction:	ACTUAL/360
Business Days:	New York

Floating Amounts:

Floating Rate Payer:	BritCo plc
Floating Rate Payer Payment Dates:	The 18th day of each month, commencing June 18, 1999, with the final payment on the termination date, accruing from and including 18 April, 2000, up to but excluding 19 May, 2000, subject to adjustment in accordance with the MODIFIED FOLLOWING Business Day Convention.

Floating Rate for initial Calculation Period:	To be determined
Floating Rate Option:	USD-LIBOR-BBA
Designated Maturity:	1 Month
Spread:	Minus 0.03000% per annum

Floating Rate Day Count Fraction:	ACTUAL/360
Reset Dates:	The first day of each Calculation Period.
Compounding:	Not applicable
Business Days:	NEW YORK, LONDON

Fee:	ABC Bank plc will pay BritCo plc USD 3,000.00 value 19 May 1999.

Calculation Agent: ABC Bank plc or as stated in the Agreement.

Details:

Payments to ABC Bank plc:
ABC Bank plc
41111451

Payments to Counterparty:
XYZ Bank plc
02837129
FAO: Dominic Taylor

3. Offices

The Office of the Floating Rate Payer for this Transaction is London EC2.

Your transaction with us in schedule 5 instruments are at present covered by the S.43 exemption from the Financial Services Act and therefore are subject to the London Code of Conduct of the Financial Services Authority.

BritCo plc hereby agrees (a) to check this Confirmation (Deal No: 0399134025) carefully and immediately upon receipt so that errors or discrepancies can be promptly identified and rectified and (b) to confirm that the foregoing correctly sets forth the terms of the agreement between ABC Bank plc and BritCo plc with respect to the particular Transaction to which this Confirmation relates, by manually signing this Confirmation and providing the other information requested herein and immediately returning an executed copy to Facsimile No: 0207 311 5267.

Please contact us immediately should the particulars of this Confirmation not be in accordance with your understanding (Tel No: 0208 123 3456).

For and on behalf of ABC Bank plc.

Name:

Title:

19 May 1999

Accepted and confirmed as of the date first written:

BritCo plc

Name:

Name:

Title:

Title:

Your reference:

Second example

ABC Bank plc

2nd Revision as of July 7, 1999

Date:

March 26, 1999

To:

BritCo plc

Our reference:

319971-jfa

Re:

Basis Swap Transaction – This Confirmation supersedes and replaces all prior communication between the parties hereto with respect to the Transaction described below.

Ladies and Gentlemen:

The purpose of this letter agreement is to set forth the terms and conditions of the Transaction entered into between ABC Bank plc (“ABC”) and BritCo plc (“Counterparty”) on the Trade date specified below (the “Transaction”). This letter constitutes a “Confirmation” as referred to in the Agreement specified below.

The definitions and provisions contained in the 1991 ISDA Definitions (as supplemented by the 1998 Supplement, the “Definitions”) as published by the International Swaps and Derivatives Association, Inc. are incorporated by reference herein. In the event of any inconsistency between the Definitions and this Confirmation, this Confirmation will govern.

For the purpose of this Confirmation, references in the Definitions or the Agreement to a “Swap Transaction” shall be deemed to be references to this Transaction.

1. This Confirmation supplements, forms part of, and is subject to, the ISDA Master Agreement dated as of December 20, 1996, (as the same may be amended or supplemented from time to time, the “Agreement”), between ABC Bank plc and Counterparty. All provisions contained in the Agreement shall govern this Confirmation except as expressly modified below.
2. The terms of the particular Transaction to which this Confirmation relates are as follows:

Notional Amount:	USD 75,000,000.00
Trade Dates:	March 25, 1999
Effective Date:	April 6, 1999
Termination Date:	April 6, 2000, subject to adjustment in accordance with the Modified Following Business Day Convention.
Reset Dates:	The first Business Day in each Calculation Period or Compounding Period, if Compounding is applicable.

Floating Amounts I

Floating Rate Payer:	ABC
Floating Rate Payer Payment Dates:	July 6, 1999, October 6, 1999, January 6, 2000 and April 6, 2000, subject to adjustment in accordance with the Modified Following Business Day Convention.
Floating Rate for initial Calculation Period:	To be determined
Floating Rate Option:	USD-Prime-H.15
Spread:	Minus 2.77%
Floating Rate Day Count Fraction:	Action/360
Reset Dates:	Each New York Business Day for each Calculation Period.

Rate Cut-off Dates **Two New York Business Days Prior to each Floating Rate Payer Payment Date**

Method of Averaging:	Weighted Average
Compounding:	Inapplicable

Floating Amounts II:

Floating Rate Payer:	Counterparty

Floating Rate Payer
 Payment Dates: July 6, 1999, October 6, 1999, January 6, 2000 and April 6, 2000, subject to adjustment in accordance with the Modified Following Business Day Convention.

Floating Rate for initial
 Calculation Period: To be determined

Floating Rate Option: USD-LIBOR-BBA
 Designated Maturity: Three months
 Spread: Minus 0.045%

Floating Rate
 Day Count Fraction: Actual/360

Business Days: London and New York

3. Accounts Details

Account Details for ABC Bank plc

02837129

Ref: Interest Rate Swaps

Accounts Details for Counterparty:

Account No: 41111450 with XYZ Bank New York

4. Offices:

The Office for ABC Bank plc for this transaction is New York.

5. Calculation Agent: ABC Bank plc

6. Representations

Each party will be deemed to represent to the other party on the date on which it enters into this Transaction that (absent a written agreement between the parties that expressly imposes affirmative obligation to the contrary for this Transaction):-

(i) Non Reliance. It is acting for its own account, and it has made its own independent decision to enter into this Transaction and as to whether this Transaction is appropriate or proper for it based upon its own judgement and upon advice from such advisers as it has deemed necessary. It is not relying on any communication (written or oral) of the other party as investment advice or as a recommendation to enter into this Transaction; it being understood that information and explanations related to the terms and conditions of this Transaction shall not be considered investment advice or a recommendation to enter into this Transaction. No communication (written or oral) received from the other party shall be deemed to be an assurance or guarantee as to the expected results of this Transaction.

(ii) Assessment and Understanding. It is capable of assessing the merits of and understanding (on its own behalf or through independent professional advice), and understands and accepts, the terms, conditions and risks of this Transaction. It is also capable of assuming, and assumes, the risks of this Transaction.

(iii) Status of Parties. The other party is not acting as a fiduciary for, an adviser to it in respect of this Transaction.

7. Please confirm that the foregoing correctly sets forth the terms of our agreement by having an authorised officer sign this Confirmation and return it by facsimile to:

Attention: Dominic Taylor, Swap documentation
 Telephone No: 0208 123 3456
 Facsimile No: 0207 311 5267

This message will be the only form of Confirmation despatched by us. If you wish to exchange hard copy form of this Confirmation, please contact us.

Yours sincerely,

ABC Bank plc

APPENDIX 39.2 Deriving the discount function

The discount function for a given maturity period, given that period's spot rate, is given by (39.29) and (39.30) below.

$$df_n = \frac{1}{(1 + rs_n t_n)}. \quad (39.29)$$

$$df_n = \frac{1}{(1 + rs_n)^{t_n}}. \quad (39.30)$$

We can obtain the expression for the discount function using a Taylor expansion of the above equations and the continuously compounded discount function expression $e^{-rs \cdot t}$, which will give us similar results in each case.

$$\begin{aligned} \frac{1}{(1 + rs \cdot t)} &\approx 1 - rs \cdot t + (rs \cdot t)^2 - (rs \cdot t)^3 + (rs \cdot t)^4 - \dots \\ &\approx 1 - \log(1 + rs \cdot t) + \frac{(\log(1 + rs \cdot t))^2}{2!} - \frac{(\log(1 + rs \cdot t))^3}{3!} + \frac{(\log(1 + rs \cdot t))^4}{4!} - \dots \\ \frac{1}{(1 + rs)^t} &\approx 1 - t \log(1 + rs) + \frac{(t \log(1 + rs))^2}{2!} - \frac{(t \log(1 + rs))^3}{3!} + \frac{(t \log(1 + rs))^4}{4!} - \dots \\ e^{-rs \cdot t} &\approx 1 - rs \cdot t + \frac{(rs \cdot t)^2}{2!} - \frac{(rs \cdot t)^3}{3!} + \frac{(rs \cdot t)^4}{4!} - \dots \end{aligned}$$

where

rs is the spot interest rate
 t is the term to maturity

(this assumes that the time now is zero and that the expiry date is t).

It is possible to obtain identical expressions in all three cases, by substituting $r = \log(1 + rs \cdot t)/t$ into the first expression and $r = \log(1 + rs)$ into the penultimate expression, which replaces the discrete spot rate rs with the continuously compounded interest rate r .

For readers unfamiliar with the Taylor expansion, this is summarised in the following appendix.

APPENDIX 39.3 Taylor's expansion

The Taylor expansion is an important function that is used in many areas on finance. For instance, the formulae for modified duration and convexity were derived using a Taylor expansion and it was also used to develop the Black-Scholes option pricing model. More recently it has been used to approximate value-at-risk for option portfolios in the form of the delta-gamma VaR calculation. The function was originally developed to perform approximations of complex calculations, and appears as the following:

$$\Delta G = \frac{dG}{dx} \Delta x + \frac{1}{2} \frac{d^2 G}{dx^2} \Delta x^2 + \frac{1}{6} \frac{d^3 G}{dx^3} \Delta x^3 + \dots \quad (39.31)$$

The first term represents the first derivative and is equivalent to the duration of a bond, while the second term, the second derivative, is equivalent to convexity for a bond. The term is very small and its accuracy is usually not required in financial engineering.

Therefore we can see how to obtain the duration value for a bond using the Taylor expansion; as the price of a bond is the sum of the present value of its cash flows, the first differential of the price/yield equation is the absolute change in the price of the bond. If this is divided by the bond's price, we obtain the relative price change or duration. The second differential gives the relative change, and divided by the price again gives us the convexity. Note that the second term in the Taylor expansion has the coefficient $\frac{1}{2}$. This is why, when applying the convexity adjustment to a bond's modified duration, we multiply the convexity value by 0.50. For option traders, the first and second derivatives of the Black–Scholes formula are the delta and gamma of the option.

Selected bibliography and references

- Bicksler, J., Chen, A., "An Economic Analysis of Interest Rate Swaps", *Journal of Finance* 41, 3, 1986, pp. 645–655.
- Brotherton-Ratcliffe, R., Iben, B., "Yield Curve Applications of Swap Products", in Schwartz, R., Smith, C. (eds.) *Advanced Strategies in Financial Risk Management*, New York Institute of Finance, 1993.
- Decovny, S., *Swaps*, FT Pitman, 1998.
- Geroulanos, P., *An Introduction to Swaps*, Securities Institute (Services) Publishing, 1999.
- Galitz, L., *Financial Engineering*, FT Pitman, 1995, Chapter 9.
- Kolb, R., *Futures, Options and Swaps*, Blackwell, 2000.
- International Swaps and Derivatives Association, *Code of Standard Working, Assumptions and Provisions for Swaps*, New York, 1991.
- Layard-Liesching, R., "Swap Fever", *Euromoney*, Supplement, January 1986, pp. 108–113.
- Li, A., Raghavan, V.R., "LIBOR-In-Arrears Swaps", *Journal of Derivatives* 3, Spring 1996, pp. 44–48.
- Marshall, J., Kapner, K., *Understanding Swap Finance*, South-Western Publishing, 1990.
- Turnbull, S., "Swaps: A Zero Sum Game", *Financial Management* 16, Spring 1987, pp. 15–21.

Questions and exercises

- How can interest-rate swaps be used to hedge interest-rate exposure?
- A Eurocurrency swaps dealer enters into a one-year quarterly-paying swap as a payer of fixed rate against three-month Libor. The first swap receipt is set at 3.30%. How can the dealer hedge against a fall in market rates using FRAs?
- An interest-rate swap with notional principal of £100 million has 16 months remaining to maturity. The swap pays fixed at 7.125% on a semi-annual basis, while the average of the one-year and two-year swap rates is currently 6.50%. The six-month Libor fixing two months ago was 6.375%. What is the value of the swap to the floating-rate payer? What is the value to the fixed-rate payer?
- Strata plc is a highly rated corporate with both fixed-rate and floating-rate liabilities. It wishes to replace some of its fixed-rate funding by floating-rate funding, provided it can do so at a cost of no more than Libor + 12.5 bps. Among Strata's fixed-rate liabilities is a £100 million Eurobond issue with five years to maturity, paying a coupon of 7.625%. Roxmore plc is a financial firm with a large amount of floating rate debt. It believes interest rates are likely to rise and would like to lock in some of its borrowing at current interest rates. Roxmore has an outstanding £100 million floating-rate note issue with five years to maturity paying a coupon of Libor + 30 bps, and believes that it could issue a Eurobond of equivalent size and maturity for a coupon of 8%. Five-year swap rates are quoted at 7.60%–7.50% by a bank swaps desk, with whom the companies will deal. This means the bank will pay fixed at 7.50%, and receive fixed at 7.60%.
 - What interest rate swaps would Strata and Roxmore have to do to meet their objectives?
 - Construct a diagram illustrating the swap, and calculate the net borrowing cost for each company.
- A corporate issues £100 million of a ten-year 7.00% bond, paying annual coupons, and swaps the proceeds into floating-rate euro. The spot sterling/euro exchange rate is 1.61, and the current swap rate is 5.85% fixed-rate sterling, against six-month euro-Libor. The swap matches the cash flows from the sterling bond, so that

the liability of the corporate is floating-rate euro, calculated on the amount of euro borrowed. The same amount is repaid on maturity. What is the net borrowing-cost on euro, given a discount rate of 4.15% for euro and 7.00% for sterling? Use a diagram to illustrate the cash flows of the swap structure.

6. Explain how a bank may use interest-rate swaps to hedge its assets and liabilities when its assets are fixed-rate and its liabilities are floating-rate.
7. A bank issues a five-year US dollar bond with a coupon of 6.50%, and raises net proceeds of \$99.65 per cent from the issue. The interest-rate liability is swapped into floating-rate funding. The swap is constructed so that the par amount is used as the notional, paying six-month Libor, for the five-year term. The five-year swap rate is 7.00% against floating, both annual/360 basis. Taking the swap rate as the discount rate, what is the net interest cost payable by the bank relative to Libor?
8. A small regional building society that has not previously transacted in swaps offers five-year mortgages at a fixed rate of 6.99%. How can it use a swap contract to hedge the exposure in its mortgage book?
9. Consider the following zero-coupon term structure:

1 year:	5.75%
2 years:	6.125%
3 years:	6.000%
4 years:	5.875%
5 years:	5.75%
6 years:	5.625%
7 years:	5.500%

Calculate the swap rate for a six-year interest-rate swap. What is the valuation of the swap if the six-year market rate subsequently rises by 50 basis points.

10. A corporate has an outstanding five-year bullet loan of £3 million, on which interest is being paid at 7.90% fixed rate. The current five-year swap rate is quoted to the company as “80-90 over gilts”; the five-year benchmark is trading at 8.00%. If the company believes that market interest rates are about to fall, what approximate gain can it make if it enters into a swap in order to change its interest-rate liability?
11. What risk is reflected in the spread of the swap rate over the equivalent-maturity government bond yield?
12. Consider the following cash and OBS rates:

3-month depo:	5.6875%
3v6 FRA:	5.75%
6v9 FRA:	5.625%
9v12 FRA:	5.875%
12v15 FRA:	6.000%
15v18 FRA:	6.125%
18v21 FRA:	6.25%
21v24 FRA:	6.50%

Assume that all the periods are for 91 days except the cash rate, which is for 92 days. Calculate the zero-coupon rates, on an annualised bond-equivalent basis for all three-month periods up to two-year maturity. What would be the price quote for a one-year swap?

13. An industrial company wishes to borrow sterling at a fixed rate of interest. An import-export company wishes to borrow euros at a fixed rate of interest. Assume that the amounts required by both companies, at the current exchange rate, are roughly equal. The companies have been quoted the following borrowing rates:

	Sterling	euro
Industrial	10%	6.50%
Import-export	12%	7.00%

Construct a swap that will provide a gain to an intermediary swap bank of a minimum of 30 basis points, but will provide both companies with lending rates that is attractive to them.

14. What does the term *warehousing* of a swap mean?
15. A junior trader on a swaps desk has the zero-coupon term structure for sterling and dollars out to five years. Should he be able to price a five-year sterling/dollar currency swap, where there is an exchange of principals at the start and at maturity?

40

Swaps II

In this chapter we review some uses of swaps as a hedging tool, and also how to hedge a swap book. This is illustrated with case studies. There is also a discussion of the convergence basis risk in the cash market, and a review of the convexity bias when analysing the impact of using FRAs, futures and swaps for hedging purposes. It is important to be aware of the convexity bias when pricing long-dated swaps against futures contracts.

40.1 Using swaps

Swaps are part of the over-the-counter (OTC) market and so they can be tailored to suit the particular requirements of the user. It is common for swaps to be structured so that they match particular payment dates, payment frequencies and Libor margins, which may characterise the underlying exposure of the customer. As the market in interest-rate swaps is so large, liquid and competitive, banks are willing to quote rates and structure swaps for virtually all customers. It may be difficult for smaller customers to obtain competitive piece quotes; in the sterling market for example, the large clearing banks are reluctant to quote rates for swaps of below £10 million or £5 million notional value, however customers such as small building societies can readily obtain quotes for swaps as small as £1 million nominal from other institutions.¹

Swap applications can be viewed as being one of two main types, asset-linked swaps and liability-linked swaps. Asset-linked swaps are created when the swap is linked to an asset such as a bond in order to change the characteristics of the income stream for investors. Liability-linked swaps are traded when borrowers of funds wish to change the pattern of their cash flows. Of course, just as with repo transactions, the designation of a swap in such terms depends on whose point of view one is looking at the swap. An asset-linked swap hedge is a liability-linked hedge for the counterparty, except in the case of swap market making banks who make two-way quotes in the instruments.

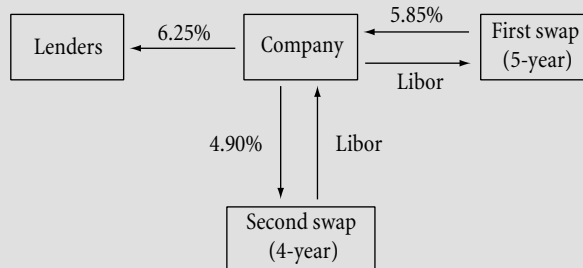
A straightforward application of an interest-rate swap is when a borrower wishes to convert a floating-rate liability into a fixed-rate one, usually in order to remove the exposure to upward moves in interest rates. For instance a company may wish to fix its financing costs. Let us assume a company currently borrowing money at a floating rate, say six month Libor +100 basis points fears that interest rates may rise in the remaining three years of its loan. It enters into a three-year semi-annual interest rate swap with a bank, as the fixed-rate payer, paying say 6.75 per cent against receiving six-month Libor. This fixes the company's borrowing costs for three years at 7.75% (7.99 per cent effective annual rate). This is shown in Figure 40.1.



Figure 40.1: Changing liability from floating- to fixed-rate.

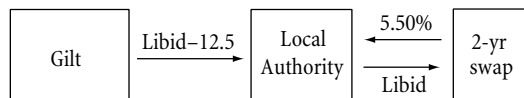
It is less common for borrowers to enter into swap contracts in order to switch from fixed-rate to floating-rate liability, but this does occur. An example would be where a bank or corporate had issued bonds of say, ten year maturity at a coupon of 8%, only for interest rates to fall shortly after issue of the debt. The company may wish to switch to floating-rate liability by receiving fixed in a swap, reducing the level of its fixed-rate borrowing. In such a scenario however the borrower is now exposed to rising rates, and so switching liabilities in this fashion is to imply that the company does not expect the yield curve to rise substantially over the medium term. If rates did then move up again however the borrower could switch liabilities again by entering into a second swap.

¹ The author has direct experience of dealing in swaps with just such customers!

EXAMPLE 40.1 Liability-linked swap, fixed- to floating- to fixed-rate exposure**Figure 40.2**

A corporate borrows for five years at a rate of $6\frac{1}{4}\%$ and shortly after enters into a swap paying floating-rate, so that its net borrowing cost is $\text{Libor} + 40\text{bps}$. After one year swap rates have fallen such that the company is quoted four-year swap rates as 4.90-84%. The company decides to switch back into fixed-rate liability in order to take advantage of the lower interest rate environment. It enters into a second swap paying fixed at 4.90% and receiving Libor. The net borrowing cost is now 5.30%. The arrangement is illustrated in Figure 40.2. The company has saved 95 basis points on its original borrowing cost, which is the difference between the two swap rates.

Asset-linked swap structures might be required when for example, investors require a fixed interest security when floating-rate assets are available. Borrowers often issue FRNs, the holders of which may prefer to switch the income stream into fixed coupons. As an example, consider a local authority pension fund holding two-year floating-rate gilts. This is an asset of the highest quality, paying Libid minus 12.5bps. The pension fund wishes to swap the cash flows to create a fixed interest asset. It obtains a quote for a tailor-made swap where the floating leg pays Libid, the quote being 5.55-50%. By entering into this swap the pension fund has in place a structure that pays a fixed coupon of 5.375%. This is shown in Figure 40.3.

**Figure 40.3: Transforming floating-rate asset to fixed-rate.**

Another application is the liability-linked floating-floating currency swap. Banks and large corporates are able to access markets in many different countries, across a range of currencies. They may therefore obtain funding in any currency where there is potential relative value (that is, potentially cheaper funding) and swap the proceeds of a foreign currency loan into their required currency. This is illustrated in Example 40.2

EXAMPLE 40.2 Floating-floating currency swap

A multinational company based in the United Kingdom is planning to raise ongoing funding and may issue a five-year sterling bond at $8\frac{1}{4}\%$, or a floating-rate US dollar bond that pays Libor plus 15 basis points. The five-year sterling swap quote is 8.10-8.05 and the same-maturity dollar swap quote is 7.70-7.65. The sterling/dollar currency basis swap is quoted at six-month sterling $\text{Libor}-7/\text{Libor}-2$ against six-month dollar Libor. The company may choose to issue the sterling fixed-coupon bond, and then swap to floating-rate liability if it feels the five-year market rate scenario is downward, or it may issue the floating-rate dollar bond and transform its liability into sterling via the currency swap.

Using the sterling bond, the company will have a net borrowing cost of Libor plus 20 basis points, which is the coupon on the bond minus the swap rate (which it receives). However it can make a greater saving by issuing debt in dollars and then transforming its liabilities using the currency swap. How? The dollar-denominated floating-rate bond pays $\text{Libor} + 15$; the company issues this and simultaneously enters into a currency

swap, receiving dollar Libor +15 and paying sterling Libor +13. This arrangement cancels out the dollar cash flows and has a net sterling borrowing cost that is lower than the plain vanilla arrangement by seven basis points. The alternatives are illustrated in Figure 40.4.

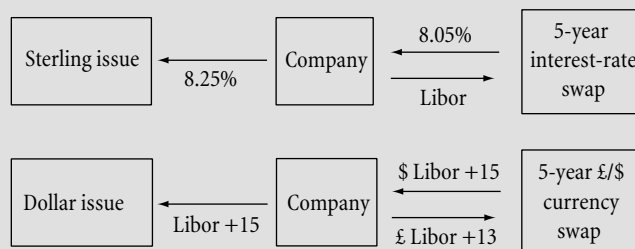


Figure 40.4: Cross-currency swap funding.

40.2 Hedging an interest-rate swap

A position in a swap may be hedged with another swap that has offsetting characteristics, or with another instrument designed to offset the effects of moves in market interest rates on the value of the swap. The hedge may be a cash instrument such as a bond, or an off-balance sheet instrument such as a FRA, strip of futures contracts or an interest-rate option. We review the different possibilities in this section.

40.2.1 Hedging using bonds and swaps

A swap book can be hedged using other swaps, futures contracts or bonds. A bond book can similarly be hedged using futures, swaps and other bonds. In some of the larger banks an integrated risk management technique is used, whereby the bank's overall net risk exposure, arising from its entire position in all instruments including swaps, FRAs and futures, is managed as a whole. This results in cheaper hedging. However it is still common for individual books to be hedged separately. If an integrated method is used, the bank must identify all the points along the term structure where it has an interest rate exposure, and then calculate the present value of a basis point (PVBp) at each of these points. That is, it must calculate the change in value of its positions along each maturity bucket that would result from a 1 basis point change in that maturity's interest rate. The bank's daily risk report will also list the bank's aggregate risk along the entire yield curve.

Let us now consider some of the points involved in hedging a swap book. When hedging a position we will want to put on another position of the same *basis point value* and in the opposite direction. Assume we have only one position on the book, a five-year sterling swap of £5 million notional, in which we pay fixed- and receive floating-rate interest. This is conceptually similar to borrowing money. As the maturity of the swap is longer than that of the longest interest-rate futures contract, which is three years, we decide to hedge the position using a UK government bond, or gilt. As we have "borrowed" funds, the hedge action must be to "lend" funds. Therefore we need to buy a gilt to hedge the swap. In summary:

<i>Swap position</i>	<i>Hedge</i>
Pay fixed	Receive fixed (buy bond)
Receive fixed	Pay fixed (sell bond)

In normal situations we will probably wish to put on the hedge using the five-year benchmark gilt. We need to establish the hedge ratio to enable us to decide how much nominal of the gilt to buy, which is done using each instrument's *basis point value* (BPV). The BPV of a swap is another term for PVBp, the change in value of a swap resulting from a 1 bp move in interest rates. To establish the nominal amount of the gilt required, the basic calculation is:

$$(\text{BPV Swap} / \text{BPV Gilt}) \times 10\,000.$$

In our example we would need to calculate the BPV for the five-year swap. One way to do this is to view the swap as a strip of futures contracts, whose BPV is known with certainty. The short sterling future traded on LIFFE is a standardised contract with a BPV or "tick value" of £12.50 (in fact short sterling futures move in minimum units of

0.005, so that a tick value is actually £6.25. However this is exactly half of a basis point). As our swap is a sterling swap it will pay semi-annually, while short sterling futures mature every quarter. The calculations are:

Convert swap nominal to futures:	$5\text{m} \times 2$	=	£10m
Futures periods:	$4 \times 5 \text{ years}$	=	20 contracts
Less “fixing” of first period of swap: -2 (if a quarterly paying swap, is -1)		=	18 contracts
BPV:	$18 \times 12.5 \times 10$	=	£2250.

The BPV of the gilt is a simple function of its modified duration (see Chapter 7) and is straightforward to calculate, although in practice will be obtained direct from a spreadsheet model or Bloomberg®.

The illustration we have used is a “rough-and-ready” hedge. The remainder of this section deals with more systematic hedging procedures.

A bond trading book will often be hedged using swaps, although it is more usual for desks to use futures. The same principles will apply as we have mentioned above. Swaps are more commonly used to hedge a new issue of bonds or as asset swaps to change an interest rate basis from fixed to floating or vice versa. Two examples of swaps traded simultaneously with a purchase or issue of bonds are shown in Example 40.3.

EXAMPLE 40.3

A subsidiary of a leading US bank conducts investment business in the United States and wishes both its income and interest payments to be on a floating rate basis.

Purchase of Deutsche Bank 6.7% 2006 USD bond, swapped with swap counterparty, resulting in floating rate income.

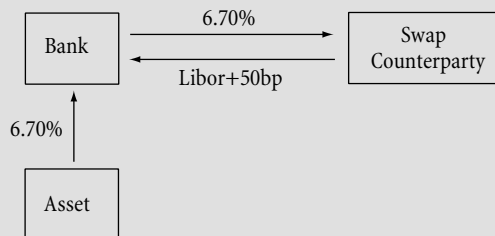


Figure 40.5

The company also raises finance in the debt markets, and wishes its interest payments to be on a floating-rate basis. It issues a one-year MTN, £15m, paying 5.3%. The bank simultaneously enters into a swap with terms on the same basis, coupon (for the fixed-rate leg) and coupon dates as the MTN, with the result that its interest rate liability is floating-rate.

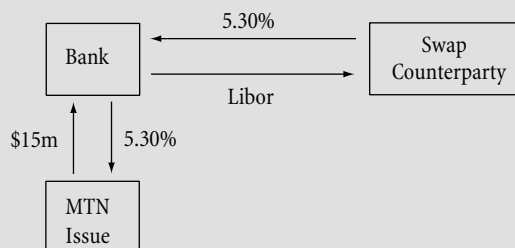


Figure 40.6

40.2.2 Hedging a swap transaction using futures contracts

In this section we use a worked example to illustrate how a swaps trader hedges a short-dated swap with derivative instruments. Consider a Eurocurrency swaps trader who enters into a one-year swap transaction, paying fixed against three-month Libor. The first swap receipt is set at 3.27%, which is the current three-month Libor. As a receiver of floating-rate Libor, the dealer is exposed to a fall in interest rates over the period of the swap, and wishes to hedge this exposure. The terms of the trade are:

Date	17 March
Swap principal	EUR 100 million
Fixed rate	3.50%
Floating rate	Three-month Libor
First fixing	3.26953%

The current prices for interest-rate futures contracts are:

June futures price	96.67 (implied rate 3.33%)
September futures price	96.55 (implied rate 3.45%)
December futures price	96.37 (implied rate 3.63%)

To hedge the exposure, the trader buys a strip of June, September and December three-month Euribor futures contracts. The one-year strip rate is derived from calculating the returns on a three-month deposit reinvested along the futures implied forward rate curve for one year. This is shown below, with the strip rate given as rf , where

$$1 + rf \times \frac{d}{360} = \left(1 + \text{fixing rate} \times \frac{d_1}{360}\right) \times \left(1 + rf_{fut1} \times \frac{d_2}{360}\right) \times \left(1 + rf_{fut2} \times \frac{d_3}{360}\right) \times \left(1 + rf_{fut3} \times \frac{d_4}{360}\right).$$

Substituting the values into the above expression, we obtain:

$$1 + rf \times \frac{364}{360} = \left(1 + 0.0326953 \times \frac{91}{360}\right) \times \left(1 + 0.0333 \times \frac{91}{360}\right) \times \left(1 + 0.0345 \times \frac{91}{360}\right) \times \left(1 + 0.0363 \times \frac{91}{360}\right).$$

This is solved to give rf equal to 3.46%. If we assume that the price of the futures contracts closed at the levels shown, the outcome of the hedge is shown in Table 40.1

June contract	
Date	16 June
Settlement price	96.35
Three-month Libor	3.6500%
June contract P/L	−32 ticks, or 96.35 − 96.67
Effective rate	0.0365 + (−0.0032) or 3.3300%
September contract	
Date	15 September
Settlement price	96.32
Three-month Libor	3.6875%
September contract P/L	−23 ticks, or 96.32 − 96.55
Effective rate	0.036875 + (−0.0023) or 3.4485%
December contract	
Date	15 December
Settlement price	96.19
Three-month Libor	3.8125%
December contract P/L	−18 ticks, or 96.19 − 96.37
Effective rate	0.038125 + (−0.0018) or 3.6325%

Table 40.1: Swap hedge using interest-rate futures contracts.

From Table 40.1 we see that as the trader has locked into the swap rate implied by each of the futures contracts at the time the hedge was put on, they achieved the anticipated 3.46% strip rate over the one-year hedge period. This example also illustrates how a trader can hedge a swap using a strip of futures, and also how the futures rates can be used to price the swap. Notice however that we did not calculate the actual number of futures contracts to put on against the 100 million swap position; this is a theoretical approach only. In practice this method will not provide a hedge of sufficient accuracy, due to convexity bias between futures prices and swap rates in the markets. To overcome this a slightly modified hedge calculation is employed.

To determine the correct number of futures contracts with which to construct the strip, the trader must carry out the following:

- calculate all the cash flows and their present values, using implied futures forward rates;
- calculate the basis point value (BPV) of the present value of the cash flows for a one tick change in each of the futures contracts;
- calculate the futures hedge ratio for each period in the strip using the BPV of the cash flow and the BPV of the future.

This procedure is shown in Tables 40.2 to 40.4, with the expected strip rate calculated as 6.82%.

Date	Days	Contract	Price	Implied rate %	Fixed cash flow	Floating cash flow	PV fixed cash flow	PV Floating cash flow
17 March	91	Stub		3.69		932,750		924,130
16 June	91	Jun	96.67	3.33		841,750		827,010
15 September	91	Sep	96.55	3.45		872,083		849,404
15 December	91	Dec	96.37	3.63		917,583		885,595
16 March	364							
			Fixed strip rate	3.57	(3,612,061)	(3,486,139)		3,486,139

Table 40.2

The change in the present value of each cash flow is calculated for a one basis point fall in each futures price. A new strip rate is calculated to each futures maturity date and is used to discount each cash flow to its new present value. In this way it is possible to observe how a one basis point change in any of the futures contracts will impact the overall position of the swap.

Contract	June fut less 1 tick; PV of cash flows	Change	Sep fut less 1 tick; PV of cash flows	Change	Dec fut less 1 tick; PV of cash flows	Change
Stub	924,130	0	924,130	0	924,130	0
Jun	829,473	2,463	827,010	0	827,010	0
Sep	849,383	(21)	851,845	2,441	849,404	0
Dec	885,573	(22)	885,573	(22)	888,013	2,417
Strip rate (3,486,052)		87	(3,486,052)	87	(3,486,052)	87
		<u>2,507</u>		<u>2,506</u>		<u>2,504</u>

Table 40.3

We are now in a position to calculate the number of contracts to put against the swap, based on the BPV of the two instruments. The hedge ratio is given by

$$\text{Hedge} = \frac{BPV_{\text{cash flow}}}{BPV_{\text{fut}}}$$

so for the June contract it is 2507/25 or 100.28, for the September contract it is 2506/25 or 100.24 and for the December contract it is 2504/25 or 100.16. In fact it is only possible to transact futures contracts in round lots, so the strip put on by the trader is comprised of 100 lots each of June, September and December contracts.

The outcome of the hedge, assuming the closing prices in the previous illustration, is shown in Table 40.4.

June contract	
Date	16 June
Settlement price	96.35
Three-month Libor	3.6500%
June contract P/L	−32 ticks, or $96.35 - 96.67$ $32 \times 100 \times \text{EUR}25 = -\text{EUR}80000$
Effective rate	$0.0365 + (-0.0032)$ or 3.3300%
September contract	
Date	15 September
Settlement price	96.32
Three-month Libor	3.6875%
September contract P/L	−23 ticks, or $96.32 - 96.55$ $23 \times 100 \times \text{EUR}25 = -\text{EUR}57500$
Effective rate	$0.036875 + (-0.0023)$ or 3.4485%
December contract	
Date	15 December
Settlement price	96.19
Three-month Libor	3.8125%
December contract P/L	−18 ticks, or $96.19 - 96.37$ $18 \times 100 \times \text{eur}25 = -\text{EUR}45000$
Effective rate	$0.038125 + (-0.0018)$ or 3.6325%

Table 40.4

The example above illustrates how the trader was able, by locking into the swap rate implied by the futures contracts at the time the hedge was initiated, to achieve the anticipated 6.82% strip rate during the one-year swap period.

While corporates tend to use FRAs and swaps to hedge interest-rate exposure, thus creating a large demand for them amongst banks, banks themselves tend to use futures contracts to hedge both FRAs and swaps. The advantages of using futures contracts are that:

- the bid-offer spread in the futures market is usually very close, and invariably tighter than in the swap market;
- in practice there is no credit risk associated with trading futures contracts, because the clearing house assumes the credit risk (as it is the central counterparty to everyone that transacts exchange-traded futures), while swap contracts carry associated credit risk;
- in the system of depositing *variation margin* at the clearing house to cover losses in daily trading, where there is a positive variation margin as a result of profitable trading, this is paid “up front”, thus offering the additional benefit of allowing reinvestment of realised profits;
- there is no regulatory capital requirement for futures positions, so from a capital point of view they are cheaper than swaps.

For these reasons it is more common to observe swaps hedged using exchange-traded futures contracts.

40.3 The convexity bias

It is common for swaps traders to calculate swaps prices from the prices of interest-rate futures contracts. The price of a futures contract is the implied three-month forward rate on expiry date of the contract. However there are

differences in the way that futures contracts and swaps behave that means swap rates derived from futures prices, or a zero-coupon yield curve constructed from futures contracts and swap rates, will not be completely accurate. The main difference between the two instruments is that futures contracts move in minimum increments of a tick, which applies to the number of months covered by the contract. Swaps on the other hand have less discrete rate movements and accrue interest on a daily basis. These differences lead to swap rates exhibiting lower convexity than futures prices, which can lead to inaccuracies in pricing. In this section we review the impact of the *convexity bias* and how it should be accounted for in futures and swaps analysis.

40.3.1 The convexity bias in futures contracts

In Chapter 35 we referred to the fact that in practice the rates implied by futures contracts are assumed to be equal to actual forward rates, but that for longer-dated futures contracts, differences in the way that forwards and exchange-traded futures are handled will result in futures rates not being equal to forward rates. In this section we review how the difference between rates implied by futures contracts and forward rates in practice must be taken into account when pricing long-dated forward instruments.

The convexity bias is the term used to explain the observation that the price of futures contracts such as the Eurodollar contract (traded on the Chicago Mercantile Exchange) should in fact be lower than their fair value, that is, the three-month interest rates implied by the contract should be higher than the three-month forward rates to which they are tied. The bias becomes more prominent for longer-dated contracts, but is negligible for contracts that have an expiry date of under two years. The presence of this bias however will influence the fair value for a swap; for example a five-year swap rate should be lower than the yield implied by the first five years of the Eurodollar contract. Where swap rates do not take the convexity bias into account, there would be an advantage to being short of the swap, against a hedge with futures contracts. Let us consider the main issues.

Interest-rate swaps and futures contracts such as Eurodollar futures are both priced under the same type of forward interest rate environment. The two instruments are fundamentally different in one key respect however; with an interest-rate swap, cash flows are exchanged (in fact a net cash payment) only once for each leg of a swap, and then only in arrears. With an exchange-traded futures contract though, profits and losses are settled every day. The difference in the way profit and loss are settled affects the values of swap and futures relative to each other. The resulting bias acts in favour of a short swap (paying floating, receiving fixed) against a long futures contract. It is therefore important to measure the extent of this bias when calculating swap rates.

40.3.2 Interest-rate swaps and futures

From our reading of the previous chapter, we are familiar with the structure of an interest-rate swap as being an exchange of fixed- versus floating-rate interest payments on a specified notional principal. The floating-leg may be on any required basis but is usually reset on a semi-annual or quarterly basis. A five-year swap that paid fixed and received quarterly floating payments would require the floating-rate to be reset 20 times during its life, once when the swap was transacted and then every three months thereafter. We may therefore view the swap as being conceptually the same as the sum of 20 separate segments, with the value of each segment being dependent on the fixed rate of the swap and on the market's expectation of what the floating rate will be on the quarterly reset date.

The basis point value (BPV) of a swap is given by (40.1) and is the amount by which the swap changes value for every basis point that the closing day's same-maturity swap rate fixes above or below the swap fixed rate. A change in value given by a change in market rates is not realised on the day however,² but is realised on the maturity of the swap.

$$BPV = 0.0001 \times \frac{d}{B} \times M \quad (40.1)$$

where

d	is the number of days in the floating period
B	is the year day base (360 or 365)
M	is the notional principal of the swap.

² The mark-to-market is unrealised P/L.

The basis point value of a futures contract is fixed however, irrespective of the maturity of the contract, and is the “tick value” of the contract itself. For example the Eurodollar futures contract has a BPV of \$25, while the short-sterling contract on LIFFE has a tick value of £6.25. In a combined position therefore, consisting of an interest-rate swap hedged with futures contracts, both instruments are sensitive to the same change in interest rates. If we wish to compare the effects of a change in the value of both instruments, the most straightforward way to do this is to use the present value for both price changes. As futures contracts are settled on a daily basis, the present value of its basis point value is unchanged, so \$25 in the case of the Eurodollar contract. The present value of the BPV of an interest-rate swap can be determined using a set of futures rates from a strip of similar maturity. How could we obtain the discount rate used?

For our illustration assume an hypothetical \$100 million five-year swap, with quarterly floating coupons, with a BPV of \$2 500, but with a forward start date five years away. We require a means of calculating the present value of the BPV. Ordinarily we would require the five-year discount rate. However the simplest way to calculate the present value is to first calculate what \$1 would grow to if we invested it at the successive rates given by the futures strip, out to five years (that is, invest for the first 91 days at the front month futures rate, then the next 91 days for the following contract’s futures rate, and so on). Let us say that this resulted in a future value of \$1.55 at the end of the five-year period. This would give us a present value of: $\$1/\$1.55 = 0.645161$. That is, \$0.645161 is the present value of \$1 to be received in five years’ time. We may use this value to calculate the present value of our hypothetical five-year swap. We said that this had a BPV of \$2 500, so the present value of this sum is obtained as follows:

$$\$2\,500 \times 0.645161 = \$1\,612.90.$$

Given these values, we can determine the number of Eurodollar futures contracts that would be required to hedge our hypothetical swap, which is $1\,612.90/25 = 64.516$ or 65 contracts. That is, 65 Eurodollar futures would have the same exposure to a change in the five-year three-month forward rate as would the \$100 million five-year swap. If a bank is short the swap, that is receiving fixed-rate and paying floating-rate, it could hedge the interest-rate exposure arising from a rise in the forward rate by selling 65 Eurodollar futures. Therefore we may set the hedge calculation for any leg of a swap whose floating rate is three-month Libor as (40.2):

$$\text{Hedge ratio} = \frac{M \times \left(0.0001 \times \frac{d}{B}\right) \times P_{\text{zero-coupon}}}{PV(BPV_{\text{fut}})} \quad (40.2)$$

where $P_{\text{zero-coupon}}$ is the price today of a bond that pays \$1 on the same day when the swap settlement is paid. In the hypothetical example we discussed, the swap settlement is five years away, and the price of such a bond was given as 0.645161. The denominator of (40.2) requires the present value of the basis point value of the futures contract, however this is for a cash flow that is received on the same day so it is unchanged from the basis point value itself.

The basis point value of the future and the swap is a measure of their interest-rate risk. A swap contract has another type of interest-rate risk however. Since any gain or loss on a swap contract is unrealised, and realised only at the end of the term, a swap may have unrealised asset value. In particular the present value of a short position in the hypothetical swap we described may be calculated using (40.3):

$$PV_{\text{swap}} = M \times \left((r_{\text{swap}} - rf) \times \frac{d}{B}\right) \times P_{\text{zero-coupon}} \quad (40.3)$$

where

r_{swap} is the swap fixed rate
 rf is the current market same-maturity forward rate.

Equation (40.3) tells us that the unrealised asset value of a swap depends both on the difference between the swap fixed rate and the market swap rate, as well as on the present value of one dollar (or pound) to be received on the swap’s maturity date. In effect (40.3) states that there are in fact two sources of interest-rate risk in a forward-starting swap. The first is the uncertainty surrounding the forward rate rf . The other concerns uncertainty about the zero-coupon bond price, which is a reflection of the term structure of forward rates from today to the swap cash settlement date. If the forward rate is below the fixed rate, for example the person who is receiving fixed and paying floating has an asset whose value is reduced by a general increase in interest rates. To protect against interest-rate risk then, the person hedging the swap must not only offset the exposure to changes in the forward rate, but also the

exposure to changes in the term structure of the zero-coupon yield curve as well. This may be done by buying or selling an appropriate quantity of zero-coupon bonds whose maturity matches that of the swap.

This is the key difference between an interest-rate swap and a futures contract. With futures such as Eurodollar futures the only source of market risk is the forward or futures rate. When the futures rate changes, the holder of a futures contract experiences a profit or loss, and collects the profit or pays out the loss the next day. The holder of a swap however faces two types of risk, arising from a change in the forward rate and a change in the term rate. What is the impact of this?

The effect on the change in value of the swap contract will differ from interest rate moves in different direction, because of the convexity effect, while the futures will experience the same change whatever direction rates move in. Put another way, and alluding to our hypothetical example again, if the prices of all futures contracts from the front month out the five-year contract moved up or down by 10 basis points, the effect on the P/L of our 65 contracts would be the same. However, the swap would behave differently. If there was a parallel shift upwards of 10 basis points, the present value of the loss on the five-year swap would be lower than the present value of the profit of there was a parallel shift downwards of the same magnitude. The precise amounts will depend on the maturity and BPV of the swap; however for the purposes of our discussion, the hedge position of futures contracts makes a net gain if there is rise in forward rates and less if forward rates fall. Remember this is from the point of view of someone who is short the swap. The opposite will apply for someone long the swap.

This relationship is illustrated graphically in Figure 40.7.

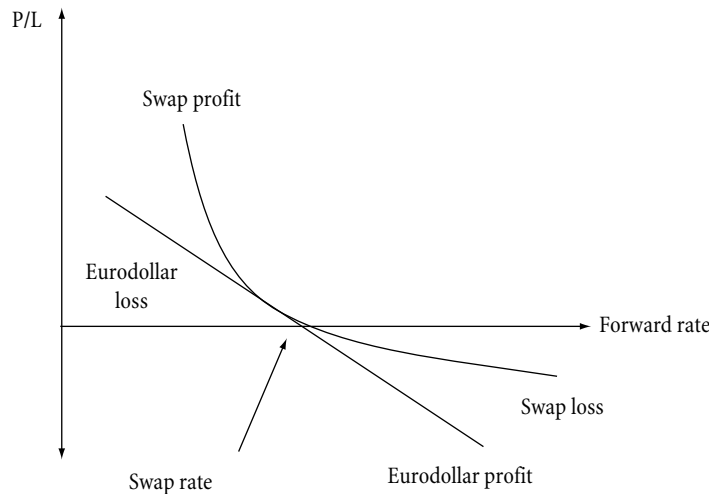


Figure 40.7: The convexity difference between swaps and futures.

As a result of the difference in convexities of the two instruments, a short swap hedged with a short position in Eurodollar futures benefits from changes in the level of interest rates. The difference in the performance of a swap and a futures contract is a function of:

- the magnitude of the change in the forward rate;
- the magnitude of the change in the term rate (or zero-coupon bond price);
- the correlation between the two rates.

As we might expect, there is a very close positive correlation between forward interest rates and zero-coupon rates, verging on unity. With virtually no exceptions, increases in the forward rate are accompanied by increases in the term rate, and vice-versa.

40.3.3 Calculating the convexity bias

Banks often calculate the value of the convexity bias by empirical analysis of their futures and swaps P/L history. There is a more systematic approach that may be used however, and this involves estimating the bias using three parameters, the volatility of the forward rate, the volatility of the corresponding term rate and the correlation

between the two rates – the three factors we noted above. The extent of the bias can be measured using (40.4), which calculates the *drift* in the spread between the futures rates and the forward rates. This drift is the amount of bias that must be accounted for when say, pricing swaps or arranging a hedge.

$$CVbias = \sigma_{rf} \times \sigma_{rs} \times \rho_{rf/rs} \quad (40.4)$$

where

$CVbias$ is the amount of drift
 σ_{rf} is the standard deviation of changes in the forward rate
 σ_{rs} is the standard deviation of changes in the zero-coupon rate
 $\rho_{rf/rs}$ is the correlation coefficient between changes in the two rates.

The drift is the number of ticks that the swap rate spread has to fall during any given period to compensate for the convexity bias. The derivation of this expression is given at Appendix 40.1.

The value of the convexity bias is a function of the convexity of the forward swap that is associated with the futures contract. This depends in turn on the price sensitivity of the zero-coupon bond that corresponds to the swap maturity. Since the price of a zero-coupon bond with say, five-years to maturity is more interest-rate sensitive than the price of a bond with less than five years to maturity, (measured by the modified duration of the bond), the value of the bias is higher the longer the maturity of the futures contract. This is why the convexity bias effect is greatest for long-dated futures and swaps positions. The highlighting of the convexity bias is a relatively recent phenomenon for this reason; as the longest-dated futures contracts have extended out to 10 years, and very long-dated swaps are now fairly common, the impact of the convexity bias has been more pronounced.

40.3.4 Impact of the convexity bias

It is common practice for swaps traders to price dollar swaps against Eurodollar futures contracts, and this is not surprising given the liquidity and transparency of the contract, as well as its narrow bid-offer spread. However the convexity bias that results from the inherent differences between the two instruments makes it important for traders to price in the effect of the bias for long-dated swaps. By adjusting the prices of futures contracts by the amount of the convexity bias before using them to calculate the implied swap rate, we will obtain a more accurate reflection of the forward rates implied by the futures prices. The extent of the bias is indicated in Table 40.5, which was calculated as the convexity bias to be applied for Eurodollar contracts used to price forward swaps during 1997.

Swap term (years)										
Years forward	1	2	3	4	5	6	7	8	9	10
Spot	0.23	1.08	2.32	3.83	5.58	7.57	9.77	12.189	14.79	17.58
1	1.99	3.49	5.23	7.21	9.44	11.88	14.55	17.42	20.48	
2	5.11	7.05	9.25	11.71	14.39	17.32	20.46	23.78		
3	9.16	11.58	14.31	17.24	20.43	23.85	27.47			
4	14.22	17.22	20.42	23.91	27.61	31.52				
5	20.48	23.94	27.71	31.73	35.95					
6	27.71	31.81	36.14	40.69						
7	36.28	40.91	45.77							
8	45.93	51.13								
9	56.76									

Table 40.5: Convexity bias in forward swaps (basis points).

Source: Acknowledged.

Any calculation of convexity bias is based on an assumption about the volatility of market interest rates. This assumption must be advertised before the values may be used. From Table 40.5 we see that the bias effect is greatest for long-dated long forward swaps. The effect is least for “spot swaps” or swaps that have an immediate effective date. Therefore at the time that these rates were effective, if we were pricing a five-year swap with a one-year forward

start date, we would adjust the one-year futures contract by 9.44 basis points before using it to calculate the swap rate.

The other effect of the convexity bias concerns the marking-to-market of the swap book. The common practice is for the mark to be based on the closing prices of the futures contracts, and for dollar swaps this may go out to long-dated swaps. However the convexity bias means that Eurodollar futures prices produce forward rates that are higher than the forward rates that should ideally be used to value swaps. Therefore some banks make an allowance for the value of the bias, which enables a more accurate picture of the value of the swap book to be drawn. However such an adjustment is based on assumptions of interest-rate volatilities, which is one reason why many banks do not make the convexity adjustment.

40.4 Swaps netting

Swap trades are bilateral contracts, and a bank that is a market maker in swaps will have counterparty credit risk and capital requirements for each swap it transacts, on an individual basis. Individual banks may treat swaps on a swap-by-swap basis or have an agreement with a particular counterparty that allows for netting of all the swaps between them. This requires the ISDA netting agreement to be signed and in place between them. Overall netting, where all of a bank's swaps are netted out and it has just one counterparty, a central clearing house, was introduced in selected currency swaps in August 1999 by the London Clearing House (LCH), which is the clearing house for the LIFFE futures exchange. The system developed by LCH is known as *SwapClear* and is similar in concept to their RepoClear netting system for repo transactions. Both concepts are similar to the well-established clearing principle used in futures exchanges, whereby all parties that transact business on a futures exchange are deemed to have dealt with one counterparty, the clearing house, and the counterparty risk assumed by the clearing house is covered by the deposit of initial margin, and variation margin on a daily basis, by each party that conducts futures business.

A similar concept applies to SwapClear. Instead of individual counterparty agreements with a large number of banks and other counterparties on the other side of a swaps book, a bank that uses SwapClear will have the LCH as its sole counterparty in swaps (for all counterparties that are also users of SwapClear). The benefits to the bank are:

- there will be a reduction in the credit exposure of the bank, as its sole exposure is to the clearing house;
- there will be a reduction in the bank's capital adequacy requirements, as lending lines are reduced;
- there will be a reduction in administrative costs, as swaps administration need concern dealings with the clearing house only.

The first two benefits could contribute potentially significant cost savings for swap banks. With only a single counterparty for its swaps business, the Tier One capital requirement for a bank will fall, and the credit risk exposure will fall dramatically. There is no formal credit rating of LCH although informally it is viewed as being triple-A rated. The credit exposure of the clearing house is managed through the *default fund*, a fund to which member banks contribute that would be used in the event that a member bank defaulted on its obligations. The initial size of this fund was £350 million.

The potential benefits to banks from entering into swaps netting should result in most banks moving over to a netting system such as SwapClear. The reduction in bid-offer spreads in swaps, and the effects as the market becomes more commoditised, will also impact on the growth of netting.

40.4.1 Electronic swaps trading

Towards the end of the 1990s more and more banks began to trade swaps over automated electronic trading platforms. The various systems in use in the market were only declared "live" in recent years, so it is perhaps premature to judge their effectiveness in terms of liquidity. However electronic trading platforms have already had an impact on business practices in other financial markets, so it is reasonable to assume that this will be the case with electronic swaps trading.³ One of the main electronic trading systems is SwapsWire, which has been developed primarily as an inter-dealer platform. There are 10 banks currently involved in its development and inception, and at the time of writing it was set up for trading US dollar and euro interest-rate swaps. Four main systems are summarised in Table 40.6.

³ We refer to checking rates and dealing all at the touch of a mouse button. This compares to the normal approach, which usually entails checking prices on Bloomberg, Telerate, or Reuters and actually conducting the bargain over the telephone.

System	Start of trading	Partner banks	Instruments traded
SwapWire	November 2000	BNP Paribas, Citigroup, Deutsche Bank, JP Morgan Chase, Morgan Stanley Dean Witter, CSFB, Goldman Sachs, Merrill Lynch, UBS Warburg	Interest-rate swaps
Blackbird	September 1999	Privately-owned	Interest-rate derivatives, currency swaps
CFQWeb.com	June 2000	ABN Amro, AIG International, Bank of America, Dresdner Kleinwort Benson, ING Barings, Standard Chartered	Foreign exchange, interest-rate and money market instruments
Treasury Connect	May 2000	Williams Capital Group, eVentures International, AIG Financial Products, Enron	Interest-rate and currency swaps, caps, floors, collars, swaptions

Table 40.6: Interest-rate swap electronic trading platforms. Source: RISK, September 2000.

Appendices

APPENDIX 40.1 Calculating the convexity bias

A method that is used to calculate the convexity bias in Eurodollar rates, the rate of *drift* relative to forward rates involves calculating the expected gain when a forward swap is hedged with Eurodollar futures, and assuming specified volatility levels for the change in forward rates and zero-coupon rates.

Swap value

The net present value of a forward swap that receives fixed and pays quarterly floating is given by (40.5):

$$PV_{\text{swap}} = M \times (r_{\text{swap}} - rf) \times \frac{d}{B} \times P_{\text{zero-coupon}}. \quad (40.5)$$

where r_{swap} and rf are expressed in basis points. Multiplying the expression by \$1 million and dividing by 90, and then rearranging gives us an expression setting the new present value in terms of the \$25 tick value of a Eurodollar futures contract, shown as (40.6):

$$PV_{\text{swap}} = \left(\frac{M}{\$1\text{m}} \right) \times (r'_{\text{swap}} - rf') \times \frac{d}{90} \times \$25 \times P_{\text{zero-coupon}} \quad (40.6)$$

At the time the forward swap is transacted the difference between the two rates is zero as the net present value of the swap is zero. As interest rates change the rate rf and the value of $P_{\text{zero-coupon}}$ both change, which affects the present value of the swap.

Swap P/L and hedge ratio

For a change of Δrf in the forward rate and $\Delta P_{\text{zero-coupon}}$ in the price of the zero-coupon bond, the mark-to-market profit on a forward swap is given by (40.7):

$$\Delta PV = -\left(\frac{M}{\$1\text{m}} \right) \times \left(\frac{d}{90} \right) \times \$25 \times \Delta rf \times (P_{\text{zero-coupon}} + \Delta P_{\text{zero-coupon}}). \quad (40.7)$$

The change in the value of one Eurodollar futures contract is equal to the tick value multiplied by the change in the forward rate, therefore the number of contracts required to hedge against \$1 million notional of the swap is given by (40.8):

$$\text{HedgeRatio} = -\left(\frac{M}{\$1\text{m}} \right) \times \left(\frac{d}{90} \right) \times P_{\text{zero-coupon}}. \quad (40.8)$$

The negative sign in (40.8) indicates that the hedge against the short swap position (receive fixed, pay floating) is a short sale of Eurodollar futures. Given this hedge ratio the profit on the short Eurodollar futures position is expressed as (40.9):

$$\text{profit or loss} = \left(\frac{M}{\$1m} \right) \times \left(\frac{d}{90} \right) \times P_{\text{zero-coupon}} \times (\Delta rf + \text{drift}) \times \$25 \quad (40.9)$$

where *drift* is the systematic change in the Eurodollar futures rate relative to the forward rate required to compensate for the convexity difference between the swap contract and the futures contract.

To preserve no-arbitrage pricing, the expected profit from such a hedge must be zero. That is, the expected profit on the swap must offset precisely the expected profit on the Eurodollar position. This principle enables us to calculate the drift. The expression $(M/\$1m) \times (d/90) \times \$25$ is used to calculate the profit for both swaps and futures, therefore it cancels out. Thus we may set the following expression, which recognises that there is no profit advantage (in theory) between the short swap and futures hedge, so if we arrange the profit expressions as equal to zero, we may rearrange to solve for drift. This is shown as (40.10):

$$E[\Delta rf \times (P_{\text{zero-coupon}} + \Delta P_{\text{zero-coupon}})] = E[P_{\text{zero-coupon}} \times (\Delta rf + \text{drift})] \quad (40.10)$$

where $E[]$ represents the market's expectation today of the value of profit. As $P_{\text{zero-coupon}}$ is known, we may solve for drift by dividing the expression by it, which gives us (40.11):

$$E(\text{drift}) = E\left[\Delta rf \times \left(\frac{\Delta P_{\text{zero-coupon}}}{P_{\text{zero-coupon}}}\right)\right] \quad (40.11)$$

This is the expression for the calculation of the convexity bias. Assuming that the average change in forward rates and zero-coupon rates is zero, we may combine the expression with the standard formula for correlation to give us an expression which may be used to approximate the amount of the convexity bias, shown as (40.12):

$$E(\text{drift}) = \sigma(\Delta rf) \times \sigma\left(\frac{\Delta P_{\text{zero-coupon}}}{P_{\text{zero-coupon}}}\right) \times \rho\left(\Delta rf, \frac{\Delta P_{\text{zero-coupon}}}{P_{\text{zero-coupon}}}\right) \quad (40.12)$$

No assumption is made about the distribution of rate changes. The drift is expressed in basis points per period.

APPENDIX 40.2 Calculating futures strip rates and implied swap rates

A futures strip is a position that contains one each of the futures contracts in a sequence of contract months. For example the one-year short sterling strip in August 1999 would consist of one each of the Sep99, Dec99, Mar00 and Jun00 contracts. The two-year strip would contain these but be followed by one each of the Sep00, Dec00, Mar01 and Jun01 contracts. The three-month forward rates that are implied by the prices of these contracts, together with the initial cash market deposit rate for the *stub* period may be used to obtain the future value of an investment made today. The expression to calculate this is given at (40.13):

$$FV_N = \left(1 + r_0 \times \left(\frac{d_0}{B}\right)\right) \times \left(1 + rf_1 \times \left(\frac{d_1}{B}\right)\right) \times \dots \times \left(1 + rf_n \times \left(\frac{d_n}{B}\right)\right) \quad (40.13)$$

where

FV_N	is the future value of £1 invested today for N years
r_0	is the cash market deposit rate (Libor) for the period from today to the expiry date of the first futures contract
rf_1	is the forward rate implied by the price of the front-month futures contract
rf_n	is the forward rate implied by the price of the last futures contract in the strip
d_i	is the number of days in each futures period, where $i = 0, 1, \dots, n$
B	is the year day-base (360 or 365).

The future value of an investment made at futures rates can then be used to calculate implied zero-coupon bond yields. The continuously compounded yield r_{CC} is given by (40.14) while the price $P_{\text{zero-coupon}}$ of a zero-coupon bond of maturity N years is given by (40.15).

$$r_{CC} = \ln\left(\frac{FV_N}{N}\right). \quad (40.14)$$

$$P_{\text{zero-coupon}} = \frac{1}{FV_N}. \quad (40.15)$$

The forward rates implied by a strip of futures contracts may be used to calculate implied swap rates. As we saw in Chapter 39 a plain vanilla interest-rate swap is priced on the basis that it consists of conceptually a short position in a fixed-coupon bond and a long position in a floating-rate bond, that is, a pay fixed- and receive floating-rate swap. The theoretical swap rate is therefore the average of the forward rates given by the futures strip from rf_1 to rf_N where N is the maturity of the swap. The most straightforward way to calculate this is using the discount factors of each spot rate, rather than the forward rates, and this was described in the main body of the text in Chapter 39.

Selected bibliography and references

Block, T., *Pricing and hedging interest-rate swaps*, Mars Business Associates, 1994.

French, K., "A Comparison of Futures and Forwards Prices", *Journal of Financial Economics* 12, November 1983, pp. 311–342.

Henna, P., *Interest-rate Risk Management using Futures and Swaps*, Probus, 1991.

Jarrow, R., Oldfield, G., "Forward Contracts and Futures Contracts", *Journal of Financial Economics* 9, December 1981, pp. 373–382.

Park, H., Chen, A., "Differences between Futures and Forward Prices: A Further Investigation of Marking to Market Effects", *Journal of Futures Markets* 5, February 1985, pp. 77–88.

Questions and exercises

1. Strata plc may borrow funds at a fixed rate of 8.00% and Libor + 10 basis points, while Roxmore may borrow at a fixed rate of 8.925% and Libor + 50 basis points. Strata would prefer to borrow on a floating-rate basis while Roxmore would prefer fixed-rate financing. Construct a swap, traded via an intermediary bank, that will suit both companies and also net the bank a gain of at least 50 basis points (on an annualised basis).
2. The following discount function has been constructed given the quoted sterling par swap rates.

Years	Par swap rate %	Spot rate %	Discount factor	Forward rate %
1	4.15	4.15	0.9601536	4.15
2	4.50	4.51	0.9155915	4.87
3	4.85	4.87	0.8669779	5.61
4	5.05	5.08	0.8200785	5.72
5	5.85	5.98	0.7478282	9.66
6	6.14	6.31	0.6927900	7.94
7	6.25	6.42	0.6468577	7.10
8	6.40	6.59	0.5999833	7.81
9	6.60	6.84	0.5511096	8.87
10	6.85	7.18	0.4998654	10.25
			<u>7.3012357</u>	

- (a) A customer requests a quote for a zero-coupon product that doubles her investment on maturity (that is, deposit of 100 and redemption of 200 on maturity). What maturity do you offer and what is the hedge that you establish in the par swaps?
- (b) A second customer requests a price in a 2v3 FRA, for which you quote 5.65-5.55. The client sells £10 million of the FRA. Construct a suitable hedge for this instrument, using the par swaps listed above.

41

Bond Futures

The most widely used risk management instrument in the bond markets is the government bond futures contract. This is usually an exchange-traded standardised contract that fixes the price today at which a specified quantity and quality of a bond will be delivered at a date during the expiry month of the futures contract. Unlike short-term interest-rate futures, which only require cash settlement, and which we encountered in the section on money markets, bond futures require the actual physical delivery of a bond when they are settled.

In this chapter we review bond futures contracts and their use for trading and hedging purposes.

41.1 Introduction

The concept of a bond futures contract is probably easier to grasp intuitively than a short-dated interest-rate future. This reflects the fact that a bond futures contract represents an underlying physical asset, the bond itself, and a bond must be delivered on expiry of the contract. In this way bond futures are similar to commodity futures, which also require physical delivery of the underlying commodity.

A *futures contract* is an agreement between two counterparties that fixes the terms of an exchange that will take place between them at some future date. They are standardised agreements as opposed to OTC ones, when traded on an exchange, so they are also referred to as *exchange traded futures*. In the UK financial futures are traded on LIFFE, which opened in 1982. LIFFE is the biggest financial futures exchange in Europe in terms of volume of contracts traded. There are four classes of contract traded on LIFFE: short-term interest rate contracts, long-term interest rate contracts (bond futures), currency contracts and stock index contracts. We discussed interest-rate futures contracts, which generally trade as part of the money markets, in an earlier chapter. In this section we will look at bond futures contracts, which are an important part of the bond markets; they are used for hedging and speculative purposes. Most futures contracts on exchanges around the world trade at three-month maturity intervals, with maturity dates fixed at March, June, September and December each year. This includes the contracts traded on LIFFE. Therefore at pre-set times during the year a contract for each of these months will *expire*, and a final *settlement* price is determined for it. The further out one goes the less liquid the trading is in that contract. It is normal to see liquid trading only in the *front month* contract (the current contract, so that if we are trading in April 2000 the front month is the June 2000 future), and possibly one or two of the next contracts, for most bond futures contracts. The liquidity of contracts diminishes the further one trades out in the maturity range.

When a party establishes a position in a futures contract, it can either run this position to maturity or close out the position between trade date and maturity. If a position is closed out the party will have either a profit or loss to book. If a position is held until maturity, the party who is long the future will take delivery of the underlying asset (bond) at the settlement price; the party who is short futures will deliver the underlying asset. This is referred to as *physical settlement* or sometimes, confusingly, as *cash settlement*.

There is no counterparty risk associated with trading exchange-traded futures, because of the role of the *clearing house*, such as the London Clearing House. This is the body through which contracts are settled. A clearing house acts as the buyer to all contracts sold on the exchange, and the seller to all contracts that are bought. So in the London market the LCH acts as the counterparty to all transactions, so that settlement is effectively guaranteed. The LCH requires all exchange participants to deposit *margin* with it, a cash sum that is the cost of conducting business (plus brokers commissions). The size of the margin depends on the size of a party's net *open* position in contracts (an open position is a position in a contract that is held overnight and not closed out). There are two types of margin, *maintenance margin* and *variation margin*. Maintenance margin is the minimum level required to be held at the clearing house; the level is set by the exchange. Variation margin is the additional amount that must be deposited to cover any trading losses and as the size of the net open positions increases. Note that this is not like margin in say, a repo transaction. Margin in repo is a safeguard against a drop in value of collateral that has been supplied against a loan of cash. The margin deposited at a futures exchange clearing house acts essentially as "good faith" funds, required to provide comfort to the exchange that the futures trader is able to satisfy the obligations of the futures contract.

BOND FUTURES AND OPTIONS							
France							
■ NOTIONAL EURO BOND FUTURES (MATIF) €100,000							
	Open	Sett price	Change	High	Low	Est. vol.	Open int.
Mar	83.93	84.02	+0.16	84.17	83.79	140,548	82,363
Jun	-	83.92	+0.14	-	-	-	-
Germany							
■ NOTIONAL EURO BOND FUTURES (EUREX) €100,000 100ths of 100%							
	Open	Sett price	Change	High	Low	Est. vol.	Open int.
Mar	102.57	102.74	+0.20	102.88	102.38	517,406	669,640
Jun	101.75	101.91	+0.19	102.00	101.57	1,395	36,667
■ NOTIONAL EURO BOND (BOBL) FUTURES (EUREX) €100,000 100ths of 100%							
	Open	Sett price	Change	High	Low	Est. vol.	Open int.
Mar	102.89	102.94	+0.06	102.98	102.73	240,927	397,530
Italy							
■ NOTIONAL ITALIAN GOVT. BOND (BTP) FUTURES (LIFFE)* Lira 200m 100ths of 100%							
	Open	Sett price	Change	High	Low	Est. vol.	Open int.
Mar	101.55	101.66	+0.22	101.79	101.50	133	4189
Spain							
■ NOTIONAL SPANISH BOND FUTURES (MEFF) €100,000							
	Open	Sett price	Change	High	Low	Est. vol.	Open int.
Mar	86.42	86.40	+0.03	86.66	86.22	6,522	14,268
UK							
■ NOTIONAL 5 YEAR GILT FUTURES (LIFFE) £100,000 100ths of 100%							
	Open	Sett price	Change	High	Low	Est. vol.	Open int.
Mar	102.17	-	+0.09	-	-	0	5
Jun	102.11	-	+0.09	-	-	0	0
■ NOTIONAL UK GILT FUTURES (LIFFE)* £100,000 100ths of 100%							
	Open	Close	Change	High	Low	Est. vol.	Open int.
Mar	108.82	109.04	+0.14	109.37	108.73	13692	78494
Jun	-	109.54	+0.14	-	-	0	0
All Open interest figs. are for previous day.							
■ LONG GILT FUTURES OPTIONS (LIFFE) £100,000 100ths of 100%							
Strike Price	Feb	Mar	Apr	Feb	Mar	Apr	
10850	0.54	1.35	2.11	0	0.81	1.07	
10900	0.04	1.08	1.82	0	1.04	1.28	
10950	0	0.85	1.56	0.46	1.31	1.52	
11000	0	0.66	1.33	0.96	1.62	1.79	
11050	0	0.50	1.12	1.46	1.96	2.08	
11100	0	0.37	0.93	1.96	2.33	2.39	
Est. vol. total, Calls 0 Puts 0. Previous day's open int., Calls 1310 Puts 1800							
US							
■ US TREASURY BOND FUTURES (CBT) \$100,000 32nds of 100%							
	Open	Sett price	Change	High	Low	Est. vol.	Open int.
Mar	89-24	90-11	+0-20	90-12	89-20	344,584	633,543
Jun	89-21	90-01	+0-20	90-02	89-11	16,434	58,562
Sep	-	89-24	+0-20	-	-	2	625

Japan

■ NOTIONAL LONG TERM JAPANESE GOVT. BOND FUTURES (LIFFE) ¥100m 100ths of 100%

	Open	Sett price	Change	High	Low	Est. vol.	Open int.
Mar	133.71	133.92	-	133.97	133.68	2752	n/a
Jun	132.17	132.40	-	132.44	132.20	1059	n/a

Euro

■ € 5 Year BOND FUTURES (MATIF) €100,000

	Open	Sett price	Change	High	Low	Est. vol.	Open int.
Mar	-	93.43	+0.04	-	-	-	-

■ NOTIONAL EURO BOND OPTIONS €100,000 (EUREX)

Strike Price	CALLS			PUTS		
	Feb	Mar	Apr	Feb	Mar	Apr
101.5	-	-	-	0.01	0.44	1.00
102.0	0.68	1.29	1.14	0.01	0.61	1.19
102.5	0.15	0.91	-	0.01	0.82	-
103.0	0.01	0.71	-	0.37	1.03	-

Est. vol. total, Calls 49,860 Puts 34,994. Previous day's open int., Calls 57,37 Puts n/a.

■ NOTIONAL EFB SWAP FUTURES (LIFFE)* 5yr 4.0% €100,000 100ths of 100%

	Open	Sett price	Change	High	Low	Est. vol.	Open int.
Mar	-	96.03	+0.05	-	-	0	0

■ NOTIONAL EFB SWAP FUTURES (LIFFE)* 10yr 4.5% €100,000 100ths of 100%

	Open	Sett price	Change	High	Low	Est. vol.	Open int.
Mar	-	99.98	+0.24	-	-	0	0

* Listed on LIFFE CONNECTTM. All open interest figs. are for previous day.

Figure 41.1: Bond futures price quotes, 25 January 2000.

©Financial Times, 25 January 2000. Reproduced with permission.

41.1.1 Bond futures contracts

We have noted that futures contracts traded on an exchange are standardised. This means that each contract represents exactly the same commodity, and it cannot be tailored to meet individual customer requirements. In this section we describe two very liquid and commonly traded contracts, starting with the US T-Bond contract traded on the Chicago Board of Trade (CBOT). The details of this contract are given at Figure 41.2.

CBOT T-Bond Futures Contract

Unit of Trading	US Treasury bond with notional value of \$100,000 and a coupon of 8%
Deliverable grades	US T-bonds with a minimum maturity of 15 years from first day of delivery month
Delivery months	March, June, September, December
Delivery date	Any business day during the delivery month
Last trading day	12:00 noon, seventh business day before last business day of delivery month
Quotation	Per cent of par expressed as points and thirty-seconds of a point, e.g. 108–16 is 108 16/32 or 108.50
Minimum price movement	1/32
Tick value	\$31.25
Trading hours	07.20 – 14.00 (trading pit) 17.20 – 20.05 22.30 – 06.00 hours (screen trading)

Figure 41.2: CBOT US T-Bond futures contract. Source: Acknowledged.

The terms of this contract relate to a US Treasury bond with a minimum maturity of 15 years and a *notional* coupon of 8%. We introduced the concept of the notional bond in the chapter on repo markets. A futures contract specifies a notional coupon to prevent delivery and liquidity problems that would arise if there was shortage of bonds with exactly the coupon required, or if one market participant purchased a large proportion of all the bonds in issue with the required coupon. For exchange traded futures, a short future can deliver any bond that fits the maturity criteria specified in the contract terms. Of course a long future would like to be delivered a high-coupon bond with significant accrued interest, while the short future would want to deliver a low-coupon bond with low interest accrued. In fact this issue does not arise because of the way the *invoice amount* (the amount paid by the long future to purchase the bond) is calculated. The invoice amount on the expiry date is given at (41.1):

$$Inv_{amt} = P_{fut} \times CF + AI \quad (41.1)$$

where

Inv_{amt} is the invoice amount
 P_{fut} is the price of the futures contract
 CF is the conversion factor
 AI is the bond accrued interest.

Any bond that meets the maturity specifications of the futures contract is said to be in the *delivery basket*, the group of bonds that are eligible to be delivered into the futures contract. Every bond in the delivery basket will have its own *conversion factor*, which is used to equalise coupon and accrued interest differences of all the delivery bonds. The exchange will announce the conversion factor for each bond before trading in a contract begins; the conversion factor for a bond will change over time, but remains fixed for one individual contract. That is, if a bond has a conversion factor of 1.091252, this will remain fixed for the life of the contract. If a contract specifies a bond with a notional coupon of 7%, like the long gilt future on LIFFE, then the conversion factor will be less than 1.0 for bonds with a coupon lower than 7% and higher than 1.0 for bonds with a coupon higher than 7%. A formal definition of conversion factor is given below.

Conversion factor

The conversion factor (or price factor) gives the price of an individual cash bond such that its yield to maturity on the delivery day of the futures contract is equal to the notional coupon of the contract. The product of the conversion factor and the futures price is the forward price available in the futures market for that cash bond (plus the cost of funding, referred to as the gross basis).

LIFFE Long Gilt contract

Unit of Trading	UK gilt bond having a face value of £100,000, a notional coupon of 7% and a notional maturity of 10 years, changed from contract value of £50,000 from the September 1998 contract)
Deliverable grades	UK gilts with a maturity ranging from 8¾ to 13 years from the first day of the delivery month (changed from 10–15 years from the December 1998 contract)
Delivery months	March, June, September, December
Delivery date	Any business day during the delivery month
Last trading day	11:00 hours two business days before last business day of delivery month
Quotation	Per cent of par expressed as points and hundredths of a point, for example 114.56 (changed from ticks and 1/32nds of a point, as in 117-17 meaning 114 17/32 or 114.53125, from the June 1998 contract)
Minimum price movement	0.01 of one point (one tick)
Tick value	£10
Trading hours	08:00–18:00 hours All trading conducted electronically on LIFFE CONNECT™ platform

Figure 41.3: LIFFE long gilt future contract specification. Source: LIFFE.

Although conversion factors equalise the yield on bonds, bonds in the delivery basket will trade at different yields, and for this reason they are not “equal” at the time of delivery. Certain bonds will be cheaper than others, and one bond will be the *cheapest-to-deliver* bond. The cheapest-to-deliver bond is the one that gives the greatest return from a strategy of buying a bond and simultaneously selling the futures contract, and then closing out positions on the expiry of the contract. This so-called *cash-and-carry trading* is actively pursued by proprietary trading desks in banks. If a contract is purchased and then held to maturity the buyer will receive, via the exchange’s clearing house the cheapest-to-deliver gilt. Traders sometimes try to exploit arbitrage price differentials between the future and the cheapest-to-deliver gilt, known as *basis trading*. This was discussed in Chapter 34. The mathematical calculation of the conversion factor for the gilt future is given at Appendix 41.1.

We summarise the contract specification of the long gilt futures contract traded on LIFFE at Figure 41.3. There is also a medium gilt contract on LIFFE, which was introduced in 1998 (having been discontinued in the early 1990s). This trades a notional five-year gilt, with eligible gilts being those of 4 to 7 years’ maturity.

41.2 Futures pricing

41.2.1 The theoretical principle

Although it may not appear so on first sight, floor trading on a futures exchange is probably the closest one gets to an example of the economist’s perfect and efficient market. The immediacy and liquidity of the market will ensure that at virtually all times the price of any futures contract reflects fair value. In essence because a futures contract represents an underlying asset, albeit a synthetic one, its price cannot differ from the actual cash market price of the asset itself. This is because the market sets futures prices such they are arbitrage-free. We can illustrate this with an hypothetical example.

Let us say that the benchmark 10-year bond, with a coupon of 8% is trading at par. This bond is the underlying asset represented by the long bond futures contract; the front month contract expires in precisely three months. If we also say that the three-month Libor rate (the repo rate) is 6%, what is fair value for the front month futures contract?

For the purpose of illustration let us start by assuming the futures price to be 105. We could carry out the following arbitrage-type trade:

- buy the bond for £100;
- simultaneously sell the future at £105;
- borrow £100 for three months at the repo rate of 6%.

As this is a leveraged trade we have borrowed the funds with which to buy the bond, and the loan is fixed at three months because we will hold the position to the futures contract expiry, which is in exactly three months’ time. At expiry, as we are short futures we will deliver the underlying bond to the futures clearing house and close out the loan. This strategy will result in cash flows for us as shown below.

Futures settlement cash flows

Price received for bond	= 105.00
Bond accrued	= 2.00 (8% coupon for three months)
Total proceeds	= 107.00

Loan cash flows

Repayment of principal	= 100.00
Loan interest	= 1.500 (6% repo rate for three months)
Total outlay	= 101.50

The trade has resulted in a profit of £5.50, and this profit is guaranteed as we have traded the two positions simultaneously and held them both to maturity. We are not affected by subsequent market movements. The trade is an example of a pure arbitrage, which is risk-free. There is no cash outflow at the start of the trade because we borrowed the funds used to buy the bond. In essence we have locked in the forward price of the bond by trading the future today, so that the final settlement price of the futures contract is irrelevant. If the situation described above were to occur in practice it would be very short-lived, precisely because arbitrageurs would buy the bond and sell the

future to make this profit. This activity would force changes in the prices of both bond and future until the profit opportunity was removed.

So in our illustration the price of the future was too high (and possibly the price of the bond was too low as well) and not reflecting fair value because the price of the synthetic asset was out of line with the cash asset.

What if the price of the future was too low? Let us imagine that the futures contract is trading at 95.00. We could then carry out the following trade:

- sell the bond at 100;
- simultaneously buy the future for 95;
- lend the proceeds of the short sale (100) for three months at 6%.

This trade has the same procedure as the first one with no initial cash outflow, except that we have to cover the short position in the repo market, through which we invest the sale proceeds at the repo rate of 6%. After three months we are delivered a bond as part of the futures settlement, and this is used to close out our short position. How has our strategy performed?

Futures settlement cash flows

Clean price of bond	= 95.00
Bond accrued	= 2.00
Total cash outflow	= 97.00

Loan cash flows

Principal on loan maturity	= 100.00
Interest from loan	= 1.500
Total cash inflow	= 101.500

The profit of £4.50 is again a risk-free arbitrage profit. Of course our hypothetical world has ignored considerations such as bid-offer spreads for the bond, future and repo rates, which would apply in the real world and impact on any trading strategy. Yet again however the futures price is out of line with the cash market and has provided opportunity for arbitrage profit.

Given the terms and conditions that apply in our example, there is one price for the futures contract at which no arbitrage profit opportunity is available. If we set the future price at 99.5, we would see that both trading strategies, buying the bond and selling the future or selling the bond and buying the future, yield a net cash flow of zero. There is no profit to be made from either strategy. So at 99.5 the futures price is in line with the cash market, and it will only move as the cash market price moves; any other price will result in an arbitrage profit opportunity.

41.2.2 Arbitrage-free futures pricing

The previous section demonstrated how we can arrive at the fair value for a bond futures contract provided we have certain market information. The market mechanism and continuous trading will ensure that the fair price is achieved, as arbitrage profit opportunities are eliminated. We can determine the bond future's price given:

- the coupon of the underlying bond, and its price in the cash market;
- the interest rate for borrowing or lending funds, from the trade date to the maturity date of the futures contract. This is known as the *repo* rate.

For the purpose of deriving this pricing model we can ignore bid-offer spreads and borrowing and lending spreads. We set the following:

r	is the repo rate
rc	is the bond's running yield
P_{bond}	is the price of the cash bond
P_{fut}	is the price of the futures contract
t	is the time to the expiry of the futures contract.

We can substitute these symbols into the cash flow profile for our earlier trade strategy, that of buying the bond and selling the future. This gives us:

Futures settlement cash flows

$$\begin{aligned}\text{Clean price for bond} &= P_{fut} \\ \text{Bond accrued} &= rc \cdot t \cdot P_{bond} \\ \text{Total proceeds} &= P_{fut} + (rc \cdot t \cdot P_{bond})\end{aligned}$$

Loan cash flows

$$\begin{aligned}\text{Repayment of loan principal} &= P_{bond} \\ \text{Loan interest} &= r \cdot t \cdot P_{bond} \\ \text{Total outlay} &= P_{bond} + (r \cdot t \cdot P_{bond}).\end{aligned}$$

The profit from the trade would be the difference between the proceeds and outlay, which we can set as follows:

$$\text{Profit} = P_{fut} + rc \cdot t \cdot P_{bond} - (P_{bond} + r \cdot t \cdot P_{bond}).$$

We have seen how the futures price is at fair value when there is no profit to be gained from carrying out this trade, so if we set profit at zero, we obtain the following:

$$0 = P_{fut} + rc \cdot t \cdot P_{bond} - (P_{bond} + r \cdot t \cdot P_{bond}).$$

Solving this expression for the futures price P_{fut} gives us:

$$P_{fut} = P_{bond} + P_{bond}t(r - rc).$$

Rearranging this we get:

$$P_{fut} = P_{bond} (1 + t(r - rc)). \quad (41.2)$$

If we repeat the procedure for the other strategy, that of selling the bond and simultaneously buying the future, and set the profit to zero, we will obtain the same expression for the futures price as given in (41.2) above.

It is the level of the repo rate in the market, compared to the running yield on the underlying bond, that sets the price for the futures contract. From the examples used at the start of this section we can see that it is the cost of funding compared to the repo rate that determines if the trade strategy results in a profit. The expression $(r - rc)$ from (41.2) is the net financing cost in the arbitrage trade, and is known as the *cost of carry*. If the running yield on the bond is higher than the funding cost (the repo rate) this is positive funding or *positive carry*. Negative funding (*negative carry*) is when the repo rate is higher than the running yield. The level of $(r - rc)$ will determine whether the futures price is trading above the cash market price or below it. If we have positive carry (when $rc > r$) then the futures price will trade below the cash market price, known as trading at a *discount*. Where $r > rc$ and we have negative carry then the futures price will be at a premium over the cash market price. If the net funding cost was zero, such that we had neither positive or negative carry, then the futures price would be equal to the underlying bond price.

The cost of carry related to a bond futures contract is a function of the yield curve. In a positive yield curve environment the three-month repo rate is likely to be lower than the running yield on a bond so that the cost of carry is likely to be positive. As there is generally only a liquid market in long bond futures out to contracts that mature up to one year from the trade date, with a positive yield curve it would be unusual to have a short-term repo rate higher than the running yield on the long bond. So in such an environment we would have the future trading at a discount to the underlying cash bond. If there is a negative sloping yield curve the futures price will trade at a premium to the cash price. It is in circumstances of changes in the shape of the yield curve that opportunities for relative value and arbitrage trading arise, especially as the bond that is cheapest-to-deliver for the futures contract may change with large changes in the curve.

A trading strategy that involved simultaneous and opposite positions in the cheapest-to-deliver bond (CTD) and the futures contract is known as *cash-and-carry* trading or *basis trading* and was discussed in Chapter 34. However, by the law of no-arbitrage pricing, the payoff from such a trading strategy should be zero. If we set the profit from such a trading strategy as zero, we can obtain a pricing formula for the fair value of a futures contract, which

summarizes the discussion above, and states that the fair value futures price is a function of the cost of carry on the underlying bond. This is given at (41.3):

$$P_{fut} = \frac{(P_{bond} + AI_0) \times (1 + rt) - \sum_{i=1}^N C_i (1 + rt_{i,del}) - AI_{del}}{CF} \quad (41.3)$$

where

AI_0	is the accrued interest on the underlying bond today
AI_{del}	is the accrued interest on the underlying bond on the expiry or delivery date (assuming the bond is delivered on the final day, which will be the case if the running yield on the bond is above the money market rate)
C_i	is the i th coupon
N	is the number of coupons paid from today to the expiry or delivery date
r	is the repo rate
t	is the time period (in years) over which the trade takes place
CF	is the bond conversion factor
$t_{i,del}$	is the period from receipt of i th coupon to delivery.

41.3 Hedging using futures

41.3.1 The theoretical position

Bond futures are used for a variety of purposes. Much of one day's trading in futures will be speculative, that is, a punt on the direction of the market. Another main use of futures is to hedge bond positions. In theory if hedging a cash bond position with a bond futures contract, if cash and futures prices move together then any loss from one position will be offset by a gain from the other. When prices move exactly in lock-step with each other, the hedge is considered perfect. In practice the price of even the cheapest-to-deliver bond (which one can view as being the bond being traded – implicitly – when one is trading the bond future) and the bond future will not move exactly in line with each other over a period of time. The difference between the cash price and the futures price is called the *basis*. The risk that the basis will change in an unpredictable way is known as *basis risk*.

The futures basis

The *basis* refers to the difference in price between the future and the deliverable cash bond. The basis is of considerable significance. It is often used to establish the fair value of a futures contract, as it is a function of the cost of carry. The *gross basis* is defined (for deliverable bonds only) as follows:

$$\text{gross basis} = \text{clean bond price} - (\text{futures price} \times \text{conversion factor}).$$

Futures are a liquid and straightforward way of hedging a bond position. By hedging a bond position the trader or fund manager is hoping to balance the loss on the cash position by the profit gained from the hedge. However the hedge will not be exact for all bonds except the cheapest-to-deliver (CTD) bond, which we can assume is the futures contract underlying bond. The basis risk in a hedge position arises because the bond being hedged is not identical to the CTD bond. The basic principle is that if the trader is long (or net long, where the desk is running long and short positions in different bonds) in the cash market, an equivalent number of futures contracts will be sold to set up the hedge. If the cash position is short the trader will buy futures. The hedging requirement can arise for different reasons. A market maker will wish to hedge positions arising out of client business, when they are unsure when the resulting bond positions will be unwound. A fund manager may for example, know that they need to realise a cash sum at a specific time in the future to meet fund liabilities, and sell bonds at that time. The market maker will want to hedge against a drop in value of positions during the time the bonds are held. The fund manager will want to hedge against a rise in interest rates between now and the bond sale date, to protect the value of the portfolio.

When putting on the hedge position the key is to trade the correct number of futures contracts. This is determined by using the *hedge ratio* of the bond and the future, which is a function of the volatilities of the two instruments. The amount of contracts to trade is calculated using the hedge ratio, which is given by:

$$\text{Hedge ratio} = \frac{\text{Volatility of bond to be hedged}}{\text{Volatility of hedging instrument}}.$$

Therefore one needs to use the volatility values of each instrument. We can see from the calculation that if the bond is more volatile than the hedging instrument, then a greater amount of the hedging instrument will be required. Let us now look in greater detail at the hedge ratio.

There are different methods available to calculate hedge ratios. The most common ones are the conversion factor method, which can be used for deliverable bonds (also known as the *price factor* method) and the modified duration method (also known as the basis point value method).

Where a hedge is put on against a bond that is in the futures delivery basket it is common for the conversion factor to be used to calculate the hedge ratio. A conversion factor hedge ratio is more useful as it is transparent and remains constant, irrespective of any changes in the price of the cash bond or the futures contract. The number of futures contracts required to hedge a deliverable bond using the conversion factor hedge ratio is determined using the following equation:

$$\text{Number of contracts} = \frac{M_{\text{bond}} \times CF}{M_{\text{fut}}} \quad (41.4)$$

where M is the nominal value of the bond or futures contract.

The conversion factor method may only be used for bonds in the delivery basket. It is important to ensure that this method is only used for one bond. It is an erroneous procedure to use the ratio of conversion factors of two different bonds when calculating a hedge ratio. This will be considered again later.

Unlike the conversion factor method, the modified duration hedge ratio may be used for all bonds, both deliverable and non-deliverable. In calculating this hedge ratio the modified duration is multiplied by the dirty price of the cash bond to obtain the *basis point value* (BPV). As we discovered in Chapter 7, the BPV represents the actual impact of a change in the yield on the price of a specific bond. The BPV allows the trader to calculate the hedge ratio to reflect the different price sensitivity of the chosen bond (compared to the CTD bond) to interest rate movements. The hedge ratio calculated using BPVs must be constantly updated, because it will change if the price of the bond and/or the futures contract changes. This may necessitate periodic adjustments to the number of lots used in the hedge. The number of futures contracts required to hedge a bond using the BPV method is calculated using the following:

$$\text{Number of contracts} = \frac{M_{\text{bond}}}{M_{\text{fut}}} \times \frac{BPV_{\text{bond}}}{BPV_{\text{fut}}} \quad (41.5)$$

where the BPV of a futures contract is defined with respect to the BPV of its CTD bond, as given by (41.6):

$$BPV_{\text{fut}} = \frac{BPV_{\text{CTDbond}}}{CF_{\text{CTDbond}}}. \quad (41.6)$$

The simplest hedge procedure to undertake is one for a position consisting of only one bond, the cheapest-to-deliver bond. The relationship between the futures price and the price of the CTD given by (41.3) indicates that the price of the future will move for moves in the price of the CTD bond, so therefore we may set:

$$\Delta P_{\text{fut}} \cong \frac{\Delta P_{\text{bond}}}{CF} \quad (41.7)$$

where CF is the CTD conversion factor.

The price of the futures contract, over time, does not move tick-for-tick with the CTD bond (although it may on an intra-day basis) but rather by the amount of the change divided by the conversion factor. It is apparent therefore that to hedge a position in the CTD bond we must hold the number of futures contracts equivalent to the value of bonds held multiplied by the conversion factor. Obviously if a conversion factor is less than one, the number of futures contracts will be less than the equivalent nominal value of the cash position; the opposite is true for bonds that have a conversion factor greater than one. However the hedge is not as simple as dividing the nominal value of the bond position by the nominal value represented by one futures contract (!); this error is frequently made by junior traders and those new to the desk.

To measure the effectiveness of the hedge position, it is necessary to compare the performance of the futures position with that of the cash bond position, and to see how much the hedge instrument mirrored the performance of the cash instrument. A simple calculation is made to measure the effectiveness of the hedge, given by (41.8), which is the percentage value of the hedge effectiveness:

$$\text{Hedge effectiveness} = -\left(\frac{\text{Fut } p/l}{\text{Bond } p/l}\right) \times 100. \quad (41.8)$$

41.3.2 Hedging a bond portfolio

The principles established above may be applied when hedging a portfolio containing a number of bonds. It is more realistic to consider a portfolio holding not just bonds that are outside the delivery basket, but are also not government bonds. In this case we need to calculate the number of futures contracts to put on as a hedge based on the volatility of each bond in the portfolio compared to the volatility of the CTD bond. Note that in practice, there is usually more than one futures contract that may be used as the hedge instrument. For example in the sterling market it would be more sensible to use LIFEE's medium gilt contract, whose underlying bond has a notional maturity of 4–7 years, if hedging a portfolio of short- to medium-dated bonds. However for the purposes of illustration we will assume that only one contract, the long bond, is available.

To calculate the number of futures contracts required to hold as a hedge against any specific bond, we use the expression at (41.9).

$$\text{Hedge} = \frac{M_{\text{bond}}}{M_{\text{fut}}} \times \text{Vol}_{\text{bond}/\text{CTD}} \times \text{Vol}_{\text{CTD}/\text{fut}} \quad (41.9)$$

where

M	is the nominal value of the bond or future
$\text{Vol}_{\text{bond}/\text{CTD}}$	is the relative volatility of the bond being hedged compared to that of the CTD bond
$\text{Vol}_{\text{CTD}/\text{fut}}$	is the relative volatility of the CTD bond compared to that of the future.

It is not necessarily straightforward to determine the relative volatility of a bond vis-à-vis the CTD bond. If the bond being hedged is a government bond, we can calculate the relative volatility using the two bonds' modified duration. This is because the yields of both may be safely assumed to be strongly positively correlated. If however the bond being hedged is a corporate bond and/or non-vanilla bond, we must obtain the relative volatility using regression analysis, as the yields between the two bonds may not be strongly positively correlated. This is apparent when one remembers that the yield spread of corporate bonds over government bonds is not constant, and will fluctuate with changes in government bond yields. To use regression analysis to determine relative volatilities, historical price data on the bond is required; the daily price moves in the target bond and the CTD bond are then analysed to assess the slope of the regression line. In this section we will restrict the discussion to a portfolio of government bonds.

If we are hedging a portfolio of government bonds we can use (41.10) to determine relative volatility values, which is based on the modified duration of each of the bonds in the portfolio.

$$\text{Vol}_{\text{bond}/\text{CTD}} = \frac{\Delta P_{\text{bond}}}{\Delta P_{\text{CTD}}} = \frac{MD_{\text{bond}} \times P_{\text{bond}}}{MD_{\text{CTD}} \times P_{\text{CTD}}} \quad (41.10)$$

where MD is the modified duration of the bond being hedged or the CTD bond, as appropriate. This preserves the terminology we introduced in Chapters 7–10.¹

¹ In certain textbooks and practitioner research documents, it is suggested that the ratio of the conversion factors of the bond being hedged (if it is in the delivery basket) and the CTD bond can be used to determine the relative volatility of the target bond. This is a specious argument. The conversion factor of a deliverable bond is the price factor that will set the yield of the bond equal to the notional coupon of the futures contract on the delivery date, and it is a function mainly of the coupon of the deliverable bond. The price volatility of a bond on the other hand, is a measure of its modified duration, which is a function of the bond's duration (that is, the weighted average term to maturity). Therefore using conversion factors to measure volatility levels will produce erroneous results. It is important not to misuse conversion factors when arranging hedge ratios.

Once we have calculated the relative volatility of the bond being hedged, equation (41.11) (obtained from (41.7) and (41.10)) tells us that the relative volatility of the CTD bond to that of the futures contract is approximately the same as its conversion factor. We are then in a position to calculate the futures hedge for each bond in a portfolio.

$$Vol_{CTD/fut} = \frac{\Delta P_{CTD}}{\Delta P_{fut}} \approx CF_{CTD}. \quad (41.11)$$

Table 41.1 shows a portfolio of five UK gilts on 20 October 1999. The nominal value of the bonds in the portfolio is £200 million, and the bonds have a market value excluding accrued interest of £206.84 million. Only one of the bonds is a deliverable bond, the 5¾% 2009 gilt which is in fact the CTD bond. For the Dec99 futures contract the bond had a conversion factor of 0.9124950. The fact that this bond is the CTD explains why it has a relative volatility of 1. We calculate the number of futures contracts required to hedge each position, using the equations listed above. For example, the hedge requirement for the position in the 7% 2002 gilt was calculated as follows:

$$\frac{5,000,000}{100,000} \times \frac{2.245 \times 101.50}{7.235 \times 99.84} \times 0.9124950 = 14.39.$$

The volatility of all the bonds is calculated relative to the CTD bond, and the number of futures contracts determined using the conversion factor for the CTD bond. The bond with the highest volatility is not surprisingly the 6% 2028, which has the longest maturity of all the bonds and hence the highest modified duration. We note from Table 41.1 that the portfolio requires a hedge position of 2091 futures contracts. This illustrates how a “rough-and-ready” estimate of the hedging requirement, based on nominal values, would be insufficient as that would suggest a hedge position of only 2000 contracts.

CTD	5.75% 2009	Modified duration 7.234565567					
Conversion factor	0.9124950	Price 99.84					
Bond	Nominal amount (£m)	Price	Yield %	Duration	Modified duration	Relative volatility	Number of contracts
UKT 8% 2000	12	102.17	5.972	1.072	1.011587967	0.143090242	15.67
UKT 7% 2002	5	101.50	6.367	2.388	2.245057208	0.315483336	14.39
UKT 5% 2004	38	94.74	6.327	4.104	3.859791022	0.50626761	175.55
UKT 5.75% 2009	100	99.84	5.770	7.652	7.234565567	1	912.50
UKT 6% 2028	45	119.25	4.770	15.031	14.34666412	2.368603078	972.60
Total	200						2090.71

Table 41.1: Bond futures hedge for hypothetical gilt portfolio, 20 October 1999.

The effectiveness of the hedge must be monitored over time. No hedge will be completely perfect however, and the calculation illustrated above, as it uses modified duration value, does not take into account the convexity effect of the bonds. The reason why a futures hedge will not be perfect is because in practice, the price of the futures contract will not move tick-for-tick with the CTD bond, at least not over a period of time. This is the basis risk that is inherent in hedging cash bonds with futures. In addition, the calculation of the hedge is only completely accurate for a parallel shift in yields, as it is based on modified duration, so as the yield curve changes around pivots, the hedge will move out of line. Finally, the long gilt future is not the appropriate contract to use to hedge three of the bonds in the portfolio, or over 25% of the portfolio by nominal value. This is because these bonds are short- or medium-dated, and so their price movements will not track the futures price as closely as longer-dated bonds. In this case, the more appropriate futures contract to use would have been the medium gilt contract, or (for the first bond, the 8% 2000) a strip of short sterling contracts. Using shorter-dated instruments would reduce some of the basis risk contained in the portfolio hedge.

41.4 The margin process

Institutions buying and selling futures on an exchange deal with only one counterparty at all times, the exchange clearing house. The clearing house is responsible for the settlement of all contracts, including managing the delivery process. A central clearing mechanism eliminates counterparty risk for anyone dealing on the exchange, because the clearing house guarantees the settlement of all transactions. The clearing house may be owned by the exchange itself, such as the one associated with the Chicago Mercantile Exchange (the CME Clearinghouse) or it may be a separate entity, such as the London Clearing House, which settles transactions on LIFFE. The LCH is also involved in running clearing systems for swaps and repo products in certain currencies.

One of the key benefits to the market of the clearing house mechanism is that counterparty risk, as it is transferred to the clearing house, is virtually eliminated. The mechanism that enables the clearing house to accept the counterparty risk is the *margining* process that is employed at all futures exchanges. A bank or local trader must deposit margin before commencing dealing on the exchange; each day a further amount must be deposited or will be returned, depending on the results of the day's trading activity.

The exchange will specify the level of margin that must be deposited for each type of futures contract that a bank wishes to deal in. The *initial margin* will be a fixed sum per lot, so for example if the margin was £1000 per lot an opening position of 100 lots would require margin of £100,000. Once initial margin has been deposited, there is a mark-to-market of all positions at the close of business; exchange-traded instruments are the most transparent products in the market, and the closing price is not only known to everyone, it is also indisputable. The closing price is also known as the *settlement price*. Any losses suffered by a trading counterparty, whether closed out or run overnight, are entered as a debit on the party's account and must be paid the next day. Trading profits are credited and may be withdrawn from the margin account the next day. This daily process is known as *variation margining*. Thus the margin account is updated on a daily basis and the maximum loss that must be made up on any morning is the maximum price movement that occurred the previous day. It is a serious issue if a trading party is unable to meet a margin call. In such a case, the exchange will order it to cease trading, and will also liquidate all its open positions; any losses will be met out of the firm's margin account. If the level of funds in the margin account is insufficient, the losses will be made good from funds paid out of a general fund run by the clearing house, which is maintained by all members of the exchange.

Payment of margin is made by electronic funds transfer between the trading party's bank account and the clearing house. Initial margin is usually paid in cash, although clearing houses will also accept high quality securities such as T-bills or certain government bonds, to the value of the margin required. Variation margin is always cash. The advantage of depositing securities rather than cash is that the depositing firm earns interest on its margin. This is not available on a cash margin, and the interest foregone on a cash margin is effectively the cost of trading futures on the exchange. However if securities are used, there is effectively no cost associated with trading on the exchange (we ignore of course infrastructure costs and staff salaries).

The daily settlement of exchange-traded futures contracts, as opposed to when the contract expires or the position is closed out, is the main reason why futures prices are not equal to forward prices for long-dated instruments.

Appendices

APPENDIX 41.1 The conversion factor for the long gilt future

Here we describe the process used for the calculation of the *conversion factor* or *price factor* for deliverable bonds of the long gilt contract. The contract specifies a bond of maturity $8\frac{3}{4}$ -13 years and a notional coupon of 7%. For each bond that is eligible to be in the delivery basket, the conversion factor is given by the following expression: $P(7)/100$ where the numerator $P(7)$ is equal to the price per £100 nominal of the deliverable gilt at which it has a gross redemption yield of 7%, calculated as at the first day of the delivery month, less the accrued interest on the bond on that day. This calculation uses the formulae given at (41.12) and the expression used to calculate accrued interest. The analysis is adapted, with permission, from LIFFE's technical document.

The numerator $P(7)$ is given by (41.12):

$$P(7) = \frac{1}{1.035^{t/s}} \left(c_1 + \frac{c_2}{1.035} + \frac{C}{0.07} \left(\frac{1}{1.035} - \frac{1}{1.035^n} \right) + \frac{100}{1.035^n} \right) - AI \quad (41.12)$$

where

- c_1 is the cash flow due on the following quasi-coupon date, per £100 nominal of the gilt. c_1 will be zero if the first day of the delivery month occurs in the ex-dividend period or if the gilt has a long first coupon period and the first day of the delivery month occurs in the first full coupon period. c_1 will be less than $C/2$ if the first day of the delivery month falls in a short first coupon period. c_1 will be greater than $C/2$ if the first day of the delivery month falls in a long first coupon period and the first day of the delivery month occurs in the second full coupon period
- c_2 is the cash flow due on the next but one quasi-coupon date, per £100 nominal of the gilt. c_2 will be greater than $C/2$ if the first day of the delivery month falls in a long first coupon period and in the first full coupon period. In all other cases, $c_2 = C/2$.
- C is the annual coupon of the gilt, per £100 nominal
- t is the number of calendar days from and including the first day of the delivery month up to but excluding the next quasi-coupon date
- s is the number of calendar days in the full coupon period in which the first day of the delivery month occurs
- n is the number of full coupon periods between the following quasi-coupon date and the redemption date
- AI is the accrued interest per £100 nominal of the gilt.

The accrued interest used in the formula above is given according to the following procedures.

If the first day of the delivery month occurs in a standard coupon period, and the first day of the delivery month occurs on or before the ex-dividend date, then:

$$AI = \frac{t}{s} \times \frac{C}{2}. \quad (41.13)$$

If the first day of the delivery month occurs in a standard coupon period, and the first day of the delivery month occurs after the ex-dividend date, then:

$$AI = \left(\frac{t}{s} - 1 \right) \times \frac{C}{2} \quad (41.14)$$

where

- t is the number of calendar days from and including the last coupon date up to but excluding the first day of the delivery month:
- s is the number of calendar days in the full coupon period in which the first day of the delivery month occurs.

If the first day of the delivery month occurs in a short first coupon period, and the first day of the delivery month occurs on or before the ex-dividend date, then:

$$AI = \frac{t^*}{s} \times \frac{C}{2}. \quad (41.15)$$

If the first day of the delivery month occurs in a short first coupon period, and the first day of the delivery month occurs after the ex-dividend date, then:

$$AI = \left(\frac{t^* - n}{s} \right) \times \frac{C}{2} \quad (41.16)$$

where

- t^* is the number of calendar days from and including the issue date up to but excluding the first day of the delivery month
- n is the number of calendar days from and including the issue date up to but excluding the next quasi-coupon date.

If the first day of the delivery month occurs in a long first coupon period, and during the first full coupon period, then:

$$AI = \frac{u}{s_1} \times \frac{C}{2}. \quad (41.17)$$

If the first day of the delivery month occurs in a long first coupon period, and during the second full coupon period and on or before the ex-dividend date, then:

$$AI = \left(\frac{p_1}{s_1} + \frac{p_2}{s_2} \right) \times \frac{C}{2}. \quad (41.18)$$

If the first day of the delivery month occurs in a long first coupon period, and during the second full coupon period and after the ex-dividend date, then:

$$AI = \left(\frac{p_2}{s_2} - 1 \right) \times \frac{C}{2} \quad (41.19)$$

where

- u is the number of calendar days from and including the issue date up to but excluding the first day of the delivery month
- s_1 is the number of calendar days in the full coupon period in which the issue date occurs
- s_2 is the number of days in the next full coupon period after the full coupon period in which the issue date occurs
- p_1 is the number of calendar days from and including the issues date up to but excluding the next quasi-coupon date
- p_2 is the number of calendar days from and including the quasi-coupon date after the issue date up to but excluding the first day of the delivery month which falls in the next full coupon period after the full coupon period in which the issue date occurs.

Selected bibliography and references

- Choudhry, M., *An Introduction to Bond Markets*, SI (Services) Publishing, 1999, pp. 229–242.
- Fabozzi, F., *Fixed Income Mathematics*, Probus, 1993.
- Figlewski, S., *Hedging with Financial Futures for Institutional Investors*, Ballinger, 1986.
- Kolb, R., *Interest Rate Futures: A Comprehensive Introduction*, RF Dame, 1982.
- Resnick, B., “The Relationship between Futures Prices for US Treasury Bonds”, *Review of Research in Futures Markets* 3, 1984, pp. 84–104.
- Stultz, R., “Optimal Hedging Policies”, *Journal of Financial and Quantitative Analysis*, 19 June 1984, pp. 127–140.

Questions and exercises

- How does a margining mechanism at a futures exchange protect firms that trade on the exchange? Under what circumstances will a trading firm receive a margin call?
- What is *basis risk* in the context of establishing a hedge position using bond futures?

3. The long gilt future on LIFFE has expiry months in March, June, September and December each year. Which contract should be used to hedge an exposure that expires in May? In June? In January?
4. A bond with a modified duration of 12.454 is hedged using a futures contract for which the underlying CTD bond has a modified duration of 7.459. What does this imply for the hedge that is put on?
5. A bond portfolio with a nominal value of EUR250 million has a portfolio modified duration of 9.798. The instrument chosen to hedge the portfolio is a futures contract with a notional underlying value of EUR100 000. The CTD bond for the future has a modified duration of 7.117 and a conversion factor of 1.002343. Does the portfolio manager have sufficient information to hedge the portfolio effectively? If so, what hedge is put on?
6. Consider the following portfolio:

Bond	Nominal amount (£m)	Maturity year	Price	Yield %	Modified duration
1	39	2001	101.17	5.070	1.879
2	33	2007	103.15	6.450	5.079
3	161	2009	98.79	6.751	6.859
4	211	2010	99.90	6.987	7.959
5	90	2030	121.20	7.112	15.141

Calculate the hedge for the portfolio, using the following two instruments:

- long bond future (notional 10–15 years' maturity), CTD bond price 99.50, modified duration 8.766, conversion factor 1.0007571
- medium bond future (notional 5–7 years' maturity), CTD bond price 101.23, modified duration 4.104, conversion factor 0.9897429

State the exact number of each contract that will form part of the hedge.

42 Options I

Option contracts are a separate subject in their own right, even more so than the other instruments we discuss in this book. As a risk management tool, they allow banks and corporates to hedge market exposure but also to gain from upside moves in the market; this makes them unique amongst hedging instruments. Options have special characteristics that make them stand apart from other classes of derivatives. As they confer a right to conduct a certain transaction, but not an obligation, their payoff profile is different from other financial assets, both cash and off-balance sheet. This makes an option more of an insurance policy rather than a pure hedging instrument, as the person who has purchased the option for hedging purposes need only exercise it if required. The price of the option is in effect the insurance premium that has been paid for peace of mind. Of course options are also used for purposes other than hedging. As part of speculative and arbitrage trading, and option market makers generate returns from profitably managing the risk on their option books.

The advance in financial engineering since the early 1980s is demonstrated most spectacularly in the growth of options, which are very flexible and versatile financial instruments. Options contracts were originally introduced on agricultural commodities over two hundred years ago, but financial options date from the 1970s. A seminal paper by the late Fischer Black¹ and Myron Scholes, which appeared in the *Journal of Political Economy* in 1973 was the essential catalyst, since as a result of this research the market now had a straightforward way to price options.² There are now several other option pricing models used in the markets in addition to Black–Scholes, some developed specifically for the new exotic option structures that have been introduced. When originally traded they were essentially vanilla instruments, however the flexibility of options was such that they were soon combined into exotic structures. The range of combinations of options that can be dealt today, and the complex structured products that they form part of is constrained only by imagination and customer requirements. Virtually all participants in capital markets will have some requirement that may be met by the use of options. The subject is a large one, and there are a number of specialist texts devoted to them. In this chapter we introduce the basics of options; subsequent chapters will review option pricing, the main sensitivity measures used in running an option book, and the uses to which options may be put. We cannot avoid extensive use of mathematics, mainly when discussing the pricing of options, but this is kept to a minimum, with proofs and assumptions relegated to the appendices or omitted altogether. Readers who do not wish to study this may skip Chapter 43 and move on to subsequent chapters which consider practical issues in option trading and the use of options for hedging. Readers who wish to explore the subject further may wish to consult the texts listed in the selected bibliography. Key reference articles and publications are also listed in the bibliography.

42.1 Introduction

An option is a contract in which the buyer has the right, but not the obligation, to buy or sell an underlying asset at a pre-determined price during a specified period of time. The seller of the option, known as the *writer*, grants this right to the buyer in return for receiving the price of the option, known as the *premium*. An option that grants the right to buy an asset is a *call option*, while the corresponding right to sell an asset is a *put option*. The option buyer has a long position in the option and the option seller has a short position in the option.

Before looking at the other terms that define an option contract, let us discuss the main feature that differentiates an option from all other derivative instruments, and from cash assets. Because options confer on a buyer the right to effect a transaction, but not the obligation (and correspondingly on a seller the obligation, if requested by the buyer, to effect a transaction), their risk/reward characteristics are different to other financial products. The payoff

¹ Fischer Black passed away in August 1995.

² Ironically when the Black–Scholes model first appeared, some academic writers and commentators were highly critical of it, saying that it was unsatisfactory and relied on flawed assumptions. However its wide acceptance was mainly responsible for the rapid growth in the use of financial options from the 1980s onwards, coupled with advances in information technology that allowed pricing models to rapidly assimilate data and calculate option premiums. It is a market standard and is widely used today, while variants of the basic model are used to price non-vanilla options.

profile from holding an option is unlike that of any other instrument. We now consider the payoff profiles for a vanilla call option and a gilt futures contract. Suppose that a trader buys one lot of the gilt futures contract at 114.00 and holds it for one month before selling it. On closing the position the profit made will depend on the contract sale price, if it is above 114.00 the trader will have made a profit and if below 114.00 she will have made a loss. On one lot this represents a £1000 gain for each point above 114.00. The same applies to someone who had a short position in the contract and closed it out – if the contract is bought back at any price below 114.00 the trader will realise a profit. The profile is shown in Figure 42.1.

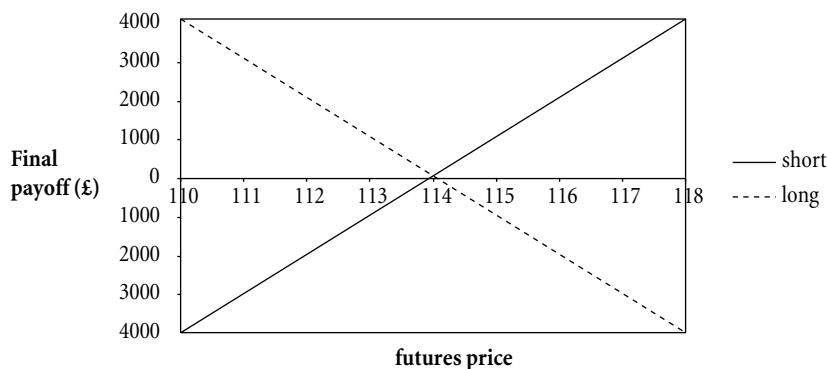


Figure 42.1: Payoff profile for a bond futures contract.

This profile is the same for other derivative instruments such as FRAs and swaps, and of course for cash instruments such as bonds or equity. The payoff profile therefore has a *linear* characteristic, and it is linear whether one has bought or sold the contract.

The profile for an option contract differs from the conventional one. Because options confer a right to one party but not an obligation (the buyer), and an obligation but not a right to the seller, the profile will differ according to whether one is the buyer or seller. Suppose now that our trader buys a call option that grants the right to buy a gilt futures contract at a price of 114.00 at some point during the life of the option, her resulting payoff profile will be like that shown in Figure 42.2.

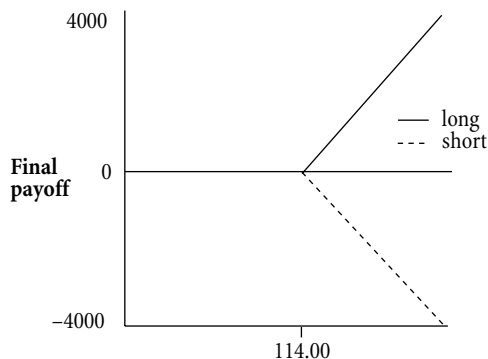


Figure 42.2: Payoff profile for call option contract.

If during the life of the option the price of the futures contract rises above 114.00, the trader will exercise her right to buy the future, under the terms of the option contract. This is known as *exercising* the option. If on the other hand the price of the future falls below 114.00, the trader will not exercise the option and, unless there is a reversal in price of the future, it will eventually expire worthless, on its maturity date. In this respect it exactly like an equity or bond warrant. The seller of this particular option has a very different payout profile. If the price of the future rises above 114.00 and the option is exercised, the seller will bear the loss equal to the profit that the buyer is now benefiting from. The seller's payoff profile is also shown in Figure 42.2, as the dashed line. If the option is not exercised and

expires, for the seller the trade will have generated premium income, which is revenue income that contributes to the P/L account.

This illustrates how unlike every other financial instrument, the holders of long and short positions in options do not have the same symmetrical payoff profile. The buyer of the call option will benefit if the price of the underlying asset rises, but will not lose if the price falls (except the funds paid for purchasing the rights under the option). The seller of the call option will suffer loss if the price of the underlying asset rises, but will not benefit if it falls (except realising the funds received for writing the option). The buyer has a right but not an obligation, while the seller has an obligation if the option is exercised. The premium charged for the option is the seller's compensation for granting such a right to the buyer.

Let us recap on the basic features of the call option. A call option is the right to buy, without any obligation, a specified quantity of the underlying asset at a given price on or before the expiry date of the option. A long position in a call option allows the holder, as shown in Figure 42.2, to benefit from a rise in the market price of the underlying asset. If our trader wanted to benefit from a fall in the market level, but did not want to short the market, she would buy a *put* option. A put option is the right to sell, again without any obligation, a specified quantity of the underlying asset at a given price on or before the expiry date of the option. Put options have the same payoff profile as call options, but in the opposite direction. Remember also that the payoff profile is different for the buyer and seller of an option. The buyer of a call option will profit if the market price of the underlying asset rises, but will not lose if the price falls (at least, not with regard to the option position). The writer of the option will not profit whatever direction the market moves in, and will lose if the market rises. The compensation for taking on this risk is the premium paid for writing the option, which is why we likened options to insurance policies at the start of the chapter.

Originally options were written on commodities such as wheat and sugar. Nowadays these are referred to as *options on physicals*, while options on financial assets are known as financial options. Today one is able to buy or sell an option on a wide range of underlying instruments, including financial products such as foreign exchange, bonds, equities, and commodities, other underlying assets such as energy and rainfall, and derivatives such as futures, swaps, equity indices and other options.

42.1.1 Option terminology

We now consider the basic terminology used in the options markets. A *call* option grants the buyer the right to buy the underlying asset, while a *put* option grants the buyer the right to sell the underlying asset. There are therefore four possible positions that an option trader may put on, long a call or put and short a call or put. The payoff profiles for each type are shown at Figure 42.3.

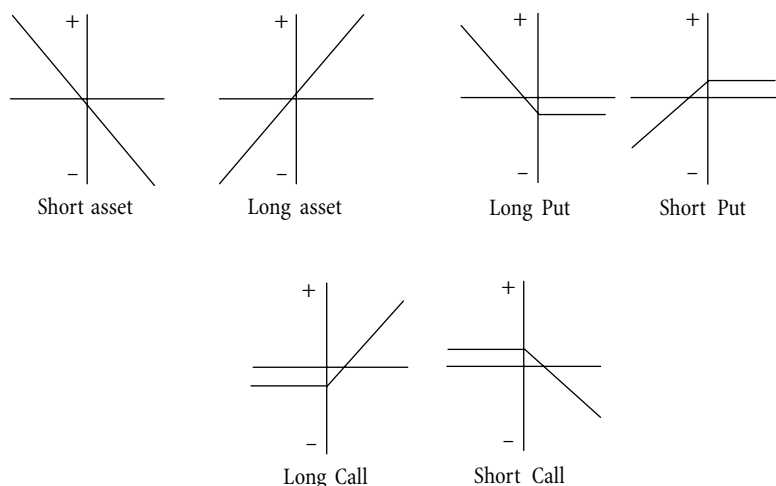


Figure 42.3: Basic option payoff profiles.

The *strike price* describes the price at which an option is exercised. For example a call option to buy ordinary shares of a listed company might have a strike price of £10.00. This means that if the option is exercised, the buyer

will pay £10 per share. Options are generally either *American* or *European* style, which defines the times during the option's life when it can be exercised. There is no geographic relevance to these terms, as both styles can be traded in any market. There is also another type, *Bermudan* style options, which can be exercised at pre-set dates.³ For reasons that we shall discuss later, it is very rare for an American option to be exercised ahead of its expiry date, so this distinction has little impact in practice, although of course the pricing model being used to value European options must be modified to handle American options. The holder of a European option cannot exercise it prior to expiry, however if they wish to realise its value they will sell it in the market.

The *premium* of an option is the price at which the option is sold. Option premium is made up of two constituents, *intrinsic value* and *time value*.

The intrinsic value of an option is the value of the option if it is exercised immediately, and it represents the difference between the strike price and the current underlying asset price. If a call option on a bond futures contract has a strike price of 100.00 and the future is currently trading at 105.00, the intrinsic value of the option is 5.00, as this would be the immediate profit gain to the option holder if it were exercised. Since an option will only be exercised if there is benefit to the holder from so doing, its intrinsic value will never be less than zero. So in our example if the bond future was trading at 95.00 the intrinsic value of the call option would be zero, not -5.00. For a put option the intrinsic value is the amount by which the current underlying price is below the strike price. When an option has intrinsic value it is described as being *in-the-money*. When the strike price for a call option is higher than the underlying price (or for a put option is lower than the underlying price) and has no intrinsic value it is said to be *out-of-the-money*. An option for which the strike price is equal to the current underlying price is said to be *at-the-money*. This term is normally used at the time the option is first traded, in cases where the strike price is set to the current price of the underlying asset.

The time value of an option is the amount by which the option value exceeds the intrinsic value. An option writer will almost always demand a premium that is higher than the option's intrinsic value, because of the risk that the writer is taking on. This reflects the fact that over time the price of the underlying asset may change sufficiently to produce a much higher intrinsic value. During the life of an option, the option writer has nothing more to gain over the initial premium at which the option was sold; however until expiry there is a chance that the writer will lose if the markets move against them, hence the inclusion of a time value element. The value of an option that is out-of-the-money is composed entirely of time value.

Table 42.1 summarises the main option terminology that we have just been discussing.

Call	The right to buy the underlying asset
Put	The right to sell the underlying asset
Buyer	The person who has purchased the option and has the right to exercise it if they wish
Writer	The person who has sold the option and has the obligation to perform if the option is exercised
Strike price	The price at which the option may be exercised, also known as the <i>exercise price</i>
Expiry date	The last date on which the option can be exercised, also known as the maturity date
American	The style of option; an American option can be exercised at any time up to the expiry date
European	An option which may be exercised on the maturity date only, and not before
Premium	The price of the option, paid by the buyer to the seller
Intrinsic value	The value of the option if it was exercised today, which is the difference between the strike price and the underlying asset price
Time value	The difference between the current price of the option and its intrinsic value
In-the-money	The term for an option that has intrinsic value
At-the-money	An option for which the strike price is identical to the underlying asset price
Out-of-the-money	An option that has no intrinsic value

Table 42.1: Basic option terminology.

³ I'm told because Bermuda is mid-way between Europe and America. But then, why "Asian" for average-rate options?! (In fact, a former colleague has since suggested because they originated in Far East option markets, written on volatile currencies like the Indonesian ringgit, and the average rate was more relevant to the buyer's business.)

42.2 Option instruments

Options are traded both on recognised exchanges and in the over-the-counter (OTC) market. The primary difference between the two types is that exchange-traded options are standardised contracts and essentially plain vanilla instruments, while OTC options can take on virtually any shape or form. Options traded on an exchange are often options on a futures contract, so for example gilt option on LIFFE in London is written on the exchange's gilt futures contract. The exercise of a futures options will result in a long position in a futures contract being assigned to the party that is long the option, and a short position in the future to the party that is short the option. Note that exchange-traded options on US Treasuries are quoted in option ticks that are half the bond tick, that is 1/64th rather than 1/32nd. The same applied to gilt options on LIFFE until gilts themselves switched to decimal pricing at the end of 1998.

Like OTC options, those traded on an exchange can be either American or European style. For example, on the Philadelphia Currency Options Exchange both versions are available, although on LIFFE most options are American style. Exchange-traded options are available on the following:

- **ordinary shares:** major exchanges including the New York Stock Exchange, LIFFE, Eurex, the Chicago Board Options Exchange (CBOE), and SIMEX in Singapore trade options on corporation ordinary shares;
- **options on futures:** most exchanges trade an option contract written on the futures that are traded on the exchange, which expires one or two days before the futures contract itself expires. In certain cases such as those traded on the Philadelphia exchange, cash settlement is available, so that if for example the holder of a call exercises, she will be assigned a long position in the future as well as the cash value of the difference between the strike price and the futures price. One of the most heavily traded exchange-traded options contracts is the Treasury bond option, written on the futures contract, traded on the Chicago Board of Trade options exchange;
- **stock index options:** these are equity market instruments that are popular for speculating and hedging, for example the FTSE-100 option on LIFFE and the S&P500 on CBOE. Settlement is in cash and not the shares that constitute the underlying index, much like the settlement of an index futures contract.
- **bond options:** options on bonds are invariably written on the bond futures contract, for example the aforementioned Treasury bond option or LIFFE's gilt option. Options written on the cash bond must be traded in the OTC market;
- **interest-rate options:** these are also options on futures, as they are written on the exchange's 90-day interest-rate futures contract;
- **foreign currency options:** this is rarer among exchange-traded options, and the major exchange is in Philadelphia. Its sterling option contract for example is for an underlying amount of £31,250.

Option trading on an exchange is similar to that for futures, and involves transfer of margin on a daily basis. Individual exchanges have their own procedures; for example on LIFFE the option premium is effectively paid via the variation margin. The amount of variation margin paid or received on a daily basis for each position reflects the change in the price of the option. So if for example an option were to expire on maturity with no intrinsic value, the variation margin payments made during its life would be equal to the change in value from the day it was traded to zero. The option trader does not pay a separate premium on the day of the day the position is put on. On certain other exchanges though it is the other way around, and the option buyer will pay a premium on the day of purchase but then pay no variation margin. Some exchanges allow traders to select either method. Margin is compulsory for a party that writes options on the exchange.

A list of derivatives exchanges is given at Appendix 42.1

The other option market is the OTC market, where there is a great variety of different instruments traded. As with products such as swaps, the significant advantage of OTC options is that they can be tailored to meet the specific requirements of the buyer. Hence they are ideally suited as risk management instruments for corporate and financial institutions, because they can be used to structure hedges that match perfectly the risk exposure of the buying party. Some of the more ingenious structures are described in a later chapter on exotic options.

42.3 Options and payoff profiles

42.3.1 Synthetic structures

The use of options to structure particular cash flow patterns has been referred to as the “building block approach”. Market participants use combinations of calls, puts, futures and the underlying asset to create synthetic positions. For example, consider a long position in the underlying asset together with a long position in a put option on the underlying. What would be the combined effect of such a position? If we look at the option payoff profiles shown earlier at Figure 42.3, we see that if we were to combine a long in the asset with a long in a put on the asset, our payoff profile would resemble that of a long call on the asset. Therefore if a bank wished to synthetically create such an exposure, it could use either structure; the effect would be the same. The rationale behind many option trading strategies is based on an understanding of the make-up and use of such synthetic positions. Several textbooks (for example Galitz (1993), Hull (1997), Briys *et alia* (1999)) have used a particular terminology to depict the net result of synthetic positions, which we will summarise here. A position with a negative-sloping profit profile, such as a short position in a futures contract, is denoted by $\{-1\}$, while a positive sloping profit profile is denoted by $\{1\}$. A flat payoff profile (neither profit nor loss) is denoted by $\{0\}$. Using this terminology, a position in a long call option would have the designation $\{0, 1\}$, which means that if the option expires out-of-the-money there is a flat gain (neither profit nor loss) and if it expires in-the-money there is a profit gain. The basic instruments have the following designations:

Long call: $\{0, 1\}$
 Short call: $\{0, -1\}$
 Long put: $\{-1, 0\}$
 Short put: $\{1, 0\}$

We always assume all options considered have identical strike prices and maturity dates. The example given above would be:

Long the underlying asset and long a put option is equal to long a (synthetic) call
 $\{1, 1\} + \{-1, 0\} = \{0, 1\}$

The basic synthetic positions are given in Table 42.2.

Long the underlying asset and short a call option is equal to short a synthetic put
$\{1, 1\} + \{0, -1\} = \{1, 0\}$
Short the underlying asset and short a put option is equal to short a synthetic call
$\{-1, -1\} + \{1, 0\} = \{0, -1\}$
Short the underlying asset and long a call option is equal to long a synthetic put
$\{-1, -1\} + \{0, 1\} = \{-1, 0\}$
Long a call option and short a put option is equal to long the synthetic underlying asset
$\{0, 1\} + \{1, 0\} = \{1, 1\}$
Short a call option and long a put option is equal to short the synthetic underlying asset
$\{0, -1\} + \{-1, 0\} = \{-1, -1\}$

Table 42.2: Basic synthetic positions.

42.3.2 Basic trading strategies

The ability to create combinations based on a mixture of calls, puts and the underlying gives rise to the various standard option strategies seen in the markets. These include trades such as vertical spreads, ratio spreads, and calendar spreads among others. Other structures are designed as volatility trades and arbitrage trades. The main structures are considered later, but we can introduce them briefly here. A *vertical spread* is the purchase of an option and the sale of another with the same expiry date, but a different strike price. If the strategy is designed to produce an outflow of cash, the holder is said to be “long the spread”. The opposite is termed being “short the spread”. A

calendar spread is composed of a long position in an option and a short position with a shorter maturity date. A *diagonal spread* is the result of a long position in an option and a short, with both a shorter maturity date and a different strike price.

Volatility trades are based on the volatility of the underlying asset, and include *straddles* and *butterflies*. These are considered later. Where a volatility strategy produces a cash outflow, the holder is said to be “long the strategy”, and “short the strategy” otherwise.

The price of an option is a function of six different factors, which are:

- the strike price of the option;
- the current price of the underlying;
- the time to expiry;
- the risk-free rate of interest that applies to the life of the option;
- the volatility of the underlying asset’s price returns;
- the value of any dividends or cash flows paid by the underlying asset during the life of the option.

To illustrate we show the prices of various European options written on the ten-year gilt in October 1999, on a day it was quoted at £99.85. Table 42.3 shows the price of call and put options on the bond at various strike prices, with term to maturity of three months (that is, 0.25 years), risk-free interest rate of 5.60% and volatility level of 6.91%. Prices are mid-prices.

	Underlying price S £99.85, Strike price X								
	97.50	98.00	98.50	99.00	99.50	100.00	100.50	101.00	101.50
Call option, C	7.194	6.908	6.629	6.358	6.095	5.839	5.590	5.349	5.115
Put option, P	3.489	3.696	3.910	4.132	4.361	4.598	4.843	5.095	5.354

Table 42.3: Option premia for different strike prices.

A trader who thought that ten-year yields, or the yield curve as whole, were due to fall in level but only by a relatively small amount might buy the 100 call for a price of 5.839. If on the expiry date the bond was trading at £101.50, the profit is the underlying minus the strike price, net of the option premium paid. On the other hand if the trader felt that the bond would not rise as high as £101.50 she might sell 101.50 call, which will generate premium income of 5.354. This will be realised profit as long as the bond does not reach that price by the expiry date.

42.4 Option pricing parameters

42.4.1 Pricing inputs

Let us consider again the parameters of option pricing. Possibly the two most important are the current price of the underlying and the strike price of the option. The intrinsic value of a call option is the amount by which the strike price is below the price of the underlying, as this is the payoff if the option is exercised. Therefore the value of the call option will increase as the price of the underlying increases, and will fall as the underlying price falls. The value of a call will also decrease as the strike price increases. All this is reversed for a put option.

Generally for bond options a higher time to maturity results in higher option value. All other parameters being equal, a longer-dated option will always be worth at least as much as one that had a shorter life. Intuitively we would expect this because the holder of a longer-dated option has the same benefits as someone holding a shorter-dated option, in addition to a longer time period in which the intrinsic value may increase. This rule is always true for American options, and usually true for European options. However certain factors, such as the payment of a coupon during the option life, may cause a longer-dated option to have only a slightly higher value than a shorter-date option.

The risk-free interest-rate is the rate applicable to the period of the option’s life, so for our table of gilt options in the previous section, the option value reflected the three-month rate. The most common rate used is the T-bill rate, although for bond options it is more common to see the government bond repo rate being used. A rise in interest rates will increase the value of a call option, although not always for bond options. A rise in rates lowers the price of a

bond, because it decreases the present value of future cash flows. However in the equity markets it is viewed as a sign that share price growth rates will increase. Generally however the relationship is the same for bond options as equity options. The effect of a rise in interest rates for put options is the reverse: they cause the value to drop.

A coupon payment made by the underlying during the life of the option will reduce the price of the underlying asset on the ex-dividend date. This will result in a fall in the price of a call option and a rise in the price of a put option.

42.4.2 Bounds in option pricing

The upper and lower limits on the price of an option are relatively straightforward to set because prices must follow the rule of no-arbitrage pricing. A call option grants the buyer the right to buy a specified quantity of the underlying asset, at the level of the strike price, so therefore it is clear that the option could not have a higher value than the underlying asset itself. Therefore the upper limit or *bound* to the price of a call option is the price of the underlying asset. Therefore, $C \leq S$ where C is the price of a call option and S is the current price of the underlying asset. A put option grants the buyer the right to sell a specified unit of the underlying at the strike price X , therefore the option can never have a value greater than the strike price X . So we may set $P \leq X$ where P is the price of the put option. This rule will still apply for a European put option on its expiry date, so therefore we may further set that the option cannot have a value greater than the present value of the strike price X on expiry. That is, $P \leq Xe^{-rT}$ where r is the risk-free interest rate for the term of the option life and T is the maturity of the option in years.

The minimum limit or bound for an option is set according to whether the underlying asset is a dividend-paying security or not. For a call option written on a non-dividend paying security the lower bound is given by:

$$C \geq S - Xe^{-rT}.$$

In fact as we noted early in this chapter a call option can only ever expire worthless, so its intrinsic value can never be worth less than zero. Therefore $C > 0$ and we then set $C \geq \max(S - Xe^{-rT}, 0)$. This reflects the law of no-arbitrage pricing. For put options on a non-dividend paying stock the lower limit is given by $P \geq Xe^{-rT} - S$ and again the value is never less than zero so we may set $P \geq \max(Xe^{-rT} - S, 0)$.

As we noted above, payment of a dividend by the underlying asset affects the price of the option. In the case of dividend paying stocks the upper and lower bounds for options are

$$C \geq S - D - Xe^{-rT}$$

and

$$P \geq D + Xe^{-rT} - S$$

where D is the present value of the dividend payment made by the underlying asset during the life of the option.

We can now look at option pricing in depth, and this is considered in the next chapter.

Appendices

APPENDIX 42.1 World Derivatives Exchanges

“Softs” are commodities such as cotton.

Exchange	Futures	Options
American Stock Exchange		Stock indices
Australian Options Market		Stock indices Precious metals
Chicago Board Options Exchange		Interest rates Stock indices
Chicago Board of Trade	Interest rates Stock indices Precious metals Softs and agriculture	Interest rates Precious metals Softs and agriculture
Chicago Mercantile Exchange (CME)	Softs and agriculture	
Chicago Rice and Cotton Exchange	Softs and agriculture	
Chicago Mercantile Exchange (Index and Options Market)	Stock indices	Interest rates Stock indices Foreign exchange Softs and agriculture
CME-International Monetary Market	Interest rates Foreign exchange	
Coffee, Sugar & Cocoa Exchange	Stock indices	Stock indices
Commodity Exchange Inc.	Interest rates Precious metals Base metals	
Copenhagen Stock Exchange	Interest rates Stock indices	Interest rates Stock indices
European Options Exchange NV		Interest rates Stock indices Foreign exchange Precious metals
Financial Instrument Exchange	Interest rates Foreign exchange	Interest rates Foreign exchange
The Futures & Options Exchange–London	Softs and agriculture	Softs and agriculture
Hong Kong Futures Exchange Ltd.	Stock indices Precious metals Softs and agriculture	

Exchange	Futures	Options
Kansas City Board of Trade	Stock indices Softs and agriculture	Softs and agriculture
London Grain Futures Market	Softs and agriculture	Softs and agriculture
London International Financial Futures Exchange	Interest rates Stock indices Foreign exchange	Interest rates Foreign exchange
London Metal Exchange	Precious metals	Precious metals
Marché à Terme International de France (MATIF)	Interest rates Stock indices Softs and agriculture	Interest rates Softs and agriculture
Marché des Options Négociables de la Bourse de Paris (MONEP)		Stock indices
Mid-American Commodity Exchange	Interest rates Foreign exchange Precious metals Softs and agriculture	Precious metals Softs and agriculture
Minneapolis Grain Exchange	Softs and agriculture	Softs and agriculture
The Montreal Exchange	Interest rates	Interest rates Precious metals
Nagoya Grain & Sugar Exchange	Softs and agriculture	
Nagoya Stock Exchange	Stock indices	Stock indices
New York Cotton Exchange	Softs and agriculture	Softs and agriculture
New York Futures Exchange	Interest rates Stock indices	Stock indices
New York Stock Exchange		Stock indices
Osaka Securities Exchange	Stock indices	Stock indices
Osaka Sugar Exchange	Softs and agriculture	
Osaka Textile Exchange	Softs and agriculture	
Pacific Stock Exchange		Stock indices
Philadelphia Board of Trade	Stock indices Foreign exchange	
Philadelphia Stock Exchange		Stock indices Foreign exchange
Rotterdam Energy Futures Exchange	Energy, oil and gas	

Exchange	Futures	Options
Singapore International Monetary Exchange	Interest rates Stock indices Foreign exchange Precious metals	Interest rates Foreign exchange
Swiss Options and Financial Futures Exchange	Stock indices	Stock indices
Sydney Futures Exchange Ltd.	Interest rates Stock indices Foreign exchange Softs and agriculture	Interest rates Stock indices Foreign exchange
Tokyo International Financial Futures Exchange	Interest rates Foreign exchange	
Toronto Futures Exchange	Interest rates Stock indices	Interest rates Precious metals

Selected Bibliography and References

- Chance, D., *An Introduction to Options and Futures Markets*, Dryden Press, 1989.
 Eales, B., *Financial Engineering*, MacMillan, 1996.
 Galitz, L., *Financial Engineering*, FT Pitman, 1993.
 Grabbe, J.O., *International Financial Markets*, 2nd edition, Elsevier, 1991.
 Levy, H., *Introduction to Investments*, 2nd edition, South-Western Publishing, 1999, Chapters 22–25.
 London International Financial Futures and Options Exchange, *LIFFE Options*, LIFFE, 1999.
 Livingstone, M., *Money and Capital Markets*, 2nd edition, New York Institute of Finance, 1993, Chapter 19.
 McMillan, L., *Options as a Strategic Investment*, New York Institute of Finance, 1986.

Questions and exercises

- What is the difference between writing a put option and buying a call option? Illustrate the potential profit profile for each instrument.
- A building society wishes to hedge its holding of sterling bonds and is considering both futures and options. Discuss the pros and cons of both methods.
- What are the constituents of the value of an option?
- How is a synthetic long call position set up?
- What is the combined effect of a long call option and short put option on the same underlying asset, with identical maturities and strike prices?
- What parameters are used in valuing an option?
- A non-dividend paying asset has a current price of £10. What is the lower bound on the price of a three-month call option written on this asset struck at £8? The three-month risk-free interest rate is 6.00%.
- The asset in the previous question has a semi-annual coupon of 5%. If its ex-dividend date falls during the life of the option, what would its lower bound be then?

43

The Dynamics of Asset Prices

The pricing of derivative instruments such as options is a function of the movement in the price of the underlying asset over the lifetime of the option, and in fact valuation models describe an environment where the price of an option is related to the behaviour process of the variables that drive asset prices. This process is described as a *stochastic* process, and pricing models describe the stochastic dynamics of asset price changes, whether this is change in share prices, interest rates, foreign exchange rates or bond prices. To understand the mechanics of option pricing therefore, we must familiarise ourselves with the behaviour of functions of *stochastic variables*. The concept of a stochastic process is a vital concept in finance theory. It describes random phenomena that evolve over time, and these include asset prices. For this reason an alternative title for this chapter could be *An Introduction to Stochastic Processes*.

This is a text on bonds after all, not mathematics, and it is outside the scope of this book comprehensively to derive and prove the main components of dynamic asset pricing theory. There are a number of excellent textbooks that the reader is encouraged to read which provide the necessary detail, in particular Ingersoll (1987), Baxter and Rennie (1996), Neftci (1996) and James and Webber (2000). Another recommended text that deals with probability models in general, as well as their application in derivatives pricing, is Ross (2000). In this chapter we review the basic principles of the dynamics of asset prices, which are then put into context in the following chapter, which looks at the Black–Scholes model. The main principles are then considered again in the context of yield curve modelling, in Part VIII.

43.1 The behaviour of asset prices

The first property that asset prices, which can be taken to include interest rates, are assumed to follow is that they are part of a *continuous* process. This means that the value of any asset can and does change at any time and from one point in time to another, and can assume any fraction of a unit of measurement. It is also assumed to pass through every value as it changes, so for example if the price of a bond moves from 92.00 to 94.00 it must also have passed through every point in between. This feature means that the asset price does not exhibit *jumps*, which in fact is not the case in many markets, where price processes do exhibit jump behaviour. For now however we may assume that the price process is continuous.

43.1.1 Stochastic processes

Models that seek to value options or describe a yield curve also describe the dynamics of asset price changes. The same process is said to apply to changes in share prices, bond prices, interest rates and exchange rates. The process by which prices and interest rates evolve over time is known as a *stochastic process*, and this is a fundamental concept in finance theory.¹ Essentially a stochastic process is a time series of random variables. Generally the random variables in a stochastic process are related in a non-random manner, and so therefore we can capture in a *probability density function*. A good introduction is given in Neftci (1996), and following his approach we very briefly summarise the main features here.

Consider the function $y = f(x)$; given the value of x we can obtain the value of y . If we denote the set W as the state of the world, where $w \in W$, the function $f(x, w)$ has the property that given a value $w \in W$ it becomes a function of x only. If we say that x represents the passage of time, two functions $f(x, w_1)$ and $f(x, w_2)$ will be different because the second element w in each case is different. With x representing time, these two functions describe two different processes that are dependent on different states of the world W . The element w represents an underlying random process, and so therefore the function $f(x, w)$ is a *random function*. A random function is also called a *stochastic process*, one in which x represents time and $x \geq 0$. The random characteristic of the process refers to the entire process, and not any particular value in that process at any particular point in time.

Examples of functions include the *exponential* function denoted by $y = e^x$ and the *logarithmic* function $\log_e(y) = x$.

¹ A formal definition of a stochastic process is given in Appendix 43.1.

The price processes of shares and bonds, as well as interest rate processes, are stochastic processes. That is, they exhibit a random change over time. For the purposes of modelling, the change in asset prices is divided into two components. These are the *drift* of the process, which is a *deterministic* element,² also called the mean, and the random component known as the *noise*, also called the volatility of the process.

We introduce the drift component briefly as follows. For an asset such as an ordinary share, which is expected to rise over time (at least in line with assumed growth in inflation), the drift can be modelled as a geometric growth progression. If the price process had no “noise”, the change in price of the stock from over the time period dt can be given by

$$\frac{dS_t}{dt} = \mu S_t \quad (43.1)$$

where the term μ describes the growth rate. Expression (43.1) can be rewritten in the form

$$dS_t = \mu S_t dt \quad (43.2)$$

which can also be written in integral form. For interest rates, the movement process can be described in similar fashion, although as we shall see interest rate modelling often takes into account the tendency for rates to return to a mean level or range of levels, a process known as *mean reversion*. Without providing the derivation here, the equivalent expression for interest rates takes the form

$$dr_t = \alpha(\mu - r_t)dt \quad (43.3)$$

where α is the mean reversion rate that determines the pace at which the interest rate reverts to its mean level. If the initial interest rate is less than the drift rate, the rate r will increase, while if the level is above the drift rate it will tend to decrease.

For the purposes of employing option pricing models the dynamic behaviour of asset prices are usually described as a function of what is known as a *Weiner process*, which is also known as *Brownian motion*. The noise or volatility component is described by an *adapted* Brownian or Wiener process, and involves introducing a random increment to the standard random process. This is described next.

43.1.2 Wiener process or Brownian motion

The stochastic process we have briefly discussed above is known as Brownian motion or a Wiener process. In fact a Wiener process is only a process that has a mean of 0 and a variance of 1, but it is common to see these terms used synonymously. Wiener processes are a very important part of continuous-time finance theory, and interested readers can obtain more detailed and technical data on the subject in Neftci (1996) and Duffie (1996)³ among others. It is a well-researched subject.

One of the properties of a Wiener process is that the sample pathway is continuous, that is, there are no *discontinuous* changes. An example of a discontinuous process is the Poisson process. Both are illustrated in Figures 43.1 and 43.2 below.

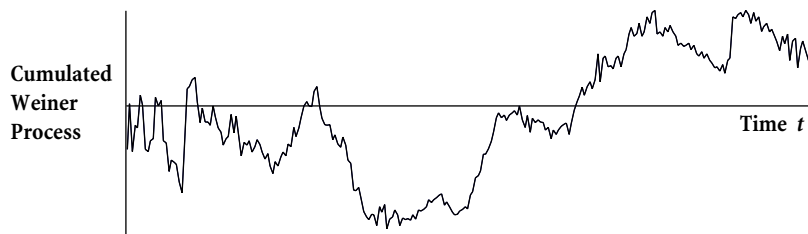


Figure 43.1: An example of a Wiener process.

² There are two types of model: *deterministic*, which involves no randomness so the variables are determined exactly; and *stochastic*, which incorporates the random nature of the variables into the model.

³ Duffie's text requires a very good grounding in continuous-time mathematics.

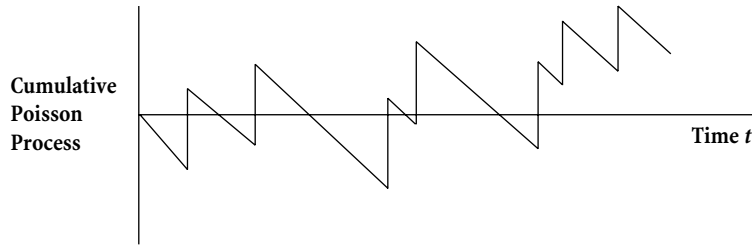


Figure 43.2: An example of a Poisson process.

In the examples illustrated, both processes have an expected change of 0 and a variance of 1 per unit of time. There are no discontinuities in the Wiener process, which is a plot of many very tiny random changes. This is reflected in the “fuzzy” nature of the sample path. However the Poisson process has no fuzzy quality and appears to have a much smaller number of random changes. We can conclude that asset prices, and the dynamics of interest rates, are more akin to a Wiener process. This, therefore is how asset prices are modelled. From observation we know that, in reality asset prices and interest rates do exhibit discontinuities or *jumps*, however there are other advantages to assuming a Wiener process, and in practice because continuous-time stochastic processes can be captured as a combination of Brownian motion and a Poisson process, analysts and researchers use the former as the basis of financial valuation models.

The first step in asset pricing theory builds on the assumption that prices follow a Brownian motion. The properties of Brownian motion W state that it is continuous, and the value of W_t ($t > 0$) is normally distributed under a probability measure P as a *random* variable with parameters $N(0, t)$. An incremental change in the asset value over time dt , which is a very small or *infinitesimal* change in the time, given by $W_{s+dt} - W_s$, is also normally distributed with the parameters $N(0, t)$ under P . Perhaps the most significant feature is that the change in value is independent of whatever the history of the price process has been up to time s . If a process follows these conditions it is Brownian motion. In fact asset prices do not generally have a mean of 0, because over time we expect them to rise. Therefore modelling asset prices incorporates a *drift* measure that better reflects asset price movement, so that an asset movement described by

$$S_t = W_t + \mu t \quad (43.4)$$

would be a Brownian motion with a drift given by the constant μ . A second parameter is then added, a *noise* factor, which scales the Brownian motion by another constant measure, the standard deviation σ . The process is then described by

$$S_t = \sigma W_t + \mu t \quad (43.5)$$

which can be used to *simulate* the price path taken by an asset, as long as we specify the two parameters. An excellent and readable account of this is given in Baxter and Rennie (1996, Chapter 3), who also state that under (43.5) there is a possibility of achieving negative values, which is not realistic for asset prices. However using the exponential of the process given by (43.5) is more accurate, and is given by (43.6):

$$S_t = \exp(\sigma W_t + \mu t). \quad (43.6)$$

Brownian motion or the *Weiner process* is employed by virtually all option pricing models, and we introduce it here with respect to a change in the variable W over an interval of time t . If W represents a variable following a Wiener process and ΔW is a change in value over a period of time t , the relationship between ΔW and Δt is given by (43.7):

$$\Delta W = \varepsilon \sqrt{\Delta t} \quad (43.7)$$

where ε is a random sample from a normal distribution with a mean 0 and a standard deviation of 1. Over a short period of time the values of ΔW are independent and therefore also follow a normal distribution with a mean 0 and a standard deviation of $\sqrt{\Delta t}$. Over a longer time period T made up of N periods of length Δt , the change in W over the period from time 0 to time T is given by (43.8):

$$W(T) - W(0) = \sum_{i=1}^N \varepsilon_i \sqrt{\Delta t}. \quad (43.8)$$

The successive values assumed by W are serially independent so from (43.8) we conclude that changes in the variable W from time 0 to time T follow a normal distribution with mean 0 and a standard deviation of $\sqrt{T}$. This describes the Weiner process, with a mean of zero or a zero drift rate and a variance of T . This is an important result because a zero drift rate implies that the change in the variable (for which now read asset price) in the future is equal to the current change. This means that there is an equal chance of an asset return ending up 10% or down 10% over a long period of time.

The next step in the analysis involves using stochastic calculus. Without going into this field here, we summarise from Baxter and Rennie (1996) and state that a stochastic process X will incorporate a *Newtonian* term that is based on dt and a Brownian term based on the infinitesimal increment of W that is denoted by dW_t . The Brownian term has a “noise” factor of σ_t . The infinitesimal change of X at X_t is given by the differential equation

$$dX_t = \sigma_t dW_t + \mu_t dt \quad (43.9)$$

where σ_t is the *volatility* of the process X at time t and μ_t is the drift of X at time t . For interest rates that are modelled on the basis of mean reversion, the process is given by

$$dr_t = \sigma_t dW_t + \alpha(u_t - r_t)dt \quad (43.10)$$

where the mean reverting element is as before. Without providing the supporting mathematics, which we have not covered here, the process described by (43.10) is called an Ornstein–Uhlenbeck process, and has been assumed by a number of interest rate models.

One other important point to introduce here is that a random process described by (43.10) operates in a continuous environment. In continuous-time mathematics the *integral* is the tool that is used to denote the sum of a infinite number of objects, that is where the number of objects is *uncountable*. A formal definition of the integral is outside the scope of this book, but accessible accounts can be found in the texts referred to previously. A basic introduction is given at Appendix 43.4. However the continuous stochastic process X described by (43.9) can be written as an integral equation in the form

$$X_t = X_0 + \int_0^t \sigma_s dW_s + \int_0^t \mu_s ds \quad (43.11)$$

where σ and μ are processes as before. The volatility and drift terms can be dependent on the time t but can also be dependent on X or W up to the point t . This is a complex technical subject and readers are encouraged to review the main elements in the referred texts.

43.1.3 The Martingale property

Continuous time asset pricing is an important part of finance theory and involves some quite advanced mathematics. An excellent introduction to this subject is given in Baxter and Rennie (1996) and Neftci (1996). A more technical account is given in Williams (1991). It is outside the scope of this book to derive, prove and detail the main elements. However we wish to summarise the essential property, and begin by saying that in continuous time, asset prices can take on an unlimited number of values. Stochastic differential equations are used to capture the dynamics of asset prices in a generalised form. So for example, as we saw in the previous section an incremental change in the price of an asset S at time t could be given by

$$dS = \mu S dt + \sigma S dW(t) \quad (43.12)$$

where

dS	is an infinitesimal change in the price of asset S
$\mu S dt$	is the predicted movement during the infinitesimal time interval dt
$\sigma S dW(t)$	is an unpredictable random shock.

Martingale theory is a branch of mathematics that classifies the *trend* in an observed time series set of data. A stochastic process is said to behave like a martingale if there are no observable trends in its pattern. The Martingale

property is often used in conjunction with a Wiener process to describe asset price dynamics. The notion of the martingale property is that the best approximation of a set of integrable random variables M at the end of a time period t is M_0 , which essentially states that the most accurate way to predict a future asset price is to use the price of the asset now. That is, using the price today is the same as using all available historical information, as only the newest information regarding the asset is relevant.

We do not describe or prove this property here but the Martingale property is used to derive (43.13), the price of an asset at time t .

$$P_t = \exp(\sigma W_t - \frac{1}{2}\sigma^2 t). \quad (43.13)$$

A martingale is an important type of stochastic process and the concept of a martingale is fundamental to asset pricing theory. A process that is a martingale is one in which the expected future value, based on what is known up to now, is the same as today's value. That is a martingale is a process in which the *conditional* expected future value, given current information, is equal to the current value. The martingale representation theorem states that given a Wiener process, and the fact that the path of the Wiener process up to that point is known, then any martingale is equal to a constant plus a stochastic integral, with respect to the Wiener process. This can be written as

$$E_t[S_T] = S_t \text{ for } t \leq T. \quad (43.14)$$

Therefore a stochastic process that is a martingale has no observable *trend*. The price process described by (43.9) is not a martingale unless the drift component μ is equal to zero, otherwise a trend will be observed. A process that is observed to trend upwards is known as a *submartingale*, while a process that on average declines over time is known as a *supermartingale*.

What is the significance of this? Here we take it as given that because price processes can be described as *equivalent martingale measures* (which we do not go into here) they enable the practitioner to construct a risk-free hedge of a market instrument. By enabling a no-arbitrage portfolio to be described, a mathematical model can be set up and solved, including risk-free valuation models.

The background and mathematics to martingales can be found in Harrison and Kreps (1979) and Harrison and Pliska (1981) as well as Baxter and Rennie (1996). For a description of how, given that price processes are martingales, we are able to price derivative instruments see James and Webber (2000, Chapter 4).

43.1.4 Generalised Wiener process

The standard Wiener process is a close approximation of the behaviour of asset prices but does not account for some specific aspects of market behaviour. In the first instance the prices of financial assets do not start at zero, and their price increments have positive mean. The variance of asset price moves is also not always unity. Therefore the standard Wiener process is replaced by the generalised Wiener process, which describes a variable that may start at something other than zero, and also has incremental changes that have a mean other than zero as well as variances that are not unity. The mean and variance are still constant in a generalised process, which is the same as the standard process, and a different description must be used to describe processes that have variances that differ over time; these are known as stochastic integrals.

We now denote the variable as X and for this variable a generalised Wiener process is given by (43.15):

$$dX = a dt + b dW \quad (43.15)$$

where a and b are constants. This expression describes the dynamic process of the variable X as a function of time and dW . The first term $a dt$ is known as the deterministic term and states that the expected drift rate of X over time is a per unit of time; the second term $b dW$ is the stochastic element and describes the variability of the move in X over time, and is quantified by b multiplied by the Wiener process. When the stochastic element is zero, $dX = a dt$, or put another way

$$dX/dt = a.$$

From this we state that at time 0, $X = X_0 + at$. This enables us to describe the price of an asset, given its initial price, over a period of time. That is, the value of X at any time is given by its initial value at time 0, which is X_0 , together with its drift multiplied by the length of the time period. We can restate (43.15) to apply over a long time period Δt , shown as (43.16):

$$\Delta X = a\Delta t + b\epsilon\sqrt{\Delta t}. \quad (43.16)$$

As with the standard Wiener process ΔX has a normal distribution with mean $a\Delta t$ and standard deviation $b\sqrt{\Delta t}$.

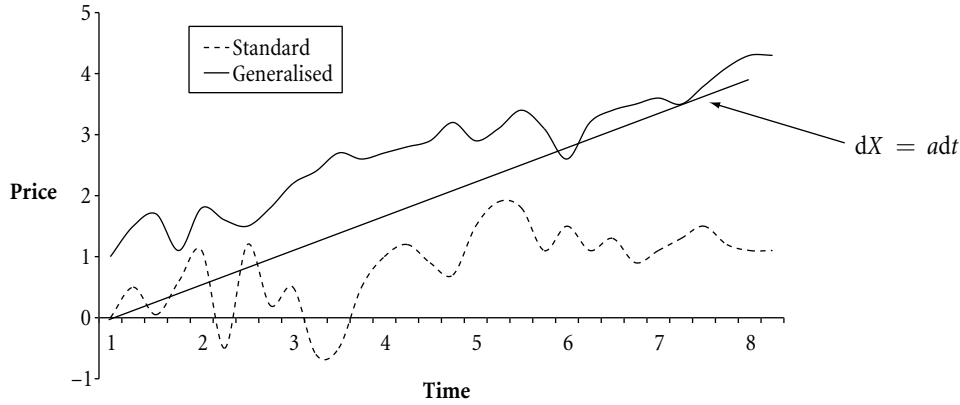


Figure 43.3: Standard and generalised Wiener processes.

The generalised Wiener process is more flexible than the standard one but is still not completely accurate as a model of the behaviour of asset prices. It has normally distributed values, which means that there is a probability of observing negative prices. For assets such as equities, this is clearly unrealistic. In addition the increments of a Wiener process are additive whereas the increments of asset prices are more realistically multiplicative. In fact as the increments of a Wiener process have constant expectation, this implies that the percentage incremental change in asset prices, or the percentage rate of return on the stock, would be declining as the stock price rises. This is also not realistic. For this reason a geometric process or geometric Brownian motion has been introduced,⁴ which is developed by an exponential transformation of the generalised process. From (43.16), a one-dimensional process is a geometric Brownian motion if it has the form e^X , where X is a one-dimensional generalised Brownian motion with a deterministic initial value of $X(0)$.

Another type of stochastic process is an Itô process. This is a generalised Wiener process where the parameters a and b are functions of the value of the variable X and time t . An Itô process for X can be written as (43.17).

$$dX = a(X, t)dt + b(X, t)dW \quad (43.17)$$

The expected drift rate and variance of an Itô process are liable to change over time; indeed the dependence of the expected drift rate and variance on X and t is the main difference between it and a generalised Wiener process. The derivation of Itô's formula is given at Appendix 43.3.

43.1.5 A model of the dynamics of asset prices

The above discussion is used to derive a model of the behaviour of asset prices sometimes referred to as *geometric Brownian motion*. The dynamics of the asset price X are represented by the Itô process shown at (43.18), where there is a drift rate of a and a variance rate of $b^2 X^2$,

$$dX = aX dt + bX dW. \quad (43.18)$$

so that

$$\frac{dX}{X} = a + b dW.$$

The uncertainty element is described by the Wiener process element, with

$$dW = \epsilon\sqrt{dt}$$

⁴ See for instance, Nielsen (1999).

where ε is the error term, a random sample from the standardised normal distribution, so that $\varepsilon \sim N(0,1)$. From this, and for over a longer period of time Δt we can write

$$\frac{\Delta X}{X} = a\Delta t + b\varepsilon\sqrt{\Delta t}.$$

Over this longer period of time, for application in a discrete-time environment, if we assume that volatility is zero, we have

$$\Delta X = a\Delta t + b\varepsilon\sqrt{\Delta t} \quad (43.19)$$

and

$$dX = a dt \text{ and } \frac{dX}{dt} = a.$$

Return is given by $X = X_0 e^{at}$.

The discrete time version of the asset price model states that the proportional return on the asset price X over a short time period is given by an expected return of $a\Delta t$ and a stochastic return of $b\varepsilon\Delta t$. Therefore the returns of asset price changes $\Delta X/X$ is normally distributed with a mean of $a\Delta t$ and a standard deviation of $b\sqrt{\Delta t}$. This is the distribution of asset price returns and is given by (43.20):

$$\frac{\Delta X}{X} \sim N(a\Delta t, b\sqrt{\Delta t}). \quad (43.20)$$

EXAMPLE 43.1

A conventional bond has an expected return of 5.875% and a standard deviation of 12.50% per annum. The initial price of the bond is 100. From (43.20) the dynamics of the bond price are given by:

$$dP/P = 0.05875 dt + 0.125 dW$$

and for a time period Δt by $dP/P = 0.05875\Delta t + 0.125\varepsilon\sqrt{\Delta t}$.

If the short time interval Δt is four weeks or 0.07692 years, assuming $\varepsilon = 1$, then the increase in price is given by:

$$\begin{aligned} \Delta P &= 100(0.05875(0.07692) + 0.125\varepsilon\sqrt{0.07692}) \\ &= 100(0.00451905 + 0.0346681\varepsilon). \end{aligned}$$

So the price increase is described as a random sample from a normal distribution with a mean of 0.452 and a volatility of 3.467. Over a time interval of four weeks $\Delta P/P$ is normal with:

$$\Delta P/P \sim N(0.00452, 0.001202).$$

43.1.6 The distribution of the risk-free interest rate

The continuously compounded rate of return is an important component of option pricing theory. If r is the continuously compounded rate of return, we can use the lognormal property to determine the distribution that this follows. At a future date T the asset price S may be written as (41.12):

$$S_T = S_t e^{r(T-t)} \quad (43.21)$$

$$\text{and } r = \frac{1}{T-t} \ln\left(\frac{S_T}{S_t}\right).$$

Using the lognormal property we can describe the distribution of the risk-free rate as:

$$r \sim N\left(\left(\mu - \frac{1}{2}\sigma^2\right), \frac{\sigma}{\sqrt{T-t}}\right). \quad (43.22)$$

43.2 Stochastic calculus models: Brownian motion and Itô calculus

We noted at the start of the chapter that the price of an option is a function of the price of the underlying stock and its behaviour over the life of the option. Therefore this option price is determined by the variables that describe the process followed by the asset price over a continuous period of time. The behaviour of asset prices follows a stochastic process, and so option pricing models must capture the behaviour of stochastic variables behind the movement of asset prices. To accurately describe financial market processes a financial model will depend on more than one variable. Generally a model is constructed where a function is itself a function of more than one variable. Itô's lemma, the principal instrument in continuous time finance theory, is used to differentiate such functions. This was developed by a mathematician, K. Itô, in 1951. Here we simply state the theorem, as a proof and derivation are outside the scope of the book. Interested readers may wish to consult Briys *et al.* (1998), and Hull (1997) for a background on Itô's lemma; we also recommend Neftci (1996). Basic background on Itô's lemma is given in appendices 43.2 and 43.3.

43.2.1 Brownian motion

Brownian motion is very similar to a Wiener process, which is why it is common to see the terms used interchangeably. Note that the properties of a Wiener process requires that it be a martingale, while no such constraint is required for a Brownian process. A mathematical property known as the *Lévy theorem* allows us to consider any Wiener process W_t with respect to an information set F_t as a Brownian motion Z_t with respect to the same information set.

We can view Brownian motion as a continuous time *random walk*, visualised as a walk along a line, beginning at $X_0 = 0$ and moving at each incremental time interval dt either up or down by an amount $\sqrt{dt}$. If we denote the position of the walk as X_n after the n th move, the position would be

$$X_n = X_{n-1} \pm \sqrt{dt}, \quad n = 1, 2, 3, \dots \quad (43.23)$$

where the $+$ and $-$ signs occur with an equal probability of $1/2$. This is a simple random walk. We can transform this into a continuous path by applying linear interpolation between each move point, so that

$$\bar{X}_t = X_n + (t - ndt) \cdot (X_{n+1} - X_n), \quad ndt \leq t \leq (n+1)dt. \quad (43.24)$$

It can be shown (but not here) that the path described at (43.24) has a number of properties, including that the incremental change in value each time it moves is independent of the behaviour leading up to the move, and that the mean value is 0 and variance is finite. The mean and variance of the set of moves is independent of dt .

What is the importance of this? Essentially this: the probability distribution of the motion can be shown, as dt approaches 0, to be normal or *Gaussian*.

43.2.2 Stochastic calculus

Itô's theorem provides an analytical formula that simplifies the treatment of stochastic differential equations, which is why it is so valuable. It is an important rule in the application of stochastic calculus to the pricing of financial instruments. Here we briefly describe the power of the theorem.

The standard stochastic differential equation for the process of an asset price S_t is given in the form

$$dS_t = a(S_t, t)dt + b(S_t, t)dW_t \quad (43.25)$$

where $a(S_t, t)$ is the drift coefficient and $b(S_t, t)$ is the volatility or *diffusion* coefficient. The Wiener process is denoted dW_t and is the unpredictable events that occur at time intervals dt . This is sometimes denoted dZ or dz .

Consider a function $f(S_t, t)$ dependent on two variables S and t , where S follows a random process and varies with t . If S_t is a continuous-time process that follows a Wiener process W_t , then it directly influences the function $f()$ through the variable t in $f(S_t, t)$. Over time we observe new information about W_t as well the movement in S over each time increment, given by dS_t . The sum of both these effects represents the *stochastic differential* and is given by the stochastic equivalent of the chain rule known as *Itô's lemma*. So for example, if the price of a stock is 30 and an incremental time period later is $30\frac{1}{2}$, the differential is $\frac{1}{2}$.

If we apply a Taylor expansion in two variables to the function $f(S_t, t)$ we obtain

$$df_t = \frac{\partial f}{\partial S_t} dS_t + \frac{\partial f}{\partial t} dt + \frac{1}{2} \frac{\partial^2 f}{\partial S_t^2} b_t^2 dt. \quad (43.26)$$

Remember that ∂t is the partial derivative while dt is the derivative.

If we substitute the stochastic differential equation (43.25) for S_t we obtain *Itô's lemma* of the form

$$df_t = \left(\frac{\partial f}{\partial S_t} a_t + \frac{\partial f}{\partial t} + \frac{1}{2} \frac{\partial^2 f}{\partial S_t^2} b_t^2 \right) dt + \frac{\partial f}{\partial S_t} b_t dW_t. \quad (43.27)$$

What we have done is taken the stochastic differential equation ("SDE") for S_t and transformed it so that we can determine the SDE for f_t . This is absolutely priceless, a valuable mechanism by which we can obtain an expression for pricing derivatives that are written on an underlying asset whose price can be determined using conventional analysis. In other words, using Itô's formula enables us to determine the SDE for the derivative, once we have set up the SDE for the underlying asset. This is the value of Itô's lemma.

The SDE for the underlying asset S_t is written in most textbooks in the following form

$$dS_t = \mu S_t dt + \sigma S_t dW_t \quad (43.28)$$

which has simply denoted the drift term $a(S_t, t)$ as μS_t and the diffusion term $b(S_t, t)$ as σS_t . In the same way Itô's lemma is usually seen in the form

$$dF_t = \left[\frac{\partial F}{\partial S_t} \mu S_t + \frac{\partial F}{\partial t} + \frac{1}{2} \frac{\partial^2 F}{\partial S_t^2} \sigma^2 S_t^2 \right] dt + \frac{\partial F}{\partial S_t} \sigma S_t dW_t \quad (43.29)$$

although the noise term is sometime denoted dZ . Further applications are illustrated in the box.

EXAMPLE 43.2(i) Lognormal distribution

A variable (such as an asset price) may be assumed to have a *lognormal distribution* if the natural logarithm of the variable is normally distributed. So if an asset price S follows a stochastic process described by

$$dS = \mu S dt + \sigma S dW \quad (43.30)$$

how would we determine the expression for $\ln S$? This can be achieved using Itô's lemma.

If we say that $F = \ln S$, then the first derivative

$$\frac{dF}{dS} = \frac{1}{S} \text{ and as there is no } t \text{ we have } \frac{dF}{dt} = 0.$$

The second derivative is $\frac{d^2 F}{dS^2} = \frac{-1}{S^2}$.

We substitute these values into Itô's lemma given at (43.29) and this gives us

$$d \ln S = \left(\mu - \frac{\sigma^2}{2} \right) dt + \sigma dW. \quad (43.31)$$

So we have moved from dF to dS using Itô's lemma, and (43.31) is a good representation of the asset price over time.

43.2(ii) The bond price equation

The continuously compounded gross redemption yield at time t on a default-free zero-coupon bond that pays £1 at maturity date T is x . We assume that the movement in x is described by

$$dx = a(\alpha - x)dt + sx dZ$$

where a, α and s are positive constants. What is the expression for the process followed by the price P of the bond? Let us say that the price of the bond is given by

$$P = e^{-x(T-t)}.$$

We have dx , and we require dP . This is done by applying Itô's lemma. We require

$$\begin{aligned}\frac{\partial P}{\partial x} &= -(T-t)e^{-x(T-t)} = -(T-t)P \\ \frac{\partial^2 P}{\partial x^2} &= -(T-t)e^{-x(T-t)} = (T-t)^2 P \\ \frac{\partial P}{\partial t} &= xe^{-x(T-t)} = xP\end{aligned}$$

From Itô's lemma

$$dP = \left[\frac{\partial P}{\partial x} a(\alpha - x) + \frac{\partial P}{\partial t} + \frac{1}{2} \frac{\partial^2 P}{\partial x^2} s^2 x^2 \right] dt + \frac{\partial P}{\partial x} sx dZ$$

which gives

$$dP = \left[-(T-t)Pa(\alpha - x) + xP + \frac{1}{2}(T-t)^2 Ps^2 x^2 \right] dt - (T-t)Psx dZ$$

which simplifies to

$$dP = \left[-a(\alpha - x)(T-t) + x + \frac{1}{2}s^2 x^2 (T-t)^2 \right] P dt - sx(T-t)P dZ.$$

So using Itô's lemma we have transformed the SDE for the bond yield into an expression for the bond price.

43.2.3 Stochastic integrals

Whilst in no way wishing to trivialise the mathematical level, we will not consider the derivations here, but simply state that the observed values of the Brownian motion up to the point at time t determine the process immediately after, and that this process is Gaussian. Stochastic integrals are continuous path martingales. As described in Neftci (1996), the integral is used to calculate sums where we have an infinite or uncountable number of items, in contrast with the Σ sum operator which is used for a finite number of objects. In defining integrals we begin with an approximation, where there is a countable number of items, and then set a limit and move to an uncountable number. A basic definition is given in Appendix 43.4. Stochastic integration is an operation that is closely associated with Brownian paths; a path is partitioned into consecutive intervals or increments, and each increment is multiplied by a random variable. These values are then summed to create the stochastic integral. Therefore the stochastic integral can be viewed as a random walk Brownian motion with increments that have varying values, a random walk with non-homogeneous movement.

43.2.4 Generalised Itô formula

It is possible to generalise Itô's formula in order to produce a multi-dimensional formula, which can then be used to construct a model to price interest-rate derivatives or other asset-class options where there is more than one variable. To do this we generalise the formula to apply to situations where the dynamic function $f(\cdot)$ is dependent on more than one Itô process, each expressed as standard Brownian motions.

Consider $W_T = (W_t^1, \dots, W_t^n)$ where $(W_t^i)_{t \geq 0}$ are independent standard Brownian motions and W_T is an n -dimensional Brownian motion. We can express Itô's formula mathematically with respect to p Itô processes $(X_t^1, \dots, X_t^p)$ as:

$$X_T^i = X_0^i + \int_0^t K_s^i ds + \sum_{j=1}^n \int_0^t H_s^{ij} dX_s^j \quad (43.32)$$

Where the function $f(\cdot)$ contains second-order partial derivatives with respect to x and first-order partial derivatives with respect to t , which are a continuous function in (x, t) , the generalised Itô formula is given by

$$\begin{aligned}
f(t, X_t^1, \dots, X_t^p) &= f(0, X_0^1, \dots, X_0^p) + \int_0^t \left(\frac{\partial f}{\partial s} \right) (s, X_s^1, \dots, X_s^p) ds \\
&\quad + \sum_{i=1}^p \int_0^t \left(\frac{\partial f}{\partial x_i} \right) (s, X_s^1, \dots, X_s^p) dX_s^i \\
&\quad + \frac{1}{2} \sum_{i,j=1}^p \int_0^t \left(\frac{\partial^2 f}{\partial x_i \partial x_j} \right) (s, X_s^1, \dots, X_s^p) d(X^i, X^j)_s
\end{aligned} \tag{43.33}$$

with

$$\begin{aligned}
dX_s^i &= K_s^i ds + \sum_{j=1}^n H_s^{im} dW_s^j \\
d(X^i, X^j)_s &= \sum_{m=1}^n H_s^{im} H_s^{jm} ds.
\end{aligned} \tag{43.34}$$

43.2.5 Information structures

A key element of the description of a stochastic process is a specification of the level of information on the behaviour of prices that is available to an observer at each point in time. As with the martingale property, a calculation of the expected future values of a price process requires information on current prices. Generally financial valuation models require data on both the current and the historical security prices, but investors are only able to deal on the basis of current known information, and do not have access to future information. In a stochastic model, this concept is captured via the process known as *filtration*.

A filtration is a family $F = (F_t), t \in T$ of variables $F_t \subset F$ which is increasing in level in the sense that $F_s \subset F_t$ whenever $s, t \in T, s \leq t$. Hence a filtration can be viewed as a dynamic information structure, and F_t represents the information available to the investor at time t . The behaviour of the asset price is seen by the increase in filtration, which implies that more and more data is assimilated over time, and historical data is incorporated into the current price, rather than disregarded or forgotten. A filtration $F = (F_t)$ is said to be *augmented* if F_t is augmented for each time t . This means that only F_0 is augmented. A stochastic process W is described as being *adapted* to the filtration F if for each fixed $t \in T$, the random variable X :

$$X_t : w \mapsto X(w, t) = X_t(w) = X_t(w) : \Omega \rightarrow \mathbb{R}^K$$

is measurable with respect to F_t .

This is an important description as it means that the value X_t of X at t is dependent only on information that is available at time t . It might also mean that an investor with access to the information level F is able to observe or make inferences on the value of X at each point in time. The augmented filtration generated by X is the filtration $F^X = (F_t^X), t \in T$. Any stochastic process X is adapted to the augmented filtration F^X that it generates. If a stochastic process is measurable as a mapping or vector then it is a measurable process, however this does not impact significantly on finance theory so we shall ignore it.

43.3 Perfect capital markets

One of the assumptions of derivative pricing is that the financial markets are assumed to be near-perfect, for example akin to Fama's semi-strong or strong-form market. The term *complete* market is also used. Essentially the market is assumed to be a general stochastic economy where transactions may take place at any time, and interest rates behave under *Gaussian uncertainty*. We shall look at this briefly later in this section. Generally pricing models assume that there is an almost infinite number of tradable assets in the market, so that markets are assumed to be complete. This includes the assumptions that there is frictionless continuous trading, with no transaction costs or taxation.

Let us consider then the key assumptions that form part of the economy of for example, the Black–Scholes option pricing model.

43.3.1 Stochastic price processes

The uncertainty in asset price dynamics is described as having two sources, both represented by independent standard Brownian motions. These are denoted

$$(W_1, W_2, t \in [0, T])$$

on a probability space denoted by (Ω, F, P) .

The flow of information to investors is described by the filtration process. The two sources of risk in the Black–Scholes model are the risk-carrying underlying asset, and the cash deposit which, though paying a riskless rate of interest, is at risk from the stochastic character of the interest rate itself.

43.3.2 Perfect markets

The assumption of complete capital markets states that, as a result of arbitrage-free pricing, there is an unique probability measure Q , which is identical to the historical probability P , under which the continuously discounted price of any asset is a Q -martingale. This probability level Q then becomes the *risk-neutral* probability.

43.3.3 Uncertainty of interest rates

All derivative valuation models describe a process followed by market interest rates. As the future level of the yield curve or spot rate curve is uncertain, the key assumption is that interest rates are follow a normal distribution, and follow a Gaussian process. Thus the interest rate is described as being a Gaussian interest rate uncertainty. Only the short-term risk-free interest rate, for which we read the T-bill rate or (in certain situations) the government bond repo rate, is captured in most models. Following Merton (1973), Vasicek (1977), Cox, Ingersoll and Ross (1985), and Jamshidian (1991), the short-dated risk-free interest r applicable to the period t is said to follow a Gaussian diffusion process under a constant volatility. The major drawback under this scenario is that under certain conditions it is possible to model a term structure that produces negative forward interest rates. However in practice this occurs only under certain limited conditions, so the validity of the models is not diminished. The future path followed by r_t is described by the following stochastic differential equation:

$$dr_t = a_t [b_t - r_t] dt + \sigma_t dW_t \quad (43.35)$$

where a and b are constant deterministic functions and σ_t is the instantaneous standard deviation of r_t . Under (43.35) the process describing the returns generated by a risk-free zero-coupon bond $P(t, T)$ that expires at time T and has a maturity of $T-t$ under the risk-neutral probability Q is given by (43.36):

$$\frac{dP(t, T)}{P(t, T)} = r_t dt - \sigma_P(t, T) dW_t \quad (43.36)$$

where σ_P is the standard deviation of the price returns of the $(T-t)$ bond and is a deterministic function defined by:

$$\sigma_P(t, T) = \sigma_t \cdot \int_t^T \exp\left(-\int_t^u a(s) ds\right) du.$$

In the Black–Scholes model the value of a \$1 (or £1) deposit invested at the risk-free zero-coupon interest rate r and continuously compounded over a period t will have grown to the value given by the expression below, where M_t is the value of the deposit at time t .

$$M_t = \exp\left(\int_0^t r(u) du\right). \quad (43.37)$$

43.3.4 Asset price processes

All valuation models must capture a process describing the dynamics of the asset price. This was discussed at the start of the chapter and is a central tenet of derivatives valuation models. Under the Black–Scholes model for example, the price dynamics of a risk-bearing asset S_t under the risk-neutral probability function Q is given by

$$\frac{dS_t}{S_t} = r_t dt + \sigma_S (\rho dW_1 + \sqrt{1 - \rho^2} dW_2(t)) \quad (43.38)$$

where σ_S is the standard deviation of the asset price returns. The correlation between the price dynamics of the risk-bearing asset and the dynamics of interest rates changes is given by $\rho, \rho \in [0, 1]$ while $W_2(t)$ is a standard Brownian motion that describes the dynamics of the asset price, and not that of the interest rates which are captured by $W_1(t)$ (and from which it is independent).

Under these four assumptions, the price of an asset can be described in present value terms relative to the value of the risk-free cash deposit M_t and, in fact the price is described as a Q -martingale. A European-style contingent liability with maturity date t is therefore valued at time 0 under the risk-neutral probability as

$$V_0 = E^Q \left[\frac{h_t}{M_t} \right]$$

where

V_0 is the value of the asset at time 0
 h_t is the stochastic payoff at maturity date t , where h_t is a measurable stochastic process

and $E^Q[\cdot]$ is the expectation of the value under probability function Q .

In the following chapter we tie in the work on dynamics of asset prices into option valuation models.

Appendices

APPENDIX 43.1 An introduction to stochastic processes

A stochastic process can be described with respect to the notion of a vector of variables. If we set the following parameters

Ω is the set of all possible states ε
 Ψ is a class of partitions of Ω

$X(\omega)$ is said to be a *random variable* when it is a measurable application from (Ω, Ψ) to $\mathbb{R}$. A vector of random variables $X(\omega) = [X_1(\omega), \dots, X_n(\omega)]$ is an application that can be measured from (Ω, Ψ) into $\mathbb{R}^n$. Therefore we have a vector of random variables that is similar to n ordinary variables defined under the same probability function.

A stochastic process is an extension of the notion of a vector of variables when the number of elements becomes infinite. It is described by

$$\{X_t(\omega)\}, t \in T$$

which is a set of random variables where the index varies in a finite or infinite group, and is denoted by $X(t)$.

APPENDIX 43.2 Itô's lemma

If f is a continuous and differentiable function of a variable x , and Δx is a small change in x , then using a Taylor expansion the resulting change in f is given by (43.39):

$$\Delta f = \left(\frac{df}{dx} \right) \Delta x + \frac{1}{2} \left(\frac{d^2 f}{dx^2} \right) \Delta x^2 + \frac{1}{6} \left(\frac{d^3 f}{dx^3} \right) \Delta x^3 + \dots \quad (43.39)$$

If f is dependent on two variables x and y then the Taylor expansion of Δf becomes (43.40):

$$\Delta f \approx \left(\frac{\partial f}{\partial x} \right) \Delta x + \left(\frac{\partial f}{\partial y} \right) \Delta y + \frac{1}{2} \left(\frac{\partial^2 f}{\partial x^2} \right) \Delta x^2 + \frac{1}{2} \left(\frac{\partial^2 f}{\partial y^2} \right) \Delta y^2 + \left(\frac{\partial^2 f}{\partial x \partial y} \right) \Delta x \Delta y + \dots \quad (43.40)$$

The limiting case where Δx and Δy are close to zero will transform (43.40) to (43.41):

$$df \approx \left(\frac{\partial f}{\partial x} \right) dx + \left(\frac{\partial f}{\partial y} \right) dy. \quad (43.41)$$

Consider now a derivative asset $f(x, \tau)$ whose value is dependent on time and on the asset price x . If we assume that x follows the general Itô process

$$dx = a(x,t)dt + b(x,t)dW \quad (43.42)$$

where a and b are functions of x and t and dW is a Weiner process. The asset price x is described by a drift rate of x and a standard deviation of b . Using Itô's lemma it can be shown that a function f of x and t will follow the following process:

$$df = \left(\frac{\partial f}{\partial x} a + \frac{\partial f}{\partial t} + \frac{1}{2} \left(\frac{\partial^2 f}{\partial x^2} \right) b^2 \right) dt + \frac{\partial f}{\partial x} b dW \quad (43.43)$$

and where dW is the Weiner process; therefore f follows an Itô process and its drift and standard deviation are described by the expressions below:

$$\frac{\partial f}{\partial x} a + \frac{\partial f}{\partial t} + \frac{1}{2} \left(\frac{\partial^2 f}{\partial x^2} \right) b^2 \quad \text{and} \quad \left(\frac{\partial f}{\partial x} \right) b.$$

This may also be stated as:

$$\Delta x = a(x,t) \Delta t + b \varepsilon \sqrt{\Delta t}$$

where the term ε is normally distributed with a mean of 0, so that $E(\varepsilon) = 0$, and a variance of 1, so that $E(\varepsilon^2) - E(\varepsilon)^2 = 1$. In the limit case (43.40) becomes (43.44), which is Itô's lemma.

$$df = \left(\frac{\partial f}{\partial x} \right) dx + \left(\frac{\partial f}{\partial t} \right) dt + \frac{1}{2} \left(\frac{\partial^2 f}{\partial x^2} \right) b^2 dt. \quad (43.44)$$

The expression at (43.44) is Itô's lemma and if we substitute (43.42) for dx , it can be transformed to (43.45).

$$df = \left[\left(\frac{\partial f}{\partial x} \right) a + \left(\frac{\partial f}{\partial t} \right) + \frac{1}{2} \left(\frac{\partial^2 f}{\partial x^2} \right) b^2 \right] dt + \left(\frac{\partial f}{\partial x} \right) b dW. \quad (43.45)$$

The derivation of Itô's formula is given at Appendix 43.3.

APPENDIX 43.3 Derivation of Itô's formula

Let X_t be a *stochastic* process described by

$$dX_t = \mu_t dt + \sigma_t dW_t \quad (43.46)$$

where W_t is a random variable and Brownian motion and dW_t is an incremental change in the Brownian motion W_t , equal to $Z_t \sqrt{dt}$, $Z_t \sim N(0,1)$. Then suppose that we have a function $Y_t = f(X_t, t)$ and we require the differential dY_t . Applying a Taylor expansion of Y_t we would obtain

$$dY_t = \frac{\partial f}{\partial X_t} dX_t + \frac{\partial f}{\partial t} dt + \frac{1}{2} \left[\frac{\partial^2 f}{\partial X_t^2} dX_t^2 + 2 \frac{\partial^2 f}{\partial X_t \partial t} dX_t dt + \frac{\partial^2 f}{\partial t^2} dt^2 \right] + \dots \quad (43.47)$$

In (43.46) if we square dX_t we obtain

$$dX_t^2 = \mu_t^2 dt^2 + 2\sigma_t \mu_t dW_t dt + \sigma_t^2 dW_t^2. \quad (43.48)$$

The first two terms in (43.48) are of a higher order and of minimal impact when dt is sufficiently small, and may be ignored. It can be shown that the variance of the $(dW_t)^2$ term will tend towards zero when the increment dt is sufficiently small. At this point it no longer has the property of a random variable and becomes more a constant with expected value

$$E(Z^2 dt) = dt. \quad (43.49)$$

It can then be shown that for sufficiently small dt

$$dX_t^2 \Rightarrow \sigma_t^2 dW_t^2 \Rightarrow \sigma_t^2 dt.$$

The differential dY_t has an element that tends towards $\frac{1}{2} \frac{\partial^2 f}{\partial X_t^2} \sigma_t^2 dt$ for sufficiently small dt but cannot be dropped as were the higher-order terms of (43.48) as it is of order dt . So the first-order differential of Y_t is

$$dY_t = \frac{\partial f}{\partial X_t} dX_t + \frac{\partial f}{\partial t} dt + \frac{1}{2} \sigma_t^2 \frac{\partial^2 f}{\partial X_t^2} dt \quad (43.50)$$

and now if we insert dX_t from (43.46) into (43.49) we will obtain

$$dY_t = \left(\mu_t \frac{\partial f}{\partial X_t} + \frac{\partial f}{\partial t} + \frac{1}{2} \sigma_t^2 \frac{\partial^2 f}{\partial X_t^2} \right) dt + \sigma_t \frac{\partial f}{\partial X_t} dW_t. \quad (43.51)$$

If the reader has followed this through he or she has arrived at Itô's lemma. We can apply this immediately. Consider a process

$$X_t = X_0 + \int_0^t \mu_v dv + \int_0^t \sigma_s dW_s \quad (43.52)$$

for which the differential form is

$$dX_t = \mu_t dt + \sigma_t dW_t. \quad (43.53)$$

If we set the function $f(X)$ equal to X_t the results of applying the Itô lemma terms are

$$\frac{\partial f}{\partial X_t} = 1; \quad \frac{\partial f}{\partial t} = 0 \text{ and } \frac{\partial^2 f}{\partial X_t^2} = 0.$$

Therefore using Itô's lemma we obtain

$$dX_t = \mu_t dt + \sigma_t dW_t \quad (43.54)$$

which is what we expect. What we have here is a stochastic differential equation at (43.54) for which the solution is (43.52).

APPENDIX 43.4 The integral

Suppose we have a deterministic function $f(x)$ of time, with $x \in [0, T]$ that corresponds to a curve of $f(x)$ over the period from 0 to T , and we wish to calculate the area given by the function from time t_0 to t_T . This can be done by integrating the function over the time interval $[0, T]$, given by

$$\int_0^T f(s) ds. \quad (43.55)$$

To calculate the integral we split the area given by the function in the time period into a series of *partitions* or intervals, described by

$$t_0 = 0 < t_1 < t_2 < \dots < t_n = T. \quad (43.56)$$

The approximate value of the area required is

$$\sum_{i=1}^n f\left(\frac{t_i + t_{i-1}}{2}\right)(t_i - t_{i-1}); \quad (43.57)$$

however, if we decrease the interval space such that it approaches 0, described by

$$\max_i |t_i - t_{i-1}| \rightarrow 0$$

the area under the space is given by the integral at (43.49), as the approximating sum approaches the area defined by the limit

$$\sum_{i=1}^n f\left(\frac{t_i + t_{i-1}}{2}\right)(t_i - t_{i-1}) \rightarrow \int_0^T f(s) ds. \quad (43.58)$$

Selected bibliography and references

- Baxter, M., Rennie, A., *Financial Calculus*, Cambridge University Press, 1996.
- Boyle, P., "Option valuation using a three jump process", *International Options Journal* 3, 1986, pp. 7–12.
- Cootner, P. (ed.), *The Random Character of Stock Market Prices*, MIT Press, 1964.
- Cox, D., Miller, H., *The Theory of Stochastic Processes*, Chapman & Hall, 1965.
- Duffie, D., *Dynamic Asset Pricing Theory*, Princeton University Press, 1996.
- Fama, E., "The Behaviour of Stock Prices", *Journal of Business* 38, January 1965, pp. 34–105.
- Feller, W., *Probability Theory and Its Applications*, vols. 1 and 2, John Wiley and Sons, 1950.
- Debreu, G., "Representation of a preference ordering by a numerical function", in Thrall, R., Coombs, C., Davis, R. (eds.), *Decision Processes*, Wiley, 1954.
- Harrison, J., Kreps, D., "Martingales and arbitrage in multi-period securities markets", *Journal of Economic Theory* 20, 1979, pp. 381–408.
- Harrison, J., Pliska, S., "Martingales and stochastic integrals in the theory of continuous trading", *Stochastic Processes and Their Applications* 11, 1981, pp. 216–260.
- Ingersoll, J., Jr, *Theory of Financial Decision Making*, Rowman & Littlefield, 1987.
- Itô, K., "On Stochastic Differential Equations", *American Mathematical Society* 4, 1951, pp. 1–51.
- James, J., Webber, N., *Interest Rate Modelling*, John Wiley and Sons 2000, Chapters 3–5, 7–9, 15–16.
- Jamshidian, F., "Bond and option valuation in the Gaussian interest rate model", *Research in Finance* 9, 1991, pp. 131–170.
- Merton, R., "Theory of Rational Option Pricing", *Bell Journal of Economics and Management Science* 4, Spring 1973, pp. 141–183.
- Nielson, L.T., *Pricing and Hedging of Derivative Securities*, Oxford University Press, 1999.
- Neftci, S., *An Introduction to the Mathematics of Financial Derivatives*, Academic Press, 1996.
- Rendleman, R., Bartter, B., "Two State Option Pricing", *Journal of Finance* 34, 1979, pp. 1092–1110.
- Ross, S., *Probability Models*, 7th edition, Harcourt Academic Press, 2000.
- Vasicek, O., "An equilibrium characterisation of the term structure", *Journal of Financial Economics* 5, 1977, pp. 177–188.
- Williams, D., *Probability with Martingales*, Cambridge University Press, 1991.

Questions and exercises

- List the factors that influence option prices.
- A variable X is described by the price process $dX = adt + b dW$ where a and b are constant and are 0 and 1 for one year and then 1 and 2 for the second year. If the starting value of the variable is 2, what is the probability distribution of the value of the variable at the end of the second year?
- Define a Wiener process.
- What is a martingale?
- A zero-coupon bond has a return of r with maturity t and follows the process given by:

$$dr = a(r_t - r_0)dt + \sigma(r)dW$$

Write down the process followed by the bond price.

- Discuss the importance of the Itô process to finance theory.
- What stochastic process is described by the following Wiener process:

$$dr_t = a_t(b_t - r_t)dt + \sigma_t dW_t$$

where a and b are positive constants?

8. What is the terminal value of £1 invested at the risk-free zero-coupon interest-rate of r over a time period beginning at 0 and ending at t ? Assume continuous compounding.
9. The dynamics of an asset price S over time period t can be described by the model below. What is being measured by the coefficient ρ ?

$$\frac{dS_t}{S_t} = r_t dt + \sigma_S \left(\rho dW_t + \sqrt{1 - \rho^2} dW_2(t) \right).$$

44

Options II: Pricing and Valuation

In this chapter we build on the introduction to asset price dynamics discussed in the previous chapter to describe the pricing of option contracts. The content is essentially a summary of the research into finance theory and option pricing carried out by a number of leading academics since the first presentation by Black and Scholes and Merton in 1973.

44.1 Option pricing

When we have discussed all the interest rate products described in this book so far, both cash and derivatives, it has been possible to determine pricing due of rigid mathematical principles, and also because on maturity of the instrument there is a defined procedure that takes place such that one is able to calculate a fair value. This does not apply to options because there is uncertainty as to what the outcome will be on expiry; an option seller does not know whether the option will be exercised or not. This factor makes options more difficult to price than other financial market instruments.

In this section we review the parameters used in the pricing of an option, and introduce the Black–Scholes model and binomial model. Readers may therefore wish to skip this section and move on to the section dealing with the use of options in bond market trading. Those who are interested in reading through this should first ensure that they are familiar with the basic statistical concepts reviewed earlier in this book, including the normal distribution and cumulative probability distribution. To recap, the basic concepts are summarised in Appendix 37.1.

Pricing an option is a function of the probability that it will be exercised. Essentially the premium paid for an option represents the buyer's *expected profit* on the option. Therefore, as with an insurance premium, the writer of an option will base his price on the assessment that the payout on the option will be equal to the premium, and this is a function on the probability that the option will be exercised. Option pricing therefore bases its calculation on the assessment of the probability of exercise and deriving from this an expected outcome, and hence a fair value for the option premium. The expected payout, as with an insurance company premium, should equal the premium received.

The following factors influence the price calculated for an option.

- **the behaviour of financial prices:** one of the key assumptions made by the Black–Scholes model (B–S) is that asset prices follow a lognormal distribution. Although as we discovered in Chapter 37 this is not strictly accurate, it is close enough of an approximation to allow its use in option pricing. In fact observation shows that while prices themselves are not normally distributed, asset returns are, and we define returns as $\ln(P_{t+1}/P_t)$ where P_t is the market price at time t and P_{t+1} is the price one period later. The distribution of prices is called a lognormal distribution because the logarithm of the prices is normally distributed; the asset returns are defined as the logarithm of the price relatives and are assumed to follow the normal distribution. The expected return as a result of assuming this distribution is given by $E[\ln(P_t/P_0)] = rt$ where $E[\]$ is the expectation operator and r is the annual rate of return.
- **the strike price:** the difference between the strike price and the underlying price of the asset at the time the option is struck will influence the size of the premium, as this will impact on the probability that the option will be exercised. An option that is deeply in-the-money has a greater probability of being exercised;
- **volatility:** the volatility of the underlying asset will influence the probability that an option is exercised, as a higher volatility indicates a higher probability of exercise. This is considered in detail below;
- **the term to maturity:** a longer-dated option has greater time value and a greater probability of eventually being exercised;
- **the level of interest rates:** the premium paid for an option in theory represents the expected gain to the buyer at the time the option is exercised. It is paid up-front so it is discounted to obtain a present value. The discount rate used therefore has an effect on the premium, although it is less influential than the other factors presented here.

The volatility of an asset measures the variability of its price returns. It is defined as the annualised standard deviation of returns, where variability refers to the variability of the returns that generate the asset's prices, rather than the prices directly. The standard deviation of returns is given by (44.1):

$$\sigma = \sqrt{\frac{\sum_{i=1}^N (x_i - \mu)^2}{N - 1}} \quad (44.1)$$

where x_i is the i th price relative, μ the arithmetic mean of the observations and N is the total number of observations. The value is converted to an annualised figure by multiplying it by the square root of the number of days in a year, usually taken to be 250 working days. Using this formula from market observations it is possible to calculate the *historic volatility* of an asset. The volatility of an asset is one of the inputs to the B-S model. Of the inputs to the B-S model, the variability of the underlying asset, or its volatility is the most problematic. The distribution of asset prices is assumed to follow a lognormal distribution, because the logarithm of the prices is normally distributed (we assume lognormal rather than normal distribution to allow for the fact that prices cannot – as could be the case in a normal distribution – have negative values): the range of possible prices starts at zero and cannot assume a negative value.

Note that it is the asset price *returns* on which the standard deviation is calculated, and not the actual prices themselves. This is because using prices would produce inconsistent results, as the actual standard deviation itself would change as price levels “grew” or increased.

However, calculating volatility using the standard statistical method gives us a figure for *historic volatility*. What is required is a figure for *future volatility*, since this is relevant for pricing an option expiring in the future. Future volatility cannot be measured directly, by definition. Market makers get around this by using an option pricing model “backwards”, as shown in Figure 44.1

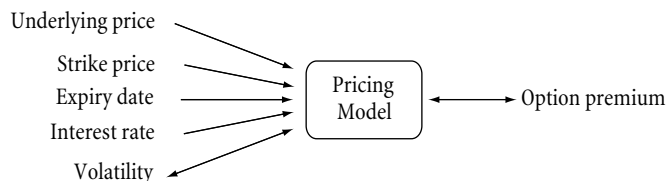


Figure 44.1: Option implied volatility and pricing model.

An option pricing model calculates the option price from volatility and other parameters. Used in reverse the model can calculate the volatility implied by the option price. Volatility measured in this way is called *implied volatility*. Evaluating implied volatility is straightforward using this method and generally more appropriate than using historic volatility, as it provides a clearer measure of an option's fair value. Implied volatilities of deeply in-the-money or out-of-the-money options tend to be relatively high.

We are now in a position to review the main parameters of option pricing more formally. All models make an assumption and include as one of their elements the behaviour of asset prices. By saying that an asset price follows a particular process, we are able to quantify a probability that a price will be at a certain state at some point in the future, and hence will lead to the exercise of an option or not. Therefore we first must look at the dynamics of asset prices, and how this is applied in derivative pricing.

44.2 Pricing derivative securities

Most option pricing models are based on one of two methodologies, although both types employ essentially identical assumptions. The first method is based on the resolution of the partial differentiation equation of the asset price model, corresponding to the expected payoff of the option security. This is the foundation of the Black-Scholes model. The second type of model uses the martingale method, and was first introduced by Harrison and Kreps (1979) and Harrison and Pliska (1981), where the price of an asset at time 0 is given by its discounted expected future payoffs, under the appropriate probability measure, known as the risk-neutral probability. There is a third type that assumes lognormal distribution of asset returns but follows the two-step binomial process.

In order to employ the pricing models, we accept a state of the market that is known as a *complete market*,¹ one where there is a viable financial market. This is where the rule of no-arbitrage pricing exists, so that there is no opportunity to generate risk-free arbitrage due to the presence of say, incorrect forward interest rates. The fact that there is no opportunity to generate risk-free arbitrage gains means that a zero-cost investment strategy that is initiated at time t will have a zero maturity value. The martingale property of the behaviour of asset prices states that an accurate estimate of the future price of an asset may be obtained from current price information. Therefore the relevant information used to calculate forward asset prices is the latest price information. This was also a property of the semi-strong and strong-form market efficiency scenarios described by Fama (1970).

44.2.1 The Black–Scholes model: basic concepts

If we assume a financial asset is specified by its terminal payoff value, when pricing an option we require the fair value of the option at the initial time when the option is struck, and this value is a function of the expected terminal payoff of the option, discounted to the day when the option is struck. There are two methods by which this process may be estimated, and the first description was by Black and Scholes in 1973; subsequent approaches use a Martingale property. We summarise both processes in this section.

The Black–Scholes model describes a process to calculate the fair value of a European call option under certain assumptions, and apart from the price of the underlying asset S and the time t all the variables in the model are assumed to be constant. The following assumptions are made:

- there are no transaction costs, and the market allows short selling;
- trading is continuous;
- underlying asset prices follow geometric Brownian motion, with the variance rate proportional to the square root of the asset price;
- the asset is a non-dividend paying security;
- the interest rate during the life of the option is known and constant;
- the option can only be exercised on expiry.

The B–S model is neat and intuitively straightforward to explain, and one of its many attractions is that it can be readily modified to handle other types of options such as foreign exchange or interest-rate options. The assumption of the behaviour of the underlying asset price over time is described by (43.15), which was introduced earlier as a generalised Weiner process; it is reproduced below with the asset price represented by S . Here we use a to denote the expected return on the underlying asset and b to denote the standard deviation of its price returns:

$$\frac{dS}{S} = a dt + b dW.$$

In this section we follow an approach described in Galitz (1995) and present an intuitive explanation of the B–S model, in terms of the normal distribution of asset price returns. From the definition of a call option, we can set the expected value of the option at maturity T as:

$$E(C_T) = E[\max(S_T - X, 0)] \quad (44.2)$$

where

S_T is the price of the underlying asset at maturity T
 X is the strike price of the option.

As we saw in Chapter 42, from (44.2) we know that there are only two possible outcomes that can arise on maturity, either the option will expire in-the-money and the outcome is $S_T - X$, or the option will be out-of-the-money and the outcome will be 0. If we set p as the probability that on expiry $S_T > X$, equation (44.2) can be rewritten as (44.3):

¹ First proposed by Arrow and Debreu (1953, 1954).

$$E(C_T) = p \times (E[S_T \mid S_T > X] - X) \quad (44.3)$$

where $E[S_T \mid S_T > X]$ is the expected value of S_T given that $S_T > X$. Equation (44.3) gives us an expression for the expected value of a call option on maturity. Therefore to obtain the fair price of the option at the time it is struck, the value given by (44.2) must be discounted back to its present value, and this is shown as (44.4):

$$C = p \times e^{-rt} \times (E[S_T \mid S_T > X] - X) \quad (44.4)$$

where r is the continuously compounded risk-free rate of interest, and t is the time from today until maturity. Therefore to price an option we require the probability p that the option expires in-the-money, and we require the expected value of the option given that it does expire in-the-money, which is the last term of (44.4). To calculate p we assume that asset prices follow a stochastic process, which enables us to model the probability function. We will consider this later.

Black and Scholes showed that the price of an option is based on the resolution of the following partial differential equation:

$$\frac{1}{2} \sigma^2 S^2 \left(\frac{\partial^2 C}{\partial S^2} \right) + rS \left(\frac{\partial C}{\partial S} \right) + \left(\frac{\partial C}{\partial t} \right) - rC = 0 \quad (44.5)$$

under the appropriate parameters. We have not demonstrated the process by which this equation is arrived at. The parameters refer to the payoff conditions corresponding to a European call option, which we considered above. We do not present a solution to the differential equation at (44.5), which is beyond the scope of the book, but we can consider now how the probability and expected value functions can be solved. For a fuller account readers may wish to refer to the original account by Black and Scholes; other good accounts are given in Ingersoll (1987), Neftci (1996), and Nielsen (1999) among others.

To find the probability p that the underlying asset price at maturity exceeds X is equal to the probability that the return over the time period the option is held will exceed a certain critical value. Remember that we assume normal distribution of asset price returns. As asset returns are defined as the logarithm of price relatives, we require p such that:

$$p = \text{prob}[S_T > X] = \text{prob} \left[\text{return} > \ln \left(\frac{X}{S_0} \right) \right] \quad (44.6)$$

where S_0 is the price of the underlying asset at the time the option is struck. Generally the probability that a normal distributed variable x will exceed a critical value x_c is given by (44.7):

$$p[x > x_c] = 1 - N \left(\frac{x_c - \mu}{\sigma} \right) \quad (44.7)$$

where μ and σ are the mean and standard deviation of x respectively and $N(\cdot)$ is the cumulative normal distribution. We know from our earlier discussion of the behaviour of asset prices that an expression for μ is the natural logarithm of the asset price returns; we already know that the standard deviation of returns is $\sigma\sqrt{t}$. Therefore with these assumptions, we may combine (44.6) and (44.7) to give us (44.8), that is,

$$\text{prob}[S_T > X] = \text{prob} \left[\text{return} > \ln \left(\frac{X}{S_0} \right) \right] = 1 - N \left(\frac{\ln \left(\frac{X}{S_0} \right) - \left(r - \frac{\sigma^2}{2} \right) t}{\sigma\sqrt{t}} \right). \quad (44.8)$$

Under the conditions of the normal distribution, the symmetrical shape means that we can obtain the probability of an occurrence based on $1 - N(d)$ being equal to $N(-d)$. Therefore we are able to set the following relationship:

$$p = \text{prob}[S_T > X] = N \left(\frac{\ln \left(\frac{S_0}{X} \right) + \left(r - \frac{\sigma^2}{2} \right) t}{\sigma\sqrt{t}} \right). \quad (44.9)$$

Now we require a formula to calculate the expected value of the option on expiry, the second part of the expression at (44.4). This involves the integration of the normal distribution curve over the range from X to infinity. This is not shown here, however the result is given at (44.10):

$$E[S_T \mid S_T > X] = S_0 e^{rt} \frac{N(d_1)}{N(d_2)} \quad (44.10)$$

where

$$d_1 = \frac{\ln\left(\frac{S_0}{X}\right) + \left(r + \frac{\sigma^2}{2}\right)t}{\sigma\sqrt{t}} \quad \text{and} \quad d_2 = \frac{\ln\left(\frac{S_0}{X}\right) + \left(r - \frac{\sigma^2}{2}\right)t}{\sigma\sqrt{t}} = d_1 - \sigma\sqrt{t}.$$

We now have expressions for the probability that an option expires in-the-money as well as the expected value of the option on expiry, and we incorporate these into the expression at (44.4), which gives us (44.11):

$$C = N(d_2) \times e^{-rt} \times \left[S_0 e^{rt} \frac{N(d_1)}{N(d_2)} - X \right]. \quad (44.11)$$

Equation (44.11) can be rearranged to give (44.12), which is the famous and well-known Black–Scholes option pricing model for a European call option:

$$C = S_0 N(d_1) - X e^{-rt} N(d_2) \quad (44.12)$$

where

S_0	is the price of the underlying asset at the time the option is struck
X	is the strike price
r	is the continuously compounded risk-free interest rate
t	is the maturity of the option

and d_1 and d_2 are as before.

What the expression at (44.12) states is that the fair value of a call option is the expected present value of the option on its expiry date, assuming that prices follow a lognormal distribution.

$N(d_1)$ and $N(d_2)$ are the cumulative probabilities from the normal distribution of obtaining the values d_1 and d_2 , given above. An approximation of the cumulative normal distribution is given at Appendix 44.3.

The term e^{-rt} is the present value of one unit of cash received t periods from the time the option is struck. Where $N(d_1)$ and $N(d_2)$ are equal to 1, which is the equivalent of assuming complete certainty, the model is reduced to:

$$C = S - X e^{-rt}$$

which is the expression for Merton's lower bound for continuously compounded interest rates, and which we introduced in intuitive fashion in Chapter 42. Therefore under complete certainty the B–S model reduces to Merton's bound.

44.2.2 Black–Scholes: deriving the general model

Given the assumptions of the B–S model, the fair value of an option is a function of the price of the underlying at the time of expiry, which is the only unknown, and over a short time period Δt , denoted by $C(S, t)$, it is possible to construct a portfolio consisting of the underlying asset, an option on the asset and a risk-free security. The B–S model expresses the expected return on the option in terms of the option price function and a partial differential equation of the price function. The hedged position described by the B–S model consists of a long position in one share of the underlying asset and a sale of $[\partial C(S, t)/\partial S]^{-1}$ options against this one long share. If the price of the share changes by a small amount ΔS , the price of the option changes by an amount $[\partial C(S, t)/\partial S]^{-1} \Delta S$. Therefore the change in value in the long position, which is the underlying share, is offset approximately by the change in value in the number of options. This hedge can be maintained continuously so that the return on the hedged position

becomes completely independent of the changes in the value of the underlying asset. That is, the return on the hedge position becomes certain.

The value of assets, V , in a hedged position that contains one share per a short position of $[\partial C(S,t)/\partial S]^{-1}$ options is given by (44.13):

$$V = S - \frac{C(S,t)}{\left(\frac{\partial C(S,t)}{\partial S}\right)}. \quad (44.13)$$

Over the short time period Δt the change in the value of the position is given by (44.14):

$$V = \Delta S - \frac{\Delta C(S,t)}{\left(\frac{\partial C(S,t)}{\partial S}\right)} \quad (44.14)$$

where $\Delta C(S, t)$ is given by $C(S + \Delta S, t + \Delta t) - C(S, t)$.

It can be shown by using stochastic calculus for $\Delta C(S, t)$, which is not demonstrated here, that we obtain

$$\Delta C(S,t) = \frac{\partial C(S,t)}{\partial S} \Delta S + \frac{1}{2} \sigma^2 S^2 \frac{\partial^2 C(S,t)}{\partial S^2} \Delta t + \frac{\partial C(S,t)}{\partial t} \Delta t. \quad (44.15)$$

The change in the value of the equity in the hedged position can be found by substituting $\Delta C(S, t)$ from (44.14) into (44.13), which gives us (44.16):

$$\frac{-\left(\frac{1}{2} \sigma^2 S^2 \frac{\partial^2 C(S,t)}{\partial S^2} + \frac{\partial C(S,t)}{\partial t}\right) \Delta t}{\frac{\partial C(S,t)}{\partial S}}. \quad (44.16)$$

Since the return on the shares in the hedged position is now certain, it must be equal to $r\Delta t$ where r is the risk-free interest rate. Therefore the change in the value of the underlying shares must be equal to the value of the asset multiplied by $r\Delta t$. This is an important result; what the model is stating now is that, as the return of the underlying asset is now certain, it must be equal to the return from holding the risk-free asset, which has a return of $r\Delta t$. This enables us to complete the model. The return on the underlying equity is given by (44.17):

$$\frac{-\left(\frac{1}{2} \sigma^2 S^2 \frac{\partial^2 C(S,t)}{\partial S^2} + \frac{\partial C(S,t)}{\partial t}\right) \Delta t}{\left(\frac{\partial C(S,t)}{\partial S}\right)} = \left(S - \frac{C(S,t)}{\left(\frac{\partial C(S,t)}{\partial S}\right)}\right) r \Delta t. \quad (44.17)$$

If we remove the time factor and rearrange (44.17) we obtain the B-S partial differential equation, shown earlier but repeated again here as (44.18):

$$\frac{1}{2} \sigma^2 S^2 \left(\frac{\partial^2 C}{\partial S^2}\right) + rS \left(\frac{\partial C}{\partial S}\right) + \left(\frac{\partial C}{\partial t}\right) - rC = 0. \quad (44.18)$$

The partial differential equation is solved under the boundary condition which expresses the value of the call option on its expiry date T , which is

$$C(S,T) = \max(S_T - X, 0) \quad (44.19)$$

where X is the strike price of the option.

For a European put option the equation must be solved under the opposite boundary condition, for the expiry date of the option, given at (44.20):

$$P(S,T) = \max(X - S_T, 0). \quad (44.20)$$

The solution of the differential equation is not presented here. To solve it under the boundary condition (44.19) Black and Scholes substitute the following:

$$C(S, t) = e^{r(t-T)} y \left(\frac{2}{\sigma^2} \left(r - \frac{1}{2} \sigma^2 \right) \left(\ln \left(\frac{S}{X} \right) - \left(r - \frac{1}{2} \sigma^2 \right) (t - T) \right) - \frac{2(t-T)}{\sigma^2} \left(r - \frac{1}{2} \sigma^2 \right)^2 \right). \quad (44.21)$$

Using this substitution, the differential equation becomes

$$\frac{\partial y}{\partial t} = \frac{\partial^2 y}{\partial S^2}. \quad (44.22)$$

In fact, (44.22) is the simplest form of the heat transfer equation used in physics, which can be used to find the value of $y(S, t)$ that satisfies the following system:

$$\begin{aligned} \partial y / \partial t &= \partial^2 y / \partial S^2 \text{ for } 0 \leq S \leq L \\ y(0, t) &= y(L, t) = 0 \text{ for } t > 0 \\ y(S, 0) &= y_0(S) \text{ for } 0 \leq S \leq L. \end{aligned}$$

The first condition is the heat transfer equation that states that the variable S lies between 0 and maximum value L . The second condition is the lower bound that in fact states that the lower limit for the option value is 0. The last condition is the boundary condition that sets the value of the option at maturity date t .

The boundary condition at (44.19) is rewritten as (44.23):

$$y(u, 0) = \begin{cases} 0 & u < 0 \\ X \left(\exp \left(-\frac{1}{2} u \sigma^2 / \left(r - \frac{1}{2} \sigma^2 \right) \right) - 1 \right) & \text{otherwise.} \end{cases} \quad (44.23)$$

The solution to (44.22) is the solution to the heat transfer equation, first given in Churchill (1963) and we show the result only here, as (44.24):

$$y(u, s) = \frac{1}{\sqrt{2\pi}} \int_{-u/\sqrt{2s}}^{\infty} X \left(\exp \left(\frac{\frac{1}{2}(u + q\sqrt{2s})\sigma^2}{r - \frac{1}{2}\sigma^2} \right) - 1 \right) \exp \left(-\frac{1}{2} q^2 \right) dq. \quad (44.24)$$

If we then substitute from (44.24) into (44.21) we obtain the Black–Scholes formula for the price of a European call option where we now set T as the option maturity term, given by (44.25):

$$C(S, T) = SN(d_1) - Xe^{-rT}N(d_2) \quad (44.25)$$

where

$$\begin{aligned} d_1 &= \frac{1}{\sigma\sqrt{T}} \left[\ln \left(\frac{S}{X} \right) + \left(r + \frac{1}{2} \sigma^2 \right) T \right] \\ d_2 &= d_1 - \sigma\sqrt{T} \end{aligned}$$

and $N()$ is the cumulative normal distribution function, given by (44.26). This is described in Appendix 44.3.

$$N(d) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^d \exp \left(-\frac{x^2}{2} \right) dx. \quad (44.26)$$

The option value given by the B–S model is not related to the expected return of the underlying asset itself. This reflects the assumption that the expected return on the asset is already fully priced into the stock itself, which is a condition of semi-strong and strong-form economies. The price of the option is a function of the underlying asset price, the time to expiry, the rate of interest and the standard deviation of the asset price returns.

Note that the partial derivative $\partial C(S, t) / \partial S$, which is equal to $N(d_1)$, gives the ratio of options to underlying assets held in our hedged position, which we noted at the start. It is also the option *delta*. This is discussed later.

44.2.3 The put-call parity relationship

Up to now we have concentrated on calculating the price of a call option. However the previous section introduced the boundary condition for a put option, so it should be apparent that this can be solved as well. In fact the price of a call option and a put option are related via what is known as the *put-call parity theorem*. This is an important relationship and obviates the need to develop a separate model for put options.

Consider a portfolio Y that consists of a call option with a maturity date T and a zero-coupon bond that pays X on the expiry date of the option. Consider also a second portfolio Z that consists of a put option also with maturity date T and one share. The value of portfolio A on the expiry date is given by (44.27):

$$MV_{Y,T} = \max(S_T - X, 0) + X = \max(X, S_T). \quad (44.27)$$

The value of the second portfolio Z on the expiry date is:

$$MV_{Z,T} = \max(X - S_T, 0) + S_T = \max(X, S_T). \quad (44.28)$$

Both portfolios have the same value at maturity. Therefore they must also have the same initial value at start time t , otherwise there would be an arbitrage opportunity. Prices must be arbitrage-free, therefore the following put-call relationship must hold:

$$C_t - P_t = S_t - Xe^{-r(T-t)}. \quad (44.29)$$

If the relationship at (44.28) did not hold, then arbitrage would be possible. So, using this relationship, the value of a European put option is given by the B-S model as shown below, at (44.30):

$$P(S, T) = -SN(-d_1) + Xe^{-rT}N(-d_2). \quad (44.30)$$

EXAMPLE 44.1 The Black-Scholes model

Here we illustrate a simple applications of the B-S model. Consider an underlying asset, usually assumed to be non-dividend paying equity, with a current price of 25, and volatility of 23%. The short-term risk-free interest rate is 5%. An option is written with strike price 21 and a maturity of three months. Therefore we have $S = 25$, $X = 21$, $r = 5\%$, $T = 0.25$, $\sigma = 23\%$.

To calculate the price of the option, we first calculate the discounted value of the strike price, as follows:

$$Xe^{-rT} = 21e^{-0.05(0.25)} = 20.73913.$$

We then calculate the values of d_1 and d_2 :

$$d_1 = \frac{\ln(25/21) + (0.05 + (0.5)(0.23)^2)0.25}{0.23\sqrt{0.25}} = \frac{0.193466}{0.115} = 1.682313$$

$$d_2 = d_1 - 0.23\sqrt{0.25} = 1.567313.$$

We now insert these values into the main price equation,

$$C = 25N(1.682313) - 21e^{-0.05(0.25)}N(1.567313).$$

Using the approximation of the cumulative normal distribution at the points 1.68 and 1.56, the price of the call option is:

$$C = 25(0.9535) - 20.73913(0.9406) = 4.3303.$$

What would be the price of a put option on the same stock? The values of $N(d_1)$ and $N(d_2)$ are 0.9535 and 0.9406, therefore the put price is calculated as $P = 20.7391(1 - 0.9406) - 25(1 - 0.9535) = 0.06943$.

If we use the call price and apply the put-call parity theorem, the price of the put option is given by:

$$\begin{aligned}
P &= C - S + Xe^{-rT} \\
&= 4.3303 - 25 + 21e^{-0.05(0.25)} \\
&= 0.069434.
\end{aligned}$$

This is exactly the same price that was obtained by the application of the put option formula in the B-S model above.

As we noted early in this chapter, the premium payable for an option will increase if the time to expiry, the volatility or the interest rate is increased (or any combination is increased). Thus if we maintain all the parameters constant but price a call option that has a maturity of six months or $T = 0.5$, we obtain the following values:

$$d_1 = 1.3071, \text{ giving } N(d_1) = 0.9049$$

$$d_2 = 1.1445, \text{ giving } N(d_2) = 0.8740.$$

The call price for the longer-dated option is 4.7217.

The Black–Scholes model as a spreadsheet

In Appendix 44.6 we show the spreadsheet formulae required to build the Black–Scholes model into Microsoft® Excel. The user must ensure that the Analysis Tool-Pak add-in is available, otherwise some of the function references may not work. By setting up the cells in the way shown, the fair value of a vanilla call or put option may be calculated. The put-call parity is used to enable calculation of the put price.

44.3 Simulation methods

To calculate the price of the option we require the expected value of the payoff of the option at expiry. This value is not known, so as we demonstrated in the previous section, it is necessary to assume, and then model, a behaviour pattern for asset prices and then calculate the present value of this expected payoff. Another method that can be used to obtain an expected value is through simulation. The best known method is Monte Carlo simulation. One approach² is to simulate price outcomes using the distribution of asset values at the option's expiry date. This distribution is determined by the process which generates future movements in the asset price. If the process is specified, then it is possible to simulate its values. Each time that a simulation is run, the computer generates a terminal value of the financial asset. If we repeat the simulation procedure a large number of times, say 1000 times or more, we obtain the distribution of terminal asset values. We can then determine the asset's expected terminal value.

Simulating the price paths of an asset is conducted in the following way. Given a random variable with a pattern μdx , if we draw n times on a computer $X_1, X_2, \dots, X_n$ enough times so that X_n follows the same random pattern, (that is, n must be a large number), following the law of large numbers the derivative asset price F can be expressed as (44.31):

$$\lim_{N \rightarrow \infty} \left(\frac{1}{N} \right) \sum_{n=1}^N f(X_n) = \int f(x) \mu(dx). \quad (44.31)$$

The Monte Carlo method may be implemented as follows. We construct a sequence of numbers $(U_n)_{n \geq 1}$ which correspond to a uniform sequence of independent random variables on the interval $[0, 1]$. Then we obtain a function to which $(u_1, \dots, u_p)$ corresponds such that the law of the random variable $F(u_1, \dots, u_p)$ is the unknown law of μdx . The sequence of random independent variables $(X_n)_{n \geq 1}$ with

$$X_n = F[U_{(n-1)p+1}, \dots, U_{np}] \quad (44.32)$$

follows the rule μ .

Monte Carlo simulation is usually used to price European options only and not American options.

² Described by Boyle (1976, 1986, 1988).

44.4 Valuation of bond options

In introducing and explaining the most common method used to price call options, we have focused on the valuation of equity options. This may appear to be not considerably relevant to this book, however the basic methodology may be applied, in fairly straightforward fashion, to interest-rate products such as bond options. As we noted at the start, the price of any financial asset is given by the present value of its expected cash flows on maturity. Therefore when valuing a bond we determine its cash flows, which (for a plain vanilla bond) comprise the regular coupon payments and the final redemption payment on maturity. The price of such an instrument is then given by the sum of all the cash flows, each discounted to their present values today. There are a variety of other bonds in the market as well, many of which do not have certain cash flows and therefore present additional concerns in their pricing. In this section we illustrate the application of the B-S model to the pricing of an option on a zero-coupon bond and a plain vanilla fixed-coupon bond. The following section introduces some other models that may be used for interest-rate options.

For a zero-coupon bond the theoretical price of a call option written on the bond is given by (44.33):

$$C = PN(d_1) - Xe^{-rT}N(d_2) \quad (44.33)$$

where P is the price of the underlying bond and all other parameters remain the same. If the option is written on a coupon-paying bond, it is necessary to subtract the present value of all coupons paid during the life of the option from the bond's price. Coupons sometimes lower the price of a call option because a coupon makes it more attractive to hold a bond rather than an option on the bond. Call options on bonds are often priced at a lower level than similar options on zero-coupon bonds.

EXAMPLE 44.2 B-S model and bond option pricing

Consider a European call option written on a bond that has the following characteristics:

Price	£98
Coupon	8.00% (semi-annual)
Time to maturity	5 years
Bond price volatility	6.02%
Coupon payments	£4 in three months and nine months
Three-month interest rate	5.60%
Nine-month interest rate	5.75%
One-year interest rate	6.25%

The option is written with a strike price of £100 and has a maturity of one year. The present value of the coupon payments made during the life of the option is £7.78, as shown below.

$$4e^{-0.056 \times 0.25} + 4e^{-0.0575 \times 0.75} = 3.9444 + 3.83117 = 7.77557.$$

This gives us $P = 98 - 7.78 = £90.22$.

Applying the B-S model we obtain:

$$d_1 = (\ln(90.22/100) + 0.0625 + 0.001812)/0.0602 = -0.6413$$

$$d_2 = d_1 - (0.0602 \times 1) = -0.7015$$

$$C = 90.22N(-0.6413) - 100e^{-0.0625}N(-0.7015) \\ = 1.1514.$$

Therefore the call option has a value of £1.15, which will be composed entirely of time value. Note also that a key assumption of the model is constant interest rates, yet is being applied to a bond price – which is a function of an interest rate – that is considered to follow stochastic price processes.

44.5 Interest-rate options and the Black model

In 1976 Fischer Black presented a slightly modified version of the B–S model, using similar assumptions, to be used in pricing forward contracts and interest-rate options. The Black model is used in banks today to price instruments such as swaptions in addition to bond and interest-rate options like caps and floors.

In this model the spot price $S(t)$ of an asset or a commodity is the price payable for immediate delivery today (in practice, up to two days forward) at time t . This price is assumed to follow a geometric Brownian motion. The theoretical price for a futures contract on the asset, $F(t, T)$ is defined as the price agreed today for delivery of the asset at time T , with the price agreed today but payable on delivery. When $t = T$, the futures price is equal to the spot price. A futures contract is cash settled every day via the clearing mechanism, whereas a forward contract is a contract to buy or sell the asset where there is no daily mark-to-market and no daily cash settlement.

Let us set f as the value of a forward contract, u as the value of a futures contract and C as the value of an option contract. Each of these contracts is a function of the futures price $F(t, T)$, as well as additional variables. So we may write at time t the values of all three contracts as $f(F, t)$, $u(F, t)$ and $C(F, t)$. The value of the forward contract is also a function of the price of the underlying asset S at time T and can be written $f(F, t, S, T)$. Note that the value of the forward contract f is not the same as the price of the forward contract. The forward price at any given time is the delivery price that would result in the contract having a zero value. At the time the contract is transacted, the forward value is zero. Over time both the price and the value will fluctuate. The futures price on the other hand, is the price at which a forward contract has a zero current value. Therefore at the time of the trade the forward price is equal to the futures price F , which may be written as:

$$f(F, t, F, T) = 0. \quad (44.34)$$

Equation (44.34) simply states that the value of the forward contract is zero when the contract is taken out and the contract price S is always equal to the current futures price, $F(t, T)$.³

The principal difference between a futures contract and a forward contract is that a futures contract may be used to imply the price of forward contracts. This arises from the fact that futures contracts are repriced each day, with a new contract price that is equal to the new futures price.⁴ Hence when F rises, such that $F > S$, the forward contract has a positive value and when F falls the forward contract has a negative value. When the transaction expires and delivery takes place, the futures price is equal to the spot price and the value of the forward contract is equal to the spot price minus the contract price or the spot price.

$$f(F, T, S, T) = F - S. \quad (44.35)$$

On maturity the value of a bond or commodity option is given by the maximum of zero, and the difference between the spot price and the contract price. Since at that date the futures price is equal to the spot price, we conclude that:

$$C(F, T) = \begin{cases} F - S & \text{if } F \geq S_T \\ 0 & \text{else.} \end{cases} \quad (44.36)$$

The assumptions made in the Black model are that the prices of futures contracts follow a lognormal distribution with a constant variance, and that the Capital Asset Pricing Model applies in the market. There is also an assumption of no transaction costs or taxes. Under these assumptions, we can create a risk-free hedged position that is composed of a long position in the option and a short position in the futures contract. Following the B–S model, the number of options put on against one futures contract is given by $\partial C(F, t) / \partial F$, which is the derivative of $C(F, t)$ with respect to F . The change in the hedged position resulting from a change in price of the underlying is given by (44.37):

$$\partial C(F, t) - [\partial C(F, t) / \partial F] \partial F. \quad (44.37)$$

³ This assumption is held in the market but as we have discussed in previous chapters, does not hold good over long periods, due chiefly to the difference in the way futures and forwards are marked-to-market, and because futures are cash settled on a daily basis while forwards are not.

⁴ An excellent description of this process is contained in Rubinstein (1999).

Due to the principle of arbitrage-free pricing, the return generated by the hedged portfolio must be equal to the risk-free interest rate, and this together with an expansion of $\partial C(F, t)$ produces the following partial differential equation:

$$\frac{\partial C(F, t)}{\partial t} = rC(F, t) - \frac{1}{2}\sigma^2 F^2 \frac{\partial^2 C(F, t)}{\partial F^2} \quad (44.38)$$

which is solved by setting the following:

$$\frac{1}{2}\sigma^2 F^2 \frac{\partial^2 C(F, t)}{\partial F^2} - rC(F, t) + \frac{\partial C(F, t)}{\partial t} = 0. \quad (44.39)$$

The solution to the partial differential equation (44.38) is not presented here. The result gives the fair value of a commodity option or option on a forward contract as shown by (44.40):

$$C(F, t) = e^{-rT} (FN(d_1) - S_T N(d_2)) \quad (44.40)$$

where

$$d_1 = \frac{1}{\sigma\sqrt{T}} \left(\ln\left(\frac{F}{S_T}\right) + \left(\frac{1}{2}\sigma^2\right)T \right)$$

$$d_2 = d_1 - \sigma\sqrt{T}.$$

There are a number of other models that have been developed for specific contracts, for example the Garman and Kohlhagen (1983) and Grabbe (1983) models, used for currency options, and the Merton, Barone-Adesi and Whaley or BAW model (1987) used for commodity options. For the valuation of American options, on dividend-paying assets, another model has been developed by Roll, Geske and Whaley. More recently the Black-Derman-Toy model (1990), based on a binomial approach, has been used to price exotic options. A detailed discussion of all of these, though very interesting, is outside the scope of this book. Readers may be interested in Appendices 35.1 and 35.2, which discuss the interest rate parity theorem and its application to the pricing of forwards, as this is relevant to our discussion on the price bias of futures against forwards.

In Chapter 46 we introduce models more relevant to pricing bond options, and which do not necessarily assume a constant risk-free interest rate during the life of the option.

44.6 Critique of the Black-Scholes model

The introduction of the B-S model was one of the great milestones in the development of the global capital markets, and it remains an important pricing model today. Many of the models introduced later for application to specific products are still based essentially on the B-S model. Nevertheless, other academics have highlighted some weaknesses in the model that stem from the nature of the main assumptions behind the model itself, which we will summarise here. The main critique of the B-S model centres on:

- assumption of frictionless markets; this is at best only approximately true for large market counterparties;
- constant interest rate; this is possibly the most unrealistic assumption. Interest rates over even the shortest time frame (the overnight rate) fluctuate considerably. In addition to a dynamic short rate, the short-end of the yield curve often moves in the opposite direction to moves in underlying asset prices, particularly so with bonds and bond options;
- lognormal distribution; this is accepted by the market as a reasonable approximation but not completely accurate, and also missing out most extreme moves or market shocks;
- European option only; although it is rare for American options to be exercised early, there are situations when it is optimal to do so, and the B-S model does not price these situations;
- for stock options, the assumption of a continuous constant dividend yield is clearly not realistic, although the trend in the US markets is for ordinary shares to cease paying dividends altogether.

These points notwithstanding, the Black–Scholes model paved the way for the rapid development of options as liquid tradeable products and is widely used today.

44.6.1 Stochastic volatility

The B–S model assumes a constant volatility and for this reason, and because it is based on mathematics, often fails to pick up on market “sentiment” when there is a large downward move or shock. This is not a failing limited to the B–S model however. For this reason however it undervalues out-of-the-money options, and to compensate for this market makers push up the price of deep in- or out-of-the-money options, giving rise to the volatility *smile*. This is considered in the next chapter.

The effect of stochastic volatility not being catered for then is to introduce mis-pricing, specifically the under-valuation of out-of-the-money options and the overvaluation of deeply in-the-money options. This is because when the price of the underlying asset rises, its volatility level also increases. The effect of this is that assets priced at relatively high levels do not tend to follow the process described by geometric Brownian motion. The same is true for relatively low asset prices and price volatility, but in the opposite direction. To compensate for this stochastic volatility models have been developed, such as the Hull–White model (1987).

44.6.2 Implied volatility

The volatility parameter in the B–S model, by definition, cannot be observed directly in the market as it refers to volatility going forward. It is different to historic volatility which can be measured directly, and this value is sometimes used to estimate implied volatility of an asset price. Banks therefore use the value for *implied volatility*, which is the volatility obtained using the prices of exchange-traded options. Given the price of an option and all the other parameters, it is possible to use the price of the option to determine the volatility of the underlying asset implied by the option price. The B–S model however cannot be rearranged into a form that expresses the volatility measure σ as a function of the other parameters. Generally therefore a numerical iteration process is used to arrive at the value for σ given the price of the option, usually the Newton–Raphson method, which is summarised in Appendix 44.5.

The market uses implied volatilities to gauge the volatility of individual assets relative to the market. Volatility levels are not constant, and fluctuate with the overall level of the market, as well as for stock-specific factors. When assessing volatilities with reference to exchange-traded options, market makers will use more than one value, because an asset will have different implied volatilities depending on how in-the-money the option itself is. The price of an at-the-money option will exhibit greater sensitivity to volatility than the price of a deeply in- or out-of-the-money option. Therefore market makers will take a combination of volatility values when assessing the volatility of a particular asset.

44.7 The Barone–Adesi and Whaley model

An extension of the B–S and Merton models is given by the BAW model (1987) and following the earlier presentations assumes a no-arbitrage scenario, so that the relationship between the spot price of an asset and its forward price for delivery at time T is given by:

$$F = Se^{rT}$$

where r is the financing rate applicable for a loan run over the period T . (Note that it is not the risk-free short-term rate, which we denote as b in this section.) The dynamics of the process followed by the asset price is given by:

$$\frac{dS}{S} = a dt + \sigma dW \quad (44.41)$$

where a is the constant relative price variation of an asset with a standard deviation of σ . Given this the following is used to describe the price process exhibited by the futures price, where b is a constant:

$$\frac{dF}{F} = (a - r)dt + \sigma dW. \quad (44.42)$$

Therefore assuming no-arbitrage pricing, an option contract on the asset can be given by the following partial differential equation, which was described by Merton (1973), which is then solved for the option price (the solution is not shown here),

$$\frac{1}{2}\sigma^2S^2\frac{\partial^2C}{\partial S^2} + rS\frac{\partial C(S,t)}{\partial S} - bC(S,t) + \frac{\partial C(S,t)}{\partial t} = 0 \quad (44.43)$$

where r is the funding cost for holding the asset as before and b is the short-term risk-free interest rate applicable over the same time period T . Both interest rates are constant rates over the period T . If the funding cost is equal to the risk-free rate, the equation is identical to the B–S model. At a funding cost of zero the equation is identical to the Black model. Under the upper bound that exists at the maturity point T , often described as Merton's bound (and which was discussed earlier), Merton presented the price of a European call option as (44.44):

$$C(S,T) = Se^{(r-b)T}N(d_1) - Xe^{-bT}N(d_2) \quad (44.44)$$

where

$$d_1 = \frac{1}{\sigma\sqrt{T}}\left(\ln\left(\frac{S}{X}\right) + \left(r + \frac{\sigma^2}{2}\right)T\right)$$

$$d_2 = d_1 - \sigma\sqrt{T}.$$

The relationship between spot and futures prices is modified for dividend paying securities. Where the underlying asset is a fixed-interest bond, the formula is changed to (44.45):

$$F = (S - c)e^{rT} \quad (44.45)$$

where c is a positive value if the asset returns an income and a negative value if the asset incurs a cost. The formula is demonstrated at Example 44.3.

EXAMPLE 44.3(i) Theoretical futures price

The spot price of a commodity is 101.95. The three-month repo rate on the asset is 5.65%. Therefore the price of a three-month futures contract is given by:

$$F = 101.95e^{(0.0565)(0.25)}$$

$$= 103.4003.$$

44.3(ii) Theoretical futures price on underlying coupon-paying bond

A two-year sterling Eurobond is priced at 113.50 and has a coupon of 10%, payable in six months' time. The one-year funding cost for the bond is 6.50%, while the six-month funding rate is 6.20%. What is the price of an OTC futures contract that expires in one year?

The bond pays a coupon on 10% during the life of the futures contract, which must be discounted to the present day. This is calculated as $I = 10e^{-0.062(0.5)} = 9.69948$.

The forward price is calculated as: $F = (113.50 - 9.69948)e^{0.065(1)} = 110.7717$.

44.8 Valuation of American options

After the first presentation of the B–S model in 1973, several developments to the basic model followed, including extensions to cover the pricing of American options. In this section we review some of the literature on American option pricing models, of which the most important contributions were by Merton (1973) and Geske (1979). Merton originally considered the valuation of American options in his 1973 article, including the problems associated with obtaining analytical solutions for American options on underlying assets that pay discrete dividend payments. The research by Geske presented a valuation formula for the pricing of “options on options”, or options on a company's ordinary shares, or the stock options used as a form of remuneration for a company's employees. In addition to the various proposals for American option pricing that have been presented, banks also use binomial models to price American options. At other times banks will use a model such as the BAW model to obtain a close approximation of an American option on assets that pay a non-discrete (continuous) dividend.

44.8.1 The issue of early exercise

A complete model for the valuation of an American option on an underlying asset that pays certain, discrete dividends has yet to be developed. When considering the pricing of an American option, the key issues are (i) the possibility of early exercise and (ii) the payment of dividends by the underlying asset during the option's life. In fact if an asset does not pay a dividend during the life of the option, there is no incentive to exercise the option early, because that would only realise intrinsic value for the holder and eliminate the time value. Therefore in this situation, as there is no possibility of early exercise, the option may be priced as a European one. This is proved in Merton (1973), who shows that if the asset does not pay a dividend, the holder of an American option will not exercise it ahead of expiry.

Holders of exchange-traded American options will sometimes exercise them early if there is a dividend payment about to be paid, as there is a benefit in so doing rather than wait until expiry (and lose the dividend). In this scenario, the optimum day to exercise the option is the last day before the underlying asset goes ex-dividend. For pricing options under these conditions, banks will use an adapted version of the B-S model, as described in Roll (1977), Geske (1979) and Whaley (1981). Another reason that an American option might be exercised early is if the "cost of carry" (funding cost) for the asset differs negatively from the risk-free interest rate. Essentially then except in the case of non-dividend paying assets, the B-S model will not adequately price American options, and in fact along with its assumptions the model's authors explicitly aimed it at European options. This notwithstanding, often a version of B-S adjusted for dividend paying securities is used to price American options. The adjustment takes the form of a constant proportional dividend yield, as proposed by BAW (1987) and the Cox, Ross and Rubinstein (1979) binomial method.

It is possible to discern exactly under what conditions early exercise might be advantageous for a holder. Where the underlying asset pays "continuous" dividends, on a constant basis, it can be shown (we do not show it here) that for the holder to have no incentive to exercise the option ahead of expiry, the following condition must be satisfied:

$$X > \frac{d}{r}$$

where d is the dividend yield. That is, if the strike price (of a call option) is greater than the ratio of the dividend rate and the funding rate (for that is what we may call it), then there is no incentive to exercise early.

Where the underlying asset pays a known dividend amount at discrete time intervals, of which the best example is a fixed-coupon bond, then for a holder to have no incentive to exercise early, the net present value of all the expected future dividends must be lower than the present value of the earnings that could be achieved by investing the amount equal to the strike price over t years. If this is not the case, there is no incentive to hold the option and it should be exercised. More formally, we can write

$$X > \sum_{i=1}^n \frac{P(t_i)}{1 - P(t_n)} d(t_i)$$

where $P(t)$ is the price of a zero-coupon bond that matures in t years, and d is the amount of dividend paid at each dividend date t_i for $i=1$ up to n .

If the dividend is known and quantified as $D(S, t)$ it can be shown that the partial differential equation used in the B-S model is adjusted, with the instantaneous rate of return now being given by:

$$(\mu - D(S, t)/S)dt$$

and Merton showed that for a call option with value $C(S, t, X)$ the partial differential equation becomes

$$\frac{1}{2}\sigma^2 S^2 \frac{\partial^2 C}{\partial S^2} + (rS - D) \frac{\partial C}{\partial S} + \frac{\partial C}{\partial t} - rC = 0 \quad (44.46)$$

which is solved under the following boundary conditions:

$$C(0, t, X) = 0$$

$$C(S, 0, X) = \max(0, S - X)$$

$$C(S, t, X) \geq \max(0, S - X).$$

The last condition is to ensure no-arbitrage pricing and indicates the value of an American option at any point in its life, and includes the possibility that at each point t the holder may exercise early. Therefore there may be a point in the life of the option, given by the value of the asset price $P(t)$ that where $S > P(t)$ it may be more worthwhile to exercise the option ahead of its expiry. For a call option the value of exercising early is given by $(S - X)$, so therefore an additional condition is necessary for the solution of the above equation, which is that

$$C(P(t), t, X) = P(t) - X = h$$

where $C(P(t), t, X)$ must satisfy the partial differential equation for $0 \leq S \leq P(t)$. However $P(t)$ is an unknown function of time which renders the solution to this difficult. Merton showed that the function $P(t)$ has to be determined as a function of the actions of the holder of the call option, who is assumed to maximise the value of the option at each instant. However the solution to this form of the equation provides the value of an American call option where the underlying asset makes discrete-time dividend payments. A solution for a perpetual-dated option is provided by Merton in his 1973 paper.

44.8.2 Valuation of American options

The valuation of an American option written on a dividend paying asset we summarise here is a formula first presented by Whaley (1981). We illustrate the application with reference to the following three hypothetical portfolios:

1. a long position in a European call option with a strike price of X and a maturity date of T ;
2. a long position in a European call option with a strike price of S_{xd} and a maturity date of $t - \varepsilon$,
($\varepsilon > 0, \varepsilon \cong 0$) where S_{xd} is the ex-dividend asset price at which level the option will be exercised early;
3. a short position in a European call option on the call option described in (1.) with a strike price of $S_{xd} + D - X$ and a maturity date of $(t - \varepsilon)$.

We note that the payoff profile on portfolios (2) and (3) is identical to that obtainable with an American call option; therefore the principle of no-arbitrage pricing implies that the value of an American call option with identical terms will be equal to that of the portfolios above. Therefore we can apply the B-S model in valuing the first two options and the Geske model (1979) for the third option, and use these to value an American option. This is done by applying a formula presented by Whaley for the valuation of American call options written on assets paying certain dividends. The formula, with reference to the hypothetical portfolios above, is:

$$\text{American call} = \text{call (a)} + \text{call (b)} - \text{call (c)}, \text{ or } C = c_a + c_b - c_c.$$

The individual options are priced using:

$$c_a = SN(a_1) - Xe^{rT}N(a_s) \quad (44.47)$$

$$c_b = SN(d_1) - (S_{xd} + D)e^{-rT}N(d_2) \quad (44.48)$$

$$c_c = SN(a_1, b_1, \sqrt{t/T}) - Xe^{-rT}N(a_2, b_2, \sqrt{t/T}) - (S_{xd} + D - X)e^{-rT}N(b_2) \quad (44.49)$$

where

$$a_1 = \frac{\ln(S/X) + (r + (\sigma^2/2))T}{\sigma\sqrt{T}} \text{ and } a_2 = a_1 - \sigma\sqrt{T}$$

$$b_2 = \frac{\ln(S/S_{xd}) + (r + (\sigma^2/2))t}{\sigma\sqrt{t}} \text{ and } b_1 = b_2 - \sigma\sqrt{t}$$

$$d_1 = \frac{\ln(S/(S_{xd} + D)) + (r + (\sigma^2/2))t}{\sigma\sqrt{t}} \text{ and } d_2 = d_1 - \sigma\sqrt{t}$$

and where $N(a, b, \rho)$ is the bivariate cumulative normal probability density function. It was shown by Whaley that using the property $N(a, -b, -\rho) = N(a) - N(a, b, \rho)$ and tidying the terms S and X the following formula is obtained:

$$c(S, T, X) = S[N(b_1) + N(a_1, b_1, -\sqrt{t/T})] + De^{-rT}N(b_2) - Xe^{-rT}[e^{r(T-t)}N(b_2) + N(a_2, -b_2, -\sqrt{t/T})]. \quad (44.50)$$

We require the price of the underlying asset at the critical point above which the option will be exercised early; this is found by applying a numerical search procedure to satisfy the following equation:

$$C(S_{xd}, T, X) = S_{xd} + D - X. \quad (44.51)$$

44.9 Describing stochastic volatilities

The development of option models following on from Black–Scholes has mainly involved research into applying the basic model to a greater range of instruments, and also to expand outside of the restrictions of the model given by its range of assumptions. One of the limiting assumptions of the B–S model is the one regarding constant volatility. As we saw in the relevant section in this chapter, the model assumes lognormal distribution of asset price returns, and a constant volatility value for the price of the underlying asset. However volatility levels fluctuate with moves in the market can lead to inaccurate valuation under certain conditions, as reviewed in Rubinstein (1985, 1994) and Koch (1992) among others.

One of the features of asset prices, especially equity and foreign exchange prices but also bond prices, is that they do not move in continuous lines but may jump across a price range (known as the *gap* in technical analysis). This was described in Cox and Ross (1976) and Merton (1976), who used a feature known as a jump diffusion process to price options. A pure diffusion process is where asset prices follow smooth and continuous changes from one period to the next, with no gaps between successive prices. This would mean that a plot of the prices recorded by an asset would follow a continuous line across prices in serial fashion. A jump process on the other hand is where an asset price follows a step pattern, for example a bond futures price recorded on the close of business at 104.50 and opening at 105.00 at the start of trading the next day.

Other writers have presented models to account for moving volatility levels (that is, stochastic volatility) which are adjustments to the B–S model to include stochastic volatility. These include Scott (1987), Hull and White (1987), Stein and Stein (1991) and Heston (1993). However by definition stochastic volatility or a jump process cannot be described by a risk-neutral probability function, which makes this type of risk difficult to hedge against. For this reason some of the pricing models that have been presented refer to a numerical solution to price the option, rather than an analytical solution.

For reference in this section we present two of the models that have been developed to account for stochastic volatility and jump processes.

44.9.1 The jump diffusion model

A model capturing the jump behaviour of asset prices was presented by Merton (1976), to describe the behaviour of asset prices that moves in non-continuous “jumps”, for example between the close of one trading session and the start of the next. The model combined the jump process and the diffusion process, hence its name. The jump process was defined in a way that limited the jumps in price to a lognormal distribution, so that the value of a European call option could be given as (44.52):

$$C = \sum_{n=0}^{\infty} \frac{e^{-\lambda(1+h)T} [\lambda(1+h)T]^n}{n!} (SN(d_1) - Xe^{-rT}N(d_2)) \quad (44.52)$$

where

$$d_1 = \frac{\ln(S/X) + (r' + (\sigma'^2/2))T}{\sigma'\sqrt{T}} \quad d_2 = \frac{\ln(S/X) + (r' - (\sigma'^2/2))T}{\sigma'\sqrt{T}}$$

and

$$r' = r - \lambda h + \frac{n \ln(1+h)}{T} \quad \sigma'^2 = \sigma^2 + \frac{n}{T} \sigma_j^2$$

where

- λ is the jump rate
- h is the average size of the jump, measured as a proportional increase in asset price
- σ_j^2 is the variance in the distribution of the jumps.

The calculation of λ is based on a Poisson distribution, which is described in Appendix 44.7.

44.9.2 The Hull–White model

The Hull–White model (1988), defines two processes, one describing the dynamics of the underlying asset price, and one describing the dynamics of the asset price volatility, which is a separate variable to the price variable. The two processes are defined as:

$$dS_t = \mu(S_t, \sigma_t, t) S_t dt + \sigma_t S_t dW_t^1 \quad (44.53)$$

$$dv_t = (\sigma_t, t) v_t dt + \delta(\sigma_t, t) v_t dW_t^2 \quad (44.54)$$

where

- S_t is the price of the underlying asset at time t
- v_t is the variance of asset price returns (that is, σ_t^2)

and W_t^1 and W_t^2 are two standard Weiner processes, which are related by the correlation coefficient ρ .

Following Black–Scholes, the price of a call option involves the setting up of a risk-free portfolio, which is composed of the call option, the underlying asset and another call option with an identical strike price and longer time to maturity.

The model assumes that:

- the two Weiner processes are independent;
- the volatility has no systematic risk, that is, there is no risk premium associated with increased volatility or volatility risk;
- the variance v is not influence by the underlying asset price, so the volatility is not correlated with the asset price.

All three assumptions are limiting factors and may present drawbacks when applying the model. Given a boundary condition on the maturity date defined as

$$C(T, S) = \begin{cases} S_T - X & \text{if } S_T \geq X \\ 0 & \text{if } S_T < X \end{cases}$$

and setting

$$v_t^*, T = \left(\frac{1}{T-t} \right) \int_t^T v_s ds$$

the model states that the value of a European call option is given by (44.55):

$$C = \int_0^\infty C_{BS}(t, S_t, v_t^*, T) dF(v_{t,T}^*/S_t, \sigma_t^2) \quad (44.55)$$

where C_{BS} is the price of the call option calculated using the standard Black–Scholes model corresponding to the variance measure $v_{t,T}^*$, and F is the conditional distribution of $v_{t,T}^*$ under the Q risk-neutral probability function, given the asset price and asset price variance at time t .

The value of the call option $C_{BS}(t, S_t, v_{t,T}^*)$ is given by (44.56):

$$C_{BS}(t, S_t, v_{t,T}^*) = S_t N(d_1) - X e^{-rt} N(d_2) \quad (44.56)$$

$$\text{where } d_1 = \frac{\ln(S/X) + (r + \frac{1}{2}v_{t,T}^*)T}{\sqrt{v_{t,T}^*}\sqrt{T}}, \quad d_2 = d_1 - \sqrt{v_{t,T}^*}.$$

The distribution of the variance $v_{t,T}^*$ can be calculated only under conditions where δ is a constant and where $v_{t,T}^*$ is distributed lognormally. Under these conditions, using a Taylor expansion of (44.55) the presentation by Hull and White gives the following as the value of the call option given an additional stochastic variable of the asset price volatility value.

$$\begin{aligned} C(t, S_t, \sigma_t^2) = & SN(d_1) - Xe^{-rt}N(d_2) + \frac{1}{2}S\sqrt{t}N(d_1)(d_1d_2 - 1)\left(2\sigma^4\left(e^h - h - \frac{1}{h^2} - \frac{1}{2}\right)\right) \\ & + \frac{1}{6}SN(d_1)((d_1d_2 - 3)(d_1d_2 - 1) - (d_1^2 + d_2^2)) \\ & + \frac{1}{8}\sigma^6\left(\frac{\sigma^6}{3h^3}\right)(e^{3h} - (9 + 18h)e^h + (8 + 24h + 18h^2 + 6h^3)) \end{aligned} \quad (44.57)$$

where $h = \varepsilon^2 t$.

In Briys *et al.* (1998) there is a comparison of the values produced for a European call option, with three different strikes (one each of in-the-money, at-the-money and out-of-the-money), three different volatility levels (10%, 25% and 50%) and three different terms to maturity (0.1 years, six months and 1 year). The differences between the valuation of a call option using the standard B-S model and the Hull-White model appear mainly when the option is out-of-the-money, and increases as the time to maturity increases. When the option is in-the-money there is very little difference in valuation, except when for the longest-dated option. In every case where there is a difference, the value calculated by the B-S model is lower, significantly so for the highest volatility level.

44.10 A final word on (and summary of) the models

We have only discussed the B-S model and certain variants of it in this chapter. Other pricing models have been developed that follow on from the pioneering work done by Messrs. Black and Scholes. The B-S model is essentially the most straightforward and the easiest to apply, and subsequent research has focused on easing some of the restrictions of the model in order to expand its applicability. Areas that have been focused on include a relaxation of the assumption of constant volatility levels for asset prices, as well as work on allowing for the valuation of American options and options on dividend-paying stocks. However often in practice some of the newer models require input of parameters that are difficult to observe or measure directly, which limits their application as well. Often there is a difficulty in calibrating a model due to the lack of observable data in the marketplace. The issue of calibration is an important one in the implementation of a pricing model, and involves inputting actual market data and using this as the parameters for calculation of prices. So for instance a model used to calculate the prices of sterling market options would use data from the UK market, including money market, futures and swaps rates to build the zero-coupon yield curve, and volatility levels for the underlying asset or interest rate (if it is a valuing options on interest-rate products, such as caps and floors). What sort of volatility is used? In some banks actual historical volatilities, more usually volatilities implied by exchange-traded option prices. Another crucial piece of data for multi-factor models (following Heath-Jarrow-Morton and other models based on this) is the correlation coefficients between forward rates on the term structure. This is used to calculate volatilities using the model itself.

The issue of calibrating the model is important, because incorrectly calibrated models will produce errors in option valuation. This can have disastrous results, which may be discovered only after significant losses have been incurred. If data is not available to calibrate the model, it may be that a simpler one needs to be used. The lack of data is not an issue for products priced in say, dollar, sterling or euro, but may be in other currency products if data is not so readily available. This might explain why the B-S model is still widely used today, although that is not to deny the increasing use of models such as the Black-Derman-Toy (1990) and Brace-Gatarek-Musiela (1994) models for more exotic option products.

Many models, because of the way that they describe the price process, are described as Gaussian interest rate models. The basic process is described by an Itô process

$$dP_T/P_T = \mu_T dT + \sigma_T dW_T$$

where P_T is the price of a zero-coupon bond with maturity date T and W is a standard Weiner process. The basic statement made by Gaussian interest-rate models is that:

$$P_T(t) = E_T \exp\left(-\int_t^T r(s)ds\right).$$

Models that capture the process in this way include Cox–Ingersoll–Ross and Harrison and Pliska. We are summarising here only, but essentially such models state that the price of an option is equal to the discounted return from a risk-free instrument. This is why the basic B–S model describes a portfolio of a call option on the underlying stock and a cash deposit invested at the risk-free interest rate. This was reviewed in the chapter. We then discussed how the representation of asset prices as an expectation of a discounted payoff from a risk-free deposit does not capture the real-world scenario presented by many option products. Hence the continuing research into developments of the basic model.

Following on from B–S, under the assumption that the short-term spot rate drives bond and option prices, the basic model can be used to model an interest-rate term structure, as given by Vasicek and Cox–Ingersoll–Ross. The short-term spot rate is assumed to follow a diffusion process

$$dr = \mu dt + \sigma dW$$

where dW is a standard Weiner process. From this it is possible the complete term structure based on the short-term spot rate and the volatility of the short-term rate. This approach is modified by Heath–Jarrow–Morton, which is reviewed later.

This chapter has summarised some of the models used to price options and the underlying assumptions of these models. In Part VIII we review again certain interest-rate models in the context of yield curve modelling.

Appendices

APPENDIX 44.1 Summary of basic statistical concepts

The arithmetic mean μ is the average of a series of numbers. The *variance* is the sum of the squares of the difference of each observation from the mean, and from the variance we obtain the standard deviation σ which is the square root of the variance. The *probability density* of a series of numbers is the term for how likely any of them is to occur. In a normal probability density function, described by the normal distribution, the probability density is given by

$$\frac{1}{\sqrt{2\pi}e^{x^2/2}}.$$

Most option pricing formulas assume a normal probability density function, specifically that movements in the natural logarithm of asset prices follow this function. That is,

$$\ln\left(\frac{\text{today's price}}{\text{yesterday's price}}\right)$$

is assumed to follow a normal probability density function. This relative price change is equal to:

$$1 + r \times \frac{\text{days}}{\text{year}}$$

where r is the rate of return being earned on an investment in the asset. The value

$$\ln\left(1 + r \times \frac{\text{days}}{\text{year}}\right)$$

is equal to $r \times (\text{days} / \text{year})$ where r is the continuously compounded rate of return. Therefore, the value

$$\ln(\text{today's price} / \text{yesterday's price})$$

is equal to the continuously compounded rate of return on the asset over a specified holding period.

APPENDIX 44.2 Lognormal distribution of returns

In the distribution of asset price returns, returns are defined as the logarithm of price relatives and are assumed to follow the normal distribution, given by

$$\ln\left(\frac{P_t}{P_0}\right) \sim N(rt, \sigma\sqrt{t}) \quad (44.58)$$

where

- P_t is the price at time t
- P_0 is the price at time 0
- $N(m, s)$ is a random normal distribution with mean m and standard deviation s
- r is the annual rate of return
- σ is the annualised standard deviation of returns.

From (44.58) we conclude that the logarithm of the prices is normally distributed, due to (44.59) where P_0 is a constant

$$\ln(P_t) \sim \ln(P_0) + N(rt, \sigma\sqrt{t}). \quad (44.59)$$

We conclude that prices are normally distributed and are described by the relationship

$$\frac{P_t}{P_0} \sim e^{N(rt, \sigma\sqrt{t})}$$

and from this relationship we may set the expected return as rt .

APPENDIX 44.3 Approximation of the cumulative normal distribution

This section is reproduced, with kind permission of John Wiley and Sons, from Briys *et al.* (1998). In order to price options or other contingent claim securities, it is necessary to calculate the cumulative standard normal distribution described by (44.60):

$$N(d) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^d \exp\left(-\frac{x^2}{2}\right) dx. \quad (44.60)$$

Two methods may be used to calculate this. The first approximation, which is accurate to a precision level of 10^{-3} , is as follows. If $x \geq 0$, then with the coefficients:

$$\begin{aligned} a_1 &= 0.196854 \\ a_2 &= 0.115194 \\ a_3 &= 0.000344 \\ a_4 &= 0.019527 \end{aligned}$$

the formula is given by (44.61):

$$N(x) \approx 1 - \frac{1}{2} \left(1 + a_1x + a_2x^2 + a_3x^3 + a_4x^4\right)^{-4}. \quad (44.61)$$

The second approach has a precision level of 10^{-7} and is approximated as shown. If $x > 0$ then with the following coefficients:

$$\begin{aligned} p &= 0.2316419 \\ b_1 &= 0.319381530 \\ b_2 &= -0.356563782 \\ b_3 &= 1.781477937 \\ b_4 &= -1.821255978 \end{aligned}$$

$$b_3 = 1.330274429$$

$$c = 1/(1 + px),$$

the formula obtained is (44.62).

$$N(x) \approx 1 - (1/\sqrt{2\pi}) \exp(x^2/2)(b_1c + b_2c^2 + b_3c^3 + b_4c^4 + b_5c^5). \quad (44.62)$$

APPENDIX 44.4 Interest-rate parity theorem

The interest-rate parity theorem states that the forward rate is equal to the spot rate compounded by the differential between the foreign and domestic currency interest rates. Using continuously compounded interest rates, the forward exchange rate is described by:

$$f = Se^{(r-r^*)T}$$

so therefore the formula for the pricing of a European call on a spot currency can be re-written as:

$$C = e^{-rT} (fN(d_1) - XN(d_2)). \quad (44.63)$$

APPENDIX 44.5 The Newton-Raphson method

This is a well-known iterative method for solving an equation of the form $f(x) = 0$. Consider an approximation on a curve where $x = \alpha$. The point $P(\alpha, f(\alpha))$ on the curve is close to the solution, and is a tangent drawn as near as possible to the point of the solution itself. The tangent is approximated at point β , which is a nearer result than α . The gradient of the tangent at P is given by $f'(\alpha)$, but the gradient of the line is given by:

$$\frac{f(\alpha)}{\alpha - \beta}.$$

$$\text{Therefore, } f'(\alpha) = \frac{f(\alpha)}{\alpha - \beta} \text{ giving } \alpha - \beta = \frac{f(\alpha)}{f'(\alpha)} \text{ so that } \beta = \alpha - \frac{f(\alpha)}{f'(\alpha)}.$$

The process can be repeated by drawing a tangent to the curve at a closer point than β in order to find a close value. The process can be summarised by an iterative formula:

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}.$$

The term x_0 is taken as the initial approximation to the solution.

APPENDIX 44.6 Black–Scholes model in Microsoft® Excel

Cell	C	D	
8	Underlying price, S	100	
9	Volatility %	0.0691	
10	Option maturity years	0.25	
11	Strike price, X	99.50	
12	Risk-free interest rate %	0.05	
13			
14			
15			Cell formulae
16	$\ln(S/X)$	0.005012542	=LN (D8/D11)
17	Adjusted return	0.0000456012500	= ((D12-D9)^2/ 2)*D10
18	Time adjusted volatility	0.131434394	=(D9*D10)^0.5
19	d_2	0.038484166	=(D16+D17)/D18
20	$N(d_2)$	0.515349233	=NORMSDIST (D19)
21			
22	d_1	0.16991856	=D19+D18
23	$N(d_1)$	0.56746291	=NORMSDIST (D22)
24	e^{-rt}	0.9875778	=EXP (-D10*D12)
25			
26	CALL	6.106018498	=D8*D23-D11*D20*D24
27	PUT	4.370009648	\$ =D26-D8+D11*D24

§ By put-call parity, $P = C - S + Xe^{-rt}$

APPENDIX 44.7 The Poisson distribution

The Poisson distribution is a standard distribution in statistics. If we denote $(N_t)_{t \geq 0}$ as a standard Poisson process with an intensity λ then at all times when $t > 0$ the random variable will follow a distribution given by the following, where the probability of N is:

$$P(N_t = n) = \frac{e^{-\lambda t}}{n!} (\lambda t)^n.$$

The expected value of N_t is given by $E(N_t) = \lambda t$ and the variance is given by:

$$\text{Var}(N_t) = E(N_t^2) - (E(N_t))^2 = \lambda t.$$

Selected bibliography and references

Arrow, K., *Essays in the theory of Risk Bearing*, North-Holland, 1970.

Black, F., Scholes, M., “The Pricing of Options and Corporate Liabilities”, *Journal of Political Economy* 81, May–June 1973, pp. 637–659.

Black, F., Derman, E., Toy, W., “A One-Factor Model of Interest Rates and its Application to Treasury Bond Options”, in Hughston, L. (ed.), *Vasicek and Beyond*, Risk Publications, 1996.

Bookstaber, R., *Option Pricing and Strategies in Investing*, Addison-Wesley, 1982.

- Boyle, P., "Option valuation using a three jump process", *International Options Journal* 3, 1986, pp. 7–12.
- Boyle, P., "A lattice framework for option pricing with two-state variables", *Journal of Financial and Quantitative Analysis* 23, March 1988, pp. 1–12.
- Brace, A., Gatarek, D., Musiela, M. "The Market Model of Interest Rate Dynamics", in Hughston, L. (ed.), *Vasicek and Beyond*, Risk Publications, 1996.
- Briys, E, et al., *Options, Futures and Exotic Derivatives*, John Wiley and Sons, 1998.
- Cox, D., Miller, H., *The Theory of Stochastic Processes*, Chapman & Hall, 1965.
- Cox, J., Ross, S., "The valuation of options for alternative stochastic processes", *Journal of Financial Economics* 3, 1976, pp. 145–166.
- Cox, J., Ross, S., Rubinstein, M., "Option pricing: A Simplified Approach", *Journal of Financial Economics* 7, October 1979, pp. 229–264.
- Fama, E., "The Behaviour of Stock Prices", *Journal of Business* 38, January 1965, pp. 34–105.
- Debreu, G., "Representation of a preference ordering by a numerical function", in Thrall, R., Coombs, C., Davis, R. (eds.), *Decision Processes*, Wiley, 1954.
- Geske, R., "The valuation of compound options", *Journal of Financial Economics* 7, 1979, pp. 63–81.
- Geske, R., "A Note on an Analytic Valuation Formula for Unprotected American Call Options on Stocks with Known Dividends", *Journal of Financial Economics* 9, June 1979, pp. 375–380.
- Harrison, J., Kreps, D., "Martingales and arbitrage in multi-period securities markets", *Journal of Economic Theory* 20, 1979, pp. 381–408.
- Harrison, J., Pliska, S., "Martingales and stochastic integrals in the theory of continuous trading", *Stochastic Processes and Their Applications* 11, 1981, pp. 216–260.
- Heston, S., "A closed form solution for options with stochastic volatility with application to bond and currency options", *Review of Financial Studies* 6, 1993, pp. 327–343.
- Heston, S., "Invisible parameters in option prices", *Journal of Finance* 48 (3), 1993, pp. 933–947.
- Hull, J., White, A., "The pricing of options on assets with stochastic volatilities", *Journal of Finance* 42, 1987, pp. 281–300.
- Hull, J., White, A., "An analysis of the bias caused by a stochastic volatility in option pricing", *Advances in Futures and Options Research* 3, 1988, pp. 29–61.
- Itô, K., "On Stochastic Differential Equations", *American Mathematical Society* 4, 1951, pp. 1–51.
- Klemkosky, R., Resnick, B., "Put-Call Parity and Market Efficiency", *Journal of Financial Economics* 34, December 1979, pp. 1141–1155.
- Merton, R., "Theory of Rational Option Pricing", *Bell Journal of Economics and Management Science* 4, Spring 1973, pp. 141–183.
- Merton, R., "Option pricing when underlying stock returns are discontinuous", *Journal of Financial Economics* 3, Jan–Mar 1976, pp. 125–144.
- Neilsen, L.T., *Pricing and Hedging of Derivative Securities*, OUP, 1999.
- Rendleman, R., Bartter, B., "Two State Option Pricing", *Journal of Finance* 34, 1979, pp. 1092–1110.
- Roll, R., "An Analytical Formula for Unprotected American Call Options on Stocks with Known Dividends", *Journal of Financial Economics* 5, 1977, pp. 251–258.
- Rubinstein, M., "Nonparametric tests of alternative option pricing models", *Journal of Financial Economics* 40, 1985, pp. 455–480.
- Rubinstein, M., *Rubinstein on Derivatives*, RISK, 1999.
- Scott, L., "Option pricing when the variance changes randomly", *Journal of Financial and Quantitative Analysis* 22, 1987, pp. 419–438.
- Smith, C., "Option Pricing: A Review", *Journal of Financial Economics* 3, 1976, pp. 3–54.
- Stein, E., Stein, J., "Stock price distributions with stochastic volatility: an analytic approach", *Review of Financial Studies* 4, 1991, pp. 113–135.
- Whaley, R., "On the Valuation of American Call Options on Stocks with Known Dividends", *Journal of Financial Economics* 9, 1981, pp. 207–211.

Questions and exercises

1. An at-the-money call option with a strike price of £100 has a price of £6. The call option has three months to expiry. The same-maturity put option is priced at £4. Draw payoff profiles for the following:
 - (a) buying the call option
 - (b) writing the call option
 - (c) writing the call option and buying the underlying security
 - (d) buying the put option and buying the underlying security
 - (e) buying the call option and selling short the underlying security
2. Explain how we obtain the put-call parity relationship for options.
3. List the factors that influence option prices.
4. Explain the impact of each of the following on the value of a call option:
 - (a) the price of the underlying asset
 - (b) the strike price
 - (c) the time to expiry
 - (d) the volatility of the underlying asset price returns
5. A six-month call option has a strike price of £90 while the underlying asset is priced at £100. The risk-free six-month interest-rate is 6.50%. From Merton's bound, what is the lowest possible price of the call option?
6. How is the B-S model used to price bond options? What process carried out in the valuation is specific to bond options?
7. Discuss the anomaly involved in applying the B-S model to the valuation of bond options.
8. In the Black-Scholes model what does the term e^{-rT} calculate? What are $N(d_1)$ and $N(d_2)$ approximating?
9. Is there a difference between the price of a futures contract and the value of a futures contract?
10. An asset price is currently £10. Over the next three one-month periods it is believed that the price will move upwards by 5% or downwards by 4%. The risk-free interest rate is 6%. What would be the value of a three-month call option on the asset that was written with a strike price of £11?
11. Prove that $SN(d_1) = Xe^{-rT}N(d_2)$.
12. Is there a difference between a futures price and a forward price in theory? In practice?
13. What can we conclude from the interest-rate theorem?
14. A non-dividend paying asset is priced today at 95.00. The seven-month funding cost for the asset is 8.25%. What is the price of a seven-month forward contract written on the asset?
15. Assume that the asset above paid a coupon of 5% every six months. The six-month funding rate is 8.00% and the one-year funding cost is 8.50%. Calculate the price of a one-year forward contract written on the asset.
16. Describe the Barone-Adesi-Whaley model. How does this reduce, following Merton, to the B-S model and the Black model?
17. Discuss how the B-S model differs from the Black model.
18. Under what circumstances is it ever optimal to exercise an American option ahead of the expiry date?
19. Discuss the assumptions behind the application of the B-S model and how they might hinder its effectiveness.
20. A non-dividend paying security has a price of £98 and the volatility of its price returns is 12%. The strike price of a six-month option written on it is £96 while the risk-free interest rate is 8%. Calculate the following:
 - (a) the price of a European call option

- (b) the price of a European put option
 - (c) confirm that the put-call parity relationship holds.
21. Calculate the implied volatility of the underlying asset for a one-year call option (the security is non-dividend paying) where the current underlying price is \$10, the option is priced at \$1.50, the strike price is \$9 and the risk-free interest rate is 6.00% annualised.
 22. What is the jump diffusion model? In theory how does it allow for the more accurate valuation of an option based on the actual price process followed by asset prices?
 23. What are the key factors that must be modelled in an environment of stochastic volatility if we wish to price options?

45 Options III: The Binomial Pricing Model

45.1 The binomial option pricing model

We have already introduced the binomial approach to pricing securities, in the chapters on callable bonds and convertibles. The methodology may be applied to options as well, and in fact there are a number of option variants that cannot be priced accurately by the B-S model, so that it is necessary to use the binomial model instead. Under certain scenarios the binomial model approximates to the B-S model, depending on the number of lattices used in the model and certain other factors. The binomial model, which is also known as the *lattice approach*, is perhaps a more academic approach than the continuous-time models pioneered by Black–Scholes, although it appeared later. However under certain conditions it is preferred as a valuation tool, most commonly for the pricing of options on equities and equity indices, that is, for products where a dividend payment is associated with the underlying asset.

The binomial approach was first presented by Cox, Ross and Rubinstein in 1979. Although first applied to equity options, it has since been extended to products such as callable bonds and convertibles and bond options, and to model the term structure of interest rates. A subsequent paper by Ho and Lee (1986) used the binomial approach to value zero-coupon bonds, and therefore can be used to describe the term structure of interest rates. Other models developed on a similar basis include Black, Derman and Toy (BDT, 1990) and Hull and White (1993). The BDT model incorporates a lattice approach to construct a one-factor model of the short-term interest rate, and the parameters used in deriving the model include the volatility values for all zero-coupon rates along the term structure, in addition to the current zero-coupon term structure. The Hull–White model (1993) involves a process that uses trinomial trees to derive a one-factor model of the short-term spot rate, where the short-term rate is assumed to follow a Markov process. Both models are widely used by banks for pricing bond and interest-rate options.

For readers unfamiliar with the concept of the binomial lattice we present introductory notes in Appendix 45.1

45.1.1 Introducing the model

Consider an option on a non-dividend paying asset. The price S of the asset can move to one of only two outcomes, one upward from the current price and one downwards. This change in price occurs over a discrete time interval, and there is a probability associated with each price move, so the price move upwards has a probability of p and the probability of a move downwards is $(1 - p)$. An option written on the asset, with a maturity date of T will move in these discrete steps as well, according to the change in the underlying asset price.

Figure 45.1 illustrates these points.

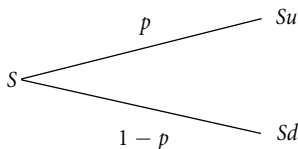


Figure 45.1: Binomial approach to the dynamics of the asset price.

In Figure 45.1 the change in price is deemed to occur during a discrete time interval Δt , and the parameters on probability and final price outcome, p , u and d are functions of the mean and standard deviation of the returns recorded by S during the time period Δt . Therefore, the standard deviation of the return on S will increase as the spread between Su and Sd increases for a given time interval.

The price of the stock at each node is given by $Su^j d^{i-j}$ for points where j moves from 0 to i . The process can be carried on for any number of time intervals, for example Figure 45.2 shows the binomial tree for three time periods.

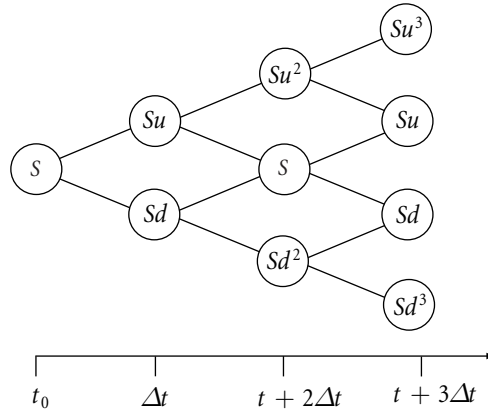


Figure 45.2: Binomial approach to asset price changes over three time periods.

Using this framework, the value of a call option C_{ij} at time $t + i\Delta t$ will be given by starting from the maturity date of the instrument at time T , at which point the payoff from the instrument is known, and working backwards down the binomial lattice to today at time t . It is easy to see why this approach also works with fixed income instruments such as callable bonds and convertibles, and why once the probability values for each leg of the lattice are known, the current value of the bond can be calculated.

Coming back to options, on the maturity date of the option at time T , the value of a call option is set by the following upper and lower bounds, $C_T = \max(S_T - X, 0)$.

This may also be written as $C_{N,j} = \max(Su^j d^{N-j} - X, 0)$.

From the previous chapter we know that the value of an option is the discounted present value of its expected payoff, with the risk-free interest rate that applies to the life of the option being used to discount the payoff. Therefore in the same way we can calculate the value of the call option at each point in the binomial tree, and this is given by (45.1):

$$C_{i,j} = e^{-r\Delta t} (pC_{i+1,j+1} + (1-p)C_{i+1,j}) \quad (45.1)$$

for $0 \leq i \leq N-1$ and $0 \leq j \leq i$. If we wish to value a put option the same formula can be used.

Equation (45.1) will value a European option; for an American call option the formula is:

$$C_{i,j} = \max(Su^j d^{i-j} - X, e^{-r\Delta t} (pC_{i+1,j+1} + (1-p)C_{i+1,j})). \quad (45.2)$$

45.1.2 Adjusting the model for dividend-paying assets

If an option is written on a dividend-paying asset it can be adjusted to take into the account the change in the asset price at the point the asset goes ex-dividend. At the time period $t + i\Delta t$ one instant before the ex-dividend date, the price of the underlying asset is given by:

$$Su^j d^{i-j} \text{ for } j = 0, 1, 2, \dots, i \quad (45.3)$$

At the instant $t + i\Delta t$ after the ex-dividend date, the price of the underlying asset is given by:

$$S(1 - r_i)u^j d^{i-j} \text{ for } j = 0, 1, 2, \dots, i \quad (45.4)$$

Therefore if we set r as the total dividend yield for all ex-dividend dates, then expression (45.4) is the price of the underlying stock. Therefore using this expression we can set the value of the asset at each node on the binomial tree as before, and solve for the price of the call option.

45.2 The binomial approach for interest-rate options

45.2.1 The Ho–Lee model

An application of the binomial lattice approach to the pricing of options on interest-rate assets was presented by Ho and Lee (1986). The option can be written on a bond, an interest-rate futures contract, a bond option, in fact any interest-rate asset.

Consider an option C with a time to expiry T and a payoff on expiry of $f(i)$, with $0 < i < T$, so that

$$C(T, i) = f(i).$$

The option has a minimum value L and maximum value H so that at time n and state i the following relationship holds:

$$L(n, i) \leq C(n, i) \leq H(n, i).$$

The model states that the option will receive $X(n, i)$ at time n and state i where $1 < n < T$.

Following Black–Scholes, under a scenario whereby a risk-free portfolio is constructed, consisting of the call option and a risk-free zero-coupon bond, using the principle of no-arbitrage pricing Ho and Lee show that the following is true:

$$C(n, i) = (\pi(C(n+1, i+1) + X(n+1, i+1)) + (1-\pi)(C(n+1, i) + X(n+1, i)))P_i^{(n)}(1) \quad (45.5)$$

where $P_i^{(n)}(1)$ is the price of a one-period zero-coupon discount bond at the point (n, i)

The expression at (45.5) will derive the arbitrage-free price of the option at the point one period before the expiry date, or $C^*(T-1, i)$. Therefore the valuation of the option is constrained by the following bounds:

$$C(T-1, i) = \max(L(T-1, i), \min(C^*(T-1, i), H(T-1, i))) \quad (45.6)$$

and using these values it is possible to work backwards along the binomial tree and calculate the option price at time t_0 , which is the fair price of the option.

Several writers have highlighted inaccuracies that can result from application of the Ho–Lee model. As with other models described under the Gaussian spot rate process, the modelling of the short-rate to describe the entire term structure can give rise to negative forward rates, for which see Ritchken and Boenawen (1990) and Heath, Jarrow and Morton (1992). In addition the parameters of the model, namely the risk-neutral probability π and the interest-rate spread can give rise to a discount function (calculated from the one-period zero-coupon bonds) that is not bounded by $[0, 1]$; this is also detailed in Ritchken and Boenawen.

45.2.2 The Hull–White model

A lattice approach is described by Hull and White (1993) that uses a trinomial process to construct a one-factor of the term structure, based on the short-dated spot rate. In Briys *et al* (1998) the model is described as being efficient and straightforward to implement. The short-term interest rate r for the period t is assumed to follow a Gaussian process also described in Vasicek (and hence known as the “extended Vasicek model”), so that $r(t)$ is given by:

$$dr = \mu(\theta, r, t)dt + \sigma dW(t) \quad (45.7)$$

where the mean is known and the standard deviation is constant. The value of θ is not known.

The short-term interest rate r is the rate at time 0 is equivalent to the continuously-compounded yield on a risk-free zero-coupon bond that matures at Δt . In the trinomial tree the values of r have a discrete interval along the nodes of $T_\theta + j\Delta t$. The values of Δt are, as they are given by the nodes of the tree, spaced equally along the lattice. As stated by the model, if the binomial tree is constructed up to the point in time where $n\Delta t$ ($n \geq 0$) is in line with the observed $R(i)$ and the interest rate at time $i\Delta t$ is the rate applicable for the period between $i\Delta t$ and $(i+1)\Delta t$ then the tree will reflect the value of $R(i)$ for $i < n+1$. The value $R(i)$ is the yield at time 0 on a zero-coupon bond that matures at time $i\Delta t$. The user then selects the value of $\theta(n\Delta t)$ that allows the tree to be consistent with $R(n+2)$, which then allows the calculation of the drift rates of the short-term rate r at time $n\Delta t$. This is given by (45.8):

$$\mu_{n,j} = \mu(\theta(n\Delta t, r_0 + j\Delta r, n\Delta t)) \quad (45.8)$$

where

$\mu(i, j)$ is the drift rate for the short-term rate r at node (i, j) with $r_j = r_0 + j\Delta r$
 (i, j) is a node on the trinomial tree for the values of $t = i\Delta t$ and $r_j = r_0 + j\Delta r$.

The final stage of the process is to calculate the probabilities along each node, and use these to calculate the short-dated interest rate at each forward point of the tree. Hull and White state the following probabilities:

$$\begin{aligned} P_1(n, j) &= \frac{\sigma^2 \Delta t}{2\Delta r^2} + \frac{\eta^2}{2\Delta r^2} + \frac{\eta}{2\Delta r} \\ P_2(n, j) &= 1 - \frac{\sigma^2 \Delta t}{\Delta r^2} - \frac{\eta^2}{\Delta r^2} \\ P_3(n, j) &= \frac{\sigma^2 \Delta t}{2\Delta r^2} + \frac{\eta^2}{2\Delta r^2} - \frac{\eta}{2\Delta r} \end{aligned} \quad (45.9)$$

where $\eta = \mu_{n,j}\Delta t + (j - k)\Delta r$ and $P_k^{(i,j)}$ is the probability corresponding to the upper, middle and lower lattices originating from node (i, j) .

45.3 Comparison with B–S model

It can be shown that a binomial model with a very large number of lattices, each of which runs over a shorter time period, approximates closely to the B–S model. This is dependent however on the possible upward and downward price movements being set appropriately. That is, when using a binomial model to calculate an option price, the possible price outcomes must be chosen so that the final valuation is close to the B–S model. For the outcomes of the two models to converge, the possible upward and downward price movements should be chosen to match the parameters that are assumed by the B–S model. Thus the parameters that are set are:

$$u = e^{\sigma\sqrt{\Delta t}} \quad d = \frac{1}{u} \quad p = \frac{e^{r\Delta t} - d}{u - d}$$

where u and d are the multiplicative upward and downward movements.

The option price that would be obtained from one- or two-step binomial lattices would be very inaccurate. In practice the binomial approach usually involves running the process over a minimum of 20 or 30 time periods, by which number the option maturity period is divided. Each time step has a price movement, and for 30 time periods there would be a possible 31 possible price outcomes. The number of price paths would be very large in this case, in fact it would be 2^{30} . For further discussion of binomial models refer to Hull (1997).

Appendices

APPENDIX 45.1 Introduction to binomial lattice approach

The primary assumption underpinning the binomial model is that at any one time an asset price can only move up or down by a pre-specified amount. For instance, consider an hypothetical asset with a current price of 1. Assume that the price of the asset may rise by a factor of 1.0100 or fall by a factor of 1/1.01 or 0.990099 each month. After a two month period the price would have moved along one of four possible routes, and there would be three finite outcomes. This is shown in Figure 45.3.

At the end of the third month there would be six possible routes and four possible outcomes. It is possible to build up a lattice tree with as many routes as required, and in fact after a large number of steps the binomial outcomes begin to approximate to a normal distribution. If we know the relative probabilities of an asset price following the “up” route or “down” route, we are able to calculate the probability and the value of the expected outcome at the end of the process. With this data, we would be able to price an option that was written on the asset in question for maturity at the end of the period represented by the end of the binomial process. Therefore we require the probabilities along each route.

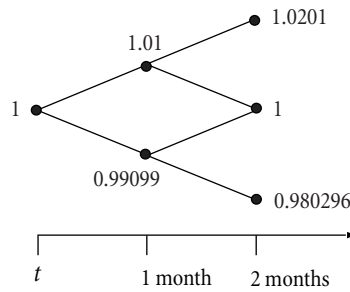


Figure 45.3: Binomial lattice of outcomes.

It is possible to calculate the probabilities given risk-free interest rate that applies over the time period of the option's life. The return generated by a cash deposit, invested at the risk-free interest rate, is set equal to the expected outcome of owning the option underlying asset. This enables us to calculate the probabilities along the binomial tree. To illustrate this, suppose that the probability of an upward move in the price of an asset at the end of one month is p ; this means that the probability of a downward move in the price is $1 - p$. Assume that the one-month risk-free interest-rate is 5.00% annualised. Given this data, we can set the expected price outcome as

$$p \times 1.01 + (1 - p) \times 0.99099 = 0.99099 + 0.19901 \times p.$$

The return generated by a cash deposit for a one-month period is given by:

$$\left(1 + 0.05 \times \frac{1}{12}\right) = 1.004167.$$

Now that we have the return generated by the risk-free deposit, we may set this as equal to the outcome generated by an investment in the underlying asset for one month, so we have $0.99099 + 0.019901 \times p = 1.004167$.

We then solve for p which gives us p equal to 0.7068992. Therefore we have the probabilities for the first two routes of the binomial lattice. This may now be used to price an option on the underlying asset. Consider a one-month call option on 1 unit of the asset with a strike price of 1.004 (that is, it is out-of-the-money). There is a probability of 0.7069 that the price outcome is 1.01. This would make the option worth $1.01 - 1.004$ or 0.006 at that point. There is also a probability of 0.2931 that the price of the asset will be at 0.99099; at this point the option will be out-of-the-money and so will expire worthless. This is shown at Figure 45.4.

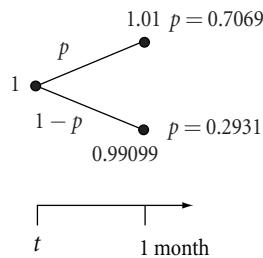


Figure 45.4: Binomial option pricing.

Therefore the expected value of the option on the expiry date is $(0.7068992 \times 0.006) + (0.2931 \times 0) = 0.0042414$. This needs to be discounted back to its present value today, when the option is struck. The present value is $0.0042414 / (1 + 0.05 \times (1/12))$ or 0.0042238. The fair value of the call option on the asset is therefore 0.4224%.

The assumption that allows us to apply the binomial model is similar to the B-S approach that equates the return from investing in the underlying asset to the return from a risk-free cash deposit. For the binomial model approach, to calculate the probabilities of upward and downward movements, we equate the expected outcome arising from an investment in the underlying asset with the outcome generated by a risk-free deposit. This is explained as follows: consider a hedged portfolio that consists of a long position in the underlying asset and a short

position in the call option. The hedge ratio is calculated by setting the amount of the asset purchased at G , so that there are two possible outcomes: $G \times 1.01 - 0.005$ or $G \times 0.99099$.

However the investment strategy is risk-free and the process must be arbitrage-free, so that the two outcomes must be identical, allowing us to set $G \times 1.01 - 0.005 = G \times 0.99099$. We then solve for G and this is 0.2512437. The final outcome is therefore 0.2512437×0.99099 or 0.248756. This is the hedge ratio. If the price of the call option on one unit of the asset is C , the initial investment in the portfolio will be $(0.2512 - C)$. With a risk-free interest rate of 5%, we can set:

$$(0.2512437 - C) \times \left(1 + 0.05 \times \frac{1}{12}\right) = 0.248756.$$

We solve this for the option price C and this provides the same answer as before.

APPENDIX 45.2 The Binomial distribution

A distribution is described as binomial when it:

- consists of a discrete number of trials or processes;
- only two outcomes are possible in each trial (for example, heads or tails in a coin toss);
- the probability of the outcomes in the trial do not change;
- the trials are independent.

Given these conditions we can use the Binomial formula to calculate directly the probabilities of an outcome. This is written as (45.10):

$$P(r) = {}^nC_r p^r q^{(n-r)} \quad (45.10)$$

where

$${}^nC_r = \frac{n!}{r!(n-r)!}$$

- p is the probability of the outcome from the trial under review
- q is $(1 - p)$, the probability of the specified outcome in the trial not occurring
- n is the number of trials
- r is the specified number of outcomes we are looking for.

Selected bibliography and references

- Cox, J., Ross, S., Rubinstein, M., "Option pricing: a simplified approach", *Journal of Financial Economics* 7, September 1979, pp. 229–263.
- Black F., Derman, E., Toy, W., "A One-Factor model of interest rates and its applications to Treasury bond options", *Financial Analysts Journal* 46, 1990, pp. 33–39.
- Hull, J., *Options, Futures and other Derivatives*, 3rd edition, Prentice-Hall Inc., 1997, Chapter 17.
- Hull, J., White, A., "One-factor interest rate models and the valuation of interest rate derivative securities", *Journal of Financial and Quantitative Analysis* 28, June 1993, pp. 235–254.
- Hull, J., White, A., "Efficient procedures for valuing European and American path-dependent options", *Journal of Derivatives* 1, Autumn 1993, pp. 21–23.
- Ho, T., Lee, S., "Term structure movements and pricing interest rate contingent claims", *Journal of Finance* 41, 1986, pp. 345–370.
- Heath, D., Jarrow, R., Morton, A., "Bond pricing and the term structure of interest rates: a new methodology for contingent claims valuation", *Econometrica* 60(1), 1992, pp. 77–105.
- Ritchken, P., Boenawen, K., "On arbitrage-free pricing of interest rate contingent claims", *Journal of Finance* 55(1), 1990, pp. 259–264.

46

Options IV: Pricing Models for Bond Options

In Chapter 44 we introduced and summarised the main methods by which option contracts are priced. We noted that the primary pricing model, the Black–Scholes formula, could be applied to options on bonds with a minor adjustment connected with subtracting the present value of expected coupons from the underlying asset price. In this chapter we present some models specifically applied to interest-rate options. These are the Vasicek (1977), Cox–Ingersoll–Ross (1985) and Heath–Jarrow–Morton (1992) models. These models are primarily associated with yield curve modelling, but as they seek to describe the process for the evolution and change in interest rates, they are very pertinent to bond option pricing. As usual here we seek only to introduce and describe the models; readers interested in the derivation and proof of the models may wish to consult the specific texts listed in the bibliography. There is also a recommended article (Reitano 1997) that summarises in very readable fashion the main interest-rate models.

46.1 Introduction

Put simply the Black–Scholes model of option pricing describes the following process: assuming that asset prices evolve according to a random process, and under a constant short-term interest rate, a market participant can construct a portfolio of assets (shares and risk-free bonds) that replicates the payoff profile of an option contract. Using the no-arbitrage rationale of asset pricing, the option price must be equal to the current price of the replicating portfolio, whose price is known.

When this approach is applied to bond options however, certain complications arise. The evolution of a bond price does not fit simply into the dynamic price process used to describe share price movements. For one thing, the price of a bond displays a pull-to-par effect, as it must reach 100 on maturity. Share prices do not have this constraint. In addition this par price constraint means that the volatility of the bond price reduces as the bond approaches maturity. The basic Black–Scholes analysis therefore cannot be applied in the bond option market without modification. Another complication is the assumed constant level of the short rate. While this may not have much economic impact in terms of a share option, assuming that short rates are constant but that the bond price follows a random, dynamic process is contradictory. Therefore when applying the Black–Scholes analysis to bond options, analysts assume that *interest rates*, rather than bond prices, follow a random evolutionary path. This allows bond prices to converge to par, and bond price volatilities would reduce to zero, and interest rates would not be assumed to be constant. But what maturity interest rate is assumed to follow a stochastic process? In fact researchers know that interest rates along the term structure are closely correlated, and while certain models focus only on the short rate, it is increasingly common to find market participants making assumptions about the evolution of the entire term structure of interest rates when pricing bond options. This also requires users to make assumptions about the *correlation* of one interest rate to another, and to the term structure as a whole. It is clear therefore that for bond options a modification to the basic Black–Scholes analysis is required.

A number of academics and practitioners have adjusted or modified the basic B–S model or derived alternative formulas for the pricing of interest-rate options and bond options. The primary difference is that for such instruments, interest rates are not constant and so are not captured accurately by the basic models. Nevertheless the adjusted models also assumed a constant risk-free rate during the life of a bond option, with the exception of Merton (1973). Since 1989 other models have emerged that allow for the valuation of options under a stochastic interest-rate process. Merton’s model has been applied to the pricing of options on money market instruments, T-bills and government (that is, risk-free) bonds. As it allows for stochastic interest rates, it is more appropriate than the B–S model for the valuation of such options. For completeness a summary of the model is given at Appendix 46.1.

46.2 Pricing bond options

In order to successfully price a bond option any model has to capture the process of changes in interest rates under conditions of both certainty and uncertainty. This is a dynamic process, which is why earlier models were not completely appropriate.

46.2.1 Interest-rate processes under conditions of certainty

To specify a process of interest-rate change under conditions of certainty, consider the following. An investor borrows £1 at time t , and will repay this at time T where T is the redemption date for the loan. The repayment amount on T is $M(t, T)$, so that the average interest rate during the period $T - t$ is $R(t, T)$. The maturity amount is given by (46.1):

$$M(t, T) = e^{R(t, T)(T-t)}. \quad (46.1)$$

Under conditions of certainty the interest rates are known over the period $T - t$, so the function for M at each instant is given by (46.2):

$$M(t, s) = M(t, u)M(u, s) \quad \text{for all } t < u < s. \quad (46.2)$$

The value of $M(t, T)$ at maturity is £1 as it is the principal repayment, so using the fact that $M(t, T) = 1$, in conjunction with (46.2), it can be shown that there is an instantaneous interest rate $r(t)$ such that the amount $M(t, T)$ can be given by (46.3). This reflects the fact that interest rates must be arbitrage-free. The amount $M(t, T)$ is therefore

$$M(t, T) = \exp\left(\int_t^T r(s)ds\right) \quad (46.3)$$

so that we may then set

$$M(t, T) = \frac{1}{T-t} \left(\int_t^T r(s)ds \right). \quad (46.4)$$

Using this result, consider a zero-coupon bond of nominal value £1 paying £1 with no uncertainty on its redemption date. This means that $P(t, T) = 1$. The price of the bond at the start of the trade at time t , given that the bond is default-free and all future interest rates are known, is therefore given by (46.5):

$$P(t, T) = \exp\left(-\int_t^T r(s)ds\right). \quad (46.5)$$

This is an important result and is used in the fitting of the zero-coupon yield curve or term structure of interest rates. Its derivation is explained very accessibly in Ross (1999), an excellent all-round text.

46.2.2 Interest-rate processes under conditions of uncertainty

Under conditions of uncertainty, the instantaneous interest rate $r(t)$ is a stochastic process during the period between t and $t + dt$. For a risk-free asset such as a government bond, its price is given by (46.6):

$$P(0, t) = \exp\left(\int_0^t r(s)ds\right). \quad (46.6)$$

To model in an environment of interest-rate uncertainty, it is necessary to construct a probability function. This is done in the following way. Assume a process $p(t, u)_{0 \leq t \leq u}$ that satisfies the boundary condition $P(u, u) = 1$. In the same way as the process followed by asset prices, it can be shown that there is a unique probability p^* equivalent to the probability $\hat{p}$ that can be described by the following process:

$$\hat{p}(t, u) = \exp\left(-\int_0^t r(s)ds\right) p(t, u) \quad (46.7)$$

which is a Martingale for each u in the time interval $[0, T]$. Under this assumption, with the new probability p^* it can be shown that

$$\hat{p}(t, u) = E^*[\hat{p}(u, u)|M_t] = E^*\left[\exp\left(-\int_0^u r(s)ds\right)|M_t\right] \quad (46.8)$$

which can be set out as:

$$p(t, u) = E^* \left[\exp \left(- \int_t^u r(s) ds \right) \middle| M_t \right]. \quad (46.9)$$

Comparing this with the expression at (46.5), this states that the price of a default-free zero-coupon bond is a function only of the process $p(t, u)_{0 \leq t \leq u}$ with the probability of p^* . This assumption allows us to identify the probability density function for p^* , which is given the notation L_T with respect to p . In this situation it is possible to show that there is a stochastic process $q(t)_{0 \leq t \leq T}$ that applies for all points in the period $[0, T]$, which is given by (46.10):

$$L_T = \exp \left(\int_0^T q(s) dW_s - \frac{1}{2} \int_0^T q^2(s) ds \right). \quad (46.10)$$

The expression at (46.10) can be used in conjunction with the property described at (46.11) to obtain a probability function describing the price at time t of a zero-coupon bond with a maturity date u . This is given at (46.12), and this probability is known as the *risk-neutral probability* or *risk-neutral density function*, and is used as a dynamic interest-rate process component in option pricing or yield curve modelling.

$$E^* [X \mid M_t] = E[XL_T \mid M_t] / L_t \quad (46.11)$$

$$p(t, u) = E \left[\exp \left(- \int_t^u r(s) ds + \int_t^u q(s) dW_s - \frac{1}{2} \int_t^u q^2(s) ds \mid M_t \right) \right]. \quad (46.12)$$

46.2.3 Using interest-rate processes to price bond options

A number of interest-rate models are based on a condition of spot interest rates where the spot rate is the underlying variable. This includes the Vasicek (1977), Hull–White (1987, 1993) and Cox–Ingersoll–Ross (1985) models. In order to generate a path for future values of the spot rate, certain parameters are input to each of these models, which suggests that the models have to be calibrated for the particular market they are being used for. This calibration is in the form of zero-coupon bond yields that are close to observed market yields. A derived term structure function is used for any market where no observable zero-coupon bond market exists. The Heath–Jarrow–Morton model is similar to the others named. Once the model has been calibrated so that accurate zero-coupon yields are generated, it is possible to use it to calculate the fair price of a bond option, under the appropriate boundary conditions. For instance a European bond call option with an expiry date of t written on a zero-coupon bond with maturity date T may be characterised by its payoff on maturity. This payoff is given by (46.13):

$$C = (P(t, T) - X)^+. \quad (46.13)$$

46.2.4 The Vasicek model

This uses the risk-neutral probability density function described above. In Vasicek's model the dynamic process for changes in the interest rate $r(t)$ is given by (46.14):

$$dr(t) = a(b - r(t))dt + \sigma dW_t \quad (46.14)$$

where a , b and σ are constants. This process is a Gaussian–Weiner process so may then be used in the pricing of options in the same way as the basic models.

46.2.5 The Cox–Ingersoll–Ross model

The Cox–Ingersoll–Ross model uses the following expression to describe the instantaneous interest-rate dynamic process:

$$dr(t) = a(b - r(t))dt + \sigma \sqrt{r(t)} dW_t \quad (46.15)$$

where a and σ are positive and b is a real number.

46.2.6 The Heath–Jarrow–Morton model

Under the process for the dynamics of the interest rate used in the Vasicek and Cox–Ingersoll–Ross models, which assumes constant volatility for all points along the term structure, it is possible to obtain negative forward rates, and so under certain circumstances both models do not reflect the actual term structure of interest rates as it is observed in the market. The possibility of generating negative forward rates exists because the price process described allows for dynamics in any direction, thus it does not rule out negative interest rates. However except under very special circumstances, negative interest rates are not a feature of debt capital markets and so this is a drawback of these models. A model proposed by Ho and Lee (1986) described a discrete-time process that reflected observed term structures more accurately. A development of the ideas used in Ho and Lee was the continuous-time model proposed by Heath, Jarrow and Morton (HJM). The HJM model is a more sophisticated interest-rate model and applies a different approach to bond option pricing compared to earlier models. The model parameters are the yields of the current term structure (that is, the underlying asset) and the volatility level of the term structure. The other interest-rate formulae model only the short-rate. The HJM approach models all the available information in the term structure, and not just the short-term interest rate; as such it has been seen to describe most of the features of term structure dynamics over time.

The continuous-time HJM model describes an environment where the returns of zero-coupon bonds of varying maturities are not perfectly correlated. The volatility of the different term interest rates are calculated from data reflected in actual changes in the term structure over time. The simple HJM model is a two-factor one; the factors are the changing level and the changing slope of the term structure. Therefore the model accounts for pivotal shifts in the term structure. The model is similar in concept to the B–S model as it requires data on the underlying term structure, and the volatilities of the different term interest rates. As the model uses more than one volatility value, which is more realistic, the model exhibits more flexibility. That is, it can reflect changes in the volatility of the term structure that have resulted from a change in absolute levels of rates, from a change in the shape of the term structure and from a change in the curvature of the term structure. The model is described in Part VIII of this book.

Readers may be interested in investigating this further, for which we recommend Heath, Jarrow and Morton (1992), Spindel (1992), as well as Reitano (1997).

46.3 Using option models to price corporate bonds¹

Bonds issued by non-sovereign borrowers, that is corporate bonds, carry an element of default risk.² The level of this risk is a function of the credit quality of the issuing institution. The compensation payable to an investor for holding bonds that carry an element of credit risk is the higher yield on these bonds, compared to the benchmark government rate. Therefore the yield (price) of a corporate bond at any time will reflect the credit quality of the bond issuer. It is increasingly common to find banks pricing corporate bonds on the basis of the probability of default of the bonds themselves, based on the credit rating of the bond. Data on expected default rates is now freely available from the major ratings agencies, and is also used as the basis of credit value-at-risk models, which measure the extent of a banks credit risk exposure. From a pricing point of view, a corporate bond market making desk will derive a zero-coupon yield curve for each category of credit rating, based on the required credit spread of each rating. This zero-coupon curve is then used to value all bonds of that particular credit rating.

The element of credit risk associated with corporate bonds means that they present additional problems in their pricing compared to risk-free bonds. Both Black and Scholes (1973) and Merton (1974) presented approaches for the pricing of corporate bonds on the basis of a contingent claims methodology. In what is termed now the “traditional approach” the original contingent claims approach modelled corporate liability, both debt and equity, as options on the value of the issuer, which was described as being in default on the maturity of the debt and if the firm has exhausted its assets. Equity holders are essentially not at risk from default because they have limited liability; therefore default risk is said to be equal to a put option on the assets of the company. As well as the work of Black and Scholes and Merton, Lee (1981) and Pitts and Selby (1983) have analysed corporate default spreads of zero-coupon bonds, and the credit-adjusted zero-coupon term structure. In all these models, default is assumed to take

¹ This section is adapted, with permission, from Briys *et al.* (1998), a particularly readable reference.

² In fact, only bonds issued by triple-AAA rated sovereign borrowers and supra-national borrowers such as the World Bank are deemed to be risk-free. All other borrowing institutions, including other sovereigns, carry an element of credit risk.

place on the expiry of the bond and as the firm exhausts its assets. The term structure is assumed to be both flat and constant, which along with the assumption of default only on maturity hinders the applicability of the models. Further models have been developed that do not have such limiting assumptions, including Longstaff and Schwartz (1995) and Briys and de Varenne (1997). In this section we briefly summarise the main features of the traditional and later models; interested readers may wish to consult some of the texts and references listed in the bibliography.

46.3.1 The price of corporate bonds

For the purposes of valuation a firm is considered insolvent if its liabilities exceed the total net worth of its assets, so on the maturity of its debt at time T , $A_T < L$ and bondholders will receive the residual after senior creditors have been recompensed, that is, $D_T = A_T$.

The value of a European call option $C_E(A_T, L)$ with strike price L that expires on T is given by the B-S model as:

$$C_E = (A_T, L) = A_T N(d_1) - Le^{-rT} N(d_2) \quad (46.16)$$

$$\text{where } d_1 = \frac{\ln(A_T/L) + (r + (\sigma_A^2/2))T}{\sigma_A \sqrt{T}} = d_2 + \sigma_A \sqrt{T}.$$

We assume the firm's debt to be in the form of a zero-coupon bond, a portfolio that was composed of this bond with payoff Le^{-rT} and a short position of a put option on the firm's equity, which is the liability exposure of the bondholders. Thus the value of the put option can be shown to be:

$$P_E(A_T, L) = -L_T N(-d_1) + Le^{-rT} N(-d_2) \quad (46.17)$$

which captures the credit risk associated with the bond.

Following this it is possible to model the value of the credit spread applicable to corporate bonds. A bank that modelled the credit structure of interest rates could use this to value corporate bonds, thus accounting for the credit risk associated with holding the corporate bond. It can be shown (for example, see Briys *et al.* 1998) that the yield of a corporate zero-coupon bond D_0 at time today ($t = 0$) that matures on T that has a par value of M is given by (46.18):

$$Y_0 = -\frac{1}{T} \ln \frac{D_0}{M} \quad (46.18)$$

where Y_0 is the yield on the bond at time 0. Using a closed-form solution following Black-Scholes the following may be obtained for D_0 :

$$Y_0 = r - \frac{1}{T} \ln \left(\frac{1}{l_0} N(-d_1) + N(d_2) \right) \quad (46.19)$$

where l_0 is the gearing ratio (debt/equity ratio), but a discounted value is used, discounted at the risk-free term rate r so it is referred to as a *quasi*-debt ratio. The spread on the corporate bond, which is defined as the difference between the yield Y_0 and the yield of the same-maturity government bond or nearest risk-free bond, is given by (46.20):

$$\begin{aligned} s_0 &= Y_0 - r \\ &= -\frac{1}{T} \ln \left(\frac{1}{l_0} N(-d_1) + N(d_2) \right) \end{aligned} \quad (46.20)$$

where s_0 is the credit spread on the corporate bond and r is the yield on the equivalent-maturity government bond. The value of the spread s_0 is a function of the quasi-debt ratio l_0 , the maturity of the bond T and the volatility of the firm's assets σ_A . By definition therefore this credit spread will increase with increasing quasi-debt ratio and increasing volatility of the company's assets. That is, if either or both of these factors increase, the default risk of the firm will rise, which will raise the credit risk premium demanded by investors for holding the firm's debt. Unlike the other two factors though the effect of the maturity of the bond T is not fixed, and will vary according to the credit rating of the firm. For low-rated or non-rated firms, increasing maturity T actually reduces credit spread, as the belief is that if the company has survived after the first say, five years its fortunes will only improve. So the yield spread should reduce after the first five years for highly leveraged firms. This is shown in Figure 46.1.

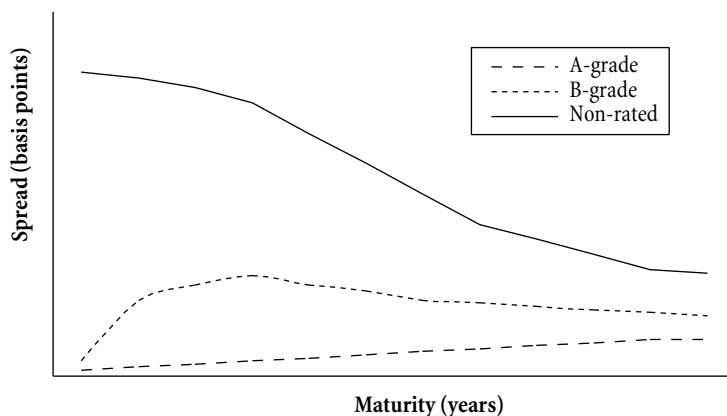


Figure 46.1: The credit structure of interest rates.

Appendices

APPENDIX 46.1 The Merton model

For completeness we present the form of the Merton model for pricing interest-rate products such as options on T-bills and risk-free bonds. The derivation is not shown.

Merton (1973) presents the formula for a call option as:

$$y(x, T) = \frac{1}{2}(x \operatorname{erfc}(h_1) - \operatorname{erfc}(h_2)) \quad (46.21)$$

where

$$h_1 = -\frac{\ln(x) + \frac{1}{2}T}{\sqrt{2T}}$$

$$h_2 = -\frac{\ln(x) - \frac{1}{2}T}{\sqrt{2T}}$$

$$T = \int_0^t (\sigma^2 + \delta^2 - 2\rho\sigma\delta) du$$

and where erfc is the error complement function: $\operatorname{erfc}(h) = 1 - \frac{2}{\sqrt{\pi}} \int_0^h e^{-w^2} dw$.

Where the risk-free interest rate r is zero, the variance of the underlying assets is 1 and the strike price X is 1, Merton's equation becomes identical to the B-S formula.

Selected bibliography

- Briys, E., de Varenne, F., "Valuing risk fixed-rate debt: an extension", *Journal of Financial and Quantitative Analysis* 32, June 1997, pp. 239–248.
- Cox, J., Ingersoll, J., Ross, S., "A theory of the term structure of interest rates", *Econometrica* 53, 1985 pp. 363–384.
- Heat, D., Jarrow, R., Morton, A., "Bond pricing and the term structure of interest rates", *Econometrica* 60 (1), 1992, pp. 77–105.
- Ho, T., Lee, S., "Term structure movements and pricing interest rate contingent claims", *Journal of Finance* 41, 1986, pp. 101–1029.
- Jarrow, R., Turnbull, S., "Pricing options on financial securities subject to default risk", Working paper, Graduate School of Management, Cornell University, 1992.
- Lando, D., "A continuous time markov model of the term structure of credit risk spreads", Working paper, Graduate School of Management, Cornell University, 1993.
- Lee, C., "The pricing of corporate debt: a note", *Journal of Finance* 36, 1981, pp. 1187–1190.

- Leland, H., “Corporate debt value, bond covenants, and optimal capital structure”, *Journal of Finance* 49, September 1994, pp. 1213–1252.
- Leland, H., “Bond prices, yield spreads, and optimal capital structure with default risk”, Finance Working Paper 240, University of California at Berkeley.
- Longstaff, F., Schwartz, E., “A simple approach to valuing risky fixed and floating rate debt and determining swap spreads”, *Journal of Finance* 50, 1995, pp. 798–819.
- Merton, R., “On the pricing of corporate debt: the risk structure of interest rates”, *Journal of Finance* 29, 1974, pp. 449–470.
- Merton, R., “Theory of rational option pricing”, *Bell Journal of Economics and Management Science* 4, 1973, pp. 141–183.
- Nielsen, L., Saa-Requejo, J., Santa-Clara, P., “Default risk and interest-rate risk: the term structure of default spreads”, Working paper, INSEAD, 1993.
- Pitts, C., Selby, M., “The pricing of corporate debt: a further note”, *Journal of Finance* 38 (4), 1983, pp. 1311–1313.
- Rabinovitch, R., “Pricing stock and bond options when the default free rate is stochastic”, *Journal of Financial and Quantitative Analysis* 24, December 1989, pp. 447–457.
- Reitano, R., “Yield Curve Risk Management”, in Fabozzi, F. (ed.), *Advances in Fixed Income Valuation, Modeling and Risk Management*, FJF Associates, 1997.
- Ross, S., *An Introduction to Mathematical Finance*, Cambridge University Press, 1999.
- Spindel, M., “Easier done than said”, *Risk* 5(9), 1992, pp. 77–80.
- Vasicek, O., “An equilibrium characterisation of the term structure”, *Journal of Financial Econometrics* 5, 1977, pp. 177–188.

Questions and exercises

1. What is the theoretical advantage of the Heath-Jarrow-Morton model over earlier models used to price bond options?
2. Describe and explain the valuation parameters in Merton’s model.
3. What feature of earlier models may result in negative forward interest rates?
4. What are the assumptions of the composition of a portfolio behind the traditional approach to valuation of bonds using a contingent claims approach? What is the role of the put-to-default bond?
5. What limitations does the traditional approach to corporate bond valuation suffer from?

47

Options V – Managing an Option Book

We continue on the theme of analysing options in this chapter with a look at how options behave in response to changes in market conditions. To start we consider the main issues that a market maker in options must consider when writing options. We then review “the Greeks”, the measures by which the sensitivity of an option book is calculated.

47.1 Behaviour of option prices

To recap from the previous chapter, the value of an option is a function of five factors:

- the price of the underlying asset;
- the strike price of the option;
- the time to expiry of the option;
- the volatility level of the underlying asset price returns;
- the risk-free interest rate applicable to the life of the option.

The Black–Scholes model assumes that the level of volatility and interest rates stays constant, so that changes in these will impact on the value of the option. On the expiry date the price of the option will be a function of the strike price and the price of the underlying asset. However for pricing purposes an option trader must take into account all the factors above. From Chapter 42 we know that the value of an option is composed of intrinsic value and time value; intrinsic value is apparent straight away when an option is struck, and a valuation model is essentially pricing time value of the option. This is considered next.

47.1.1 Assessing time value

The time value of an option reflects the fact that it is highest for at-the-money options and also higher for an in-the-money option than an out-of-the-money option. This can be demonstrated by considering the hedge process followed by a market maker in options. An out-of-the-money call option for instance, presents the lowest probability of exercise for the market maker, therefore she may not even hedge such a position. There is a risk of course that the price of the underlying will rise sufficiently to make the option in-the-money, in which case the market maker would have to purchase the asset in the market, thereby suffering a loss. This must be considered by the market maker, but deeply out-of-the-money options are often not hedged. So the risk to the market maker is lowest for this type of option, which means that the time value is also lowest for such an option.

An in-the-money call option carries a greater probability that it will be exercised. A market maker writing such an option will therefore hedge the position, either with the underlying asset, with futures contracts or via a *risk reversal*. This is a long or short position in a call that is reversed to the same position in a put by selling or buying the position forward (and vice versa). The risk with hedging using the underlying is that its price falls, causing the option to be exercised and forcing the market maker to dispose of the underlying at a loss. However this risk is lowest for a deeply in-the-money option, and this is reflected in the time value for such options, which diminishes the more the in-the-money the option is.

The highest risk lies in writing an at-the-money option. In fact the majority of OTC options are struck at-the-money. The risk level reflects the fact that there is greatest uncertainty with this option, because there is an even chance of it being exercised. The decision on whether to hedge is therefore not as straightforward. As an at-the-money option carries the greatest risk for the market maker in terms of hedging it, the time value for it is the highest.

47.1.2 American options

In the previous chapter we discussed the B–S and other models in terms of European options, and also briefly referred to a model developed for American options on dividend-paying securities. In theory an American option will have greater value than an equivalent European option, because of the early-exercise option. This added feature implies a higher value for the American option. In theory this is correct, but in practice it carries lower weight because American options are rarely exercised ahead of expiry. There are occasions when early exercise is optional

in order to benefit from an expected dividend payment, but this is not very frequent. The holder of an American option must assess if it is ever optimal to exercise it ahead of the expiry date, and usually the answer to this is “no”. This is because, by exercising an option, the holder realises only the intrinsic value of the option. However if the option is traded in the market, that is, sold than the full value will be realised, including the time value. Therefore it is rare for an American option to be exercised ahead of the expiry date; rather, it will be sold in the market to realise full value.

As the chief characteristic that differentiates American options from European options is rarely employed, in practical terms they do not have greater value than European ones. Therefore they have similar values to equivalent European options. However an option pricing model, calculating the probability that an option will be exercised, will determine under certain circumstances that the American option has a higher probability of being exercised and assign it a higher price.

Under certain circumstances it is optimal to exercise American options early. The most significant is when an option has negative time value. An option can have negative time value when for instance, a European option is deeply in-the-money and very near to maturity. The time value will be small positive, however the potential value in deferring cash flows from the underlying asset may outweigh this, leading to a negative time value. The best example of this is for a deeply in-the-money option on a futures contract. By deferring exercise, the opportunity to invest the cash proceeds from the profit on the futures contract (remember, futures are cash settled daily via the margin process) is lost and this is potential interest income foregone. In such circumstances, it would be optimal to exercise an option ahead of its maturity date, assuming it is an American one. Therefore when valuing an American option, the probability of it being exercised early is considered and if it is deeply in-the-money this probability will be at its highest.

47.2 Measuring option risk: The Greeks

It is apparent from a reading of the previous chapter that the price sensitivity of options is different to other financial market instruments. This is clear from the variables that are required when pricing an option, which we presented by way of recap at the start of this chapter. The value of an option is sensitive to changes in one or any combination of the five variables that are used in the valuation.¹ This makes risk managing an option book more complex compared to other instruments. For example the value of a swap is sensitive to one variable only, the swap rate. The relationship between the change in value of the swap and the change in the swap rate also is a linear one. A bond futures contract is priced as a function of the current spot price of the cheapest-to-deliver bond and the current money market repo rate. Options on the other hand react to moves in any of the variables used in pricing them; more importantly the relationship between the value of the option and the change in a key variable is not a linear one. The market uses a measure for each of the variables, and in some cases for a derivative of these variables, which are termed the “Greeks” as they are called after letters in the ancient Greek alphabet.² In this section we review these sensitivity measures and how they are used.

The mathematical definitions of the main Greeks are listed at Appendix 47.1.

47.2.1 Delta

The *delta* of an option is a measure of how much the value or premium of the option changes with changes in the price of the underlying asset. That is, it measures an option’s sensitivity to changes in the underlying price. We have

$$\Delta_{call} = \frac{\Delta C}{\Delta S}.$$

Mathematically, the delta of an option is the partial derivative of the option premium with respect to the underlying, given by (47.1):

$$\delta = \frac{\partial C}{\partial S} \quad \text{or} \quad \delta = \frac{\partial P}{\partial S} = e^{-rt} N(d_1) > 0. \quad (47.1)$$

¹ Of course, the strike price for a plain vanilla option is constant.

² All but one; the term for the volatility sensitivity measure, *vega*, is not a Greek letter. In certain cases one will come across the use of the term *kappa* to refer to volatility, and this is a Greek letter. However it is more common for volatility to be referred to by the term *vega*.

In fact the delta of an option is given by the $N(d_1)$ term in the B-S equation. It is closely related to but not equal to the probability that an option will be exercised. If an option has a delta of 0.65 or 65%, this means that a £100 increase in the value of the underlying will result in a £65 increase in the value of the option. Delta is probability the most important sensitivity measure for an option, as it measures the sensitivity of the option price to changes in the price of the underlying, and this is very important for option market makers. It is also the main hedge measure. When an option market maker wishes to hedge a sold option, she may do this by buying a matching option, by buying or selling another instrument with the same but opposite value as the sold option, or by buying or selling the underlying. If the hedge is put on with the underlying, the amount is governed by the delta. So for instance if the delta of an option written on one ordinary share is 0.65 and a trader writes 1000 call options, the hedge would be a long position in 650 of the underlying shares. This means that if the value of the shares rises by £1, the £650 rise in the value of the shares will offset the £650 loss in the option position. This is known as *delta hedging*. As we shall see later on, this is not a static situation, and the fact that delta changes, and is also an approximation, means that hedges must be monitored and adjusted, so-called *dynamic hedging*.

Figure 47.1 shows the value profile for two hypothetical call options each with a strike price of 100, but different times to expiry; one option is a one-month option while the other is a nine-month option. The value of the delta with each change in the underlying price is shown.³ Both options have delta of zero when they are deeply out-of-the-money, and have delta of approximately 0.5 when they are at-the-money. An option that is deeply in-the-money has a delta approaching unity. For the longer-dated option the change in delta is fairly smooth as it moves in-the-money, whereas for the shorter-dated option the change is more drastic. The delta of an option measures the extent to which the option moves with the underlying asset price; at a delta of zero the option does not move with moves in the underlying, while at a delta of 1 it will behave identically to the underlying.

A positive delta position is equivalent to being long the underlying asset, and can be interpreted as a bullish position. A rise in the asset price results in profit, as in theory a market maker could sell the underlying at a higher price, or in fact sell the option. The opposite is true if the price of the underlying falls. With a positive delta, a market maker would be over-hedge if running a delta-neutral position. Table 47.1 shows the effect of changes in the underlying price on the delta position in the option book; to maintain a delta-neutral hedge, the market maker must buy or sell delta units of the underlying asset, although in practice futures contracts may be used.

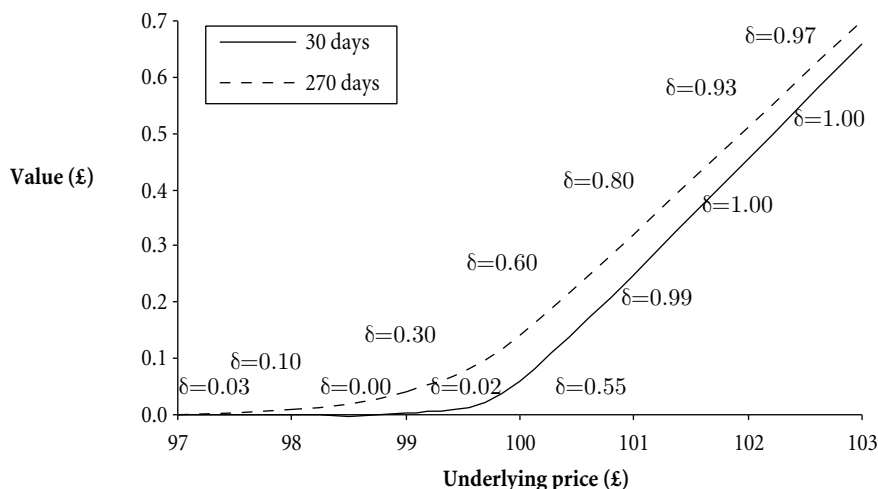


Figure 47.1: Hypothetical option delta.

³ This follows the approach described in Galitz (1993), but for a hypothetical bond option.

	Rise in underlying asset price	Fall in underlying asset price
<i>Option</i>		
Long call	Rise in delta: sell underlying	Fall in delta: buy underlying
Long put	Fall in delta: sell underlying	Rise in delta: buy underlying
Short call	Rise in delta: buy underlying	Fall in delta: sell underlying
Short put	Fall in delta: buy underlying	Rise in delta: sell underlying

Table 47.1: Delta-neutral hedging for changes in underlying price.

47.2.2 Gamma

In a similar way that the modified duration for bonds measure becomes inaccurate for larger yield changes, due to the nature of its calculation, there is an element of inaccuracy with the delta measurement and with delta hedging an option book. This is because the delta itself is not static, and changes with changes in the price of the underlying. A book that is *delta neutral* at one level may not be as the underlying price changes. To monitor this, option market makers calculate *gamma*. The gamma of an option is a measure of how much the delta value changes with changes in the underlying price. It is given by

$$\Gamma = \frac{\Delta\delta}{\Delta S}.$$

Mathematically gamma is the second partial derivative of the option price with respect to the underlying price, that is,

$$\frac{\partial^2 C}{\partial S^2} \text{ or } \frac{\partial^2 P}{\partial S^2}$$

and is given by (47.2):

$$\Gamma = \frac{N(d_1)}{S\sigma\sqrt{T}}. \quad (47.2)$$

The delta of an option does not change rapidly when an option deeply in- or out-of-the-money, so that in these cases the gamma is not significant. However when an option is close to or at-the-money, the delta can change very suddenly and at that point the gamma is very large. The value of gamma is positive for long call and put options, and negative for short call and put options. An option with high gamma causes the most problems for market makers, as the delta hedge must be adjusted constantly, which will lead to high transaction costs. The higher the gamma, the greater is the risk that the option book is exposed to loss from sudden moves in the market. A negative gamma exposure is the highest risk, and this can be hedged only by putting on long positions in other options. A perfectly hedged book is gamma neutral, which means that the delta of the book does not change.

When gamma is positive, a rise in the price of the underlying asset will result in a higher delta. Adjusting the hedge will require selling the underlying asset or futures contracts. The reverse applies if there is a fall in the price of the underlying. As the hedge adjustment is made in the same direction that the market is moving in, this adjustment is possibly easier to conceptualise for newcomers to a market making desk. When adjusting a hedge in a rising market, underlying assets or futures are sold, which in itself may generate profit. In a falling market, the delta-hedge is insufficient and must be re-balanced through purchase of the underlying.

However with a negative gamma, an increase in the price of the underlying will reduce the value of the delta, so therefore to adjust the delta hedge, the market maker must buy more of the underlying asset or futures equivalents. However, when the underlying asset price falls, the delta will rise, necessitating selling of the underlying asset to rebalance the hedge. In this scenario, irrespective of whether cash or off-balance sheet instruments are being used, the hedge involves selling assets in a falling market, which will generate losses even as the hedge is being put on. Negative gamma is therefore, as we have noted elsewhere, a high risk exposure in a rising market. Managing an option book that has negative gamma is more risky if the underlying asset price volatility is high. In a rising market the market maker becomes short and must purchase more of the underlying, which may produce losses. The same applies in a falling market. If the desk is pursuing a delta-neutral strategy, running a positive gamma position should

enable generation of profit in volatile market conditions. Under the same scenario, a negative gamma position would be risky and would be excessively costly in terms of dynamically hedging the book.

Gamma is the only one of the major Greeks that does not measure the sensitivity of the option premium, instead it measures the change in delta. Figure 47.2 illustrates the sensitivity of the gamma for change in underlying for the two hypothetical options introduced in the previous section. The relationship is similar for delta, which is not surprising. The delta of an option is its hedge ratio, and gamma is a measure of how much this hedge ratio changes for changes in the price of the underlying. This is why a gamma value results in problems in hedging an option book, as the hedge ratio is always changing. This ties in with our earlier comment that at-the-money options have the highest value, because they present the greatest uncertainty and hence the highest risk. From Figure 47.2 we see that the behaviour of gamma follows that of the delta.

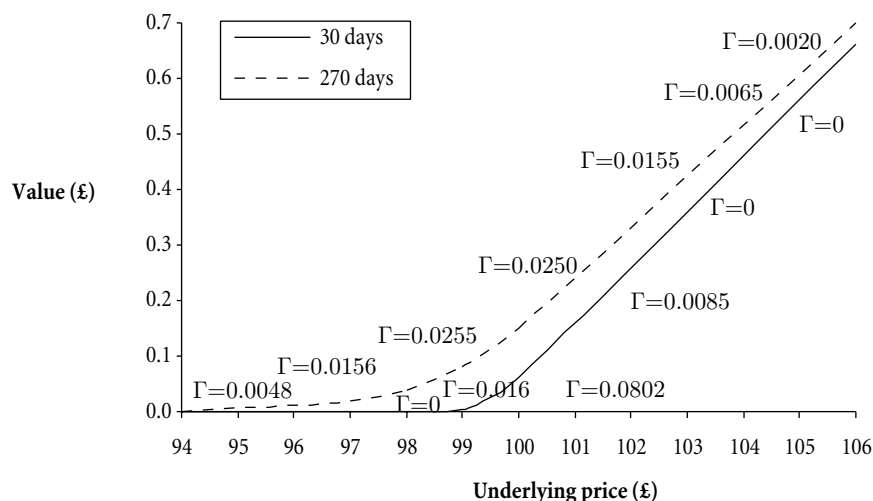


Figure 47.2: Hypothetical option gamma.

To adjust a book so that it is made gamma neutral, a market maker must put on positions in an option on the underlying or on the future. This is because the gamma of the underlying and the future is zero. It is common for market makers to use exchange-traded options. Therefore a book that needs to be made gamma-neutral must be re-balanced with options; however, by adding to its option position, the book's delta will alter. Therefore to maintain the book as delta-neutral, the market maker will have to re-balance it using more of the underlying asset or futures contracts. The calculation made to adjust gamma is a snapshot in time, and as the gamma value changes dynamically with the market, the gamma hedge must be continually re-balanced, like the delta hedge, if the market maker wishes to maintain the book as gamma-neutral.

47.2.3 Theta

The *theta* of an option measures the extent of the change in value of an option with change in the time to maturity. That is, it is

$$\Theta = \frac{\Delta C}{\Delta T} \quad \text{or} \quad -\frac{\partial C}{\partial T} \quad \text{or} \quad -\frac{\partial P}{\partial T}$$

and, from the formula for the B-S model, mathematically it is given for a call option as (47.3):

$$\Theta = -\frac{S\sigma}{2\sqrt{2\pi T}} \exp(-d_1^2/2) - Xre^{-rT}N(d_2). \quad (47.3)$$

Theta is a measure of time decay for an option. A holder of a long option position suffers from time decay because as the option approaches maturity, its value is made up increasingly of intrinsic value only, which may be zero as the option approaches expiry. For the writer of an option, the risk exposure is reduced as a result of time decay, so it is favourable for the writer if the theta is high. There is also a relationship between theta and gamma

however; when an option gamma is high, its theta is also high, and this results in the option losing value more rapidly as it approaches maturity. Therefore a high theta option, while welcome to the writer, has a downside because it is also high gamma. There is therefore in practice no gain to be high theta for the writer of an option. The theta value impacts certain option strategies. For example, it is possible to write a short-dated option and simultaneously purchase a longer-dated option with the same strike price. This is a play on the option theta; if the trader believes that the time value of the longer-dated option will decay at a slower rate than the short-dated option, the trade will generate a profit.

Figure 47.3 illustrates theta for three options. We note that in-the-money and out-of-the-money options have little time value when they are struck, so the theta sensitivity is negligible. The at-the-money option has the highest time value so its theta is more sensitive. For much of this option's life, it retains most of its time value, but once there is only one month to expiry, the time value declines rapidly.

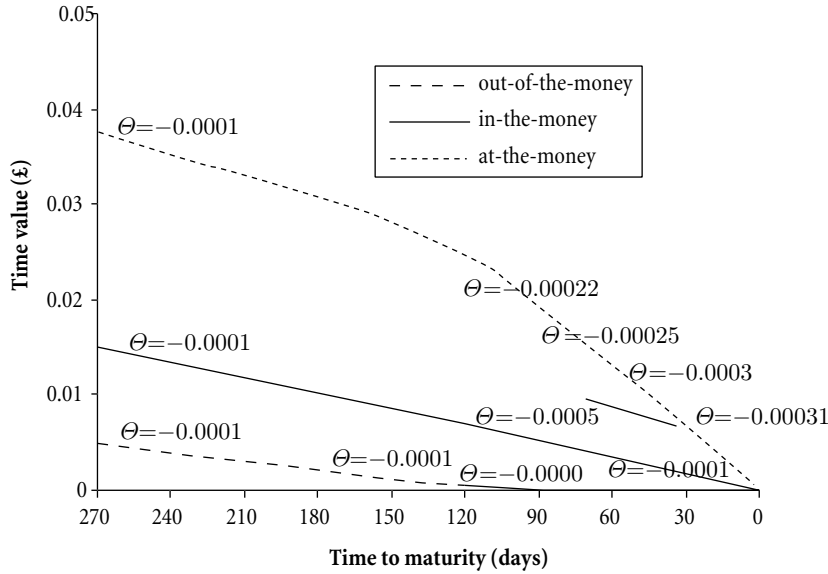


Figure 47.3: Hypothetical option theta and time decay.

47.2.4 Vega

The *vega* of an option measures how much its value changes with changes in the volatility of the underlying asset. It is also known as *epsilon* (ε), *eta* (η), or *kappa* (κ).

We define vega as: $v = \frac{\Delta C}{\Delta \sigma}$ or $v = \frac{\partial C}{\partial \sigma}$ or $\frac{\partial P}{\partial \sigma}$ and mathematically from the B-S formula it is defined as (47.4) for a call or put:

$$v = \frac{S\sqrt{T/2\pi}}{\exp(d_1^2/2)}. \quad (47.4)$$

It may also be given by (47.5):

$$v = S\sqrt{\Delta T}N(d_1). \quad (47.5)$$

An option exhibits its highest vega when it is at-the-money, and decreases as the underlying and strike prices diverge. Options with only a short time to expiry have a lower vega compared to longer-dated options. An option with positive vega generally has positive gamma. Vega is also positive for a position composed of long call and put options, and an increase in volatility will then increase the value of the options. A vega of 15 means that for a 1% increase in volatility, the price of the option will increase by 0.15. Buying options is the equivalent of buying volatility, while selling options is equivalent to selling volatility. Market makers generally like volatility and set up

their books so that they are positive vega. There are a range of trading strategies that are purely volatility trades, and these are examined in Chapter 48. The basic approach for volatility trades is that the market maker will calculate the implied volatility inherent in an option price, and then assess whether this is accurate compared to her own estimation of volatility. Just as positive vega is long call and puts, if the trader feels the implied volatility in the options is too high, she will put on a short vega position of short calls and puts, and then reverse the position out when the volatility declines.

Figure 47.4 illustrates vega for our one-month and nine-month at-the-money options. We note that vega is exhibited to a greater extent for longer-dated options. We have used the letter ν to refer to vega, but it is also common to see vega referred to by “ k ”. Table 47.2 below shows the response to a delta hedge following a change in volatility.

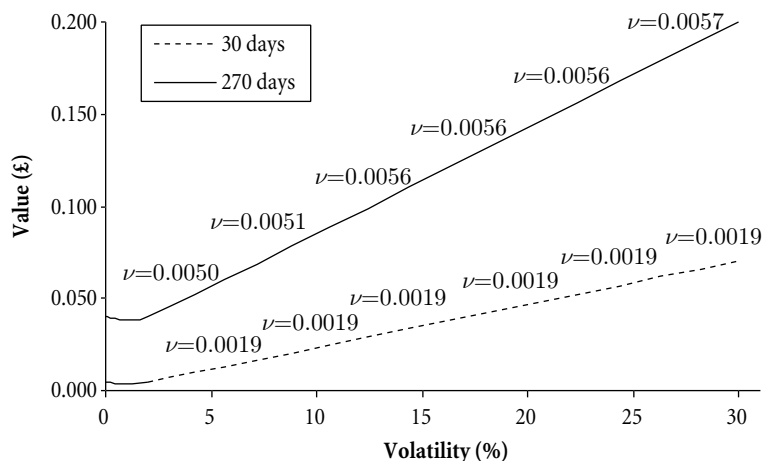


Figure 47.4: Option vega.

Option position	Rise in volatility	Fall in volatility
<i>Long call</i>		
ATM	No adjustment to delta	No adjustment to delta
ITM	Rise in delta, buy underlying	Rise in delta, sell underlying
OTM	Fall in delta, sell underlying	Fall in delta, buy underlying
<i>Long put</i>		
ATM	No adjustment to delta	No adjustment to delta
ITM	Fall in delta, sell underlying	Rise in delta, buy underlying
OTM	Rise in delta, buy underlying	Fall in delta, sell underlying
<i>Short call</i>		
ATM	No adjustment to delta	No adjustment to delta
ITM	Fall in delta, sell underlying	Rise in delta, buy underlying
OTM	Rise in delta, buy underlying	Fall in delta, sell underlying
<i>Short put</i>		
ATM	No adjustment to delta	No adjustment to delta
ITM	Rise in delta, buy underlying	Rise in delta, sell underlying
OTM	Fall in delta, sell underlying	Fall in delta, buy underlying

Table 47.2: Dynamic hedging as a result of changes in volatility.

Managing an option book involves trade-offs between the gamma and the vega, much like there are between gamma and theta. A long in options means long vega and long gamma, which is not conceptually difficult to manage, however if there is a fall in volatility levels, the market maker can either maintain positive gamma, depending on her

view of whether the fall in volatility can be offset by adjusting the gamma in the direction of the market, or she can sell volatility (that is, write options) and set up a position with negative gamma. In either case the costs associated with re-balancing the delta must compensate for the reduction in volatility.

47.2.5 Rho

The *rho* of an option is a measure of how much its value changes with changes in interest rates. Mathematically this is $\partial C/\partial r$ or $\partial P/\partial r$ and the formal definition, based on the B-S model formula, is given at (47.6) for a call option.

$$\rho = Xte^{-rT}N(d_2). \quad (47.6)$$

The level of rho tends to be higher for longer-dated options. It is probably the least used of the sensitivity measures because market interest rates are probably the least variable of all the parameters used in option pricing. Figure 47.5 illustrates rho for a three-month and nine-month call option.

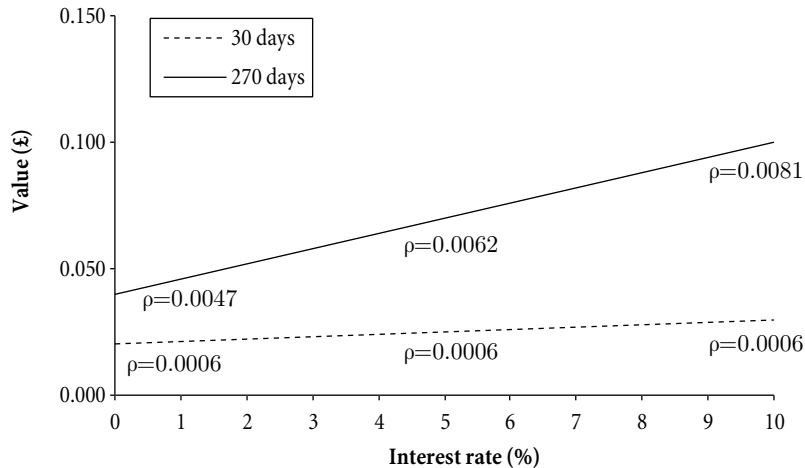


Figure 47.5: Option rho.

47.2.6 Lambda

The lambda of an option is similar to its delta in that it measures the change in option value for a change in underlying price.

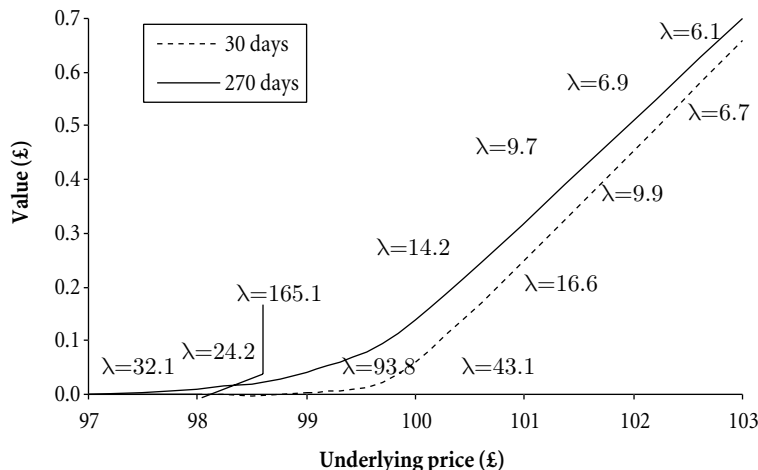


Figure 47.6: Option lambda.

However lambda measure this sensitivity as a percentage change in the price for a percentage change in the price of the underlying. Hence lambda measures the gearing or leverage of an option. This in turn gives an indication of expected profit or loss for changes in the price of the underlying. From Figure 47.6 we note that in-the-money options have a gearing of a minimum of five, and sometimes the level is considerably higher. This means that if the underlying was to rise in price, the holder of the long call could benefit by a minimum of five times more than if he had invested the same cash amount in the underlying instead of in the option.

47.2.7 Other sensitivity measures

For completeness we also describe second derivative sensitivity measures that are used for managing option books. Readers will note their esoteric names.

- **Speed:** this is the partial derivative of gamma with respect to changes in the price of the underlying asset; the change in the gamma for a change in underlying price.
- **Charm:** the partial derivative of delta with respect to changes in the time to maturity; the change in delta for a change in maturity.
- **Strangeness:** the partial derivative of delta with respect to changes in implied volatility; the change in option delta for a 1% change in the volatility.
- **Colour:** the partial derivative of delta with respect to changes in the level of the interest rate.
- **Phi:** the partial derivative of the option price with respect to change in dividend yield. This is an approximation of the change in the value of the option for a one basis point change in the level of the dividend yield, and applies to options on equities or equity indices.

This has been a brief review of the sensitivity measures used in managing option books. They are very useful to market makers and portfolio managers because they enable them to see what the impact of changes in market rates is on an entire book. A market maker need take only the weighted sum of the delta, gamma, vega and theta of all the options on the book to see the impact of changes on the portfolio. Therefore the combined effect of changes can be calculated, without having to re-price all the options on the book. The Greeks are also important to risk managers and those implementing value-at-risk systems.

47.3 The option smile

Our discussion on the behaviour and sensitivity of options prices will conclude with an introduction to the option *smile*. Market makers calculate a measure known as the volatility smile, which is a graph that plots the implied volatility of an option as a function of its strike price. The general shape of the smile curve is given at Figure 47.7. What the smile tells us is that out-of-the-money and in-the-money options both tend to have higher implied volatilities than at-the-money options. We define an at-the-money option as one whose strike price is equal to the forward price of the underlying asset.

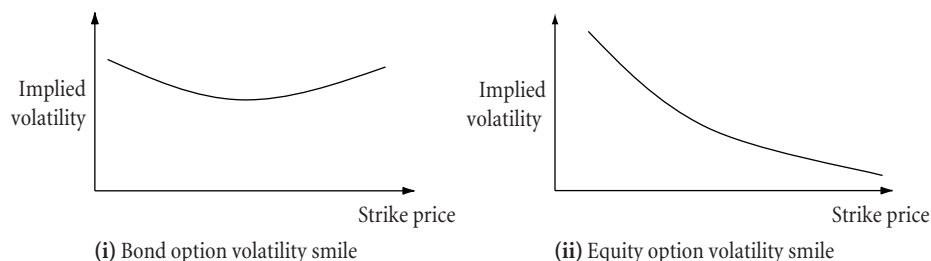


Figure 47.7: The option volatility smile curve for (i) bond options and (ii) equity options.

Under the B-S model assumptions the implied volatility should be the same across all strike prices of options on the same underlying asset and with the same expiry date. However implied volatility is usually observed in the market as a convex function of exercise price, shown in generalised form as Figure 47.7 (in practice, it is not a smooth line or even often a real smile). The observations confirm that market makers price options with strikes that are less than S , and those with strikes higher than S , with higher volatilities than options with strikes equal to S . The existence of the volatility smile curve indicates that market makers make more complex assumptions about the

behaviour of asset prices than can be fully explained by the geometric Brownian motion model. As a result, market makers attach different probabilities to terminal values of the underlying asset price than those that are consistent with a lognormal distribution. The extent of the convexity of the smile curve indicates the degree to which the market price process differs from the lognormal function contained in the B–S model. In particular the more convex the smile curve, the greater the probability the market attaches to extreme outcomes for the price of the asset on expiry, S_T . This is consistent with the observation that in reality asset price returns follow a distribution with “fatter tails” than that described by the lognormal distribution. In addition the direction in which the smile curve slopes reflects the skew of the price process function; a positively sloped implied volatility smile curve results in a price returns function that is more positively skewed than the lognormal distribution. The opposite applies for a negatively sloping curve. The existence of the smile suggest asset price behaviour that is more accurately described by non-standard price processes, such as the jump diffusion model, or a stochastic volatility, as opposed to constant volatility model.

Appendices

APPENDIX 47.1 Mathematical definition of Greek sensitivity measures given by Black–Scholes model

Call option:

$$\begin{aligned}\Delta_c &= N(d_1) \\ \Gamma_c &= \frac{\partial \Delta_c}{\partial S} = \frac{1}{S\sigma\sqrt{T}}n(d_1) \\ \Theta_c &= \frac{\partial C}{\partial T} = \frac{S\sigma n(d_1)}{2\sqrt{T}} - rXe^{-rT}N(d_2) \\ v_c &= \frac{\partial C}{\partial \sigma} = S\sqrt{T}n(d_1) \\ \rho_c &= \frac{\partial C}{\partial r} = X(T - t)e^{-rT}N(d_2).\end{aligned}$$

Put option:

$$\begin{aligned}\Delta_p &= \Delta_c - 1 \\ \Gamma_p &= \frac{\partial \Delta_p}{\partial S} = \frac{1}{S\sigma\sqrt{T}}n(d_1) \\ \Theta_p &= \frac{\partial P}{\partial T} = -\frac{S\sigma n(d_1)}{2\sqrt{T}} - rXe^{-rT}N(-d_1) + \frac{X\sigma}{\sqrt{T}}e^{-rT}n(d_2) \\ v_p &= \frac{\partial P}{\partial \sigma} = S\sqrt{T}n(d_1) \\ \rho_p &= \frac{\partial P}{\partial r} = -X(T - t)e^{-rT}N(d_2).\end{aligned}$$

Selected bibliography and references

- Butler, C., *Mastering Value-at-Risk*, FT Pitman, 1999.
- Cox, J., Rubinstein, M., *Options Markets*, Prentice-Hall, 1985.
- Dillman, S., Harding, J., “Life After Delta: the Gamma Factor”, *Euromoney*, Supplement, February 1985, pp. 14–17.
- Galitz, L., *Financial Engineering*, FT Pitman, 1993.
- Hull, J., *Options, Futures and other Derivative Securities*, 3rd edition, Prentice-Hall, 1998.
- Hull, J., White, A., “An Analysis of the Bias in Option Pricing Caused by Stochastic Volatility”, *Advances in Futures and Options Research* 3, 1988, pp. 27–61.
- Jarrow, R., Rudd, A., *Option Pricing*, Irwin, 1983.
- Rubinstein, M., “Displaced Diffusion Option Pricing”, *Journal of Finance* 38, March 1983, pp. 213–217.

Questions and exercises

1. What use is made of an option's lambda?
2. Is high theta value a positive thing for an option market maker to have? What caveat would you place on your answer?
3. The delta of an option is 0.45. Assuming one option is written on one unit of underlying assets, how would you hedge a short position in 2000 out options?
4. If price returns of a particular asset follow a stochastic volatility model, what would you expect to observe about the implied volatility of a series of options written on the asset, over a range of strike prices, that have been priced using the B-S model?
5. What is an option's colour?
6. Summarise the change in the delta of a call option for a change in each of the five option pricing parameters, assuming only one parameter changes at a time.
7. What is a volatility trade?
8. What are the risks associated with gamma in an option?
9. A three-month European call option written on a non-dividend paying asset where the risk-free interest rate is 5.50% and the underlying asset volatility is 17.8%. What is the option delta?
10. Summarise the risks associated with running an option book that is short gamma.

48

Options VI: Strategies and Uses

Options are very versatile instruments. They can be structured into multiple different combinations, to suit the needs of banks and their customers. There is a large and liquid market in certain “exotic” option products, which may have variations on their payoff profiles or strike prices, for example. Options may also be embedded into other more traditional products such as bonds or retail bank accounts. As each new theme on the basic option product is introduced, so this presents new challenges in terms of its pricing and risk management. The option trading desks of banks are frequently at the cutting edge of technological and mathematical research, as the banks seek to obtain an advantage in terms of structuring and financial engineering. There is such a diverse range of option products and combinations available that even a book dedicated to trading options would have difficulty describing and analysing all of them. In this chapter we present some of the main structures and trading strategies that have been used in the market, and also present brief descriptions of some of the exotic option products that have been introduced. At the end of this chapter we discuss an example of the use of exchange-traded options in portfolio management. In Chapter 49 we describe some of the hedge products offered by banks to their corporate customers; this is intended to demonstrate the versatility of option-based products, the design of which would appear to be limited only by users’ imagination and requirements.

48.1 Introduction

Using options as part of structured financial arrangements has been described as following a “building block” approach. This is because all option arrangements employ one or more of the four basic option types, which are long and short calls and puts. These may be combined in any combination, together with other products such as the underlying asset, zero-coupon bonds and futures, to form the more complicated structures. There are essentially four types of option combination that are used, categorised in terms of the rationale behind their use. These are:

- **spread combination:** these are composed of a long position in one option and a short in another with a different strike price and/or expiry date;
- **volatility trades:** these are trades that are designed as a play on volatility, rather than market direction;
- **strip structures:** a combination of short-dated options put on as a strip product that extends over a longer period of time;
- **arbitrage trades:** theoretically risk-free trades put on as a play to exploit mis-pricing in certain options and/or underlying assets.

Frequently a specific type of combination becomes a generic product and is given its own name.

48.2 Spreads

An *option spread* is a combination that is composed of a long position in one type of option and a short position in the same type of option, but with a different strike price, and/or a different time to expiry. A *call spread* is made up of two call options, while a *put spread* is composed of two puts. The difference in strike price and maturity between the two options is used to designate the spread as a *horizontal*, *vertical* or *diagonal* spread. This is shown at Figure 48.1.

From Figure 48.1 we see that a horizontal spread is a long position in one option and a short position in another (of the same type and on the same underlying) but with a different expiry date. A vertical spread is a long position in one option and a short in another with a different strike price, and a diagonal spread is a long position in one option and a short in another with both a different strike and expiry date.

Expiry		Jan	Feb	Mar	Apr	May	Jun
Strike							
96.00							-1
97.00						1	
98.00							
99.00							
100.00							1
101.00							-1
102.00							
103.00							
104.00		1	-1				

Figure 48.1: Vertical, horizontal and diagonal option spreads.

48.2.1 Vertical spreads

A *bull spread* is a vertical spread composed of a long position where the option has a lower strike than the short; the opposite is known as a *bear spread*. The strategy can be carried out for either call or put options; Figure 48.2 shows the payoff profile for a nine-month bull call spread strategy at different stages of its life.

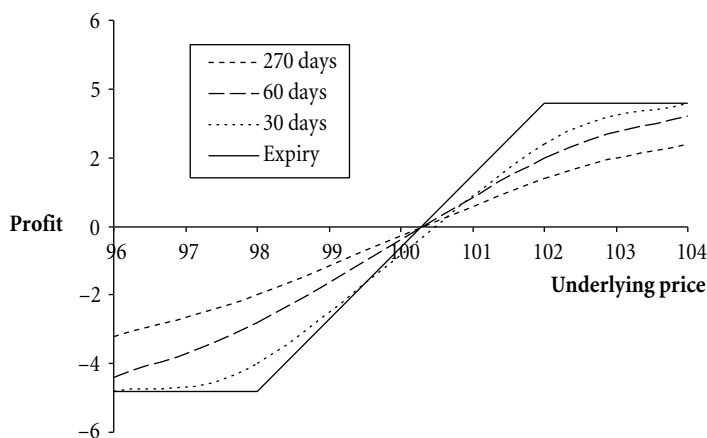


Figure 48.2: Bull call spread payoff profile.

The value of a call option moves inversely with the move in the price of the underlying. Therefore the price of the short call will always be lower than the value of the long call, thus this strategy involves an initial outlay. In our example the strike price of the long call is 98 while the strike of the short call is 102. Let us say that the price of the underlying on expiry is S_T . If the price of the underlying on expiry of the options is higher than 102, the gross profit is the difference between the two strike prices, so in our example it will be $(102 - 98) = 4$. If on expiry the underlying price lies between 98 and 102, the gross profit is the difference between S_T and 98. If the expiry asset price is below 98, the payoff is zero, although the trade will have lost the net premium on the options. A bull call spread has no downside (other than the premium outlay) but also caps the maximum gain if, as in our example, the asset price rises above 102. This contrasts with the potentially unlimited profits from buying a single call option. However the bull call has a time decay feature, as the time decay of the short call works in favour of the holder to counteract the time decay of the long call. This can be observed from Figure 48.2. Figure 48.3 illustrates the difference in payoff profiles between the bull call spread and a straight long call position.

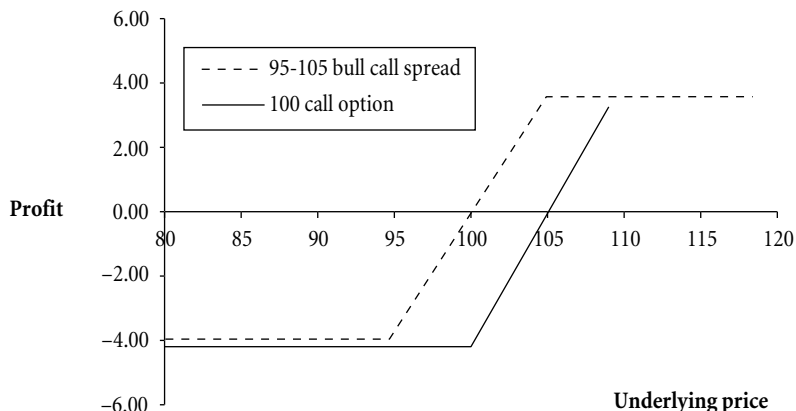


Figure 48.3: Payoff profiles for bull call spread and call position.

The strategy with the highest potential payoff is if both calls are out-of-the-money, so that both options are low cost, and have low probability of giving the highest payoff (in our example, $102 - 98$). If both calls are in-the-money the spread is higher cost and the payoff is more likely to be the lowest possible.

A *bear spread* is the mirror position of the bull spread, so that where the holder of a bull call spread is expecting the underlying asset price to rise, the holder of the bear spread is expecting the asset price to fall. A bear spread is composed of a long call and a short call as before, except the strike price of the long call is higher than the strike of the short call. This strategy delivers a net premium income for the holder, as the price of the sold call will be higher than the bought call. Using the same example as before, if the two strike prices are 102 and 98, if the expiry asset price is above 102, there is a negative payoff, and if the asset price is lower than 98 there is a zero payoff. If the expiry asset price is S_T and it lies between 98 and 102, the payoff is $-(S_T - 98)$. The total profit must include the premium income received at the start of the trade.

An hypothetical bear put spread payout profile with our two strike prices is shown at Figure 48.4.

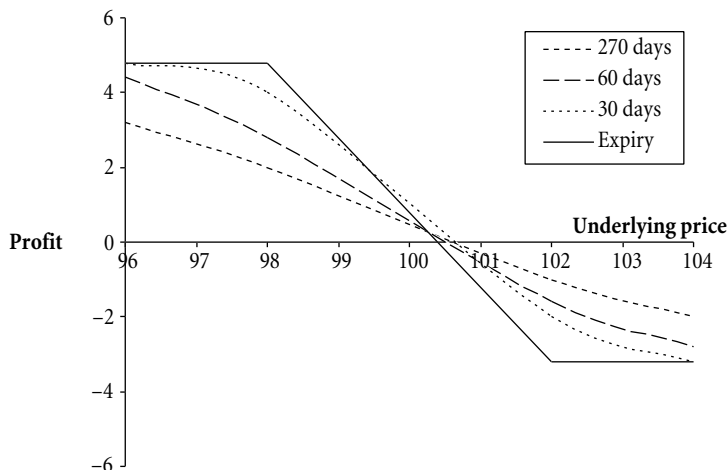


Figure 48.4: Hypothetical bear put spread.

48.2.2 Horizontal spreads

A *horizontal spread* is a position comprising long and short options that have different times to expiry. The overall position will therefore expire at the expiry date of the shorter-dated option, so the profit analysis must consider the characteristics of the position during the life of the earlier-dated option. A horizontal spread is created with at-the-

money calls, with a long position in the longer-dated option and a short position in the short-dated option. The long position will lose time value during the trade, while the short will gain time value. However the time decay is greater for the shorter-dated option, so the net spread on the position will create a gain for the holder. This illustrated in Figure 48.5. The profit profile on expiry is not straight because the long position is yet to mature. The strategy will create profit in a market where the underlying asset price is quite stable and remains within the strike prices of (in our example) 99.50 and 102.50.

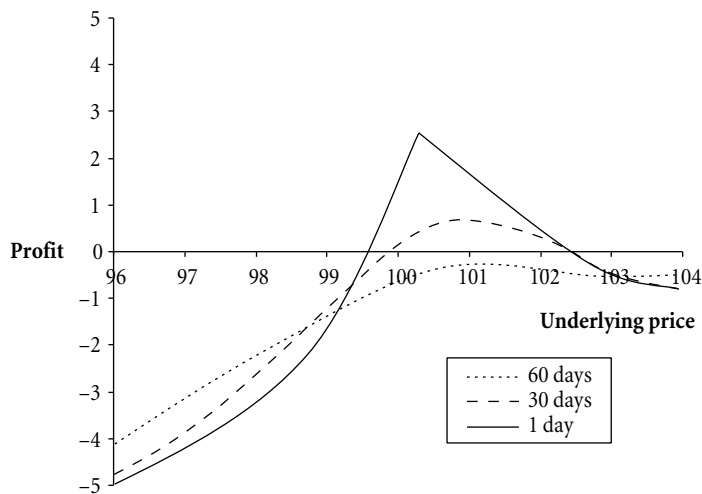


Figure 48.5: Hypothetical horizontal spread.

48.2.3 Diagonal spreads

Diagonal spreads combine the characteristics of vertical and horizontal spreads, so will reflect both time decay as well as the direction in which the market is moving. Figures 48.6 (i) and 48.6 (ii) illustrate the payoff profiles for two diagonal spreads.

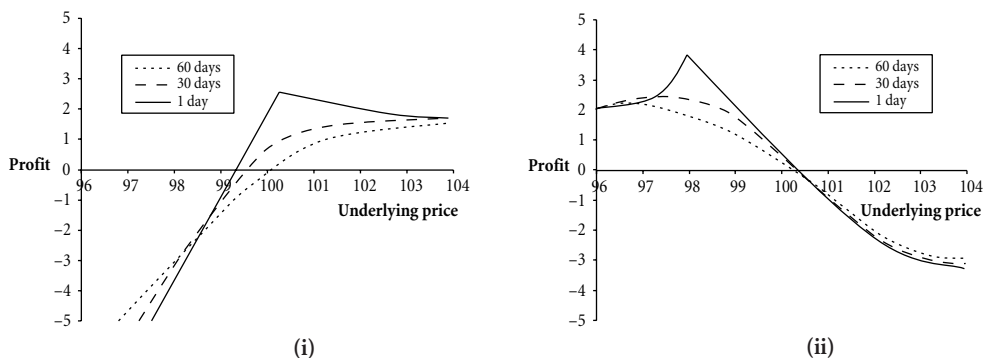


Figure 48.6: (i) and (ii) Diagonal spreads.

The first is a position composed of a short position in a short-dated 100 call and long position in a longer-dated 97.50 call. This position therefore involves a net premium outlay. It will be relatively expensive because the long call is long-dated and more so if it is in-the-money. This strategy will generate a gain if the price of the underlying asset is above 99.25 when the short-dated option expires. The maximum profit is generated if the asset price is at 100 at this point, because the profit will include the benefit of the time decay in the short call, which is shorter-dated. The second illustrates a 98 short-date call and 100.50 longer-dated call, for a net premium income. This is a bearish directional trade and will generate profit if the price of the underlying is anywhere below 100.50 on the expiry of the

shorter-dated option, while the maximum gain is achieved if its price at this point is at 98, the strike price of the short option.

48.3 Volatility trades

We noted that option traders often take a view on volatility rather than market direction, and position their book accordingly. Option strategies are available that will gain if the volatility view turns out to be correct, irrespective of the direction that the market moves in. A change in “volatility” may imply several different factors, including a sudden move in the market or a price shock, or change in central bank base rates or foreign exchange rates. Option traders refer to these as well as to changes in the implied volatility of options as given by their prices. In this section we summarise the most common volatility trades and explain their rationale; readers will note that these strategies are a little more complex than the spread trades we have just considered

48.3.1 Butterfly spreads

A *butterfly spread* is composed of option positions each of which has different strike prices. For example it can be set up by purchasing two calls, one with a low strike and the other with a high strike, and selling two calls with a strike that lies between the two other strike prices. These two short calls are usually near or at-the-money. The profit profile from such a trade is shown at Figure 48.7.

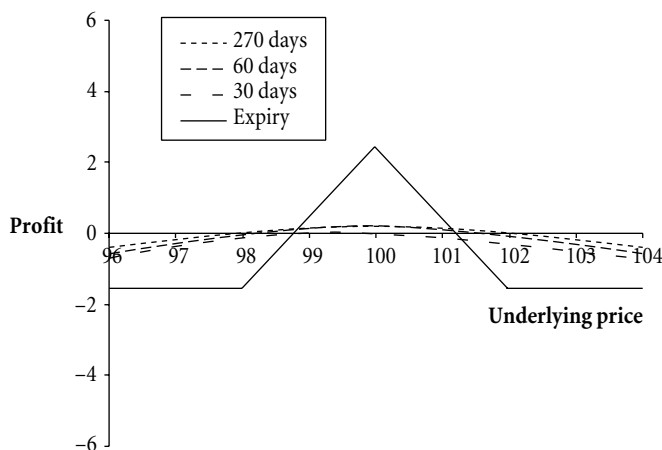


Figure 48.7: Long butterfly trade.

The three strike prices are 98.00, 100 and 102.00. The net premium will be small positive. The profit profile only emerges when the position is very close to expiry, and in fact from Figure 48.7 note that during the trade's life the position is virtually risk-free. The maximum gain from a long butterfly is achieved if the price of the underlying is at the level of the middle strike price on expiry. Generally profit is gained if the underlying price remains fairly static. There will be a loss if all the options expire out-of-the-money or in-the-money, which will occur if the underlying price is below the lowest or above the highest strike price on expiry. However this loss is limited to the premium outlay and so should not be significant. Although this is a volatility trade, its profit profile is virtually immune to changes in implied volatility levels, as illustrated in Figure 48.8. Therefore long butterfly spreads are used when traders believe that the market will remain stable and static, and are a low-cost means of profiting from low levels of market activity.

Butterfly spreads may also be out on using put options. In this case the trader buys two puts with low and high strikes, and sells two puts with the same strike, lying between the low and high strike. The profit profile is similar in shape to that of Figure 48.7

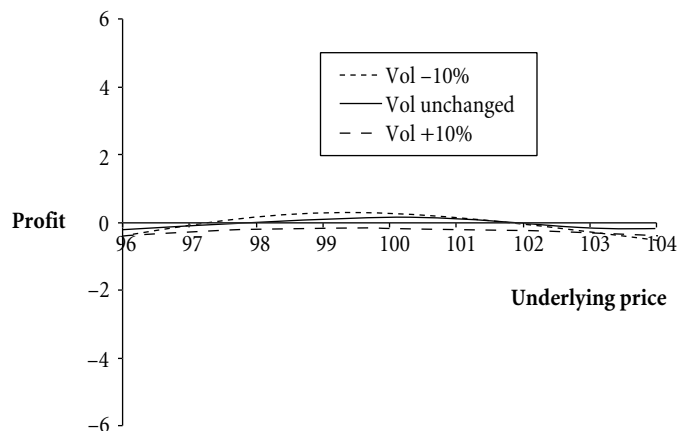


Figure 48.8: Long butterfly trade sensitivity to implied volatility.

48.3.2 Straddles

A *long straddle* is composed of a long call and a long put, both with exactly the same characteristics. The profit profile at Figure 48.9 is created by buying at-the-money options.

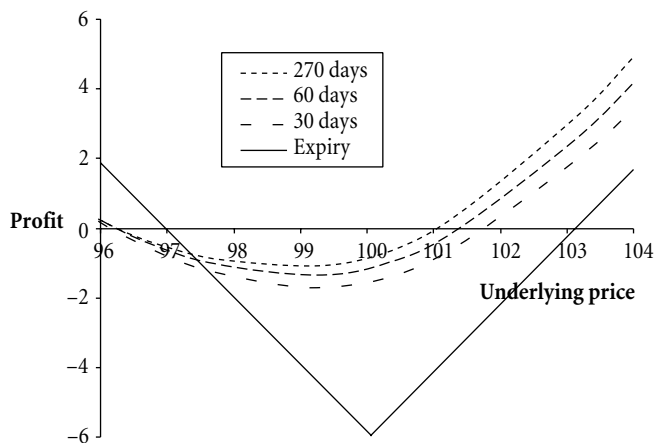


Figure 48.9: Long straddle.

This shows that there will be a gain if the underlying asset price is at any level on expiry, except at the strike price of the options. The profit gained must be greater than the premium paid for both options which, because both options are at-the-money, will be quite high. This is therefore a long volatility trade and will gain if there is a significant move, upwards or downwards, in the price of the underlying. The probability of there being such a significant shift will be priced into the options. Premium outlay aside, a long straddle is a risk-free trade and is put on by a trader expecting a significant shift in the price of the underlying but in no specific direction. A good example of this is if a company's debt is placed under debt watch but the rating agency does not specify that it is for possible downgrade or upgrade. If the credit review results in a downgrade, the price of the company's bonds will fall, while if the result is an upgrade the price of its debt will rise. A long straddle in the company's bond issue would then gain whatever the market outcome.¹ A straddle position therefore gains from a rise in implied volatility. If a trader is expecting a fall in implied volatility, they could put on a *short straddle*, with the opposite payoff profile to a long

¹ This is a typical theoretical illustration so beloved of textbooks! While it may well have taken place somewhere, the author isn't aware of it personally, but then he may have led a sheltered life. It is more suited to equity markets, but even there the price of ordinary shares that are expected to encounter a volatile period of activity are usually trading at high levels already.

straddle. This is a high-risk trade and also has a high negative gamma exposure, which is also high risk for a trader. This is illustrated at Figure 48.10.

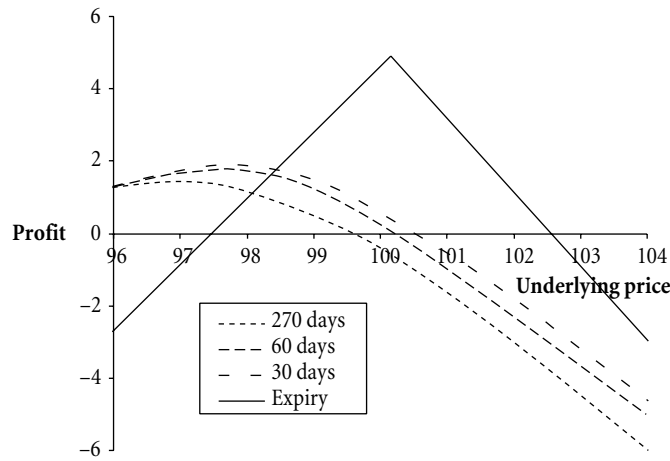


Figure 48.10: Short straddle.

Note also from Figure 48.9 that the time decay feature is very strong as the position approaches maturity, but is relatively weak for most of the life of the trade. Therefore the position will have lost little of its value up to around one month before expiry, and may be unwound for only a small loss if the trader doesn't think it will go anywhere.

48.3.3 Strangle

A version of the straddle is the *strangle*. This is also known as a *bottom vertical combination*, and is composed of a long call and a long put with identical expiry dates but different strike prices. In a long strangle the call strike price is higher than the put strike price. This generates the payoff profile shown at Figure 48.11, which shows the characteristic of a 98 put and 102 call strangle.

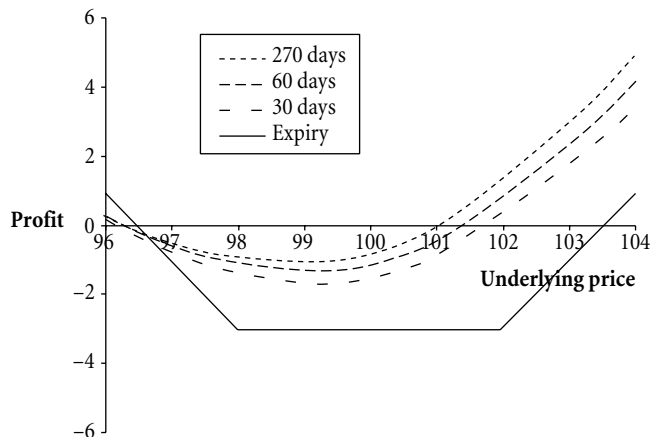


Figure 48.11: Long strangle.

With a strangle the trader believes that there will be a significant shift in the price of the underlying, in no specified direction. The profit potential is smaller than with a straddle because the asset price must change by a larger amount, however there is less downside risk if the expiry price of the underlying lies in the central range between the strike prices. The premium cost is also lower than a straddle. The exact shape of the payoff profile will

depend on the spread between the two strike prices. A larger spread has a lower downside risk, but the asset price must move by a greater amount in order to generate a profit.

A *short strangle* can be set up with short calls and puts. However as with a short straddle this has unlimited potential downside risk and so is a very high risk strategy.

48.3.4 Condors

A *condor* is similar to a butterfly spread, but has two middle strike prices instead of one. From Figure 48.12, a long condor has been constructed from a long 98 call, a sold 99 call, a sold 101 call and a long 102 call. The same profile can be created with put options. There is very little premium difference between a long call and long butterfly, and the potential upside and downside is similar for both. The advantage of using a condor is that there is a range of prices over which maximum profit is generated, rather than the single point that applies in the case of the butterfly. We also illustrate the volatility profile for the condor, which is virtually identical to the long butterfly.

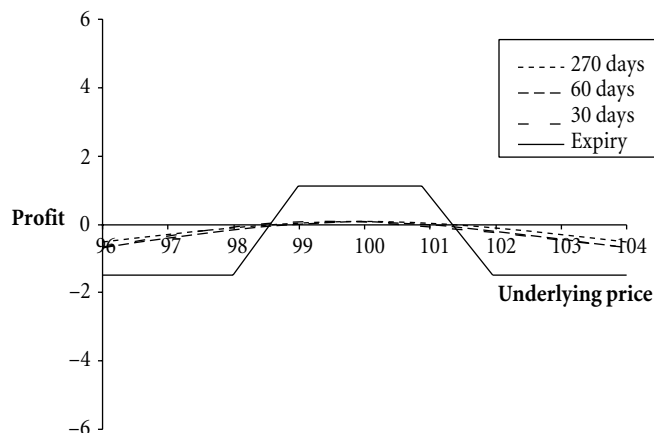


Figure 48.12: Long condor.

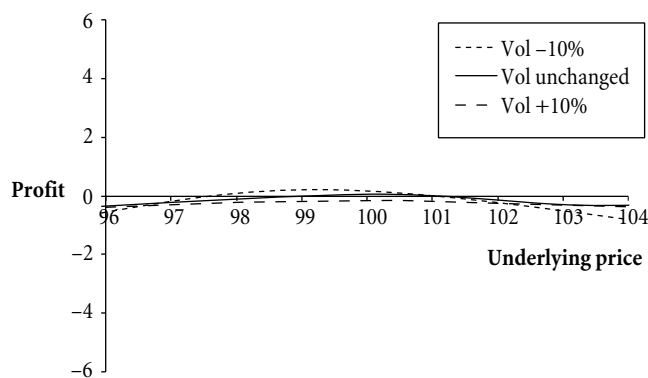


Figure 48.13: Long condor volatility sensitivity.

48.3.5 Ratio spreads

This class of option combination is quite large and there are a number of varieties of the basic combination. We introduce the main principles here. A ratio spread is the term for a position composed of a long in one type of option and a short, of different size, in the same option but at a different strike price. Where the long position is larger than the short, the combination is known as a ratio spread, while if the long position is smaller than the short position the combination is known as a *ratio backspread*. The premium paid or received will depend on the ratio of the long and short positions and the spread between the two strike prices. There are essentially four different payoff profiles

possible, depending on whether it is a spread or backspread and whether the position is put on with calls or puts. Ratio spreads are both directional trades and volatility trades.

Figure 48.14 illustrates a simple call ratio spread, constructed from a long 98 call and two 100 short calls. The volatility sensitivity of this position is evident from Figure 48.15. The trade will generate net premium income, because more options have been sold than bought. The holder of this position will gain if the underlying asset remains stable or declines in price. If the trader expected stability or a price increase they would put on a put ratio spread. A trader expecting market instability would put on a ratio backspread.

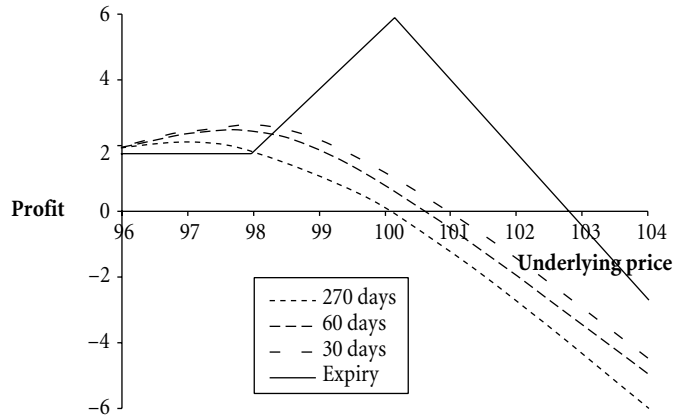


Figure 48.14: Call ratio spread.

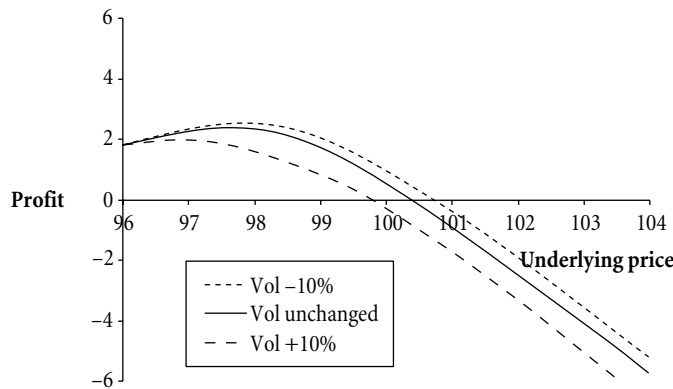


Figure 48.15: Call ratio spread volatility sensitivity.

48.4 Collars, caps and floors

An option combination that is very important in debt capital markets is the cap and floor, which is used to hedge interest-rate exposure. Caps and floors are combinations of the same types of options with identical strike prices but arranged to run over a range of time periods.

In previous chapters we reviewed the main types of instruments used to hedge interest-rate risk, and short-dated interest-rate futures and FRAs are usually used. For example a corporate that wished to protect against a rise in borrowing costs in the future could buy FRAs or sell futures, which would lock in the forward interest rate available today. As we noted at the start of Chapter 42 though, such an arrangement does not allow the hedger to gain if market rates actually go his way. He would have prevented loss, but also any extra gain. To overcome this the hedger might choose to construct the hedge using options. Note that the term *cap* and *floor* is not to be confused with floating-rate note products that have caps and/or floors set into their coupon reset terms.

48.4.1 Description

A cap is essentially a strip of options. A borrower with an existing interest-rate liability can protect against a rise in interest rates by purchasing a cap. If rates rise above the cap, the borrower will be compensated by the cap payout, however if rates fall the borrower gains from lower funding costs, and does not have an equivalent loss on the other side that he would had he taken out a conventional hedge.

A cap is composed of a series of individual options or *caplets*. The price of a cap is obtained by pricing each of the caplets individually. Each caplet has a strike interest-rate that is the rate of the cap. For example a borrower might purchase a 7.0% cap, which means that if rates rise above 7% the cap will pay out the difference between the cap rate and the actual Libor rate. A five-year cap might be composed of a strip of nine individual caplets, each providing protection for successive six-month periods. The first six-month period in the five-year term is usually not covered, because the interest rate for that period, as it begins straight away, will be known already. A caplet runs over two periods of time, the exposure period and the protection period. The exposure period runs from the date the cap is purchased to the interest reset date for the next borrowing period. At this point the protection period begins and runs to the expiry of the caplet. The protection period is usually three months, six months or one year, and will be set to the interest rate reset liability that the borrower wishes to hedge. Therefore the protection period is usually identical for all the caplets in a cap. This is illustrated for a two-year in Figure 48.16.

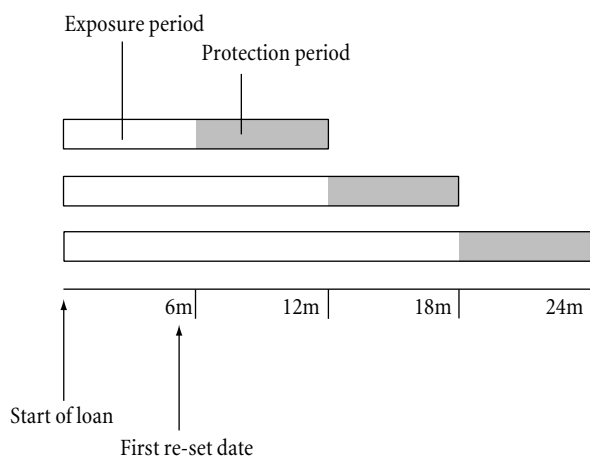


Figure 48.16: Two-year cap and caplets.

It is possible to protect against a drop in interest rates by purchasing a *floor*. This is conceptually identical to a cap but with the interest rate set as one below which the option pays out. This would be used by an institution that wished to protect against a fall in income caused by a fall in interest rate, so for example a commercial bank with a large proportion of floating-rate assets. The combination of a cap and a floor creates a *collar*, which is a corridor that fixes interest payment or receipt levels. This is sometime advantageous for borrowers because it is lower cost than a straight cap. A collar protects against a rise in rates, and provides some gain if there is a fall down to the floor rate. The cheapest structure is a collar with a narrow spread between cap and floor rates.

48.4.2 Pricing

Most banks use the Black 76 model price caps and floors. We described this model in Chapter 44. The price of the underlying and the strike price that are input to the model, which we denoted with S and X are replaced by interest rates. This is shown at (48.1):

$$S = \frac{MfTe^{-rt}}{1 + fT}$$

$$X = \frac{MxT}{1 + fT}$$
(48.1)

where

M	is the notional value of the cap (and principal amount of borrowing)
f	is the forward rate over the protection period
T	is the length of the protection period
x	is the cap rate
r	is the continuously compounded zero-coupon rate over the exposure period.

Substituting these expressions into the basic B–S equation results in the pricing formula for caplets given at (48.2):

$$CAPLET = \frac{Te^{-rt}}{1 + ft}(fN(d_1) - xN(d_2)) \quad (48.2)$$

where

$$d_1 = \frac{\ln(f/x) + (\sigma^2/2)t}{\sigma\sqrt{t}}$$

$$d_2 = d_1 - \sigma\sqrt{t}$$

t is the length of the exposure period and σ is the volatility of the forward interest rate.

A caplet is normally quoted as the percentage of the principal rather than an actual amount. The term r is the zero-coupon rate, used as a substitute for the risk-free rate.

Swaptions, which were covered in Chapter 39, are priced using a similar method to that used for pricing caps, or rather caplets.

48.5 Using options in bond markets

In this section we present some simple applications of options for trading and hedging purposes.

48.5.1 Long call strategy

A strategy of buying call options is a simple and straightforward way to put on a position where one is anticipating a fall in interest rates and a rise in the price of bonds. The trader will buy an option on an underlying bond. For example say that a trader buys a three-month call option on a 10-year government bond with a strike price of £100 per cent. The premium for the option is £5. On the expiry date if the bond is trading at above 100, the option will be exercised; if the bond is trading below 100 the option will not be exercised and will expire worthless. The trader will then have lost the premium paid for the option. We can see that for the strategy to break even the bond must be trading at £105 or above, otherwise the net profit/loss will be eaten away by the premium paid for the option.

What if the underlying bond trades at above £105 before the expiry of the option? If the option is European it can only be exercised on maturity, so this fact is not relevant. But what if the option is American style? Should the trader exercise? Remember that the value of an option is comprised of intrinsic value and time value. If an option is exercised before expiry, the holder will receive the intrinsic value of the option but not the time option (as it is being exercised there is no time value to allow for). However if the option itself is sold, rather than exercised, then the holder will receive the full worth of the option, both intrinsic value and time value. So in practice American options are rarely exercised ahead of maturity and trade in a similar fashion to European options.

48.5.2 Short call strategy

If a trader's view is that interest rates will rise or remain stable they can sell or *write* a call option. The income from such a trade is finite and limited to the premium received for selling the option. The risk profile for this strategy is the mirror image of the long call profile that would apply in our first example. If we consider a short call option with the same terms as above, the strike price is £100 and the writer will realise profit if the underlying bond does not rise above £105. This is why a trader may write the option either because their view of bond prices is bearish, or because they expect interest rates to remain where they are. The profit/loss resulting is the same for both outcomes, and is illustrated in Figure 48.17 below.

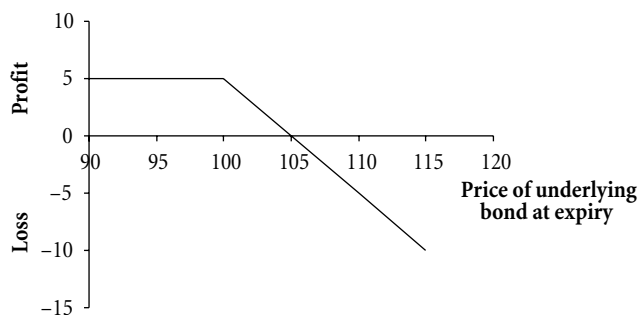


Figure 48.17: Profit/loss profile for short call strategy.

48.5.3 Long put strategy

This position is put on in anticipation of a rise in interest rates and fall in bond prices, the opposite to the long call position. The P/L profile is shown in Figure 48.18.

The opposite position for someone who is expecting rates to decrease or remain stable is a *short put* position.

For simplicity we have ignored the time value factor of options, as well as other considerations such as the time value of money (funds used to purchase options must be borrowed at a rate of interest, or forego interest if not invested and used to buy the option) and whether a coupon is paid on the underlying bond during the life of the option. These would all impact on the analysis of an option position, however this section has illustrated the principles involved.

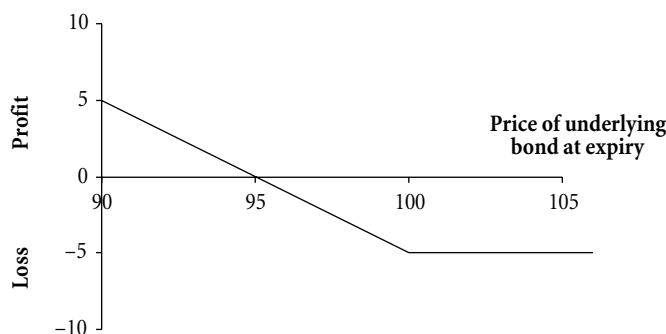


Figure 48.18: Profit/loss profile for a long put strategy.

48.5.4 Hedging using bond options

Earlier in this chapter we discussed how bond trading desks can use swaps or futures to hedge their books. Options are also used frequently to hedge bond positions. Using futures will hedge against downside risk for a bond, although of course it will also preclude any possible upside gain if the market moves in the trader's favour. A hedge using options can be used to protect against interest rate risk while still allowing for upside gains. The two most popular hedging strategies are the *covered call* and the *protective put*.

- Covered call hedge:** if a trader or investor is long of a bond, they can hedge the position by writing a call option on the bonds. The result is a short position in a call option and a long position in the underlying asset. If the price of the bond falls the investor will suffer a loss; however this will be offset by the premium income received for writing the option. The bond position is hedged therefore by the amount of premium received, which may or may not offset the total loss on the bond position. The profit/loss profile is identical in shape to a short put strategy. This hedge has provided some downside protection against a fall in the price of the bond, however there is still upside gain available, and any profit resulting from a rise in the underlying bond price will be enhanced by the amount of premium income.

- **Protective put hedge:** the protective put is a more complete hedge of a long bond position. If a trader or investor buys a put option, she is locking in receipt of the strike price of the bond at any time during the option life, minus the premium paid for buying the option.

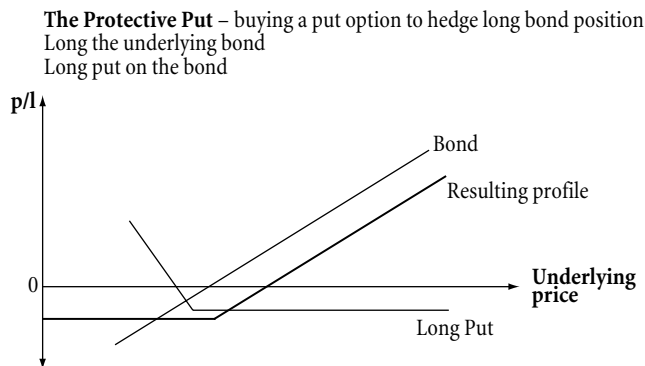


Figure 48.19: Protective Put strategy.

The trader has protection if interest rates rise and the value of the bond falls, because the put option will be in-the-money. At the same time the trader will benefit if the bond price rises, although their resulting profit will be minus the cost of the put option. Effectively the trader has formed a long call position. The option strike price sets the minimum price that is being locked in for the long bond position and the level of the downside protection. Therefore the higher the strike price is, the more the option will cost (other things being equal). The option premium that the trader is willing to pay will need to be evaluated, and is the trade-off between greater risk protection and the option cost. The profit/loss profile for a protective put is shown in Figure 48.19.

Appendices

APPENDIX 48.1 Summary of arbitrage strategies

Conversion

This is the sale of a Call option and the purchase of a put option at the same strike price, combined with a purchase of the underlying product.

This is essentially combining a synthetic sale of the underlying product with its actual purchase. This strategy will only be entered into if the synthetic sale price is higher than the actual purchase price.

Advantage

- Locked in profit of the difference between the actual purchase price and the synthetic sale price.

Disadvantage

- Rarely available once dealing costs are taken into account.

Reversal

This is the purchase of a call option and the sale of a put option at the same strike price, combined with the sale of the underlying product.

This is essentially combining a synthetic purchase of the underlying product with its actual sale. This strategy will only be entered into if the actual sale price is higher than the synthetic purchase price.

Advantage

- Locked in profit of the difference between the synthetic purchase price and the actual sale price.

Disadvantage

- Rarely available once dealing costs taken into account.

Box

This is the purchase of a call and sale of a put at one strike price, combined with the sale of a call and the purchase of a put at a higher strike price.

Essentially, this is the synthetic purchase of the underlying product at a low strike price combined with the synthetic sale of the underlying product at a higher strike price. This trade will be entered into if the net premium paid is less than the difference between the strike prices.

Advantage

- Locked in profit of the difference between the synthetic purchase price and the synthetic sale price, less the premium paid.

Disadvantage

- Rarely available once dealing costs are taken into account.

APPENDIX 48.2 Portfolio case study**Using traded options in portfolio management**

A fund manager is responsible for structuring a portfolio such that it has a particular risk/ reward profile and particular asset mix, as set down in the fund's terms and conditions. In today's competitive market place the fund manager must achieve more than just satisfying these constraints, he must perform well with respect to the fund management community as a whole. Traded options can be an effective tool in portfolio management, as they provide flexibility, ease of dealing and the ability to tailor a strategy to specific requirements.

The fund manager's main requirements are:

- Maximising returns
- Performance of the portfolio against a benchmark
- Maintaining the value of a portfolio in a declining market
- Maximising portfolio purchasing power through the timing of investment
- Ease of asset re-allocation

Traded options provide a large number of ways of achieving each of these aims. This is due to the wide variety of strike prices, terms to maturity and choice of put or call which are available on each underlying product.

Maximising returns

Most of the time a fund manager maximises returns by investing in the best performing assets available. However, at certain times, markets appear to be lacklustre giving the fund manager little scope to improve performance. Before the development of options on financial products, generally fund managers were limited to maximising returns by investing in assets which provided the highest level of coupon or dividend income. Now the options market can provide additional income, as a static market gives an option writer the opportunity to benefit from a decline in option premium caused by time decay.

A strategy often used by portfolio managers who expect little movement in a particular asset price over a period of time is to write calls on this non-performing asset. The only risk with this strategy is that if the price does move, but then the fund will benefit from the rise in the underlying asset price. If the portfolio manager does not wish to sell the underlying as a result of exercise of the option, he would simply close out the option position before exercise became likely. However, the way profit is expected to be realised is that time decay will be earned from the call, and that the underlying asset position remains unchanged.

Benchmarking

A fund manager's performance is usually measured against a benchmark, often an index. It is a very effective fund management tool to be able to synthetically invest in the benchmark itself. In a rising market any new or uninvested funds will cause the portfolio to appear to underperform. However, most fund managers need time to carry out their stock analysis and do not wish to invest directly in the cash market immediately the funds arrive. Investing in the

stock index via an option enables the fund to have a market exposure and gives the manager time to do the analysis. The traded stock index option can then be unwound at the same time as the stock investment is made, portfolio performance having been maintained.

Often the most suitable option to purchase in this circumstance is a reasonably long dated just in-the-money call option. The time decay costs to the option will be kept to a minimum because the option is long dated and therefore has relatively low theta, and since funds are available there is not the restriction on purchasing a relatively high premium option that usually occurs when trading options. In any case, there is the expectation that the premium plus a profit will be recouped when the option position is closed. The only problem with this strategy is that if the market moves up dramatically the option may become illiquid, as it will then be a deep in-the-money option. However, rolling up to a higher strike price as the market moves will maintain liquidity but does have associated dealing costs.

Portfolio protection

A fund manager is concerned that a particular market will decline and so wishes to tailor an insurance policy to when the decline is expected to occur and by how much it will fall. A dramatic decline in the market in the short term requires the purchase of short dated just out-of-the-money put options. However, a flat market, expected to drift slowly downwards over the next couple of months, may be more effectively covered by the sale of at-the-money call options with three months until expiry.

Maximising purchasing power

The fund manager believes the current market price for a particular asset is at an exceptionally low level. This price is not expected to be maintained but currently there are insufficient funds in the portfolio to make a direct investment. An inflow is expected in a few weeks' time so the current market price (plus a premium) can be locked in by purchasing an at-the-money call option which is due to expire as the funds arrive. Alternatively, a put option can be sold. This does not lock in a purchase price, instead it provides an income of premium with which to lay off some of the increased cost of purchasing the asset at the later date.

Portfolio management case study²

This case study was written at a time when gilts were still priced in "ticks" (32nds) and traded options on gilts were priced in option ticks (64ths). The prices are quoted in decimals however.

A fund manager with responsibility for a balanced UK equity and fixed income fund, current value £100 million is required to actively manage the portfolio to provide a reasonably high return on investment without being exposed to an unacceptable level of risk. The portfolio is invested mainly in blue chip stocks in the equity portion of the fund and either Gilts or at least AA rated bonds in the fixed income portion. The equity portion of the fund currently moves in line with the FTSE-100 Index and the current asset mix of the portfolio is 50:40:10, equity: bond: cash. This choice of investments gives the level of liquidity required for an actively managed fund. Also, these investments are of sufficiently high credit quality such that default risk can be essentially disregarded.

The problem is how to generate sufficiently high return from what is essentially a low risk portfolio. The option market is able to provide the income enhancement required. The current view of the fund managers is that the equity market will outperform the fixed income market in the short term. The FTSE-100 movement will be accompanied by an increase in volatility while the Gilt volatility will remain relatively unchanged. Furthermore, Gilt prices are expected to trade within a narrow range.

The FTSE-100 Index level is currently 3343.8 with historic volatility at around 9%. The Long Gilt future is trading at 106.593 with historic volatility around 7%.

The current option analysis tables for the FTSE-100 (American) options and the Long Gilt options, both expiring in two months, are as follows:

² Adapted, with permission, from a LIFFE case study, using exchange-traded options (LIFFE Gilt options technical document).

	Price	Implied Volatility	Delta	Gamma	7-day Theta ¹	Vega
FTSE-100						
3300 call	108.0	13.2	0.63	0.023	4.90	5.25
3350 call	74.0	12.2	0.53	0.022	4.65	5.60
3400 call	55.0	12.8	0.42	0.020	4.60	5.50
3450 call	35.0	12.3	0.32	0.018	3.90	5.00
3250 put	32.0	13.4	−0.26	0.017	2.85	4.65
3300 put	50.0	13.7	−0.36	0.019	3.20	5.30
3350 put	68.0	13.1	−0.46	0.021	3.05	5.60
3400 put	96.0	13.3	−0.56	0.021	2.80	5.50
Long Gilt						
105 call	2.5937	9.1	0.65	0.09	0.0781	0.1718
106 call	1.8437	8.2	0.57	0.10	0.0781	0.1875
107 call	1.3906	8.5	0.47	0.10	0.0781	0.1875
108 call	0.9531	8.3	0.36	0.10	0.0625	0.1718
105 put	1.125	9.6	−0.36	0.08	0.0781	0.1718
106 put	1.5	9.4	−0.44	0.09	0.09375	0.1875
107 put	2.0156	9.8	−0.53	0.08	0.0781	0.1875
108 put	2.5625	9.0	−0.63	0.08	0.0625	0.1718

¹ 7-day theta is the expected decline in the option value after 7 days.

Action

The fund manager of this portfolio decides to put in place an asset allocation overlay to increase the equity exposure by £5 million and reduce the fixed income exposure by £5 million. Using the options markets this can be executed by buying call options on the FTSE-100 Index and offsetting some, or all, of the cost by writing call options on the Long Gilt future. Alternatively, this can be achieved by buying put options on the Long Gilt future and offsetting some, or all, of the cost by writing put options on the FTSE-100 Index. As the fund manager expects FTSE-100 Index volatility to increase and Gilt volatility to remain unchanged, purchasing options on the index is the preferable strategy.

The implied cash amounts notionally moved from the Gilt market to the stock market must be equivalent otherwise there is an unacceptable gearing effect. This equivalence is ensured by using delta neutrality.

In addition, the fund manager decides to establish an income enhancement strategy by writing a straddle on the Gilt to take advantage of the lack of activity expected in this market.

Actions

Strategy 1: Asset allocation overlay

Notionally invest £5,000,000 in stocks by buying FTSE-100 index 3350 calls.

Cash value underlying one option

$$\begin{aligned}
 &= \text{Strike price} \times \text{Cash value per index point} \\
 &= 3350 \times £10 = £33,500.
 \end{aligned}$$

The number of contracts to trade is the Hedge Ratio

$$\begin{aligned}
 &= \text{Notional amount invested} / (\text{Cash value underlying the option} \times \text{Delta}) \\
 &= £5,000,000 / (£33,500 \times 0.53) \\
 &= 282 \text{ contracts.}
 \end{aligned}$$

Buy 282 FTSE-100 3350 calls at 74

$$\text{Number of ticks per option} = 74 / 0.5 = 148 \text{ ticks where } 0.5 = \text{tick size.}$$

Total cost of the FTSE-100 Index options

$$\begin{aligned}
 &= \text{Hedge ratio} \times \text{Number of ticks per option} \times \text{Tick value} \\
 &= 282 \times 148 \times \text{£}5 \\
 &= \text{£}208,680.
 \end{aligned}$$

Notionally realise £5,000,000 from bonds by selling Long Gilt options 107 calls

$$\begin{aligned}
 \text{Cash value underlying one option} &= \text{Strike price} \times \text{Nominal value of one future}/100 \\
 &= 107 \times \text{£}50,000/100 \\
 &= \text{£}53,500.
 \end{aligned}$$

The number of contracts to trade is the Hedge Ratio

$$\begin{aligned}
 &= \text{Notional amount invested}/(\text{Cash value underlying the option} \times \text{Delta}) \\
 &= \text{£}5,000,000/(\text{£}53,500 \times 0.47) \\
 &= 199 \text{ contracts.}
 \end{aligned}$$

Sell 199 Long Gilt 107 calls at 1.3906

$$\begin{aligned}
 \text{Number of ticks per option} &= 64 + 25 = 89 \text{ ticks} \\
 &\text{as these options are priced in 64ths.}
 \end{aligned}$$

Total income from the options on Long Gilt futures

$$\begin{aligned}
 &= \text{Hedge ratio} \times \text{Number of ticks per option} \times \text{Tick value} \\
 &= 199 \times 89 \times \text{£}7.8125 \\
 &= \text{£}138,367.
 \end{aligned}$$

Strategy 2: Income enhancement

Sell 107 calls at 1.3906 and 107 puts at 2.1093, in a delta neutral ratio (53 calls to 47 puts).

Income from call sales

$$\begin{aligned}
 &= \text{Number of contracts traded} \times \text{Number of ticks per contract} \times \text{Tick value} \\
 &= 53 \times (64 + 25) \times \text{£}7.8125 \\
 &= \text{£}36,852.
 \end{aligned}$$

Income from put sale

$$\begin{aligned}
 &= \text{Number of contracts traded} \times \text{Number of ticks per contract} \times \text{Tick value} \\
 &= 47 \times ((2 \times 64) + 7) \times \text{£}7.8125 \\
 &= \text{£}49,570.
 \end{aligned}$$

$$\text{Total income} = \text{£}36,852 + \text{£}49,570 = \text{£}86,422.$$

Outcome

Two weeks later the FTSE-100 Index level is 3423.2 and the Long Gilt future is trading at 106.8437.

The relevant option prices are as follows:

	Price	Implied	Delta volatility	Gamma	7-day decay	Vega
FTSE-100						
3350 call	116.0	13.1	0.67	0.018	4.15	5.70
Long Gilt						
107 call	1.3281	8.3	0.48	0.11	0.0937	0.1875
107 put	1.625	8.8	−0.52	0.09	0.0937	0.1875

Strategy 1: Asset allocation overlay

Current price of a FTSE-100 Index 3350 call = 116.

Number of ticks per option = $116/0.5 = 232$.

Current value of the notional stock investment

$$\begin{aligned} &= \text{Hedge ratio} \times \text{Number of ticks per option} \times \text{Tick value} \\ &= 282 \times 232 \times \text{£}5 \\ &= \text{£}327,120. \end{aligned}$$

Current price of a Long Gilt 107 call 1.3281.

Number of ticks per option = $64 + 21 = 85$.

Current value of the notional short bond position

$$\begin{aligned} &= \text{Hedge ratio} \times \text{Number of ticks per option} \times \text{Tick value} \\ &= 199 \times 85 \times \text{£}7.8125 \\ &= \text{£}132,148. \end{aligned}$$

The fund manager believes the outperformance of the stock market relative to the fixed income market has now taken place and so unwinds the asset allocation overlay.

Profit on FTSE-100 option position = $\text{£}327,120 - \text{£}208,680 = \text{£}118,440$.

Profit on Long Gilt option position = $\text{£}138,367 - \text{£}132,148 = \text{£}6,219$.

Total profit on the strategy = $\text{£}118,440 + \text{£}6,219 = \text{£}124,659$.

Strategy 2: Income enhancement

Current price of a 107 call = 1.3281.

Current value of the call position

$$\begin{aligned} &= \text{Number of contracts traded} \times \text{Number of ticks per contract} \times \text{Tick value} \\ &= 53 \times (64 + 21) \times \text{£}7.8125 \\ &= \text{£}35,195. \end{aligned}$$

Current price of a 107 put = 1.625.

Current value of the put position

$$\begin{aligned} &= \text{Number of contracts traded} \times \text{Number of ticks per contract} \times \text{Tick value} \\ &= 47 \times (64 + 40) \times \text{£}7.8125 \\ &= \text{£}38,188. \end{aligned}$$

Current value of the straddle = $\text{£}35,195 + \text{£}38,188 = \text{£}73,383$.

Current profit = $\text{£}86,422 - \text{£}73,383 = \text{£}13,039$.

The fund manager could unwind this strategy and take the current profit. However, implied volatility is expected to decline further and the inactivity in the market to continue, so the fund manager decides to continue to hold the strategy. The fund manager does not have a directional view so will rebalance the position to maintain delta neutrality. The desired option ratio is now 52 calls to 48 puts, and so the action now is to buy back 1 call at 1.3281 and sell 1 put at 1.625.

APPENDIX 48.3 Case study: replicating futures contracts with options

Replicating bond options with futures

(Successful outcome)

Day	Futures Price (32nds)	Option Price (64ths)	Option Delta	100 Long DEC 97 Calls		Delta Based Futures	
				Day's P/L (64ths)	Total P/L (64ths)	Day's P/L (64ths)	Total P/L (64ths)
15/9/97	94.21	2.06	.44	0	0	0	0
16/9	94.23	2.10	.44	+400	+400	+176	+176
17/9	95.11	2.17	.47	+700	+1100	+1760	+1936
18/9	93.22	1.38	.37	−4300	−3200	−4982	−3046
19/9	92.18	1.17	.31	−2100	−5300	−2664	−5710
22/9	93.19	1.32	.37	+1500	−3800	+2064	−3664
23/9	93.30	1.37	.38	+500	−3300	+814	−2850
24/9	95.21	2.21	.49	+4800	+1500	+4180	+1334
25/9	95.07	2.10	.46	−1100	+400	−1372	−42
26/9	95.21	2.20	.49	+1000	+1400	+1288	+1246

Explanation

On the first day, a December 97 Bond call option's delta was 0.44. With 100 long calls in the option position, the aggregate delta was 44. To replicate this with futures required 44 long futures since each long future had a delta of +1.

On the second day, the futures price has risen very slightly to 94.23, which was an increase of 2/32nds from the previous day's price. The total returns are recorded in 1/64ths (which is the minimum tick for the options in the US Treasury market), so the total return to the 44 long futures was $176/64\text{ths} = 44 \times 4/64\text{ths}$.

On the third day, futures had risen to 95.11, bringing the calls closer to the money and raising their delta to 0.47. To keep the futures replicating portfolio in line with the options, it was necessary to buy 3 additional futures contracts, bringing the total from 44 to 47. The fourth day's futures P/L, then, was calculated using 47 futures together with the price change from 95.11 to 93.22 ($4982/64\text{ths} = 47 \times 106/64\text{ths}$).

Notice that in this example, the option portfolio and the delta-based futures portfolio provided approximately the same returns.

Replicating bond options with futures

(Unsuccessful outcome)

Day	Futures Price (32nds)	Option Price (64ths)	Option Delta	100 Long JUN PAR Calls		Delta Based Futures	
				Day's P/L (64ths)	Total P/L (64ths)	Day's P/L (64ths)	Total P/L (64ths)
09/3/97	100.09	1.36	.53	0	0	0	0
10/3	100.05	1.27	.52	−900	−900	−424	−424
11/3	100.04	1.27	.52	0	−900	−104	−528
12/3	100.13	1.35	.55	+800	−100	+936	+408
13/3	100.24	1.43	.59	+800	+700	+1210	+1618
16/3	100.17	1.32	.57	−1100	−400	−826	+792
17/3	100.28	1.41	.62	+900	+500	+1254	+2046
18/3	100.25	1.41	.60	0	+500	−372	+1674
19/3	100.28	1.47	.61	+600	+1100	+360	+2034
20/3	100.21	1.33	.59	−1400	−300	−854	+1184

Explanation:

On the first day, a June par call option's delta was 0.53. With 100 long calls in the option position, the aggregate delta was 53. To replicate this with futures required 53 long futures.

On the second day, the futures price had dropped slightly from 100.09 to 100.05 (a loss of 8/64ths). The total loss on the futures position was $424/64\text{ths} = 53 \times 8/64\text{ths}$. On the second day one future was sold since the aggregate delta on the 100 par calls went down from 53 to 52.

On the third day, the futures had dropped once again by 2/64ths. The loss on the 52 futures was $104/64\text{ths} = 52 \times 2/64\text{ths}$.

Notice that by the 10th day, the delta-based futures portfolio far outperformed the options portfolio.

Selected bibliography and references

Bookstaber, R., *Option Pricing and Strategies in Investing*, Addison-Wesley, 1981.

McMillan, L., *Options as a Strategic Investment*, 2nd edition, New York Institute of Finance, 1986.

Hull, J., *Options, Futures and Other Derivatives*, 3rd edition, Chapters 8–18, Prentice-Hall, 1998.

Questions and exercises

1. What is the expected profit payoff for an option strategy consisting of the following:
 - (a) long underlying asset and short call
 - (b) long underlying asset and long put
2. If a strangle is constructed with the call strike price at a lower level than the put strike price, what is the trader's expectations of the market and volatility?
3. What strategies might be employed by an options trader that felt that the market was going to experience a sudden shock, but was unsure of the market direction itself? What strategy might they employ if they thought the market would remain roughly where it was over the next few months?
4. If a diagonal spread is constructed with a 70 long call and a 90 short call. The short call expires after the long call. Draw a diagram of the expected payoff profile. What would be the effect if the trade put on was a 90 long call and a 70 short call?
5. What is the difference between a bull call and a bull put spread?
6. What are the risks associated with a short straddle position?
7. Describe the structure of a protective put.
8. How does the protective put benefit the holder of a long position in bonds compared to a hedge consisting of futures?

49

Options VII: Exotic Options

In this chapter we present for reference a summary of some of the *exotic* option structures that have been and are traded in the market. The term exotic option is used to refer to a family of products that have been designed with non-vanilla payoff profiles, usually to meet specific client requirements. For this reason the main users of exotic options are financial institutions and corporates, while the number of banks that actively offer these products is actually quite limited, reflecting the complexity involved in pricing and managing an exotic option book.¹ Generally it is the payoff profile that has been altered in some way, however other standard features that may be adjusted in the make-up of the exotic include variations in the way the strike price is set, or setting payoff proceeds in a different currency to the currency of the underlying, or changing the payoff so that it is dependent on the prices of two underlying assets rather than one.

There are no standard categories involved with exotic options, almost due to the nature of their design, however they are generally categorised as being either variations on standard contract terms, path-dependent options or multi-factor options, or a combination of two or all three. We consider some of the products in this chapter.

At the end of the chapter we present four examples of customer uses for exotic option products, which often form part of a larger structured product. The examples illustrate the diverse nature of the products involved, as well as the uses to which they can be put.

49.1 Options with modified contract terms

49.1.1 Bermudan options

Unlike European options, which are only exercisable on expiry, or American options, which may be exercised at any point during their lifetime, a Bermudan option may be exercised at pre-specified dates during its life. They are otherwise plain vanilla options, although of course a Bermudan-type feature could be incorporated into an option with other exotic characteristics. A Bermudan option could be worth using under certain specific conditions, for example as a hedge against a long position in a bond that had embedded option features, for example it was put-able by the investors on certain dates.

49.1.2 Digital options

Digital options or *binary* options have a different payoff profile to vanilla options. Unlike a standard option, where the payoff function for a call is given by $\max(S - X, 0)$, the payout from a digital option is specified at the time the contract is struck, and will be the same irrespective of how far in-the-money the option is in on expiry. If the option expires out-of-the-money, the payout is zero as with a vanilla option. This is known as a *cash-or-nothing* option. There is another type of digital option known as a *one-touch* option, which will pay out a pre-specified amount on expiry if at any time the option was in-the-money, even if it expires out-of-the-money. Therefore one-touch options are also *path dependent*. In certain cases a one-touch option will pay out as soon as it goes in-the-money; this feature might be requested by the customer because, being a digital option, there is no time value associated with the option once it has become in-the-money.

For a cash-or-nothing digital option, following Black–Scholes the probability that the underlying asset price is above the strike price on expiry is given by $N(d_2)$. Therefore the price of such an option is given by

$$C_{\text{Dig}} = Me^{-rT}N(d_2) \quad (49.1)$$

where M is the fixed value that the option pays out if it expires in the money.

¹ While engaged in consulting work at the investment banking arm of a UK bank, the author was particularly impressed to observe a foreign exchange exotic option book that had been positioned so as to register a profit whatever direction the underlying FX rate moved in!

49.1.3 Chooser option

This is an interesting product, also known as an “as you like it” option. A chooser option’s main feature is that after an initial period, the holder can decide whether the option is a call option or a put option. After the choice has been made though, the instrument is essentially a plain vanilla option. Chooser options are valued as follows: let us call the time that the option is elected to be a call or put time t . At this point the value of the option is given by $\max(C, P)$, where C is the price of the call option and P is the price of the put option that the original option will be transformed into. If the two options have identical strike prices, we can use the put-call parity relationship to value the option. Under this,

$$\begin{aligned}\max(C, P) &= \max(C, C + Xe^{-r(T-t)} - Se^{-q(T-t)}) \\ &= C + e^{-q(T-t)} \max(Xe^{-(r-q)(T-t)} - S)\end{aligned}\quad (49.2)$$

where S is the price of the underlying at time t , X is the strike price, T is the maturity date of both options, and r is the short-term risk-free interest rate as normal. Essentially (49.2) states that the chooser option is composed of a call option with a strike price of X and an expiry date of T , and $e^{-q(T-t)}$ put options with a strike price of $Xe^{-(r-q)(T-t)}$ and maturity of t . Therefore it does not present as much of a problem to value as might first have been expected. However the description is for an essentially plain vanilla chooser option; it is more likely that the call and put components have different strike prices, in which the chooser is valued as a compound option.

49.1.4 Forward start options

A forward start option is conceptually similar to a forward start swap; the option is paid for today but does not become live until a specified date in the future. At that point the option becomes a vanilla product; as such forward start options do not present too much difficulty in terms of valuation. Hull (1997) describes a forward start call option written on a non-dividend paying asset, which becomes effective at the forward start date t_1 and matures at time t_2 . The price of the underlying asset at time 0 is S , and its price at time t_1 is S_1 . Following Black–Scholes, which states that the value of an at-the-money call option is relative to the price of the underlying asset, the value of the forward-start option at time t_1 is given by CS_1/S , where C is the value of the at-the-money call option at time 0 with maturity of $t_2 - t_1$. Under risk-neutral valuation, the value of the forward-start option at time 0 is given by (49.3):

$$C_F = e^{-rt_1} \hat{E} \left[C \frac{S_1}{S} \right] \quad (49.3)$$

where $\hat{E}$ is the expected value in a risk-neutral scenario. We know the value of C and S and also that $\hat{E}[S_1] = Se^{rt_1}$ therefore the value of the forward-start option is C . Hence the value of a forward-start option is equal to the value of an at-the-money vanilla option with an identical maturity period.

49.1.5 Contingent options

A contingent or *pay later* option is one for which no premium is paid at the time the transaction is agreed, and premium only becomes payable if the option is exercised. If the option is in-the-money at expiry it must be exercised, triggering the premium payment. This clause is included in the terms of the contract in case the intrinsic value of the option on expiry is below the premium, in which case the holder has an incentive not to exercise. However no premium is paid if the option expires worthless, and this is one reason why this product is attractive to corporate and financial firm hedgers, as it is akin to buying insurance against risk exposure that need only be paid for if a claim is made on the insurance contract.

49.2 Path-dependent options

49.2.1 Asian options

The term *Asian* option refers to a class of products that have become very popular as hedging instruments for a variety of exposures. An Asian option is a type of instrument known as a *path-dependent option*. It is also known as an *average-rate* option. In a vanilla option the payoff is the difference between the strike price and price of the underlying asset on the expiry date. With Asian options, rather than consider the price of the underlying on expiry, the average of the prices of the asset over a specified period of time is used. The most common type uses the

arithmetic average of the prices recorded for the underlying asset during the life of the option, which is then compared against the strike price when calculating the final payout on expiry.

A variant on this is the *average-strike* option. This is similar to the average-rate option, and uses an average price of the underlying, however the strike price is then set to this average rate. The payout on expiry is calculated based on the difference between the strike price and the price of the asset on the expiry date. This is illustrated in Figure 49.1.

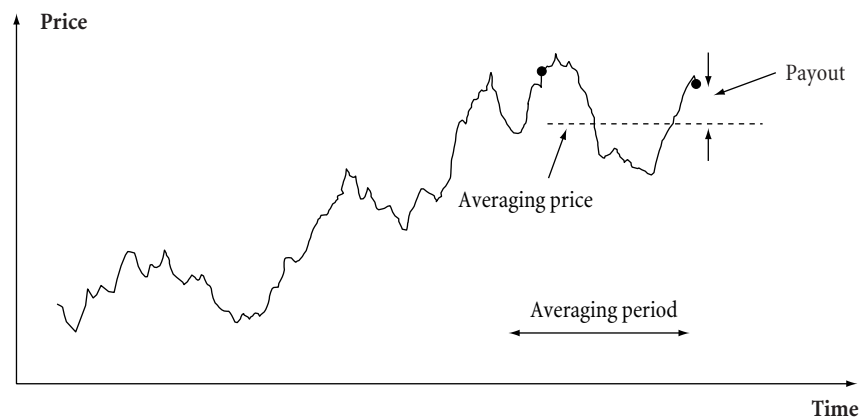


Figure 49.1: Average-strike option.

The payoff from the different types are listed below for call options, where S_T is the price of the asset on expiry, S_A is the average price for the asset calculated from all the prices recorded for the asset during the averaging period, and X is the strike price.

- Vanilla option: $\text{Payoff} = \max(0, S_T - X)$
- Asian option: $\text{Payoff} = \max(0, S_A - X)$
- Average-strike option: $\text{Payoff} = \max(0, S_T - S_A)$

In certain cases average rate options carry a lower premium than a vanilla option with (otherwise) identical terms, and are more appropriate than vanilla contracts in a number of situations. For example, consider a financial institution that has offered a deposit account to its customers that is linked to stock market equity index, but also guarantees a minimum interest rate should the stock market fall in level during the life of the deposit account. In this case to hedge its exposure on the stock-market-linked part of the account, the banks can purchase an option in which the payout is related to the average rate of the daily close on the stock market index, rather than one fixed level. This example is described below.

EXAMPLE 49.1 Structured product equity-linked swap

The option contract that is part of this structured product is a European Asian call option, which has been sold to a retail bank counterparty. The bank has offered a deposit account to retail customers that pays a minimum interest rate, but whose overall performance is linked to the FTSE-100 equity index. The trade details are:

Trade date:	1 September 1998
Effective date:	1 November 1998 (forward start)
Expiry date:	4 May 2004
Notional:	£2,100,000
Strike price:	The closing level of the FTSE-100 on 1 November 1998
Seller:	Market maker in option
Buyer:	Retail bank
Cost:	Three-month sterling Libor, payable from 1/11/98 to 4/5/04

On the expiry date, the payoff is £1 050 000 if $F_2 > F_1$, or zero if $F_2 < F_1$, where

F_1	is the closing level of the FTSE-100 on 1/11/98
F_2	is the average level of the daily closing levels of the FTSE-100 recorded from 2 May 2003 to 4 May 2004

Therefore the Asian option has a strike price that is fixed when it becomes live, and the payoff is dependent on the difference between this strike price and the average level of the index during the last year of the option's life.

The averaging method used to set the rate in an Asian option can be a geometric average or an arithmetic average, the latter is more common. For a geometric average rate option, the valuation is similar to a standard option because the assumption of lognormal distribution of asset price returns still holds; the geometric average of lognormally distributed returns is itself lognormal. With arithmetic average rate options, there is no closed-form solution available to price the option, because the distribution of the average returns cannot be modelled. Banks therefore calculate a close approximation, based on a calculation of the first two moments of the arithmetic average probability distribution, and then assuming that this value follows a normal distribution for the life of the option.

49.2.2 Lookback options

A *lookback* option is similar to an average-strike option, as its strike price is set on the expiry date of the option. In the case of a lookback though, the holder may “look back” and choose the most advantageous price that was recorded by the asset during the lookback period. A lookback call holder would pick the lowest price recorded as the strike price of the option. In theory lookback options provide a strong advantage to the holder, and in the case of an American option the holder can wait until expiry and still exercise at the best rate. The option can also never expire out-of-the-money. In fact this makes it more expensive than the average-strike option and this expense means that in practice it is rarely traded. There are also few risk profiles that can only be met through the purchase of a lookback option, so customers will use cheaper products instead.

Goldman *et al* (1979) have produced a formula for the valuation of European lookback call options, and this is given at (49.4):

$$C = Se^{-qT}N(a_1) - Se^{-qT}\frac{\sigma^2}{2(r-q)}N(-a_1) - S_{\min}e^{-rT}\left(N(a_2) - \frac{\sigma^2}{2(r-q)}e^{Y_1}N(-a_3)\right) \quad (49.4)$$

where

$$\begin{aligned} a_1 &= \frac{\ln(S/S_{\min}) + (r - q + (\sigma^2/2))T}{\sigma\sqrt{T}} & a_2 &= a_1 - \sigma\sqrt{T} \\ a_3 &= \frac{\ln(S/S_{\min}) + (-r + q + (\sigma^2/2))T}{\sigma\sqrt{T}} & Y_1 &= \frac{2(r - q - (\sigma^2/2))\ln(S/S_{\min})}{\sigma^2} \end{aligned}$$

and $S_{\min}$ is price for the underlying asset at time 0 when the option is priced, so therefore it is equal to S . If the option is revalued at any time during its life, then $S_{\min}$ is the lowest price for the underlying asset that has been recorded up to the valuation date.

A very good analysis of lookback options is presented in Briys *et al.* (1998).

49.2.3 Cliquet options

A cliquet or *ratchet* option has a feature in which the strike price is reset in line with changes in the price of the underlying asset on pre-specified dates. By “ratcheting up” the option the holder locks in gains achieved during the life of the option, at each point that the strike is reset. This product is a specialised instrument and is commonly applied to options written on equity indices, enabling the holder to lock in gains recorded by the stock market while continuing to maintain risk exposure protection.

49.2.4 Ladder options

A ladder option is similar to a cliquet option, except that the strike price is automatically reset to higher levels that have been pre-specified, irrespective of the dates on which the underlying asset reaches these levels.

49.2.5 Barrier options

A *barrier option* specifies two price levels when it is written, the strike price and a *barrier* or trigger price. The barrier price is the level at which, if it is breached by the underlying asset, the option will either be activated or extinguished. A *knock-in* barrier option will be activated once the underlying asset price reaches the barrier price, at which point the option is transformed into a plain vanilla option. A *knock-out* barrier option begins life as a vanilla option, but will be inactivated immediately if the barrier price is breached. This might be purchased by a company that wished to hedge an exposure but only up (or down) to a certain level, and specifying a knock-out level would reduce its price. A call barrier option usually has its strike price set above the barrier price, while the opposite is usually true for a put barrier.

There are a number of variations of barrier option available. In certain cases the premium payable for a barrier option is lower than the (otherwise) identical standard option, and they may be more suitable as hedging instruments, according to the nature of the risk exposure. They may be priced by adjusting the basic B-S model for the level at which the option is knocked in or out. For a *down-and-out* call option, a barrier level is set at which the option will be extinguished if the price of the underlying asset reaches the barrier, which will be below the asset price level at the time the option was struck. A *down-and-in* barrier call option becomes live if the price of the underlying asset reaches the specified barrier level. If we denote the barrier as L , for $L \leq X$ Hull (1997) gives the price of a down-and-in barrier call option on the day it is struck as (49.5):

$$C_{DI} = Se^{-qT} (L/S)^{2\lambda} N(y) - Xe^{-rT} (L/S)^{2\lambda-2} N(y - \sigma\sqrt{T}) \quad (49.5)$$

where

$$\lambda = \frac{r - q + (\sigma^2/2)}{\sigma^2} \quad y = \frac{\ln(L^2/SX)}{\sigma^2\sqrt{T}} + \lambda\sigma\sqrt{T}.$$

The value of a standard call option is equal to the value of a down-and-in call and a down-and-out call, so to calculate the price of a down-and-out call option we simply subtract the price of the down-and-in call from the price of the standard option.

Where $L \geq X$ the price of a down-and-out call option is given by (49.6):

$$C_{DO} = SN(x_1)e^{-qT} - Xe^{-rT}N(x_1 - \sigma\sqrt{T}) - Se^{-qT} (L/S)^{2\lambda} N(y_1) + Xe^{-rT} (L/S)^{2\lambda} N(y_1 - \sigma\sqrt{T}) \quad (49.6)$$

where

$$x_1 = \frac{\ln(S/L)}{\sigma\sqrt{T}} + \lambda\sigma\sqrt{T}, \quad y_1 = \frac{\ln(L/S)}{\sigma\sqrt{T}} + \lambda\sigma\sqrt{T}$$

and, of course, $C_{DI} = C - C_{DO}$.

The opposite type of barrier option product is an *up-and-out*, where (for a call) the option is extinguished if the price of the underlying asset rises to a barrier level that is above the price of the asset set when the option is struck. A call option that becomes live if a higher barrier is reached is an *up-and-in* barrier option. If we denote the barrier price as H , for $H \leq X$ the value of an up-and-out call option is zero and the price of the up-and-in call is equal to the price of the corresponding standard call option. For $H \geq X$ the price of an up-and-in call option is given by (49.7):

$$C_{UI} = SN(x_1)e^{-qT} - Xe^{-rT}N(x_1 - \sigma\sqrt{T}) - Se^{-qT} (H/S)^{2\lambda} (N(-y) - N(-y_1)) + Xe^{-rT} (H/S)^{2\lambda-2} (N(-y + \sigma\sqrt{T}) - N(-y_1 + \sigma\sqrt{T})). \quad (49.7)$$

The price of the up-and-out call option is given by $C_{UO} = C - C_{UI}$.

Since down-and-out call and up-and-out put options can be extinguished during their life, the premium payable for them is often lower than for the corresponding vanilla option. As soon as the barrier price is breached the option will die, whereas a vanilla option would continue until the expiry date. The lower price of the barrier therefore makes

it attractive for hedging customers, however it is important to ensure that the risk profile is being matched correctly. Once a barrier option is extinguished, there is no protection against whatever risk exposure was being hedged in the first place, so unless that risk exposure has also disappeared, the option holder will still be at risk. Therefore, customers should only purchase barrier options if their risk exposure is matched by the payoff profile of the barrier. An example is discussed below.

An excellent description and analysis of barrier options is given in Briys *et al.* (1998).

EXAMPLE 49.2 Risk profile matched by barrier option

A petroleum company domiciled in the UK must account for its profit in sterling, but oil prices are quoted in dollars. Consider a case where the company has sold petroleum to a customer for delivery in four months' time, which means it will receive dollars in four months' time. The current exchange rate is $\text{£}1 = \$1.6300$ spot and $\text{£}1 = \$1.6050$ four months' forward. If sterling appreciates between now and the delivery date, the petroleum exporter will receive reduced revenue. Let us assume that if sterling rose to $\text{£}1.6700$, the sale would be uneconomic for the company, given the price of the oil itself. If however sterling depreciated to $\text{£}1.5700$ the company would be satisfied with the revenue received as a result of the exchange rate, and would arrange to sell dollars via a forward contract (expiring on the delivery date) to lock in the profit. In other words, the company wishes to protect itself from a rise in sterling above $\$1.6700$, but requires no protection at an exchange rate below $\$1.5700$, because then it would simply transact a forward contract. In this case the company could purchase a four-month down-and-out barrier option, which will pay out if the exchange rate reaches $\$1.6700$, but will be extinguished if sterling falls to $\$1.5700$. This suits the risk profile of the petroleum company. The option is a vanilla option unless the exchange rate reaches the barrier price, and so will be cheaper than the corresponding vanilla option.

Like all the other classes of exotic options we have highlighted here, there are a number of variations on the basic option we have described. For detail on the many types available, we recommend a specialist options textbook. There is just one other type of barrier option that the author would like to draw attention to, and that is a *hokey-cokey option*. This is a barrier option which knock in on one side of the underlying asset, and knock out on the other side. That's another product whose name has obscure origins!

49.3 Multi-asset options

It is clear that there is a great variety of option instruments that can be created, indeed we have only touched on some of the more common exotic options that have been traded. In this section we summarise some of the even more (!) exotic options in the market today; a complete description is of course outside the scope of this book, and readers may wish to consult the bibliography.

49.3.1 Quanto options

A *quanto* option is one in which the payoff is determined in one asset but its value is determined in terms of another asset. The term comes from the name *quantity-adjusting* option. A typical example would be an option that was linked to the FTSE-100 index, with underlying asset and strike price set as levels of the FTSE-100, but where (if the option expires in-the-money) the payout is paid in euros. The market maker selling the option must therefore hedge their exposure to the FTSE-100 index, most probably by purchasing exchange-traded FTSE-100 futures contracts, but also the exposure in the sterling/euro exchange rate, because the final payoff is a function of the exchange rate as well as the level of the FTSE-100. A depreciation in the level of the euro will increase the market maker's exposure irrespective of any change in the level of the FTSE-100.

The pricing of a quanto option is complex and often market making banks will use either a binomial model or a simulation technique. One of the factors that renders valuation difficult is the need to evaluate correlation between the price volatility of two assets. In the chapter on pricing we discussed the issues in assuming constant volatilities in option pricing, and how the basic B-S model had been modified to account for stochastic volatilities, for example in the Hull-White and other models. The issue is magnified when two volatility levels and the correlation between them need to be evaluated.

49.3.2 Rainbow options

A *rainbow* option is written on two or more underlying assets. They are usually equity index options written on two or more indices. The payoff is a function of the level reached by the best-performing asset in the rainbow. So for

example a call rainbow option written on the FTSE-100, Nikkei500 and S&P500, will pay out the difference between the strike price and the level of the index that has risen by the largest amount of the three indices.

49.3.3 Basket options

Unlike a rainbow option, which considers a group of assets but ultimately pays out on the level of one, a basket option is written on a basket of underlying assets, and will pay out on a weighted-average gain of the basket as a whole. Like rainbow options they are most commonly written on a basket of equity indices, although they are also frequently written on a basket of individual equities as well. For example, a call option could be written on a basket of technology stocks, where the basket was composed of say, ten technology stocks in weighted proportions. The strike price X_{basket} is usually set at the current value of the basket (at-the-money) and the payoff profile will be

$$\max(S_{\text{basket}} - X_{\text{basket}}, 0).$$

A basket option could be purchased to provide an element of downside protection by the fund manager of a specific fund, where the more usual hedge instruments written on the overall equity index may not match the performance profile of the fund that the fund manager is running.

49.4 Pricing and hedging exotic options

49.4.1 Pricing considerations

In certain cases the valuation of exotic options presents no more problems than the valuation of the corresponding vanilla option after adjustments have been made to say, the B-S model. However, given the variable nature of the final payoff for most exotic options, valuation is more complex. Where an analytical or closed-form solution to a pricing formula is not available, for example as we saw with arithmetic-average rate Asian options, the valuation is either an approximation under certain assumptions, or a binomial approach is used.

It is possible to value any option using the binomial or trinomial method, which we discussed in Chapter 45. Where the option being valued has a path-dependent payoff, this results in a binomial lattice with a much greater number of paths. This is illustrated in Figure 49.2 below. The number of final outcomes that exist in a standard binomial model is $n + 1$, where n is the number of periods that are evaluated. However a path-dependent option will produce 2^n final outcomes. In Chapter 45 we noted that generally around 20–30 steps are carried out in binomial model valuation; for an Asian option this would generate over 1 billion terminal values, which may render the evaluation of the option price rather time-consuming.

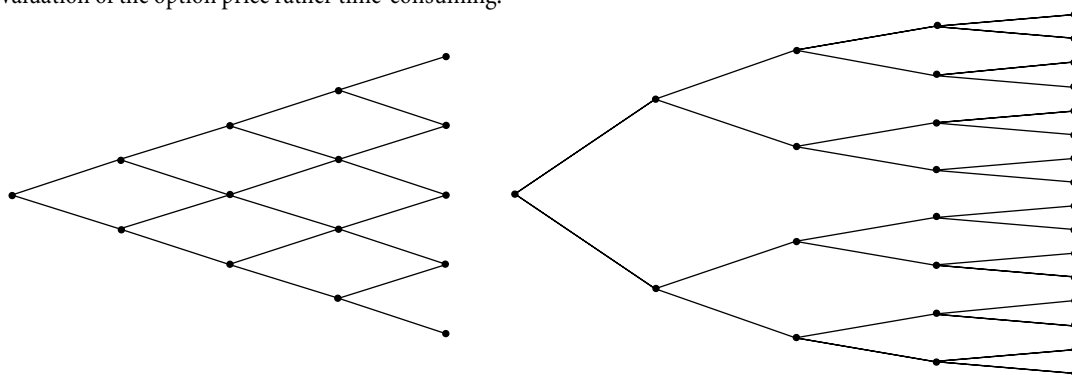


Figure 49.2: Binomial paths for vanilla option and path-dependent option.

The leading exotic option market makers now use a form of Monte Carlo or other simulation to value complex products. Again, while the price obtained will be accurate the process is time-consuming because of the number of simulations that would need to be carried out for path-dependent options.

Note that option prices are not always quoted as live prices in the unit of currency of the option. Bank customers or non-banking customers purchasing options are usually quoted actual prices, because their interest is in the cost, the strike price and the maturity period. However interbank customers and market makers are quoted volatility

prices, which is a combination of the option and the cash instrument hedge, which are both agreed between the buyer and seller.

49.4.2 Hedging considerations

A bank that makes markets in exotic options will also carefully monitor the positions for hedging purposes. The sensitivity measures reviewed in Chapter 47 are used for exotic option books as well, but the actual hedge instrument can be different. For example certain exotics are better hedged with the underlying asset rather than an offsetting option. This includes average-rate options, where the uncertainty about the value of any payoff is reduced during the life of the option, as the average rate can be estimated as the price of the underlying asset is recorded each day. As the option reaches expiry, the level of the payout is quite clear, as the average rate can be estimated quite closely. A hedge requirement on an equity Asian option could be met using the cash underlying. However a barrier option presents almost unique problems, because there is a level at which the options will be extinguished (down-and-out, up-and-out), whereas just above or below this level the option will be functioning as a vanilla option. Therefore there is a discontinuous risk associated with the option regarding the price of the underlying asset, and the delta of the option is discontinuous as well.

When hedging an option book, the delta and gamma hedge require continuous adjustment in line with the change in the price of the underlying asset. This is referred to as dynamic hedging, and involves potentially high expense because of the transaction costs associated with continually rebalancing the hedge. An approach that is used for exotic option books is *static hedging* or static options *replication*. This is the process of putting on a hedge that is composed of exchange-traded options, or a portfolio of such options, whose behaviour is approximately near to that of the exotic option or exotic option book. Let us consider static hedging for down-and-in barrier options.

For a down-and-in call option, when the strike price X is equal to the barrier price L , the option is at-the-money. To hedge a short position in this option, the market maker can sell a vanilla put option on the same underlying asset, with identical strike price and expiry date as the barrier option. At all times that the underlying asset price remains above L , then both the barrier option and the put option have no value. At the point that the price of the underlying asset reaches the barrier, or at any point subsequent to this that it is at the barrier, the values of a vanilla call and a put option on the asset are identical; to hedge the position the market maker can sell the put and buy the vanilla call. Therefore the hedge for a written down-and-in is a long position in the corresponding vanilla put option. The relationship is summarised below, for options where the strike price is equal to the barrier price.

$$C_{DI} = (X, L) = P(X) \text{ for } L = X. \quad (49.8)$$

Where $X < L$ and the strike price of the option is lower than the barrier price, a down-and-in call option is out-of-the-money. At the point that the underlying asset price hits the barrier however, the option goes live but is out-of-the-money. Therefore put-call parity will not apply, because the call and put option have non-identical strike prices. It is possible to show² that to hedge a short position in the down-and-in call, where the strike price is below the barrier, the market maker can by purchasing X/L put options that have been struck at L^2/X .

In most cases there are a number of ways that the payoff profile of an exotic option can be approximated using an exchange-traded product. Essentially hedging a derivative instrument involves putting an opposite position on in a product that mirrors the payoff profile or *boundary conditions*. In static hedging the hedge portfolio may well have a higher price than the option that is being hedged, but does not require constant re-adjustment as is the case with dynamic delta hedging. A market maker will therefore take into the account the initial set-up and transaction costs when assessing which hedge methodology to employ.

EXAMPLE 49.3 Comparison between exotic option prices

The difference between option premiums is illustrated with reference to six-month exotic call options written on the ten-year benchmark gilt in August 1999. The terms of the options are:

Asset	Ten-year benchmark (5¾% 2009)
Strike	£100
Expiry	Six months

² For example, see Briys *et al.* (1998), Chapter 18. See also Taleb (1997) for a thorough treatment of option book hedging.

Interest rate 5.625%
Volatility 6.01%

The differences between the premiums for changes in the price of the underlying are shown in Figure 49.3. As well as the vanilla call option, three other options are valued, two Asian options, and a lookback option.

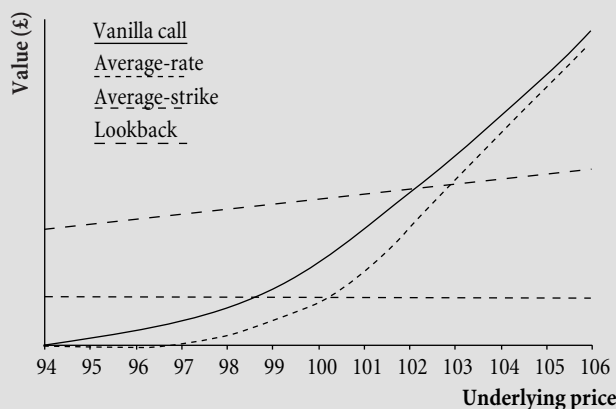


Figure 49.3: Exotic option premiums.

49.5 Using exotic options: case studies

The four examples listed here present some flavour of the types of uses to which exotic options can be put, often in conjunction with other instruments as part of a structured product. The example of the “gold producer” (in fact a jewellery company, rather than a mining company!) is particularly interesting, as it has purchased a product to hedge against adverse movements in the gold price, but at zero net premium; this is an example of a zero-cost option hedge.

49.5.1 Hedge structure for gold producer (jewellery company)

Requirement Producer of gold wishes to hedge against fall in the gold price over the next three years, and prefers to sacrifice some of the upside if the gold price rises, in order to avoid paying an up-front premium. The production volume to be hedged is 100,000 ounces per year (approx 90 million dollars total).

Structure Customer buys a strip of three gold put, dollar call options expiring annually for the next three years.

- Buy 1 year 100,000 ounce gold put, dollar call, strike \$300.00/ounce
- Buy 2 year 100,000 ounce gold put, dollar call, strike \$300.00/ounce
- Buy 3 year 100,000 ounce gold put, dollar call, strike \$300.00/ounce

To finance these they sell a strip of three knock-in gold call, dollar put options

- Sell 1 year 100,000 ounce gold call, dollar put, strike \$325.00/ounce, knock-in \$345.00/ounce
- Sell 2 year 100,000 ounce gold call, dollar put, strike \$325.00/ounce, knock-in \$345.00/ounce
- Sell 3 year 100,000 ounce gold call, dollar put, strike \$325.00/ounce, knock-in \$345.00/ounce

No net premium is paid; the price of the bought options is structured by the market maker to match that of the sold options.

Scenario	1.	<i>Gold is below \$300 per ounce at year end</i> Customer exercises gold put option and sells that year's gold production at \$300 – the customer's worst case rate scenario.
	2.	<i>Gold is between \$300 and \$345 per ounce at year end, and \$345 per ounce has not been touched at any time</i> Both bought and sold options for that year expire worthless and the customer sells that year's gold production marketplace at prevailing favourable rates.
	3.	<i>If \$345 per ounce is touched at any time</i> All unexpired gold options knock-in, which caps the customer's best rate for that year and future years to \$325 per ounce. If gold is below \$325 at year end, the customer will sell that year's gold production in the marketplace at prevailing favourable rates, but if gold is above \$325 at year end, the customer must sell the gold at \$325.
Summary		Customer is fully hedged at \$300 per ounce, with the ability to take advantage of higher prices up to \$345 per ounce.

49.5.2 Structure: Fund/Portfolio protection against market collapse

Requirement	Investor/Fund Manager has seen value of stock portfolio rise by 10% in first quarter and wishes to protect against a short term reversal. They are willing to pay premium; portfolio valuation equivalent £20 million.
Structure	Customer buys 3month put option on FT-SE index. Option Cost = 3.60% Opportunities available to investor - Buy 3m put with at the money strike (i.e. at current market rate) on a quarterly basis to synthetically create a "floor fund", i.e. portfolio with no downside. This is a high cost solution. - Buy 3m put with at the money strike for 50% of the portfolio. This has a lower cost than option 1. - Alternative: Investor with flat position, that is, cash looking to short the FT-SE, looking for retracement.
Scenarios	<i>FT-SE continues to rise</i> Put option will expire worthless. Profits re-invested to purchase further protection at higher levels. For the investor with cash – no returns. <i>FT-SE falls dramatically</i> Put option will expire in the money. Depending on the correlation of the portfolio to the INDEX and the % of the portfolio covered (i.e. 50% example) – option profits will hedge portfolio losses. For the investor with cash, option will create profitable position. <i>FT-SE remains at current levels; relatively high cost of the option will have negative impact in all structures.</i>
Summary	Investor's profits are protected (minus option costs). Particularly relevant in current market with inflated stock markets vulnerable to a sharp decline. Large profits enable investors to buy protection.

49.5.3 Structure: European equity “floor fund”

Requirement	A fund manager is looking to create fund structures that will lock-in returns, following an appreciation in asset value. (Several investment houses have launched this type of product, FT-SE linked). This enables the portfolio to “ratchet” up the base value, to avoid a significant market correction.
Structure:	Portfolio: 25% Germany, 25% France, 25% Benelux and 25% Switzerland. Fund fully invested in local equity markets indices. Initial Floor set at 95.00% fund Value for the first year. Portfolio manager will have the discretion to manage the “protection level”, which might involve selling the existing protection levels at 95.00% to purchase a higher floor. Cost of 95% protection = 4.45%. (For ease of example we will assume that the charges on the fund will amount to the remaining 0.55%. Customer buys put at 95% of current spot cost 4.45%. The customer rolls up the protection level as the fund level increases.)
Scenarios	Fund’s value appreciated by 11.50%. Fund Manager takes decision to roll up the protection level. Sell the 12 month 95. 00% put; receive 1. 15%. Buy 9 month 105. 00% put; pay 4.09%. Net gain to portfolio will be asset value 108.56, with protection “Floor Level” set at 105.00. (This procedure can be managed on a more dynamic basis, example being to sell a 115. 00% cap level on the portfolio, i.e. fund’s upside level capped at 115. 00, to generate further premium to improve the floor level.) Fund Value declines. The Fund, upto the expiry of the option (12 months), is protected at 95.00%. Any further decline in the net asset value (assuming that the components are closely correlated with the indices) will be offset by the option.
Summary	The floor and cap protection levels can be utilised by the fund manager to “iron out” market volatility.

Selected bibliography and references

- Boyle, P., Lau, S., “Bumping Up Against the Barrier with the Binomial Method”, *Journal of Derivatives* 1(4), Summer 1994, pp. 6–14.
- Briys, E., Bellalah, M., Mai, H.M., de Varenne, F., *Options, Futures and Exotic Derivatives*, John Wiley and Sons, 1998, Chapters 18, 19.
- Conze, A., Viswanathan, P., “Path Dependent Options: The Case of Lookback Options”, *Journal of Finance* 46, 1991, pp. 1893–1907.
- Geske, R., “The Valuation of Compound Options”, *Journal of Financial Economics* 7, 1979, pp. 63–81.
- Hull, J., *Futures, Options and Other Derivatives*, 3rd edition, Prentice-Hall Inc., 1997, Chapter 18.
- Hull, J., White, A., “Efficient Procedures for Valuing European and American Path-Dependent Options”, *Journal of Derivatives*, 1993, pp. 21–31.
- Kemna, A., Vorst, A., “A Pricing Method for Options Based on Average Asset Values”, *Journal of Banking and Finance* 14, March 1990, pp. 113–129.
- Rubinstein, M., “Pay Now, Choose Later”, *RISK*, February 1991.
- Rubinstein, M., “Somewhere Over the Rainbow”, *RISK*, November 1991.
- Rubinstein, M., Reiner, E., “Unscrambling the Binary Code”, *RISK*, October 1991.
- Taleb, N., *Dynamic Hedging*, Wiley, 1997.
- Turnbull, S., Wakeman, L., “A Quick Algorithm for Pricing European Average Options”, *Journal of Financial and Quantitative Analysis* 26, September 1991, pp. 377–389.

Questions and exercises

1. What is a path-dependent option?
2. How would a bank price an average-rate Asian option where the rate is an arithmetic average? Why does hedging such an option present fewer problems when using the cash underlying as a hedge instrument compared to barrier options?
3. What is a binary option?
4. What is the payoff profile of an all-or-nothing option?
5. What is the value of a down-and-in call option whose barrier is below the strike price?
6. What sort of market risk exposure do you think would be best hedged using a lookback option?
7. Write down the terms for the payoff from an average-rate option. What is the difference compared to an average-strike option.
8. What is a basket option?
9. What are the hedging implications of a short position in a down-and-in barrier option?

Part VII Approaches to Trading and Hedging

In Part VII, which is composed of only one chapter, we review some of the key issues in trading and hedging bonds in a market-making environment. It is based on the author's own experiences in the sterling bond market during 1992–1997, and some of the material dates from that period. Where necessary, the content has been revised and updated.

There is no rulebook of trading as such, but basic points can be made with regard to running a successful bond trading book. Some of these are considered here.

50 Approaches to Trading and Hedging

It is not the intention of this book to suggest trading “strategies” as such, or any particular approach to running a fixed-income market making or proprietary trading book. Rather, we will discuss certain approaches that have worked in the past and may, given the right circumstances, work again at some point in the future. It is the intention however, to focus on real-world application whilst maintaining analytical rigour. The term *trading* covers a wide range of activity. Market makers who are quoting two-way prices to market participants may be tasked with providing a customer service, building up retail and institutional volume, or they may be tasked with purely running the book at a profit and trying to maximise return on capital. The nature of the market that is traded will also impact on their approach. In a highly transparent and liquid market such as the US Treasury or the UK gilt market the price spreads are fairly narrow,¹ although increased demand has reduced this somewhat in both markets. However this means that opportunities for profitable trading as a result of mispricing of individual securities, whilst not completely extinct, are rare. It is much more common for traders in such markets to take a view on *relative value* trades, such as the yield spreads between individual securities or the expected future shape of the yield curve. This is also called *spread trading*. A large volume of trading on derivatives exchanges is done for hedging purposes, but speculative trading is also prominent. Very often bond and interest-rate traders will punt using futures or options contracts, based on their view of market direction. Ironically, market makers who have a low level of customer business, perhaps because they are newcomers to the market, for historical reasons or because they do not have the appetite for risk that is required to service the high quality customers, tend to speculate on the futures exchanges to relieve tedium, often with unfortunate results.

Speculative trading is undertaken on the basis of the views of the trader, desk or head of the department. This view may be an “in-house” view, for example the collective belief of the economics or research department, or the individual trader’s view, which will be formulated as a result of *fundamental analysis* and *technical analysis*. The former is an assessment of macroeconomic and microeconomic factors affecting not just the specific bond market itself but the economy as a whole. Those running corporate debt desks will also concentrate heavily on individual sectors and corporations and their wider environment, because the credit spread, and what drives the credit spread, of corporate bonds is of course key to the performance of the bonds. Technical analysis or *charting* is a discipline in

¹ In fact, in the late 1990s spreads in the gilt market were beginning to widen as excess demand over supply, particularly at the long end of the yield curve, drove down yields and reduced liquidity. In the Treasury market at the start of 2000 the yield curve had inverted, with the yield on the long bond at 6.05% in February that year, down over 70 basis points from the start of the year. The announcement by the US Treasury that it would buy back over \$30 billion of debt in the year also led to increased demand at the long end, helping to depress yields. The volatility level in the market was at a two-year high at that time. These developments in the two markets have led to wider price quotes and lower liquidity. A sustained public sector deficit has many implications for the debt markets, if governments start to repay national debt and cease issuing securities. This is an important topic which is currently the subject of some debate. A significant reduction in government debt levels, while advantageous in many respects, will pose new problems. This is because government bonds play an important part in the financial systems of many countries. In the first instance, government bonds are used as the benchmark against which many other instruments are priced, such as corporate bonds. An illiquid market in government debt could have serious implications for the corporate bond markets, with investors possibly becoming reluctant to invest in corporate paper unless yield levels rise. Derivatives may also suffer from pricing problems, particularly bond futures contracts. In addition, while long-horizon institutional investors such as pension funds may find themselves short of investment products, many central banks and sovereign governments are big holders of securities such as US Treasuries, gilts and bunds. A shortage of supply in these instruments, particularly Treasuries, might have implications for all these investors unless an alternative instrument is made available. The continuing inverted yield curve in the UK, which dated from July 1997, and the inversion of the US curve in February 2000, is put down partly to shortage of long-dated government stock. The OECD, as reported in *The Economist* (12 February 2000) has suggested a policy whereby governments maintain a minimum level of gross public debt, with this minimum being an amount sufficient to maintain bond market liquidity. This may not be a practical solution for large economies however, especially that of the United States. The issue of alternative benchmarks is currently being researched by the author.

its own right, and has its adherents. It is based on the belief that over time the patterns displayed by a continuous time series of asset prices will repeat themselves. Therefore detecting patterns should give a reasonable expectation of how asset prices should behave in the future. Many traders use a combination of fundamental and technical analysis, although chartists often say that for technical analysis to work effectively, it must be the only method adopted by the trader. A review of technical analysis is presented in Chapter 63.

In this chapter we introduce some common methods and approaches, and some not so common, that might be employed on a fixed interest desk.

50.1 Futures trading

Trading with derivatives is often preferred, for both speculative or hedging purposes, to trading in the cash markets mainly because of the liquidity of the market and the ease and low cost of undertaking transactions. The essential features of futures trading are volatility and leverage. To establish a futures position on an exchange, the level of margin required is very low proportional to the notional value of the contracts traded. For speculative purposes traders often carry out open, that is uncovered trading, which is a directional bet on the market. So therefore if a trader believed that short-term sterling interest rates were going to fall, they could buy a short sterling contract on LIFFE. This may be held for under a day, in which case if the price rises the trader will gain, or for a longer period, depending on their view. The tick value of a short sterling contract is £10, so if they bought one lot at 92.75 (that is, $100 - 92.75$ or 7.25%) and sold it at the end of the day for 98.85 they made a profit of £100 on their one lot, from which brokerage will be subtracted. The trade can be carried out with any futures contract; the same idea could be carried out with a cash market product or a FRA, but the liquidity, narrow price spread and the low cost of dealing make such a trade easier on a futures exchange. It is much more interesting however to carry out a spread trade on the difference between the rates of two different contracts. Consider Figures 50.1 and 50.2 which relate to the prices for the LIFFE short-sterling futures contract on 22 March 1999. The specification for this contract was summarised in Chapter 35.

Date	22/03/1999 LIFFE SHORT STERLING CONTRACT									
Term	1w	1m	2m	3m	4m	5m	6m	9m	1y	
Libor	5.561	5.439	5.407	5.359	5.344	5.321	5.304	5.295	5.300	
Futures	M99	U99	Z99	H00	M00	U00	Z00	H01	M01	U01
Price	94.88	94.97	94.75	94.85	94.77	94.68	94.56	94.58	94.56	94.57
Rate %	5.12	5.03	5.25	5.15	5.23	5.32	5.44	5.42	5.44	5.43
Expiry	16-Jun	15-Sep	15-Dec	15-Mar	21-Jun	20-Sep	20-Dec	21-Mar	20-Jun	19-Sep
Days	86	177	268	359	457	548	639	730	821	912
Yield curve %				1y				2y		2.5y
Cash	5.367	5.307	5.295	5.300	5.243	5.252	5.262	5.270	5.287	5.303
Futures	5.367	5.273	5.234	5.289	5.214	5.223	5.242	5.271	5.291	5.310

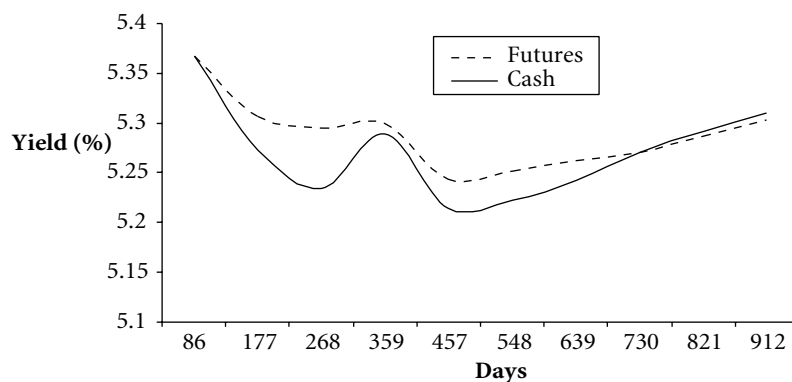


Figure 50.1: LIFFE short-sterling contract analysis 22 March 1999.

Most futures exchanges use the designatory letters H, M, U and Z to refer to the contract months for March, June, September and December. So the June 1999 contract is denoted by “M99”. From Chapter 6 we know that forward rates can be calculated for any term, starting on any date. In Figure 50.1 we see the future prices on that day, and the interest rate that the prices imply. The “stub” is the term for the interest rate from today to the expiry of the first futures contract, which is called the *front month* contract (in this case the front month contract is the June 1999 contract). Figure 50.2 lists the forward rates from the spot date to six months, one year and so on. It is possible to trade a strip of contracts to replicate any term, out to the maximum maturity of the contract. This can be done for hedging or speculative purposes. Note from Figure 50.1 that there is a spread between the cash curve and the futures curve. A trader can take positions on cash against futures, but it is easier to transact only on the futures exchange.

Date 22/03/1999 LIFFE SHORT STERLING CONTRACT										
Date	Days	Contract	Price	Rate	6m-fwd	1y-fwd	1.5y-fwd	2y-fwd	3y-fwd	4y-fwd
Spot	86	Stub	94.6317	5.3683	5.268	5.292	5.224	5.271	5.337	5.356
16-Jun-99	91	M99	94.88	5.1200	5.107	5.239	5.216	5.278	5.341	5.353
15-Sep-99	91	U99	94.97	5.0300	5.173	5.267	5.269	5.318	5.363	
15-Dec-99	91	Z99	94.75	5.2500	5.233	5.341	5.334	5.369	5.390	
15-Mar-00	98	H00	94.85	5.1500	5.223	5.390	5.366	5.392	5.399	
21-Jun-00	91	M00	94.77	5.2300	5.310	5.461	5.416	5.428	5.415	
20-Sep-00	91	U00	94.68	5.3200	5.416	5.515	5.452	5.447		
20-Dec-00	91	Z00	94.56	5.4400	5.467	5.544	5.467	5.452		
21-Mar-01	91	H01	94.58	5.4200	5.467	5.544	5.457	5.440		
20-Jun-01	91	M01	94.56	5.4400	5.472	5.541	5.447	5.428		
19-Sep-01	91	U01	94.57	5.4300	5.472	5.526	5.431			

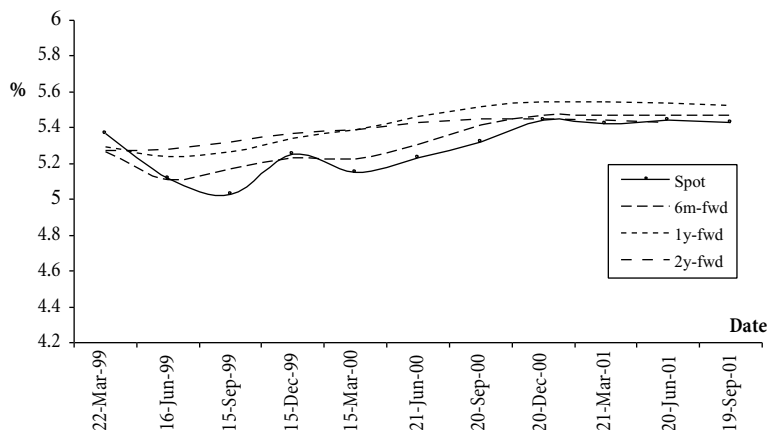


Figure 50.2: LIFFE short-sterling forward rates analysis 22 March 1999.

Short-term money market interest rates often behave independently of the yield curve as a whole. A money markets trader may be aware of cash market trends, for example an increased frequency of borrowing at a certain point of the curve, as well as other market intelligence that suggests that one point of the curve will rise or fall relative to others. One way to exploit this view is to run a position in a cash instrument such as CD against a futures contract, which is a *basis spread* trade.

However the best way to trade on this view is to carry out a spread trade, shorting one contract against a long position in another trade. Consider Figure 50.1; if we feel that three-month interest rates in June 2000 will be lower than where they are implied by the futures price today, but that September 2000 rates will be higher, we will buy the M00 contract and short the U00 contract. This is not a market directional trade, rather a view on the relative spread between two contracts. The trade must be carried out in equal weights, for example 100 lots of the June against 100

lots of the September.² If the rates do move in the direction that the trader expects, the trade will generate a profit. There are similar possibilities available from an analysis of Figure 50.2, depending on our view of forward interest rates.

Spread trading carries a lower margin requirement than open position trading, because there is no directional risk in the trade. It is also possible to arbitrage between contracts on different exchanges. If the trade is short the near contract and long the far contract, so the opposite of our example, this is known as *buying the spread* and the trader believes the spread will widen. The opposite is *shorting the spread* and is undertaken when the trader believes the spread will narrow. Note that the difference between the two price levels is not limitless, because the theoretical price of a futures contract provides an upper limit to the size of the spread or the basis. The spread or the basis cannot exceed the cost of carry, that is the net cost of buying the cash security today and then delivering it into the futures market on the contract expiry. The same principle applies to short-dated interest-rate contracts; the net cost is the difference between the interest cost of borrowing funds to buy the “security” and the income accruing on the security while it is held before delivery. The two associated costs for a short-sterling spread trade are the notional borrowing and lending rates from having bought one and sold another contract. If the trader believes that the cost-of-carry will decrease they could sell the spread to exercise this view.

The trader may have a longer time horizon and trade the spread between the short-term interest-rate contract and the long bond future. This is usually carried out only by proprietary traders, because it is unlikely that one person would be trading both three-month and 10-year (or 20-year, depending on the contract specification) interest rates. A common example of such a spread trade is a yield curve trade. If a trader believes that the sterling yield curve will steepen or flatten between the three-month and the ten-year terms, they can buy or sell the spread by using the LIFFE short-sterling contract and the long gilt contract. To be first-order risk neutral however the trade must be duration-weighted, as one short-sterling contract is not equivalent to one gilt contract. The tick value of both contracts is £10, although the gilt contract represents £100 000 of a notional gilt and the short-sterling contract represents a £500 000 time deposit. We use (50.1) to calculate the hedge ratio, with £1000 being the value of a 1% change in the value of both contracts.

$$h = \frac{(100 \times \text{tick}) \times P_b^f \times D}{(100 \times \text{tick}) \times P_{\text{short ir}}^f} \quad (50.1)$$

where

- tick is the tick value of the contract
- D is the duration of the bond represented by the long bond contract
- P_b^f is the price of the bond futures contract
- $P_{\text{short ir}}^f$ is the price of the short-term deposit contract.

The notional maturity of a long bond contract is always given in terms of a spread, for example for the long gilt it is 8¾ – 13 years. Therefore in practice one would use the duration of the cheapest-to-deliver bond.

A *butterfly spread* is a spread trade that involves three contracts, with the two spreads between all three contracts being traded. This is carried out when the middle contract appears to be mispriced relative to the two contracts either side of it. The trader may believe that one or both of the outer contracts will move in relation to the middle contract; if the belief is that only one of these two will shift relative to the middle contract, then a butterfly will be put on if the trader is not sure which of these will adjust. For example, consider Figure 50.1 again. The prices of the front three contracts are 94.88, 94.97 and 94.75. A trader may feel that the September contract is too low, and has a spread of +9 basis points to the June contract, and –22 basis points to the December contract. The trader feels that the September contract will rise, but will that be because June and December prices fall or because the September price will rise? Instead of having to answer this question, all the trader need believe is that the Jun–Sep spread will widen and the Sep–Dec spread will narrow. To put this view into effect, the trader puts on a butterfly spread, which is equal

² The author would particularly like to thank Peter Matthews, now with ABN Amro Securities (UK) Ltd, and Ed Hardman who was in the short sterling booth at GNI on LIFFE, with whom I’ve sadly lost contact, for information on this type of trading back in 1994.

to the Sep–Dec spread minus the Jun–Sep spread, which they expect to narrow. Therefore the trader buys the Jun–Sep spread and sells the Sep–Dec spread, which is also known as *selling the butterfly spread*.

50.2 Yield curves and relative value

Bond market participants take a keen interest in the yield curve, both cash and zero-coupon (spot) yield curves. In markets where an active zero-coupon bond market exists, much analysis is undertaken into the relative spreads between derived and actual zero-coupon yields. In this section we review some of the yield curve analysis used in the market.

50.2.1 The determinants of government bond yields

Market makers in government bond markets will analyse various factors in the market in deciding how to run their book. Customer business apart, decisions to purchase or sell securities will be a function of their views on:

- market direction itself, that is the direction in which short-term and long-term interest rates are headed;
- which maturity point along the entire term structure offers the best value;
- which specific issue within a particular maturity point offers the best value.

All three areas are related but will react differently to certain pieces of information. A report on the projected size of the government's budget deficit for example, will not have much affect on two-year bond yields, whereas if the expectations came as a surprise to the market it could have an adverse on long-bond yields. The starting point for analysis is of course the yield curve, both the traditional coupon curve plotted against duration and the zero-coupon curve. Figure 50.3 illustrates the traditional yield curve for gilts in October 1999.

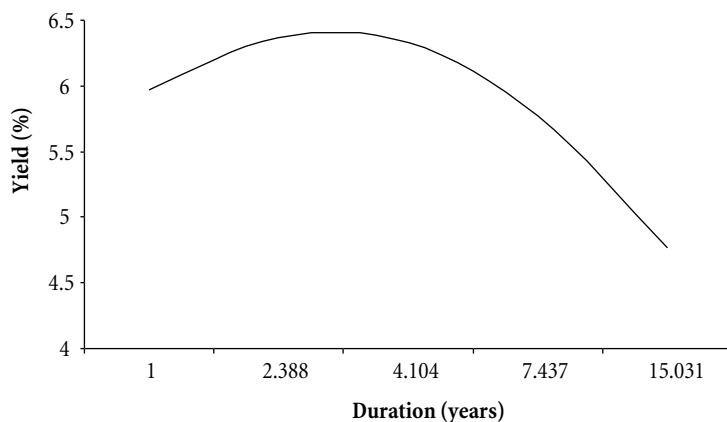


Figure 50.3: Yield and duration of gilts, 21 October 1999.

For a first-level analysis, many market practitioners will go no further than Figure 50.3. An investor who had no particular view on the future shape of the yield curve or the level of interest rates may well adopt a neutral outlook and hold bonds that have a duration that matches their investment horizon. If they believed interest rates were likely to remain stable for a time, they might hold bonds with a longer duration in a positive sloping yield curve environment, and pick up additional yield but with higher interest-rate risk. Once the decision has been made on which part of the yield curve to invest in or switch in to, the investor must decide on the specific securities to hold, which then brings us on to relative value analysis. For this the investor will analyse specific sectors of the curve, looking at individual stocks. This is sometimes called looking at the “local” part of the curve.

An assessment of a local part of the yield curve will include looking at other features of individual stocks in addition to their duration. This recognises that the yield of a specific bond is not only a function of its duration, and that two bonds with near-identical duration can have different yields. The other determinants of yield are liquidity of the bond and its coupon. To illustrate the effect of coupon on yield consider Table 50.1. This shows that, where the duration of a bond is held roughly constant, a change in coupon of a bond can have a significant effect on the bond's yield.

Coupon	Maturity	Duration	Yield
8%	20-Feb-02	1.927	5.75%
12%	5-Feb-02	1.911	5.80%
10%	20-Jun-10	7.134	4.95%
6%	1-Jul-10	7.867	4.77%

Table 50.1: Duration and yield comparisons for bonds in a hypothetical inverted curve environment.

In the case of the long bond, an investor could under this scenario both shorten duration and pick up yield, which is not the first thing that an investor might expect. However an anomaly of the markets is that, liquidity issues aside, the market does not generally like high coupon bonds, so they usually trade cheap to the curve.

The other factors affecting yield are supply and demand, and liquidity. A shortage of supply of stock at a particular point in the curve will have the affect of depressing yields at that point. A reducing public sector deficit is the main reason why such a supply shortage might exist. In addition as interest rates decline say ahead of or during a recession, the stock of high coupon bonds increases, as the newer bonds are issued at lower levels, and these “outdated” issues can end up trading at a higher yield. Demand factors are driven primarily by the investor’s views of the country’s economic prospects, but also by government legislation, for example the Minimum Funding Requirement in the UK compels pension funds to hold a set minimum amount of their funds in long-dated gilts, which has the effect of permanently keeping demand high.³

Liquidity often results in one bond having a higher yield than other, despite both having similar durations. Institutional investors prefer to hold the benchmark bond, which is the current two-year, five-year, ten-year or thirty-year bond and this depresses the yield on the benchmark bond. A bond that is liquid also has a higher demand, thus a lower yield, because it is easier to convert into cash if required. This can be demonstrated by valuing the cash flows on a six-month bond with the rates obtainable in the Treasury bill market. We could value the six-month cash flows at the six-month bill rate. The lowest obtainable yield in virtually every market⁴ is the T-bill yield, therefore valuing a six-month bond at the T-bill rate will produce a discrepancy between the observed price of the bond and its theoretical price implied by the T-bill rate; as the observed price will be lower. The reason for this is simple: because the T-bill is more readily realisable into cash at any time, it trades at a lower yield than the bond, even though the cash flows fall on the same day.

We have therefore determined that a bond’s coupon and liquidity level, as well as its duration, will affect the yield at which it trades. These factors can be used in conjunction with other areas of analysis, which we look at next, when deciding which bonds carry relative value over others.

50.2.2 Characterising the complete term structure

As many readers would have gathered, the yield versus duration curve illustrated in Figure 50.3 is an ineffective technique with which to analyse the market.

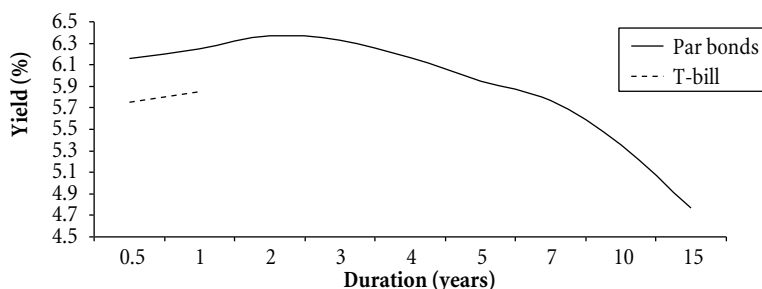


Figure 50.4: T-bill and par yield curve, October 1999.

³ The MFR is currently under review in the UK and changes to the requirements are expected during 2001.

⁴ The author is not aware of any market where there is a yield lower than its shortest-maturity T-bill yield, but that does not mean such a market doesn’t exist!

This is because it does not highlight any characteristics of the yield curve other than its general shape; this does not assist in the making of trading decisions. To facilitate a more complete picture, we might wish to employ the technique described here. Figure 50.4 shows the bond par yield curve⁵ and T-bill yield curve for gilts in October 1999. Figure 50.5 shows the difference between the yield on a bond with a coupon that is 100 basis points below the par yield level, and the yield on a par bond. The other curve in Figure 50.5 shows the level for a bond with a coupon that is 100 basis points above the par yield. These two curves show the “low coupon” and “high coupon” yield spreads. Using the two figures together, an investor can see the impact of coupons, the shape of the curve and the effect of yield on different maturity points of the curve.

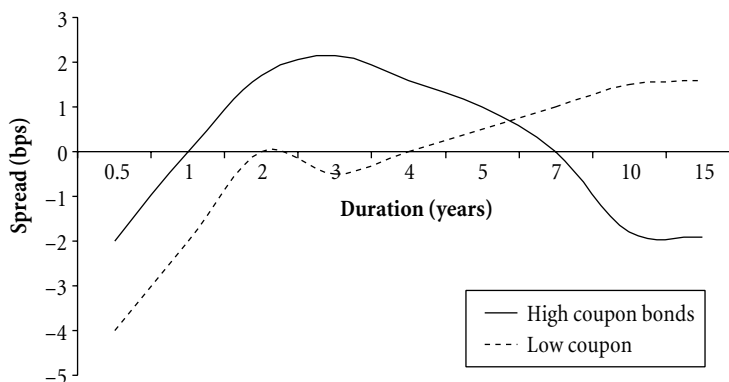


Figure 50.5: Structure of bond yields, October 1999.

50.2.3 Identifying relative value in government bonds

Constructing a zero-coupon yield curve provides the framework within which a market participant can analyse individual securities. In a government bond market, there is no credit risk consideration (unless it is an emerging market government market), and therefore no credit spreads to consider. There are a number of factors that can be assessed in an attempt to identify relative value.

The objective of much of the analysis that occurs in bond markets is to identify value, and identifying which individual securities should be purchased and which sold. At the overview level, this identification is a function of whether one thinks interest rates are going to rise or fall. At the local level though, the analysis is more concerned with a specific sector of the yield curve, whether this will flatten or steepen, whether bonds of similar duration are trading at enough of a spread to warrant switching from one into another. The difference in these approaches is one of identifying which stocks have absolute value, and which have relative value. A trade decision based on the expected direction of interest rates is based on assessing absolute value, whether interest rates themselves are too low or too high. Yield curve analysis is more a matter of assessing relative value. On (very!) rare occasions, this process is fairly straightforward, for example if the three-year bond is trading at 5.75% when two-year yields are 5.70% and four-year yields are at 6.15%, the three-year would appear to be overpriced. However this is not really a real-life situation. Instead, a trader might find himself assessing the relative value of the three-year bond compared to much shorter- or longer-dated instruments. That said, there is considerable difference between comparing a short-dated bond to other short-term securities and comparing say, the two-year bond to the thirty-year bond. Although it looks like it on paper, the space along the x-axis should not be taken to imply that the smooth link between one-year and five-year bonds is repeated from the five-year out to the thirty-year bonds. It is also common for the very short-dated sector of the yield curve to behave independently of the long end.

One method used to identify relative value is to quantify the coupon effect on the yields of bonds. The relationship between yield and coupon is given by (50.2):

$$rm = rm_p + c \cdot \max(C_{PD} - rm_p, 0) + d \cdot \min(C_{PD} - rm_p, 0) \quad (50.2)$$

⁵ See Chapter 6 for a discussion of the par yield curve.

where

- rm is the yield on the bond being analysed
- rm_p is the yield on a par bond of specified duration
- C_{PD} is the coupon on an arbitrary bond of similar duration to the part bond

and c and d are coefficients. The coefficient c reflects the effect of a high coupon on the yield of a bond. If we consider a case where the coupon rate exceeds the yield on the similar-duration par bond ($C_{PD} > rm_p$), (50.2) reduces to (50.3):

$$rm = rm_p + c \cdot (C_{PD} - rm_p). \quad (50.3)$$

Equation (50.3) specifies the spread between the yield on a high coupon bond and the yield on a par bond as a linear function of the spread between the first bond's coupon and the yield and coupon of the par bond. In reality this relationship may not be purely linear; for instance the yield spread may widen at a decreasing rate for higher coupon differences. Therefore (50.3) is an approximation of the effect of a high coupon on yield where the approximation is more appropriate for bonds trading close to par. The same analysis can be applied to bonds with coupons lower than the same-duration par bond.

The value of a bond may be measured against comparable securities or against the par or zero-coupon yield curve. In certain instances the first measure may be more appropriate when for instance, a low coupon bond is priced expensive to the curve itself but fair compared to other low coupon bonds. In that case the overpricing indicated by the par yield curve may not represent unusual value, rather a valuation phenomenon that was shared by all low coupon bonds. Having examined the local structure of a yield curve, the analysis can be extended to the comparative valuation of a group of similar bonds. This is an important part of the analysis, because it is particularly informative to know the cheapness or dearth of a single stock compared to the whole yield curve, which might be somewhat abstract. Instead we would seek to identify two or more bonds, one of which was cheap and the other dear, so that we might carry out an outright switch between the two, or put on a spread trade between them. Using the technique we can identify excess positive or negative yield spread for all the bonds in the term structure. This has been carried out for our five gilts, together with other less liquid issues as at October 1999 and the results are summarised in Table 50.2.

Coupon	Maturity	Duration	Yield %	Excess yield spread (bp)
8%	07/12/2000	1.072	5.972	−1.55
10%	26/02/2001	1.2601	6.051	4.5
7%	07/06/2002	2.388	6.367	−1.8
5%	07/06/2004	4.104	6.327	−3.8
6.75%	26/11/2004	4.233	6.351	2.7
5.75%	07/12/2009	7.437	5.77	−4.7
6.25%	25/11/2010	7.957	5.72	1.08
6%	07/12/2028	15.031	4.77	−8.7

Table 50.2: Yields and excess yield spreads for selected gilts, 22 October 1999.

From the table as we might expect the benchmark securities are all expensive to the par curve, and the less liquid bonds are cheap. Note that the 6.25% 2010 appears cheap to the curve, but the 5.75% 2009 offers a yield pick-up for what is a shorter-duration stock; this is a curious anomaly and one that had disappeared a few days later.⁶

What this section has introduced is the concept of relative value for individual securities, and how the simple duration/yield analysis can be extended to assess other determinants of a bond's yield. Other types of analysis, for example assessing the value of coupon bonds relative to the zero-coupon yield, have been assessed elsewhere in the book. For example, coupon stripping was examined in Chapter 11. We now look at the issues involved in putting on a spread trade.

⁶ In other words, we've missed the opportunity! This analysis used mid-prices, which would not be available in practice.

50.3 Yield spread trades

In the earlier section on futures trading, we introduced the concept of spread trading, which are not market directional trades but rather the expression of a view point on the shape of a yield curve, or more specifically the spread between two particular points on the yield curve. Generally there is no analytical relationship between changes in a specific yield spread and changes in the general level of interest rates. That is to say, the yield curve can flatten when rates are both falling or rising, and equally may steepen under either scenario as well. The key element of any spread trade is that it is structured so that a profit (or any loss) is made only as a result of a change in the spread, and not due to any change in overall yield levels. That is, spread trading eliminates market directional or first-order market risk.

50.3.1 Bond spread weighting

Table 50.3 shows data for our selection of gilts but with additional information on the basis point value (BPV) for each point. This is also known as the “dollar value of a basis point” or DV01.

Coupon	Maturity	Duration	Yield %	Price	BPV
8%	07/12/2000	1.072	5.972	102.17	0.01095
10%	26/02/2001	1.2601	6.051	105.01	0.01880
7%	07/06/2002	2.388	6.367	101.5	0.02410
5%	07/06/2004	4.104	6.327	94.74	0.03835
6.75%	26/11/2004	4.233	6.351	101.71	0.03980
5.75%	07/12/2009	7.437	5.77	99.84	0.07584
6.25%	25/11/2010	7.957	5.72	104.3	0.07526
6%	07/12/2028	15.031	4.77	119.25	0.17834

Table 50.3: Bond basis point value, 22 October 1999.

If a trader believed that the yield curve was going to flatten, but had no particular strong feeling about whether this flattening would occur in an environment of falling or rising interest rates, and thought that the flattening would be most pronounced in the two-year versus ten-year spread, they could put on a spread consisting of a short position in the two-year and a long position in the ten-year. This spread must be duration-weighted to eliminate first-order risk. At this stage we must point out, and it is important to be aware of, that fact that basis point values, which are used to weight the trade, are based on modified duration measures. From Chapters 7–10 we know that this measure is an approximation, and will be inaccurate for large changes in yield. Therefore the trader must monitor the spread to ensure that the weights are not going out of line, especially in a volatile market environment.

To weight the spread, we use the ratios of the BPVs of each bond to decide on how much to trade. In our example, assume the trader wants to purchase £10 million of the ten-year. In that case he must sell $((0.07584/0.02410) \times 10\,000\,000)$ or £31 468 880 of the two-year bond. It is also possible to weight a trade using the bonds' duration values, but this is rare. It is common practice to use the BPV.

The payoff from the trade will depend on what happens to the two-year versus ten-year spread. If the yields on both bonds move by the same amount, there will be no profit generated, although there will be a funding consideration. If the spread does indeed narrow, the trade will generate profit. Note that disciplined trading calls for both an expected target spread as well as a fixed time horizon. So for example, the current spread is 59.7 basis points; the trader may decide to take the profit if the spread narrows to 50 basis points, with a three-week horizon. If at the end of three weeks the spread has not reached the target, the trader should unwind the position anyway, because that was their original target. On the other hand what if the spread has narrowed to 48 basis points after one week and looks like narrowing further – what should the trader do? Again, disciplined trading suggests the profit should be taken. If contrary to expectations the spread starts to widen, if it reaches 64.5 basis points the trade should be unwound, this “stop-loss” being at the half-way point of the original profit target.

The financing of the trade in the repo markets is an important aspect of the trade, and will set the trade's break-even level. If the bond being shorted (in our example, the two-year bond) is *special*, this will have an adverse impact on the financing of the trade. The repo considerations were reviewed in Chapter 34.

50.3.2 Types of bond spreads

A bond spread has two fundamental characteristics; in theory there should be no P/L effect due to a general change in interest rates, and any P/L should only occur as a result of a change in the specific spread being traded. Most bond spread trades are yield curve trades where a view is taken on whether a particular spread will widen or narrow. Therefore it is important to be able to identify which sectors of the curve to sell. Assuming that a trader is able to transact business along any part of the yield curve, there are a number of factors to consider. In the first instance, the historic spread between the two sectors of the curve. To illustrate in simplistic fashion, if the 2–10 year spread has been between 40 and 50 basis points over the last six months but very recently has narrowed to less than 35 basis points, this may indicate imminent widening. Other factors to consider are demand and liquidity for individual stocks relative to others, and any market intelligence that the trader gleans. If there has been considerable customer interest on certain stocks relative to others, because investors themselves are switching out of certain stocks and into others, this may indicate a possible yield curve play. It is a matter of individual judgement.

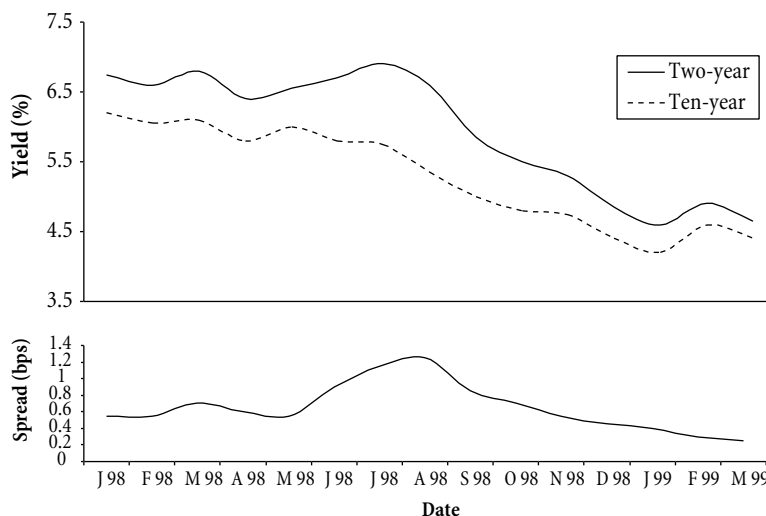


Figure 50.6: 2-year and 10-year spread, UK gilt market March 1999.

Source: Bloomberg.

An historical analysis requires that the trader identify some part of the yield curve within which he expects to observe a flattening or steepening. It is of course entirely possible that one segment of the curve will flatten while another segment is steepening, in fact this phenomenon is quite common. This reflects the fact that different segments respond to news and other occurrences in different ways.

A more exotic type of yield curve spread is a *curvature* trade. Consider for example a trader who believes that three-year bonds will outperform on a relative basis, both two-year and five-year bonds. That is, he believes that the two-year/three-year spread will narrow relative to the three-year/five-year spread, in other words that the curvature of the yield curve will decrease. This is also known as a *butterfly/barbell* trade. In our example the trader will buy the three-year bond, against short sales of both the two-year and the five-year bonds. All positions are duration-weighted. The principle is exactly the same as the butterfly trade we described in the previous section on futures trading.

50.4 Hedging bond positions

Hedging is a straightforward concept to understand or describe, however it is very important that it is undertaken as accurately as possible. Therefore the calculation of a hedge is critical. A hedge is a position in a cash or off-balance sheet instrument that removes the market risk exposure of another position. For example a long position in ten-year bonds can be hedged with a short position in 20-year bonds, or with futures contracts. That is the straightforward part; the calculation of the exact amount of the hedge is where complexities can arise. In this section we review the basic concepts of hedging, and a case study at the end illustrates some of the factors that must be considered.

50.4.1 Simple hedging approach

The hedge calculation that first presents itself is the duration-weighted approach. From the sample of gilts in Table 50.3, it is possible to calculate the amount of one bond required to hedge an amount of any other bond, using the ratio of the BPVs. This approach is very common in the market; however it suffers from two basic flaws that hinder its effectiveness. First, the approach assumes implicitly comparable volatility of yields on the two bonds, and secondly it also assumes that yield changes on the two bonds are highly correlated. Where one or both of these factors do not apply, the effectiveness of the hedge will be compromised.

The assumption of comparable volatility becomes increasingly unrealistic the more the bonds differ in terms of market risk and market behaviour. Consider a long position in two-year bonds hedged with a short-position in five-year bonds. Using the bonds from Table 50.3, if we had a position of £1 million of the two-year, we would short £628,422 of the five-year. Even if we imagine that yields between the two bonds are perfectly correlated, it may well be that the amount of yield change is different because the bonds have different volatilities. For example if the yield on the five-year bond changes only by half the amount that the two-year does, if there was a five basis point rise in the two-year, the five-year would have risen only by 2.5 basis points. This would indicate that the yield volatility of the two-year bond was twice that of the five-year bond. This suggests that a hedge calculation that matched nominal amounts, due to BPV, on the basis of an equal change in yield for both bonds would be incorrect. In our illustration, the short position in the five-year bond would be effectively hedging only half of the risk exposure of the two-year position.

The implicit assumption of perfectly correlated yield changes can also lead to inaccuracy. Across the whole term structure, it is not always the case that bond yields are even positively correlated all the time (although most of the time there will be a close positive correlation). Therefore, using our illustration again, imagine that the two-year and the five-year bonds possess identical yield volatilities, but that changes in their yields are uncorrelated. This means that knowing that the yield on the two-year bond rose or fell by one basis point does not tell us anything about the change in the yield on the five-year bond. If yield changes between the two bonds are indeed uncorrelated, this means that the five-year bonds cannot be used to hedge two-year bonds, at least not with accuracy.

50.4.2 Hedge analysis

From the foregoing we note that there are at least three factors that will impact the effectiveness of a bond hedge; these are the basis point value, the yield volatility of each bond and the correlation between changes in the two yields of a pair of bonds. Considering volatilities and correlations, Table 50.4 shows the standard deviations and correlations of weekly yield changes for a set of gilts during the nine months to October 1999. The standard deviation of weekly yield changes was in fact highest for the short-date paper, and actually declined for longer-dated paper. From the table we also note that changes in yield were imperfectly correlated. We expect correlations to be highest for bonds in the same segments of the yield curve, and to decline between bonds that are in different segments. This is not surprising, and indeed two-year bond yields are more positively correlated with five-year bonds and less so with 30-year bonds.

	Segment					
	2-year	3-year	5-year	10-year	20-year	30-year
Volatility (bp)	19.3	19.5	20.2	20.0	20.1	20.3
Correlation						
2-year	1.000	0.973	0.949	0.919	0.887	0.879
3-year	0.973	1.000	0.961	0.935	0.901	0.889
5-year	0.949	0.961	1.000	0.968	0.951	0.945
10-year	0.919	0.935	0.968	1.000	0.981	0.983
20-year	0.887	0.901	0.951	0.981	1.000	0.987
30-year	0.879	0.889	0.945	0.983	0.987	1.000

Table 50.4: Yield volatility and correlations, selected gilts October 1999.

We can use the standard relationship for correlations and the effect of correlation to adjust a hedge. Consider two bonds with nominal values M_1 and M_2 ; if the yields on these two bonds changes by Δr_1 and Δr_2 the net value of the change in position is given by:

$$\Delta PV = M_1 BPV_1 \Delta r_1 + M_2 BPV_2 \Delta r_2. \quad (50.4)$$

The uncertainty of the change in the net value of a two-bond position is dependent on the nominal values, the volatility of each bond and the correlation between these yield changes. Therefore for a two-bond position we set the standard deviation of the change in the position as (50.5):

$$\sigma_{pos} = \sqrt{M_1^2 BPV_1^2 \sigma_1^2 + M_2^2 BPV_2^2 \sigma_2^2 + 2M_1 M_2 BPV_1 BPV_2 \sigma_1 \sigma_2 \rho} \quad (50.5)$$

where ρ is the correlation between the yield volatilities of bonds 1 and 2. We can rearrange (50.5) to set the optimum hedge value for any bond using (50.6):

$$M_2 = -\frac{\rho BPV_1 \sigma_1}{BPV_2 \sigma_2} M_1 \quad (50.6)$$

so that M_2 is the nominal value of any bond used as a hedge given any nominal value M_1 of the first bond, and using each bond's volatility and the correlation. The derivation of (50.6) is given in the appendix. A lower correlation leads to a smaller hedge position, because where yield changes are not closely related, this implies greater independence between yield changes of the two bonds. In a scenario where the standard deviation of two bonds is identical, and the correlation between yield changes is 1, (50.6) reduces to:

$$M_2 = \frac{BPV_1}{BPV_2} M_1 \quad (50.7)$$

which is the traditional hedge calculation based solely on basis point values.

50.5 Introduction to bond analysis using spot rates and forward rates in continuous time⁷

This section analyses further the relationship between spot and forward rates and discusses briefly how this can be applied in bond analysis.

50.5.1 The spot and forward rate relationship

In the discussion to date, we have assumed discrete time intervals and interest rates in discrete time. Here we consider the relationship between spot and forward rates in continuous time. For this we assume the mathematical convenience of a continuously compounded interest rate.

The rate r is compounded using e^r and an initial investment M earning $r(t, T)$ over the period $T - t$, initial investment at time t and for maturity at T , where $T > t$, would have a value of $Me^{r(t, T)(T-t)}$ on maturity.⁸ If we denote the initial value M_t and the maturity value M_T then we can state $M_t e^{r(t, T)(T-t)} = M_T$ and therefore the continuously compounded yield, defined as the continuously compounded interest rate $r(t, T)$ can be shown to be

$$r(t, T) = \frac{\log(M_T/M_t)}{T - t}. \quad (50.8)$$

We can then formulate a relationship between the continuously compounded interest rate and yield. It can be shown that

$$M_T = M_t e^{\int_t^T r(s) ds} \quad (50.9)$$

⁷ This section does not predate the book, unlike the rest of this chapter, and was written specifically for the book. It follows the analysis given in Jarrow (1996), Neftci (2000) and other recent texts.

⁸ e is the mathematical constant 2.7182818... and it can be shown that an investment of £1 at time t will have grown to e on maturity at time T (during the period $T - t$) if it is earning an interest rate of $1/(T - t)$ continuously compounded.

where $r(s)$ is the instantaneous spot interest rate and is a function of time. It can further be shown that the continuously compounded yield is actually the equivalent of the average value of the continuously compounded interest rate. In addition it can be shown that

$$r(t, T) = \frac{\int_t^T r(s) ds}{T - t}. \quad (50.10)$$

In a continuous time environment we do not assume discrete time intervals over which interest rates are applicable, rather a period of time in which a borrowing of funds would be repaid instantaneously. So we define the forward rate $f(t, s)$ as the interest rate applicable for borrowing funds where the deal is struck at time t ; the actual loan is made at s (with $s > t$) and repayable almost instantly. In mathematics the period $s - t$ is described as infinitesimally small. The spot interest rate is defined as the continuously compounded yield or interest rate $r(t, T)$. In an environment of no arbitrage, the return generated by investing at the forward rate $f(t, s)$ over the period $s - t$ must be equal to that generated by investing initially at the spot rate $r(t, T)$. So we may set

$$e^{\int_t^T f(t, s) ds} = e^{r(t, T)(T - t)} \quad (50.11)$$

which enables us to derive an expression for the spot rate itself, which is

$$r(t, T) = \frac{\int_t^T f(t, s) ds}{T - t}. \quad (50.12)$$

The relationship described by (50.12) states that the spot rate is given by the *arithmetic* average of the forward rates $f(t, s)$, where $t < s < T$. How does this differ from the relationship in a discrete time environment? From Chapter 6 we know that the spot rate in such a framework is the *geometric* average of the forward rates,⁹ and this is the key difference in introducing the continuous time structure. Equation (50.12) can be rearranged to

$$r(t, T)(T - t) = \int_t^T f(t, s) ds \quad (50.13)$$

and this is used to show (by differentiation) the relationship between spot and forward rates, given below:

$$f(t, s) = r(t, T) + (T - t) \frac{dr(t, T)}{dT}. \quad (50.14)$$

If we assume we are dealing today (at time 0) for maturity at time T , then the expression for the spot rate becomes

$$r(0, T) = \frac{\int_0^T f(0, s) ds}{T} \quad (50.15)$$

so we can write

$$r(0, T) \cdot T = \int_0^T f(0, s) ds. \quad (50.16)$$

This is illustrated in Figure 50.7 which is a diagrammatic representation showing that the spot rate $r(0, T)$ is the average of the forward rates from 0 to T , using the hypothetical value of 5% for $r(0, T)$. Figure 50.7 also shows the area represented by (50.16)

⁹ To be precise, if we assume annual compounding, the relationship is one plus the spot rate is equal to the geometric average of one plus the forward rates.

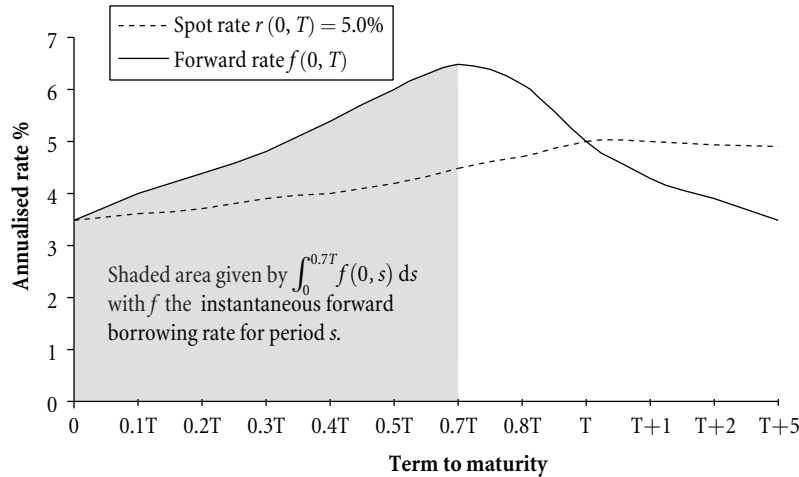


Figure 50.7: Diagrammatic representation of the relationship between spot and forward rate.
The spot rate $r(t, T)$ is the average of the forward rates between t and T .

What (50.14) implies is that if the spot rate increases, then by definition the forward rate (or *marginal* rate as has been suggested that it may be called¹⁰) will be greater. From (50.14) we deduce that the forward rate will be equal to the spot rate plus a value that is the product of the *rate* of increase of the spot rate and the time period $(T - t)$. In fact the conclusions simply confirm what we already discovered in the discrete time analysis in Chapter 6: the forward rate for any period will lie above the spot rate if the spot rate term structure is increasing, and will lie below the spot rate if it is decreasing. In a constant spot rate environment, the forward rate will be equal to the spot rate.

However it is not as simple as that. An increasing spot rate term structure only implies that the forward rate lies above the spot rate, but not that the forward rate structure is itself also *increasing*. In fact one can observe the forward rate term structure to be increasing or decreasing while spot rates are increasing. As the spot rate is the average of the forward rates, it can be shown that in order to accommodate this, forward rates must in fact be *decreasing* before the point at which the spot rate reaches its highest point. This confirms market observation. An illustration of this property is given in Appendix 50.2. As Campbell *et al.* (1997) state, this is a property of average and marginal cost curves in economics.

50.5.2 Bond prices as a function of spot and forward rates¹¹

In this section we describe the relationship between the price of a zero-coupon bond and spot and forward rates. We assume a risk-free zero-coupon bond of nominal value £1, priced at time t and maturing at time T . We also assume a money market bank account of initial value $P(t, T)$ invested at time t . The money market account is denoted M . The price of the bond at time t is denoted $P(t, T)$ and if today is time 0 (so that $t > 0$) then the bond price today is unknown and a random factor (similar to a future interest rate). The bond price can be related to the spot rate or forward rate that is in force at time t .

Consider the scenario below, used to derive the risk-free zero-coupon bond price¹².

The continuously compounded *constant* spot rate is r as before. An investor has a choice of purchasing the zero-coupon bond at price $P(t, T)$, which will return the sum of £1 at time T or of investing this same amount of cash in the money market account, and this sum would have grown to £1 at time T . We know that the value of the money market account is given by $Me^{r(t, T)(T-t)}$. If M must have a value of £1 at time T then the function $e^{-r(t, T)(T-t)}$ must give the present value of £1 at time t and therefore the value of the zero-coupon bond is given by

¹⁰ For example see Section 10.1 of Campbell, Lo and Mackinlay (1997), Chapter 10 of which is an excellent and accessible study of the term structure, and provides proofs of some of the results discussed here. This book is written in very readable style and is worth purchasing for Chapter 10 alone.

¹¹ This section follows the approach in Neftci (2000), Chapter 18, Section 3. This is also an excellent, readable text.

¹² This approach is also used in Campbell *et al.* (q.v.).

$$P(t, T) = e^{-r(t, T)(T-t)}. \quad (50.17)$$

If the same amount of cash that could be used to buy the bond at t , invested in the money market account, does *not* return £1 then arbitrage opportunities will result. If the price of the bond exceeded the discount function $e^{-r(t, T)(T-t)}$ then the investor could short the bond and invest the proceeds in the money market account. At time T the bond position would result in a cash outflow of £1, while the money market account would be worth £1. However the investor would gain because in the first place $P(t, T) - e^{-r(t, T)(T-t)} > 0$. Equally if the price of the bond was below $e^{-r(t, T)(T-t)}$ then the investor would borrow $e^{-r(t, T)(T-t)}$ in cash and buy the bond at price $P(t, T)$. On maturity the bond would return £1, which proceeds would be used to repay the loan. However the investor would gain because $e^{-r(t, T)(T-t)} - P(t, T) > 0$. To avoid arbitrage opportunities we must therefore have

$$P(t, T) = e^{-r(t, T)(T-t)}. \quad (50.18)$$

Following the relationship between spot and forward rates it is also possible to describe the bond price in terms of forward rates.¹³ We show the result here only. First we know that

$$P(t, T)e^{\int_t^T f(t, s)ds} = 1 \quad (50.19)$$

because the maturity value of the bond is £1, and we can rearrange (50.19) to give

$$P(t, T) = e^{-\int_t^T f(t, s)ds}. \quad (50.20)$$

Expression (50.20) states that the bond price is a function of the range of forward rates that apply for all $f(t, s)$, that is, the forward rates for all time periods s from t to T (where $t < s < T$, and where s is infinitesimally small). The forward rate $f(t, s)$ that results for each s arises as a result of a random or *stochastic* process that is assumed to start today at time 0. Therefore the bond price $P(t, T)$ also results from a random process, in this case all the random processes for all the forward rates $f(t, s)$.

The zero-coupon bond price may also be given in terms of the spot rate $r(t, T)$, as shown at (50.18). From our earlier analysis we know that

$$P(t, T)e^{r(t, T)(T-t)} = 1 \quad (50.21)$$

which is rearranged to give the zero-coupon bond price equation

$$P(t, T) = e^{-r(t, T)(T-t)} \quad (50.22)$$

as before.

Equation (50.22) describes the bond price as a function of the spot rate only, as opposed to the multiple processes that apply for all the forward rates from t to T . As the bond has a nominal value of £1 the value given by (50.22) is the discount factor for that term; the range of zero-coupon bond prices would give us the discount function.

What is the importance of this result for our understanding of the term structure of interest rates? First, we see (again, but this time in continuous time) that spot rates, forward rates and the discount function are all closely related, and given one we can calculate the remaining two. More significantly, we may model the term structure either as a function of the spot rate only, described as a stochastic process, or as a function of all of the forward rates $f(t, s)$ for each period s in the period $(T - t)$, described by multiple random processes. The first yield curve models adopted the first approach, while a later development described the second approach. Both approaches are examined in Part VIII.

Appendices

APPENDIX 50.1 Summary of derivation of optimum hedge equation

From equation (50.5) we know that the variance of a net change in the value of a two-bond portfolio is given by

$$\sigma_{pos}^2 = M_1^2 BPV_1^2 \sigma_1^2 + M_2^2 BPV_2^2 \sigma_2^2 + 2M_1 M_2 BPV_1 BPV_2 \sigma_1 \sigma_2 \rho. \quad (50.23)$$

¹³ For instance, see *ibid*, Section 4.2.

Using the partial derivative of the variance σ^2 with respect to the nominal value of the second bond, we obtain

$$\frac{\partial \sigma^2}{\partial^2 M_2} = 2M_2 BPV_2^2 \sigma_2^2 + 2M_1 BPV_1 BPV_2 \sigma_1 \sigma_2 \rho. \quad (50.24)$$

If (50.23) is set to zero and solved for M_2 we obtain (50.25) which is the hedge quantity for the second bond.

$$M_2 = -\frac{\rho BPV_1 \sigma_1}{BPV_2 \sigma_2} M_1. \quad (50.25)$$

APPENDIX 50.2 Illustration of forward rate structure when spot rate structure is increasing

We assume the spot rate $r(0, T)$ is a function of time and is increasing to a high point at $\bar{T}$. It is given by

$$r(0, T) = \frac{\int_0^T f(0, s) ds}{T}. \quad (50.26)$$

At its high point the function is neither increasing nor decreasing, so we may write

$$\frac{dr(0, \bar{T})}{dT} = 0 \quad (50.27)$$

and therefore the second derivative with respect to T will be

$$\frac{d^2 r(0, \bar{T})}{dT^2} < 0. \quad (50.28)$$

From (50.14) and (50.27) we may state

$$f(0, \bar{T}) = r(0, \bar{T}) \quad (50.29)$$

and from (50.28) and (50.29) the second derivative of the spot rate is

$$\frac{d^2 r(0, \bar{T})}{dT^2} = \left[\frac{df(0, \bar{T})}{dT} - \frac{dr(0, \bar{T})}{dT} \right] \frac{1}{\bar{T}} < 0. \quad (50.30)$$

From (50.27) we know the spot rate function is zero at $\bar{T}$ so the derivative of the forward rate with respect to T would therefore be

$$\frac{df(0, \bar{T})}{dT} < 0. \quad (50.31)$$

So in this case the forward rate is decreasing at the point $\bar{T}$ when the spot rate is at its maximum value. This is illustrated hypothetically at Figure 50.7 and it is common to observe the forward rate curve decreasing as the spot rate is increasing.

Selected bibliography

- Brennan, M., Schwartz, E., "A continuous time approach to the pricing of bonds", *Journal of Banking and Finance* 3, 1979, p.134 ff.
- Brennan, M., Schwartz, E., "Conditional predictions of bond prices and returns", *Journal of Finance* 35, 1980, p. 405 ff.
- Choudhry, M., *Bond Market Securities*, FT Prentice Hall, 2001.
- Campbell, J., Lo, A., MacKinley, A., *The Econometrics of Financial Markets*, Princeton, 1997.
- Fisher, L., Leibowitz, M., "Effects of alternative anticipations of yield curve behaviour on the composition of immunized portfolios and on their target returns", in Kaufmann, G., et al. (eds.), *Innovations in Bond Portfolio Management*, Jai Press, 1983.
- The Economist, "Out of Debt", *The Economist*, 12 February 2000.
- Neftci, S., *An Introduction to the Mathematics of Financial Derivatives*, 2nd edition, Academic Press, 2000.

Questions and exercises

1. What are the differences between speculation and arbitrage?
2. Why are most spread trades carried out in the exchange-traded futures and options markets rather than in the cash markets?
3. A trader expects short-term interest rates to fall relative to long-term rates. Describe one way in which they could trade this view.
4. Describe a strategy that a trader might follow if the following set of short-sterling interest-rate futures prices were observed:

September 2000	93.65
December 2000	93.66
March 2001	93.80
5. A trader observes the following short-sterling futures prices:

June	95.50
September	95.67
December	95.70

and purchases 100 butterfly spreads. One week later the prices stand as follows:

June	95.36
September	95.56
December	95.60

Calculate the position profit or loss if the trader unwinds their position at this point. Tick values is £10.
6. Discuss the key financing considerations with regard to a yield spread trade conducted with cash bonds.
7. In an interest rate environment with 5% annualised rates, consider a financial instrument with a tenor of three months. What is the instrument's rate of return under annual compounding? Monthly compounding? What is the equivalent continuously compounded rate of interest?
8. For the time period $T - t$, where $T > t$, write down the mathematical equation which states that the period $(T - t)$ multiplied by the spot interest rate applicable at time t for maturity at T , produces the sum of the set of forward interest rates from time t to T .

Part VIII **Advanced Fixed Income Analytics**

Introduction

If readers have covered all previous seven parts in this book, they should be comfortable, indeed enthusiastic about the content in Part VIII. Assuming one is fully conversant with the preceding topics, we are now in a position to carry our review of the bond markets to fundamental questions of the yield curve, the processes that determine the evolution of the interest-rate term structure, and how these processes can be modelled. In Part VIII we consider some more advanced topics in bond market theory, so readers should be familiar with the basic concepts including interest-rate risk, convexity, the pricing process for bonds with embedded options and the concept of option-adjusted spread, the fundamental of index-linked bonds, and the application of the Black–Scholes model in options pricing. The chapters in this part will review a number of interest-rate models and the assumptions underlying these models. We also look at the procedure involved in estimating and fitting the yield curve. This topic is one of the most heavily researched subjects in financial market economics, and indeed research is ongoing. There are a number of ways to estimate and fit the yield curve, and there is no one right or wrong method.

It is important, both when discussing the subject matter or writing about it, to remember to place the relevant ideas in context, otherwise there is the danger of becoming too theoretical. The aim is to confine the discussion within the boundary of user application; there is a great deal of published material that is, quiet simply rather too theoretical, not to mention highly technical. We must always try to keep in touch with the markets themselves. The chapters are written from the point of view of both the market practitioner and the research student. We emphasise again that this is a book about the bond markets, and not a mathematics textbook. There are very few derivations, and fewer (if any) proofs. This material is superbly covered in existing literature.

In the following chapters we summarise a number of interest-rate models, in a practical way that does exclude most of the mathematics. The aim is firmly to discuss application of the models, and not to derive them or prove the maths. In this way readers should be able to assess the different methodologies for themselves and decide the efficacies of each for their own purposes. As always, selected recommended texts, plus chapter references are listed at the back, and would be ideal as a starting point for further research. This serves to highlight that Part VIII of this book is very much a summary of the latest developments, rather than a fully comprehensive review of the subject. The topics would be suitable for a separate book in their right, and such a book would make an ideal companion to this one. However there is sufficient detail and exposition to leave the reader with, hopefully, a good understanding of the subject.

To begin with we review the main one-factor models that were developed initially for the term structure. This includes the Vasicek, Cox–Ingersoll–Ross and Hull–White models. In most cases the model result is given and explained, rather than the full derivation. The objective here is to keep the content accessible, and pertinent to practitioners and most postgraduate students. A subsequent book is planned that will delve deeper into the models themselves, and the latest developments in research. In Chapter 52 we look at more advanced multi-factor models, and the Heath–Jarrow–Morton model. Chapter 53 reviews some techniques used to estimate and fit the zero-coupon curve using the prices of bonds observed in the market, using an illustration from the United Kingdom gilt market that previously appeared in the Bank of England *Quarterly Bulletin* for November 1999.

In Chapter 54 we review some advanced analytical techniques for index-linked bonds. Chapter 55 is a look at some of the peculiar properties of very long-dated bond yields, including the convexity bias inherent in such yields, and their relative volatility. In Chapter 56 we review some concepts that apply to the analysis of the credit default risk of corporate bonds, and how this might be priced.

The dynamics of the yield curve

In Chapter 6 we introduced the concept of the yield curve, and reviewed some preliminary issues concerning both the shape of the curve and to what extent the curve could be used to infer the shape and level of the yield curve in the future. We do not know what interest rates will be in the future, but given a set of zero-coupon (spot) rates today we can estimate the future level of forward rates using a yield curve model. In many cases however we do not have a

zero-coupon curve to begin with, so it then becomes necessary to derive the spot yield curve from the yields of coupon bonds, which one can observe readily in the market. If a market only trades short-dated debt instruments, then it will be possible to construct a short-dated spot curve.

It is important for a zero-coupon yield curve to be constructed as accurately as possible. This is because the curve is used in the valuation of a wide range of instruments, not only conventional cash market coupon bonds, which we can value using the appropriate spot rate for each cash flow, but other interest-rate products such as swaps.

If using a spot rate curve for valuation purposes, banks use what are known as *arbitrage-free* yield curve models, where the derived curve has been matched to the current spot yield curve. The concept of arbitrage-free, also known as no-arbitrage pricing or “the law of one price” is that if one is valuing the same product or cash flow in two different ways, the same result will be obtained from either method. So as we demonstrated in Chapter 6, if one was valuing a two-year bond that was put-able by the holder at par in one year’s time, it could be analysed as a one-year bond that entitled the holder to reinvest it for another year. The rule of no-arbitrage pricing states that an identical price will be obtained whichever way one chooses to analyse the bond. When matching derived yield curves therefore, correctly matched curves will generate the same price when valuing a bond, whether a derived spot curve is used or the current term structure of spot rates.

In Chapter 44 we observed that option pricing models such as Black–Scholes assume that asset price returns follow a lognormal distribution. The dynamics of interest rates and the term structure is the subject of some debate, and the main differences between the main interest-rate models is in the way that they choose to capture the change in rates over a time period. However, although volatility of the yield curve is indeed the main area of difference, certain models are easier to implement than others, and this is a key factor a bank considers when deciding which model to use. The process of *calibrating* the model, that is, setting it up to estimate the spot and forward term structure using current interest rates that are input to the model, is almost as important as deriving the model itself. So the availability of data for a range of products, including cash money markets, cash bonds, futures and swaps, is vital to the successful implementation of the model.

As one might expect the yields on bonds are correlated, in most cases very closely positively correlated. This enables us to analyse interest-rate risk in a portfolio for example, but also to model the term structure in a systematic way. Much of the traditional approach to bond portfolio management assumed a parallel shift in the yield curve, so that if the five-year bond yield moved upwards by 10 basis points, then the 30-year bond yield would also move up by ten basis points. This underpins traditional duration and modified duration analysis, and the concept of immunisation. To analyse bonds in this way, we assume therefore that bond yield volatilities are identical and correlations are perfectly positive. Although both types of analysis are still common, it is clear that bond yields do not move in this fashion, and so we must enhance our approach in order to perform more accurate analysis.

Factors influencing the yield curve

From an earlier discussion we are aware that there are a number of factors that impact the shape and level of the yield curve. A combination of economic and non-economic factors are involved. A key factor is investor expectations, with respect to the level of inflation, and the level of real interest rates in the future. In the real world the market does not assume that either of these two factors is constant, however given that there is a high level uncertainty over anything longer than the short-term, generally there is an assumption about both inflation and interest rates to move towards some form of equilibrium in the long-term.

It is possible to infer market expectations about the level of real interest rates going forward by observing yields in government index-linked bonds, which trade in a number of countries including the US and UK. The market’s view on the future level of interest rates may also be inferred from the shape and level of the current yield curve. We know that the slope of the yield curve carries an information content. There is more than one way to interpret any given slope however, and this debate is still open.

The fact that there are a number of factors that influence changes in interest rates and the shape of the yield curve means that it is not straightforward to model the curve itself. In the following chapter we consider some of the traditional and more recent approaches that have been developed.

Approaches to modelling

The area of interest rate dynamics and yield curve modelling is one of the most heavily researched in financial economics. There are a number of models available in the market today, and generally it is possible to categorise them as following certain methodologies. By categorising them in this way, participants in the market can assess them for their suitability, as well as draw their own conclusions about how realistic they might be. Let us consider the main categories.

The process begins with the fair valuation of a set of cash flows. If we are analysing a financial instrument comprised of a cash flow stream of nominal amount C_i , paid at times $i = 1, 2, \dots, N$, then the value of this instrument is given by

$$PV = \sum_{i=1}^N C_i P(0, t_i)$$

where $P(0, t_i)$ is the price today of a zero-coupon bond of nominal value 1 maturing at each point i , or the i -period discount factor. This expression can be written as

$$PV = \sum_{i=1}^N C_i \exp(-(t_i)r(0, t_i))$$

which indicates that in a no-arbitrage environment the present value of the cash flow stream is obtained by discounting the set of cash flows and summing them. Therefore in theory it is straightforward to calculate the present value of any cash flow stream (and by implication virtually any financial instrument) using the yields observed on a set of risk-free and default-free zero-coupon bonds. In practice though the set of such zero-coupon bonds is limited and is influenced by liquidity and other market considerations. We therefore require a technique that enables us to use conventional coupon bonds and extract a set of zero-coupon discount factors from these.

One-factor, two-factor and multi-factor models

The key assumption that is made by an interest-rate model is whether it is one-factor, that is the dynamics of the yield change process is based on one factor, or multi-factor. From observation we know that in reality there are a number of factors that influence the price change process, and that if we are using a model to value an option product, the valuation of that product is dependent on more than one underlying factor. For example the payoff on a bond option is related to the underlying bond's cash flows as well as to the reinvestment rate that would be applied to each cash flow, in addition to the other factors we discussed in Chapter 44. Valuing an option therefore is a multi-factor issue. In many cases however there is a close degree of correlation between the different factors involved. If we are modelling the term structure, we can calculate the correlation between the different maturity spot rates by using a covariance matrix of changes each of the spot rates, and thus obtain a common factor that impacts all spot rates in the same direction. This factor can then be used to model the entire term structure in a one-factor model, and although two-factor and multi-factor models have been developed, the one-factor model is still commonly used. In principle it is relatively straightforward to move from a one-factor to a multi-factor model, but implementing and calibrating a multi-factor model is a more involved process. This is because the model requires the estimation of more volatility and correlation parameters, which slows down the process.

Readers will encounter the term *Gaussian* in reference to certain interest-rate models. Put simply a Gaussian process describes one that follows a normal distribution under a probability density function. The distribution of rates in this way for Gaussian models implies that interest rates can attain negative values under positive probability, which makes the models undesirable for some market practitioners. Nevertheless such models are popular because they are relatively straightforward to implement and because the probability of the model generating negative rates is low and occurs only under certain extreme circumstances.

The short-term rate and the yield curve

The application of risk-neutral valuation requires that we know the sequence of short-term rates for each scenario, which is provided in some interest-rate models. For this reason, many yield curve models are essentially models of the stochastic evolution of the short-term rate. They assume that changes in the short-term interest-rate is a *Markov* process. (It is outside the scope of this book to review the mathematics of such processes, but references are provided in subsequent chapters.) This describes an evolution of short-term rates in which the evolution of the rate

is a function only of its current level, and not the path by which it arrived there. The practical significance of this is that the valuation of interest-rate products can be reduced to the solution of a single partial differential equation.

Short-rate models are composed of two components. The first attempts to capture the average rate of change, also called the *drift*, of the short-term rate at each instant, while the second component measures this drift as a function of the volatility of the short-term rate. This is given by:

$$dr(t) = \mu(r,t)dt + \sigma(r,t)dW(t)$$

where $dr(t)$ is the instantaneous change in the short-term rate, and $W(t)$ is the stochastic process that describes the evolution in interest rates, known as a Brownian or Weiner process.

The term $\mu(r,t)$ is the value of the drift multiplied by the size of the time period. The term $\sigma(r,t)dW(t)$ is the volatility of the short-term rate multiplied by a random increment that is normally distributed. In most models the drift rate term is determined through a numerical technique that matches the initial spot rate yield curve, while in some models an analytical solution is available. Generally models assume an arbitrage-free relationship between the initial forward rate curve, the volatility $\sigma(r,t)$, the market price of interest-rate risk and the drift term $\mu(r,t)$. In models such as those presented by Vasicek (1977) and Cox–Ingersoll–Ross (1985), the initial spot rate yield curve is given by an analytical formula in terms of the model parameters, and they are known as *equilibrium* models, because they describe yield curves as being derived from an assumption of economic equilibrium, based on a given market interest rate. So the Vasicek and CIR models are models of the short-term rate, and both incorporate the same form for the drift term, which is a tendency for the short-term rate to rise when it is below the long-term mean interest rate, and to fall when it is above the long-term mean. This is known as *mean reversion*. Therefore we can describe the short-term rate drift in the form:

$$\mu = \kappa(\theta - r)$$

where r is the short-term rate as before and κ and θ are the mean reversion and long-term rate constants. In the Vasicek model, the rate dependence of the volatility is constant, in the CIR model it is proportional to the square-root of the short rate. In both models, because the dynamics of the short-rate cover all possible moves, it is possible to derive negative interest rates, although under most conditions of initial spot rate and volatility levels, this is quite rare. Essentially the Vasicek and CIR models express the complete forward rate curve as a function of the current short-term rate, which is why later models are sometimes preferred.

Arbitrage-free and equilibrium modelling

In an arbitrage-free model, the initial term structure described by spot rates today is an input to the model. In fact such models could be described not as models per se, but essentially a description of an arbitrary process that governs changes in the yield curve, and projects a forward curve that results from the mean and volatility of the current short-term rate. An equilibrium term structure model is rather more a true model of the term structure process; in an equilibrium model the current term structure is an output from the model. An equilibrium model employs a statistical approach, assuming that market prices are observed with some statistical error, so that the term structure must be estimated, rather than taken as given.

When valuing an option written on say an equity, the price of the underlying asset is the current price of the equity. When pricing an interest-rate option the underlying is obtained via a random process that describes the instantaneous risk-free zero-coupon rate, which is generally termed the short rate.

In the following chapters we explore the different models that may be used and their application.

Mathematics primer

The level of mathematics required for a full understanding of even intermediate concepts in finance is frighteningly high. To attempt to summarise even the basic concepts in just a few pages would be a futile task and might give the impression that the mathematics was being trivialised. Our intention is quite the opposite. As this is a financial markets book, and not a mathematics textbook, a certain level of knowledge has been assumed, and a formal or rigorous approach has not been adopted. Hence readers will find few derivations, and fewer proofs. What we provide here is a very brief introduction to some of the concepts; the aim of this is simply to provide a starting point for individual research. We assist this start by listing recommended texts in the bibliography.

Random variables and probability distributions

In financial mathematics random variables are used to describe the movement of asset prices, and assuming certain properties about the process followed by asset prices allows us to state what the expected outcome of events are. A random variable may be any value from a specified *sample space*. The specification of the *probability distribution* that applies to the sample space will define the frequency of particular values taken by the random variable. The cumulative distribution function of a random variable X is defined using the distribution function $f(\cdot)$ such that $\Pr\{X \leq x\} = f(x)$. A discrete random variable is one that can assume a finite or *countable* set of values, usually assumed to be the set of positive integers. We define a discrete random variable X with its own probability function $p(i)$ such that $p(i) = \Pr\{X = i\}$. In this case the probability distribution is

$$f(i) = \Pr\{X \leq i\} = \sum_{n=0}^i p(n)$$

with $0 \leq p(i) \leq 1$ for all i . The sum of the probabilities is 1.

Discrete probability distributions include the Binomial distribution and the Poisson distribution.

Continuous random variables

The next step is to move to a continuous framework. A continuous random variable X may assume any real value and its probability density function $f(x)$ is defined as

$$\begin{aligned} f(x) &= \lim_{dx \rightarrow 0} \frac{\Pr\{x \leq X \leq x + dx\}}{dx} \\ &= \frac{dF(x)}{dx}. \end{aligned}$$

The probability distribution function is given as $F(x) = \Pr\{X \leq x\} = \int_{s=-\infty}^x f(s) ds$

Continuous distributions are commonly encountered in finance theory. The *normal* or Gaussian distribution is perhaps the most important. It is described by its mean μ and standard deviation σ , sometimes called the location and spread respectively. The probability density function is

$$f(x) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}.$$

Where a random variable X is assumed to follow normal distribution it will be described in the form $X \sim N(\mu, \sigma^2)$ where $\sim$ means “is distributed according to”. Examples of the graphical representation of the normal distribution are included in Chapter 37. The standard normal distribution is written as $N(0, 1)$ with $\mu = 0$ and $\sigma = 1$. The cumulative distribution function for the standard normal distribution is given by

$$\Phi(x) = N(x) = \int_{z=-\infty}^x \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}z^2} dz$$

and it is this function that is tabulated at Appendix 37.2. The key assumption in the derivation of the Black–Scholes option pricing model is that the asset price follows a *lognormal* distribution, so that if we assume the asset price is P we write

$$\log\left(\frac{P_t}{P_0}\right) \sim N\left((r - \frac{1}{2}\sigma^2)t, \sigma^2 t\right).$$

Expected values

A probability distribution function describes the distribution of a random variable X . The expected value of X in a discrete environment is given by

$$E[X] = \bar{X} = \sum_{i=0}^{\infty} ip(i)$$

and the equivalent for a continuous random variable is

$$E[X] = \bar{X} = \int_{s=-\infty}^{\infty} sf(s)ds$$

The dispersion around the mean is given by the *variance* which is

$$Var[X] = E[(X - \bar{X})^2] = \sum_{i=0}^{\infty} (i - \bar{X})^2 pi$$

or

$$Var[X] = E[(X - \bar{X})^2] = \int_{s=-\infty}^{\infty} f(s)ds$$

in a continuous distribution. A squared measure has little application so commonly the square root of the variance, the standard deviation is used.

Regression analysis

A linear relationship between two variables, one of which is dependent, can be estimated using the least squares method. The relationship is

$$Y_i = \alpha + \beta X_i + \varepsilon_i$$

where X is the independent variable and ε is an error term capturing those explanatory factors not covered by the model. ε is described as $\varepsilon_i \sim N(0, \sigma^2)$. β is the slope of the linear regression line that describes the relationship, while α is the intercept of the y -axis. The sum of the squares of the form

$$SS = \sum_{i=1}^n (y_i - \hat{y}_i)^2$$

is minimised in order to calculate the parameters.

Where we believe the relationship is non-linear we can use a regression model of the form

$$Y_i = \alpha X_{1i} (1 + \beta X_{2i}) + \varepsilon_i.$$

This can be transformed into a form that is linear and then fitted using least squares. This is carried out by minimising squares and is described by

$$SS = \sum_{i=1}^n (y_i - \alpha(1 - e^{-\beta x_i}))^2.$$

Yield curve fitting techniques that use splines are often fitted using multiple regression methods.

Stochastic processes

This is perhaps the most difficult area of financial mathematics. Most references are also very technical and therefore difficult to access for the non-mathematician.

We begin with some definitions. A random process is usually referred to as a *stochastic* process. This is a collection of random variables $X(t)$ and the process may be either discrete or continuous. We write $\{X(t), t \in T\}$ and a sample $\{x(t), 0 \leq t \leq t_{max}\}$ of the random process $\{X(t), t \geq 0\}$ is known as the *realisation* or *path* of the process.

A *Markov* process is one where the path is dependent on the present state of the process only, so that all historical data, including the path taken to arrive at the present state, is irrelevant. So in a Markov process, all data up to the present is contained in the present state. The dynamics of asset prices are frequently assumed to follow a Markov process, and in fact it represents a semi-strong form efficient market. It is written

$$\Pr\{X(t) \leq y \mid X(u) = x(u), 0 \leq u \leq s\} = \Pr\{X(t) \leq y \mid X(s) = x(s)\}$$

for $0 \leq s \leq t$.

A *Weiner process* or *Brownian motion* for $\{X(t), t \geq 0\}$ has the following properties:

- $X(0) = 0$;
- $\{X(t), t \geq 0\}$ has independent increments, so that $X(t + b) - X(t)$ and $X(t + 2b) - X(t + b)$ are independent and follow the same distribution;
- the variable $X(t)$ has the property $X(t) \sim N(0, t)$ for all $t > 0$;
- $X(t) - X(s) \sim N(0, t - s)$ for $0 \leq s < t$.

Many interest rate models assume that the movement of interest rates over time follows a Wiener process.

Stochastic calculus

The Wiener process is usually denoted with W although Z and z are also used. For a Wiener process $\{W(t), t \geq 0\}$ it can be shown that after an infinitesimal time interval Δt we have

$$W(t + \Delta t) - W(t) \sim N(0, \Delta t).$$

If we also have $U \sim N(0, 1)$ then we may write

$$W(t + \Delta t) - W(t) = \sqrt{\Delta t} U.$$

As the time interval decreases and approaches (but does not reach) 0, then the expression above may be written $dW(t) = \sqrt{\Delta t} U$.

A Wiener process is not differentiable but a generalised Wiener process termed an Itô process is differentiable and is described in the form

$$dX(t) = a(t, X)dt + b(t, X)dW$$

where a is the drift and b the noise or volatility of the stochastic process.

Selected bibliography

Baxter, A., Rennie, D., *Financial Calculus*, Cambridge University Press, 1996.

Gujarati, D., *Basic Econometrics*, AP, 1997.

Hogg, R., Craig, A., *An Introduction to Mathematical Statistics*, 5th edition, Prentice Hall, 1995.

Kreyszig, E., *Advanced Engineering Mathematics*, Wiley, 1993.

Ross, S., *An Introduction to Probability Models*, 5th edition, Academic Press, 1997.

Ross, S., *An Introduction to Mathematical Finance*, Cambridge University Press, 1999.

51 Interest-rate Models I

In Chapter 43 we introduced the concept of stochastic processes. Most but not all interest rate models are essentially a description of the short-rate in terms of a stochastic process. Recent literature¹ has tended to categorise models into one of up to six different types, but for our purposes we can generalise them into two types. Thus we introduce some of the main models, according to their categorisation as equilibrium or arbitrage-free models. This chapter looks at the earlier models, including the first ever term structure model presented by Vasicek in 1977. The next chapter considers what have been termed “whole yield curve” models, or the Heath–Jarrow–Morton family, while Chapter 53 reviews considerations in modelling the yield curve.

51.1 Introduction

51.1.1 Bond price and yield

We first set the scene by introducing the interest rate market. In fact we have already covered spot and forward rates, and the term structure, in Chapter 6. Let us reiterate the key points here. The price of a zero-coupon bond of maturity T at time t is denoted by $P(t, T)$, so that its price at time 0 is denoted by $P(0, T)$. The process followed by the bond price is a stochastic one and therefore can be modelled, equally options that have been written on the bond can be hedged by it. If market interest rates are constant, the price of the bond at time t is given by $e^{-r(T-t)}$. This enables us to state that given a zero-coupon bond price $P(t, T)$ at time t , the yield $r(t, T)$ is given by (51.1):

$$r(t, T) = -\frac{\log P(t, T)}{T - t}. \quad (51.1)$$

Of course interest rates are not constant but (51.1) is valuable as it is used later in constructing a model. By using (51.1) we are able to produce a yield curve given a set of zero-coupon bond prices. For modelling purposes we require a definition of the *short rate*, or the current interest rate for borrowing a sum of money that is paid back a very short period later (in fact, almost instantaneously). This is the rate payable at time t for repayment at time $t + \Delta t$ where Δt is an incremental passage of time. This is given by

$$r(t, t + \Delta t) = -\frac{\log P(t, t + \Delta t)}{\Delta t} \quad (51.2)$$

and the incremental change can be steadily decreased to give the instantaneous rate, which is described by

$$r_t = -\frac{\partial}{\partial T} \log P(t, t) \quad (51.3)$$

and is identical to $r(t, t)$.

The instantaneous rate is an important mathematical construct that is widely used in the modelling process.

We can define forward rates in terms of the short rate. Again for infinitesimal change in time from a forward date T_1 to T_2 (for example, two bonds whose maturity dates are very close together), we can define a forward rate for instantaneous borrowing, given by

$$rf(t, T) = -\frac{\partial}{\partial T} \log P(t, T) \quad (51.4)$$

which is called the forward rate. We can also set

$$rf(t, t) = r_t \quad (51.5)$$

that is the forward rate for borrowing at the point $t = T$ which is identical to the short rate. The forward rate is valuable because, given the set of forward rates from t to T , we can calculate the bond price for a T -maturity date.

¹ For example, see James and Weber (2000), or Van Deventer and Imai (1997).

This is presented in a number of texts, one of the best being Jarrow (1996). Given the expressions for the bond yield and the forward rate, the bond prices can be defined in terms of either the yield,

$$P(t, T) = \exp(-(T - t)r(t, T)) \quad (51.6)$$

or the forward rates, as a stochastic integral

$$P(t, T) = \exp\left(-\int_t^T rf(t, s)ds\right). \quad (51.7)$$

This is convenient because this means that the price at time t of a zero-coupon bond maturing at T is given by (51.7), and forward rates can be calculated from the current term structure or vice-versa.

For readers unfamiliar with the basic maths, an introductory primer is given at the start of Part VIII.

51.1.2 Interest rate models

An interest rate model provides a description of the dynamic process by which rates change over time, in terms of a statistical construct, as well as a means by which interest rate derivatives such as options can be priced. It is often the practical implementation of the model that dictates which type is used, rather than mathematical neatness or more realistic assumptions. An excellent categorisation is given in James and Webber (2000) who list models as being one of the following types:

- the traditional one-, two- and multi-factor equilibrium models, known as *affine term structure* models (see James and Webber (2000) or Duffie (1996, page 136)).² These include Gaussian affine models such as Vasicek, Hull–White and Steeley, where the model describes a process with constant volatility; and models that have a square-root volatility such as Cox–Ingersoll–Ross (CIR);
- whole yield curve models such as Heath–Jarrow–Morton;
- so-called market models such as Jamshidian;
- so-called consol models such as Brennan and Schwartz.

There are also other types of models and we suggest that interested readers consult a specialist text; James and Webber is an excellent start, which also contains detailed sections on implementing models as well as a comparison of the different models themselves.

The most commonly used models are the Hull–White type models which are relatively straightforward to implement, although HJM models are also more commonly encountered. The Hull–White and *extended* CIR models incorporate a mean reversion feature that means that they can be fitted to the term structure in place at the time. The CIR model has a square root factor in its volatility component, which prevents the short-term rate reaching negative values. What criteria are used by a bank in deciding which model to implement? Generally a user will seek to implement a model that fits current market data, fits the process by which interest rates change over time and is *tractable*. This means that it should be computationally efficient, and provide explicit solutions when used for pricing bonds and vanilla options.

51.2 Interest-rate processes

Term structure models are essentially models of the interest-rate process. The problem being posed is, what behaviour is exhibited by interest rates, and by the short-term interest rate in particular? An excellent description of the three most common processes that are used to describe the dynamics of the short-rate is given in Phoa (1998), who describes:

- **the Gaussian or normal process:** random shifts in forward rates are normally distributed and any given forward rate drifts upward at a rate proportional to the initial time to the forward date. The interest-rate volatility is independent of the current interest rate, and the volatility term has the form $\sigma dW(t)$ where $W(t)$ is a generalised Weiner process or Brownian motion. An example of a Gaussian model is the Vasicek model;
- **the square root or squared Gaussian process:** the interest-rate volatility is proportional to the square root of the current interest rate, so the volatility term is given by $\sigma\sqrt{r} dW(t)$. An example of this is the Cox–Ingersoll–Ross model;

² A function $H: \mathbb{R} \rightarrow \mathbb{R}$ is *affine* if there are constants a and b such that for all values of x , $H(x) = a + bx$. This describes certain term structure models' drift and diffusion functions.

- **the lognormal process:** interest-rate volatility is proportional to the current interest rate, with the volatility term described by $\sigma r dW(t)$. An example of this is the Black–Derman–Toy model.

To illustrate the differences, this means that if the current short-rate is 8% and is assumed to have an annualised volatility of 100 basis points, and at some point in the future the short-rate moves to 4%, under the Gaussian process the volatility at the new rate will remain at 50 basis points, the square root process will assume a volatility of 82.8 basis points and the lognormal process will assume a volatility of 50 basis points.

The most straightforward models to implement are normal models, followed by square root models and then lognormal models. The process that is used will have an impact on the distribution of future interest rates predicted by the model. A generalised distribution is given at Figure 51.1.

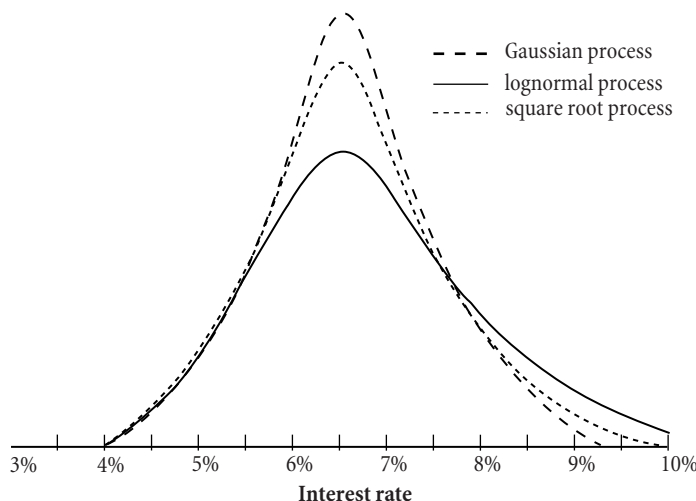


Figure 51.1: Distribution of future interest rates implied by different processes.
 Reproduced from Phoa, W., *Advanced Fixed Income Analytics*, FJF Associates, 1998.
 Used with permission.

Empirical studies have not pointed conclusively to one specific process as the most realistic. One study (BoE 1999) states that observation of interest rate behaviour in different markets suggests that when current interest rate levels are low, at 4% or below, the rate process has tended to a Gaussian process, while when rates are relatively high the process is more akin to a lognormal process. At levels between these two, it would seem an “intermediate” process is followed. These observations can be supported by economic argument however. The nominal level of interest rates in an economy has two elements, a real interest rate and an inflation component. Thus interest-rate volatility arises as a result of real interest-rate volatility and consumer prices volatility. When interest rates are low, the inflation component will be negligible, at which point only real rate volatility has an impact. However as real rates are linked to the rate of growth, it is reasonable to assume that they follow a normal distribution. An extreme case has occurred in some markets where the real rates on index-linked bonds has occasionally been recorded as negative. When interest rates are at relatively high levels, the inflation component is more significant, so that price volatility is important. However economic rationale suggests that the price of traded goods follows a lognormal distribution.

Where does this leave the thinking on interest-rate models? As we demonstrate in the next section, one of the drawbacks of Gaussian interest-rate models is that they can result in negative forward rates. Although not impossible, this is an extremely unusual, not to say rare, situation and one that is unlikely in any environment bar one with very low current interest rates. However such a phenomenon is not completely unheard of, and an environment of low interest rates is one that is best described by a Gaussian process. Negative interest rates have been recorded, for example in the Japanese government bond repo market and certain other repo markets when bonds have gone very *special*, and bear in mind that rates in Japan have been very low for some time now. Essentially then a model that permits negative interest rates is not necessarily unrealistic in an economic sense.

51.3 One-factor models

A short-rate model can be used to derive a complete term structure. We can illustrate this by showing how the model can be used to price discount bonds of any maturity. The derivation is not shown here. Let $P(t, T)$ be the price of a risk-free zero-coupon bond at time t maturing at time T that has a maturity value of 1. This price is a random process, although we know that the price at time T will be 1. Assume that an investor holds this bond, which has been financed by borrowing funds of value C_t . Therefore at any time t the value of the short cash position must be $C_t = -P(t, T)$, otherwise there would be an arbitrage position. The value of the short cash position is growing at a rate dictated by the short-term risk-free rate r , and this rate is given by

$$\frac{dC_t}{dt} = r(t)C_t.$$

By integrating this we obtain $C_t = C_0 \exp\left(-\int_0^t r(s)ds\right)$ which can be rearranged to give

$$P(0, T)/P(t, T) = \exp\left(-\int_0^t r(s)ds\right)$$

so that the random process on both sides are the same, so that their expected values are the same. This can be used to show that the price of the zero-coupon bond at any point t is given by:

$$P(t, T) = E\left[\exp\left(-\int_t^T r(s)ds\right)\right].$$

Therefore, once we have a full description of the random behaviour of the short-rate r , we can calculate the price and yield of any zero-coupon bond at any time, by calculating this expected value. The implication is clear: specifying the process $r(t)$ determines the behaviour of the entire term structure, so if we wish to build a term structure model we need only (under these assumptions) specify the process for $r(t)$.

So now we have determined that a short-rate model is related to the dynamics of bond yields and therefore may be used to derive a complete term structure. We also said that in the same way the model can be used to value bonds of any maturity. The original models were one-factor models, which describe the process for the short-rate r in terms of one source of uncertainty. This is used to capture the short-rate in the following form

$$dr = \mu(r)dt + \sigma(r)dW \quad (51.8)$$

where μ is the instantaneous drift rate and σ the standard deviation of the short rate r . Both these terms are assumed to be functions of the short-rate and independent over time. The key assumption made in a one-factor model is that all interest rates move in the same direction.

51.3.1 The Vasicek model

In the Vasicek model (1977) the instantaneous short-rate r is assumed to follow a stochastic process known as the Ornstein–Uhlenbeck process, a form of Gaussian process, described by (51.9):

$$dr = a(b - r)dt + \sigma dW. \quad (51.9)$$

This model incorporates *mean reversion*, which is a not unrealistic feature. Mean reversion is the process that describes that when the short-rate r is high, it will tend to be pulled back towards the long-term average level; when the rate is low, it will have an upward drift towards the average level. In Vasicek's model the short-rate is pulled to a mean level b at a rate of a . The mean reversion is governed by the stochastic term σdW which is normally distributed. Using (51.9) Vasicek shows that the price at time t of a zero-coupon bond of maturity T is given by:

$$P(t, T) = A(t, T)e^{-B(t, T)r(t)} \quad (51.10)$$

where $r(t)$ is the value of r at time t ,

$$B(T, t) = \frac{1 - e^{-A(t-T)}}{a} \quad (51.11)$$

$$\text{and } A(t, T) = \exp\left\{\frac{B(t, T) - (T - t)(a^2b - (\sigma^2/2))}{a^2} - \frac{\sigma^2 B(t, T)^2}{4a}\right\}. \quad (51.12)$$

It can be shown further that

$$r(t, T) = -\frac{1}{T-t} \ln A(t, T) + \frac{1}{T-t} B(t, T) r(T), \quad (51.13)$$

which describes the complete term structure as a function of $r(t)$ with parameters a , b and the standard deviation σ . The expression at (51.13) states that $r(t, T)$ is a linear function of $r(t)$, and that the value of $r(t)$ will determine the level of the term structure at time t . Using the parameters described above we can calculate the price function for a risk-free zero-coupon bond. Chan *et al.* (1992) used the following parameters: a long-run mean b of 0.07, drift rate a of 0.18 and standard deviation of 0.02. Using these parameters Figure 51.2 shows two zero-coupon bond price curves that result from two different initial short rates, $r(t) = 4\%$ and $r(t) = 10\%$. The time to maturity T is measured on the x -axis, with the price of the zero-coupon bond with that time to maturity (a redemption value of 1) is measured along the y -axis.

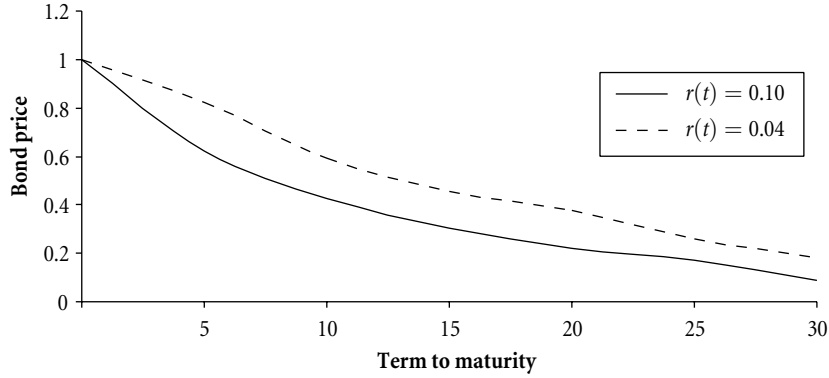


Figure 51.2: Zero-coupon bond price curves at $r(t) = 0.04$ and $r(t) = 0.10$.

For derived forward rates the bond price function $P(t, T)$ is continuously differentiable with respect to t . Therefore the model produces the following for the instantaneous forward rates:

$$\begin{aligned} f(t, T) &= -\frac{\partial}{\partial t} \ln P(t, T) \\ &= A'(T-t) + B'(T-t)r(t) = (1 - e^{-a(T-t)})b - \frac{1}{2}\nu(T-t) + e^{-a(T-t)}r(t) \\ &= f(r(t), T-t) \end{aligned} \quad (51.14)$$

where $f(r, T)$ is the function $f(r, T) = (1 - e^{-aT})b - \frac{1}{2}\nu(T) + e^{-aT}r$.

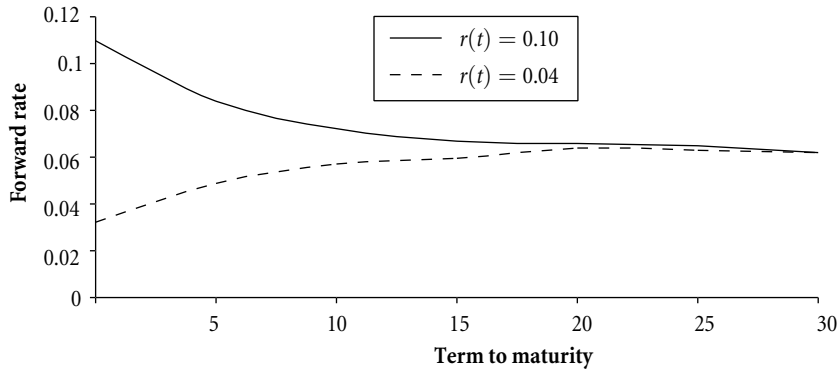


Figure 51.3: Forward rate curves with spot rate $r(t) = 0.04$ and $r(t) = 0.10$.

The forward rate is a function of the short-rate and is normally distributed. Figure 51.3 shows the forward rate curves that correspond to the price curves in Figure 51.2, under the same parameters.

An increase in the initial short-rate r will have the effect of raising forward rates, as will increasing the long-run mean value b . The effect of an increase in r is most pronounced at shorter maturities, whereas an increase in b has the greatest effect the longer the term to maturity. An equal increase or decrease in both r and b will have the effect of moving all forward rates up or down by the same amount. With these changes the forward curve moves up or down in a parallel fashion.

The derived forward rate is a decreasing function of the instantaneous standard deviation σ , one of the model parameters. The partial derivative of the forward rate with respect to the standard deviation is given at (51.15):

$$\frac{\partial f(r, T)}{\partial \sigma} = -\sigma B(T - s)^2 = -\frac{\sigma}{a^2}(-2e^{-aT} + e^{-2aT} + 1). \quad (51.15)$$

The expression at (51.15) states that the forward rate is a decreasing function of T , that is it becomes more negative as T becomes larger. The effect of the standard deviation on the forward rate is shown in Figure 51.4, which shows the two forward rate curves from Figure 51.3, with two additional forward rate curves where the standard deviation has been raised from 0.02 to 0.05.

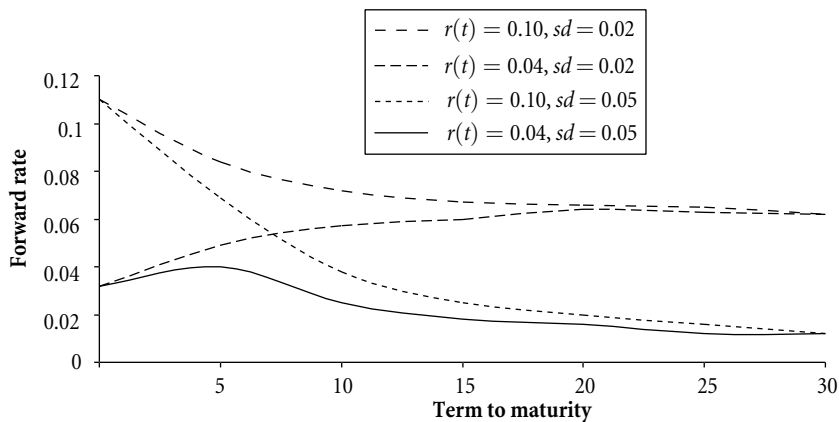


Figure 51.4: Forward rate curves with standard deviations of 0.02 and 0.05.

In describing the dynamics of the yield curve the Vasicek model only captures changes in the short rate r , and not the long-run average rate b .

A key point about this model is that as the short-rate follows a normal distribution, it has a positive probability of becoming negative at any point in time. This is common to all models that assume a Gaussian interest-rate process, and although it might be considered a significant drawback, in fact it will only be exhibited under extreme parameter values. For instance in the example at Figure 51.3 the forward rates are not unusual; however if we increase the standard deviation the effect will be to decrease forward rates, and this ultimately produces negative forward rates. For example if we calculate the forward rates for a standard deviation $\sigma = 0.09$, the result will be to produce negative rates, as shown in Figure 51.5. A negative forward rate is equivalent to a zero-coupon bond price that increases over time, which is clearly unrealistic under all but the most unusual and rare conditions. The reason that Gaussian interest-rate models can produce negative forward rates when the standard deviation is high is because the probability of achieving negative interest rates is high. Under certain parameter values, particularly under high values for the standard deviation, the probability of negative forward rates exists. However we saw that this is only under certain parameters, and in fact the presence of mean reversion makes this a low possibility.

It might be considered to be more realistic to consider that there are no constant parameters for the drift rate and the standard deviation that would ensure that the price of zero-coupon bond at any time is exactly the same as that suggested by observed market yields. For this reason a modified version of the Vasicek model has been described by Hull and White (1990), known as the Hull–White or extended Vasicek model, which we consider later.

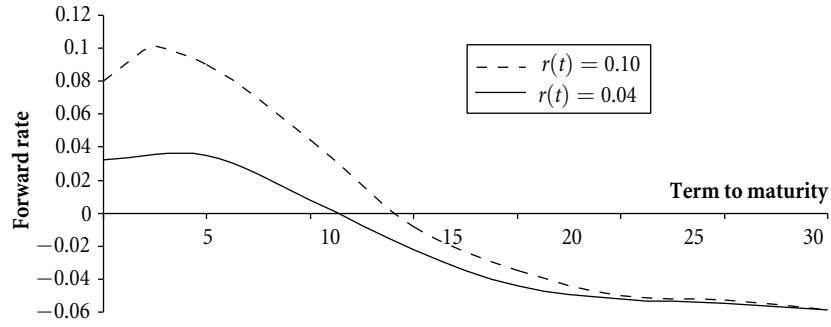


Figure 51.5: Forward rate curves under high volatility.

51.3.2 The Merton model

In Merton's model (1971) the interest-rate process is assumed to be a generalised Weiner process, described by (51.16):

$$r(t) = r_0 + \alpha t + \sigma W(t) \quad (51.16)$$

which in differential form is given by (51.17):

$$dr = \alpha dt + \sigma dW. \quad (51.17)$$

For $0 \leq t \leq T$ it can be shown that

$$r(T) = r(t) + \alpha(T - t) + \sigma(W(T) - W(t)). \quad (51.18)$$

The distribution of $r(T)$ is normal with a mean of $r(t) + \alpha(T - t)$ and standard deviation of $\sigma\sqrt{(T - t)}$.

For a fixed term to maturity T the forward rate $f(r(t), T - t)$ is an Itô process of the form:

$$\begin{aligned} df(r(t), T - t) &= dr - \alpha dt + \sigma^2(T - t)dt \\ &= \sigma^2(T - t)ds + \sigma dW. \end{aligned} \quad (51.19)$$

The continuously compounded yield at time t of a risk-free zero-coupon bond paying 1 on maturity at time T is given by:

$$\begin{aligned} R(t, T) &= \frac{1}{T - t} \ln \left(\frac{1}{P(r(t), T - t)} \right) \\ &= \frac{1}{T - t} A(T - t) + r(t) \\ &= R(r(t), T - t) \end{aligned} \quad (51.20)$$

where R is the function

$$R(r, T) = \frac{1}{T} A(T) + r = \frac{1}{2} \alpha T - \frac{\sigma^2}{6} T^2 + r. \quad (51.21)$$

The average future interest rate over the time period (t, T) is given by (51.22).

$$r_a = \frac{1}{T - t} \int_t^T r(s) ds. \quad (51.22)$$

In the Merton model forward rates will always be negative at long maturities, unlike the Vasicek model where there are a range of parameters under which the forward rates will be positive at all maturities. This is because although in both models the forward rate is negatively affected by the standard deviation of the future interest rate, which is an increasing function of the time to maturity, in the Merton model it changes in a linear fashion to infinity,

whereas in the Vasicek model it grows to a finite limit. Therefore the standard deviation is more powerful in the Merton model, and it results in the forward rates being negative at long maturities.

51.3.3 The Cox–Ingersoll–Ross model

From the previous section we see that under a model that assumes the short-rate to follow a normal distribution, there can arise instances of negative forward rates. The Cox–Ingersoll–Ross model (1985) is a one-factor model and as originally presented removed the possibility of negative rates.³ Under the CIR model the dynamics of the short-rate are described by (51.23):

$$dr = a(b - r)dt + \sigma\sqrt{r}dW \quad (51.23)$$

which like Vasicek also captures a mean-reverting phenomenon. However the stochastic term has a standard deviation that is proportional to $\sqrt{r}$. This is a significant difference because it states that as the short-rate increases, the standard deviation will decrease. This means that forward rates will be positive. In the CIR model the price of a risk-free zero-coupon bond is given by:

$$P(t, T) = A(t, T)e^{-B(t, T)r} \quad (51.24)$$

where

$$B(t, T) = \frac{2(e^{\gamma(T-t)} - 1)}{(\gamma + a)(e^{\gamma(T-t)} - 1) + 2\gamma}$$

$$A(t, T) = \left(\frac{2\gamma e^{(a+\gamma)(T-t)/2}}{(\gamma + a)(e^{\gamma(T-t)} - 1) + 2\gamma} \right)^{2ab/\sigma^2}$$

$$\gamma = \sqrt{a^2 + 2\sigma^2}.$$

The long-run interest rate $R(t, T)$ is a function of the short-rate $r(t)$, so that the short-rate only is all that is required to fit the entire term structure.

51.3.4 General comment

The Gaussian models, also called affine models (see for example, James and Webber 2000) are popular because they are straightforward to implement and they provide explicit numerical solutions when used in instrument pricing. Although Gaussian models allow negative interest rates under certain conditions, this is not necessarily a completely unrealistic trait, although some academic opinion holds that any model that allows negative interest rates cannot be correct and should not be used. Negative interest rates will only result under very specific conditions, which have a low probability (see for example Rogers 1995), and for this reason these models remain popular. However in an environment of low interest rates for instance, the CIR type models, which do not permit negative interest rates, may be preferred.

51.4 Arbitrage-free models

An equilibrium model of the term structure, of which we reviewed three in the previous section, is a model that is derived from (or consistent with) a general equilibrium model of the economy. They use generally constant parameters, including most crucially a constant volatility, and the actual parameters used are often calculated from historical time series data. Banks commonly also use parameters that are calculated from actual data and implied volatilities, which are obtained from the prices of exchange-traded option contracts.

An arbitrage-free model of the term structure on the other hand, can be made to fit precisely with the current, observed term structure, so that observed bond yields are in fact equal to the bond yields calculated by the model. So an arbitrage-free model is intended to be consistent with the currently observed zero-coupon yield curve, and the short-rate drift rate is dependent on time, because the future average path taken by the short-rate is determined by the shape of the initial yield curve. This means that in a positively sloping yield curve environment, the short rate r

³ Although formally published in 1985, the Cox–Ingersoll–Ross model was being circulated in academic circles from the mid-1970s onwards, which would make it one of the earliest interest-rate models.

will be increasing, on average, while it will be decreasing in an initial inverted yield curve environment. In a humped initial yield curve environment, the expected short-rate path will also be humped. In an equilibrium model the drift term for the short rate (that is, the coefficient of the dt term given above) is not dependent on time.

In theory, the price predicted by any model, were it to be observed in the market, would render that model to be an *arbitrage-free* one; however arbitrage-free models are so-called because they compare the model-predicted price to the actual market price. In an equilibrium model the initial term structure is a product of the model, while in an arbitrage-free model the actual term structure is an input to the model in the first place. In practice an equilibrium model may not be arbitrage-free under certain conditions; namely it may show small errors at particular points along the curve, or it may feature a large error across the whole term structure. The most fundamental issue in this regard is that the concept of the risk-free short-term interest-rate, is difficult to identify as an actual interest rate in the money market. In practice there may be more than one interest-rate that presents itself, for example the T-bill rate or the same maturity government bond repo rate, and this remains a current issue.

For these reasons practitioners may prefer to use an arbitrage-free model if one can be successfully implemented and calibrated. This is not always straightforward, and under certain conditions it is easier to implement an equilibrium multi-factor model (which we discuss in the next section) than it is to implement a multi-factor arbitrage-free model. Under one particular set of circumstances however it is always preferable to use an equilibrium model, and that is when reliable market data is not available. If modelling the term structure in a developing or “emerging” bond market, it will be more efficient to use an equilibrium model.

Some texts have suggested that equilibrium models can be converted into arbitrage-free models by making the short-rate drift rate time dependent. However this may change the whole nature of the model, presenting problems in calibration.

51.4.1 The Ho and Lee model

The Ho-Lee model (1986) was one of the first arbitrage-free models and was presented using a binomial lattice approach, with two parameters; the standard deviation of the short-rate, and the risk premium of the short-rate. We summarise it here. Following Ho and Lee, let $P_i^{(n)}(\cdot)$ be the equilibrium price of a zero-coupon bond maturing at time T under state i . That is $P_i^{(n)}(\cdot)$ is a discount function describing the entire term structure of interest rates, and will satisfy the following conditions:

$$\begin{aligned} P_i^{(n)}(0) &= 1 \\ \lim_{T \rightarrow \infty} P_i^{(n)}(T) &= 0. \end{aligned}$$

To describe the binomial lattice we denote the price at the initial time 0 as $P_0^{(0)}(\cdot) = 1$.

At time 1 the discount function is specified by two possible functions $P_1^{(1)}(0)$ and $P_0^{(1)}(0)$ which correspond respectively to the upside and the downside outcomes. Therefore at time n the binomial process is given by the discount function $P_i^{(n)}(\cdot)$ which can move upward to a function $P_{i+1}^{(n+1)}(\cdot)$ and downwards to a function $P_i^{(n+1)}(\cdot)$ for $i=0$ to n .

As described by Ho and Lee there are two functions denoted $h(T)$ and $h^*(T)$ that describe the upstate and downstate as (51.25) and (51.26) respectively, below,

$$P_{i+1}^{(n+1)}(T) = \left(\frac{P_i^{(n)}(T+1)}{P_i^{(n)}(1)} \right) h(T) \quad (51.25)$$

$$P_i^{(n+1)}(T) = \left(\frac{P_i^{(n)}(T+1)}{P_i^{(n)}(1)} \right) h^*(T) \quad (51.26)$$

with $h(0) = H^*(0) = 1$.

The two functions specify the deviations of the discount functions from the implied forward functions. To satisfy arbitrage-free conditions, they define an implied binomial probability π that is independent of time T , while the initial discount function $P(T)$ is given by:

$$\pi h(T) + (1 - \pi) h^*(T) = 1 \text{ for } n, i > 0 \quad (51.27)$$

and

$$P_i^{(n)}(T) = \left(\pi P_{i+1}^{(n+1)}(T-1) + (1-\pi) P_i^{(n+1)}(T-1) \right) P_i^{(n)}(1). \quad (51.28)$$

Equation (51.28) shows that the bond price is equal to the expected value of the bond, discounted at the prevailing one-period rate. Therefore π is the implied risk-neutral probability.

The assumption that the discount function evolves from one state to another as a function only on the number of upward and downward movements is equivalent to the assumption that a downward movement followed by an upward movement is equivalent to an upward movement followed by a downward movement. This produces the values for h and h^* given by (51.29).

$$h(T) = \frac{1}{\pi + (1-\pi)\delta^T} \text{ for } T \geq 0 \quad (51.29)$$

$$h^*(T) = \frac{\delta^T}{\pi + (1-\pi)\delta^T} \quad (51.30)$$

where δ is the interest-rate spread.

It has been shown⁴ that the model describes a continuous time process given by

$$dr = \theta(t)dt + \sigma dW(t) \quad (51.31)$$

where σ is the constant instantaneous standard deviation of the short-rate and $\theta(t)$ is a time-dependent function that describes the short-rate process and fits the model to the current observed term structure. This term defines the average direction that the short-rate moves at time t , which is independent of the short-rate. The variable $\theta(t)$ is given by:

$$\theta(t) = f(0,t) + \sigma^2 t \quad (51.32)$$

where $f(0,t)$ is the instantaneous forward rate for the period t at time 0. In fact the term $\theta(t)$ approximates to $f(0,t)$ which states that the average direction of the short-rate in the future is given by the slope of the instantaneous forward curve.

The Ho and Lee model is straightforward to implement and is regarded by practitioners as convenient because it uses the information available from the current term structure, so that it produces a model that precisely fits the current term structure. It also requires only two parameters. However it assigns the same volatility to all spot and forward rates, so the volatility structure is restrictive for some market participants. In addition the model does not incorporate mean reversion.

51.4.2 The Hull–White model

The Hull–White model (1990) is an extension of the Vasicek model designed to produce a precise fit with the current term structure of rates. It is also known as the *extended Vasicek model*, with the interest rate following a process described by (51.33):

$$dr = (\alpha - ar)dt + \sigma dW(t). \quad (51.33)$$

It is also sometimes written as

$$dr = a\left(\frac{\alpha}{a} - r\right)dt + \sigma dW(t) \quad (51.34)$$

where a is the mean reversion rate and a and σ are constants. It has been described as a Vasicek model with a time-dependent reversion level. The model is also called the *general Hull–White model*, while a special case where $a \neq 0$ is known as the simplified Hull–White model. In the Vasicek model $a \neq 0$ and $\alpha = ab$ where b is constant.

The Hull–White model can be fitted to an initial term structure, and also a volatility term structure. A comprehensive analysis is given in Pelsser (1996) as well as James and Webber (2000).

⁴ For example, see Hull (1997).

It can be shown that

$$r(t) = e^{-K(t)} \left(r_0 + \int_t^T e^K \alpha \, dt + \int_t^T e^K \sigma \, dW(t) \right) \quad (51.35)$$

where the process K is given by $K(t) = \int_t^T a \, dt$.

To calculate the price of a zero-coupon bond, the first step is to calculate the integral $I(t, T) = \int_t^T r \, ds$ which follows a normal distribution with mean $m(r(t), t; T)$ and standard deviation $\sqrt{v(t; T)}$. The price of a bond is given by (51.36):

$$\begin{aligned} P(t, T) &= E_Q[\exp[-I(t, T)] \mid F_t] \\ &= \exp(-m(r(t), t; T) + \tfrac{1}{2}v(t; T)) \\ &= P(r(t), t; T) \end{aligned} \quad (51.36)$$

where $P(r, t; T)$ is the function

$$P(r(t), t; T) = \exp(-m(r, t; T) + \tfrac{1}{2}v(t; T)). \quad (51.37)$$

The price of a zero-coupon discount at time t is defined in terms of the short-rate r at time t and the current term structure. The price function is not static, and the price of a bond at time t that matures at time T is a function of the short-rate, as we have noted, and separately of the time t .

The volatility of the bond price is given by the function $B(t; T)\sigma(t)$ where B is defined as

$$B(t, T) = \int_t^T e^{K(u)+K(t)} \, du = e^{K(t)} \int_t^T e^{-K} \, du. \quad (51.38)$$

The bond price volatility is a deterministic function of t . The “pull to par” of the zero-coupon bond is captured by the fact that the volatility reduces to zero as t approaches T , as long as σ is continuous at t . As the mean m is normally distributed, it follows that the bond price is lognormally distributed, so therefore we have the function

$$\ln P(t, T) = -A(t, T) - B(t, T)r(t)$$

where $A(t, T)$ is defined by $A(t, T) = \int_t^T e^{-K(u)} \int_t^u e^K \alpha \, dx \, du - \tfrac{1}{2}v(t, T)$.

The price function above can be continuously differentiated as a function of t . The forward rate is given by (51.39):

$$\begin{aligned} f(t, T) &= -\frac{\partial}{\partial T} \ln P(t, T) \\ &= A_T(t, T) + B_T(t, T)r(t) \\ &= e^{-K(T)} \int_t^T \alpha \, dx - \tfrac{1}{2}v_T(t, T) + e^{-K(T)+K(t)}r(t) \\ &= f(r(t); t, T) \end{aligned} \quad (51.39)$$

where $f(r(t); t, T)$ is defined by the function below:

$$f(r(t); t, T) = e^{-K(T)} \int_t^T \alpha \, dx - \tfrac{1}{2}v_T(t, T) + e^{-K(T)+K(t)}r. \quad (51.40)$$

The forward rate function f at time t is not static and is a function of the short rate r at time t , the time t and the time to maturity to time T . The Hull–White model can be calibrated in terms of the forward rate f . That is, at time t the information (parameters) required to implement this is the short rate $r(t)$, the standard deviation σ of the short-rate, the forward rate f and the standard deviations $B_T(t, T)\sigma(t)$ of the forward rates at time t . If the forward rates are known in a form that allows their first differential to be calculated with respect to t , using the other information it is possible to calculate the function B_T , the derivative of this function and thereby the value for $a(t)$, using the relationship at (51.41):

$$a(t) = -\frac{B'_T(t, T)}{B_T(t, T)} \quad (51.41)$$

which describes the volatility of the bond price as a function of the maturity date T .

The continuously compounded yield of a zero-coupon bond at time t that matures at time T is shown to be

$$\begin{aligned} R(t, T) &= \frac{1}{T-t} \left(m(r(t), t; T) - \frac{1}{2} v(t, T) \right) \\ &= \frac{1}{T-t} \ln \left(\frac{1}{P(t, T)} \right) \\ &= \frac{1}{T-t} (A(t, T) + B(t, T)r(t)) \\ &= R(r(t), t, T) \end{aligned} \quad (51.42)$$

where R is given by the function shown at (51.43):

$$R(r, t, T) = \frac{1}{T-t} (A(t, T) + B(t, T)r). \quad (51.43)$$

Like the bond price function, the yield on a zero-coupon bond is a function of the short-rate r and follows a normal distribution; the yield curve is a function of the short-rate r , the time t and the time to maturity T . The long-run average future interest over the time to maturity (t, T) is normally distributed and given by:

$$r_a = \frac{1}{T-t} \int_t^T r(s) ds = \frac{1}{T-t} I(t, T). \quad (51.44)$$

51.4.3 The Black–Derman–Toy model

In the models we have reviewed in this chapter there has only been one function of time, the parameter α . In certain models either or both of the parameters a and σ are also made to be functions of time. In their 1990 paper Black, Derman and Toy (BDT) proposed a binomial lattice model described by (51.45),

$$d \ln r = \left(\alpha + \frac{\sigma'(t)}{\sigma(t)} \ln(r) \right) dt + \sigma(t) dW \quad (51.45)$$

where $\sigma'(t)$ is the partial derivative of σ with respect to time t . The BDT model is a lognormal model, which means that the short-rate volatility is proportional to the instantaneous short-rate, so that the ratio of the volatility to the rate level is constant. The drift term is more complex than that described in the earlier models, and so the BDT model requires numerical fitting to the observed current interest-rate and volatility term structures. That is, the drift term is not calculated analytically. The short-rate volatility is also linked to the mean reversion such that where long-term rates are less volatile than the short-rate, the short-rate volatility will decrease in the long-term. A later model developed by Black and Karasinski (1992) removed the relationship between mean reversion and the volatility level. This is given at (51.46):

$$d \ln r = (\theta(t) - a(t) \ln(r)) dt + \sigma(t) dW(t). \quad (51.46)$$

As with the previous models the key factor is the short-rate. Using the binomial tree approach, a one-step tree is used to derive the current short-rate to the short-rates one period in the future. These derived rates are then used to derive rates two periods away, and so on.

51.5 Fitting the model

Implementing an interest-rate model requires the input of the term structure yields and volatility parameters, which are used in the process of calibrating the model. The process of fitting the model is called *calibration*. This can be done in at least three ways, which are:

- calibration to the current spot rate yield curve, using a pre-specified volatility level and not the volatility values given by the prices of exchange-traded options. This may result in mispriced bonds and options if the selected volatilities are not accurate;
- calibration to the current spot rate curve and using the volatilities implied by the prices of exchange-traded options; therefore the model would be implemented using volatility parameters that were exactly similar to those implied by the traded option prices. In practice this can be a lengthy process;
- calibrating the model to the current spot rate curve, using volatility parameters that are approximately close enough to result in prices that are near to those of observed exchange-traded options. This is usually the method that is adopted.

Generally volatility values for the different period interest rates are taken from the volatilities of exchange-traded options. However where great accuracy is not required, for example for regulatory capital purposes practitioners may use the first method, while for the purposes of fixed income research the third method is suitable. In both the second and third methods there is the danger that calibrating the model to option prices will result in error simply because the options are mispriced. This is quite possible if using long-dated and/or OTC options, which frequently differ in price according to which bank is pricing them.

In any case a model will usually therefore use volatility inputs from option prices for a range of options that range in maturity from the shortest period to the longest in the term structure. To test the accuracy of the model, one can use the expression at (51.47):

$$\sum_{n=1}^N (p_n - P_n)^2 \quad (51.47)$$

where p_n is the observed price of the n 'th option and P_n is the price of the option as calculated by the model, and N options have been used to calibrate the model. A model that has the lowest value given by (51.47) can be considered to be the most accurate. In deciding which option products should be used to calibrate the model, care should be taken to use instruments that are most similar to the instrument that is being priced by the model.

The different models can lend themselves to a particular calibration method. In the Ho–Lee model, only parallel yield curve shifts are captured and the current yield curve is a direct input; therefore a constant volatility parameter is used. This implies that all the forward rate implied volatilities are identical. In practice this is not necessarily realistic, as long-dated bond prices often experience lower volatility than short-dated bond prices. The model also assumes that volatility is a decreasing function of the time to maturity, which may also be unrealistic. Models that incorporate mean reversion can be implemented with more realistic volatility parameters, as it is the mean reversion effect that results in long-dated bonds having lower volatilities. Therefore a mean-reverting model can be implemented more accurately using the second or third methods described above.

To recap on the issues involved in fitting the extended Vasicek model or Hull–White model; this describes the short rate process as following the form

$$dr = (\alpha - ar)dt + \sigma dW(t). \quad (51.48)$$

In implementing this model, there are three possible approaches. The model could be calibrated by keeping α and a constant and calibrating the standard deviation parameter. This means that the model is fitted to the current yield curve and the volatility value is adjusted to that required to produce the observed curve. However this may result in high volatility values, which rise by a squared function, and therefore will not be realistic. The second method is to calibrate α , keeping the other two parameters constant. This is adjusting the mean reversion rate in order to fit the derived curve with the observed curve. The resulting derived yield curve will be a function of the current short-rate and the mean reversion rate. This method is sometimes applied in practice, although it can result in inaccurate volatility levels for long-dated bonds, because large adjustments in the mean reversion rate are needed to fit the derived curve to the long-dated part of the observed curve. The third approach would be to calibrate ar , keeping the other parameters constant. This produces a stable yield curve and is most commonly followed by practitioners in the market.

51.6 Summary

In this chapter we have considered both equilibrium and arbitrage-free interest-rate models. These are one-factor Gaussian models of the term structure of interest rates. We saw that in order to specify a term structure model, the respective authors described the dynamics of the price process, and that this was then used to price a zero-coupon bond. The short-rate that is modelled is assumed to be a risk-free interest rate, and once this is modelled we can derive the forward rate and the yield of a zero-coupon bond, as well as its price. So it is possible to model the entire forward rate curve as a function of the current short-rate only, in the Vasicek and Cox–Ingersoll–Ross models, among others. Both the Vasicek and Merton models assume constant parameters, and because of equal probabilities of forward rates and the assumption of a normal distribution, they can, under certain conditions relating to the level of the standard deviation, produce negative forward rates.

The models are based on the fact that the price of a bond, which exhibits a pull-to-par effect, and the forward rate, are both Itô processes. For the bond price the relative drift is the interest rate, and is deterministic, as is the forward rate. The bond price, yield and forward rate are functions of the current short-rate, and follow a normal distribution. An increase in the short-rate will result in a rise in the forward rates, and this is more pronounced for the shortest maturity rates. The instantaneous volatility of the forward rates decreases with decreasing time to maturity, and approaches the volatility of the current short-rate at time t .

The Vasicek, Cox–Ingersoll–Ross, Hull–White and other models incorporate mean reversion. As the time to maturity increases and as it approaches infinity, the forward rates converge to a point at the long-run mean reversion level of the current short-rate. This is the limiting level of the forward rate and is a function of the volatility of the current short-rate. As the time to maturity approaches zero, the short-term forward rate converges to the same level as the instantaneous short-rate. In the Merton and Vasicek models the mean of the short-rate over the maturity period T is assumed to be constant. The same constant for the mean, or the *drift* of the interest rate, is described in the Ho–Lee model, but not the extended Vasicek or Hull–White model.

We also noted that the efficacy of a model was not necessarily solely related to how realistic its assumptions might be, but how straightforward it was to implement in practice, that is, the ease with which it could be calibrated.

Selected bibliography and references

- Baxter, M., Rennie, A., *Financial Calculus*, Cambridge University Press, 1996.
- Black, F., Derman, E., Toy, W., “A one-factor model of interest rates and its application to Treasury bond options”, *Financial Analysts Journal*, Jan–Feb 1990, pp. 33–39.
- Black, F., Karasinski, P., “Bond and option prices when short rates are lognormal”, *Financial Analysts Journal*, Jul–Aug 1991, pp. 52–59.
- Chan, K., *et al.*, “An empirical comparison of alternative models of the short-term interest rate”, *Journal of Finance* 47, 1992, pp. 1209–1227.
- Cox, J., Ingersoll, J., Ross, S., “A theory of the term structure of interest rates”, *Econometrica* 53, 1985, pp. 385–407.
- Duffie, D., *Dynamic Asset Pricing Theory*, 2nd edition, Princeton University Press, 1996.
- Ho, T., Lee, S., “Term structure movements and pricing interest rate contingent claims”, *Journal of Finance* 41, 1986, pp. 1011–1029.
- Hull, J., White, A., “Pricing interest-rate derivative securities”, *Review of Financial Studies* 3, 1990, pp. 573–592.
- James, J., Webber, N., *Interest Rate Modelling*, Wiley, 2000.
- Jamshidian, F., “Bond, Futures and Option Valuation in the Quadratic Interest Rate Model”, *Applied Mathematical Finance* 3, 1996, pp. 93–115.
- Jarrow, R., *Modelling Fixed Income Securities and Interest Rate Options*, McGraw-Hill, 1996.
- Merton, R., “Optimum consumption and portfolio rules in a continuous time model”, *Journal of Economic Theory* 3, 1971, pp. 373–413.
- Merton, R., “Theory of option pricing”, *Bell Journal of Economics and Management Science* 4, 1973, pp. 141–183.
- Pelsser, A., “Efficient methods for valuing and managing interest rate and other derivative securities”, PhD thesis, Erasmus University, Rotterdam, 1996.
- Phoa, W., *Advanced Fixed Income Analytics*, FJF Associates, 1998.
- Pliska, S., “A stochastic calculus model of continuous trading: optimal portfolios”, *Mathematics of Operations Research* 11, 1986, pp. 371–382.

- Rogers, L., “Which model for the term-structure of interest rates should one use?” in Davis, M., *et al.* (eds.), *Mathematical Finance*, Springer-Verlag, 1995, pp. 93–115.
- Rebonato, R., *Interest Rate Option Models*, 2nd edition, Wiley, 1998.
- Steeley, J., “Estimating the Gilt-Edged Term Structure: Basis Splines and Confidence Intervals”, *Journal of Business Finance and Accounting* 18, 1991, pp. 513–530.
- Van Deventer, J., Imai, K., *Financial Risk Analysis*, McGraw-Hill, 1997.
- Vasicek, O., “An equilibrium characterization of the term structure”, *Journal of Financial Economics* 5, 1977, pp. 177–188.

Questions and exercises

1. In the Vasicek model what is the effect of increasing the current short-rate? Of increasing the standard deviation?
2. For the one-factor Ho–Lee model, construct a binomial tree with the following parameters: a is 0.07, standard deviation is 0.06, the instantaneous short-rate is 5.50% with a drift of $0.02t$. If one time period is six months, use the tree to calculate the price of a government zero-coupon bond with a maturity of 1.5 years.
 - (a) Write down the process that describes the dynamics of the forward rate $f(t, T)$ in the Vasicek model.
3. Repeat the question above for the Cox–Ingersoll–Ross model and the extended Vasicek model.
4. You are given the following values: a is 0.25, b is 0.05 and the standard deviation is 0.095. Use the Vasicek model to calculate the value of a six-month European call option written on a zero-coupon bond that matures in one year at par value of 100, when the option is struck at 94 and the current short-rate is 6.50%.
5. Certain interest-rate models assume that short-rates (as well as bond price and yields, and forward rates) follow a normal distribution and that all possible values of the rate have equal probability of occurring. What effect does this have in practice under certain conditions that may render the model unrealistic?
6. The current short-rate is 10%, and is assumed to have an annualised volatility of 1%. At a future date the short-rate moves to 5%. What assumption is made about the short-rate volatility in
 - (a) the Vasicek model
 - (b) the Cox–Ingersoll–Ross model
 - (c) the Black–Derman–Toy model?
7. Calculate the price of a risk-free zero-coupon bond that expires in five years’ time, when the current short-rate is 6.50%, a is 0.30, b is 0.08 and the standard deviation of the short-rate is 5%. Use the Vasicek model, the Ho–Lee model and the Cox–Ingersoll–Ross model and compare the results.
8. Explain the difference between an equilibrium model and an arbitrage-free model.
9. What are advantages and disadvantages of a one-factor model compared to a two-factor or multi-factor model?

52 Interest-rate Models II

In this chapter we consider multi-factor and whole yield curve models. As we noted in the previous chapter, short-rate models have certain drawbacks, which though not necessarily limiting their usefulness, do leave room for further development. The drawbacks are that, as the single short-rate is used to derive the complete term structure, in practice this can be unsuitable for the calculation of bond yields. When this happens it becomes difficult to visualise the actual dynamics of the yield curve, and the model no longer fits observed changes in the curve. This means that the accuracy of the model cannot be observed. Another drawback is that in certain equilibrium model cases the model cannot be fitted precisely to the observed yield curve, as they have constant parameters. In these cases calibration of the model is on a “goodness of fit” or best fit approach.

In response to these issues interest-rate models have been developed that model the entire yield curve. In a whole yield curve, the dynamics of the entire term structure are modelled. The Ho–Lee model is a simple type of whole curve model, which allows random parallel shifts in the yield curve. More advanced models, the Heath–Jarrow–Morton family of models, are discussed in this chapter, as are factors involved in their implementation.

52.1 Introduction

A landmark development in interest-rate modelling has been the specification of the dynamics of the complete term structure. In this case the volatility of the term structure is given by a specified function, which may be a function of time, term to maturity or zero-coupon rates. A simple approach is described in the Ho–Lee model, in which the volatility of the term structure is a parallel shift in the yield curve, the extent of which is independent of the current time and the level of current interest rates. The Ho–Lee model is not widely used, although it was the basis for the Heath–Jarrow–Morton (HJM) model, which is widely used. The HJM model describes a process whereby the whole yield curve evolves simultaneously, in accordance with a set of volatility term structures. The model is usually described as being one that describes the evolution of the forward rate, however it can also be expressed in terms of the spot rate or of bond prices (see for example James and Webber (2000), Chapter 8). For a more detailed description of the HJM framework refer to Baxter and Rennie (1996), Hull (1997), Rebonato (1998), Bjork (1998) and James and Webber (2000). Baxter and Rennie is very accessible, while Neftci (1996) is an excellent introduction to the mathematical background.

In seeking to develop a model for the entire term structure, the requirement is to model the behaviour of the entire forward yield curve, that is the behaviour of the forward short-rate $f(t, T)$ for all forward dates T . Therefore we require the random process $f(T)$ for all forward dates T . Given this, it can be shown that the yield R on a T -maturity zero-coupon bond at time t is the average of the forward rates at that time on all the forward dates s between t and T , given by (52.1):

$$R(t, T) = \frac{1}{T - t} \int_t^T f_t(s) ds. \quad (52.1)$$

To model the complete curve it is necessary to specify a drift rate and volatility level for $f(t, T)$ for each T .

52.2 The Heath–Jarrow–Morton model

A landmark development in the longstanding research into yield curve modelling was presented by David Heath, Robert Jarrow and Andrew Morton in their 1989 paper, which formally appeared in volume 60 of *Econometrica* (1992).¹ The paper considered interest-rate modelling as a stochastic process, but applied to the entire term structure rather than only the short-rate. The importance of the Heath–Jarrow–Morton (HJM) presentation is this: that in a market that permits no arbitrage, where interest rates including forward rates assumed to follow a Weiner process, the drift term and the volatility term in the model’s stochastic differential equation are not independent from each other, and in fact the drift term is a deterministic function of the volatility term. This has significant practical implications for the pricing and hedging of interest-rate options.

¹ In the author’s opinion perhaps the most technical of the quantitative academic research journals!

The general form of the HJM model is very complex, not surprisingly as it is a multi-factor model. We begin by describing the single-factor HJM model. This section is based on Chapter 5 of Baxter and Rennie, *Financial Calculus*, Cambridge University Press (1996), and follows their approach with permission. The reference was noted in Chapter 51, this work is an accessible and excellent text and is highly recommended.

52.2.1 The single-factor HJM model

In the previous chapter, and indeed in previous analysis, we have defined the *forward rate* as the interest rate applicable to a loan made at a future point in time and repayable instantaneously. We assume that the dynamics of the forward rate follow a Wiener process. The *spot rate* is the rate for borrowing undertaken now and maturing at T , and we know from previous analysis that it is the geometric average of the forward rates from 0 to T that is

$$r(0, T) = T^{-1} \int_0^T f(0, t) dt. \quad (52.2)$$

We also specify a money market account that accumulates interest at the continuously compounded spot rate r .

A default-free zero-coupon bond can be defined in terms of its current value under an *initial probability measure*, which is the Wiener process that describes the forward rate dynamics, and its price or present value under this probability measure. This leads us to the HJM model, in that we are required to determine what is termed a “change in probability measure”, such that the dynamics of the zero-coupon bond price are transformed into a *martingale*. This is carried out using Itô’s lemma and a transformation of the differential equation of the bond price process. It can then be shown that in order to prevent arbitrage there would have to be a relationship between drift rate of the forward rate and its volatility coefficient.

First we look at the forward rate process. We know from the previous chapter for $[0, T]$ at time t that the stochastic evolution of the forward rate $f(t, T)$ can be described as

$$df(t, T) = a(t, T)dt + \sigma(t, T)dW_t \quad (52.3)$$

or alternatively in integral form as

$$f(t, T) = f(0, T) + \int_0^t a(s, T)ds + \int_0^t \sigma(s, T)dW_s \quad (52.4)$$

where a is the drift parameter, σ the volatility coefficient and W_t is the Wiener process or Brownian motion. The terms dz or dZ are sometimes used to denote the Wiener process.

In (52.3) the drift and volatility coefficients are functions of time t and T . For all forward rates $f(t, T)$ in the period $[0, T]$ the only source of uncertainty is the Brownian motion. In practice this would mean that all forward rates would be perfectly positively correlated, irrespective of their terms to maturity. However if we introduce the feature that there is more than one source of uncertainty in the evolution of interest rates, it would result in less than perfect correlation of interest rates, which is what is described by the HJM model.

Before we come to that however we wish to describe the spot rate and the money market account processes. In (52.4) under the particular condition of the maturity point T as it tends towards t (that is $T \rightarrow t$), the forward rate tends to approach the value of the short rate (spot rate), so we have

$$\lim_{T \rightarrow t} f(t, T) = f(t, t) = r(t)$$

so that it can be shown that

$$r(t) = f(0, t) + \int_0^t a(s, t)ds + \int_0^t \sigma(s, t)dW_s. \quad (52.5)$$

The money market account is also described as a Wiener process. We denote by $M(t, t) \equiv M(t)$ the value of the money market account at time t , which has an initial value of 1 at time 0 so that $M(0, 0) = 1$. This account earns interest at the spot rate $r(t)$ which means that at time t the value of the account is given by

$$M(t) = e^{\int_0^t r(s)ds} \quad (52.6)$$

that is the interest accumulated at the continuously compounded spot rate $r(t)$. It can be shown by substituting (52.5) into (52.6), that

$$M(t) = \exp\left(\int_0^t f(0,s)ds + \int_0^t \int_0^s a(u,s)duds + \int_0^t \int_0^s \sigma(u,s)dW_u ds\right). \quad (52.7)$$

To simplify the description we write the double integrals in (52.7) (!) in the form given below, which is

$$\int_0^t \int_s^t a(s,u)duds + \int_0^t \int_s^t \sigma(s,u)dudW_s.$$

For reasons of space the description of the process by which this simplification is achieved is relegated to a page on the author's Web site.

Using the simplification above, it can be shown the value of the money market account, which is growing by an amount generated by the continuously compounded spot rate $r(t)$, is given by

$$M(t) = \exp\left(\int_0^t f(0,u)du + \int_0^t \int_s^t a(s,u)duds + \int_0^t \int_s^t \sigma(s,u)dudW_s\right). \quad (52.8)$$

The expression for the value of the money market account can be used to determine the expression for the zero-coupon bond price, which we denote $P(t,T)$. The money market account earns interest at the spot rate $r(t)$, while the bond price is the present value of 1 discounted at this rate. Therefore the inverse of (52.8) is required, which is

$$M^{-1}(t) = e^{-\int_0^t r(u)du}. \quad (52.9)$$

Hence the present value at time 0 of the bond $P(t,T)$ is

$$P(t,T) = e^{-\int_0^t r(u)du} P(t,T)$$

and it can be shown that as a Wiener process the present value is given by

$$P(t,T) = \exp\left(-\int_0^T f(0,u)du - \int_0^t \int_s^T \sigma(s,u)dudW_s - \int_0^t \int_s^T a(s,u)duds\right). \quad (52.10)$$

52.2.2 Transforming the probability measure

Since the pioneering work of Harrison and Pliska (1981), which recognised that the absence of arbitrage was linked to the existence of a martingale probability measure, the valuation of derivatives has been deemed to require a probability measure that would transform the underlying security process into a martingale. This is the case here, what is required is a change in probability measure such that $P(t,T)$ becomes a martingale.

This is done by using Itô's lemma² to transform the stochastic differential equation of the price process, and then determine the change in the Brownian differential dW so that there remains no drift term. The first step is to consider the differential of $P(t,T)$. We express this in the form

$$P(t,T) = e^{-X_t} \quad (52.11)$$

where X_t is a Wiener process described by

$$X_t = \int_0^T f(0,u)du + \int_0^t \int_s^T \sigma(s,u)dudW_s + \int_0^t \int_s^T a(s,u)dudt. \quad (52.12)$$

The differential of X_t is written as

$$\begin{aligned} dX_t &= \int_t^T \sigma(t,u)dudW_t + \int_t^T a(t,u)dudt \\ &= v(t,T)dW_t + \int_t^T a(t,u)dudt \end{aligned} \quad (52.13)$$

where $v(t,T) \equiv \int_t^T \sigma(t,u)du$ represents the volatility element of the X_t stochastic process. It can be shown by applying Itô's lemma to (52.11) that

² See Chapter 43.

$$dP(t, T) = P(t, T) \left(-v(t, T) dW_t - \int_t^T a(t, u) du dt + \frac{v^2}{2}(t, T) dt \right). \quad (52.14)$$

To obtain a new probability measure such that $P(t, T)$ is transformed into a martingale, following Baxter and Rennie we effect a change in dW_t such that (52.14) may be expressed as

$$dP(t, T) = -P(t, T) v(t, T) d\tilde{W}_t. \quad (52.15)$$

It can then be shown that the solution of (52.15) under such conditions is indeed a martingale, described by

$$P(t) = P_0 \exp \left(\int_0^t v(\tau, T) d\tilde{W}_\tau - \frac{1}{2} \int_0^t v^2(\tau, T) d\tau \right). \quad (52.16)$$

To reiterate, then, we require a transformation of (52.14) so that it becomes a Weiner process with no drift term, in other words a relationship between $d\tilde{W}_t$ and dW_t . It has been shown that this exists in the form

$$d\tilde{W}_t = dW_t + \frac{1}{v(t, T)} \int_t^T a(t, u) du dt - \frac{v(t, T)}{2} dt \equiv dW_t + \gamma_t dt \quad (52.17)$$

and where the change of measure γ_t is given by

$$\gamma_t = \frac{1}{v(t, T)} \int_t^T a(t, u) du - \frac{v(t, T)}{2}. \quad (52.18)$$

52.2.3 The principle of no-arbitrage

The demonstration of the no-arbitrage condition in the evolution of the HJM model is perhaps its most significant aspect, as it demonstrated that for arbitrage to be avoided, the volatility function must be related to the drift parameter. This is effected through a constraint that is the change of measure element we introduced just now: again, following Baxter and Rennie and to summarise the original paper, in order to prevent arbitrage, a bond of maturity less than T must have the same change of measure γ_t . The change of measure must by implication therefore not be a function of and be independent from T . It can be shown that if we multiply (52.18) by $v(t, T)$ and then differentiate it with respect to T we obtain (52.19):

$$\begin{aligned} a(t, T) &= \frac{\partial v(t, T)}{\partial T} (v(t, T) + \gamma_t) \\ &= \sigma(t, T) (v(t, T) + \gamma_t). \end{aligned} \quad (52.19)$$

The expressions (52.18) and (52.19) represent the two fundamental constraints of the single-factor HJM model. (52.18) is the result of the change in the drift term required by the transformation of $P(t, T)$ into a martingale, while (52.19) comes from the need to incorporate a no-arbitrage condition. This is the model in essence, an expression for the value of the drift parameter for $f(t, T)$ in the context of the $\tilde{W}_t$ Brownian motion. This impacts as follows: in (52.3) the Brownian motion term dW_t is replaced by $d\tilde{W}_t - \gamma_t dt$ and $a(t, T)$ is replaced with the constraint given by (52.19). This results in

$$\begin{aligned} df(t, T) &= \sigma(t, T) (d\tilde{W}_t - \gamma_t dt) + \sigma(t, T) (v(t, T) + \gamma_t) dt \\ &= \sigma(t, T) d\tilde{W}_t + \sigma(t, T) v(t, T) dt. \end{aligned} \quad (52.20)$$

In conclusion, then, in the single-factor HJM model under the martingale measure the coefficient of the drift must be equal to (52.21):

$$\sigma(t, T) v(t, T) = \sigma(t, T) \int_t^T \sigma(t, u) du. \quad (52.21)$$

In an important application of the HJM model Jarrow and Turnbull (1996) express the price of a zero-coupon risk-free bond as a function of the spot rate $r(t)$, given by

$$P(t, T) = \frac{P(0, T)}{P(0, t)} \exp \left(-(r(t) - f(0, t)) X(t, T) - \frac{\sigma^2}{4\lambda} X^2(t, T) (1 - e^{-2\lambda t}) \right) \quad (52.22)$$

where $X(t, T) \equiv (1 - e^{-\lambda(T-t)})/\lambda$ and σ and λ are positive constants of the volatility coefficient $\sigma(t, T)$ which is of the form $\sigma \exp(-\lambda(T-t))$.

Thus at time t the price of any bond of maturity T is given by the ratio of prices of bonds of maturity t and T , which are the first part of the (52.22) and which are observed in the market, and the random variable given by the exponential element of (52.22). If we express the latter as $A(t, T)$ we have

$$P(t, T) = \frac{P(0, T)}{P(0, t)} A(t, T) \quad (52.23)$$

and under conditions of zero volatility where $\sigma(t, T) = 0$ it can be shown that this element disappears and

$$\begin{aligned} P(t, T) &= \frac{P(0, T)}{P(0, t)} = \frac{e^{-rT}}{e^{-rt}} \\ &= e^{-r(T-t)} \end{aligned} \quad (52.24)$$

which is exactly what we expect.

52.3 Multi-factor term structure models

Previously we considered one-factor models used to varying degrees in the market; these describe only a single kind of change in the yield curve, the parallel shift. In practice there are a range of changes that may occur to the curve, including non-parallel (pivotal) shifts and changes in the slope of the curve. Certain two-factor and multi-factor models have been developed that seek to describe the different type of yield curve shifts. An early two-factor model was that presented by Brennan and Schwartz (1982) which described the stochastic process of the short-rate r and the yield of the long-dated government bond R . In the model these two factors move independently of each other, thus permitting both parallel and pivotal changes in the yield curve. The Brennan–Schwartz model is categorised as a *consol* model in James and Webber (2000). A modified version of the Brennan–Schwartz model³ has been developed in which the two variable factors are the price of the long bond $P = 1/R$ and the spread between the long-dated yield and the short-rate, which is $S = R - r$. Both factors are assumed to follow a random stochastic process described by (52.25):

$$\begin{aligned} dP &= \mu_P P dt + \sigma_P P dW_t^{(1)} \\ dS &= \mu_S dt + \sigma_S dW_t^{(2)} \end{aligned} \quad (52.25)$$

where $E[dW_t^{(1)} dW_t^{(2)}] = \rho dt$, and the values of μ_P and μ_S are set to be arbitrage-free in terms of the price of bonds of different maturities. So the price of the long-dated bond follows a lognormal process, while the spread S follows a Gaussian process. This means that the spread can be either positive or negative, which permits both a positive sloping or an inverted yield curve.

The modified Brennan–Schwartz model is used in the markets and describes a realistic process for changes in the yield curve and are relatively straightforward to implement; only two variables are required to model the entire term structure.

The Heath–Jarrow–Morton model (1992) is a general approach which is a multi-factor whole yield curve model, where arbitrary changes in the entire term structure can be one of the factors. In practice because of the mass of data that is required to derive the yield curve, the HJM model is usually implemented by means of Monte Carlo simulation, and requires powerful computing systems. The model is described in the next section.

52.3.1 The multi-factor Heath–Jarrow–Morton model

A multi-factor model of the whole yield curve has been presented by Heath, Jarrow and Morton (1992).⁴ This is a seminal work and a ground-breaking piece of research. The approach models the forward curve as a process arising from the entire initial yield curve, rather than the short-rate only. The spot rate is a stochastic process and the derived yield curve is a function of a number of stochastic factors. The HJM model uses the current yield curve and forward

³ See Rebonato and Cooper (1996).

⁴ Heath, D., Jarrow, R., and Morton, A., “Bond pricing and the term structure of interest rates: a new methodology”, *Econometrica* 60(1), 1992, pp. 77–105.

rate curve, and then specifies a continuous time stochastic process to describe the evolution of the yield curve over a specified time period.

The model is summarised here only; readers interested in the derivation of the model are directed to the original paper or a discussion of it in Baxter and Rennie (1996), Hull (1997) or James and Webber (2000). To describe the model we use the following notation:

(t, T) is the trading interval for a fixed period from t to T , where $t > 0$

$W(t)$ is the independent Brownian motion or Wiener process that describes the interest rate process

(Ω, F, Q) is the probability space where F is the σ -algebra representing measurable events and Q is the measure of probability

$P(t, T)$ is the price at time t of a zero-coupon bond that matures at time T .

The bond has a redemption value of 1 at time T .

The instantaneous forward rate $f(t, T)$ at time t is given by (52.26):

$$f(t, T) = \frac{-\partial \ln P(t, T)}{\partial T} \quad (52.26)$$

and describes the interest rate that is applicable on a default-free loan at time t for the period from T to a point one instant later. In their paper Heath, Jarrow and Morton state that the solution to the differential equation of (52.26) results in the expression for the price of the bond, shown at (52.27):

$$P(t, T) = \exp\left(-\int_t^T f(t, s) ds\right) \quad (52.27)$$

while the spot interest rate at time t is the instantaneous forward rate at time t for maturity date t , shown by:

$$r(t) = f(t, t).$$

We now describe the model's exposition of movements in the term structure.

The HJM model describes the evolution of forward rates given an initial forward rate curve, which is taken as given. For the period $T \in [0, t]$ the forward rate $f(t, T)$ satisfies the equation at (52.28):

$$f(t, T) - f(0, T) = \int_0^t \alpha(v, T, \omega) dv + \sum_{i=1}^n \int_0^t \sigma_i(v, T, \omega) dW(t). \quad (52.28)$$

The expression describes a stochastic process composed of n independent Wiener processes, from which the whole forward rate curve, from the initial curve at time 0, is derived. Each individual forward rate maturity is a function of a specific volatility coefficient. The volatility values $(\sigma_i(t, T, \omega))$ are not specified in the model and are dependent on historical Wiener processes. From (52.28) following the HJM model the spot rate stochastic process is given by (52.29):

$$r(t) = f(0, t) + \int_0^t \alpha(v, t, \omega) dv + \sum_{i=1}^n \int_0^t \sigma_i(v, t, \omega) dW(t) \quad (52.29)$$

for the period $t \in (0, T)$.

The model then goes to show that the process of changes in the bond price are given by:

$$\ln P(t, T) = \ln P(0, T) + \int_0^t (r(v) + b(v, T)) dv - \frac{1}{2} \sum_{i=1}^n \int_0^t a_i(v, T)^2 dv + \sum_{i=1}^n \int_0^t a_i(v, T) dW(t) \quad (52.30)$$

where $a_i(t, T, \omega) \equiv -\int_t^T \sigma_i(t, v, \omega) dv$ for $i = 1, 2, \dots, n$ and

$$b(t, T, \omega) \equiv -\int_t^T \alpha(t, v, \omega) dv + \frac{1}{2} \sum_{i=1}^n a_i(t, T, \omega)^2.$$

The expression at (52.30) describes the dynamics of the bond price as continuous stochastic process with a drift of $(r, (t, \omega)) + b(t, T, \omega)$ and a volatility value of $a_i(t, T, \omega)$.

The no-arbitrage condition is set by defining the price of a zero-coupon bond that matures at time T in terms of an “accumulation factor” $B(t)$ which is the value of a money market account that is invested at time 0 and reinvested at time t at an interest-rate of $r(t)$. This accumulation factor is defined as (52.31):

$$B(t) = \exp\left(\int_0^t r(y) dy\right) \quad (52.31)$$

and the value of the zero-coupon bond in terms of this accumulation factor is $Z(t, T) = P(t, T)/B(t)$ for the period $T \in (0, t)$ and $t \in (0, T)$.

Following HJM, by applying Itô's lemma the model obtains the following result for $Z(t, T)$, shown at (52.32):

$$\ln Z(t, T) = \ln Z(0, T) + \int_0^t b(v, T) dv - \frac{1}{2} \sum_{i=1}^n \int_0^t a_i(v, T)^2 dv + \sum_{i=1}^n \int_0^t a_i(v, T) dW(t). \quad (52.32)$$

In the HJM model the processes for the bond price and the spot rate are not independent of each other. As an arbitrage-free pricing model it differs in crucial respects from the equilibrium models presented in the previous chapter. The core of the HJM model is that given a current forward rate curve, and a function capturing the dynamics of the forward rate process, it models the entire term structure.

A drawback of the model is that requires the input of instantaneous forward rates, which cannot necessarily be observed directly in the market. Models have been developed that are in the HJM approach that take this factor into account, including those presented by Brace, Gatarek and Musiela (1997) and Jamshidian (1997). This family of models is known as the LIBOR market model or the BGM model. In the BGM model there is initially one factor, the forward rate $f(t)$ which is the rate applicable from time t_k to time t_{k+1} at time t . The forward rate is described by (52.33):

$$df(t) = \theta(t) f(t) dW \quad (52.33)$$

where the market is assumed to be forward risk-neutral.

The relationship between forward rates and the price of a zero-coupon bond at time t is given by (52.34):

$$\frac{P(t, t_i)}{P(t, t_{i+1})} = 1 + \delta_i f_i(t) \quad (52.34)$$

where δ_i is the compounding period between t and t_{i+1} .

To determine the volatility of the zero-coupon bond price $v(t)$ at time t , it can be shown that applying Itô's lemma to (52.34) we obtain

$$v_i(t) - v_{i+1}(t) = \frac{\delta_i f_i(t) \theta_i(t)}{1 + \delta_i f_i(t)}. \quad (52.35)$$

It is possible to extend the BGM model to incorporate more independent factors.

52.3.2 Jump models

Further research has produced a category of models that attempt to describe the jump feature of asset prices and interest rates. Observation of the markets confirms that many asset price patterns and interest rate changes do not move continuously from one price (rate) to another, but sometimes follow a series of jumps. A good example of a jump movement is when a central bank changes the base interest rate; when this happens, the entire yield curve shifts to incorporate the effect of the new base rate. There is a considerable body of literature on the subject, and we only refer to a small number of texts here.

One type of jump model is the jump-augmented HJM model, described in Jarrow and Madan (1991), Bjork (1996) and Das (1997). This is not described here because we have not covered the necessary technical background. Another is the jump-augmented Vasicek model described by Das and Foresi (1996) and Baz and Das (1996). In this, the short rate process is captured by

$$dr_t = \alpha(\mu - r_t)dt + \sigma dW_t + J dN_t \quad (52.36)$$

where N_t is a Poisson process with a constant intensity λ and J is a random jump size.

Other jump models have been described by Attari (1996), Das and Foresi (1996) and Honore (1998).

52.4 Assessing one-factor and multi-factor models

In assessing the value of the different models that have been developed and the efficacy of each, what is important is how they can be applied in the market, rather than any notion that multi-factor models are necessarily “better” than one-factor models because they are somehow more “real-world”. What is required is a mechanism that efficiently prices bonds and interest-rate options; a term structure model attempts to accomplish this by describing the dynamics of the interest rate process and generating random interest-rate paths. The generated paths are then used to discount the cash flows from the fixed income instrument, having initially been used to generate the cash flows in the first place. In practice a one-factor model that has been accurately calibrated will value fixed income instruments efficiently. This is because of the determinants of bond pricing; to illustrate, consider a fixed income instrument with a fixed maturity date. To value such a bond at a particular time, we need only know the bond yield at that time, and this is essentially a one-factor process. Similarly for a callable bond: when generating its cash flow, we will know whether it will be called by knowing its price at a future date. Generating this cash flow from the interest-rate model is again a one-factor process. Therefore if we are pricing a bond, the dynamics of the price process can usually be adequately described by (52.37):

$$dP = \mu_P P dt + \sigma_P P dW(t) \quad (52.37)$$

which is the process followed by for example the Black–Scholes option model when used to price an interest-rate option. This model does not discount the forward price of the option, which is the second part of the B–S approach: that of assuming a single continuously compounded short-term risk-free interest-rate.

While this approach would work in practice, this would only be for a single security portfolio; it would be unwieldy and inaccurate for valuing a number of securities. As banks and market makers must value many hundreds of cash and off-balance sheet instruments, another approach is required. This other approach was considered in this chapter and involves describing the dynamics of the bond price process in the form of a term structure model. Under this situation a multi-factor model may be more suitable, particularly when used to value options.

To consider one-factor models then, we know that the yield of a bond at a future date is essentially a one-factor process, so a one-factor model may well be accurate. A one-factor model describes only parallel shift yield changes, and it assumes that bond yields and discount rates are perfectly correlated, so that it will not generate all the possible paths of the future discount rate. In practice however much yield curve movement is close to a parallel shift, so this may not be as much of a problem for a majority of situations. If a term structure model accurately reflects the random evolution of the price of a bond, and the actual current rate and forward rate volatilities of the bond are as generated by the model, then the model can be considered effective, and it will generate reasonable cash flow scenarios and with accurate probabilities. It is possible to achieve this with one-factor models. Essentially then, a bank can use a one-factor model when conditions are appropriate, and need only use a multi-factor model where the one-factor model cannot be expected to be accurate. That said, why not simply use a multi-factor model at all times? The main reason is because generating forward rates and valuations from a multi-factor model is a time-consuming process, employing considerable computing power, and as rapidity of analysis and response is of the essence in the markets, it is logical to use a slower model only when it is significantly more accurate than the one-factor model.

It is generally accepted that one-factor models can be used for most bond applications; where multi-factor models are more appropriate may be in the following situations:

- where the instrument being valued is linked to two different interest rates, for example an interest-rate quanto option, or an option with a payoff profile that is a function of the spread between two different reference rates;
- for the valuation of long-dated options or deeply in-the-money or out-of-the-money options, which are affected by the volatility smile. As a stochastic volatility factor will impact the price, a model that assumes constant volatilities would be inaccurate;
- for the valuation of securities that are to some extent reflect the slope of the yield curve, such as certain mortgage-backed bonds whose level of prepayment is sometimes a function of the slope of the yield curve;
- for the valuation of very long-dated options, where all possible paths of the future discount rate may be required.

The optimum approach would appear to be a combination of a one-factor model and a multi-factor model to suit individual requirements. However this may not be practical; it might not be ideal to have different parts of a bank using different models (although this does happen; desks across the larger investment banks sometimes use different models) and valuing instruments using different models. The key factors to focus on are accessibility, accuracy, appropriateness and speed of computation.

52.4.1 Choosing the model⁵

There are essentially two approaches to modelling the term structure that we have discussed in this and the previous chapter. The Ho–Lee and HJM models begin with the evolution of the whole yield curve, while the BDT, Hull–White and other models specify the dynamics of the short rate, and determine the parameters so that the model itself corresponds to the current term structure. We have also discussed the relative merits of the equilibrium model approach and the no-arbitrage approach. In this final section we discuss the different issues that apply in each case.

Essentially there are two dimensions to consider: risk-neutral versus realistic and equilibrium versus no-arbitrage. There are situations under which each approach may be applied with validity.

No-arbitrage, risk-neutral approach

A commonly encountered approach is the risk-neutral, no-arbitrage model. This is a no-arbitrage model used frequently to value interest-rate options, using parameters that have been interpolated from a set of current market prices rather than estimated from actual historical data. This approach is valid when there is a reliable set of observable market prices and rates. Note that two different no-arbitrage models that are applied in a risk-neutral framework will only generate identical term structures and valuations if exactly identical input parameters have been used. The actual type of model used will have a significant effect on the valuation that is achieved. If however market data is not readily observable, or not reliable, this approach can lead to inaccuracies. This can be expected in illiquid markets such as that for certain long-dated exotic options; where this occurs there is no way to estimate a correlation term structure that allows the model to interpolate between the option prices, because there are two of them or their reliability is not accepted. In this scenario, a multi-factor model that captures the correlations between interest rates of different maturities, as well as the impact of the shape of the yield curve on these correlations, may be more valid. A model with a good statistical fit to the historical correlations recorded by the option product may therefore produce more robust prices. We hesitate to say “accurate” because, in an illiquid market with only a small number of market makers, who is to say which price is the most accurate? For certain exotic options the valuation depends on each market making bank’s valuation model, and how effectively the model has been calibrated.

No-arbitrage, realistic approach

A no-arbitrage model that is implemented in a realistic approach matches precisely the term structure of interest rates that are implied by the current (or initial) observed market yields. It then derives a forward curve for the future that is dependent on the way it has modelled the dynamics of the interest-rate process, which is a measure of probability. This approach is valid when it is important that the initial yield curve must be identical to the current observed yield curve, and is often used for analysing hedging strategy or portfolio strategies. In implementing this method it is sometimes difficult to evaluate the efficacy of the model because of problems in discriminating between model error and exogenous effects. In this approach the model parameters are set to match precisely observed market yields, with no regard to historical data, and there is little degree of freedom by which one can evaluate the model results. Only in a situation where the model generated an identical true term structure, so that the time-dependent parameters resulted in no pricing error at all points along the term structure, and for all dates past and forward, could the model be described for certainty as accurate. Otherwise the difficulty in assessing the effectiveness of this approach means that it is rarely used in practice.

Equilibrium, risk-neutral approach

The second approach is an equilibrium model under risk-neutral conditions. This is also valid under certain conditions. Remember that a no-arbitrage model uses input parameters that are based on observed prices and yields. However observed bond yields reflect a number of factors that frequently distort them away from “fair value”, resul-

⁵ This section draws heavily (with permission of course) on Fitton and McNatt (1997).

ting in a discount function that is also distorted. For instance we saw in our discussions in Part I that bond prices reflect liquidity, benchmark effects, supply and demand and other factors, which can include taxation, coupon size, convexity effect and so on. The same applies in the government zero-coupon bond market. Therefore a no-arbitrage model will use parameters that have been distorted by these factors. However equilibrium models are able to capture the global behaviour of the term structure over a long-term period, which has the effect of stripping out the market distortions (they are treated as “noise”). Therefore risk-neutral equilibrium models have an advantage over no-arbitrage models as they are not as sensitive to external market factors. In addition when used to price bonds today, equilibrium models can be estimated from historical data when market-observed current prices are unavailable or unreliable. So we conclude that one of the most appropriate times to adopt the risk-neutral equilibrium approach is when observed market yields are not available or subject to excessive distortions. This brings us to the subject of *horizon pricing*, which is the estimation of prices for a bond or other instrument under some expected future market state. While parameters are usually available, and reasonably reliable (in developed markets) for use in current pricing, they will not be available for a scenario-type valuation. In this situation, no-arbitrage models cannot be used at all, because they require the input of a set of market yields, which would not be available for horizon pricing as they would be unknown. Therefore in this case we use an equilibrium model, otherwise no analysis would be possible.

Equilibrium, realistic approach

In fact the inappropriateness of the no-arbitrage realistic approach means that only the equilibrium realistic approach is available. This methodology is used where speed of computation and accuracy of term structure generation and valuation are not of prime importance. It is used most frequently for risk management, regulatory and testing purposes; this includes value-at-risk calculations, VaR stress testing, capital adequacy calculations and other scenario purposes. In this approach and with equilibrium models the derived current term structure that is generated will not match the actual current term structure precisely. This has led to some analysts suggesting that any testing performed with the model will not be perfectly realistic. However the main purpose behind scenario analysis is to assess the impact of different situations; there is nothing illogical with comparing the effect of a theoretical future yield curve on an asset book held today and valued using today’s actual yield curve. An equilibrium model is a statistical model of the behaviour of the term structure of rates; therefore using it implies an acceptance that its derived curve will differ from the observed curve. Using a no-arbitrage model would imply that the current term structure model was completely accurate. Therefore for risk management and capital purposes it is common to encounter the equilibrium model, realistic approach.

52.4.2 Choosing the model: second-time around

It is important to remain focused on the practical requirements of interest rate modelling. Market participants are more concerned with the ease with which a model can be implemented, and its accuracy with regard to pricing. In practice different models are suited to different applications, so the range of products traded by a market practitioner will also influence which model is chosen. For instance the extended Vasicek model can be fitted very accurately to the initial term structure, and its implementation is relatively straightforward, being based on a lattice structure. It is also able to accurately price most products, however like all one-factor models it is not a valid model to use when pricing instruments that are sensitive to two or more risk factors, for example quanto options. The extended CIR model is also tractable, although it has a more restricted set of term structures compared to the extended Vasicek model, as a result of the limitations imposed by the $\sqrt{r_t}$ term on the volatility parameter. Both types of models are unable to capture the dynamics of the whole yield curve, for which HJM models must be used.

A drawback of these models is that although they fit the initial term structure, due to their structure they may not continue to calculate prices as the term structure evolves. In practice the models must be re-calibrated frequently to ensure that they continue to describe term structure volatilities that exist in the market.

In selecting the model, a practitioner will select the market variables that are incorporated in the model; these can be directly observed such as zero-coupon rates or forward rates, or swap rates, or they can be indeterminate such as the mean of the short rate. The practitioner will then decide the dynamics of these market or *state variables*, so for example the short rate may be assumed to be mean reverting. Finally the model must be calibrated to market prices, so the model parameter values input must be those that produce market prices as accurately as possible. There are a number of ways that parameters can be estimated; the most common techniques of calibrating to time

series data such as interest rate data are *general method of moments* and the *maximum likelihood* method. For information on these estimation methods refer to the bibliography.

Models exhibit different levels of sensitivity to changes in market prices and rates. The extent of a model's sensitivity will also influence the frequency with which the model must be re-calibrated. For example the Black–Derman–Toy model is very sensitive to changes in market prices; because it is a log- r model changes in the process of the underlying variable are larger as they are log- r , than those in the process for r_t itself. Some practitioners believe that as they take bond prices and the term structure as given, arbitrage models suffer from an inherent weakness. Liquidity and other considerations frequently result in discrepancies between market yields and theoretical value, and such discrepancies would feed through into an arbitrage model. This drawback of arbitrage models means that users must take care about term structure inputs, and the curve fitting techniques and *smoothing* techniques that are used become critical to model effectiveness. This is discussed in the next chapter.

Other considerations are detailed below.

- **Model inputs.** Arbitrage models use the term structure of spot rate as an input, and this data is straightforward to obtain. Equilibrium models require a measure of the investors market risk premium, which is rather more problematic. Practitioners analyse historical data on interest rate movements, which is considered less desirable.
- **Using models as part of bond trading strategy.** A key element of market makers' and proprietary traders' strategy is relative value trading, which includes simultaneous buying and selling of certain bonds against others, or classes of bonds against other classes. A yield curve spread trade is a typical relative value trade. How does one determine relative value?⁶ Using an interest-rate model is the answer. For such purposes though, only equilibrium models can be used. By definition since arbitrage models take bond prices and the current term structure as given, they clearly cannot be used to assess relative value. This is because the current price structure would be assumed to be correct. If one were to use such a model for a yield curve trade, it would imply a zero profit potential. Therefore only equilibrium models can be used for such purposes.
- **Model consistency.** As we have noted elsewhere, using models requires their constant calibration and re-calibration over time. For instance, an arbitrage model makes a number of assumptions about the interest rate drift rate and volatility, and in some cases the mean reversion of the dynamics of the rate process. Of course these values will fluctuate constantly over time, so that the estimate of these model parameters used one day will not remain the same over time. So the model will be inconsistent over time and must be re-calibrated to the market. Equilibrium models use parameters that are estimated from historical data, and so there is no unused daily change. Model parameters remain stable. Over time therefore these models remain consistent, at least with themselves. However given the points we have noted above, market participants usually prefer to use arbitrage models and re-calibrate them frequently.

We have only touched on the range of considerations that must be followed when evaluating and implementing an interest rate model. This is a complex subject with a number of factors to consider, and ongoing research in the area serves to reinforce the fact that it is an important and very current topic.

Selected bibliography and references

- Attari, M., "Discontinuous Interest Rate Processes: An Equilibrium Model for Bond Option Prices", Working Paper, University of Iowa, 1996, pp. 1–32.
- Baxter, M., Rennie, A., *An Introduction to Derivative Pricing*, Cambridge UP, 1998, Chapter 5.
- Baz, J., Das, S., "Analytical Approximations of the Term Structure for Jump Diffusion Processes: A Numerical Analysis", *Journal of Fixed Income*, June 1996, pp.78–86.
- Bjork, T., "Interest rate theory", *CIME Lectures*, Working Paper, Stockholm School of Economics, pp. 1–90, 1996.
- Black, F., Karasinski, P., "Bond an option pricing when short rates are lognormal", *Financial Analysts Journal*, Jul–Aug 1991, pp. 52–59.

⁶ Some traders determine relative value by conducting the analysis inside their head! Nowadays one needs to back up ones gut feeling with formal analysis. A term structure model will assist.

- Brennan, M., Schwartz, E., "An equilibrium model of bond pricing and a test of market efficiency", *Journal of Financial and Quantitative Analysis* 17(3), September 1982, pp. 301–329.
- Brace, A., Gatarek, D., Musiela, M., "The market model of interest-rate dynamics", *Mathematical Finance* 7(2), 1997, pp. 127–155.
- Cheyette, O., "Term structure dynamics and mortgage valuation", *Journal of Fixed Income*, March 1992, pp. 28–41.
- Das, S., "A Direct Discrete-Time Approach to Poisson–Gaussian Bond Option Pricing in the Heath–Jarrow–Morton model", Working Paper, Harvard Business School, 1997, pp. 1–44.
- Das, S., Foresi, S., "Exact Solutions for Bond and Option Prices with Systematic Jump Risk", *Review of Derivatives Research* 1, 1996, pp. 1–24.
- Fitton, P., McNatt, J., "The Four Faces of an Interest Rate Model", in Fabozzi, F. (ed.), *Advances in Fixed Income Valuation Modelling and Risk Management*, FJF Associates, 1997.
- Duffie, D., Kan, R., "A yield-factor model of interest rates", *Mathematical Finance* 6(4), 1996, pp. 379–406.
- Harrison, M., Pliska, S., "Martingales and stochastic integrals in the theory of continuous trading", *Stochastic Processes and their Applications II*, 1981, pp. 215–260.
- Heath, D., Jarrow, R., Morton, A., "Bond pricing and the term structure of interest rates: a discrete time approximation", *Journal of Financial and Quantitative Analysis* 25(4), December 1990, pp. 419–440.
- Heath, D., Jarrow, R., Morton, A., "Bond pricing and the term structure of interest rates: a new methodology", *Econometrica* 60(1), 1992, pp. 77–105.
- Ho, T.S.Y., Lee, S-B., "Term Structure Movements and Pricing Interest Rate Contingent Claims", *Journal of Finance* 41, 1986, pp. 1011–1029.
- Honore, P., "Five Essays on Financial Econometrics in Continuous-Time Models", PhD Thesis, Aarhus School of Business, 1998.
- Hughston, L. (ed.), *Vasicek and Beyond: Approaches to Building and Applying Interest Rate Models*, Risk Publications, 1996.
- Hull, J., White, A., "Bond option pricing based on a model for the evolution of bond prices", *Advances in Futures and Options Research* 6, 1993, pp. 1–13.
- Hull, J., White, A., "Numerical procedures for implementing term structure models II: two-factor models", *Journal of Derivatives* 2(2), Winter 1994, pp. 37–48.
- Jamshidian, F., "LIBOR and swap market models and measures", *Finance and Stochastics* 1, 1997, pp. 293–330.
- Jarrow, R., Madan, D., "Option Pricing Using the Term Structure of Interest Rates to Hedge Systematic Discontinuities in Asset Returns", Working Paper, Cornell University, 1991, pp. 1–47.
- Rebonato, R., *Interest Rate Option Models*, 2nd edition, John Wiley & Sons, 1998.
- Rebonato, R., Cooper, I., "The limitations of simple two-factor interest-rate models", *Journal of Financial Engineering*, March 1996.
- Ritchken, P., Sankarasubramanian, L., "Volatility structures of forward rates and the dynamics of the term structure", *Mathematical Finance* 5, 1995, pp. 55–72.
- Uhrig, M., Walter, U., "A new numerical approach for fitting the initial yield curve", *Journal of Fixed Income*, March 1996, pp. 82–90.

References on estimation methods

- Andersen, T., Lund, J., "Estimating Continuous-Time Stochastic Volatility Models of the Short-Term Interest Rate", *Journal of Econometrics* 77, 1997, pp. 343–377.
- Brown, R., Schaefer, S., "The Term Structure of Real Interest Rates and the Cox, Ingersoll and Ross Model", *Journal of Financial Economics* 35, 1994, pp. 2–42.
- Campbell, J., Lo, A., MacKinlay, C., *The Econometrics of Financial Markets*, Princeton University Press, 1997.
- Davidson, R., MacKinnon, J., *Estimation and Inference in Econometrics*, Oxford University Press, 1993.
- Duffie, D., Singleton, K., "Simulated Moments Estimation of Markov Models of Asset Prices", *Econometrica* 61, 1993, pp. 929–952.
- Hansen, L., "Large Sample Properties of Generalized Method of Moments Estimators", *Econometrica* 50, 1982, pp. 1029–1055.
- Lo, A., "Statistical Tests of Contingent Claims Asset Pricing Models", *Journal of Financial Economics* 17, 1986, pp. 143–173.

Longstaff, F., Schwartz, E., “Interest Rate Volatility and the Term Structure: A Two-Factor General Equilibrium Model”, *Journal of Finance* 47, 1992, pp. 1259–1282.

Questions and exercises

1. In the Brennan–Schwartz model, describe the process followed by the price of a zero-coupon bond $P(t, T)$.
2. Discuss the circumstances in which it is advisable to use a single-factor model rather than a multi-factor model.
3. How does the Black–Karasinski model describe the dynamics of the short-rate? How does this differ from the approach adopted by the Vasicek and Cox–Ingersoll–Ross models?
4. Under certain circumstances a no-arbitrage model is unsuitable and an equilibrium model must be used. Discuss the different situations that might lead to this.

53

Estimating and Fitting the Term Structure

The two previous chapters introduced and described a fraction of the most important research into interest rate models that has been carried out since the first model, presented by Oldrich Vasicek, appeared in 1977. These models can be used to price derivative securities, and equilibrium models can be used to assess fair value in the bond market. Before this can take place however a model must be fitted to the yield curve, or *calibrated*.¹ In practice this is carried out on two ways; the most popular approach involves calibrating the model against market interest rates given by instruments such as cash Libor deposits, futures, swaps and bonds. The alternative method is to model the yield curve from the market rates and then calibrate the model to this fitted yield curve. The first approach is common when using for example extended Vasicek models, while the second technique is more useful with whole yield curve models such as the Heath–Jarrow–Morton model.

There are a number of techniques that can be used to fit the yield curve. These include regression methods and *spline* techniques. More recent methods such as *kernel approximations* and *linear programming* are also beginning to be used by practitioners. In this chapter we provide an introduction to some of these, however a detailed exposition would warrant a book in its own right. We discuss fitting the spot and forward yield curve and review the methods used to estimate spot and forward yield curves. We then illustrate cubic spline method for fitting a yield curve from observed government bond yields. There is a large body of literature on this subject. For further information readers are recommended to review Anderson *et alia* (1996) and James and Webber (2000) for the most important research, and interested readers may also wish to consider Bliss (1996), Dahlquist and Svensson (1996) and Waggoner (1997). Alternative approaches are given in Kim (1993) and Zheng (1994).

For a number of reasons practitioners, investors, central banks and government authorities are interested in fitting the zero-coupon yield curve, or the true term structure of interest rates. The use of yield curves is standard in monetary policy analysis, and central banks are increasingly making use of forward interest rates for this purpose as well. Forward rates must be estimated from the yield curve that has been constructed from current market yields, generally T-bill and government bond yields. Particularly useful information that can be derived from government bond prices includes the yield curve for implied forward rates, as these reflect the market's expectations of the future path of interest rates.² They are also used by the market to price bonds and determine the extent of the credit spread applicable to corporate bonds. The requirements of the monetary authorities however are slightly different to those of market practitioners: central bankers and the government are not so concerned with the accuracy of the spot curve with regard to pricing securities; rather, they are interested in the information content of the fitted curve, particularly concerning implied forward rates and the market expectations of future interest rates and levels of inflation. In the second part of this chapter we review the information content of the yield curve in the UK gilt market.

53.1 Introduction

From Chapters 6, 39, and 50 we know that there is a relationship between a set of discount factors, and the discount function, the par yield curve, the zero-coupon yield curve and the forward yield curve. If we know one of these functions we may readily compute the other three. In practice although the zero-coupon yield curve is directly observable from the yields of zero-coupon government bonds, liquidity and investor preferences usually mean that a theoretical set of all these curves is derived from the yields of coupon government bonds in the market. There are a number of ways that the zero-coupon curve can be fitted, using either a discount function or the par yield curve.

¹ In fact a model needs to be calibrated to the market, but the most important item against which it must be calibrated is the current term structure.

² From our discussions in Chapter 6 and later, we of course remember that the forward rate is derived from the current spot rate term structure, and therefore although it is an expectation based on all currently known information, it is not a prediction of the term structure in the future. Nevertheless the forward rate is important because it enables market makers to price and hedge financial instruments.

The pricing of financial instruments in the debt market revolves around the yield curve. The use in the market and by the central authorities of the government term structure to ascertain the market's expectation of future interest rates is well established. This reflects the fact that the spot yield curve is the geometric average of the same maturity structure implied forward rates. Here we discuss the information content of the yield curve, and how the zero-coupon curve may be fitted best to enable analysts to extract information from the implied forward rate yield curve. This is used for a number of purposes by central government monetary authorities and by analysts and economists. In the United Kingdom for example the yield on government bonds is used as the benchmark for interest charges to local authorities and public sector bodies. Yield curve data may also be used as one of the parameters for a general interest rate model (for example, see Cox, Ingersoll, Ross (1985) for a one-factor model and Heath, Jarrow, Morton (1992) for a multi-factor model).

Although the use of yield curves is quite common as part of monetary policy analysis, central banks such as the US Federal Reserve and the Bank of England have only recently begun to use *forward* interest rates as an indicator for monetary policy purposes. We know that a forward rate is an interest rate applicable to a debt instrument whose term begins at a future date, and ends at a date beyond that. Although there is a market in forward rates, the prices at which forward instruments are quoted are derived from spot interest rates. That is, *implied* forward rates are calculated from the spot yield curve, which is in turn modelled from the prices of instruments in the market, usually government bills and bonds. This implies that the shape and position of the spot curve reflects market belief on future interest rates, which is why it is used to calculate forward rates. The information content and predictive power of a spot term structure is based on this belief. Forward rates may be estimated using any one of a number of models. They can be interpreted as reflecting the market's expectations of future *short-term* interest rates, which in turn are indicators of expected inflation levels. The same information is contained in the spot yield curve, however monetary authorities often prefer to use forward rates as they are better applied to policy analysis. Whereas the spot yield curve is the expected average of forward rates, the forward rate curve reflects the expected future time period of one short-term forward rate. This means that the forward curve can be split into short-term and long-term segments in a more straightforward fashion than the spot curve.

As it is used as a predictive indicator, the spot yield curve needs to be fitted as accurately as possible. This is an area that has been extensively researched (see McCulloch 1975, Deacon and Derry 1994, Schaefer 1981, Waggoner 1997, Nelson and Siegel 1987, Svensson 1994, 1995 *inter alia*). Invariably researchers use the government debt market as the basis for modelling the term structure. This is because the government market is the most liquid debt market in any country, and also because (in a developed economy) government securities are default-free, so that government borrowing rates are considered risk-free. Whatever method is used to fit the term structure, it should aim to meet the following criteria when the main use is for government policy, rather than the pricing of financial instruments:

- the method should attempt to fit implied forward rates, because the primary objective is to derive the forward curve and not market spot yields;
- the resulting derived forward curve should be as smooth as possible, again because the aim is to provide information on future the level and direction of interest rates, and expectations on central bank monetary policy, rather than an accurate valuation of financial instruments along the maturity term structure;
- it should have as few market assumptions as possible.

There are a number of curve fitting methods that may be employed. In the United Kingdom gilt market the Bank of England previously used an in-house model³ but has since adopted a modified technique proposed by Svensson (1995) and subsequently Fisher, Nychka and Zervos (1995), Waggoner (1997) and Anderson and Sleath (1999). The Waggoner method is discussed in a later section. In the UK the introduction of a market in government zero-coupon bonds (see Chapter 11) has enabled the accuracy of a fitted spot term structure to be compared to actual market spot rates; there is also useful information to be gleaned from using data from the gilt repo market when comparing the accuracy of the short-end of the fitted curve, as discussed by Anderson and Sleath (1999). We set the scene below.

³ See Matronikola (1991).

53.2 Bond market information

53.2.1 Basic concepts

Central banks and market practitioners use interest rates prevailing in the government bond market to extract certain information, the most important of which is implied forward rates. These are an estimate of the market's expectations about the future direction of short-term interest rates. They are important because they signify the market's expectations about the future path of interest rates, however they are also used in derivative pricing and to create synthetic bond prices from the extent of credit spreads of corporate bonds.

Forward rates may be calculated using the discount function or spot interest rates. If spot interest rates are known then the bond price equation can be set as:

$$P = \frac{C}{(1 + rs_1)} + \frac{C}{(1 + rs_2)^2} + \dots + \frac{C + M}{(1 + rs_n)^n} \quad (53.1)$$

where

- C is the coupon
- M is the redemption payment on maturity (par)
- rs_t is the *spot interest rate* applicable to the cash flow in period t ($t = 1, \dots, n$).

The bond price equation is usually given in terms of *discount factors*, with the present value of each coupon payment and the maturity payment being the product of multiplying them by their relevant discount factors. This allows us to set the price equation as shown by (53.2),

$$P = \sum_{t=1}^n Cdf_t + Mdf_n \quad (53.2)$$

where df_t is the t -period discount factor ($t = 1, \dots, m$) given by (53.3):

$$df_t = \frac{1}{(1 + rs_t)^t}, \quad t = 1, \dots, m. \quad (53.3)$$

A discount factor is a value for a discrete point in time, whereas markets often prefer to think of a continuous value of discount factors that applies a specific discount factor to any time t . This is known as the *discount function*, which is the continuous set of discrete discount factors and is indicated by $df_t = \delta(t_t)$.

The discount function relates the current cash bond yield curve with the spot yield curve and the implied forward rate yield curve. From (53.3) we can set:

$$df_t = (1 + rs_t)^{-t}.$$

As the spot rate rs is the average of the implied short-term forward rates $rf_1, rf_2, \dots, rf_t$ we state

$$\begin{aligned} 1/df_1 &= (1 + rs_1) = (1 + rf_1) \\ 1/df_2 &= (1 + rs_2)^2 = (1 + rf_1)(1 + rf_2) \\ 1/df_t &= (1 + rs_t)^t = (1 + rf_1)(1 + rf_2) \dots (1 + rf_t). \end{aligned} \quad (53.4)$$

From (53.4) we see that $1 + rs_t$ is the geometric mean of $(1 + rf_1), (1 + rf_2), \dots, (1 + rf_t)$.

Implied forward rates indicate the expected short-term (one-period) future interest rate for a specific point along the term structure; they reflect the spread on the marginal rate of return that the market requires if it is investing in debt instruments of longer and longer maturities.

In order to calculate the range of implied forward rates we require the term structure of spot rates for all periods along the continuous discount function. This is not possible in practice because a bond market will only contain a finite number of coupon-bearing bonds maturing on discrete dates. While the coupon yield curve can be observed, we are then required to "fit" the observed curve to a continuous term structure. Note that in the UK gilt market for example, there is a zero-coupon bond market, so that it is possible to observe spot rates directly, but for reasons of liquidity, analysts prefer to use a fitted yield curve (the *theoretical curve*) and compare this to the observed curve.

53.2.2 Estimating yield curve functions

The traditional approach to yield curve fitting involves the calculation of a set of discount factors from market interest rates. From this a spot yield curve can be estimated. The market data can be money market interest rates, futures and swap rates and bond yields. In general though this approach tends to produce “ragged” spot rates and a forward rate curve with pronounced jagged knot points, due to the scarcity of data along the maturity structure.⁴ A refinement of this technique is to use polynomial approximation to the yield curve.

The McCulloch method (1971, 1975) describes the discount function as a linear combination of a specified number of approximating functions, so for example if there are k such functions on which j coefficients are estimated, the discount function that is generated by the set of approximations is a k th degree polynomial. The drawback of this approach is that unless the market observations are spaced at equal intervals through the maturity range, such a polynomial will fit the long end of the curve fairly inaccurately. To account for this McCulloch proposed using piecewise polynomial functions or splines to approximate the discount function. A polynomial spline can be thought of as a number of separate polynomial functions, joined smoothly at a number of join, break or *knot* points. In mathematics the term “smooth” has a precise meaning, but in the context of a piecewise r -degree spline it is generally taken to mean that the $(r - 1)$ th derivative of the functions either side of each knot point are continuous. McCulloch originally used a quadratic spline to estimate the discount function. This results however in extreme bumps or “knuckles” in the corresponding forward rate curve, which makes the curve unsuitable for policy analysis. To avoid this effect, it is necessary to increase the number of estimating functions and to use a *cubic spline*. This was presented by McCulloch in his second paper, and his specification is summarised in Appendix 53.1.

One of the main criticisms of cubic and polynomial functions is that they produce forward rate curves that exhibit unrealistic properties at the long end, usually a steep fall or rise in the curve. A method proposed by Vasicek and Fong (1982) avoids this feature, and produces smoother forward curves. Their approach characterises the discount function as exponential in shape, which is why splines, being polynomials, do not provide a good fit to the discount function, as they have a different curvature to exponential functions. Vasicek and Fong instead propose a transform to the argument T of the discount function $v(T)$. This transform is given by

$$T = -(1/\alpha)\ln(1 - x), \text{ where } 0 \leq x \leq 1 \quad (53.5)$$

and has the effect of transforming the discount function from an approximately exponential function of T to an approximately linear function of x . Polynomial splines can then be employed to estimate this transformed discount function. Using this transform it is straightforward to impose additional constraints on the discount function. The parameter α constitutes the limiting value of the forward rates, and can be fitted to the data as part of the estimation. Vasicek and Fong use a cubic spline to estimate the transformed discount function. In terms of the original variable T this is equivalent to estimating the discount function by a third-order exponential spline, that is between each pair of knot points $v(T)$ takes the form:

$$v(T) = b_0 + b_1 e^{-2\alpha T} + b_3 e^{3\alpha T}. \quad (53.6)$$

However, Shea (1985) has indicated that in practice exponential splines do not produce more stable estimates of the term structure than polynomial splines. He also recommended using basis splines or *B-splines*, functions that are identically zero over most of the approximation space, to prevent loss of accuracy due to the lack of observations at the long end of the curve.

53.3 Curve-fitting techniques: parametric

As we reviewed in Chapters 51–52 there are a number of models that one may use to fit the spot rate term structure. One-factor models (see Vasicek 1997, Dothan 1978, Cox, Ingersoll, Ross 1985) model the one-period short rate to obtain a forward yield curve. The simplest method uses a binomial model of probabilities to model the forward rate. Multi-factor models (see Heath, Jarrow, Morton 1992) express analytically the entire yield curve in terms of two forward rates (or “spanning” rates). For an analysis of the information content of both methodologies see Edmister and Madan (1993), who conclude that the multi-factor models provide more accurate results. Essentially the information content of the yield curve is best estimated using a multi-factor model, and is more accurate at the

⁴ For a good account of why this approach is not satisfactory see James and Webber (2000, Chapter 15).

longer end of the curve whatever methodology is used. Edmister and Madan also conclude that modelling the short-end of the curve suffers from distortions resulting from government intervention in short-term interest rates.

The Bank of England uses a variation of the Svensson yield curve model, a one-dimensional *parametric* yield curve model. This is similar to the Nelson and Siegel model and defines the forward rate curve $f(m)$ as a function of a set of unknown parameters, which are related to the short-term interest rate and the slope of the yield curve. The model is summarised in Appendix 53.2. Anderson and Sleath (1999) assess parametric models, including the Svensson model, against spline-based methods such as those described by Waggoner (1997), and we summarise their results later in this chapter.

53.3.1 Parametric techniques

Curve-fitting techniques generally fit into two classes, as described for example in Chapter 15 of James and Webber (2000), *parametric* methods and spline-based methods. Parametric techniques are so-called because they model the forward rate using a parametric function. An early parametric technique was that described by Nelson and Siegel (1987), which models the forward rate curve. Given the relationship between spot and forward rates, such an approach is identical to modelling a spot rate curve by taking a geometric average of the forward rates curve. A fairly flexible function for the forward rate is described in the Nelson-Siegel approach, known as a Laguerre function (plus a constant) and is given by

$$f(T) = \beta_0 + \beta_1 e^{-T/\tau_1} + \frac{\beta_2}{\tau_1} T e^{-T/\tau_1} \quad (53.7)$$

where T is the variable being calculated and $\beta_0, \beta_1, \beta_2$, and τ_1 are the parameters required to be estimated. Remembering that the spot rate is an average of the forward rates, that is

$$rs = \frac{\int_0^T f(u) du}{T} \quad (53.8)$$

then (53.7) implies that the spot rate is given by

$$rs(T) = \beta_0 + (\beta_1 + \beta_2) \frac{\tau_1}{T} (1 - e^{-T/\tau_1}) - \beta_2 e^{-T/\tau_1}. \quad (53.9)$$

To illustrate implementation, we adapt with permission the Anderson and Sleath (1999) evaluation of the Nelson-Siegel method; we set parameter values of:

$$\beta_0 = 5.0 \quad \beta_1 = -1 \quad \tau_1 = 1$$

and denote the remaining parameter as a , which reduces (53.9) to

$$rs(T) = 5 + (-1 + a) \frac{1 - e^{-T}}{T} - a e^{-T}. \quad (53.10)$$

Setting β_0 as 5.0 means that the spot rate has been set to a common value of 5.0%. As an exercise we evaluate the possible results with the same parameters used by Anderson and Sleath in their analysis with the exception of the initial spot rate, and change the values for the term to maturity to 10, 20, 30 and 1000 years and the value of a to $-5, -3, -1, 0, 1, 3$, and 5. Our results are given in Table 53.1. As the value for T increases to very high values the convergence of spot rates to the initial value proceeds only slowly. However, our results illustrate the process.

Maturity (T) years	a values						
	-5	-3	-1	0	1	3	5
10	4.4003	4.6002	4.8001	4.9	5.0000	5.1999	5.3998
20	4.7000	4.8000	4.9000	4.9500	5.0000	5.1000	5.2000
30	4.8000	4.8667	4.9333	4.9667	5.0000	5.0667	5.1333
1000	4.9940	4.9960	4.9980	4.9990	5.0000	5.0200	5.0040

Table 53.1: Spot rate values using Nelson-Siegel model and user-specified parameters.

An evaluation fitting the Nelson–Siegel curve to actual gilt yields from June 1997 is described in the next section.

Another parametric method is described by Svensson (1994, 1995). This adds an extra coefficient to the Nelson–Siegel model and has been described as an extended Nelson–Siegel model. The extra parameter introduces greater flexibility, so that the resulting curve can model forward curves that have more than one “hump”. It is given by (53.11):

$$f(T) = \beta_0 + \beta_1 e^{-T/\tau_1} + \beta_2 \frac{T}{\tau_1} e^{-T/\tau_1} + \beta_3 \frac{T}{\tau_1} e^{-T/\tau_2}. \quad (53.11)$$

In the Svensson model there are six coefficients $\beta_0, \beta_1, \beta_2, \beta_3, \tau_1, \tau_2$ that must be estimated. The model was adopted by central monetary authorities such as the Swedish Riksbank and the Bank of England (who subsequently adopted a modified version of this model, which we describe shortly, following the publication of the Waggoner paper by the Federal Reserve Bank of England). In their 1999 paper Anderson and Sleath evaluate the two parametric techniques we have described, in effort to improve their flexibility, based on the spline methods presented by Fisher, Nychka and Zervos (1995) and Waggoner (1997).

53.3.2 Parameterised yield curves

The technique for curve fitting presented by Nelson and Siegel and variants on it described by Svensson (1994), Wiseman (1994) and Bjork and Christensen (1997) have a small number of parameters, and generally one obtains a relatively close approximation to the yield curve with them. As we saw above, the Nelson and Siegel curve contains four parameters while the Svensson curve has six parameters. The curve presented by Wiseman contains $2 \times (n + 1)$ parameters, given by $\{\beta_j, k_j\}_{j=0, \dots, n}$. The curve is $f_2(\tau)$:

$$f_2(\tau) = \sum_{j=0}^n \beta_j e^{-k_j \tau}. \quad (53.12)$$

The original Nelson and Siegel curve does not produce close approximations for all types of yield curves, because the small number of parameters limits flexibility. It can be used to model the spot rate or the forward rate curve, but does not produce accurate results if used to model the discount curve. An example of a fitted Nelson and Siegel curve is shown at Figure 53.1 for UK gilt yields from June 1997. The table of actual gilt yields is shown as well.

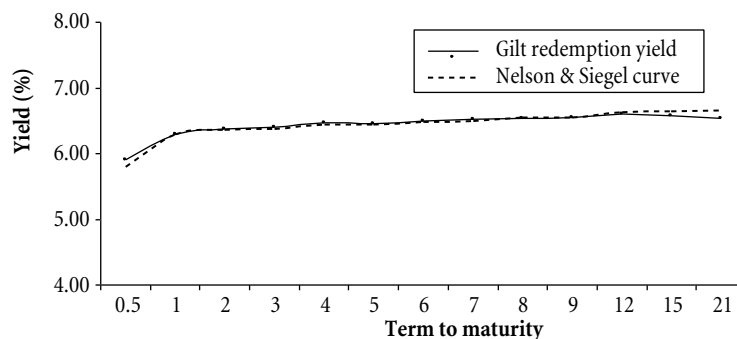


Figure 53.1: A Nelson and Siegel fitted yield curve and gilt redemption yield curve.

Term to maturity	Gilt Redemption Yield %	Term to maturity	Gilt Redemption Yield %
0.5	5.90	7	6.52
1	6.29	8	6.54
2	6.37	9	6.55
3	6.40	12	6.60
4	6.47	15	6.58
5	6.45	21	6.54
6	6.50		

Table 53.1: Gilt redemption yields. Source: Butler Gilts.

The fitted curve is a close approximation to the redemption yield curve, and is also very smooth. However the fit is inaccurate at the very short end, indicating an underpriced six-month bond, and also does not approximate the long end of the curve. For this reason B-spline methods are more commonly used.

53.4 The cubic spline method for estimating and fitting the yield curve

In mathematical applications a *spline* is *piecewise polynomial*, this being a function that is composed of a number of individual polynomial segments that are joined at user-specified points known as *knot points*. The function is twice-differentiable at each knot point, which produces a smooth curve at each connecting knot point. The commonest approach uses regression methods to fit the spline function, and an excellent and accessible account of this technique is given in Suits *et al* (1978); the article is summarised in Choudhry (2001). In this section we summarise with permission the spline approach described by Waggoner in his ground-breaking article published by the Federal Reserve Bank of Atlanta in 1997.

Spline methods were introduced in Chapter 6. They are commonly used to derive spot and forward rate curves and the discount function from the observed yields of bonds in the market. A popular method is that proposed by McCulloch (1975), which uses regression cubic splines to derive the discount function. Waggoner (1996) has written that this method however, while accurate and stable, produces forward rate curves that oscillate. In fact this property is exhibited by virtually all curve fitting techniques, but the objective for the analyst is to produce curves with the smallest amount of oscillation. A technique posited by Fisher, Nychka and Zervos (1995) used a cubic spline that incorporated a “roughness penalty” when extracting the forward rate curve. This approach produces a decreased level of oscillation but also reduces the fit of the curve to the actual observed yields. A later technique modified this method by using a “variable roughness penalty” (Waggoner 1997) and this approach is described here.

53.4.1 Using a cubic spline: the Waggoner model

A cubic spline approach can be used as the functional form for the discount function or the forward rate curve. We define a function g on the interval $[t_1, t_N]$ as a cubic spline with node points $t_1 < t_2 < \dots < t_n$ if it is a cubic polynomial on each of the subintervals $[t_{j-1}, t_j]$ for $1 < j < n$ and if it can be continuously differentiated over the interval $[t_1, t_N]$. The node points are $\tau_1 < \tau_2 < \dots < \tau_N$ which are the cash flow and maturity dates of the set of bonds (assuming the bonds are semi-annual coupon instruments). Following Waggoner (1997) we set $\tau_0 \equiv 0$ so that the curve is derived from the point zero to the point of the longest-dated bond in the sample. It is possible to use all the node points in the interval to produce the yield curve, however the more points there are in a cubic spline, the greater the tendency for the derived forward curve to oscillate, more so at longer maturities. We wish to minimise the level of oscillation, because for monetary policy purposes the curve is used to provide information on expected future interest rates. A fluctuating yield curve would imply oscillations in expected future prices, and this can produce illogical results, particularly at the long end of curve. For example, a curve may imply that while the current yield of a six-month T-bill is £97.50, the price of six-month bill in one year's time will be £98, while the price of such a bill in two years' time will be £95. This is not an unreasonable expectation. However the same implications for six-month bill prices in 25, 26 and 27 years' time is less reasonable. Therefore the fitted curve should smooth out the forward rates at longer maturities, which calls for a reduced level of oscillation. The McCulloch technique uses regression splines to reduce forward rate fluctuation, while the Fisher *et al* and the Waggoner approach uses a smoothed spline and a modified smoothed spline.

In a regression spline a smaller number of node points are used in order to reduce the level of oscillation. This affects the flexibility of the cubic spline over the interval that is being considered; there is a trade-off between accuracy and the level of oscillation. By reducing node points at the longer end but keeping more at the short end, oscillation is reduced but the curve retains accuracy at the short end. In practice it is common for node points to be set in one of the ways shown at Figure 53.2, but obviously there are any number of ways that node points may be set.

1w	1m	3m	6m	9m	1y	2y	3y	4y	5y	6y	7y	8y	9y	10y
3m	6m	1y	2y	3y	4y	5y	7y	10y	15y	20y	25y	30y		

Figure 53.2: Suggested node points.

Once we have chosen the node points we set the yield curve ψ as the cubic spline that minimises the function at (53.13):

$$\sum_{i=1}^N (P_i - \hat{P}_i(\psi))^2. \quad (53.13)$$

The technique proposed by McCulloch (1975) used a regression cubic spline to approximate the discount function, and he suggested that the number of node points that are used be roughly equal to the square root of the number of bonds in the sample, with equal spacing so that an equal number of bonds mature between adjacent nodes. A number of writers have suggested that this approach produces accurate results in practice.⁵ The discount function is constrained to set $v(0) = 1$. Given these parameters the discount function chosen is the one that minimises the function at (53.14). As this is a discount function and not a yield curve, (53.14) can be solved using the least squares method.

$$\sum_{i=1}^N (P_i - \hat{P}_i^v(v))^2. \quad (53.14)$$

For a smoothed spline, the level of oscillation is controlled by setting a “roughness penalty” in the function, and not by reducing the number of node points. The yield curve ψ is chosen that minimises the objective function at (53.15):

$$\sum_{i=1}^N (P_i - \hat{P}_i^*(\psi))^2 + \lambda \int_0^{\tau_N} (\psi''(t))^2 dt. \quad (53.15)$$

for all the cubic splines over the node points $\tau_0 < \tau_1 < \tau_2 < \dots < \tau_N$. In minimising this function there is a trade-off between the goodness of fit, which is given by the first term, and the degree of smoothness, which is measured by the second term. This trade-off is known as the “roughness penalty” and is given by λ , which is a positive constant. If λ is set to zero the function reverts to a regression spline, and as it increases λ approaches a linear function. The flexibility of the spline is a function of both the spacing between the node points and the magnitude of λ , although as λ increases the impact of the node spacing decreases. For large values of λ the flexibility of the spline is essentially similar across all terms. This is not necessarily ideal because as we saw from Figure 53.1 we require the spline to be more flexible at the short end, and less so at the long end. Therefore Waggoner (1997) has proposed a modified smoothed spline. For a modified smoothed spline the objective function at (53.16) is minimised over the whole term covering the node points $\tau_0 < \tau_1 < \tau_2 < \dots < \tau_N$.

$$\sum_{i=1}^N (P_i - \hat{P}_i^*(\psi))^2 + \int_0^{\tau_N} \lambda(t) (\psi''(t))^2 dt. \quad (53.16)$$

The approach used by Fisher (1995) is a smoothed cubic spline that approximates the forward curve. The number of nodes to use is recommended as approximately one-third of the number of bonds used in the sample, spaced apart so that there is an equal number of bonds maturing between adjacent nodes. This is different to the theoretical approach, which is to have node points at every interval where there is a bond cash flow, however in practice using the smaller number of nodes as proposed by Fisher produces essentially an identical forward rate curve, but with fewer calculations required. The resulting forward rate curve is the cubic spline that minimises the function at (53.17):

$$\sum_{i=1}^N (P_i - \hat{P}_i^f(f))^2 + \lambda \int_0^{\tau_N} (f''(s))^2 ds. \quad (53.17)$$

⁵ For example see Bliss (1997).

The value of λ is obtained by a method known as generalised cross-validation (GCV). It is the value that minimises the expression at (53.18):

$$\gamma(\lambda) = \frac{rss(\lambda)}{(N - \theta ep(\lambda))^2} \quad (53.18)$$

where

N is the number of bonds in the sample

$rss(\lambda)$ is the residual sum of squares, given by

$$rss(\lambda) = \sum_{i=1}^N (P_i - \hat{P}_i^f(f\lambda))^2 \text{ where } f_\lambda \text{ is the forward rate curve that minimises the expression,}$$

$ep(\lambda)$ is the effective number of parameters

θ is the cost or tuning parameter.

The higher the value for θ , the more rigid is the resulting spline. Fisher *et al.* and Waggoner both set θ equal to 2. Expression (53.17), for a fixed term λ , can be solved using a non-linear least squares method. The GCV method can be implemented by using a method known as a *line search*.

Following Fisher *et al.*, Waggoner (1997) proposes using a cubic spline to approximate the forward rate function, with the number of nodes again being approximately one-third of the number of bonds in the sample, and spaced so that there is an equal number of bonds maturing between adjacent nodes. The Waggoner approach is termed the “variable roughness method” (VRP). The cubic spline forward rate curve is selected that will minimise the function at (53.19):

$$\sum_{i=1}^N (P_i - \hat{P}_i(f))^2 + \int_0^{\tau_N} \lambda(s) (f''(s))^2 ds. \quad (53.19)$$

The roughness penalty λ is set as follows:

$$\lambda(t) = \begin{cases} 0.1 & 0 \leq t \leq 1 \\ 100 & 1 \leq t \leq 10 \\ 100,000 & 10 \leq t \end{cases}$$

where t is measured in years. The VRP method is non-linear and can be solved using the non-linear least squares method.

53.4.2 The Anderson–Sleath model

In this section we summarise a paper by Anderson and Sleath which first appeared in the Bank of England *Quarterly Bulletin* in November 1999. The main objective of this work was to evaluate the relative efficacy of parametric versus spline-based methods. In fact different applications call for different methods; the main advantage of spline methods is that individual functions in between knot points may move in fairly independent fashion, which makes the resulting curve more flexible than that possible using parametric techniques. In section 53.5.1 we reproduce their results with permission, which shows that a shock introduced at one end of the curve produces unsatisfactory results in the parametric curve.

The Anderson–Sleath model, which is the method adopted by the Bank of England, is a modification of the Waggoner approach in a number of significant ways. The $\lambda(t)$ function of Waggoner was adapted thus:

$$\log \lambda(m) = L - (L - S)e^{-m/\mu} \quad (53.20)$$

where the parameters to be estimated are L, S, μ . In addition the difference in bond market and theoretical prices is weighted with the inverse of the modified duration of the bond. This accounts for observed pricing errors for bonds that are more volatile than others.

The model therefore minimises the expression at (53.21):

$$X = \sum_{i=1}^N \left(\frac{P_i - \pi_i(c)}{MD_i} \right)^2 + \int_0^M \lambda_r(m) (f''(m))^2 dm \quad (53.21)$$

where P and MD are the price and modified duration of bond i , c is the parameter vector of the polynomial spline being estimated and M is the time to maturity of the longest-dated bond.

The outstanding feature of the Anderson–Sleath approach is their adaptation of both spline and parametric techniques.

53.4.3 Applications

Each of the methods described in this section can be used to fit the zero-coupon curve with validity. In practice results produced by each method imply that certain techniques are more suitable than others under specific conditions. Generally the incorporation of a “roughness” penalty that varies across maturities produces more accurate pricing of short-dated bonds, and this is the case in the Fisher and Waggoner methods. The McCulloch technique is reasonably accurate and, as it is a linear method, is more straightforward to implement than the other techniques. It produces a similar curve to the VRP method in terms of goodness of fit and smoothness. Therefore in most cases it is reasonable to use this method. The advantage of the VRP method is that it allows the user to select the degree of smoothing.

In deciding which method to use, practitioners will need to consider the effectiveness of each approach with regard to flexibility, simplicity and consistency. The requirements of central monetary authorities differ in some detail respects to investment and commercial banks, as we noted at the start of the chapter. Generally however curves should fit as wide a range of term structures as possible, and be tractable, or straightforward to compute. They should also be consistent with a yield curve model. For example the approach presented by Bjork and Christensen (1997) is compatible with the Hull–White or extended Vasicek yield curve model. In the same paper it is stated that the Nelson and Siegel technique is not consistent with any common term structure model. James and Webber (2000) state that the simplicity of the Nelson and Siegel approach, which is an advantage of the technique, is also its main drawback. In the same review it is concluded that B-spline methods are the most flexible and consistent, along with that described by Bjork and Christensen.

53.5 The Anderson–Sleath evaluation

53.5.1 Fitting the spot curve

In this section we summarise, with permission, results obtained in highly innovative research by Anderson and Sleath (1999), comparing the different methods. The accuracy of any of the techniques is usually tested by using a goodness of fit measure, for example if we fit the curve using n bonds we wish to minimise the measure given by (53.22):

$$X_P = \sum_{i=1}^n \left(\frac{P_i - \prod_i(\rho)}{MD_i} \right)^2 \quad (53.22)$$

where

- P_i is the market price of the i th bond
- MD_i is the modified duration of the i th bond
- $\prod_i(\rho)$ is the fitted price of the i th bond

A popular technique is the spline-based method of curve fitting, which we introduced in Chapter 6. Unlike other methods (such as the parametric Svensson method) which specify a single short-rate to describe the instantaneous forward rate curve, spline-based methods fit a curve to observed data that is composed of a number of sections, but with constraints to ensure that the curve is smooth and continuous. As this is one of the aims we stated at the beginning, this is an advantage of the spline-based method, as it allows individual sections of the curve to move independently of each other. This is demonstrated with Figures 53.3 and 53.4, which show a hypothetical yield curve that has been fitted, from an assumed set of bond prices, using the cubic spline method and a parametric method such as Svensson. The change of the long bond yield has a significant effect on the Svensson curve, notably at the

short end of the curve. The spline curve however undergoes only a slight change in response to the change in yield, and only at the long end.

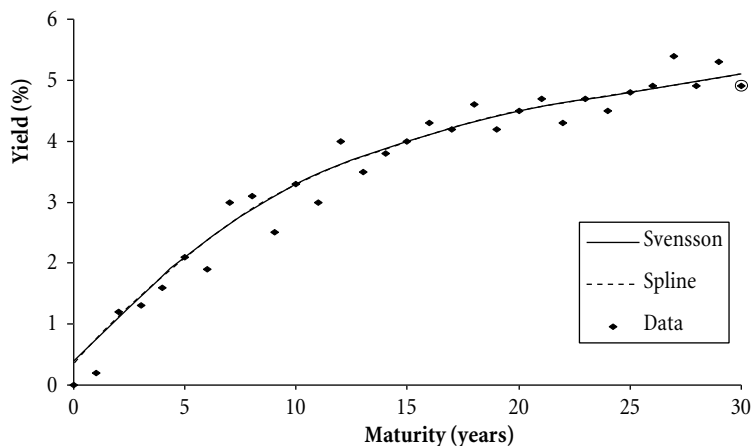


Figure 53.3: Yield curves fitted using cubic spline method and Svensson parametric method, hypothetical bond yields. Reproduced with permission from the Bank of England *Quarterly Bulletin*, November 1999.

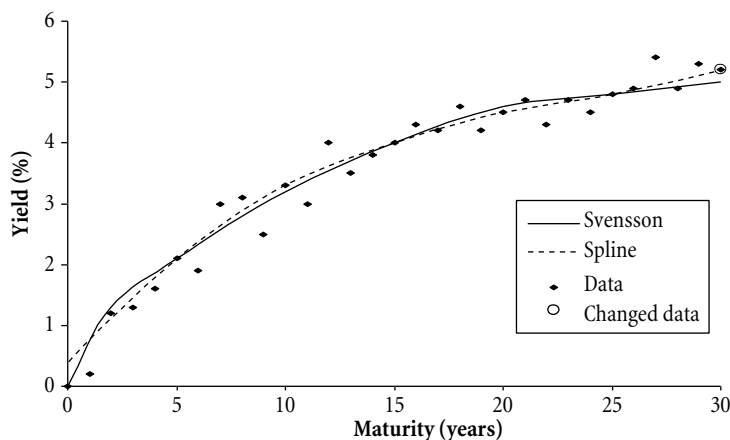


Figure 53.4: Effect on fitted yield curves of change in long-dated bond yield. Reproduced with permission from the Bank of England *Quarterly Bulletin*, November 1999.

The effect of a change in a yield on the Svensson curve is amplified because the technique specifies a constraint that results in yields converging to a constant level. This assumption is based on the belief that forward rates reflect market expectations of the future level of short rates, and following this the 30-year forward rate will be expected to be not significantly different from the 25-year or 20-year forward rate. This causes the forward rate after about 10 years to converge to a constant level.

We can compare fitted yield curves to an actual spot rate curve wherever there is an active government (risk-free) zero-coupon market in operation. In the UK a zero-coupon bond market was introduced in December 1997. In theory any derived spot rate curve can be compared to the actual spot rate curve, this comparison serving to provide an instant check of the accuracy of the yield curve model. In practice however, discrepancies between the observed and fitted curves may not have that much significance, because of the way that strip yields behave in practice. In the UK market there is a certain level of illiquidity associated with strip yields at certain points of the term structure; the UK market also exhibits a common trait of strip markets everywhere that the longest-dated issue traded dear to the

yield curve. Another factor is that coupon strips trade cheaper to principal strips; which yield should be used in the comparison?⁶ In Figure 53.5 we compare the theoretical spot curve fitted using the Svensson method to the observed coupon strip curve in July 1998, at a time when the UK gilt yield curve was inverted.

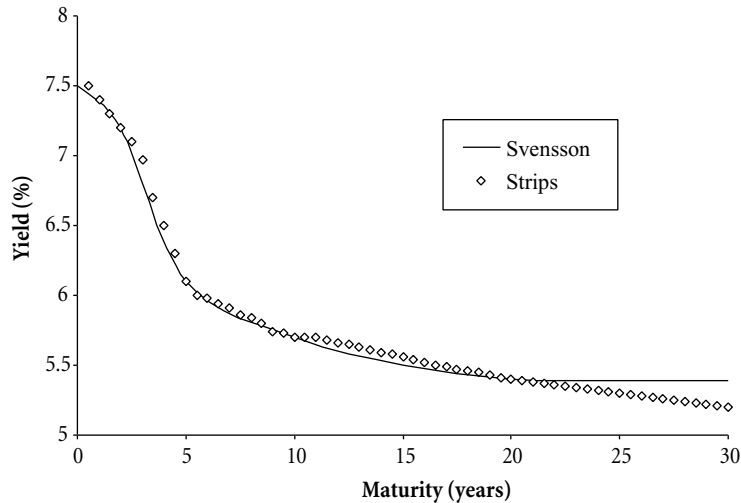


Figure 53.5: Comparison of fitted spot yield curve to observed spot yield curve.

Reproduced with permission from the Bank of England

Quarterly Bulletin, November 1999.

The fitted curve exhibits the constant long yield that we observed in the hypothetical yield curve in Figure 53.1, while the strip curve trades expensive at the long end, which as we noted is a common observation. Nevertheless for the purposes of accurate fitting the parametric method exhibits a significant difference to the observed curve. A cubic spline-based fitted curve such as that proposed by Waggoner (1997) produces a more realistic curve, as shown by Figure 53.6.

This reflects the properties of the spline curve, including the fact that forward rates are described by a series of segments that are in effect connected together. This has the effect of localising the influence of individual yield movements to only the relevant part of the yield curve; it also allows the curve to match more closely the observed yield curve. The goodness of the spline-based method is measured using (53.23):

$$X_S = X_P + \int_0^M \lambda_t(m) (f''(m))^2 dm \quad (53.23)$$

where $f''(m)$ is the second derivative of the fitted forward curve and M is the maturity of the longest-dated bond. The term $\lambda_t(m)$ is the “roughness penalty”. Figure 53.6 shows that the spline-based method generates a more realistic curve, that better mirrors the strip yield curve seen in Figure 53.5.

⁶ This is the observation that, due to demand and liquidity reasons, zero-coupon bonds sourced from the principal cash flow of a coupon bond trades at a lower yield than equivalent-maturity zero-coupon bonds sourced from the coupon cash flow of a conventional bond.

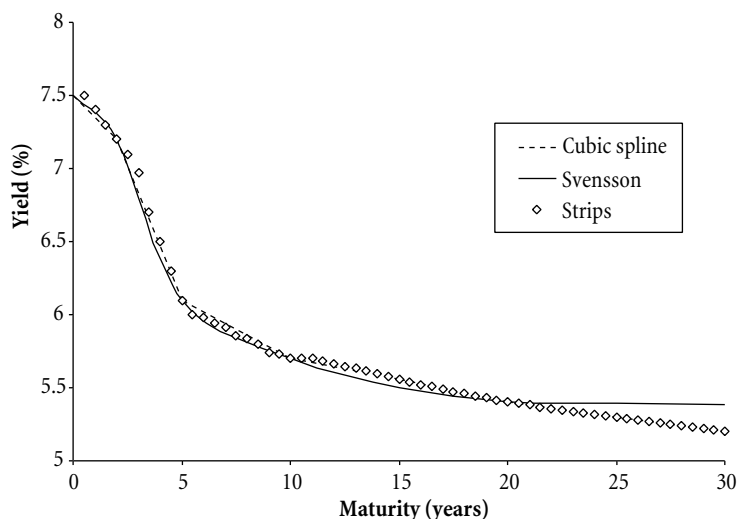


Figure 53.6: Fitted yield curves and observed strip yield curve, July 1998.

Reproduced with permission from the Bank of England
Quarterly Bulletin, November 1999.

53.5.2 Repo and estimating the short-end of the yield curve

For the purposes of conducting monetary policy and for central government requirements, little use is made of the short-end of the yield curve. This is for two reasons; one is that monetary and government policy is primarily concerned with medium-term views, for which a short-term curve has no practical input, the second is that there is often a shortage of data that can be used to fit the short-term curve accurately. In the same way that the long-term term structure must be fitted using risk-free instruments, the short-term curve can only be estimated using Treasury bills. The T-bill can be restricted to only a small number of participants in some markets; moreover the yield available on T-bills reflects its near cash, risk-free status, and so may not be the ideal instrument to use when seeking to extract market views on forward rates. So for liquidity purposes the existence of an alternative instrument to T-bills would be useful. In most respects the government repurchase market or repo market is a satisfactory substitute for T-bills, although there is an element of counterparty risk associated with repo that does not apply to T-bills, they can be considered to be essentially risk-free instruments, more so if margin has been taken by the party lending cash. We can therefore consider general collateral repo to be essentially a liquid, short-term and risk-free instrument.⁷

The fitted spot curve can differ considerably if yields on short-term repo is included. The effect is shown in Figure 53.7, which is reproduced from Anderson and Sleath (1999). Note that this is a short-term spot curve only; the maturity extends out to only two years. Two curves have been estimated; the cubic spline-based yield curve using the repo rate and without the repo rate. The curve that uses repo data generates a curve that is much closer to the money market yield curve than the one that does not. The only impact is at the very short end. After about one year, both approaches generate very similar curves.

⁷ See Chapter 34 for more information on the repo markets and the UK gilt repo market.

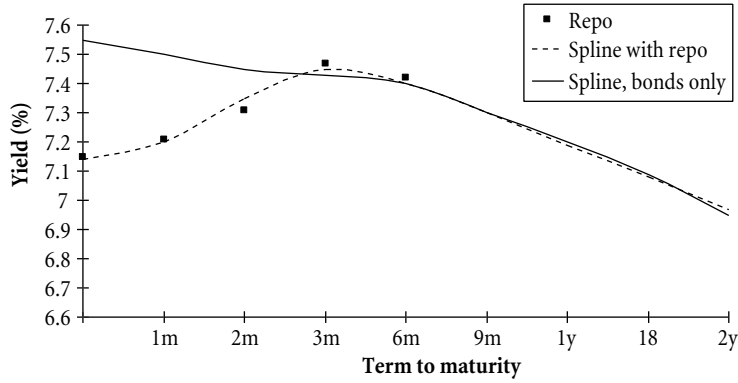


Figure 53.7: Fitting short-term yield curves using government repo rates.
 Reproduced with permission from the Bank of England
Quarterly Bulletin, November 1999.

For an account of the impact of “special” repo rates on term structure modelling see Barone and Risa (1994) and Duffie (1993), which are available from the respective institution web sites.

Appendices

APPENDIX 53.1 The McCulloch cubic spline model

This was first described by McCulloch (1975) and is referred to in Deacon and Derry (1994). We assume the maturity term structure is partitioned into q knot points with $q_1, \dots, q_q$ where $q_1 = 0$ and q_q is the maturity of the longest-dated bond. The remaining knot points are spaced such that there is, as far as possible, an equal number of bonds between each pair of knot points. With $j < q$, we employ the following functions:

- for $m < q_{j-1}$

$$f_j(m) = 0 \quad (53.24)$$

- for $q_{j-1} \leq m \leq q_j$

$$f_j(m) = \frac{(m - q_{j-1})^3}{6(q_j - q_{j-1})} \quad (53.25)$$

- for $q_j \leq m \leq q_{j+1}$

$$f_j(m) = \frac{c^2}{6} + \frac{cf}{2} + \frac{f^2}{2} - \frac{f^3}{6(q_{j+1} - q_j)} \quad (53.26)$$

where

$$c = q_j - q_{j-1}$$

$$f = m - q_j$$

- for $q_{j+1} \leq m$

$$f_j(m) = (q_{j+1} - q_{j-1}) \left(\frac{2q_{j+1} - q_j - q_{j-1}}{6} + \frac{m - q_{j+1}}{2} \right) \quad (53.27)$$

- for $j = q$

the function $f_q(m) = m$ for all values of m .

APPENDIX 53.2 Parametric and cubic spline yield curve models

In the Nelson and Siegel method (1987), we may model the implied forward rate yield curve along the entire term structure using the following function:

$$rf(m, \beta) = \beta_0 + \beta_1 \exp\left(\frac{-m}{t_1}\right) + \beta_2 \left(\frac{m}{t_1}\right) \exp\left(\frac{-m}{t_1}\right) \quad (53.28)$$

where $\beta = (\beta_0, \beta_1, \beta_2, t_1)'$ is the vector of parameters describing the yield curve, and m is the maturity at which the forward rate is calculated. There are three components, the constant term, a decay term and a term reflecting the “humped” nature of the curve. The shape of the curve will gradually lead into an asymptote at the long end, the value of which is given by β_0 , with a value of $\beta_0 + \beta_1$ at the short end.

Svensson (1994) presents a modification of this, by means of an adjustment to allow for the humped characteristic of most yield curves. This is fitted by adding an extension, as shown by (53.29):

$$rf(m, \beta) = \beta_0 + \beta_1 \exp\left(\frac{-m}{t_1}\right) + \beta_2 \left(\frac{m}{t_1}\right) \exp\left(\frac{-m}{t_1}\right) + \beta_3 \left(\frac{m}{t_2}\right) \exp\left(\frac{-m}{t_2}\right). \quad (53.29)$$

So we note that the Svensson curve is modelled using six parameters, the additional inputs being β_3 and t_2 .

A different approach is adopted by smoothing cubic spline models. A generic spline is a segmented polynomial, or a curve that is constructed from individual polynomial segments that are joined together at user-specified “knot points”. That is, the x -axis is divided into selected segments (the knot points). The segments can be at equal intervals or otherwise. At the knot points the curve and its first derivative are continuous at all points along the curve. Generally the market uses cubic functions, resulting in a cubic spline. A cubic spline is given by (53.30):

$$S(x) = \alpha x^3 + \beta x^2 + \gamma x + \delta + \sum_{i=1}^{N-1} \eta_i |x - k_i|^3 \quad (53.30)$$

for a range of constants $\alpha, \beta, \gamma, \delta$, and η and where $k_i, i = [0, N]$ is the set of knot points. The expression at (53.30) is the most common one used for a cubic spline, however in practice it is unwieldy for the purposes of calculation. Therefore splines are usually constructed as a linear combination of cubic basis splines or B-splines. This is a general transformation which removes the numerical problems associated with (53.30). A B-spline of order n can be written in the form:

$$B_{i,n}(x) = \frac{x - k_i}{k_{i+n-1} - k_i} B_{i,n-1}(x) + \frac{k_{i+n} - x}{k_{i+n} - k_{i+1}} B_{i+1,n-1}(x) \quad (53.31)$$

where $B_{i,1}(x) = 1$ if $k_i \leq x < k_{i+1}$, and $B_{i,1}(x) = 0$ otherwise. This approach was described in Lancaster and Šalkauskas (1986). When a large number of knot points are used a cubic spline can be used for interpolation, however as noted by Anderson and Sleath (1999) this approach is not used for monetary policy purposes, because it does not produce a smooth curve.

Selected bibliography and references

- Anderson, N., Breedon, F., Deacon, M., Derry, A., Murphy, M., *Estimating and Interpreting the Yield Curve*, Wiley, 1996.
- Anderson, N., Sleath, J., “New estimates of the UK real and nominal yield curves”, *Bank of England Quarterly Bulletin*, November 1999, pp. 384–392.
- Bank of England, *Quarterly Bulletin*, August 1999.
- Barone, E., Risa, S., “Valuation of Floaters and Options on Floaters under Special Repo Rates”, Instituto Mobiliare Italiano, 1994.
- Bjork, T., Christensen, B., “Interest Rate Dynamics and Consistent Forward Rate Curves”, University of Aarhus Working Paper, 1997.
- Bliss, R., “Testing term structure estimation methods”, *Advances in Futures and Options Research* 9, 1997, pp. 197–231.
- Brenner, R., Jarrow, R., “A Simple Formula for Options on Discount Bonds”, *Advances in Futures and Options Research* 6, 1993, pp. 45–51.
- Choudhry, M., *Bond Market Securities*, FT Prentice Hall, 2001.
- Cox, J., Ingersoll, J., Ross, S., “A Theory of the Term Structure of Interest Rates”, *Econometrica* 53, 1985, pp. 385–407.

- Dahlquist, M., Svensson, L., “Estimating the Term Structure of Interest Rates for Monetary Policy Analysis”, *Scandinavian Journal of Economics* 98, 1996, pp. 163–183.
- Dothan, U., “On the term structure of interest rates”, *Journal of Financial Economics* 6, 1978, pp. 59–69.
- Deacon, M., Derry, A., “Estimating the term structure of interest rates”, *Bank of England Working Paper*, No. 24, July 1994.
- Duffie, D., *Special Repo Rates*, Graduate School of Business, Stanford University, 1993.
- Edmister, R.O., Madan, D.P., “Informational Content in Interest Rate Term Structures”, *The Review of Economics and Statistics* 75 (4), November 1993, pp. 695–699.
- Fisher, M., Nychka, D., Zervos, D., “Fitting the term structure of interest rates with smoothing splines”, *Finance and Economic Discussion Series* 95–1, Federal Reserve Board, 1995.
- Harrison, J., Pliska, S., “Martingales and Stochastic Integrals in the Theory of Continuous Trading”, *Stochastic Processes and their Applications* 11, 1981, pp. 215–260.
- Heath, D., Jarrow, R., Morton, A., “A New Methodology for Contingent Claims Valuation”, *Econometrica* 60, 1992, pp. 76–106.
- James, J., Webber, N., *Interest Rate Modelling*, Wiley, 2000.
- Kim, J., “A Discrete-Time Approximation of a One-Factor Markov Model of the Term Structure of Interest Rates”, Graduate School of Industrial Administration, Carnegie-Mellon University.
- Lancaster, P., Salkauskas, K., *Curve and surface fitting: an introduction*, Academic Press, 1986.
- Mastronikola, K., “Yield curves for gilt-edged stocks: a new model”, *Bank of England Discussion Paper (Technical Series)*, No. 49, 1991.
- McCulloch, J., “Measuring the term structure of interest rates”, *Journal of Business* 44, January 1971, pp. 19–31.
- McCulloch, J., “The Tax Adjusted Yield Curve”, *Journal of Finance* 30, June 1975, pp. 811–829.
- Nelson, C., Siegel, A., “Parsimonious modelling of yield curves”, *Journal of Business*, Volume 60, 1987, p. 473.
- Schaefer, S., “Measuring a tax-specific term structure of interest rates in the market for British government securities”, *The Economic Journal*, Volume 91, June 1981, pp. 415–438.
- Shea, G., “Interest-rate term structure estimation with exponential splines: a note”, *Journal of Finance* 40(1), March 1985, pp. 319–325.
- Svensson, L., “Estimating and Interpreting Forward Interest Rates: Sweden 1992–94”, *International Monetary Fund Working Paper* 114, 1994.
- Svensson, L., “Estimating forward interest rates with the extended Nelson & Siegel method”, *Sveriges Riksbank Quarterly Review*, 1995:3, p. 13.
- Waggoner, D., “The robustness of yield and forward rate extraction methods”, unpublished working paper, Federal Reserve Bank of Atlanta, September 1996.
- Waggoner, D., “Spline methods for extracting interest rate curves from coupon bond prices”, *Working Paper*, No. 97–10, Federal Reserve Bank of Atlanta, 1997.
- Vasicek, O., “An Equilibrium Characterization of the Term Structure”, *Journal of Financial Economics* 5, 1977, pp. 177–188.
- Vasicek, O., Fong, H.G., “Term structure modelling using exponential splines”, *Journal of Finance* 37(2), May 1982, pp. 339–356.
- Wiseman, J., “The Exponential Yield Curve Model”, European Fixed Income Research, Technical Specification, JP Morgan, 1994.

Questions and exercises

1. Discuss the differing requirements that central monetary authorities and market practitioners have for theoretical spot and forward curves.
2. What are the advantages of using a cubic spline-based technique for fitting the term structure?

3. Consider the following hypothetical set of risk-free coupon bonds:

Maturity (years)	Bond (annual coupon)	Price	Yield %
1	7.50% 2001	102.235	5.15
2	8% 2002	105.095	5.25
3	5.75% 2003	100	5.75
4	6% 2004	100	6.00
5	9% 2005	111.96	6.15
7	8.25% 2007	108.155	6.75
8	6.50% 2008	99.39	6.60
10	5.75% 2010	93.23	6.70

Write down (but do not solve) the expression that must be solved to fit the term structure in this yield curve environment using the

- regression-based method;
 - the Fisher method;
 - the VRP method as proposed by Waggoner.
4. From Nelson and Siegel's original model of the forward rate curve, derive the expression for the spot rate.

54

Advanced Analytics for Index-Linked Bonds

Bonds that have their part or all of their cash flows linked to an inflation index form an important segment of several government bond markets. In the United Kingdom the first index-linked bonds were issued in 1981 and at the end of 1999 they accounted for approximately 15% of outstanding nominal value in the gilt market. Index-linked bonds were only recently introduced in the US Treasury market (see Chapter 12) but are more established in Australia, Canada, the Netherlands, New Zealand and Sweden. There is no uniformity in market structure and as such there are significant differences between the index-linked markets in these countries. There is also a wide variation in the depth and liquidity of these markets.

Index-linked bonds or inflation-indexed bonds present additional issues in their analysis, due to the nature of their cash flows. Measuring the return on index-linked bonds is less straightforward than with conventional bonds, and in certain cases there are peculiar market structures that must be taken into account as well. For example, in the US market index-linked Treasuries (known as “TIPS” from Treasury inflation-indexed securities) there is no significant lag between the inflation link and the cash flow payment date. In the UK there is an eight-month lag between the inflation adjustment of the cash flow and the cash flow payment date itself, while in New Zealand there is a three-month lag. The existence of a lag means that inflation protection is not available in the lag period, and that the return in this period is exposed to inflation risk; it also must be taken into account when analysing the bond.

From market observation we know that index-linked bonds can experience considerable volatility in prices, similar to conventional bonds, and therefore there is an element of volatility in the real yield return of these bonds. Traditional economic theory states that the level of real interest rates is constant, however in practice they do vary over time. In addition there are liquidity and supply and demand factors that affect the market prices of index-linked bonds. In this chapter we present analytical techniques that can be applied to index-linked bonds, the duration and volatility of index-linked bonds and the concept of the real interest rate term structure.

54.1 Index-linked bonds and real yields

The real return generated by an index-linked bond, or its real yield, is usually defined as yield on risk-free index-linked bonds, or in other words the long-term interest rate on risk-free funds minus the effect of inflation. There may also be other factors involved, such as the impact of taxation. Therefore the return on an index-linked bond should in theory move in line with the real cost of capital. This will be influenced by the long-term growth in the level of real gross domestic product in the economy. This is because in an economy experiencing rapid growth, real interest rates are pushed upwards as the demand for capital increases, and investors therefore expect higher real yields. Returns are also influenced by the demand for the bonds themselves.

The effect of general economic conditions and the change in these over time results in real yields on index-linked bonds fluctuating over time, in the same way nominal yields fluctuate for conventional bonds. This means that the price behaviour of indexed bonds can also be fairly volatile.

The yields on indexed bonds can be used to imply market expectations about the level of inflation. For analysts and policy makers to use indexed bond yields in this way, it is important that a liquid secondary market exists in the bonds themselves. For example the market in Australian index-linked bonds is relatively illiquid, so attempting to extract an information content from their yields may not be valid. Generally though the real yields on indexed bonds reflect investors' demand for an inflation premium, or rather a premium for the uncertainty regarding future inflation levels. This is because holders of indexed bonds are not exposed to inflation-eroded returns; therefore if future inflation was expected to be zero, or known with certainty (whatever its level) there would be no requirement for an inflation premium, because there would be no uncertainty. In the same way, the (nominal) yields on conventional bonds reflect market expectations on inflation levels. Therefore higher volatility of the expected inflation rate will lead to a higher inflation risk premium on conventional bonds, and a lower real yield on indexed bonds relative to nominal yields. It is the uncertainty regarding future inflation levels that creates a demand for an inflation risk yield

premium, rather than past experience of inflation levels. However investor sentiment may well demand a higher inflation premium in a country with a poor record in combating inflation.

Traditionally information on inflation expectations has been obtained by survey methods or theoretical methods. These have not proved reliable however, and were followed only because of the absence of an inflation indexed futures market.¹ Certain methods for assessing market inflation expectations are not analytically valid; for example the suggestion that the spread between short-term and long-term bond yields cannot be taken to be a measure of inflation expectation, because there are other factors that drive this yield spread, and not just inflation risk premium. We reviewed these factors in Chapter 6. Equally, the spread between the very short-term (overnight or one week) interest rate and the two-year bond yield cannot be viewed as purely driven by inflation expectations. Using such approaches to glean information on inflation expectations is logically unsound. One approach that is valid, as far as it goes, would be to analyse the spread between historical real and nominal yields, although this is not a forward-looking method. It would however indicate the market's inflation premium over a period of time. The best approach though is to use the indexed bond market; given a liquid market in conventional and index-linked bonds it is possible to derive estimates of inflation expectations from the yields of both sets of bonds. This is reviewed later in the chapter.

54.2 Duration and index-linked bonds

In earlier chapters we reviewed the basic features of index-linked bonds and their main uses. We also discussed the techniques used to measure the yield on these bonds. The largest investors in indexed bonds are long-dated institutions such as pension fund managers, who use them to match long-dated liabilities that are also index-linked, for example a pension contract that has payments linked to the inflation index. It is common though for investors to hold a mixture of indexed and conventional bonds in their overall portfolio.

The duration of a bond is used as a measure of its sensitivity to changes in interest rate. The traditional measure, if applied to indexed bonds, will result in high values due to the low coupon on these bonds and the low real yield. In fact the longest-duration bonds in most markets are long-dated indexed bonds. The measure, if used in this way however, is not directly comparable to the duration measure for a conventional bond. Remember that the duration of a conventional bond measures its sensitivity to changes in (nominal) yields, or put another way to changes in the combined effect of inflation expectations and real yields. The duration measure of an indexed bond on the other hand, would be a measure of its sensitivity to changes in real yields only, that is to changes in real interest rate expectations. Therefore it is not valid to compare traditional duration measures between conventional bonds and indexed bonds, because one would not be comparing like-for-like. This has important implications for the portfolio level. If a portfolio is composed of both conventional and indexed bonds, how does one measure its combined duration? The traditional approach of combining the duration values of individual bonds would have no meaning in this context, because the duration measure for each type of bond is measuring something different. For example, consider a situation where there are two portfolios with the same duration measure. If one portfolio was composed of a greater amount by weighting of index-linked bonds, it would have a different response to changes in market yields, especially so if investors' economic expectations shifted significantly, compared to the portfolio with a lower weighting in indexed bonds. Therefore a duration-based approach to market risk would no longer be adequate as a means of controlling portfolio market risk.

Therefore the key focus of fund managers that run combined portfolios of conventional and indexed bonds is to manage the duration of the conventional and indexed bonds on a separate basis, and to be aware of the relative weighting of the portfolio in terms of the two bond types. A common approach is to report two separate duration values for the portfolio, which would measure two separate types of risk exposure. One measure would be the *portfolio real yield duration*, which is the value of the combined durations of both the conventional and indexed bonds. This measure is an indication of how the portfolio value will be affected by a change in market real yields, which would impact both indexed and conventional bond yields. The other measure would be the *portfolio inflation duration*, which is duration measure for the conventional bonds only. This duration measure indicates the sensitivity of the portfolio to a change in market inflation expectations, which have an impact on nominal yields but

¹ The New York Coffee, Sugar and Cocoa Exchange traded a futures contract on the US consumer prices index (CPI) in the 1980s.

not real yields. Portfolio managers also follow a similar approach with regard to interest-rate volatility scenarios. Therefore if carrying out a parallel yield curve shift simulation, which in terms of a combined portfolio would actually correspond to a real-yield simulation, the portfolio manager would also need to undertake a simulation that mirrored the effect of a change in inflation expectations, which would have an impact on nominal yields only.

The traditional duration approach can be used with care in other areas. As we saw in Chapter 11 in the section on index-linked gilts, the Bank of England monetary policy committee is tasked with keeping inflation at a level of 2.5%. If therefore the ten-year benchmark gilt is trading at a yield of 6.00% while the ten-year index-linked gilt is trading at a real yield of 3.00%, this implies that the market expectation of average inflation rates during the next ten years is 3.00%. This would suggest that the benchmark gilt is undervalued relative to the indexed gilt. To effect a trade that matched the market maker's view, one would short the ten-year index-linked gilt and buy the conventional gilt. If the view turned out to be correct and market inflation expectations declined, the trade would generate a profit. If on the other hand real interest-rate expectations changed, thus altering real yields, there would be no effect. The other use of the traditional duration approach is with regard to hedging. Indexed bonds are sometimes difficult to hedge because of the lack of suitable hedging instruments. The most common hedging instrument is another indexed bond, and the market maker would use a duration weighting approach to calculate the nominal value of the hedging bond.

In the traditional approach the duration value is calculated using nominal cash flows, discounted at the nominal yield. A more common approach is to assume a constant average rate of inflation, and adjust cash flows using this inflation rate. The real yield is then used to discount the assumed future cash flows. There are a number of other techniques that can be used to calculate a duration value, all requiring the forecasting of the level of future cash flows and discounting using the nominal yield. These include:

- as above, assuming a constant average inflation rate, which are then used to calculate the value of the bond's coupon and redemption payments. The duration of the cash flow is then calculated by observing the effect of a parallel shift in the zero-coupon yield curve. By assuming a constant inflation rate and constant increase in the cash flow stream, a further assumption is made that the parallel shift in the yield curve is as a result of changes in real yields, not because of changes in inflation expectations. Therefore this duration measure becomes in effect a real yield duration;
- a repeat of the above procedure, with the additional step, after the shift in the yield curve, of recalculating the bond cash flows based on a new inflation forecast. This produces a duration measure that is a function of the level of nominal yields. This measure is in effect an inflation duration, or the sensitivity to changes in market inflation expectations, which is a different measure to the real yield duration;
- an assumption that the inflation scenario will change by an amount based on the historical relationship between nominal yields and the market expectation of inflation. This is in effect a calculation of nominal yield duration, and would be a measure of sensitivity to changes in nominal yields.

Possibly the most important duration measure is the real yield duration, which is more significant in markets where there is a lag between the indexation and cash flow dates, due to the inflation risk exposure that is in place during the lag period. This is the case in both the UK and Australia, although as we noted the lag is not significant in the US market. It is worth noting that index-linked bonds do not have stable nominal duration values, that is, they do not exhibit a perfectly predictable response to changes in nominal yields. If they did, there would be no advantage in holding them, as their behaviour could be replicated by conventional bonds. For this reason, index-linked bonds cannot be hedged perfectly with conventional bonds, although this does happen in practice on occasions when no other hedging instrument is available.

One final point regarding duration is that it is possible to calculate a *tax-adjusted duration* for an index-linked bond in markets where there is a different tax treatment to indexed bonds compared to conventional bonds. In the US market the returns on indexed and conventional bonds are taxed in essentially the same manner, so that in similar fashion to Treasury strips, the inflation adjustment to the indexed bond's principal is taxable as it occurs, and not only on the maturity date. Therefore in the US indexed bonds do not offer protection against any impact of after-tax effects of high inflation. That is, Tips real yields reflect a premium for only pre-tax inflation risk. In the UK market however, index-linked gilts receive preferential tax treatment, so their yields also reflect a premium for after-

tax inflation risk. In practice this means that the majority of indexed gilt investors are those with high marginal tax rates.² This factor also introduces another element in analysis; if the demand for indexed or conventional bonds were to be a function of expected after-tax returns, this would imply that pre-tax real yields should rise as expected inflation rates rise, in order to maintain a constant after-tax real yield. This has not been observed explicitly in practice, but is a further factor of uncertainty about the behaviour of real yields on index-linked bonds.³

B Corp YA

Enter all values and hit <GO>.

INDEX-LINKED YIELD ANALYSIS

TSY I/L

UKT2 1/2 05/20/09

217.2713/217.3338 (1.97/1.97) BGN @11/08

SETTLEMENT DATE

11/10/1999

ECONOMIC FACTORS

ASSUMED ANNUAL INFLATION RATE(%)

3.00

NEXT INTEREST PAYMENT

2.6044

BASE

78.7579

CURRENT

166.2

RPI

DATE

2/82

9/99

CLEAN PRICE

217.33384900

REAL YIELD

1.967

MONEY YIELD

4.972

YIELD VALUE OF A

0.02

REAL YIELD ADJ DURATION

8.386

INFLATION ADJ DURATION

8.262

INTEREST YIELD

2.397

AFTER TAX YIELD (INCOME = 95.00 & CAP GAINS =) = 1.114

NUMBER OF BONDS

1000

REINVESTMENT RATE

4.972

EFFECTIVE YLD

4.972

PAYMENT

INCOME

PRINCIPAL

2173338.49

ACCRUED INTEREST

24628.57

SCHEDULED

TOTAL PAYMENT

2197967.06

PROJ REDEMPTION VALUE

2752739.03

PROJ COUPON PAYMENTS

600311.00

INTEREST ON INTEREST

156192.91

TOTAL INCOME

3509242.94

PROFIT

1311275.89

Copyright 1999 BLOOMBERG L.P.

Frankfurt 1-606-2040

Hong Kong 1-606-2000

London 1-330-7500

New York 1-212-318-2000

Princeton 609-279-3000

Singapore 226-2000

Sydney 2-9777-8888

Tokyo 3-3201-8900

Sao Paulo 11-9048-4500

1999-059-9 09-Nov-99 10-23-02

Bloomberg

Copyright 1999 BLOOMBERG L.P. Frankfurt 119-720410 Hong Kong 2339-2722-4200 London 171-330-7500 New York 212-318-2000
 Princeton 609-279-3000 Singapore 226-2000 Sydney 2-5777-6686 Tokyo 3-3201-8900 Sao Paulo 11-3948-4500
 1593-359-0 09-Nov-99 10:23:32

Bloomberg

Figure 54.1: Example of index-linked yield analysis, UK 2½% Treasury 2009 (assumed annual inflation rate 3.00%, base inflation index 78.7579, current index 166.2), showing real yield and money yield, 9 November 1999.
 ©Bloomberg LP Reproduced and used with permission.

54.3 Estimating the real term structure of interest rates

In Chapter 11 we considered some approaches used to measure inflation expectations, with reference to UK index-linked gilts. To recap, these measures included:

- the “simple” approach, where the average expected inflation rate is calculated using the Fisher identity, so that the inflation estimate is regarded as the straight difference between the real yield on an index-linked bond, at an assumed average rate of inflation, and the yield on a conventional bond of similar maturity;
- the “break-even” inflation expectation, where average inflation expectations are estimated by comparing the return on a conventional bond against that on an indexed bond of similar maturity, but including an application of the compound form of the Fisher identity. This has the effect of decomposing the nominal rate of return on the bond into components of real yield and inflation;
- a variation of the break-even approach, but matching stocks by duration rather than by maturity.

The drawbacks of each of these approaches were discussed in Chapter 11. A rather more valid and sound approach is to construct a term structure of the real interest rates, which would indicate, in exactly the same way that the conventional forward rate curve does for nominal rates, the market’s expectations on future inflation rates. In countries where there are liquid markets in both conventional and inflation-indexed bonds, we can observe a nominal yield curve and a real yield curve. It then becomes possible to estimate both a conventional term structure and a real term structure; using these allows us to create pairs of hypothetical conventional and indexed bonds that

² For example, see Brown and Schaefer (1996).

³ For further detail on this phenomenon, see Roll (1996).

have identical maturity dates, for any point on the term structure.⁴ We could then apply the break-even approach to any pair of bonds to obtain a continuous curve for both the average and the forward inflation expectations. To maximise use of the available information we can use all the conventional and indexed bonds that have reasonable liquidity in the secondary market.

In this section we review one method that can be used to estimate and fit a real term structure.

54.3.1 The term structure of implied forward inflation rates

In previous chapters we reviewed the different approaches to yield curve modelling used to derive a nominal term structure of interest rates. We saw that the choice of yield curve model can have a significant effect on the resulting term structure; in the same way, the choice of model will impact the resulting real rate term structure as well. One approach has been described by McCulloch (1975), while in the UK market the Bank of England uses a modified version of the approach posited by Waggoner (1997) which we discussed in the previous chapter. McCulloch's approach involves estimating a discount function by imposing a constraint on the price of bonds in the sample to equal the sum of the discounted values of the bonds' cash flows. The Waggoner approach uses a cubic spline-based method, like McCulloch, with a roughness penalty that imposes a trade-off between the smoothness of the curve and its level of forward rate oscillation. The difference between the two approaches is that with McCulloch it is the discount function that is specified by the spline function, whereas in the Waggoner model it is the zero-coupon curve. Both approaches are valid, in fact due to the relationship between the discount function, zero-coupon rate and forward rate, both methods will derive similar curves under most conditions.

Using the prices of index-linked bonds it is possible to estimate a term structure of real interest rates. The estimation of such a curve provides a real interest counterpart to the nominal term structure that was discussed in the previous chapters. More important it enables us to derive a real forward rate curve. This enables the real yield curve to be used as a source of information on the market's view of expected future inflation. In the UK market there are two factors that present problems for the estimation of the real term structure; the first is the eight-month lag between the indexation uplift and the cash flow date, and the second is that fact that there are fewer index-linked bonds in issue, compared to the number of conventional bonds. The indexation lag means that in the absence of a measure of expected inflation, real bond yields are dependent to some extent on the assumed rate of future inflation. The second factor presents practical problems in curve estimation; in December 1999 there were only 11 index-linked gilts in existence, and this not sufficient for most models. Neither of these factors present an insurmountable problem however, and it is still possible to estimate a real term structure.

54.3.2 Estimating the real term structure⁵

There are a number of techniques that can be applied in estimating the real term structure. One method was described by Schaefer (1981). The method we describe here is a modified version of the cubic spline technique described by Schaefer. This is a relatively straightforward approach. The adjustment involves simplifying the model, ignoring tax effects and fitting the yield-to-maturity structure. A reduced number of nodes defining the cubic spline is specified than is the case with the conventional term structure, because of the fewer number of index-linked bonds available, and usually only three node points are used. Our approach therefore estimates three parameters, defining a spline consisting of two cubic functions, using 11 data points. The approach is defined below.

In the first instance, we require the real redemption yield for each of the indexed bonds. This is the yield that is calculated by assuming a constant average rate of inflation, applying this to the cash flows for each bond, and computing the redemption yield in the normal manner. The yield is therefore the market-observed yield, using the price quoted for each bond. These yields are used to define an initial estimate of the real yield curve, as they form the initial values of the parameters that represent the real yield at each node point. The second step is to use a non-linear technique to estimate the values of the parameters that will minimise the sum of the squared residuals between the observed and fitted real yields. The fitted yield curve is viewed as the real par yield curve; from this curve we calculate the term structure of real interest rates and the implied forward rate curve, using the technique described

⁴ We are restricted however to the longest-dated maturity of either of the two types of bonds.

⁵ This section follows the approach (with permission) from Deacon and Derry (1994), a highly accessible account. This is their Bank of England working paper, "Deriving Estimates of Inflation Expectations from the Prices of UK Government Bonds".

in Chapter 53. In estimating the real term structure in this way, we need to be aware of any tax effects. In the UK market, there is a potentially favourable tax effect, which may not apply in say, the US Tips market. Generally for UK indexed gilts, high marginal taxpayers are the biggest holders of index-linked bonds because of the ratio of capital gain to income, and their preference is to hold shorter-dated indexed bonds. On the other hand pension funds, which are exempt from income tax, prefer to hold longer-dated indexed gilts. The approach we have summarised here ignores any tax effects, but to be completely accurate any tax impact must be accounted for in the real term structure.

54.3.3 Fitting the discount function

The term structure method described by McCulloch (1971) involved fitting a discount function, rather than a spot curve, using the market prices of a sample of bonds. This approach can be used with only minor modifications to produce a real term structure.

Given the bond price equation at (54.1):

$$P_i = C_i \int_0^{T_i} df(\mu) d\mu + M_i df(T_i) \quad (54.1)$$

where P_i, C_i, T_i, M_i are the price, coupon, maturity and principal payment of the i th bond, we set the set of discrete discount factors as the discount function df , defined as a linear combination of a set of k linearly independent underlying basis functions, given by (54.2):

$$df(T) = 1 + \sum_{j=1}^k a_j f_j(T) \quad (54.2)$$

where $f_j(T)$ is the j th basis function and a_j is the corresponding coefficient, with $j = 1, 2, \dots, k$. It can be shown (see Deacon and Derry (1994)) that for index-linked bonds equation (54.2) can be adapted by a scaling factor Δ_i that is known for each bond, once an assumption has been made about the future average inflation rate, to fit a discount function for indexed bonds. We estimate the coefficients a_j from:

$$y_i = \sum_{j=1}^k a_j x_{ij}$$

where

$$\begin{aligned} y_i &= P_i - \Delta_i C_i T_i - \Delta_i M_i \\ x_{ij} &= \Delta_i C_i \int_0^{T_i} f_j(\mu) d\mu + \Delta_i M_i f_j(T_i) \\ u &= (1 + \pi^e)^{-1/2} \\ \Delta_i &= \begin{cases} [u^{tdj}]_i \cdot \frac{RPID_j}{RPiB_i} & \text{if } RPID_j \text{ is known} \\ [u^{tdl-L/6}]_i \cdot \frac{RPIL}{RPiB_i} & \text{otherwise} \end{cases} \end{aligned}$$

where P_i, C_i, T_i, M_i are as before, but this time representing the index-linked bond. The scaling factor Δ_i is that for the i th bond, and depends on the ratio of the retail price index (RPI) at the time compared to the RPI level in place at the time the bond was issued, known as the *base* RPI.⁶ If in fact the RPI that is used to index any particular cash flow is not known, it must be estimated using the latest available RPI figure, in conjunction with an assumption about the path of future inflation, using π^e .

54.3.4 Deriving the term structure of inflation expectations

Using any of the methods described in Chapter 53 or the discount function approach summarised above, we can construct curves for both the nominal and the real implied forward rates. These two curves can then be used to infer

⁶ Due to the lag in the UK gilt market, for index-linked gilts the base RPI is actually the level recorded for the month eight months before the issue date.

market expectations of future inflation rates. The term structure of forward inflation rates is obtained from both these curves by applying the Fisher identity:

$$1 + \frac{f}{2} = (1 + i)^{1/2} \left(1 + \frac{r}{2}\right) \quad (54.3)$$

where f is the implied nominal forward rate, r is the implied real forward rate and i is the implied forward inflation rate. As with the term structure of real spot rates, the real implied forward rate curve is dependent on an assumed rate of inflation. To make this assumption consistent with the inflation term structure that is calculated, we can use an iterative procedure for the assumed inflation rate. Essentially this means that the real yield curve is re-estimated until the assumed inflation term structure and the estimated inflation term structure are consistent. Real yields are usually calculated using either a 3% or a 5% flat inflation rate. This enables us to estimate the real yield curve, from which the real forward rate curve is derived. Using (54.3) we can then obtain an initial estimate of the inflation term structure. This forward inflation curve is then converted into an average inflation curve, using (54.4):

$$i_i = \prod_j^k (1 + if_i)^{-1/k} - 1 \quad (54.4)$$

where

if_i is the forward inflation rate at maturity i
 i_i is the average inflation rate at maturity i .

From this average inflation curve, we can select specific inflation rates for each index-linked bond in our sample. The real yields on each indexed bond is then re-calculated using these new inflation assumptions. From these yields the real forward curve is calculated, enabling us to produce a new estimate of the inflation term structure. This process is repeated until there is consistency between the inflation term structure used to estimate the real yields and that produced by (54.3).

Using the modified Waggoner method described in Chapter 53, the nominal spot yield curve for the gilt market in July 1999 is shown at Figure 54.2. The real term structure is also shown, which enables us to draw the implied forward inflation expectation curve, which is simply the difference between the first two curves.

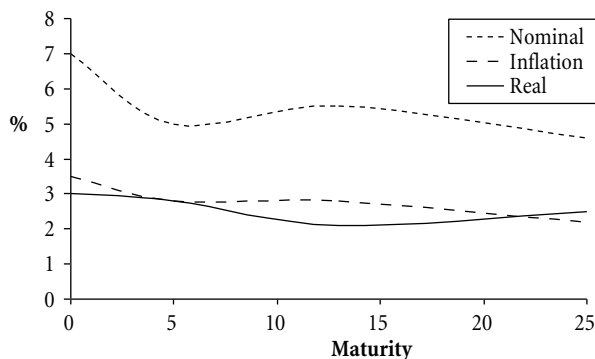


Figure 54.2: UK market nominal and real term structure of interest rates, July 1999. Yield source: BoE.

54.3.5 Application

Real yield curves are of some use to investors, for a number of reasons. These include applications that arise in insurance investment management and corporate finance, such as the following:

- they can be used to value inflation-linked liabilities, such as index-linked annuity contracts;

- they can be used to value inflation-linked revenue streams, such as taxes that are raised in line with inflation, or for returns generated in corporate finance projects; this makes it possible to assess the real returns of project finance or government revenue;
- they can be used to estimate the present value of a company's future staff costs, which are broadly linked to inflation.

Traditionally, valuation methods for such purposes would use nominal discount rates and an inflation forecast, which would be constant. Although the real term structure also includes an assumption element, using estimated market real yields is equivalent to using a nominal rate together with an implied market inflation forecast, which need not be constant. This is a more valid approach; a project financier in the UK in July 1999 can obtain more meaningful estimates on the effects of inflation using the rates implied in Figure 54.2, rather than an arbitrary, constant inflation rate. The inflation term structure can be used in other ways as well; for example, an investor in mortgage-backed bonds, who uses a prepayment model to assess the prepayment risk associated with the bonds, will make certain assumptions about the level of prepayment of the mortgage pool backing the bond. This prepayment rate is a function of a number of factors, including the level of interest rates, house prices and the general health of the economy. Rather than use an arbitrary assumed prepayment rate, the rate can be derived from market inflation forecasts.

In essence, the real yield curve can and should be used for all the purposes for which the nominal yield curve is used. Provided that there are enough liquid index-linked bonds in the market, the real term structure can be estimated using standard models, and the result is more valid as a measure of market inflation expectations than any of the other methods that have been used in the past.

Selected bibliography and references

- Arak, M., Kreicher, L., "The real rate of interest: inferences from the new UK indexed gilts", *International Economic Review* 26(2), June 1985, pp. 399–408.
- Bootle, R., *Index-Linked Gilts*, 2nd edition, Woodhead-Faulkner, 1991.
- Brown, R., Schaefer, S., "Ten years of the real term structure: 1984–1994", *Journal of Fixed Income*, March 1996, pp. 129–141.
- Brynjolfsson, J., Fabozzi, F. (eds.), *Handbook of Inflation-Indexed Bonds*, FSF Associates, 1999.
- Deacon, M., Derry, A., "Deriving Estimates of Inflation Expectations from the Prices of UK Government Bonds", *Bank of England Working Paper* No. 23, 1994.
- Deacon, M., Derry, A., *Inflation-Indexed Securities*, Prentice Hall, 1998.
- McCulloch, J., "Measuring the term structure of interest rates", *Journal of Business*, Vol. XLIV, January 1971, pp. 19–31.
- McCulloch, J., "The tax-adjusted yield curve", *Journal of Finance* 30(3), June 1975, pp. 811–830.
- Roll, R., "US Treasury inflation-indexed bonds: the design of a new security", *Journal of Fixed Income*, December 1996, pp. 321–335.
- Schaefer, S., "Measuring a tax-specific term structure of interest rates in the market for British government securities", *The Economic Journal* 91, June 1981, pp. 415–438.

Questions and exercises

1. Discuss the issues involved in estimating and fitting a real term structure of interest rates. What purposes is the real term structure used for?
2. How can the McCulloch technique for fitting the zero-coupon term structure be adapted for use in estimating a real term structure?
3. From the real yields reported in a national newspaper of your choice (for example, the *Financial Times* for index-linked gilt yields), fit the discount function for the complete sample of bonds, using the method described in the text.

55

Analysing the Long Bond Yield

A common observation in government bond markets is that the longest-dated bond trades expensive to the yield curve. It also exhibits other singular features that has been the subject of recent research for example by Phoa (1998), which we review in this chapter. The main feature of long bond yields is that they reflect a convexity effect. Analysts have attempted to explain the convexity effects of long bond yields in a number of ways. These are discussed first. We then consider the volatility and convexity bias that is observed in long bond yields.

55.1 Theories of long-dated bond yields

In both the United States and United Kingdom government markets extremely long-dated bonds have been actively traded. These include “century” bonds in the US and undated or irredeemable bonds in the UK, including gilt issues such as the “consols” or War Loan. At what yields should very long-dated bonds trade? Under the conventional hypothesis reviewed in Chapter 6, investors might believe that if the yield on the 30-year government bond is 6.00%, the yield on a hypothetical 100-year government bond will be higher, say 6.25%, the higher rate signifying the term premium payable on the longer bond. In fact, this is extremely unlikely, and it has been shown that, for such a term structure to be observed, we would require forward interest rates to be very high. Expected rates will not therefore be as high as the forward rates. We explore the issues in this section.

Theories of very long-dated interest rates have been proposed that on observation, would appear to hold; these include:

- that very long-dated yields are not an unbiased average of expected future interest rates, but rather can be estimated using a weighting of various interest-rate scenarios; at sufficiently long maturities the highest interest-rate scenarios do not impact the long-dated yield (Dybvig and Marshall 1996);
- extremely long-dated zero-coupon and forward rates can never decline, even when expected long-term future interest rates fall; therefore this limits the extent to which very long-dated bond yields are effected by a change in the current interest-rate environment (Dybvig, Ingersoll and Ross 1996).

The very long-dated zero-coupon yield is taken to be the infinite maturity zero-coupon yield, that is the limiting yield of a risk-free zero-coupon bond whose maturity approaches infinity. Although it might appear so, the infinite maturity yield is not identical to the yield on an irredeemable bond, which pays coupons during its life and so has a shorter-dated yield. It is also not identical to the long-term interest rate, which is defined as the expected long-term rate of return on bonds, or the expected rate of return on a bond with infinite duration. The long rate is a measure of the expected future rate of *return*, rather than a present bond *yield*. The two interest-rate hypotheses above are general and apply to both conventional and index-linked bonds. They use the principal of no-arbitrage pricing, in terms of a trading strategy, in their derivation, which we do not present here. They do however have a practical significance in terms of the valuation of long-dated bonds.

55.1.1 Long-dated yields

In an environment of interest-rate uncertainty, from previous chapters we know the price today of a zero-coupon bond of maturity T to be a function of the expectation of future short rates, which at time t are not known; this is given at (55.1):

$$P(t, T) = \exp\left(-\int_t^T r(s) ds\right). \quad (55.1)$$

Expression (55.1) states that price of a zero-coupon bond is equal to the discount factor from time t to its maturity date, or the average of the discount factors under all interest-rate scenarios, weighted by their probabilities. It can be shown that the T -maturity forward rate at time t is given by

$$f(t, T) = \frac{E_t \left[r_T \exp \left(- \int_t^T r(s) ds \right) \right]}{E_t \left[\exp \left(- \int_t^T r(s) ds \right) \right]} \quad (55.2)$$

which expresses the T -term forward rate in terms of the dynamics of the T -maturity short-rate r_T under all possible interest-rate scenarios, that is, along all possible random interest-rate paths. The weightings are in terms of the probabilities of each interest-rate path occurring and the discount factors from the period t to T that occur in each scenario. So for instance consider an environment where there are only two possible interest-rate scenarios, each with a probability of $p(1)$ and $p(2)$. Phoa (1998) states that the T -maturity forward rate is given by the weighted average, shown at (55.3) where Df is the discount factor.

$$f(t, T) = \frac{p(1)Df_1(t, T) \cdot r_{T,(1)} + p(2)Df_2 \cdot r_{T,(2)}}{p(1)Df_1(t, T) + p(2)Df_2(t, T)}. \quad (55.3)$$

The effect of weighting using discount factors is to make the lower level interest-rate scenario more significant, because the discount factors are higher under these scenarios. This means that a lower interest-rate scenario has more influence on the forward rate than an higher-rate scenario, and this influence steadily increases as the forward rate term grows in maturity, since the difference between the discount factors increases. This is an important result.

55.1.2 Long-dated forward rates

Following Phoa, we can illustrate this with an hypothetical example. Consider a binomial interest-rate environment under which there are the following interest-rate scenarios:

- the short-rate is at a constant level of 8.00%, with a probability of 70%;
- the short-rate is at a constant level of 4.00%, with a probability of 30%.

The expected future short-rate at any point in the future will be 8%, given the probabilities, however the forward rate will be lower than 8%, because it is calculated by weighting each interest-rate scenario by the relevant discount factors. This is illustrated in Figure 55.1.

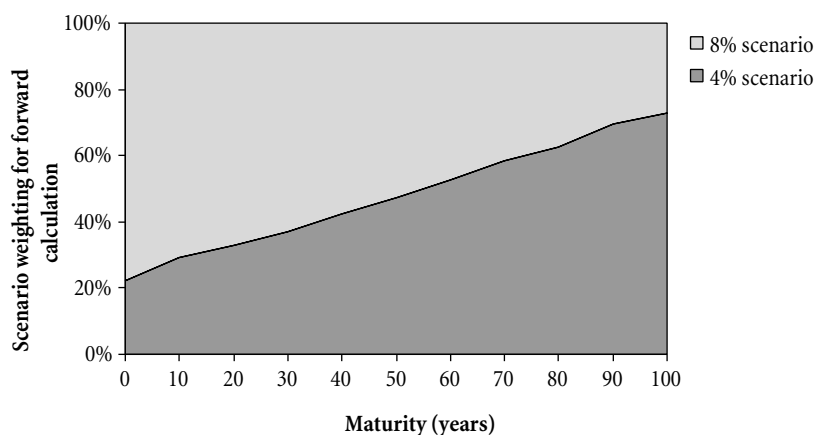


Figure 55.1: Forward rate calculation weighted by discount factors.

The weight attached to the lower 4% interest-rate scenario increases with increasing term-to-maturity, while the weight on the higher rate will diminish. Therefore the 30-year forward rate will be below 8% while the 100-year forward rate will be around half of the short-rate for the most probable scenario. Put another way, over a long period only the lowest interest rate scenario is relevant, which is the theorem posited by Dybvig and Marshall. This tendency for the forward rate curve to fall with very long maturities is as a result of a *convexity bias* in the behaviour of the yield curve, which we consider later. This effect influences zero-coupon yields, which also exhibit a tendency to gravitate towards the lowest interest-rate scenario. Consider now another hypothetical example, where the current short-rate is 6%, and that there are now three (and only three) possible interest-rate scenarios, which are:

- that the short-rate increases from 6% to a long-term rate of 10%;
- that the short-rate increases from 6% to a long-term rate of 8%;
- that the short-rate decreases from 6% to long-term rate of 4%.

The probabilities of each of these occurrences are 10%, 80% and 10% respectively, that is the most likely scenario is a rise in the short-rate from 6% to 8%. For each scenario we assume that the short-rate approaches the expected long-term level in exponential fashion. The expected interest-rate scenario therefore is a rise from 6% to 8%. From Figure 55.2 we see that the forward rate curve behaves differently to expected future short-rate levels. The forward rates peak at around 12–14 years, and then steadily decline as the term to maturity increases. The zero-coupon yield curve, which can be derived from the forward yield curve, has a different shape and starts to decline from the 20-year term period.

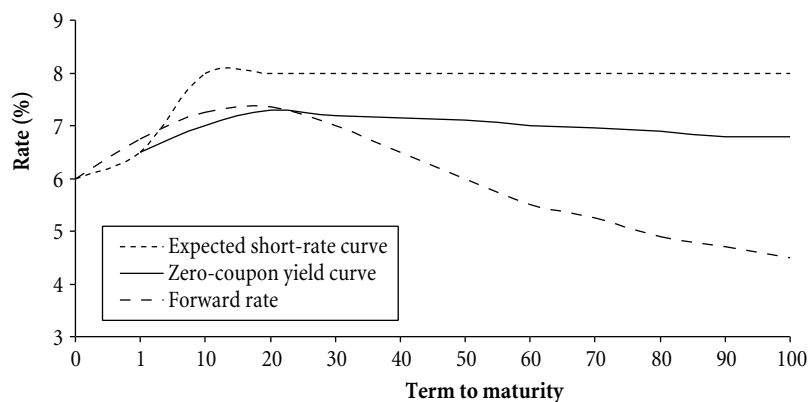


Figure 55.2: The theoretical behaviour of the long-bond yield.

Figure 55.2 suggests that the unbiased expectations hypothesis, which we reviewed in Chapter 6 and which states that forward rates are equal to the expected level of future short-term rates, is incorrect, and so it is not valid to calculate par and zero-coupon yield curves using the expected short-rate curve. Instead the forward rate curve should be used. Figure 55.3 illustrates the extent of the error that might be made using the expected short-rate curve to calculate the zero-coupon yield curve, which is magnified over longer terms to maturity.

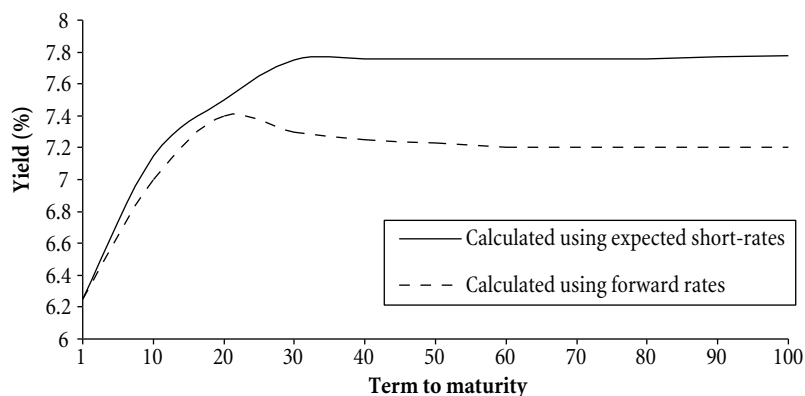


Figure 55.3: Zero-coupon yield curves calculated using expected short-rates and forward rates.

From Figure 55.3 we see that to price a very long-dated bond off the yield of the 30-year government bond would lead to errors. The unbiased expectations hypothesis suggests that 100-year bond yields are essentially identical to 30-year yields, however this is in fact incorrect. The theoretical 100-year yield in fact will be approximately 20–25 basis points lower. This reflects the convexity bias in longer-dated yields. In our illustration we used an hypothetical

scenario where only three possible interest-rate states were permitted. Dybvig and Marshall showed that in a more realistic environment, with forward rates calculated using a Monte Carlo simulation, similar observations would result. Therefore the observations have a practical relevance.

This is an important result for the pricing of longer-dated bonds. Certain corporate bonds including those issued by Walt Disney, Coca-Cola and British Gas to name three instances in recent years, have been very long-dated bonds, from 50 to 99 years' maturity. The analysis above suggests that such bonds priced at a spread over the 30-year government bond are theoretically undervalued. While investor sentiment would appear to demand a yield premium for buying such long-dated bonds, the theoretical credit-risk spread for a 100-year corporate bond is essentially the same as that for a 30-year corporate bond. For instance if a 30-year corporate bond has a default probability of 1% each year, while a 100-year corporate bond has a default probability of 1% for the first 30 years and a subsequent default probability of 3% for the remainder of its life, the longer-dated bond appears at first sight to hold considerable more credit risk. However it can be shown that the yield premium an investor should demand for holding the longer-dated bond will not be much more than 15–20 basis points. This is because the impact of future loss scenarios is weighted by the discount factor that applies from today to the loss date; the influence of each discount factor steadily diminishes over an increasing term to maturity.

55.2 Pricing a long bond

In a conventional positive yield curve environment it is common for the 30-year government bond to yield say 10–20 basis points above the ten-year bond. This might indicate to investors that a 100-year bond should yield approximately 20–25 basis points more than the 30-year bond. Is this accurate? As we noted in the previous section, such an assumption would not be theoretically valid. Marshall and Dybvig have shown that such a yield spread would indicate an undervaluation of the very long-dated bond, and that should such yields be available an investor, unless he or she has extreme views on future interest rates, should hold the 100-year bond.

This is intuitively apparent. In the first instance, long-dated forward rates have very little influence on the prices of bonds, and therefore for there to be a yield spread of say 20 basis points between 30 years and 40 years, forward rates would have to be very high. This reflects the relationship between spot and forward rates, the former being an average of the latter to the longest maturity. Similarly, expected future short-rates are assumed to be composed of the market's expectation of these rates and a premium for interest-rate risk. For there to be a high enough expectation such that there is a yield premium of 20 basis points between 30 and 100 years would require very high expectations about the future level of short-rates, or a very high risk premium. We now consider this in greater detail.

55.2.1 The impact of forward rates on the long-bond yield

From Part VI of this book we know that at an investment of £1 at a continuously compounded interest rate of r will have a value at time t given by e^{-rt} , so that the value of a coupon C at this time is given by Ce^{-rt} .

This enables us to set the value of a bond with a coupon of C maturing at time T and redemption value of M as (55.4):

$$\int_0^T MCe^{-rt} ds + Me^{-rT} \quad (55.4)$$

which can also be given as

$$\frac{MC}{r}(1 - e^{-rT}) + Me^{-rT}. \quad (55.5)$$

We assume that the zero-coupon rate r term structure is flat until the time s and that forward rates are flat at f . The value of £1 to be received at time $t > s$ is given by:

$$e^{-rs - f(t-s)} \quad (55.6)$$

while the price of a bond maturing at T is given by (55.7):

$$P = \int_0^t MCe^{-rt} ds + \int_t^T MCe^{-rs - f(t-s)} ds + Me^{-rs - (T-t)f}. \quad (55.7)$$

Equation (55.7) can be integrated to give:

$$\frac{MC}{r}(1 - e^{-rt}) + \frac{MC}{f}e^{-f(T-t)} + Me^{-rt-f(T-t)}. \quad (55.8)$$

This can be illustrated with an example. Consider a situation where the zero-coupon rates term structure is flat at 6% for 30 years and that forward rates are flat at f for terms from 30 years to 100 years. This results in the price of a 30-year bond with a coupon of 6% and a redemption value of £100 having a price of par, shown below.

$$P = 100 = \frac{6}{0.06}(1 - e^{-0.06 \times 30}) + 100e^{-0.06 \times 30}.$$

Now let us imagine that the yield on a 100-year government bond with a coupon of 6.20% is 6.20%. This fits investor expectations that the very long-dated bond should have a yield premium of approximately 20 basis points. This would set the price of the 100-year bond as:

$$P = 100 = \frac{6.20}{0.06}(1 - e^{-0.06 \times 30}) + e^{-0.06 \times 30} \left(\frac{6.20}{f} \right) (1 - e^{-70f}) + e^{-0.06 \times 30} 100e^{-f \times 70}$$

where the price of the bond is par. The forward rate given by the expression above must be greater than 6.20%, and is higher because long-dated forward rates have very little influence on the price of a coupon bond. The size of the coefficient $e^{-0.06 \times 30}$, in this instance, indicates the extent of the impact of the forward rate on the price of the bond. In fact in a term structure environment that is flat or only very slightly positive out to 30 years, the zero-coupon term structure beyond this term is flat.

Let us look now at the T -period forward rate again as a function of the range of spot rates from the time t today to point T in more detail than in Section 55.1. If $P(t, T)$ is the price today of a zero-coupon bond that has a redemption value of £1 at time T , then this price is given in terms of the instantaneous structure of forward rates by (55.9):

$$P(t, T) = \exp\left(-\int_t^T f(t, s) ds\right) \quad (55.9)$$

where the forward rate $f(t, T)$ is given by:

$$f(t, T) = -\frac{\partial \ln P(t, T)}{\partial T}. \quad (55.10)$$

However, the price of the zero-coupon bond is also given in terms of the spot rate as the expression at (55.1), where E_t is the expectation under the risk-free probability function. Therefore forward rates are related to the expected level of the instantaneous spot rates, and if we differentiate the expression at (55.10) we obtain a result that states that the forward rate is a weighted average of the range of spot rates in the period t to T . This is given at (55.11), which we encountered earlier as (55.2):

$$f(t, T) = \frac{\exp\left(-\int_t^T r(s) ds\right)}{E_t\left[\exp\left(-\int_t^T r(s) ds\right)\right]}. \quad (55.11)$$

In spot-rate scenario where the expected future rate is high, the interest rate $r(T)$ will exert very little influence, while it exerts more weight at lower levels. Therefore the forward rate will be lower than the expected spot rate, and this is described below, where

$$E_t\left[r(T) \mid \int_t^T r(s) ds\right]$$

is an increasing function in $\int_t^T r(s) ds$.

Therefore we may write for long-term forward rates $f(t, T) \leq E_t[r(T)]$, where interest rates are assumed to not be deterministic. This result has an important effect on the pricing of very long-dated bonds. Since forward rates lie below the level of expected short-term rates, for a very long-dated bond to trade at a yield of 6.20% means that the average level of future short-rates over the life of the bond would have to be higher than 6.20%. If this is not the case,

a yield premium of 20–25 basis points between the long-bond and a very long-dated bond would indicate an unrealistically low price for the latter instrument. And crucially, for there to be a spread of this magnitude for up to say, 50 years beyond the benchmark long bond, we would observe unrealistically high forward rates and an exploding forward rate curve.

55.3 Further views on the long-dated bond yield

In the previous section we described a theorem from Dybvig, Ingersoll and Ross stating that extremely long-dated bond yields can not decline. This carries implications about the level of interest-rate risk attached to the very long-dated yield. We present a summary of their results here.

Assume that along all random interest-rate paths ω , the short-rate gravitates towards a long-term equilibrium level of r_ω^∞ , which is dependent on the path ω . A long-term level r can result if the set of interest-rate paths ω for which $r_\omega^\infty \leq r$ has a positive probability. Consider then the lowest possible value r^∞ of the long-term equilibrium level r_ω^∞ . The result from the previous section, that very long-dated forward rates do not reflect the unbiased expectations hypothesis but rather a disproportionate weighting of the lowest yields, implies that long-dated rates are determined by r^∞ . That is, as the maturity approaches infinity, both the forward rate and the zero-coupon rate are essentially equal to the lowest possible long-term interest rate r^∞ . Over time a particular long-term interest-rate level r_ω^∞ that was previously possible may become impossible, so that r^∞ may rise over time. However a previously unattainable level r_ω^∞ will remain impossible, and if it is possible today it will have been possible before. Therefore r^∞ cannot fall over time, which indicates that very long-dated forward rates and zero-coupon rates cannot fall. This means that long-dated yields are essentially given by the lowest interest rate scenario, and will remain sticky at this level. It also means that there is a limit to the extent to which long-dated yields will be affected by changes in the expectations of future interest rates. The yield on a 100-year bond yield is essentially determined by the lowest yield scenario, and a fall in expected future short-term rates will have very little impact indeed.

We can illustrate this with the same example as before. Consider now that there is a 50 basis point decline in the short-rate, and that the probabilities of the three interest-rate scenarios are now:

- that there is a zero probability that the short-rate increases from 5.50% to a long-term rate of 10%;
- that there is a 80% probability that the short-rate increases from 5.50% to a long-term rate of 8%;
- that there is a 20% probability that the short-rate decreases from 5.50% to long-term rate of 4%.

The change in the short-rate will result in a 50 basis point decline in all the expected future interest rates. However this will not result in a uniform fall in all bond yields. The impact on the zero-coupon curve and the forward rate curve is shown in Figure 55.4.

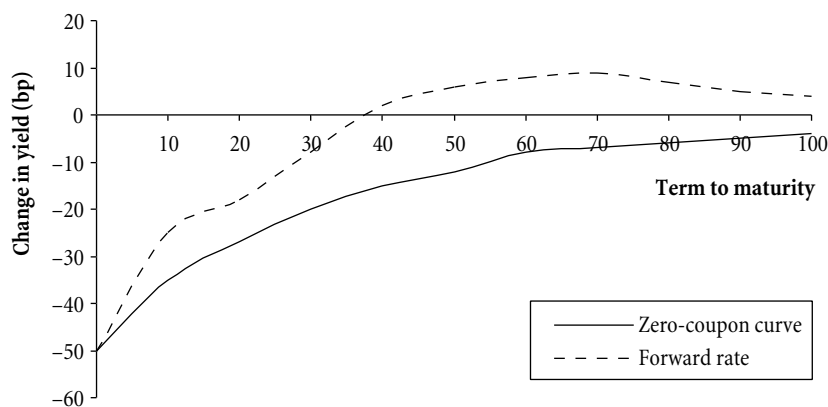


Figure 55.4: The effect of a decline in expected short rates on zero-coupon and forward curves.

From Figure 55.4 we observe that at the very short-end, yields fall by 50 basis points. However the 100-year spot rate falls by only approximately 4 basis points, while the 100-year forward rate actually rises. This is because under the probabilities used in our scenario the 6% scenario has a higher weight at these forward dates.

What are the practical implications of these results? We may conclude that if there is a rally in the government bond market, very long-dated bond yields should be virtually unaffected. More important though, the results indicate that any term structure model that allows very long-dated yields to fall is inconsistent with the Dybvig–Ingersoll–Ross theorem, and is therefore invalid because it would permit arbitrage. Such a model would also price 100-year bonds incorrectly (although it may well price 30-year bonds correctly). The theorem is still consistent with the concept of mean reversion, and a term structure model that assumes that long-term yields will revert to a constant long-term level will fit in with the theorem. Dybvig *et al* state that the long-term level must be the “lowest possible level” of the average level of interest rates, but calculating this level is problematic. The issues involved in accurately pricing a very long-dated bond, and the fact that term structure modelling out to this maturity is not yet consistently applied, may explain why, despite there being no theoretical basis for it, the yields on 50-, 90- and 100-year corporate bonds sometimes lie some way above the 30-year risk-free yield.

55.4 Analysing the convexity bias in long-bond yields

In theory the results implied by our discussion of convexity in Chapters 9–10 imply that if there are two fixed income portfolios that have identical durations and yields, the portfolio with the higher convexity will outperform the other under conditions of a parallel yield curve shift. In fact in practice this will not be the case, as such portfolios will have lower yields, which reflects the price paid for convexity in the market.¹ We can observe this yield/convexity trade-off in the government bond yield curve: in a positive sloping yield curve environment, the yield on the longest-dated (usually 30-year) bond is almost always lower than the 15- or 20-year bond yield. This is explained by the fact that the longer-dated bond has higher convexity and that the value of this convexity is the difference in the yields. This *convexity bias* is evident in other markets, and in an earlier chapter we reviewed the convexity bias that exists between the swap yield curve and the yield curve implied by long-dated interest-rate futures contracts. For example in one study² the ten-year interest-rate swap rate was found to be significantly lower than the rate implied by the equivalent strip of ten-year Eurodollar futures contracts. This reflects the fact that the swap instrument has convexity while the futures position does not. Therefore it is theoretically possible to benefit from a position where the trading book is short the swap (receiving fixed) and short the futures strip, as the combined effect is to be long convexity.

55.4.1 Estimating the convexity bias

Phoa (1998) presents an approximation of the convexity bias as follows. Consider a conventional fixed coupon bond, which has a yield at a future time t of r and a price at this time of $P(r)$. The convexity bias is estimated using

$$E[r] - r_{fwd} \approx (C/D)\sigma^2 t \quad (55.12)$$

where:

$E[r] - r_{fwd}$	is the difference between the forward yield and the expected future yield (which is the convexity adjustment to the bond yield)
C	is the convexity divided by two
D	is the duration of the forward bond position
σ	is the basis point volatility of bond yields.

The volatility value used can be estimated in two ways. We can estimate volatility separately, and then use to this to calculate what the approximate convexity adjustment should be. Or we may observe the convexity bias directly and derive a volatility value from this. This would require an examination of market swap rates and bond yields, and use these to estimate the volatility implied by these rates.

Selected bibliography and references

Burghardt, G., Hoskins, B., “A question of bias”, *Risk*, March 1995.

Dhillon, U., Lasser, D., “Term premium estimates from zero-coupon bonds: new evidence on the expectations hypothesis”, *Journal of Fixed Income*, June 1998, pp. 52–58.

¹ For example, see Lacey and Nawalkha (1993).

² See Burghardt and Hoskins (1995).

- Dybvig, P., Ingersoll, J., Ross, S., “Long forward and zero-coupon rates can never fall”, *Journal of Business* 69(1), 1996.
- Dybvig, P., Marshall, W., “Pricing long bonds: Pitfalls and Opportunities”, *Financial Analysts Journal*, Jan–Feb 1996, pp. 32–39.
- Lacey, N., Nawalkha, S., “Convexity, risk, and returns”, *Journal of Fixed Income*, December 1993, pp. 129–145.
- Phoa, W., *Advanced Fixed Income Analytics*, FJF Associates, 1998, Chapter 4.

56

The Default Risk of Corporate Bonds

In Part III we looked at the range of corporate bond instruments that are held by investors. Institutions are interested in holding non-government bonds because of the higher yield that these bonds offer, relative to government bonds. The existence of a credit default risk on such bonds means that bondholders must ensure that the return is satisfactory and compensates them for the risk of the bond portfolio. This can be done by measuring the risk premium obtainable from the corporate bond, the total return that is expected from holding the bond, and assessing whether this is sufficient to compensate for all the risks associated with the bond, excluding the interest-rate risk. These risks must be identified and quantified, and the higher the risk, the higher the risk premium should be. A common measure for the risk premium is the *option-adjusted spread* (OAS). It is basically the spread of the corporate bond over the equivalent-maturity risk-free bond. A bond's OAS measures the constant spread that must be added to the current short-term interest rate to make the theoretical price of the corporate bond, as calculated by the pricing model, identical to the observed market price. This means that it is a quantification of the excess return of the bond over the short-rate. There is no one measure of OAS however, and it means different things depending on what type of bond it is applied to. This means that if it is used to measure the yield premium on a corporate bond that reflects a particular bond's credit risk, any specific limitations of the measure must be accounted for. There are other measures that may be considered however, in terms of the default risk of a bond, and these are considered in this chapter. We also present a theoretical default spread model.

56.1 Corporate bond default spread risk

56.1.1 Spread risk

The general rule of corporate bonds is that they are priced at a spread to the government yield curve. This price is a yield spread for conventional bonds or on an OAS basis for callable or other option-embedded bonds. If an OAS calculation is undertaken in a consistent framework, price changes that result in credit events will result in changes in the OAS. Therefore we can speak in terms of a sensitivity measure for the change in value of a bond or portfolio in terms of changes to a bond's OAS measure. One of these measures is the *spread duration*. The spread duration of a bond is the sensitivity of its OAS to a change in yield of one basis point. For a conventional bond, the spread duration is essentially its modified duration, because a change in the OAS would have identical effect on the price of such a bond as a similar magnitude change in the yield on the equivalent government bond. For the same reason the spread duration of a callable bond is essentially identical to its modified duration. However the spread duration for an asset-backed security such as a mortgage-backed bond is not equal to its modified duration. This is because a change in the OAS will not have the same effect necessarily as a similar change in government bond yields.

The effect of a change in OAS for mortgage-backed bonds can be explained thus. For instance a rise in yields will lead to a rise in the level of mortgage rates, which will have the effect of decreasing prepayment rates (see Chapter 26). This will change the expected cash flow profile of the bond. However a change in the OAS of the bond will only have an effect on the bond's expected cash flows if it also leads to a rise in the prevailing mortgage interest rate.

The spread duration of a bond can be applied to calculate the break-even spread change. Remember that investors who are looking to outperform government bonds will set up portfolios to include corporate bonds, whose yields are higher than those of government bonds. However it is important for them to determine the extent to which yield spreads can widen before the additional income from the higher-yield corporate bonds is offset by the negative price effect of these bonds with regards to the price of government bonds. This measure will indicate the extent of the risk profile of their portfolios. One approach is to calculate break-even spreads for a holding period of up to one year using (56.1).

$$\text{Breakeven spread} = \frac{\text{Income excess}}{\text{Spread duration}} = \frac{\text{Holding period} \times \text{spread}}{\text{Spread duration}}. \quad (56.1)$$

EXAMPLE 56.1 Spread duration

An investor's corporate bond portfolio has an identical duration of a benchmark portfolio of government bonds, and an OAS of 50 basis points. Assume that the portfolio has a spread duration of five. During a 12-month holding period, the excess income of the portfolio of the compared to government bonds is 0.25%. How much can the OAS widen before the corporate bond portfolio begins to underperform the government portfolio?

Break-even spread shift = $0.25/5 = 5$ basis points.

Therefore if spreads widen by five basis points or more over the 12-month period, or if the OAS of the portfolio widens beyond 55 basis points, the portfolio will underperform the government portfolio.

Note that this is an approximation that is valid for short-term holdings only.

56.1.2 Spread risk and government bond yields

The risk premium available on a corporate bond reflects the total risk exposure of the bond, over and above the interest-rate risk which is expected to be identical in theory to the interest-rate risk on an otherwise risk-free bond. This means that the discussion of spread risk above implies that it is independent of interest-rate risk. In practice this is not so. Observation shows that the yield spread of corporate bonds is positively correlated to the outright government bond yield: when yields increase, the yield spread often decreases, while when yields fall the yield spread usually increases. Empirically though this effect can only be measured for specific issuers, and not for a class of identical credit-quality bonds. This is because the group of same-rated bond issuers is constantly fluctuating, and measuring the change in yield spread for a group of say, single-A rated borrowers will reflect changes to the group of issuers as some are re-rated, and others enter or leave the group. In most OAS calculations the relationship between outright yield levels and corporate yield spreads is not taken into account, resulting in an OAS spread of a corporate bond being equal to its nominal spread over the government yield curve.

To assess the impact of changing yield spreads therefore, it is necessary to carry out a simulation on the effect of different yield curve assumptions. For instance we may wish to analyse one-year holding period returns on a portfolio of investment-grade corporate bonds, under an assumption of widening yield spreads. This might be an analysis of the effect on portfolio returns if the yield spread for triple-B rated bonds widened by 20 basis points, in conjunction with a varying government bond yields. This requires an assessment of a different number of scenarios, in order to capture this interest-rate uncertainty.

56.2 Default risk and default spreads**56.2.1 The theoretical default spread**

We have stated that the yield premium required on a corporate bond accounts for the default risk exposure of such a bond. The level of yield spread is determined by the expected default loss of the bond, and assumes that investors can assess the level of the default risk. This makes it possible to calculate the level of the theoretical default spread.

We set p_t as the probability that a bond will default in year t , and is the probability in year t , while r_t is the expected recovery rate on the bond should it default. The default probability is assumed to fluctuate over time, while the recovery rate remains constant. Therefore the probability that the bond will not have defaulted up to the beginning of year t is given by:

$$s_t = \prod_{\tau=1}^{t-1} (1 - p_\tau) \quad (56.2)$$

while the probability that the bond will default in year t is given by:

$$p(t) = p_t s_t. \quad (56.3)$$

If the bond has a maturity of T , there are $T + 1$ scenarios, represented by default in years 1 to T or survival until maturity. The final scenario has a probability of

$$Q = s_{T+1} = 1 - \sum_{\tau=1}^T p(\tau). \quad (56.4)$$

Therefore using the assumed recovery rate, we may calculate the cash flows of the bond under each of the possible scenarios. For any given yield r , we can then calculate the present value of the bond's cash flows for each of these scenarios. These are denoted by $PV_1, PV_2, \dots, PV_T$ and PV_{T+1} . Let $p(r)$ be the probability-weighted average for all the possible scenarios, shown by (56.5):

$$p(r) = \left(\sum_{t=1}^T p_r \cdot PV_t \right) + Q \cdot PV_{T+1}. \quad (56.5)$$

This means that $p(r)$ is the expected value of the present value of the bond's cash flows, that is, the expected yield gained by buying the bond at the price $p(r)$ and holding it to maturity is r . If our required yield is r , for example this is the yield on the equivalent-maturity government bond, then we are able to determine the coupon rate C for which $p(r)$ is equal to 100. The default-risk spread that is required for a corporate bond means that C will be greater than r . Therefore the theoretical default spread is $C - r$ basis points. If there is a zero probability of default, then the default spread is zero and $C = r$.

Generally the theoretical default spread is almost exactly proportional to the default probability, assuming a constant default probability. Generally however the default probability is not constant over time, nor do we expect it to be. In Figure 56.1 we show the theoretical default spread for triple-B rated bonds of various maturities, where the default probability rises from 0.2% to 1% over time. The longer-dated bonds therefore have a higher annual default risk and so their theoretical default spread is higher. Note that after around 20 years the expected default probability is constant at 1%, so the required yield premium is also fairly constant.

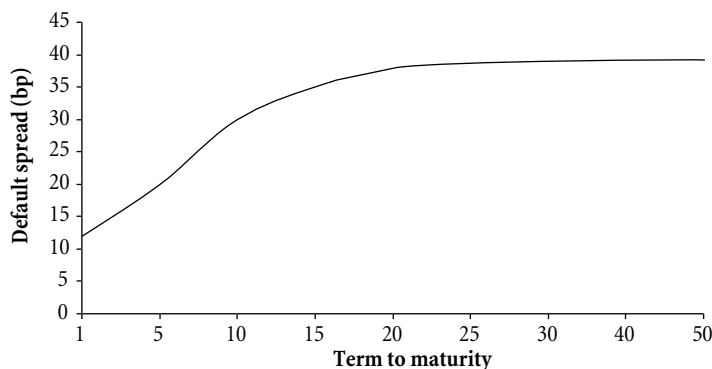


Figure 56.1: Theoretical default spread on BBB-rated corporate bond.

For lower-rated and non-rated bonds, the observed effect is the opposite to that of an investment-grade corporate. Over time the probability of default decreases, therefore the theoretical default spread decreases over time. This means that the spread on a long-dated bond will be lower than that of a short-dated bond, because if the issuer has not defaulted on the long-dated bond in the first few years of its existence, it will then be viewed as a lower risk credit, although the investor may well continue to earn the same yield spread.

Default probabilities are not known with certainty, and credit rating agencies suggest that higher-risk bonds have more uncertain default probabilities. The agencies publish default rates for each rating category (which are used in credit value-at-risk calculations), but the default probability values assume wider spreads for lower-rated bonds. For example a long-dated triple-B bond may have a default probability of between 0.5% and 2%, whereas a medium-dated single-B rated bond may have default probabilities of between 5% and 15%. This uncertainty will influence the calculation of the theoretical default yield spread. To estimate this, one approach involves the use of a probability distribution of the default probability, and applying the analysis using a range of possible default probabilities, rather than a single default probability. This results in a range of theoretical yield spreads. The result of this approach, somewhat surprisingly, is that the greater the range of uncertainty about the future default probability, the lower the theoretical default spread. This result has a significant impact on the yield spreads of high-risk or “junk” bonds. The reason behind this is that an assumption of lower default probabilities results in the generation of scenarios with higher cash flows, and the scenarios generated by these lower default probability assumptions carry a correspondingly higher weight. The default-adjusted yield being earned under a given default assumption is essen-

tially the coupon rate minus the loss rate, where the loss rate is the product of the annual default probability and the recovery rate. Therefore the assumption of a low default probability corresponds to a lower default-adjusted yield, and has a higher weight in determining the theoretical default spread than does a high default probability assumption. A greater level of uncertainty about the level of the default probability means that more extreme high and low default probability assumptions are being used, and as the low assumptions carry greater weight in the calculation, the theoretical default spread emerges as lower.

56.2.2 The default spread in relation to the outright government bond yield

In the previous section we noted that in practice there is a positive correlation between the extent of the default yield spread and outright yield levels. We may wish to analyse the effect of a correlation that results in a higher level of default in a lower yield environment, or a recessionary environment when interest rates are lower. In fact the outcome will depend on whether the default probability rate that is used is assumed to rise or fall over time. If the default probability rises over time, as it does for an investment-grade bond, then the theoretical yield spread has a negative correlation with the outright yield level, whereas for a lower-rated bond or junk bond, where the default probability falls the further we move into the future, the theoretical yield spread is positively correlated with the outright yield level. This is illustrated in Figure 56.2.

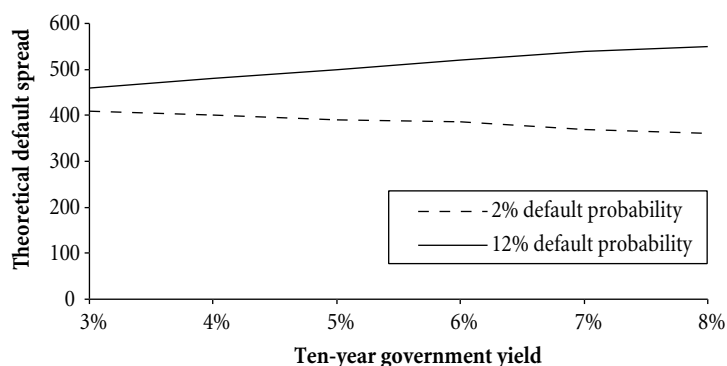


Figure 56.2: Correlation of theoretical yield spread with outright government bond yield, ten-year corporate bonds.

Portfolio managers must also take account of a further relationship between default risk and interest-rate risk. That is, that if two corporate bonds have the same duration but one bond has a higher default probability, it essentially has a “shorter” duration because there is a greater chance that it will experience premature cash flows, in the event of default. This means that an investor who holds bonds that carry an element of default risk should in theory take this default risk into consideration when calculating the duration of his or her portfolio. In practical terms this only has an effect with un-rated or junk bonds, which have default probabilities much greater than 1%. Figure 56.3 shows how the theoretical duration of a bond decreases as its assumed default probability increases.

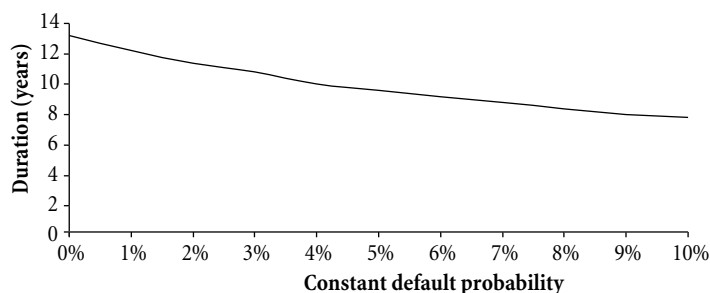


Figure 56.3: Duration of a 30-year bond relative to default probability.

Selected bibliography and references

Cohler, G., Feldman, M., Lancaster, B., “Price of risk constant (PORC): Going beyond OAS”, *Journal of Fixed Income*, March 1997.

Duffee, G., “Treasury yields and corporate bond yield spreads: an empirical analysis”, *Federal Reserve Board*, May 1996.

Duffee, G., “Estimating the price of default risk”, *Federal Reserve Board*, July 1996.

Kopprasch, R., “Option-adjusted spread analysis: going down the wrong path?”, *Financial Analysts Journal*, May–Jun 1994, pp. 121–135.

Part XI Introduction to Credit Derivatives

Credit derivatives are instruments that were originally developed to enable financial institutions to lay off and hedge credit risk exposure. During the late 1990s there was a marked rise in both volumes and liquidity of credit derivatives. The global credit derivatives market has expanded rapidly and what was once viewed as an exotic instrument is now used by a wide range of financial institutions and corporates. The majority of market participants remain banks however, and they are predominantly end-users of the instruments rather than market makers. The level of liquidity will vary considerably according to the market and type of instrument that is traded.

In Part XI of the book we introduce the subject of credit derivatives, and their application in the bond markets. A wide range of instruments fall into the category of “credit derivatives”. One of the more commonly used instruments is the *total return swap*, together with the *default swap* and the *credit linked note*. The key issues concerning credit derivatives are the need to ensure that one has a robust way to price them, and an effective way to manage a credit book. However as credit derivatives are still a relatively new instrument, banks are still developing both their systems, risk management processes and pricing models as they become more familiar with the product. The following three chapters review the main instruments as well as the pricing methodology used by some banks in the market. A more rigorous technical approach is contained in Choudhry *et al.* (2001).

57 Portfolio Management I

In undertaking the management of a portfolio, a fund manager is responsible for achieving the objectives that have been set for the fund. In the first instance the fund manager will identify the client's utility function, and then proceed to manage the portfolio so as to maximise this utility function. To introduce the main concepts, we examine initially overviews of passive and active management of a portfolio, the basic logic of which applies to both equity and bond portfolios.

57.1 Generic portfolio management

57.1.1 Passive portfolio management

The theoretical construct of this strategy assumes the following conditions:

- that the client is concerned purely with risk and return, has an infinite time horizon and that his level of risk appetite has been measured;
- there is no taxation effect, all securities are liquid;
- that all investors may borrow and lend funds at an identical risk-free interest rate.

There are two main types of passive strategy, the buy-and-hold strategy and the indexation strategy. In buy-and-hold the portfolio manager purchases securities and holds on to them for the time horizon of the investment (here assumed to be to infinity), or in the case of fixed income instruments, until maturity, at which point they are replaced with similar securities. The returns generated from a buy-and-hold strategy are the income receipts from the instruments held in the portfolio, which for a bond portfolio would be coupon receipts. As the view of the market is that security prices reflect fair value at all times, it is not important which specific securities are held in the portfolio. It is necessary though to hold a sufficient number of securities such that diversifiable risk is eliminated. Risk that can be diversified away is eliminated, in theory, by adopting an index-matching strategy. This involves the construction of an index fund, which is designed to duplicate the performance of the market itself or a market index. Several types of index-matching approaches are available. Complete indexing is the construction of an index fund that matches precisely the underlying market portfolio, which for equities might be the FTSE-100 or S&P500, while for bonds it might be the Lehman Brothers bond index. Complete indexing carries potentially high transaction costs, especially so if the index being replicated is composed of a large number of securities. There are also costs associated with matching the index over time, as constituents of the market index drop out and are replaced. Over time, the average maturity of a bond index will decline unless bonds that are approaching maturity are replaced by new bonds, further adding to transaction costs.

For these reasons complete indexing is rarely if ever, attempted and other techniques are adopted to try and replicate the performance of the market as closely as possible. The first of these is known as *stratified sampling*, which involves the construction of an index fund that is based on a sample of securities from the total population that comprises the index. The complete set of securities is divided into sectors (or *strata*), usually according to the maturity of the bonds, and an overall sample proportion is selected. The sample proportion of securities that have the highest correlation with the market index then become the constituents of the index fund. This approach minimises initial set-up costs and subsequent rebalancing costs, but results in higher risk of *tracking error*, the error that the index does not replicate the market. Another technique is known as *factor matching* or *risk matching*. This is a more general approach than stratified sampling. In the latter approach, securities are selected on the basis of a single factor, which is the industry sector (for equities) or maturity range. Factor matching on the other hand involves the construction of an index fund using securities that have been selected on the basis of more than one factor. The first factor is usually the level of systematic risk, which it is possible to calculate for bonds as well as equities. Other factors are then used to select securities. The resulting index fund then might be composed of say 50 securities that match the market in terms of the five factors selected and also have the highest correlation with the market. A third approach is known as *comingling*. This is a technique that involves the use of mixed funds such as

mutual funds (unit trusts) and investment trusts, rather than individual securities, in an effort to match the returns of the market. This approach is sometimes adopted by smaller funds as a compromise between the first two techniques.

In addition to the transaction costs associated with setting up and rebalancing, there are other issues in managing an index fund. The most important of these are the treatment of coupon payments. The total return on an index includes both the capital gain on securities (not relevant in passive bond fund management) and the income gain from coupon or dividend receipts. In order to match the performance of the index in terms of income, the index fund would require an identical pattern of income payments to the index. It would also need to make the same reinvestment assumptions. This is unlikely in practice. Other errors arise when index constituents fall out of or are added to the index, which results in major price fluctuations for the securities concerned. This adds to the tracking error. These issues often result in a passive index fund that underperforms the index itself. However this is not the key test of the effectiveness of an index fund; the main point of comparison is how well an index fund performs in relation to an actively managed fund. Passive fund management is a popular strategy, especially with regard to equity funds, and it is also theoretically valid in an environment where there is consensus about the market portfolio's return and risk.

Under certain circumstances a passive fund manager may make more active adjustments to the fund. This may occur for instance if the client requirements change, or when the consensus about the market's risk and return alters. These may result in a new combination of the risk-free asset and market portfolio, or a rebalancing of the existing index fund.

57.1.2 Active portfolio management

For a number of reasons investors who have an appetite for a certain amount of risk prefer an active portfolio management strategy. Active management is not recommended on the whole for investors that are risk averse. A portfolio will be actively managed whenever it is believed that the market contains mispriced securities, in which case a consensus on the returns from a market portfolio will not exist. Expectations of price movements are very important in active management. This is not the case with passive management, where expectations are less important but where risk aversion dominates behaviour. It is possible to construct an optimum active portfolio in the same way that an optimum passive portfolio is constructed; the difference is that whereas in the latter case the portfolio is composed of securities that form the market consensus on estimates of risk and return, the active portfolio is composed of assets that have been selected based on the fund manager's own estimate of relative risks and returns. This means that the fund manager uses their own estimates of risk and return to construct (what is believed to be) an efficient portfolio. In practice however an active portfolio is not constructed in this way; rather, it is set up based on three distinct phases, which are *asset allocation*, *security selection* and *market timing*.

At the asset allocation stage the fund manager decides on the share of the fund that will be invested in different asset classes. That is, how much of the total portfolio will be allocated to equities, bonds, money markets, property, cash and so on. The optimal asset allocation decision is governed by modern portfolio theory, which is not discussed here; however it can be summarised by (57.1):

$$\text{Max}_{\theta_s} \bar{u} = \theta_s \bar{r}_s + (1 - \theta_s) \bar{r}_b - \frac{1}{R} (\theta_s^2 \sigma_s^2 + 2\theta_s (1 - \theta_s) \sigma_{sb} + \sigma_b^2 (1 - \theta_s)^2) \quad (57.1)$$

and if we differentiate the right-hand side of (57.1) with respect to θ_s , setting the result to zero and solving for θ_s we obtain (57.2), which is the theoretical optimal proportion of the portfolio that should be invested in shares.

$$\theta_s^* = \frac{\sigma_b^2 - \sigma_{sb}}{(\sigma_s^2 - 2\sigma_{sb} - \sigma_b^2)} + \frac{(\bar{r}_s - \bar{r}_b)}{2(\sigma_s^2 - 2\sigma_{sb} + \sigma_b^2)} R \quad (57.2)$$

where

θ_s, θ_b is the proportion of the portfolio invested in shares in bonds

$\bar{r}_s, \bar{r}_b$ is the expected return on shares and bonds

σ_s^2, σ_b^2 is the variance of returns on shares and bonds

R is the degree of risk tolerance, where $R = 1/(\text{Slope of tangent to indifference curve})$ and where the indifference curve measuring the extent of an investor's risk tolerance is given by

$$\bar{r}_p = \bar{u} + \frac{1}{R} \sigma_p^2 \quad (57.3)$$

where

$\bar{r}_p$ is the expected return on the portfolio
 σ_p^2 is the variance of return on the portfolio
 $\bar{u}$ is the constant expected utility level.

Asset allocation will depend on the degree of client risk tolerance and the fund manager's estimates of the risks and returns on shares, bonds and other assets. The asset allocation decision is quite important, as it dominates the performance of most portfolios. This is because the returns on securities within each asset category often have a high degree of positive correlation. This would tend to imply that selecting the best-performing asset category is more important for performance than selecting the best-performing securities within each asset category. In practice of course fund managers will be restricted to the type of assets they can invest in, and in fact bond and equity funds are managed separately, so in effect the asset allocation decision is often made by the client investor before funds are received at the fund manager level.

Once the asset allocation decision has been made, the fund manager will proceed to the second stage, that of stock or security selection. We will consider this from the point of view of a bond portfolio.

57.2 Active bond portfolio management

57.2.1 Basic principles

Active bond portfolio management occurs where there is a view that certain bonds in the market are mispriced or there is more than one expectation about the risk and returns on bonds. Once the asset allocation decision has been made, active management revolves around the activities of security selection and the timing at which bonds are bought and sold, known as market timing. The key difference between equity and bond active management is that, while the emphasis with equities arguably is on security selection, with bonds the emphasis is on market timing. There are certain similarities however. Security selection is important whenever fund managers are prepared to accept the overall consensus for the market as a whole, but believe that certain individual securities are mispriced. An overvalued security has an expected return that is less than, or a risk estimate that is greater than, the market consensus estimate, and vice-versa for an undervalued security. In terms of the capital asset pricing model and what is known as the *security market line*, a security is said to be mispriced when it lies off the security market line or has a non-zero *alpha* value (see Appendix 57.1 for a general discussion on the capital asset pricing model). A security that is fairly priced has an equilibrium expected return given by (57.4):

$$\bar{r}_i^* = r_f + (\bar{r}_m - r_f) \beta_i \quad (57.4)$$

and so lies on the security market line. The difference between the actual expected return $\bar{r}_i$ and the equilibrium return is known as the alpha value of the security, given by:

$$\alpha_i = \bar{r}_i - \bar{r}_i^*. \quad (57.5)$$

When a security's alpha-value is positive, it will be undervalued, and when it is negative the security will be overvalued. The alpha-value for the portfolio is the weighted-value average of the individual securities in the portfolio, given by (57.6):

$$\alpha_p = \sum_{i=1}^N \theta_i \alpha_i. \quad (57.6)$$

Selection of bonds for the portfolio will be based on the fact that, compared to the market portfolio, the active portfolio will have a lower than proportionate weighting in overvalued bonds, as they are expected to fall in price,

and a higher than proportionate weighting in undervalued bonds, which are expected to rise in price. That is, the portfolio will have a low weighting in negative-alpha bonds and a high weighting in positive-alpha bonds.

The timing of security selection is also important. A fund manager engages in market timing when they do not accept the consensus market portfolio, and is either more bullish or bearish than the market. A key view to have regarding this then is the expectation of changes in interest rates. Put simply, if a fund manager is expecting a bull market due to a fall in base interest rates, they will increase the duration of the portfolio, by replacing low-duration bonds with high-duration bonds, or increasing the proportion of the fund invested in high-duration bonds. This is known as duration switching. The opposite is undertaken if a rise in interest rates is expected.

57.2.2 Portfolio adjustment

Adjusting a bond portfolio involves the purchase and sale of bonds, which is known as switching or swapping bonds. However nowadays it is uncommon for market participants to use the term “swap” with regard to a bond swap, to avoid confusion with interest-rate and asset swaps. Essentially there are two kinds of bond switches, anomaly switches and policy switches. An *anomaly switch* is a switch between two bonds with very similar characteristics but whose yields are out of line with each other. A *policy switch* is a switch between two dissimilar bonds because of an anticipated change in the structure of the market, for example because of an expected change in credit ratings, which will lead to a change in the relative yields of the two bonds. In theory policy switches carry greater potential returns together with greater risk.

The simplest example of an anomaly switch is a substitution switch, which involves the exchange of two bonds which are similar in terms of maturity, coupon and credit rating, and virtually every other characteristic, but which are currently trading at different yields. Although in theory two such similar bonds should trade at the same yield, there may be liquidity or other reasons that they do not, and there may be yield pick-up available for little or no extra risk, or there may be an arbitrage opportunity available. A substitution switch between two bonds of different maturities and/or coupon must be duration-weighted, so that the switch does not alter the current interest-rate exposure. Duration weighting was explained in Part I of this book. A switch that is undertaken between two similar bonds that trade at different yields, where the bond with the higher yield is switched into, is known as a pure yield pick-up switch.

Policy switches are designed to take advantage of an anticipated change, which might be a change in the general level of interest rates, a change in the shape of the yield curve, a change in a bond's credit rating or a change in the relationship between different issuer sectors. A change in interest rates is the key change that a market timing switch is designed to take advantage of, but the other changes can also be exploited profitably if they are anticipated in sufficient time. Other switches may be undertaken if the shape of the yield curve is expected to change, either a fundamental shift such as a change from positive to inverted, or a change that removes or creates humps in the yield curve. An example of a change in sector relationships is a change in the tax treatment of two different sectors. For instance, the domestic bond market may be subject to withholding tax on the coupon payments, whereas the international bond market may not. If it is anticipated that withholding tax will be applied to one sector or removed from another, then a switch can be undertaken that will benefit from the expected change in yields once the new tax legislation has been announced.

We have presented here only a general discussion of portfolio management and the following two chapters discuss some of the most common strategies in more detail.

Appendices

APPENDIX 57.1 The Capital Asset Pricing Model

The capital asset pricing model (CAPM) is an equilibrium model of asset pricing, based on what is known as utility maximisation and a given portfolio opportunity. Put another way, equilibrium asset prices are determined in a way that balances the supply of assets with their demand. It is a significant part of modern finance theory and associated with the eminent economist William Sharpe (1964).

Suppose that there are $h = 1, H$ investors and $I = 1, N$ risky securities and risk-free debt, and that ω_{ih} is the proportion of the total market invested by investor h in security i . Similarly $\omega_{N+1,h}$ is the proportion of the total market taken up by investor h in risk-free debt. Finally, φ_h is the proportion of total wealth represented by the wealth of investor h . Now suppose that each individual investor h has a utility function of the form:

$$\bar{u}_h = \bar{u}_h(\bar{r}_{ph}, \sigma_{ph}^2) \quad (57.7)$$

where

$$\bar{r}_{ph} = \left(\frac{1}{\varphi_h} \right) \left(\sum_{i=1}^N \omega_{ih} \bar{r}_i - \omega_{N+1,h} r_f \right) \quad (57.8)$$

is the expected return on the portfolio of investor h and

$$\sigma_{ph}^2 = \left(\frac{1}{\varphi_h} \right)^2 \left(\sum_{i=1}^N \sum_{j=1}^N \omega_{ih} \omega_{jh} \sigma_{ij} \right) \quad (57.9)$$

is the variance of the return on investor h 's portfolio. Investor h 's budget constraint is given by:

$$\left(\frac{1}{\varphi_h} \right) \left(\sum_{i=1}^N \omega_{ih} - \omega_{N+1,h} \right) = 1. \quad (57.10)$$

The objective of investor h is to maximise (57.10). The first-order conditions for a maximum are given by (57.11), where λ_h is what is known as a Lagrange multiplier for investor h ,

$$\begin{aligned} \frac{\partial \bar{u}_h}{\partial \bar{r}_{ph}} \frac{\partial \bar{r}_{ph}}{\partial \omega_{ih}} + \frac{\partial \bar{u}_h}{\partial \sigma_{ph}^2} \frac{\partial \sigma_{ph}^2}{\partial \omega_{ih}} + \lambda_h \left(\frac{1}{\varphi_h} \right) &= \frac{\partial \bar{u}_h}{\partial \bar{r}_{ph}} \left(\frac{1}{\varphi_h} \right) \bar{r}_i + \frac{\partial \bar{u}_h}{\partial \sigma_{ph}^2} \left(2(1/\varphi_h)^2 \sum_{j=1}^N \omega_{jh} \sigma_{ij} \right) + \lambda_h \left(\frac{1}{\varphi_h} \right) \\ &= 0, \quad (i = 1, N) \end{aligned} \quad (57.11)$$

and

$$\frac{\partial \bar{u}_h}{\partial \bar{r}_{ph}} \frac{\partial \bar{r}_{ph}}{\partial \omega_{N+1,h}} + \frac{\partial \bar{u}_h}{\partial \sigma_{ph}^2} \frac{\partial \sigma_{ph}^2}{\partial \omega_{N+1,h}} - \lambda_h \left(\frac{1}{\varphi_h} \right) = \frac{\partial \bar{u}_h}{\partial \bar{r}_{ph}} [-(1/\varphi_h) r_f] - \lambda_h \left(\frac{1}{\varphi_h} \right) = 0. \quad (57.12)$$

By substituting (57.12) into (57.11) we can eliminate λ_h , shown below:

$$\frac{\partial \bar{u}_h}{\partial \bar{r}_{ph}} (\bar{r}_i - r_f) + \frac{\partial \bar{u}_h}{\partial \sigma_{ph}^2} \left(2 \left(\frac{1}{\varphi_h} \right)^2 \sum_{j=1}^N \omega_{jh} \sigma_{ij} \right) = 0, \quad (i = 1, N). \quad (57.13)$$

Equation (57.13) is an equilibrium relationship that must hold for all investors $h = 1, H$ and for all securities $i = 1, N$. As (57.13) holds for all securities, it will also hold for ratios of pairs of securities. Taking two securities i and k , we obtain (57.14):

$$\frac{(\partial \bar{u}_h / \partial \bar{r}_{ph})(\bar{r}_i - r_f)}{(\partial \bar{u}_h / \partial \bar{r}_{ph})(\bar{r}_k - r_f)} = \frac{-(\partial \bar{u}_h / \partial \sigma_{ph}^2) \left(2(1/\varphi_h)^2 \sum_{j=1}^N \omega_{jh} \sigma_{ij} \right)}{-(\partial \bar{u}_h / \partial \sigma_{ph}^2) \left(2(1/\varphi_h)^2 \sum_{j=1}^N \omega_{jh} \sigma_{kj} \right)} \quad (57.14)$$

which simplifies to

$$\frac{\bar{r}_i - r_f}{\sum_{j=1}^N \omega_{jh} \sigma_{ij}} = \frac{\bar{r}_k - r_f}{\sum_{j=1}^N \omega_{jh} \sigma_{kj}}. \quad (57.15)$$

In market equilibrium the following relationship must hold for all securities,

$$\sum_{h=1}^H \omega_{ij} = \theta_j, \quad j = 1, N \quad (57.16)$$

and summing (57.15) across all investors $h = 1, H$ while applying (57.16) gives us

$$\frac{\bar{r}_i - r_f}{\sum_{j=1}^N \theta_j \sigma_{ij}} = \frac{\bar{r}_k - r_f}{\sum_{j=1}^N \theta_j \sigma_{kj}} = \gamma \quad (57.17)$$

where γ is a common ratio for all securities.

If we multiply both the numerator and the denominator of (57.17) by θ_k and summing over all securities $k = 1, N$ gives us

$$\frac{\sum_{k=1}^N (\bar{r}_k \theta_k - r_f \theta_k)}{\sum_{k=1}^N \sum_{j=1}^N \theta_k \theta_j \sigma_{kj}} = \frac{\bar{r}_m - r_f}{\sigma_m^2} = \gamma \quad (57.18)$$

where

$\bar{r}_m$ is the expected return on the market portfolio
 σ_m^2 is the variance of the return on the market portfolio.

Equation (57.18) states that if we sum over all securities and investors, and under conditions of market equilibrium, our end result will be the market portfolio. Substituting (57.18) into (57.17) and rearranging gives us the capital asset pricing model, shown below.

$$\bar{r}_i = r_f + \left(\frac{\bar{r}_m - r_f}{\sigma_m} \right) \left(\frac{\sigma_{im}}{\sigma_m} \right) \quad (57.19)$$

which may be written as

$$\bar{r}_i = r_f + (\bar{r}_m - r_f) \beta_i \quad (57.20)$$

where

$$\sigma_{im} = \sum_{j=1}^N \theta_j \sigma_{ij} \quad (57.21)$$

describes the covariance of the return on the i th security relative to the return on the market, and

$$\beta_i = \sigma_{im} / \sigma_m^2 \quad (57.22)$$

is the *beta* of the i th security.

Equations (57.19) and (57.20) essentially state that the expected return on the i th security is a function of (the price of) time, the price of risk and the magnitude of risk. Equation (57.20) uses the covariance of the i th security with the market relative to the total market risk, as measured by the variance of the market. This measure is known as the beta of the security. This is observed more clearly if one plots the *capital market line* (CML) against the *security market line* (SML). The CML is a line plotting expected return against total risk, while the SML is a line plotting expected return against market risk or beta (the graphs are not shown here). In equilibrium all securities will be priced so that they lie on the SML.

The CAPM can be used to determine the required rate of return on a security, and as such is widely used in corporate finance and project appraisal. For example, if the risk-free interest rate is 5.0% and the expected return on the market is 12.0%, using CAPM we calculate:

$$\begin{aligned} \bar{r}_i &= 0.05 + (0.12 - 0.05) \beta_i \\ &= 0.05 + 0.07 \beta_i \end{aligned}$$

where the market risk premium is $(\bar{r}_m - r_f) = 7\%$. If a particular security has the same beta as the market, it will have the same required return, which here would be 12.0%. If the security has a beta greater than that of the market, it is known as a volatile stock, as its price volatility is higher than that of the market. Volatile stocks have a higher required rate of return, because they have higher undiversifiable risk.

It is possible to calculate the beta coefficient for a bond by regressing the rate of return on the bond against the rate of return in the market. Although such a measure is a measure of the undiversifiable risk in a bond, it does not measure interest-rate risk. The version of the CAPM that captures interest-rate risk is given by:

$$\bar{r}_i = r_f + (\bar{r}_m - r_f)\beta_i \quad (57.23)$$

where $\beta_i = D_i/D_m$ and D_i is the duration of the i th bond and D_m is the duration of the market portfolio of bonds.

Note that this measure only accounts for parallel shifts in the yield curve, so practitioners using the CAPM may wish to opt to use (57.24):

$$\bar{r}_i = r_f + \beta_i(\bar{r}_m - r_f) + \kappa_i(\bar{r}_l - r_f) \quad (57.24)$$

where $(\bar{r}_l - r_f)$ is the difference between the expected long-term and short-term interest rates, and hence a measurement of the slope of the yield curve.

Selected bibliography and references

Elton, E., Gruber, M., "Optimum portfolios from simple ranking devices", *Journal of Portfolio Management*, Spring 1978, pp. 15–19.

Fisher, R., Jordan, R., *Security Analysis and Portfolio Management*, Prentice-Hall, 1987.

Fong, H.G., Fabozzi, F., *Fixed Income Portfolio Management*, Dow-Jones Irwin, 1985.

Markowitz, H., *Portfolio Selection*, Yale University Press, 1959.

Sharpe, W. F., "Capital Asset Prices: A Theory of Market Equilibrium Under Conditions of Risk", *Journal of Finance*, September 1964, pp. 425–442.

Questions and exercises

1. Discuss the rationale behind active and passive portfolio management.
2. Consider the following data:

Expected return on shares:	22%
Expected return on bonds:	8%
Variance of return on shares:	9%
Variance of return on bonds:	4%
Correlation between share and bond returns:	0.6

Calculate the optimal mix of shares and bonds for investors with a risk tolerance of:

 - (a) 1
 - (b) 2.5
 - (c) 3.0
3. What are the three main segments of an active portfolio management strategy?
4. For what reasons might a bond fund manager consider switching bond positions?
5. What practical reasons cause a passive index fund to underperform the market index?

58 Portfolio Management II

In the previous chapter we noted that bond portfolio management can follow either a passive or active strategy. In between both domains there are also *structured* portfolio strategies. In this chapter we review the essentials of active management of a bond portfolio.

58.1 Overview

Active portfolio management can be broken down into four basic categories. The first type might be described as an expectations approach, which is based on the expected direction of interest rates, and which aims to gain from correctly calling changes in interest rates. The second approach is a yield curve approach, which is aimed at achieving gains resulting from changes in the shape of the yield curve. The third, related approach is the yield spread strategy, which is an attempt to make gains from changes in yield spread between individual bonds or sectors of the market. The fourth approach might be termed the fair value approach, and is concerned with selecting, holding and trading individual securities. In practice active bond funds will combine all four approaches. In the same way as in the equity market, active funds attempt to outperform the market, and as we saw in the previous chapter this presupposes that the market is not perfectly efficient and that it can be outperformed by adopting an active strategy. The existence of pricing inefficiency therefore allows an active strategy to be pursued with positive results. If on the other hand the fund manager believes that the market is efficient then they are more likely to adopt a structured strategy, which manifests itself most clearly in the form of indexing. Let us consider now the different approaches.

58.1.1 Forecasting interest rates

A fund manager that wishes to position a portfolio to reflect her own view on the future level of interest rates will adjust the portfolio's sensitivity to changes in interest rates. Put simply this would involve lengthening the portfolio's duration if interest rates are expected to decrease, and shortening duration if rates are expected to increase. If the fund performance is measured relative to a specified benchmark or index, the duration of the portfolio is adjusted in line with that of the benchmark. To adjust duration, bonds in the portfolio are sold and replaced with other bonds whose duration is in line with the direction that the fund manager wishes to go in. To reduce transaction costs however, the fund manager may decide to use bond futures contracts, rather than sell out of current bonds. For this approach to work means, quite obviously, that the fund manager must forecast accurately, and consistently, the direction and level of future interest rate changes. Over the long term, such accurate forecasting is unlikely. Hence this is a high-risk approach and one that is almost akin to gambling. It is therefore rare, if not unheard of for any bond fund to be managed purely on this basis.

That said, it is common to observe fund managers engaging in punts on interest rate changes to enhance the performance of their portfolio, or to improve an otherwise mediocre performance by the fund. Fund performance is measured on a quarterly, or even monthly basis by external commentators and analysts, and therefore a poorly performing fund manager may be tempted to bet on the direction of interest rates, using futures, to make up some ground. Consistent appearance in the "bottom 10" performing funds in one's category will act as a deterrent to further investment, not to mention an hindrance to the fund manager's job prospects. Investors sometimes attempt to limit the extent to which managers engage in punts by stipulating that the fund's duration cannot differ significantly from that of the benchmark index.

Another method by which fund managers can undertake interest rate strategies is by using very rate-sensitive cash instruments. These include mortgage-backed bonds and bonds with embedded option features. The prepayment rate on mortgages is interest-rate-sensitive, while callable bonds exhibit negative convexity as their yield approaches a certain level.

58.1.2 Yield curve forecasting

The yield curve is a static representation of the (dynamic) term structure of interest rates. A shift in the yield curve will occur for a number of reasons, connected not just with the market's view on interest rates but also factors such

as liquidity and supply and demand. From Part I we are familiar with the main types of shifts in the yield curve being one of the following:

- upward and downward *parallel shifts*;
- flattening and steepening yield curve twists;
- changes in the humped shape of the curve, sometimes called *butterfly twists*.

Jones (1991)¹ has suggested that these forms of yield curve shift are not mutually exclusive. For instance, the most common changes are a combination of a downward shift and steepening, or an upward shift and flattening. In the same article it was also suggested, from observation of the US Treasury market, that returns generated from bondholdings were predominantly due to parallel shifts and steepening or flattening of the curve, while only a small proportion of the total return resulted from changes in the humped nature of the curve. The conclusion was that a fund manager adopting a yield curve strategy would have to accurately forecast the direction of the parallel curve shift, as well as the change in the curve spread. This places the approach, in analytical terms, in the same class as interest rate forecasting.

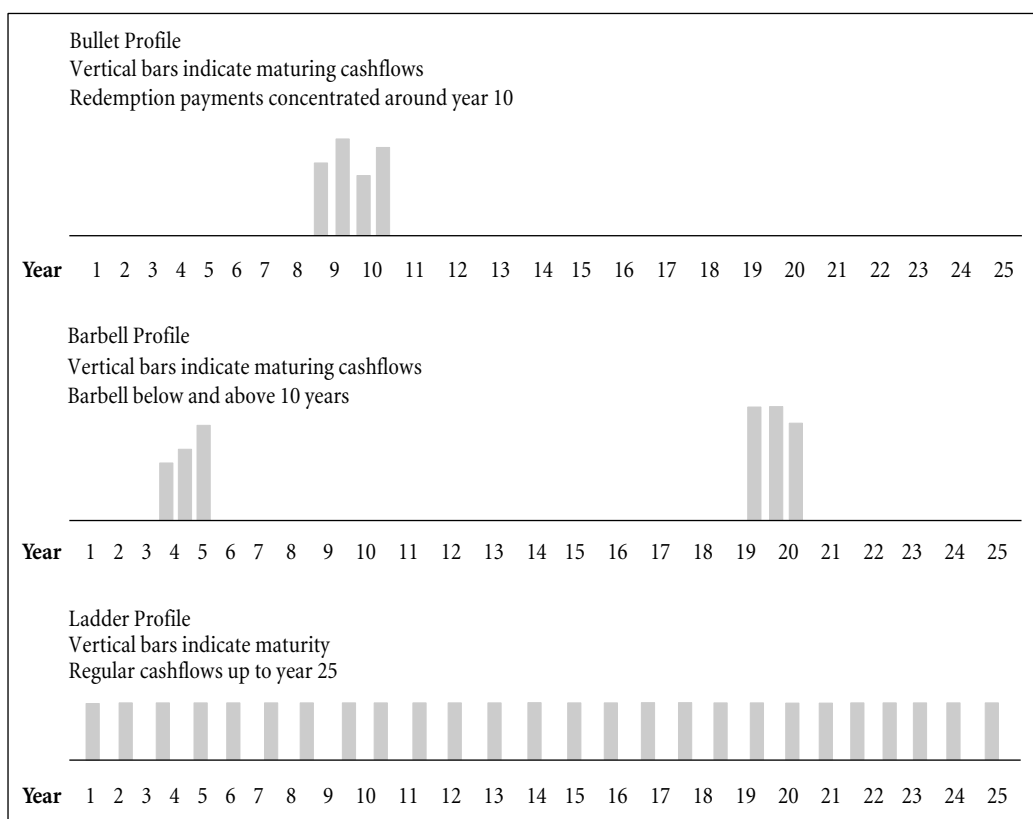


Figure 58.1: Approaches to yield curve positioning. Adapted with permission from *Bond Portfolio Management*, Frank Fabozzi, FJF Associates, 1996.

If a fund manager wishes to exploit the movements in yields over the short-term, the main source of total return that will be generated is the change in the prices of bonds. Therefore the duration of the portfolio will influence heavily the return recorded by the portfolio. Put simply, a portfolio that is composed of bonds that mature in

¹ See Jones, F., "Yield curve strategies", *Journal of Fixed Income*, September 1991, pp. 43–51.

anything from one to three years will generate a return, measured over the three-year period, that is not affected by changes in interest rates that occur in three years' time. The return of a portfolio of bonds that all mature in ten years from now is sensitive to a change in yields three years from now, because the portfolio will then be composed of seven-year bonds. The other possibility, of a portfolio that is composed of bonds that mature every year from now to ten years will generate a different return again over the three-year period, because some of its bonds will be sensitive to changes in interest rates three years from now. This suggests that over a short-term investment period, say anything from six months out to three years, the maturity profile of the bonds in the portfolio is very influential. Therefore a fund manager following a yield curve strategy will position the portfolio in terms of the maturities of the bonds in the relevant sector. Following Fabozzi (1996), the three main approaches are illustrated in Figure 58.1.

These approaches are known as *bullet strategies*, *barbell strategies* and constant or *ladder strategies*. A portfolio managed along the lines of the bullet strategy is composed of bonds that all mature around the same time; this is frequently undertaken in conjunction with the barbell or butterfly strategy, which involves bonds maturing at two different points along the term structure. For example to gain from an anticipated change in the shape of the yield curve the portfolio manager might short seven-year bonds while going long of three-year and 10-year bonds.

58.1.3 Yield spread forecasts

Positioning a portfolio to take advantage of changes in yield spreads between individual bonds and bond sectors is more common than the first two approaches. The bond market can be classified in terms of issuer, credit rating, coupon and maturity. A yield spread strategy involves constructing a portfolio so that changes in yield spreads between bonds and bond sectors produces gains. This can be done either as an outright trade, that is switching out of the bond whose spread relative to the government benchmark is expected to worsen, and into the bond whose spread is expected to tighten, or as a relative value-type trade where the first bond is shorted and the second one purchased. In either case, switching or spreading, the amounts of bonds traded must be duration-weighted. This ensures that any gain results from a change in the relative spreads, rather than in the change in the yield levels themselves. This only applies in the case of a parallel yield curve shift.

The most common types of yield spread strategy is based on a view of the expected change in the credit quality of the bond issuer. The credit spread of a bond over the government benchmark will fluctuate in line with the economic prospects of the issuing company as well as the health of the macro-economy itself; generally spreads widen in a recessionary environment and narrow during times of economic growth. This reflects the belief that in a contracting economy, firms will experience a reduced level of business, thus depressing revenue levels, which will make debt servicing more difficult unless the company has a high level of reserves. The opposite applies in a growing economy. Yield spreads are also related to the general level of interest rates, and the general observation is that spreads narrow in a high interest rate environment.

Given that the main determinant of the return from corporate bonds is the credit quality of the bond, portfolio management strategy for a corporate bond fund is essentially a view on credit spreads. Several studies² have considered this. The return generated by such a portfolio is a function of three factors:

- the yield spread between the corporate bond and the equivalent-maturity government bond;
- fluctuations in the yield spread during the investment term; and
- the change in the credit rating of the issuer during the investment term.

A change in the credit quality is one of the main factors influencing the yield spread. The downgrading of the credit quality of an issuer will result in widening spreads for its bonds, which will result in a capital loss for holders of these bonds. Sometimes a firm's debt issue can suffer a credit downgrade because the industrial or commercial sector it is in suffers a collective downgrade; a good example of this was the widening of spreads for bonds issued by United Kingdom merchant banks after the collapse of Barings, another UK merchant bank, in 1995. On the other hand, if a company is upgraded in terms of its credit standing, its bonds will benefit from a rise in price (tightening of yield spread). The return generated by a bond fund therefore will depend on the composition of upgraded and downgraded bonds that make up the portfolio (Crabbe 1995). Therefore a fund manager will assess the possibility of

² For example, see Crabbe (1995) and Malvey (1997).

an unfavourable change in credit quality for specific issuers when deciding on which bonds to invest in. This is often done using a credit rating probability matrix. Such a matrix is shown in Figure 58.2, which shows the probability of a ratings event for bonds of different credit grade during 1998. The values are one-year probabilities. Using this table we observe that there is a higher probability attached to a ratings downgrade rather than an upgrade.

Credit rating	Probability of upgrade or downgrade						
	AAA	AA	A	BBB	BB	B	C
AAA	92.8	6.56	0.64	0	0	0	0
AA	2.16	92.19	5.37	0.28	0	0	0
A	0.24	3.23	91.61	4.66	0.21	0.05	0
BBB	0	0.34	6.83	85.42	6.32	0.95	0.14

Figure 58.2: Credit event probabilities, 1998. Source: JP Morgan, CSFB.

If a fund manager accepts the probability values, then it is possible for them to calculate the expected return from holding a bond of each different credit rating. This involves engaging in the following:

- estimate the yield spread for each bond over the relevant government bond;
- calculate, using the probability values obtained from one or more of the ratings agencies, the price at the end of the investment term of each bond; using these prices the total return over the investment period can be estimated;
- calculate the expected incremental return for each bond of differing credit quality, using the weighted returns of each bond (calculated using the probability values).

On the other hand if the portfolio manager does not agree with the probability levels given by the ratings agencies, they can set up the fund investments to reflect this. This would be an example of a bet on credit spreads.

The spread between conventional bonds and bonds with embedded options such as callable bonds is also sensitive to changes in interest rates. It will change as a result of anticipated and actual changes in the direction and volatility of interest rates. Generally an expected fall in the level of interest rates will increase the spread between the callable and non-callable bonds of one issuer, because this makes it more likely that the issuer will exercise the call option. For the same reason the yield spread will narrow if interest rates are expected to increase. The opposite applies in the case of puttable bonds. In addition the value of the option element of the callable or puttable bond will increase if there is a rise in the interest rate volatility, and so this will also lead to an increase in the spread between a callable bond and a conventional bond. The fund manager therefore can position the portfolio to take advantage of these expected changes in interest rates.

It is common for analysts to measure the yield spread for bonds that contain an option feature in terms of their *option-adjusted spread* (OAS). The approach involves positioning a portfolio such that it gains from changes in the option-adjusted spread during the investment term. It is important to remember that just as the yield measure given by a bond does not give an indication of the true total return that is gained from holding it over an investment period, the measure of a bond's OAS is also no guarantee of the return that will be achieved. Studies in the performance of portfolio strategies that sought to maximise the OAS of bond holdings³ however have found that the relative performance of mortgage-backed securities for example, have been positively correlated to their OAS during the investment period. It has also been suggested⁴ that an OAS-based approach will generate a better return than the conventional yield spread.

58.1.4 Individual securities forecast

To some extent it is a misnomer to suggest that there is a separate strategy that focuses on individual bonds, because all the aforementioned strategies will eventually break down to the selection of specific bonds. However picking individual bonds is often done on a relative-value basis, where fund managers attempt to identify mispriced bonds,

³ For example see Malvey, Mandl and Varadhachary (1989) and Hayre and Lauterbach (1990).

⁴ *Ibid.*

either as undervalued or overvalued. A common approach involves assessing the valuation of a bond as underpriced as a result of its yield being higher than comparable bonds of similar credit quality, or identifying a bond whose yield is expected due to an anticipated change in the issuing company's credit rating.

The simplest action is for the fund manager to switch out of one bond and into the mispriced bond if its yield is expected to fall. However often yields are out of line with fair value because of structural imperfections in the market, or due to liquidity and supply factors. Switching into a bond that is not virtually identical to the bond that has been sold will introduce these other factors into the determination of the total return achieved. Frequently the yield difference between comparable bonds of similar maturity is due to the convexity of one bond being higher (so its yield is lower), and from observation of market yields we know that over the time the higher-convexity bond will not outperform the other bond, precisely because its yield is always lower. This is essentially the price of convexity.

Another approach is to target bonds issued by companies that have just recently been downgraded or upgraded. This is because the performance of such bonds has been found⁵ to be superior to bonds in the next grading category in an initial period after the grading change. For example a 1995 study by the investment bank Lehman Brothers observed that bonds that were downgraded bonds underperformed the banks own triple-and double-B bond indices in the period *before* they were downgraded, but then outperformed the index in the six-month period after they were downgraded. Similarly bonds that were upgraded were found to outperform the index in the three-month period before the upgrade, but then to underperform in the six-month period after the rating change. Both observations can be explained by the fact that the market priced in the ratings change in anticipation of the change itself. The results of the study therefore indicate that a bond that has been downgraded should be purchased immediately or shortly after the actual rate change has been announced, which is referred to as *buying the downgrade*, and that upgraded bonds should be sold or shorted after the upgrade has been announced, known as *selling the upgrade*.

58.2 Structured portfolio strategies

A structured portfolio strategy is one where the fund manager attempts to match the performance of a selected bond index. It is therefore more closely related to a passive approach rather than an active strategy, but the requirements of matching the fund to the index call for dynamic rebalancing of the portfolio as required, so therefore it is also related to the active approach. There are three structured portfolio strategies; these are:

- the *indexing* strategy, which is where the fund manager attempts to match the performance of a specific bond index;
- the *immunisation* strategy, which is a combination of passive and active approaches, and seeks to minimise the interest-rate and reinvestment risk of the portfolio over the required investment term. This is most commonly associated with investors such as pension funds, who seek to match medium- and long-dated liabilities with equivalent-maturity assets;
- the *cash flow matching* strategy, which involves the construction of a portfolio that seeks to fund future liabilities from the returns generated by the portfolio, as well as its total asset value. The portfolio matches the liability completely, so that if there is a finite number of liabilities, the portfolio will have a value of zero immediately after the last liability has been discharged.

Let us consider the main points of each approach.

58.2.1 Indexation

Indexing a bond portfolio follows exactly the same principles as for an equity portfolio or *tracker fund*, where the portfolio is constructed so that its performance matches the performance of a selected index. In indexation the performance of the portfolio is measured in terms of the total rate of return that is achieved over the required investment horizon. Indexation is popular with investors for a number of reasons. In recent years the performance of active fund managers has been below that of the relevant index, while their charges are higher, leading some investors to switch to indexed funds. An indexed fund will charge a lower management fee than an active fund. Clients also have more comfort that their fund will be managed according to strict guidelines when it is indexed. For instance if the fund guidelines state that the duration of a portfolio must not exceed a certain value, an active fund

⁵ *Ibid.*

manager can remain within this guideline (which is designed to restrict interest-rate exposure) and still pursue a range of strategies that are high-risk. In an indexed fund, the fund manager is limited to constructing the portfolio in the way that most closely replicates the structure and performance of the index. The main challenge associated with indexation is replicating the index itself while keeping transaction costs at a minimum. The exact method used to achieve this will depend on the benchmark that is selected. However an index that is composed of a large number of bonds will be expensive to replicate.

Proponents of active strategies argue that although an indexed fund will match the performance of an index, if it is properly constructed, the performance of the index itself may not be the best that can be achieved. This is of course the primary rationale behind active strategies. A fund that matches an index will not necessarily provide a sufficient return for an investor either; for example a pension fund will have long-dated obligations that must be met, and merely indexing the fund does not guarantee that the fund value will be sufficient to discharge all its obligations. That is, the performance of the index is not related to the fund's liabilities. Certain investors also feel that indexation is unacceptably restrictive, as it limits the fund to assets that are constituents of the index only. This means that otherwise attractive investment opportunities that lie outside the index cannot be considered. However if an investor simply wishes to match the index, or wishes to make certain that his investments do not underperform the index, than indexation is a straightforward and sensible strategy to adopt.

There are a number of issues to consider when choosing which index a fund should match. These include:

- the extent to which the investor is risk-averse or risk-seeking; for example depending on the level of risk tolerance an investor has, the index chosen may be a government bond index, an emerging-market index or an index that mixes government and corporate bonds;
- the investment objectives of the fund. If an investor wishes to have a steady return over a period of time, an index that has historically high volatility of returns will not be suitable.

The investment objectives are the primary determinant of index choice. Depending on the objective, a fund may be constructed to match a generalist market index, a specialised index such as a high-yield or emerging markets index, or a tailor-made index. Once the index has been selected, the portfolio can be constructed that tracks the selected index. This is why such funds are often known as *tracker funds*. Over time the difference between the performance of the index and that of the bond fund is known as *tracking error*. Such discrepancy arises because of three factors, which are:

- the effect of transaction costs on the total return of the bond fund;
- differences between the constituents of the bond fund and the composition of the index;
- differences between the prices used to calculate the returns of the index, which may well be mid-market prices, and the actual market prices paid and received by the bond fund.

The total return recorded by the bond fund must account for transaction costs and bid-offer spreads, while the index is not subject to such effects. This is the primary cause of tracking error, and usually results in some slight underperformance of the bond fund. The other cause as we note above, is that unless the fund is composed of every one of the index's bonds, in the same proportion, it will not be an exact match and therefore will experience discrepancies in return. It is very expensive to replicate an index precisely, because a generalist index may contain several thousand different individual bonds, so this problem is difficult to remove. There is no real solution in fact; a fund that uses the fewest securities to replicate an index will suffer from the lowest tracking error due to the effect of transaction costs, but the highest tracking error due to the mismatch between the composition of the fund and that of the index. A fund that replicated the index exactly however, would have the lowest tracking error due to mismatch of the highest error as a result of the effects of transaction costs. The fund manager must decide on the amount of the trade-off between these two factors.

In undertaking indexation a fund can use two approaches, which are commonly known as *stratified sampling* and *optimisation*. We discuss portfolio optimisation in some depth in the next section. The stratified sampling approach involves dividing the index into sectors or *cells*, where each cell represents a particular characteristic of the index. Generally an index will be divided into cells on the basis of the following factors:

- duration;
- coupon rate;
- term to maturity;
- market sectors, that is government bonds, corporate bonds, asset-backed securities and so on;
- credit-risk factors; this is usually done by breaking down bonds into their respective credit rating categories;
- the presence of any embedded option features.

Once this has been done, specific bonds from each cell are selected that are designed to replicate the whole cell. Usually only one or two bonds are selected from each cell. The market value of each bond purchased from a cell will be a function of that cell's weighting in the index as a whole. So for example if the index is composed of 60% by market value of government bonds, the bond fund will be composed of the same proportion of government bonds by market value. Note that this composition, like that of the index itself, is not static. To achieve proper indexation, the portfolio will have to be rebalanced to account for changes in the weightings in the index as market values of constituent bonds change. The more frequently this is done, the more the fund will suffer from the effects of transaction costs and paying the bid-offer spread.

The number of cells that will be used in the indexation is a function of the cash value of the fund that is being invested. Generally the smaller the value of the fund, the fewer cells will be used. This is because the more cells are used, the greater the cost of buying the issues to represent each cell, and this will increase the tracking error. On the other hand reducing the number of cells will increase tracking error due to the mismatch between the composition of the fund and that of the index. However the rule is that only large funds, say of £1 billion or more, can afford to match an index accurately, and therefore these funds have a larger number of cells.

58.2.2 Portfolio optimisation

Optimising a portfolio as part of a structured strategy is undertaken in order to achieve the objective of matching the index. The process is usually carried out using a mathematical model. Optimisation models are also used by portfolio managers who seek to generate sufficient funds to pay off future liabilities, and this is known as *liability funding*. An optimisation model is used in order to achieve the best possible allocation of (limited) funds so that the index is replicated as accurately as possible. The key assumption behind an optimisation model is that there are a range of solutions that are acceptable to the investor, but that there is only one solution that will optimise the value of the investor's objectives. To do this the portfolio manager will set up a mathematical model. There is more than one type of model available.

One of the most commonly-used models is based on linear programming. This sets a linear objective function and linear constraints on the portfolio composition. The constraints can include such conditions as a limit on the portfolio duration, or a sector allocation limit, or a minimum holding in government bonds, for example. The portfolio itself may have an objective of say, being required to meet the obligations of future liabilities, at the lowest possible cost. The linear programming method will allocate the assets to the portfolio. For example consider that the complete set of bonds that may be invested in is 100 issues, which are what the index is composed of. These are a combination of government and corporate bonds. This means that there are 100 different decision variables that the portfolio must consider. If we set the following,

- m_i = is the nominal value to purchase of a particular constituent bond i (where $i = 1, 2, \dots, 100$),
 P_i = is the price of bond i per cent of par value,

the total cost of the portfolio is given by:

$$m_1 P_1 + m_2 P_2 + \dots + m_{100} P_{100}. \quad (58.1)$$

Expression (58.1) represents the objective function, which is linear. The portfolio manager must aim to minimise the objective function, as costs must be kept to a minimum. The constraints that must be followed will depend on the investment criteria of the fund. For example the fund might state that no more than £20 million of the fund total value may be invested in high-yield bonds, or that a minimum of £100 million must be invested in government bonds such as gilts at all times. If we assume that of the 100 bonds, bonds 1–60 are government bonds and bonds 95–100 are high-yield bonds, we can express these constraints in the following way:

$$m_{95}P_{95} + m_{96}P_{96} + \cdots + m_{100}P_{100} \leq 20,000,000$$

$$m_1P_1 + m_2P_2 + \cdots + m_{60}P_{60} \geq 100,000,000.$$

A variation on the linear programming approach is known as *mixed integer* programming. This allows for odd-lot transaction sizes.

58.3 Immunisation

This is the traditional approach used to construct a portfolio so that it generates a certain return over the required investment period, and in theory is unaffected by changes in interest rates. It was first presented by Reddington (1952) and later by Fisher and Weil (1971). The principle behind immunisation is that it produces a portfolio that balances changes in its market value during its term, and at maturity, with the return generated by the reinvestment of coupons and each bond's maturity value. Immunisation balances interest-rate risk and coupon reinvestment risk. This is undertaken by controlling the duration of the portfolio, which is set to the investment period required by the investor. This period is usually the date of the investor's liability. If the duration of the portfolio is set equal to the investor's required time horizon, the effects of changes in portfolio value as a result of moves in interest rates can offset the effects on reinvestment of coupon. This is illustrated in the following section, which is the traditional approach and follows Fabozzi (1996).

58.3.1 Simple model of immunisation

A life company has a future obligation as a result of selling a life assurance policy. This future obligation has a known value M , and the present value of the obligation is given by (58.2):

$$PV_0 = \frac{M}{(1+r)^N} \quad (58.2)$$

where r is the appropriate discount rate. The life company hopes to discharge its obligation as a result of its investment in a bond, the value of which is currently PV_B and which is equal to PV_0 . From Chapter 3 we know that if the bond pays an annual coupon of C its present value is given by (58.3):

$$PV_B = \sum_{n=1}^N \frac{C_n}{(1+r)^n} \quad (58.3)$$

where N is the maturity of the bond.

Assume that there is a change in the yield of the bond from r to $r + \Delta r$. Under a first-order approximation as described by duration, the value of the life company's obligation is given by:

$$PV_0 + \Delta PV_0 \approx PV_0 + \frac{dPV_0}{dr} \Delta r = PV_0 + \Delta r \left(\frac{-NM}{(1+r)^{N+1}} \right) \quad (58.4)$$

while the new value of the bond is given by:

$$P_B + \Delta P_B \approx P_B + \frac{dP_B}{dr} \Delta r = P_B + \Delta r \sum_{n=1}^N \frac{-nC_n}{(1+r)^{n+1}} \quad (58.5)$$

where we now use P to denote the price of the bond.

As the two expressions are identical, a change in interest rates should not have an impact on the ability of the bond to discharge its obligation. If we set the expressions as equal we obtain the following:

$$P_B + \Delta r \sum_{n=1}^N \frac{-nC_n}{(1+r)^{n+1}} = PV_0 + \Delta r \left(\frac{-NM}{(1+r)^{N+1}} \right). \quad (58.6)$$

Using (58.2) and (58.3) we can obtain from (58.6) the following expression:

$$\frac{1}{P_B} \sum_{n=1}^N \frac{nC_n}{(1+r)^n} = N. \quad (58.7)$$

The expression at (58.7) means that in an environment where we can reinvest coupons at a uniform rate, that is the interest-rate term structure is flat or there are only parallel shifts in the yield curve, then for the market value of the bond to be equal to the present value of the future obligation M at all times the duration of the asset must be equal to the maturity of the obligation. This is the Macaulay duration of the bond. A portfolio that matched the duration of its assets to the maturity term of its liability is described as being immunised. However this traditional description of immunisation has a number of limitations. In the first instance note that the analysis above was a first-order approximation only. The effect of changes under a first-order approximation is not identical to the actual effects of yield changes in practice. The other critical factor is that the rule holds only in circumstances of a flat term structure or parallel changes in the yield curve. From Chapters 9–10 we know this is not realistic. However this does not mean that there are no circumstances in which this approach may be adopted.

58.3.2 Illustration

We can illustrate classical immunisation using a numerical example. A life office wishes to immunise a 10-year obligation with a present value of £100. The current market discount rate is 6.00%, which means that the obligation currently has a future value of £179.085.⁶ The life company can match this 10-year liability by using one or more of three bonds, which are detailed in Table 58.1.

	Bond 1	Bond 2	Bond 3
Coupon	6.75%	7.00%	5.50%
Term to maturity (years)	10	16	25
Price	105.1521	91.2452	101.3973
Nominal value of £100 market value	95.1	91.244	101.397

Table 58.1: Hypothetical set of bonds.

The bonds all yield 6.00%. These prices are used to ascertain how much nominal value of each bond we need to purchase in order to have £1000 market value of each bond. Providing that the gross redemption yield does not change, the life office will be able to reinvest each coupon at 6.00%. Therefore under this assumption we can calculate the maturity value of each of the bonds, as well as the value of each bond at the end of the 10-year investment period; for example bond 2 has a terminal value of £196.270.⁷ However the nominal value of this bond purchased today is £91.244, which is equal to the market value of £100, so that the terminal value of the life company's holding is £179.084. This is its target requirement and allows it to discharge its obligation. The results for each of the bonds are shown in Table 58.2.

	Bond 1	Bond 2	Bond 3
Bond price	100	104.163	98.863
Coupon reinvestment	88.331	92.107	77.767
Total	188.331	196.27	176.63
Nominal holding	95.1	91.24	101.4
Actual terminal value	179.085	179.084	179.085

Table 58.2

According to this analysis then the life company can purchase £100 market value-worth of any of the three bonds and be able to meet its liability in ten years' time, assuming that it can reinvest all the coupons during the ten-year period at the current yield of 6.00%. What would happen if there was a change in yield? Table 58.3 shows the effect of a 1% drop in yields on the terminal values of the three bonds, assuming that the yield change is immediate and also that it then stays at this level for the remainder of the investment term.

⁶ Given in normal fashion by $100 \times (1 + 0.06)^{10}$.

⁷ This is obtained in the usual way using $\sum_{n=0}^9 7.00 \times (1.06)^n + \left(\sum_{n=1}^5 \frac{7.00}{(1.06)^n} + \frac{100}{(1.06)^{10}} \right) = 92.107 + 104.163 = 196.270$.

	Bond 1	Bond 2	Bond 3
Bond price	100	108.607	111.216
Reinvested coupons	84.272	87.894	74.21
Total	184.272	196.501	185.426
Nominal value	95.10	91.24	101.40
Terminal value	175.243	179.296	188.21

Table 58.3: Impact of 1% fall in yields.

From Table 58.3 we see that in this new scenario the first bond will no longer meet the liability of the life company, while bond 3 will over-fund the liability. The impact on bond 2 is essentially unchanged. From a reading of the earlier analysis we will not be surprised at this result, because this bond has a duration of 10 years. The results are illustrated graphically in Figure 58.3.

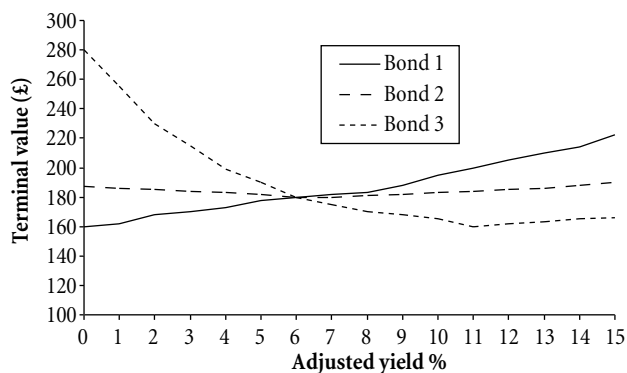


Figure 58.3: Immunisation properties of three hypothetical bonds.

To obtain a better immunisation we use the convexity property of the bonds. Remember that the duration of a portfolio is the weighted average duration of the assets in the portfolio. The life company in our hypothetical example could therefore construct a portfolio with a duration of 10 years, rather than simply hold one bond with a duration of 10 years. It can be shown that if the life company invests £66.51 in bond 1, £33.49 in bond 3 it will have a portfolio of 10 years duration. Figure 58.4 illustrates the performance of the portfolio compared with that of bond 2 on its own. The result is no surprise; the terminal value of the portfolio outperforms that of bond 2 whatever happens to interest rates, because the convexity of the portfolio is greater than that of bond 2; however this is a theoretical example. In practice the yield gain on bond 2 on its own will offset the higher performance of the portfolio. This result was first presented by Reddington (1952).

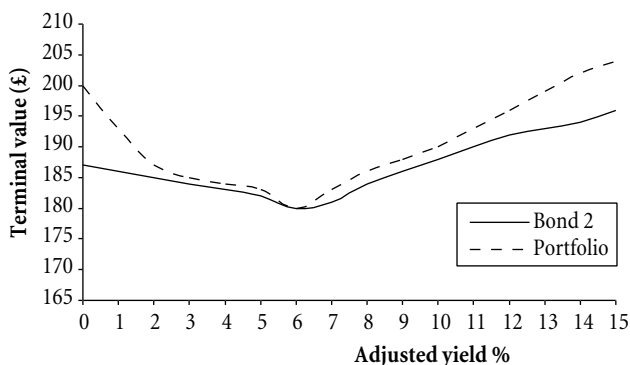


Figure 58.4: Convexity effect on terminal value: bond 2 versus portfolio.

To use convexity more accurately we require the second-order derivative of the duration function. This adjusts for the errors made when using duration. To improve the immunisation quality of a portfolio, we can match changes in its terminal value not only to first-order changes in yields but also to second-order changes. From the earlier analysis we can write

$$N(N+1) = \sum_{n=1}^N \frac{n(n+1)C_n}{(1+r)^n} \quad (58.8)$$

which is a measure of the adjustment effect under the second-order approximation. Consider now another set of hypothetical bonds, one of which is bond 2 from the illustration above, which has a duration of 10 years. The bonds are listed in Table 58.4.

	Bond 1	Bond 2	Bond 3	Bond 4
Coupon	5.00%	7.00%	3.50%	11.00%
Term to maturity	25	16	14	10
Price	82.795	109.596	76.763	136.811
Nominal value for £100 market value	120.781	91.244	130.273	73.099
Duration	12.715	10.000	10.9176	7.087
Second-order duration derivative	228.9125	136.4781	149.6513	67.91876

Table 58.4: Further set of hypothetical bonds.

The second-order duration calculation for each bond is given by (58.9):

$$\frac{1}{P} \sum_{n=1}^N \frac{n(n+1)C_n}{(1+r)^n}. \quad (58.9)$$

To use this second-order estimation, we construct the portfolio such that both its duration and second-order derivative are equal to the liability figure. It can be shown that the proportions of the bonds that meets this requirement are:

- bond 1 equal to -0.561214
- bond 2 equal to 1.641316
- bond 3 equal to -0.07971.

This means that if we construct a bullet/barbell portfolio composed of a long position in bond 2 and short positions in bonds 1 and 3 we will have constructed an immunised portfolio that responds better to changes in interest rates with regard to its terminal value. From Figure 58.5 we can see that this portfolio meets the requirements of the life company more accurately than bond 2; however although the portfolio immunises the liability more precisely, a fund manager may prefer to hold bond 2 because of its convexity value.

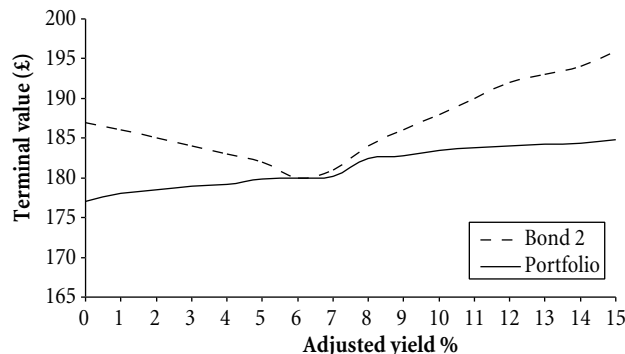


Figure 58.5: Second-order immunisation.

58.3.3 Recap of main points

We saw initially that setting a portfolio or individual bond holding to match the liability target, using its duration measure, would effectively immunise the portfolio against changes in interest rates, but only if this change was a parallel shift. Essentially the rule is that to immunise a portfolio target value, the assets used to match the liabilities must have a duration equal to the target maturity, and that the initial present value of the cash flows from the assets must be equal to the present value of the future liability. Using an asset that had a shorter duration than the target maturity would expose the portfolio to reinvestment risk, while using an asset with a longer duration will expose the terminal value to interest-rate risk. The analysis used Macaulay duration to illustrate immunisation. In fact in practice it is more common to find that an asset's modified duration is used, so that it is the modified duration of the bond that is matched to the effective duration of the liability.

58.3.4 Dynamic immunisation

The earlier analysis only considered a one-off change in interest rates, which was assumed to remain constant up to the maturity date of the liability. This is slightly unrealistic. In practice interest rates and bond yields will change frequently during the investment term, and this will alter the duration of the immunised portfolio. By definition the duration of the portfolio will also change over time, irrespective of any change to yield levels. The rate at which portfolio duration changes over time will also be a function of the shape and level of the yield curve. To allow for the effects of change in portfolio duration, it is necessary to rebalance the portfolio from time to time. There are two considerations:

- adjusting the portfolio balance so that its duration matches the target duration after there have been significant changes in yield level or the shape of the yield curve;
- adjusting the portfolio over time, as its duration steadily decreases.

For instance if a portfolio is constructed with a duration of 10 years, matching a 10-year liability, after six months the target liability will have a maturity of 9.5 years, whereas the portfolio will almost certainly have a different duration. It will need to be rebalanced, by switching into appropriate securities, to an adjusted duration value of 9.5 years. The same issue will arise every six months.

The frequency of portfolio adjustment will depend on the extent to which the portfolio is diverging from target maturity and the amount of transactions costs that will be incurred in rebalancing the portfolio.

58.4 Extending traditional immunisation theory

Fabozzi (1996) notes that the traditional analysis will only hold under certain conditions. These include:

- changes in the yield curve are parallel shifts only;
- the portfolio is valued at a fixed horizon date, and once the portfolio has been established there are no further investments into it;
- the target value of the investment is the value of the portfolio at the target maturity date, and there are no changes in the forward reinvestment rates.

The first assumption is rarely observed in practice so the portfolio is not immunised against non-parallel yield curve shifts.

Various studies have considered how to extend traditional immunisation theory to account for non-parallel yield curve shifts. One approach⁸ presents a strategy that is able to handle an arbitrary change in interest rates so that it is not necessary to specify any alternative duration measures. The immunisation risk measure is then minimised, subject to a constraint that the duration of the portfolio must be equal to the investment term. One method by which immunisation risk can be minimised is by setting a bullet portfolio. If we consider two portfolios, one with a bullet structure and the other with a barbell portfolio, the barbell portfolio will carry greater immunisation risk. This is not surprising. Assume that both portfolios have a duration value that is equal to the required investment horizon, so that they are both immunised against the effects of parallel yield curve shifts. This would have been

⁸ See Fong and Vasicek (1984).

achieved by balancing the effect of changes in interest rates (reinvestment rates) on cash flows received during the investment term against the effect of changes in the market value of the portfolio. Under a non-parallel yield curve shift however, the portfolios will behave in different ways. Imagine a scenario where short-dated interest rates decrease while long-dated rates increase, that is a steepening of the yield curve. Both portfolios would suffer a loss at the end of the investment term that is below the terminal value, since there will be a loss in market value in addition to lower reinvestment rates. The loss in value would be higher for the barbell portfolio however. This is because the lower reinvestment rates are experienced by the bonds in the barbell portfolio for a longer period than by those in the bullet portfolio, resulting in a higher opportunity loss. In addition the maturity of the bonds in the barbell portfolio at the end of the investment term is longer-dated than those in the bullet portfolio, so that the same interest rate increase causes a greater capital loss. Therefore the bullet portfolio has a lower level of interest-rate exposure irrespective of the change in the term structure, when compared to the barbell portfolio.

For portfolio managers immunisation risk is essentially reinvestment risk. This means that the portfolio with the lowest reinvestment risk will carry the lowest immunisation risk. Where there is a relatively high dispersion of cash flows around the target investment date, which would be the case with a barbell portfolio, the portfolio will exhibit a higher reinvestment risk. The opposite occurs when cash flows are concentrated around the target date, as occurs in the case of a bullet portfolio. The simplest way to illustrate this is by using a portfolio of zero-coupon bonds. A zero-coupon bond carries no reinvestment risk, therefore a portfolio consisting entirely of zero-coupon bonds would carry no immunisation risk. Thus a portfolio that consisted of bonds that replicated zero-coupon bonds would minimise the immunisation risk of the portfolio. Once a portfolio is constructed with coupon-bearing bonds, the fund manager needs to select bonds that present the lowest immunisation risk. We have seen that this occurs with bonds that have their cash flows as close as possible to the investment horizon date.

Fabozzi (1996) expresses this requirement more formally. We know that the target value of an immunised portfolio is the minimum level of the terminal value of the portfolio at the investment maturity date as long as there is a parallel shift in interest rates. If there is a non-parallel shift, then the target accumulated value is not necessarily the minimum level of the investment value. It has been demonstrated⁹ that if the yield curve changes in an arbitrary non-parallel way, the relative change in the value of a portfolio is a function of two different terms. The first term is dependent on the structure of the portfolio, while the second term is dependent of the change in interest rates. It is this second term that characterises the nature of the change in the yield curve. The shift itself is arbitrary, so therefore this term is unknown and cannot be controlled by the portfolio manager. The first can be manipulated however, since it is related to the structure of the portfolio itself. It is given by:

$$I = \frac{PVC_1(1-n)^2 + PVC_2(2-n)^2 + \dots + PVC_N(N-n)^2}{PV_{port}} \quad (58.10)$$

and is a measure of the immunisation risk of a portfolio where:

- PVC_N is the present value of the cash flow in period n discounted at the current redemption yield
- n is the term of the investment horizon
- N is the time to receipt of the last portfolio cash flow
- PV_{port} is the initial investment value.

This measure of immunisation risk I is in line with our discussion of the relative risk of bullet and barbell portfolios mentioned earlier. For a barbell portfolio, the portfolio cash flows are dispersed over a longer time horizon, and this results in a high value for I . The opposite occurs with a bullet portfolio.

58.5 Multiple liabilities immunisation

The analysis so far has considered only a single liability with a fixed maturity date. In practice institutional investors such as insurance companies and pension funds will deal with a large number of liabilities, all of which must be met from the investment portfolio and which will occur over a number of target dates. That is, the investor will need to match multiple liabilities. Many retail investment contracts contain multiple payout dates, for example payments made on a pension contract, and annuity payments made under a permanent health insurance or unemployment

⁹ *Ibid.*

insurance policy. There are two basic approaches that can be used to meet multiple liabilities. These are an extension of the single liability immunisation we discussed first, and a cash flow matching approach. These are reviewed next.

58.5.1 Multiple liability immunisation

This approach was first presented by Fong and Vasicek (1984). They showed that under certain conditions it is possible to immunise a portfolio to meet multiple liabilities, under parallel yield curve shifts. The conditions are that the present values of both liabilities and assets must be equal, and that the durations of the portfolio and the liabilities must be equal. The duration of the liabilities is given by (58.11):

$$D_L = \frac{(1)PVL_1 + (2)PVL_2 + \cdots + (n)PVL_n}{PV_L} \quad (58.11)$$

where

- PVL_n is the present value of the liability at time n
- n is the time of the last liability payment
- PV_L is the total present value of the liabilities.

Another condition is that the distribution of durations of individual bonds must have a wider range than the distribution of the liabilities. More formally we can say that the mean deviation of the portfolio cash flows must be equal to or higher than the mean deviation of the liabilities at each payment date. This means that if the final target date for the longest-dated liability is say 30 years, we are able to immunise the portfolio without having to set its duration to 30 years. What is important is that the portfolio duration matches the weighted average of all the liability durations. As a result it is not necessary to construct a portfolio with a very long duration, which is an important result because, as we saw on Chapter 7 the limiting value for duration means that very few bonds have a duration greater than about 18 years.¹⁰ By using a weighted-average duration of the liabilities, the fund manager is able to immunise the portfolio by setting its duration to a lower figure than the longest-dated liability. It is also necessary for the portfolio cash flows to have a greater amount of dispersion than the liabilities. This means that within the portfolio there must be an asset with a duration value that is less than or equal to the duration of the shortest-dated liability, so that the portfolio can be used to discharge the liability as it becomes due.

We have stated that the present value of the assets must match that of the liabilities. The calculation of present value uses a discount rate selected by the user, and the most appropriate rate must be chosen. Moreover, it is necessary to use the correct range of discount rates, because it would be inappropriate to use a single rate to discount multiple liabilities with a range of maturity dates. Some of the approaches that have been considered include using the government yield plus an appropriate spread, using the government zero-coupon curve plus an appropriate spread, or using a yield curve that has been derived from a portfolio of assets. This last involves obtaining the discount rates from a yield curve constructed using the yields of the assets in the portfolio itself.¹¹ In fact the first approach is invalid, for the same reasons that we discussed in Chapter 6, and therefore portfolio managers will use the zero-coupon curve. The third option may be considered valid but has no relationship to the nature of the liabilities. The only valid approach is to use the government zero-coupon yield curve. The selection of the discount rates is important, because they have an impact on the valuation of the liabilities and hence the composition of the portfolio, as well as the surplus (the difference between the liability value and the asset value). The surplus of a portfolio is used in performance measurement so will be important for the fund manager.

58.5.2 Cash flow matching

The cash flow matching approach is another way of constructing a portfolio to meet multiple liabilities, and separate to an immunisation strategy. It is a popular approach as it involves selecting assets that precisely match liabilities. The basic approach is that a bond is selected that has a maturity date that matches the maturity of the longest-dated liability. The present value of this liability is invested in the bond. The coupon receipts from this bond are used to reduce the present value of the remaining liabilities, before a second bond is selected whose maturity date matches

¹⁰ We ignore certain specialised instruments such as inverse floaters, index-linked bonds and certain spin-offs from mortgage-backed securities.

¹¹ For example, see Choie (1992).

that of the second-longest-dated liability. This second liability value would have been adjusted for the coupon receipts of the first bond, which will impact the market value purchased of the second bond. The remaining liabilities are then adjusted using the coupon payments of this second bond, and the process is repeated. The process is continued until all the liabilities have been matched by coupon and redemption payments of the bonds selected for the portfolio. This approach is straightforward and intuitively attractive. In the basic approach we have described only coupon payments that occur ahead of the maturity date of a liability can be used to discharge that liability. Under certain circumstances the approach can be modified so that cash flows that fall both ahead of and after a liability date are used to satisfy the liability.¹²

It is common for fund managers to combine the immunisation approach with the cash flow matching technique. This is known as *combination matching* or *horizon matching*. The approach involves duration matching the portfolio to the weighted average duration of the liabilities, but additionally cash matching the portfolio for a selected shorter period, say for the first five or 10 years. This has the advantage of satisfying liquidity requirements in the first few years of the portfolio, and also allows for the effect of the shape of the yield curve. For both positive and negative sloping yield curves, the most pronounced slope is usually observed in the first few years of the maturity structure. Therefore cash flow matching the first few years of the portfolio will reduce the exposure of the portfolio to non-parallel shifts in the yield curve.

Selected bibliography and references

- Choie, K., "Caveats in immunisation of pension liabilities", *Journal of Portfolio Management*, Winter 1992, pp. 54–69.
- Crabbe, E., "A framework for corporate bond strategy", *Journal of Fixed Income*, June 1995, pp. 15–25.
- Fabozzi, F., *Bond Markets: Analysis and Strategies*, McGraw-Hill, 1993.
- Fabozzi, F., *Bond Portfolio Management*, FJF Associates, 1996.
- Fabozzi, T.D., Tong, T., Zhu, Y., "Extensions of dedicated bond portfolio techniques", in Fabozzi, F. (ed.), *The Handbook of Fixed Income Securities*, 3rd edition, Dow-Jones Irwin, 1991.
- Fisher, L., Weil, R., "Coping with risk of interest rate fluctuations: returns to bondholders from naïve and optimal strategies", *Journal of Business*, October 1971, pp. 408–431.
- Fong, H., G., Vasicek, O., "A risk minimizing strategy for portfolio immunisation", *Journal of Finance*, December 1984, pp. 1541–1546.
- Hayre L., Lauterbach, K., "Stochastic valuation of debt securities", in Fabozzi, F. (ed.), *Managing Institutional Assets*, Harper & Row, 1990.
- Malvey, J., "Global corporate bond portfolio management", in Fabozzi, F. (ed.), *The Handbook of Fixed Income Securities*, 5th edition, McGraw-Hill, 1997.
- Malvey, J., Mandl, H., and Varadhachary, U., *Corporate Bond Strategy Playbook: Part II*, 1989.
- Reddington, F., "Review of the principles of life insurance valuations", *Journal of the Institute of Actuaries* 78, 1952, pp. 286–340; reprinted in Hawanini, G. (ed.), *Bond Duration and Immunization: Early Developments and Recent Contributions*, Garland Press, 1972.

Questions and exercises¹³

1. Discuss the relative merits of passive versus active portfolio management strategies. Why might an investor wish to place his assets in a tracker fund?
2. What is a structured portfolio strategy?
3. Indexation is essentially a structured portfolio strategy, but has elements of both active and passive approaches included in it. Discuss.
4. Consider the two portfolios below, 1 and 2, both of which have a market value of £1 billion. Assume that bonds are trading at par, and that the portfolio durations are identical.

¹² For example, see Fabozzi, Tong and Zhu (1991).

¹³ The style of these questions borrows heavily from Fabozzi (1996) and AIMR exams, but with a sterling slant!

Portfolio 1			Portfolio 2		
Bond	Maturity (years)	Market value	Bond	Maturity (years)	Market value
1	2.9	240	1	5.1	280
2	3.3	260	2	6	260
3	10.4	190	3	4.9	190
4	10.1	310	4	5.3	270

- (a) Identify the bullet portfolio and the barbell portfolio.
 - (b) Will each portfolio respond in the same way if there is the change in interest rates, given that the portfolio durations are the same?
 - (c) If you anticipated an upward move in interest rates during the next six months, and that was the term of your investment horizon, which portfolio would you invest in?
5. Speaking off-the-record to a financial journalist, a fund manager states the following:

“We shall be entering into a barbell position with new inward funds and funds from maturing bonds being used to buy five-year and 25-year government bonds. Therefore our position in ‘govies’ will increase by proportion and our position in corporates will decrease. The duration of the portfolio will remain approximately similar at around 8.52 years. We’ll retain the barbell set-up over the next few months as I expect the yield curve to flatten during this period, but the return from the long-dated bonds will compensate from the rise in shorter-dated bonds. If there is a rise in interest rates over the medium-term, as the economy picks up steam, we may consider investing in asset-backed bonds such as mortgage-backed securities, for which we will switch out of government bonds. Generally we don’t hold this type of paper because of prepayment risk levels when interest rates are falling. We expect to maintain our current holding of corporates, because there is no value in current spreads.”

What points do you glean from a reading of the fund strategy?

6. What is tracking error? How can it be reduced? Is it possible to eliminate it completely?
7. The fixed interest pages of a financial journal contain the following:

“Farhana Rahman, fund manager of the ABC Bank’s £1 billion bond portfolio, is moving out the current ladder strategy and into a barbell strategy. This will involve the sale of the four- and five-year bonds, both corporate and government issues, and the purchase of two-year and 30-year bonds, again both government and corporate bonds. Ms Rahman believes that the very short-end of the yield curve will outperform the five-year end of the curve, as she thinks that interest rates will rise most sharply in the five-year area. She also expects that the yield curve will flatten with short-term interest rates increasing by about 25 basis points, while the long-end of the curve should stay stable at around 7.00% over the next few months.

The fund is engaging in a barbell approach to reduce the risk exposure at the long-end of the curve and to maintain the current portfolio duration at 7.56 years, which is slightly longer than the duration of the benchmark index. In addition Ms Rahman stated that the fund was switching out of five-year corporate bonds because she didn’t believe that there was any value in corporate spreads at the moment. However if spreads widen out at some point in the next few months she may increase the proportion of the fund invested in the corporate sector. The current investment shares are 54.1% government bonds, both sterling and euro, 23.9% corporate bonds rated from single-A to triple-A, 9.8% to asset-backed floating-rate notes and 5.2% in dollar-denominated Eurobonds. She may increase the weighting allocated to FRNS in the very short term.”

What is your view of this strategy?

8. An investor wishes to invest £1000 in a bond portfolio that is composed of the following two bonds:

	Bond 1	Bond 2
Coupon	8.00%	12.00%
Term to maturity	6	25
GRY	9.00%	9.00%

- (a) calculate the duration of each bond (you may wish to review Chapter 7 again)
 - (b) how much funds should the investor place in bonds 1 and 2 in order to create a bond portfolio with a duration of nine years?
 - (c) assess the impact on the terminal value of the portfolio if the investor has a target maturity of nine years and requires a sum of £2 172 at the end of this period. Show how a combined portfolio of both bonds will immunise the investor's requirement more accurately than a holding in just one of the bonds.
9. Explain the principles of immunisation. What is meant by dynamic rebalancing of a portfolio, and why is it necessary?
10. A fund manager increases its allocation to corporate bonds in the belief that yield spreads will decrease as the economy recovers from recession, as interest rates rise and, in the more confident environment, credit ratings are upgraded. What strategy is the fund manager adopting? On what occasions might these generalisations not apply?
11. A money market trader at a bank invests in Japanese bank sterling Certificates of Deposit that the market demands a yield premium for, but which the trader believes are not risky. This is in effort to pick up yield. What strategy would this be described as?
12. At the morning "shout", the head of the fund management desk informs junior staff of the following points:
- (a) to reduce the exposure to widening credit spreads, which are anticipated as the economy slows down, the sector allocation to corporate bonds will be reduced by around 12–14%, with the released funds being used to switch into government bonds. For example the single-A rated PowerGroup plc five-year bond was purchased 12 months ago at a spread of 64 basis points to the five-year government bond, and is now trading at 44 basis points over the bond. The fund will lock in realised gains by switching out of corporates now. The duration of the portfolio is slightly below that of the benchmark index, by about 0.34 years
 - (b) after the switch the fund allocation will be 52% government bonds, 11% corporates, 7% asset-backed FRNs, 18% foreign-currency supranational Eurobonds, 6% convertible bonds, 4% corporate zero-coupon bonds and 2% high-yield corporate bonds. The allocation to convertibles may be increased.

Discuss this strategy.

13. Explain what is meant by cash flow matching. Why do you think this might be a popular fund management technique?
14. A life assurance company has a range of liabilities that mature every six months starting from ten years, until 30 years. The present values of each liability are all different. How can a portfolio be constructed that would match the liabilities and also minimise immunisation risk?
15. A fund reports a shift in its strategy as follows:
- "In the short-term we will shorten the duration of the portfolio from its current 7.81 years as we believe that interest rates are set to rise and that the economy will begin to experience strong growth. The duration shortening will be accomplished by a short-term holding of cash following the sale of long-dated government bonds. The asset allocation is over 70% government bonds, with only 19% allocated to corporate bonds."

- (a) under what circumstances might a fund shorten duration in anticipation of a change in interest rates?
- (b) comment on the decision to shorten duration by maintaining sale proceeds as cash, rather than switching into very short-dated bond
- (c) what is the fund expecting to happen to corporate credit spreads, given its views on the economy as a whole?

59 Portfolio Management III

The performance of a portfolio is crucial to understanding how the fund manager is performing relative to the market as a whole, and how he or she is performing within the relevant sector. In this chapter we introduce some performance evaluation techniques. Performance measurement involves the calculation of the return that has been realised by the portfolio over a specified time period, which may be the required investment horizon of the investor, or more frequently. The time interval over which performance is measured is known as the evaluation period.

59.1 Introduction

To begin with the measurement of performance involves calculating return. As there are a number of methods used to do this, it is sometimes difficult to compare the performance of different fund managers. Some of the measures of return are introduced here.

The rate of return achieved by an investment is the cash return, expressed in terms of the market value of the investment at the beginning of the evaluation period. This is given by (59.1):

$$R_p = \frac{MV_1 - MV_0 + CF}{MV_0} \quad (59.1)$$

where

- R_p is the return on the portfolio
- MV_1 is the market value of the portfolio at the end of the evaluation period
- MV_0 is the market value of the portfolio at the beginning of the evaluation period
- CF is the cash distribution paid by the portfolio to the investors during the evaluation period.

EXAMPLE 59.1

At the beginning of the evaluation period the market value of a portfolio is £297.76 million, and at the end the value is £312.43 million. During this period the portfolio paid out £22.53 million to investors. The return is calculated as:

$$\frac{312.43 - 297.76 + 22.53}{297.76} \text{ or } 12.547\%.$$

The simple measure of return gives no indication of how the fund has performed relative to other evaluation periods, or relative to other funds. To do this it is necessary to calculate return over a shorter standard period, such as a month or quarter. This measure is known as a *sub-period return*. To obtain the return for the complete evaluation period we then average the sub-period returns. For example if the evaluation period is one year and four quarterly sub-period returns are calculated, the latter must be averaged to obtain the evaluation period return. There are three methods by which averages are calculated, the arithmetic average rate, the time-weighted rate of return and the cash-weighted rate of return. These were reviewed in Chapter 2.

59.2 Performance evaluation

To assess how effective a fund manager has been in running a portfolio, we are interested to know (i) how the manager performed after one allows for the level of risk associated with the strategy used and (ii) by what methods did the fund manager achieve the reported return figure. By answering these questions we can rate a fund manager's performance relative to an external benchmark. When evaluating and comparing performance, we require the process to be as accurate as possible, and this may have some influence on the type of return measure we use. Whatever measure is adopted, the return should make allowances for the timing of bond cash flows, so that the return is as accurate as possible.

The first step in assessing portfolio performance involves specifying against which benchmark the fund manager will be measured. This benchmark can be a market index, such as one that is published by an investment bank (for example, the various Lehman Brothers bond indices), or it can be another portfolio. If another portfolio is used it will be what is known as a *normal portfolio*, which is a tailor-made benchmark that is composed of the securities that form the universe from which the fund manager selects her securities, weighted in the way that they are weighted in the fund manager's portfolio. A normal portfolio therefore is a customised index. Constructing a normal portfolio is a complex task, and is usually done as a joint effort between the fund manager and the investing client.

59.2.1 Single performance evaluation measures

Three commonly-used measures of performance are the *Sharpe ratio*, the *Treynor index* and the *Jensen index*. All these methods assume that there is a linear relationship between the return on the portfolio and the return on a general market index.

The Sharpe ratio is a risk/reward ratio. The risk of the portfolio is measured by its standard deviation of returns; the measure itself is given by (59.2):

$$Sharpe = \frac{R_p - R_f}{\sigma_p} \quad (59.2)$$

where R_f is the risk-free return. This is usually the return on a government bond or T-bill. The expression at (59.2) indicates that the Sharpe ratio is a measure of the excess return on a portfolio relative to the total variance of its returns. A ratio of 0.5 is considered fair return for risk incurred. For an investor it is more useful as a relative measure, in comparing the ratio of one investment to that of another. For bank trading desks it is a useful measure of the return generated against the risk incurred, for which the return and volatility of individual trading books can be compared to that on the risk-free instrument (or a bank book trading only T-bills).

The Treynor index is also a measure of the excess return of a portfolio relative to the risk taken on by the fund manager. The excess return is defined as the difference between the portfolio's return and the risk-free return achieved over the same evaluation period. The risk measure that is used is the portfolio's beta. This is considered to be appropriate because in theory in a well-diversified portfolio the unsystematic risk should be zero. The Treynor index is given by (59.3):

$$Treynor = \frac{R_p - R_f}{\beta_p}. \quad (59.3)$$

The Jensen index adopts another approach based on the capital asset pricing model, to determine whether the fund manager has outperformed the market index. From Chapter 57 we know that the CAPM is

$$E[R_p] - R_f = \beta_p (E(R_{Market}) - R_f) + \varepsilon \quad (59.4)$$

where

$$\begin{aligned} E[R_p] & \text{ is the expected return on the portfolio} \\ \varepsilon & \text{ is a random error term.} \end{aligned}$$

If the excess return produced by the fund manager is not greater than the excess return given by (59.5), then the manager has not added any value to the fund's performance. This is because the beta of the portfolio represents an expectation of performance relative to the market, so that a random portfolio should record at least this level of performance. In Jensen's index, an additional factor is added to represent the performance of the portfolio that diverges from the beta measure. This is the portfolio *alpha*, and is a measure of the fund manager's performance. The alpha is given by (59.5), and is solved using regression analysis. We hope to review this process in a later edition of this book.

$$R_p - R_f = \alpha_p + \beta_p ((R_{Market}) - R_f) + \varepsilon_p. \quad (59.5)$$

The alpha term is the return realised by the fund manager. The Jensen measure is the measure of the risk of the portfolio, and a positive alpha indicates that the fund manager outperformed the index, while a negative measure

indicates underperformance. The evaluation assumes that the portfolio is well-diversified, so that there is no unsystematic risk remaining, as with the Treynor index.

Selected bibliography and references

Bailey, J., Tierney, J., “Gaming Manager Benchmarks”, *Journal of Portfolio Management*, Summer 1993, pp. 37–40.

Jensen, M., “The performance of mutual funds in the period 1945–1964”, *Journal of Finance*, May 1968, pp. 389–416.

Sharpe, W., “Mutual fund performance”, *Journal of Business*, January 1966, pp. 119–138.

Treynor, J., “How to rate management of investment funds”, *Harvard Business Review*, Jan–Feb 1965, pp. 63–75.

Questions and exercises

- The performance of five portfolios in the last year is shown below.

Portfolio	Return	Standard deviation	Beta
1	25	10	1.5
2	15	8	0.8
3	20	6	1.1
4	19	7	0.9
5	20	5	1.5

- The risk-free rate of interest during the year was 10.00% and the return on the market was 19.00%.
 - rank the portfolios using the Sharpe ratio, Treynor’s index and Jensen’s index
 - from your calculations, how well was portfolio 5 managed?
- What is the difference between performance measurement and performance evaluation?
- An analyst is given the following information on a bond portfolio:

Bond	Asset allocation	Return	Specific risk	Duration (years)
1	20%	9%	12%	5.11
2	25%	10%	10%	6.998
3	35%	11%	11%	9.05
4	20%	12%	13%	11.03

- If the risk-free rate of interest is 7.00%, the return of the market portfolio of bonds is 10.00%, the total risk is 12.00% and duration is 10 years, evaluate the performance of the fund manager.
- The Treynor index is the most accurate measure of fund performance. Discuss.

60 Portfolio Yield Measurement

In this chapter we consider the approaches that are used to approximate the yield on a portfolio of bonds, using the yields of the constituent bonds. The process is not necessarily straightforward, but must be applied correctly as otherwise there will be errors in the analysis of portfolio performance and trades such as relative value and butterfly/barbell trades.

60.1 Portfolio yield

We know that the yield on a bond is the discount rate that equates the present value of the bond's cash flows to the current price of the bond, which price is usually taken to be the clean price. Following this we can say that the yield on a portfolio of bonds is the discount rate that equates the present value of the portfolio's cash flows to the current market value of the portfolio. This can be taken to be the definition of portfolio yield.

Readers are familiar with the formula used to calculate the yield on an individual bond. For a conventional bond with a fixed term to maturity and N cash flows during its life, where the n th payment is the amount C_n and will occur in N interest periods, the price of the bond is given by (60.1) for an annual coupon bond.

$$P + AI = \sum_{n=1}^N C_n \cdot (1 + r)^{-N} \quad (60.1)$$

where AI is the accrued interest on the bond at the calculation date. For a semi-annual paying bond with m interest payments per year, the formula is:

$$P + AI = \sum_{n=1}^N C_n \cdot \left(1 + \frac{1}{2}r\right)^{-Nm}. \quad (60.2)$$

As the right-hand side of (60.1) and (60.2) is the present value of the bond's cash flows, the yield r is the discount rate that makes the present value of the bond's cash flows equate the current market price of the bond.

Consider now where we have a portfolio of K individual bonds. Let P_i and A_i be the current market price and accrued interest of the i th bond, and let Q_i be the quantity of the i th bond denominated in terms of par value. Also assume that $f_i(\cdot)$ denote the present value function of the i th bond so that $f_i(r)$ is the present value of the bond's future cash flows discounted at the interest rate r . The yield on the i th bond is then given by

$$P_i + A_i = f_i(r_i), \dots \quad i = 1, 2, \dots, K \quad (60.3)$$

and again following this analysis we can say that the yield on a portfolio of bonds is the level of r that satisfies equation (60.4):

$$\sum_{i=1}^K Q_i \cdot (P_i + A_i) = \sum_{i=1}^K Q_i \cdot f_i(r). \quad (60.4)$$

The market value of the portfolio is given by the left-hand side of (60.4), and as it is the market value it includes the total value of accrued interest. The right-hand side of the equation is the total present value of the portfolio's future cash flows, where all payments are discounted at the same interest rate r . Therefore (60.4) states that the yield on a portfolio of bonds is the discount rate that equates the present value of the portfolio's future cash flows to its current market value. However although (60.4) is indeed an expression of portfolio yield and is, given the usual assumptions that we reviewed in Chapter 4, analytically tenable, it is not of much use in practice because it requires all the individual yield valuation functions $f_1(\cdot), f_2(\cdot), \dots, f_K(\cdot)$. A more useful measure would be an average of the yields of the individual bonds $r_1, r_2, \dots, r_K$, as defined by (60.3), which would be an approximation of portfolio yield. This is reviewed in the next section.

60.2 Value-weighted portfolio yield

60.2.1 Approximating the yield

Consider a portfolio of K different bonds. An approximation of the portfolio yield is to calculate an “average” as shown by (60.5):

$$R = \frac{1}{K} \sum_{i=1}^K r_i \quad (60.5)$$

where r_i is the yield on the i th bond as before and R is the approximate yield on the portfolio. This measure is not logically tenable however because it provides an equal weighting to every bond in the portfolio, irrespective of the asset allocation of individual bonds. Therefore to arrive at a more realistic yield it is necessary to weight individual bond yields in line with their weighting in the portfolio; a common way that this is calculated is given by (60.6), which weights each bond yield by the total market value of that bond:

$$R = \frac{\sum_{i=1}^K Q_i \cdot (P_i + A_i) \cdot r_i}{\sum_{i=1}^K Q_i \cdot (P_i + A_i)} \quad (60.6)$$

where all the terms are as before.

Although (60.6) is only an approximation of portfolio yield, it is commonly used in practice.

60.2.2 A closer approximation

In the previous section we illustrated the portfolio yield calculation that is commonly used in the markets, although it remains an approximate measure. A closer approximation may be obtained however by weighting individual bonds in terms of the aggregate value of a basis point rather than total market value. This is considered a more logical approach following the formal definition given at (60.4). This equation is the formal definition of portfolio yield, but it presents problems in its calculation because of the need to approximate r from the yields $r_1, r_2, \dots, r_K$ for each of the bonds in the portfolio. An estimation may be obtained if we consider that the portfolio yield is made up of a number of yield increments $\Delta r_1, \Delta r_2, \dots, \Delta r_K$ such that

$$r_1 + \Delta r_1 = r_2 + \Delta r_2 = \dots = r_K + \Delta r_K \quad (60.7)$$

and that

$$\sum_{i=1}^K Q_i \cdot (P_i + A_i) = \sum_{i=1}^K Q_i \cdot f_i(r_i + \Delta r_i). \quad (60.8)$$

The portfolio yield R is therefore any of the yields $r_i + \Delta r_i$ for $i = 1$ or $2 \dots$ or K . Following (60.8) we state that this yield satisfies (60.4).

In order to calculate the values of the Δr_i s we apply a first-order approximation to the right-hand side of (60.8), which is:

$$\sum_{i=1}^K Q_i \cdot (P_i + A_i) = \sum_{i=1}^K Q_i \cdot f_i(r_i) + \sum_{i=1}^K Q_i \cdot f'_i(r_i) \cdot \Delta r_i \quad (60.9)$$

where $f'_i(r_i)$ is the derivative of the yield-dependent value function $f_i(\cdot)$ evaluated at the yield r_i . Following (60.3) we multiply both sides of both equations by Q_i and sum from $i = 1$ to $i = K$ and obtain

$$\sum_{i=1}^K Q_i \cdot (P_i + A_i) = \sum_{i=1}^K Q_i \cdot f_i(r_i) \quad (60.10)$$

which gives us an adjusted version of (60.9), shown as (60.11):

$$\sum_{i=1}^K Q_i \cdot f'_i(r_i) \cdot \Delta r_i = 0. \quad (60.11)$$

Let us now denote BPV_i as the value of a basis point for the i th bond so that $BPV_i = f'_i(r_i)$. Equation (60.11) then becomes

$$\sum_{i=1}^K Q_i \cdot BPV_i \cdot \Delta r_i = 0. \quad (60.12)$$

We have already stated that $R = r_i + \Delta r_i$ for $i = 1, 2, \dots, K$ so therefore (60.12) can be re-stated as

$$\sum_{i=1}^K Q_i \cdot BPV_i \cdot (R - r_i) = 0 \quad (60.13)$$

or, put another way, as

$$R \cdot \sum_{i=1}^K Q_i \cdot BPV_i = \sum_{i=1}^K Q_i \cdot BPV_i \cdot r_i. \quad (60.14)$$

Therefore we can approximate the yield on a portfolio as (60.15), which is closer to the true definition than (60.6).

$$R = \frac{\sum_{i=1}^K Q_i \cdot BPV_i \cdot r_i}{\sum_{i=1}^K Q_i \cdot BPV_i}. \quad (60.15)$$

Using (60.15) to approximate portfolio yield is similar in approach to (60.6) but with the exception that the yield on each individual bond is weighted by the aggregate value of a basis point of the position in that bond, given by $Q_i \cdot BPV_i$ rather than by the aggregate market value of the position itself.

We consider that a better approximation of portfolio yield can be obtained using the basis point-weighted average of the yields of the individual bonds rather than as a value-weighted average of the yields themselves. What are effects of using the different approaches in practice? In practice approximating the yield on a bond portfolio with a value-weighted average results in an insufficient weight being given to yields on bonds with longer durations (and larger basis point values). Therefore the conventional approximation tends to understate the true portfolio yield. Note that this effect occurs when the yield curve is positively sloped, as the yields on the longer-dated bonds will be higher. In an inverted yield curve environment the conventional measure will overstate the portfolio yield. By calculating portfolio yield using basis point value-weighted averages, sufficient weight is given to the price sensitivity of longer-dated bonds in a portfolio. This has relevance for a number of applications, for instance when optimising an immunised portfolio.

Selected bibliography and references

Brennan, M., Schwartz, E., "Duration, bond pricing and portfolio management", in Kauffman, G., Bierwag, G., Toevs, A. (eds.), *Innovations in Bond Portfolio Management*, Jai Press, 1983.

61

Bond Indices

Bond indices serve a number of purposes. Although there are a large number of different indices in existence, used for a variety of purposes, they generally have a lower profile amongst retail investors compared to equity indices. Amongst institutional investors however they are very important, and serve the same purpose as equity indices. The performance of portfolio managers is measured against specified indices, as they are used as benchmarks. In this chapter we introduce the basic principles of constructing and calculating bond indices. A selection of leading bond market indices is given at Appendix 61.1.

61.1 Overview

A bond index is used to meet the following objectives:

- to act as a benchmark against which portfolio performance can be measured;
- to serve as an indicator of market performance, such as the behaviour of bond yields but also the duration and average maturity of a group of bonds;
- to be used to compare performance across different markets.

Different types of user will have different requirements for indices. Initially indices were used to compare the performance of different government bond markets, but gradually their use expanded across market sectors. For instance domestic investors generally require an index that represents the entire market and so would tend to use an “all-market” or “all-bond” index. International investors tend to use indices that are composed of liquid benchmark bonds.

61.1.1 General index

The composition of general indices or “all-bond” indices tend to be the complete universe of available bonds. This can be many hundreds of bonds. As investors have the opportunity to invest in any or all bonds, a portfolio manager’s decision to hold or sell specific bonds is rewarded if the result is outperformance. Generally all-bond indices are calculated only for the government bond market, because price transparency and liquidity may hinder the analysis of other bonds. A general index is most suitable for domestic investors, as these are most likely to hold a wider range of securities, and often hold illiquid securities.

61.1.2 Tracker index

The tracker index is a subset of the general index. The selection of bonds for a tracker index takes into account the following criteria:

- liquidity and issue size; the largest size bonds are generally very liquid and often the benchmark in the sector. They are sometimes taken to be representative of the sector. It is more common to use size in terms of market value, which is straightforward to calculate, and also allows the comparison of issues with different coupons and terms to maturity, which cannot be done using nominal values;
- trading volume; in some cases trading turnover of a bond is considered, and bonds with low turnover excluded;
- transparency of prices;
- credit rating;
- yield volatility; in some cases (for example in the case of the French government bond index) bonds that have historically high yield volatilities are excluded from the index.

The bonds that are used to construct a tracker index are selected from those within the general index, and bonds that are deemed illiquid are usually removed. As seen from the points noted above, bonds below a certain level of issue size, credit rating and trading volume may also be excluded.

61.1.3 The “bell-weather” index

Many institutional investors will hold only the liquid benchmark bonds in their portfolios. For such investors an index that covers only the most liquid issues will be sufficient. For instance, in the US Treasury and the Japanese government bond market, a large volume of trading is conducted in the latest benchmark or “on-the-run” bond. To cater for this a *bell-weather* index can be used, which is essentially but not completely, the liquid current benchmarks from the government market. This leads to some inconsistencies in the comparison of index values, because over time the constituent bonds are always changing, which can effect on the continuity of the index values.

As well as turnover values, one of the measures of bond liquidity is the number of market makers that are prepared to make prices in the issue. This applies to non-government bonds such as Eurobonds. This may be a more suitable indicator of liquidity than the nominal size, turnover or bid-offer spread of a bond.

61.2 Maturity of an index

The time horizon of an index is of critical importance. This is because the evaluation of a portfolio manager against an index will be of little value if the index itself has a significantly longer or shorter maturity horizon than the portfolio itself. The investment horizon of portfolios differs according to the type of fund that they are. In any case the best practice is for indices and bond returns to be calculated for bonds that have very similar maturity profiles. The actual maturity measure that is used for individual bonds differs according to the type of bond that is being analysed. The measures used include:

- actual maturity term;
- average life;
- equivalent life;
- life to call, and life to put;
- modified duration;
- operative life.

These measures were all reviewed in Part I of this book, except for *operative life*. The operative life of a bond is the average life but adjusted for any option feature, on the assumption that both issuers and investors are rational entities. So for instance for a callable bond the traditional “yield-to-worst”¹ measure is used, which assumes that the bond will be called at its earliest call date if it is trading over par. In the case of a puttable bond, a “yield-to-best” calculation is performed.

In certain cases indices use the duration measure for the life of individual bonds. However the duration of a bond is a static value and fluctuates with changes in the bond’s yield, so it is not necessarily valid to use this measure. In addition the duration measure does not decrease in a smooth or continuous fashion from the current date to maturity, which would result in individual bonds moving from maturity bands in rapid jumps, or when there has been no change in yield. That said, the choice of which maturity measure to use when constructing indices lies in most cases between operative life or duration. The key points to bear in mind are:

- duration provides a more accurate measure of the period of the outstanding cash flows for the security than does operative life;
- as duration is defined, it is a snapshot of one point in time, and will change for all changes in a bond’s yield (note that this also occurs in operative life in the case of bonds with embedded option features). The effect of this is that a bond can move from one maturity sub-group into another, and then back again as yields moved in the market. This would clearly be unsatisfactory;
- the duration of a bond increases on each coupon date;
- in practice, market participants often simply group together bonds that have similar maturity dates!

¹ This is the Bloomberg LP term.

However if we wish to preserve analytical rigour in the index the best measure to use when grouping bonds together is operative life. The most common maturity segments used are 1–3 years, 3–5 years, 5–7 years, 7–10 years and over 10 years.

61.3 Responding to events

The composition of an index, based as it is on actual bonds, must make allowances for various events that occur in the markets. In this section we summarise the main treatment of events in the market.

61.3.1 Cash flows

Constituent bonds in an index, unless they are zero-coupon bonds, will pay periodic coupon during their life. All bonds will pay out a redemption payment on maturity. The general rule is that coupon receipts are not considered when the clean price of an index is calculated, although redemption cash flows are. An investor is faced with the problem of reinvestment of coupon cash flows. The index treatment varies; it may be any of the following:

- ignoring the cash flows completely;
- place cash flows into what is termed a “cash” stock that does not have a coupon;
- account fully for the cash flows when making the return calculation;
- reinvest the coupons in the index constituent in accordance with the constituent weights.

Not all these approaches are valid, and again to maintain analytical rigour index managers generally apply the first or last options.

61.3.2 Constituent bonds

Individual bonds may present problems for various reasons. For instance if there is no price available for a particular bond, it may be retained in the index and its last used price used until the new price is known. However this can only be for a short time. Other approaches include removing the bond from the index, either permanently, or temporarily until prices are available again. To maintain consistency, the most appropriate action is a combination of these approaches, that is, to use the previous price for up to five days, and then temporarily suspending the security from the index. It should not be re-entered into the index until the next time that all constituents are reviewed, to prevent distortions in index calculations.

If the issuer of an individual security is in default, a bond can be priced in the normal way, however it is much more likely that prices are no longer available. The last recorded price may well reflect market expectations of default, however the recovery value notwithstanding, the bonds have little worth and must be replaced in the index.

61.4 Composition of the index

The universe of individual bonds that may be selected in an index is generally defined as every single bond in that currency and category, all of which should have no exotic features attached to them. In general the criteria used in deciding which bonds to include are first that, the aim is to compare only similar instruments, so that yields on conventional bonds should not be compared to those on index-linked bonds, and secondly that future cash flows from constituent bonds should be measurable, and known by the calculating authorities.

The general approach that is followed is that:

- bonds with under one year to maturity are excluded;
- all conventional plain-vanilla bonds are included;
- zero-coupon bonds are sometimes included, or may be placed in a separate sub-index;
- irredeemable bonds are placed in a separate sub-index;
- floating-rate notes are excluded;
- index-linked bonds are excluded, and placed in a specialised index-linked index;
- callable bonds are excluded unless they form a significant proportion of the sector; the same applies to puttable bonds;
- bonds with sinking funds are excluded;

- convertible bonds are excluded;
- bonds with attached warrants are excluded;
- mortgage-backed bonds and other asset-backed bonds with uncertain cash flow patterns are excluded;
- dual-currency bonds are excluded, as their return includes an element of currency rate fluctuation;
- small size and illiquid bonds are excluded.

61.5 Calculation of index value²

Once the composition of the index has been agreed, it is possible to calculate the index value once yields are recorded on a daily basis. The primary calculation is the *clean price index*, given by (61.1):

$$PI_0 = 100$$

$$PI_t = PI_{t-1} \frac{\sum_i P_{i,t}^* \times N_{i,t-1}}{\sum_i P_{i,t-1} \times N_{i,t-1}} \quad (61.1)$$

where

$P_{i,t}^*$ is the clean price of the i th bond at time t
 N is the nominal value of the bond outstanding.

The gross price index is given by (61.2), where $AI_t = PI_{t-1} \frac{\sum_i A_{i,t} \times N_{i,t-1}}{\sum_i P_{i,t}^* \times N_{i,t-1}}$ and so

$$GI_t = PI_t \times (1 + AI_t) \quad (61.2)$$

where A is the accrued interest to the next settlement date.

For indices composed of bonds that do not have an ex-dividend period, such as Eurobonds, the *total return index* is used.

$$TR_0 = 100$$

$$TR_t = TR_{t-1} \frac{\sum_i (P_{i,t}^* + A_{i,t} + G_{i,t}) \times N_{i,t-1}}{\sum_i (P_{i,t-1} + A_{i,t-1}) \times N_{i,t-1}} \quad (61.3)$$

where $G_{i,t}$ is the value of any coupon payment received from the i th bond at time t or since time $(t - 1)$.

In a market where there is an ex-dividend period, the expression at (61.3) is modified as shown at (61.4).

$$TR_0 = 100$$

$$TR_t = TR_{t-1} \frac{\sum_i (P_{i,t}^* + A_{i,t} + CP_{i,t} + G_{i,t}) \times N_{i,t-1}}{\sum_i (P_{i,t-1} + A_{i,t-1} + CP_{i,t-1}) \times N_{i,t-1}} \quad (61.4)$$

where C and P are the coupon rate and price respectively.

The index calculations shown will produce continuous results during the calculation period and at the end of each period. Note that the average life of an index will by definition reduce gradually each calculation period so that for example, at the end of each month the index will be one month shorter than at the beginning of the month. The average life will also change more significantly if there is a change in constituents during the calculation period.

Earlier in this chapter we noted that certain investors have a requirement for composite indices, for instance a bond index that contains a maturity term that is different to the one computed at the previous calculation date. For

² This section borrows heavily (with permission) from Brown, P., *Constructing and Calculating Bond Indices*, Probus Books, 1994, Chapter 8.

instance, an investor may be using an index for a 3–5 year period and another for a 5–7 period, but now wishes to use a 3–7 year index. To calculate this we can use two approaches; the first involves calculating the required index in its own right, using the combined constituent bonds. This is problematic however as it involves a large amount of additional computations and in addition, the date may not be available. To avoid this, the second approach involves combining the two separate indices to approximate the value for the required index. This requires the two constituent indices to be weighted. To do this we can use:

$$S_{I,t} = \sum_{i \in I} N_{i,t} \quad (61.5)$$

$$W_{I,t} = \sum_{i \in I} P_{i,t} \times N_{i,t} \quad (61.6)$$

where

$S_{I,t}$ is the normal weight of the individual index I at t
 $W_{I,t}$ is the market weight of index I at time t .

The combined index value is then calculated using the expressions below, beginning with the clean price index.

$$PI_{1+2,0} = 100$$

$$PI_{1+2,t} = PI_{1+2,t-1} \frac{W_{1,t} \times \frac{PI_{1,t}}{PI_{1,t-1}} + W_{2,t} \times \frac{PI_{2,t}}{PI_{2,t-1}}}{W_{1,t} + W_{2,t}} \quad (61.7)$$

where

$PI_{g,t}$ is the clean price index of sub-group g at time t
 $W_{g,t}$ is the market weight of the sub-group g at time t .

The gross price index is given by (61.8) and the total return index by (61.9).

$$GI^{1+2,t} = PI_{1+2,t} \frac{W_{1,t} \times \frac{GI_{1,t}}{PI_{1,t}} + W_{2,t} \times \frac{GI_{2,t}}{PI_{2,t}}}{W_{1,t} + W_{2,t}}. \quad (61.8)$$

$$TR_{1+2,0} = 100$$

$$TR_{1+2,t} = TR_{1+2,t-1} \frac{W_{1,t} \times \frac{TR_{1,t}}{TR_{1,t-1}} + W_{2,t} \times \frac{TR_{2,t}}{TR_{2,t-1}}}{W_{1,t} + W_{2,t}}. \quad (61.9)$$

We have presented a brief overview of the main issues involved in constructing and calculating bond indices. In practice there will be a range of considerations and problems that may hinder calculations. It is important to remember that there is no one concrete rule that can be applied in every circumstance. Index calculation works most accurately if one included only homogeneous groups of bonds, to avoid distortions. Where there are only a very small number of bonds that are actually liquid and tradeable, again it will be difficult to obtain accurate results. In fact a meaningful index can only be produced for a market where there are a sufficient number of liquid bonds trading in the secondary market, and for which close market prices can be obtained.

Appendices

APPENDIX 61.1 Selected leading bond market indices

Index	Number of Issues	Maturity	Issue minimum size \$m	Weighting	Reinvestment Assumption
<i>US Investment grade indices</i>					
Lehman Brothers Aggregate	5000	Over one year	100	Market value	No
Merrill Lynch Composite	5000	Over one year	50	Market value	In certain bonds
Salomon Brothers Composite	5000	Over one year	50	Market value	No
Ryan Treasury	159	Over one year	All Treasury	Market value and equal	In certain bonds
<i>US High Yield indices</i>					
First Boston	423	All maturities	75	Market value	Yes
Lehman Brothers	624	Over one year	100	Market value	No
Merrill Lynch	735	Over one year	25	Market value	Yes
Salomon Brothers	299	Over seven years	50	Market value	Yes
<i>Global Government Bond Indices</i>					
Lehman Brothers	800	Over one year	200	Market value	Yes
Merrill Lynch	9736	Over one year	100	Market value	Yes
JP Morgan	445	Over one year	200	Market value	Yes
Salomon Brothers	525	Over one year	250	Market value	Yes, at local short-term rate

Source: Reilly, F., Wright, D., in Fabozzi (ed), *The Handbook Of Fixed Income Securities*, 5th edition, Dow-Jones Irwin 1997. Used with permission.

Selected bibliography and references

McClellan, S., *The European Bond Markets*, 5th edition, ISMA, 1992.

Ryan, R., *Yield Curve Dynamics*, Glenlake Publishing Company, 1997, Chapter 10.

Salomon Brothers, *The New Eurodollar Bond Index*, Salomon Brothers, 1994.

62 International Investment

The practice of investing across national borders is not new. There are a number of reasons why institutions may wish to invest in international markets, and these reasons are the subject of this chapter. This is a large subject and we do not pretend to cover even the fundamentals; this is merely an introduction to the main issues.

62.1 Arguments for investing in international bonds

The rationale for investing across national borders has a number of strands. A primary reason is diversification, although this is keenly debated. With the increasing integration of global debt markets, it may be that investing across borders offers no diversification of risk but exposes investors to political and currency risk. We will examine these issues later. First we discuss the attraction of overseas investment in terms of higher returns.

62.1.1 The rate of return argument

The rate of return in bond markets across the world differs markedly. This is the primary argument for investing across borders. If we consider the issues from the point of view of an investor in the United States, there have been periods when international bonds recorded higher returns than bonds in the US, while the opposite was true during other periods. For example during 1985–1987 international bonds outperformed US bonds, while the reverse was true during 1981–1984 and 1988–1989. However it is important to analyse the nature of the reasons behind such performance. During 1985–1987 the US dollar depreciated strongly, while 1981–1984 was marked by declining US interest rates. In considering why foreign bonds might outperform domestic bonds, the investor must assess prospects for movements in domestic interest rates versus overseas rates, and the prospects of the domestic currency versus foreign currencies.

A historical analysis helps to illustrate the rate of return argument. Table 62.1 shows rates of return from selected bond markets, using figures recorded by the Salomon Brothers¹ world bond market performance index, which reports figures relative to the US market. The table reports data for the period 1980–1997. Note that fluctuations in foreign exchange rates accounted for a large part of the difference in returns. However the figures cannot be taken to imply any future rate of return and there are specific, often unique reasons why returns over one time period are as they are.

	Total return %	Return vs US %	Components of return	
			Domestic return %	FX return (%)
Canada	9.8	–0.6	11.2	–1.2
France	11.2	0.7	11.4	–0.3
Germany	9.8	–0.6	7.5	2.1
Japan	13	2.6	7.8	4.8
Netherlands	10.6	0.1	8.5	1.9
United Kingdom	10.7	0.29	12	–1.2
United States	10.4	–	10.4	–
International index	11.8	1.4	9	2.5

Table 62.1: Rates of return in selected bond markets 1980–1997. Source: Citigroup.

The foreign currency element of international investing is very significant, as it is an important component of the volatility of overseas bond returns.

¹ The index is calculated and reported by Salomon Smith Barney, part of Travelers Citigroup.

62.1.2 Diversification

The other key factor behind the rationale for international investing is that of diversification. In theory the inclusion of foreign bonds in a portfolio should lead to a reduction of the variability of returns, compared to a portfolio that is invested solely in domestic bonds. This is because foreign bonds do not move exactly in line with any selected domestic market, that is the returns are not perfectly positively correlated. This is a difficult argument to refute, if only because at any one time different countries will occupy different stages of the business cycle. There are also structural factors such as the make-up of a particular bond market and other issues such as government regulations and taxes that will produce returns that are not perfectly positively correlated. Therefore investing part of a portfolio in overseas securities will reduce the volatility of the portfolio's returns. Table 62.2 shows the correlation coefficients of monthly changes in total return for selected bond markets during 1980–1997. There are some interesting points to note about the figures. For instance the correlation between returns in the US and Canada is very high, which reflects the state of integration between the economies of the two countries. The lowest level of correlation is between certain countries in Europe and Japan, while the correlation between countries of the European Union is also relatively high.

	US	Canada	France	Germany	Japan	Netherlands	United Kingdom
United States	1.00						
Canada	0.71	1.00					
France	0.34	0.38	1.00				
Germany	0.50	0.50	0.61	1.00			
Japan	0.38	0.32	0.31	0.52	1.00		
Netherlands	0.52	0.47	0.54	0.88	0.44	1.00	
United Kingdom	0.39	0.38	0.37	0.48	0.32	0.43	1.00

Table 62.2: Bond market returns, coefficients of correlation. Source: Citigroup.

It is expected that correlation levels will converge as the global economy becomes more integrated, and as inflation levels and hence interest rates also begin to decline across countries. The introduction of the euro should result in bond returns in the 11 countries of euroland recording virtually perfectly correlated bond market returns in due course. However the established macroeconomic paradigm that is accepted as essentially global orthodoxy, namely the granting of independence to central banks, and the targeting of inflation, should also hasten the time when bond market returns are more closely correlated.² This will erode the diversification argument if applied to developed solely country markets.

62.2 International portfolio management

The same principles of portfolio management apply when the portfolio is invested across markets, although there are additional practical considerations because market structures differ. In general fund managers are restricted in the extent to which they can invest in foreign markets, and therefore specific foreign market bond funds are usually set up to cater for investors who wish to have exposure to different markets. The international fund manager is also usually required to follow a set of guidelines when deciding where and in what instruments investments should be made. Guidelines usually stipulate that an external benchmark be used against which the fund is compared. There are a number of global indices and benchmarks available for fund managers. Often investors may set a simple target of outperforming the domestic benchmark, however international benchmarks are usually selected in order that both performance and risk exposure can be measured. Table 62.3 shows the composition of two popular global government bond indices, the Salomon Brothers index and the JP Morgan global government bond index, during 1999. There are some detail differences between the two indices, for example the allocation of the Japan sector. The Salomon index is market-weighted, which means that it is set and calculated on the basis of outstanding nominal value in each market; the JP Morgan index uses only actively-traded bonds within each of the markets.

² For instance, central banks in countries including Australia, Canada, New Zealand, Sweden and the United Kingdom have adopted inflation targets, while the European Central Bank target inflation as one of a range of economic indicators.

Country	JP Morgan index		Salomon Brothers	
	Weight %	Duration	Weight %	Duration
Japan	21.3	6.3	28.2	5.8
Germany	18.1	4.7	14.4	4.7
France	11.9	5.1	9.4	5.2
United Kingdom	12.6	6.2	8.3	6.4
Italy	5.4	3.8	7.8	3.1
Holland	7.1	5.4	6.5	6.1
Belgium	4.9	4.7	4.1	5.4
Spain	4.7	3.9	3.5	3.3
Canada	5.4	5.8	6.7	5.7
Sweden	3.1	3.9	3.1	4.2
Denmark	2.9	5.1	3.2	4.8
Australia	2.6	4.8	2.8	4.8
Austria	–	–	2.0	3.9

Table 62.3: Global government bond indices (excluding United States), 1999.
Source: www.bondresearch.com.

In addition to selecting a benchmark, fund guidelines usually explicitly state which markets and what instruments can be invested in, and what credit ratings can be held, as well as describing the rules regarding hedging, use of derivatives, foreign currency hedging, duration limits, and other related factors.

An actively managed international portfolio is usually run under strict guidelines relating to which sectors can be allocated an amount over or under the normal sector allocation. Guidelines are frequently set in terms of the four currency “blocs”; these are:

- the dollar bloc, which is the US, Canada, Australia and New Zealand;
- the European Union bloc, which is subdivided into the euro bloc and sterling;
- Japan;
- Emerging markets.

By focusing on blocs like this, investors focus on yield spreads and price movements relative to other blocs, rather than amongst individual countries. Since generally the bond markets within each bloc usually trade in line with each other, one approach that a portfolio manager can use when seeking to outperform an index is to allocate assets to selected markets within a bloc without breaking any rules about the overall bloc allocation.

Selected bibliography and references

Steward, C., Greshin, A., “International bond investing and portfolio management”, in Fabozzi, F. (ed.), *The Handbook of Fixed Income Securities*, 5th edition, McGraw-Hill, 1997.

Part X Technical Analysis

In this part, which comprises just the single chapter, we introduce the fundamentals of technical analysis. This is by no means a complete treatment, rather a general introduction to the subject. The chapter bibliography lists starting points for further study.

63 Technical Analysis

Participants in the capital and money markets have a wide range of information to track, from a variety of sources. This entails monitoring economic data, currency data, market data and market indices across different countries. There are essentially two main schools of thought on how to analyse data, which are fundamental analysis and technical analysis or *charting*. Fundamental analysis is concerned broadly with reviewing the macro factors that are believed to affect markets and asset prices. A fundamental analysis of the virtues of investing in the UK gilt market will assess among other things, the economic fundamentals of the UK economy, including size of public sector debt and budget deficit, inflation and interest rate levels, and forecasts of these for the future. The investor would decide whether it was worth buying gilts partly as a result of their assessment of the prospects for the UK economy. Technical analysis seeks to analyse and interpret price movements themselves. Traders and investors often combine analysis from both schools of thought when making their buy or sell decision. In this chapter we review the fundamentals of technical analysis and its application to the markets. Note that the principals remain the same irrespective of what market one is trading in, whether it is bonds, equities, indices or a combination of these and other instruments. Therefore when we consider the main points we will review price patterns from a range of assets.

Technical analysis is now widely used in the futures markets, and since futures prices influence cash market prices, they are also important in the cash markets as well. Traders review daily, hourly and even five-minute charts, although they will always bear the long-term charts in mind as well. Although there is no one way to analyse a chart, in very simple terms the rationale is straightforward, in the belief that if a trader knows that selling has occurred before at a particular level, they are unlikely to buy there and that point then becomes a *resistance* level. In the same way if a particular level has proved to be good buying point in the past, traders will probably buy at that level again, and this point then becomes a *support* level. In essence then price charts an illustration of market psychology. The view of many traders however is that charting is most useful when they give the same indications as fundamental analysis, so that the levels can then be used to provide market buy and sell points. This is why for example, a country's government bond yields will fall in response to healthy economic indicators, and are illustrated in charts as well as being indicated by fundamental analysis. The key point then is that while it is a useful exercise to study and understand technical analysis, in most cases it should be used in conjunction with other indicators such as fundamental analysis. There is also the investment horizon to consider, as "day traders" may rely more on charting than longer-term investors or relative value traders.¹

63.1 Introduction

All technical analysis is subject to the user's interpretation. Technical analysts or chartists look for recurring patterns in the graphs of price movements. Some users of technical analysis claim they can call future price movements on past price patterns alone, even if they are unaware of the underlying asset! A wide range of graphs are used in technical analysis. These include:

- line graphs;
- bar graphs;
- candlestick graphs;
- point and figure charts;
- market profiles;
- moving averages;
- "oscillators", such as relative strength indicators, stochastics, and moving average convergence and divergence indicators.

¹ Although we have stated that market participants often combine elements of technical and fundamental analysts, true chartists maintain that in order for it to be effective, technical analysis should be used on its own.

The above is only a sample. Given the large number of charting techniques available, it is probably not possible or even desirable to use every technique. Some chartists prefer to stick to the simplest methods such as monitoring trendlines, support and resistance levels, and combine this with fundamental analysis and their own intuition. It is a matter of personal choice.

At the risk of being accused of stating the obvious, we shall begin with the following beliefs of achieving profitable trading: first, to achieve profit one must buy low and sell high (or sell high and then buy low), and within this one may buy “breakouts” and sell “breakdowns”. Both points imply that the market concerned is in a trading range, therefore there must be some movement in the market one is trading in order to make profits. This sounds (and indeed, is) obvious however it is worth bearing in mind that trading in a static or flat market will not generate great profit. Buying a breakout implies that once the market level moves beyond old high points, prices will move higher still, so that one takes profit at higher prices. The reverse is true when selling a breakdown. However there must be an element of market volatility to facilitate this.

63.1.1 Basic principles of technical analysis

Consider below the basic principles behind charting, which the technical analyst will believe about any developed market.

- **Market price discounts all information.** This means that everything that is known about a market at any given time is reflected in its price. Therefore all present information or rumour about political, economic, fundamental, psychological or other factor relevant to the price of an asset is already discounted. If any new information comes out, the market will *immediately* factor it into the price, after which the information will then no longer be relevant to the process of forecasting. This is because at any time, market price action discounts all known information.²
- **History repeats itself.** Technical analysts believe that the markets are composed of people whose collective influence affects prices and direction. The psychological nature of the financial markets means that, like history generally, the future is a repetition of the past.
- **Market prices moves in trends, which persist over a period of time.** When a price moves in a particular direction, whether up or down, it will continue to trend. The quote made most frequently to junior traders is “The trend is your friend,” or “Don’t fight the market.” This reflects the fact that markets move in a trend, and it is not recommended to trade against a trend, as the odds are you will suffer a loss.

The techniques and tools discussed in this chapter can be applied to both day-trading, short-term trading and longer term trading. Many of the tools were originally developed for application to trade durations of longer than a day. However developing technology allowed their use for intra-day trading. For long-term charts traders commonly use bar charts, and combine daily bar charts with weekly and monthly charts for longer term trend analysis.

63.1.2 The trade decision

Again we may be accused of stating the obvious, but it is important to begin from first principles. As technical analysis is often associated with short-term trading, the act of formulating one’s ideas and then executing them requires discipline. The trade decision must be made and executed with discipline.

Speaking coldly, there are three parts to trade. These are the trade decision, the trade execution and trade management. The trade decision is the part of the trade where the trader studies the fundamental and technical factors behind a market or markets. Once these factors are understood, or believed understood, the trader must decide on whether to buy, sell, or stay out of the market.³ Once the decision has been taken on a trade, the trader must consider *market entry* and then *market exit*. Market entry decisions usually require more analysis because there will be a number of factors that will influence whether to enter the markets. One may want to be long or short of a market due to fundamental or technical reasons or a combination of both. The decision to exit a market probably requires less analysis if the reason for exit is that one is experiencing a loss. If a position is losing money, it

² This is a reiteration of Fama’s semi-strong form market structure, which we reviewed in the preface. A strong-form market would also discount information that was yet to be released.

³ Also referred to as “sitting on your hands” (!).

will have to be cut sooner or later, so the level of analysis is low. However the decision on market exit when one wants to take a profit calls for some level of analysis as well; often the timing on unwinding a profitable trade is a crucial one.

There is of course no correlation between the time spent on the trade decision and level of profit. The decision-making part of the trade does not create profit. It is important however because hopefully it will assist the trader in entering the market on the right side. The third element of the trade decision, trade management, is the key to success or failure of a trade. In the author's experience, many traders excel at the mismanagement of a trade. It is a combination of analysis, intuition and experience that should see consistent performance over time. There is also an element of luck required, which a book cannot supply. Let us begin though with issues that *can* be explained in a book.

63.2 Trading market profile

63.2.1 Assumption of normality

Before we look at chart construction and the principles of technical analysis, we should review some assumptions of the pattern of market prices. This is a very important topic and assumptions on how market prices behave form a cornerstone of the mathematics behind say, option pricing and yield curve modelling. For example finance theory makes great use of the concept of stochastic processes, which describe random phenomena that evolve over time. This includes asset prices and interest prices, as well returns achieved in the market. We need not cover this here, but it is worth discussing the assumptions behind market prices.

The principle behind much of technical analysis is that frequently-occurring real world events follow a *normal distribution*. This distribution was introduced in Chapter 37. Essentially as a large number of frequent occurrences, such as the height of people, follows a normal distribution, we may assume that market prices will also follow a normal distribution. This assumption is behind much past and current finance theory, although as Chapter 37 showed, it is not strictly correct. Nevertheless we can illustrate the principle with some hypothetical examples. Consider Figure 63.1 which purports to show market prices following a certain pattern.

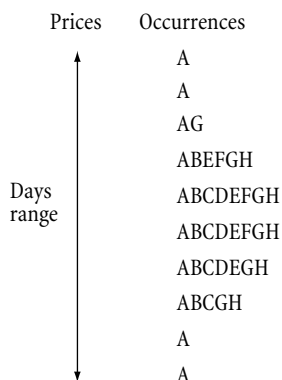


Figure 63.1: Normal day price profile.

A trader who assumed that the normal distribution of prices will be observed after a specified period of time will plot a chart of prices to date and then trade accordingly. Let us assume that the trader specified the time period as one day. Consider a range of prices plotted after the first half of the day as shown in Figure 63.2.

If market prices formed the pattern shown in Figure 63.2 in the first half of the trading day, if we believe returns follow the normal distribution we will expect the remainder of the day's prices to complete the pattern, as shown in Figure 63.3, and trade accordingly. An actual plot of prices over any length of time will not follow a pure normal distribution however, and the analysis we have described does not consider trading volumes, which indicate the depth of a market. We cannot use an assumption of normality to formulate trading strategy therefore, especially as the predictive capability of assuming so is only valid if a perfect curve is in the process of being formed. This is rare, and more so when a market is going against a previous trend or moving outside a range. We shall look at these aspects next.

Price	
114.10	
114.20	X
114.30	XXX
114.40	XXXX
114.50	XXXXXX
114.60	XXXXXXXX
114.70	
114.80	
114.90	
115.00	
115.10	

Figure 63.2: A hypothetical first half of a trading day.

Price	
114.10	
114.20	X
114.30	XXX
114.40	XXXX
114.50	XXXXXX
114.60	XXXXXXXX
114.70	OOOOOO
114.80	OOOO
114.90	OOO
115.00	O
115.10	

Figure 63.3: A normal price distribution.

63.3 Dow theory

One of the earliest proponents of the art of technical trading was Charles Dow, who was editor of *The Wall Street Journal* in the late 19th century. The six basic principles of Dow theory are:

- the averages discount everything;
- the market has three trends, primary, secondary and minor;
- major moves in trend occur in three basic phases;
- the average must confirm each other;
- trading volumes must confirm any trend;
- any trend is assumed to be intact until a clear signal is indicated to show it that it has been reversed.

Let us examine these points in more detail.

- **The average discounts everything.** This is related to our earlier principle that the current price of an asset discounts all information. We can apply this to a stock market index: In Dow's original theory the Industrial average and the Transportation indices (we could now apply this to the index level for any market, whether FT-SE or S&P500 or any such index) reflects the activities of all market participants, this means that all factors that have effected the price of any market stock have been discounted, or built into the price of that stock.
- **The market has three trends.** *Primary* trends represent the overall direction of the market, for example over a time period of over one year. *Secondary* trends are a temporary interruption of the primary trend and are viewed as market corrections. These generally occur over a time span of between three weeks and three months. *Minor* trends reflect short-term fluctuations within a secondary trend. These can be in the direction of either the secondary or the primary trend and last from one day to three weeks.
- **Major trend moves occur in three phases.** In Dow's theory, in a move out of a bear market the first move up is caused by far-sighted investors accumulating stock while the majority are still selling stock, believing the market is headed further downwards. The media and market analysts will echo the negative sentiment. The second move arrives in the form of price acceleration out of a base level when professionals buy the market after a trend has emerged. Finally the third move is caused by speculative buying after the previous upward move. This third move can sometimes be the final move of the bull market. In a downward move out of a bull market, the sentiments are reversed.
- **Signal of a new trend is confirmed by the averages.** As implied by the original theory, the Dow Jones Industrial average and the Dow Jones Transportation average must confirm each other by both rising above or falling below a secondary market reaction. If one of the indices does not, there is no confirmation of a new trend in the market. In this respect, Dow theory can be applied to any market, for example an indication of a rise in the

UK gilt market cannot be confirmed if yield spreads narrow against one country's equivalent bond market but widen against another. For example Figure 63.4 shows the gilt yield spread for the ten-year benchmark narrowed against three different ten-year benchmark bonds. This suggested at the time that a trader could indeed confirm a broad gilt market rally, although if a comparison against other markets had shown a widening spread this would not be possible.

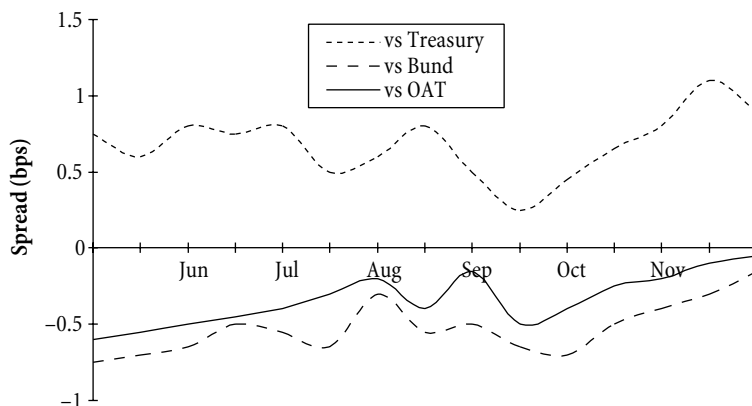


Figure 63.4: Ten-year gilt spread against ten-year Treasury, Bund and OAT during 1999.
Yield source: Bloomberg.

- **Trading volumes must confirm the trend.** The theory suggest that it is reasonable for trading volume to expand in the primary direction of the market trend and decrease in the corrective phase. This would indicate that the weight of volume was enforcing the general trend. If it does not, the trend cannot be confirmed.
- **The trend is assumed to be in effect until a definite signal otherwise is indicated.** This is stating in effect that we do not call the end of trend simply because we feel it has “done enough” or is too high or low.

The principles behind Dow theory can be applied to analysis using charts. Let us now consider the basics of chart construction.

63.4 Chart construction

There is a large literature on technical analysis and using chart patterns. Charts were described in detail initially by Gann (1940) and by Edwards and Magee (1948). These authors characterised charts as one of three types: *reversal* patterns, *congestion* patterns and the patterns that connected these two, known as *trend* patterns. The charts were applied to short-term trading periods on the assumption that patterns over a very long time were replicated in shorter time frames. Within a trading range one may observe *consolidation* patterns.

The primary tool in technical analysis is the graph or *chart*. There are a wide range of charts available. Let us begin by describing the most commonly occurring ones.

A *line graph* is, at its name suggests, a continuous line which connects the prices for each time interval during the period covered by the chart. The user specifies which price is recorded, whether bid, offer or mid-price. Often however price at which the last traded took place is recorded. The time intervals can be minutes, days, months, quarters or years, and the period graphed can range from intra-day to several years. The time intervals are more recent as you move from left to right across the horizontal or *x*-axis, and prices are recorded on the vertical or *y*-axis. Whereas a line graph has one price for each time interval with a line joining those from different intervals, a *bar graph* has several prices per interval which are connected by a vertical bar. The top of the bar is the highest price recorded in the interval and the bottom of the bar is the lowest price in that interval. A line to the left of the bar shows where the price was at the start of the time interval and a line to the right is where the price stood at the end of the interval.

A line chart is straightforward and is illustrated at Figure 63.5.

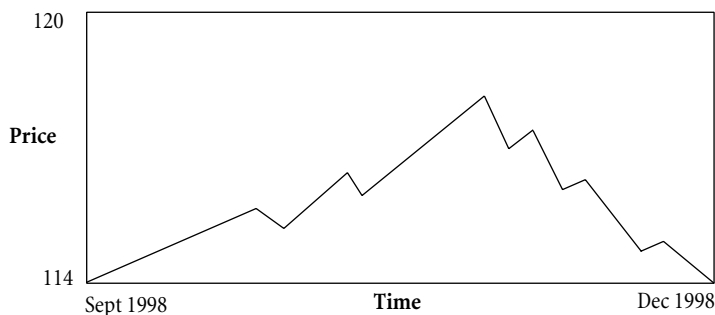


Figure 63.5: Closing prices line chart.

Figure 63.6 is an illustration of a bar chart.

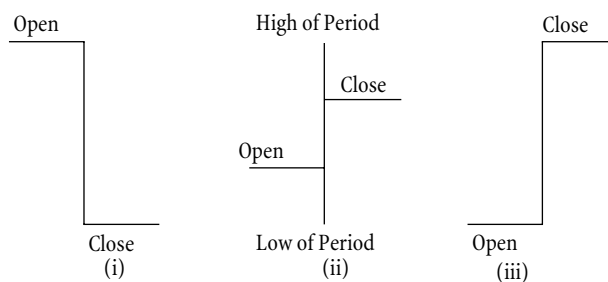


Figure 63.6: Bar chart.

The first diagram (i) in Figure 63.6 shows a market closing on the low. This is typical of a bearish market. Diagram (ii) shows a market that has experienced little change on the day, which indicated a *range-bound* market or a pause in the trend. Diagram (iii) shows a market closing on the high, which indicates a positive or bull market.

A *point and figure* chart is different to the two graphs described above in that it does not have an explicit, linear time axis. There is therefore no time frame but what is known as a *reversal box*. The chart is composed of alternating “O”s and “X”s. The Os and Xs (or boxes) each represent a unit of price, called the box size. Falling prices are represented by columns of Os, while rising prices are indicated by columns of Xs.

1.57							
1.56							X
1.55	X						X
1.54	X	O	X				X
1.53	X	O	X	O	X		X
1.52	X	O	X	O	X	O	X
1.51	X	O	X	O	X	O	X
1.50	X	O	X	O	X	O	X
1.49	X	O	X	O	X	O	X
1.48	X	O	X	O	X	O	X
1.47		O		O	X	O	X
1.46				O		O	

X = Up move

O = Down Move

Figure 63.7: Point and figure chart.

Only whole numbers of boxes are filled, fractions of a box are ignored. The number of boxes that the price must retrace in order to trigger a new column to the right of the current position is called the reversal criterion. The price

stays in the same column until a large enough reversal occurs. Point and figure charts greatly compress the price data and are thought to provide more precise buy and sell signals. These signals should then be observed more easily than those in other types of charts. An illustration of a point-and-figure chart is given at Figure 63.7, above.

Another popular charting technique is *candlestick graphs*. These were originally developed in commodity markets in Japan and are similar to bar charts. The main difference is that the bar is filled in to form a box or “candlestick” and the shade of the candlestick will depend on whether the closing price is above or below the open. A close at the same level as the open is marked by the lack of a candlestick. The chart is illustrated at Figure 63.8.

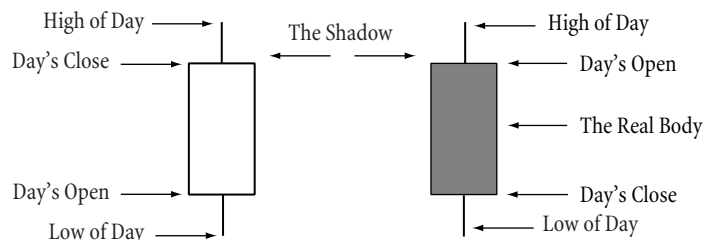


Figure 63.8: Japanese candlestick chart.

Many charts can also be configured to track volume of trading and level of *open interest*. This is the number of contracts (in a futures market) that are run as overnight positions rather than closed out before the close of business. The level of open interest can be used to gauge the true hedging requirement in a market for example, as well as indicate more long-term trades such as spread positions. We will consider volume and open interest later in the chapter.

63.5 Trend analysis

63.5.1 Definition

The starting point for charting is trend analysis. In the first instance the technical analyst is attempting to detect a trend in the price action. In a bull market the price action moves upward in a series of rising, roughly jagged shapes, while in a bear market the price action is moving in a series of declining jagged lines. Peaks and troughs in the graph are discernible. When the peaks and troughs are within a defined price range, the market is said to be *congested* or in a trading *range*. Many of the charting tools used attempt to detect and display the price trend. Figure 63.9 illustrates the two trends and the range.

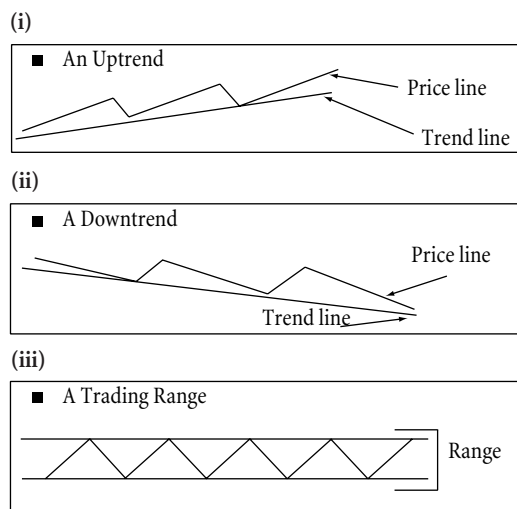


Figure 63.9: Trading trends: (i) up trend, (ii) down trend, (iii) trading range.

An upward trend line (or uptrend line) is a straight line drawn from left to right, underneath a minimum of two lowest prices in the period; the second low price must be higher than the first. A downward trend line (or downtrend line) is the opposite of this. The more points there are in a trend line, the more significant it is, and these points enable the trader to identify an area to enter or re-join a trend. The time period represented by the trend line is significant as well, for example a one-year trend line carries more weight than a one-week trend line.

Generally analysts classify trends into three different types. These are *major*, which span a period of six months or more, *intermediate*, for periods between three weeks and six months, and *minor*, which are from two to three weeks. Shorter length trends are by definition part of a longer-term trend and can continue for almost indefinite periods, so that we could chart a very long-dated trend over the last eighty or ninety years, or even longer. A commonly observed long-term trend that is a graph of inflation levels or equity yields versus bond yields.

There are also three time horizons normally associated with trends. These are *short*, which is from one minute to two weeks, *medium*, which is from three weeks to three months and *long term*, which is from six months to 10 years. The time horizon is specified by the user to suit their requirements. For example option market makers usually use a medium horizon, while fund managers would use more usually a long time horizon.

In an upward trend, rising peaks and troughs in the price line are used to mark points in the price range known as *support* and *resistance*. The point in the price range where buyers entered the market in greater numbers than sellers is known as the *support* level. It is an actual price level at which the downward move in the asset price is expected to be halted for a certain length of time. Equally in a downward trend the falling price peaks and troughs define the areas of support and resistance, with the area where sellers are expected to outnumber buyers known as the *resistance* level.

Figures 63.10 illustrates support and resistance levels in a hypothetical market.

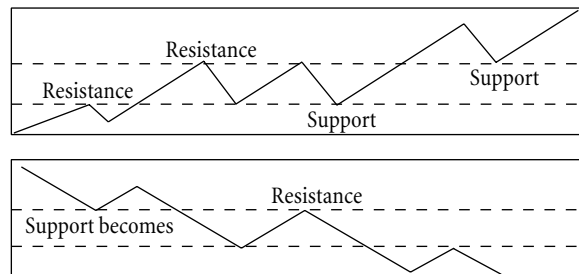


Figure 63.10: Support and resistance levels.

63.5.2 Trends and trading ranges

The point at which a market moves out of a congestion zone or trading range is significant and it is important for the chartist to identify this. The level of significance is a function of the following:

- the longer the market has traded in a range between support and resistance levels the more significant the trading range itself becomes, because participants have become used to the range while the market has stayed within it, and are generally caught off their guard when the market does break out;
- the volume of trading as the market breaks out of a range. This is important because volumes expand as a level is taken out, so an increase in volume levels can help confirm a break out of the trading range;
- the time at which any break out occurred. A very recent break may not confirm a move out of a range, as often a price will be recorded outside a range only for the market to fall back into the congestion zone;
- the distance away from the range that prices have moved. The further the market moves away from a support or resistance level, the more participants will believe that the break out is valid.

When a market breaks a support or resistance level, providing that it is a valid break, the move will be a rapid one, that is, the market should not hesitate. If prices do not continue in the direction of the break, participants may well pause before entering the market, as the break has not been confirmed. A psychological point concerning support and resistance levels is the effect of round numbers. The point at which an asset price is a whole number

(that is, no cents or pence) is often used as entry or exit points by junior traders. For this reason experienced traders often force a move through a round number price, which is why in practice round numbers do not often represent effective support or resistance levels. Some observers suggested that an exception to this was the recent example of the performance of the euro against the dollar in the months following its introduction in January 1999. Following initial weakness in the currency markets, there was a psychological weakness barrier at euro parity to the US dollar, that is a €/ \$ exchange rate of 1.00. Although the euro approached parity it did not break through. However although it later stabilised, the euro remained in a range slightly above \$1.00, and was at \$1.03 in November 1999; technical analysts were again suggesting that the euro would break through this level, at which point there would be a substantial move below \$1.00. Therefore stops at this level were not being recommended.⁴

63.5.3 Breaking through trend lines

A general rule on trend lines is that major lines must stay below the bulk of the price action. An exception to this is when the market makes a “wild gap” on very thin volume and then recovers back and over the original trend line in a very short time. In this case the trend line is deemed to be intact and the preceding price action is ignored. Apart from this, there are no rigid rules regarding trend line violation.

The *angle* of a trend line is a key feature to any trend that is approaching a mature stage. The general rule is that the trend line is at its steepest angle as a market is approaching the end of a bull or bear run, and shortly to change direction. This is an important characteristic because generally the fundamental and prevalent market sentiment towards the end of a trend is either very bullish in an upward trend or very bearish in a downward trend.

One of the earliest exponents of technical analysis was W. D. Gann,⁵ who favoured a relative trend line angle of 45 degrees. His name is preserved in the use of the term “Gann line” in charting. Generally a trend line that is at an angle steeper than 45 degrees is unsustainable in the long term and a correction towards 45 degrees is to be expected. A break out of a trading range is often seen at a steeper angle than 45 degrees as large numbers of traders enter the market and follow the market. Once this price action has reverted to normal trading levels the Gann line will approach the conventional steepness. The final point to note on trend line angles is that as a market enters the final phase of a bull or bear market, we will often observe near-panic buying or selling, which represents the “last gasp” of the market (this stage may take several months). The ending of the trend is often viewed as a steep angle, with the subsequent reversal also seen as a steep angle trend line. This also implies that the strongest angle of a trend is very often at the end of the trend.

The *fan principle* is one technique used to assess the approaching end of a trend. It is based on three trend lines; when one line is broken, a second line is drawn at a shallower angle. When the second line is broken, a third line is drawn, again at a shallower angle. If this angle is subsequently broken, this is an indication that the market is shortly about to experience a complete reversal in its trend. Figure 63.11 illustrates the principle.

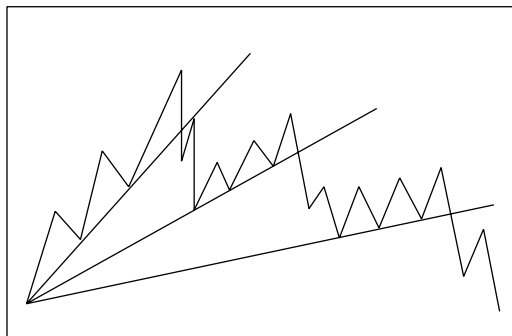


Figure 63.11: The Fan Principle.

Analysts often draw a line that is parallel to the trend line, with the intention of enclosing the price action within the two lines. The two lines together are known as a *trend channel*. The parallel line is called the *return line*. A trend

⁴ True to prediction, once the euro broke parity it traded as low as 83 cents.

⁵ Gann's seminal work is noted in the bibliography.

channel is a useful tool for intra-day traders, who then trade against the return line. Trading in the range marked by the trend channel is often for small gains in prices, say two or three *ticks*, often also called *scalping* the market. This requires instantaneous decision-making and trade execution, which is typically best suited to a *local* on an exchange floor. The tactic is risky as it sometimes requires trading in the opposite direction to the trend, which is generally not advisable. Note that a failure to reach the return line is sometimes interpreted as a signal that the market is about to experience a reversal (which adds to the uncertainty!).

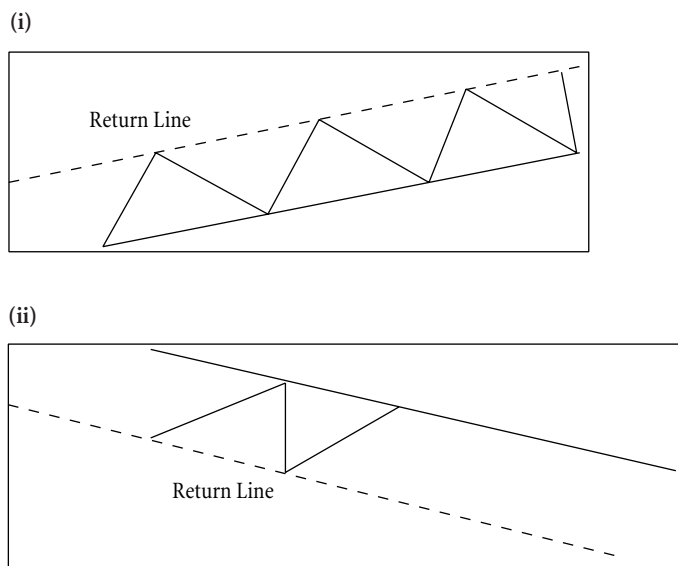


Figure 63.12: Trend channels: (i) upward trend, (ii) downward trend.

63.5.4 Retracement lines

Trend lines are often used in conjunction with *retracement lines* to indicate trading ranges. A retracement line marks the point where the price action has moved back from a point that is a specified range away from a certain trend line. The markets generally use conventional retracement ranges. The first important retracement level is 33% away from the prior move, and is known as the *minimum retracement* level. The next retracement level is 50%; this is identified by Gann theory, Elliot, Dow and others. (We consider these other theories later in this chapter.) The maximum retracement level is at 66% and a break of this level is an indication that the market will experience a complete 100% retracement of the prior move. The point at which the market reaches 100% retracement is often important as well as it then becomes a support or resistance level, depending on the direction of the retracement.

The key retracement levels are shown in Figure 63.14 together with an example from the UK gilt futures contract on LIFFE, with retracement levels indicated, which is an example of a *Fibonacci* graph.

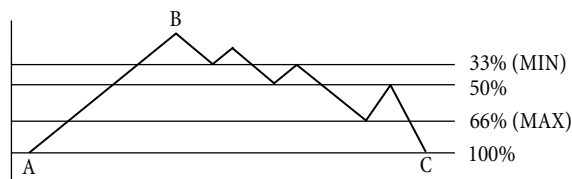


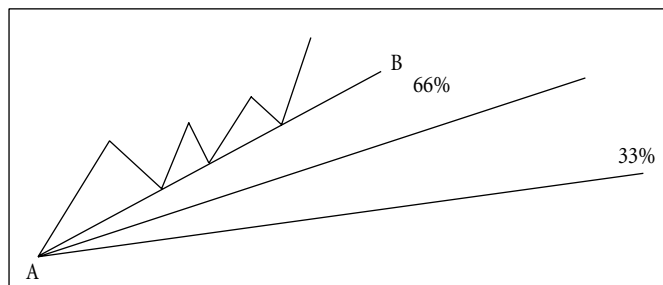
Figure 63.13: Key retracement levels.



Figure 63.14: Gilt future Fibonacci graph.

©Bloomberg L.P. Used with permission

After the retracement levels have been established, the analyst draws *speed lines* to identify areas of possible support or resistance. The lines are based on the percentage retracements of the previous trend, usually 33% or 66%.



Line A–B is used to calculate the speedlines.

Figure 63.15: Generating a speed line.

63.5.5 Price gaps

A *price gap* or simply *gap* occurs when the market opens sharply higher or lower than the previous day's closing price, and therefore misses the price range indicated by the closing price and the new opening price. This is indicated by a gap between the continuation line on a bar or a line chart. A market needs to be closed for a period of time (say overnight or over a weekend or public holiday) for there to be a gap. The rule is that a market will always close the gap at some point, meaning that a sharply higher opening would eventually give way to a downward price move until the gap with the (lower) previous closing price was closed.⁶ The period during which a trader waits for the gap to be closed is usually one of high volatility.

There are three main types of gap, detailed below.

- **Breakaway gap.** This gap signals the beginning of a new trend, normally within the completion of a major *reversal* pattern. For this reason, breakaway gaps are not always filled.

⁶ In the author's experience, this rule is indeed followed.

- **Runaway or measuring gap.** This gap is where the market is in an established trend and the gap moves in the directions of the trend. It is always filled. It also has predictive value because it often occurs at the 50% level within the trend.
- **Exhaustion gap.** This appears near the end of a trend. It is always filled. It represents a market that is “exhausted” as the price quickly falls back against the previous trend and into the gap.

A diagrammatic representation of the different gaps is shown at Figure 63.16, with an actual representation shown at Figure 63.17.

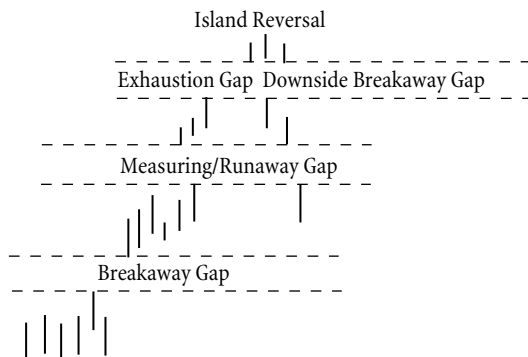


Figure 63.16: Examples of price gaps.

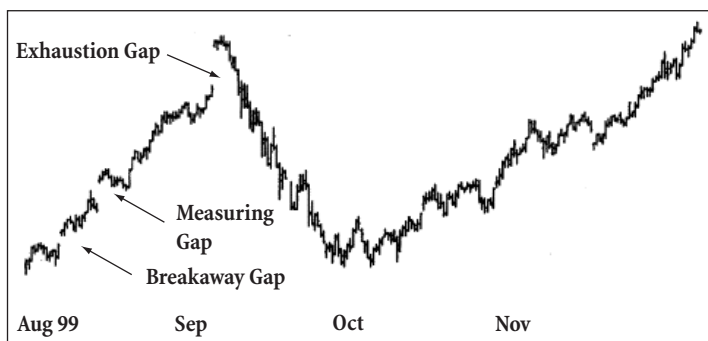


Figure 63.17: Illustration of price gaps. Long gilt future. Price source: Reuters.

It is important to look at the “big picture” by using as long-term a chart as possible. Key support and resistance levels are more apparent over a long term. Although day traders often base ideas on price gaps, it is worth remembering that although the market will always fill a gap, there is no indication when this will happen, and certainly no guarantee that it will take place on the same day.

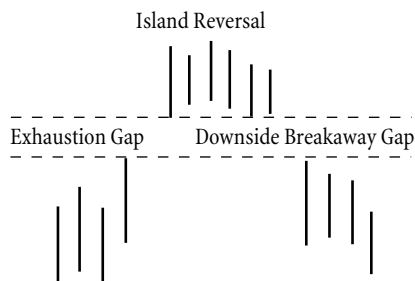


Figure 63.18: The island reversal.

Price gaps are used in conjunction with *island reversals*. An island reversal occurs at the top or the bottom of a market. The first part of the formation of an island reversal is an exhaustion gap followed by a period of “sideways” price action (so-called because the market is going nowhere). The market then experiences a gap in the opposite direction of the previous trend, which would be a (downside) breakaway gap. The resulting chart pattern appears detached from all the other price action. The island reversal is illustrated diagrammatically at Figure 63.18.

63.6 Reversal patterns

A reverse in a current trend occurs over a period of time, which can range from one day to several weeks, and this transition period is usually illustrated by a *reversal pattern*. Traders try to identify reversal patterns because they have predictive value. Certain patterns are observed more frequently than others and therefore are more well known. The types of reversal patterns found are listed below.

1. Head-and-shoulders, tops and bottoms.
2. Rounded tops or bottoms.
3. Double tops and bottoms.
4. Triple tops and bottoms.
5. Broadening formations.
6. Triangles tops and bottoms.

There is a seventh reversal pattern known as a *diamond formation*. These are quite rare to observe however, and even after considerable analysis, a pattern may not be seen. However attempting to find a diamond reversal often results in the discovery of another reversal formation first.

When using reversal patterns the analyst should remember some basic principles. First, a prior trend must have been observed. Any pattern formed is more significant if it is larger and formed over a longer period of time. Finally, if price action results in a long period of time being spent around the tops or the bottoms of the pattern, the subsequent reaction in the opposite direction will be greater.

63.6.1 The head-and-shoulders reversal pattern

As well as occurring quite frequently the head-and-shoulders pattern is also one of the most well known. It is comprised of:

- the *left shoulder*: this represents the continuation of the trend in heavy trading volume, with a subsequent fall back to the reaction low level. At this point there is no indication that the market is going to break out of the trend;
- the *head*: amid continuing heavy trading volume the trend line is observed to be at an acute angle. The previous level is taken out. Tops of heads are often accompanied by a fill of an exhaustion gap and subsequent breakaway gap on the downside. On the move down usually a lower volume of trading is observed;
- the *right shoulder*: this is formed as the market finds some support close to the reaction low of the left shoulder, before rallying in thin trade. The time span between the formation of the left and right shoulders is important, a ratio where the time taken for the left shoulder to form is higher confirms the head-and-shoulder pattern. A line drawn parallel from the left shoulder to the neckline can act as a resistance level to the formation of the right shoulder;
- the *neckline*: this is the trigger point. Traders often enter the market on a clear close below the neckline, although it is also common to see a return to the neckline after the initial move away from it. This is known as a *return move* and is interpreted as a sell or buy opportunity depending on the direction of the previous trend that is now being reversed.

A diagrammatic representation of a head-and-shoulders pattern is given at Figure 63.19 while an example of an inverted head-and-shoulders is given at Figure 63.20.

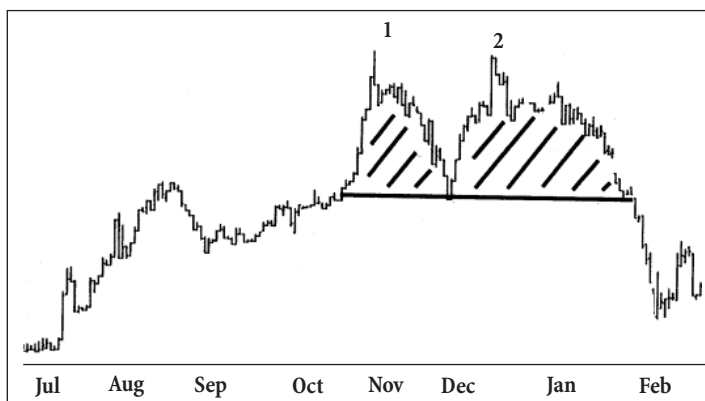


Figure 63.22: Double top pattern. Price source: Bloomberg.

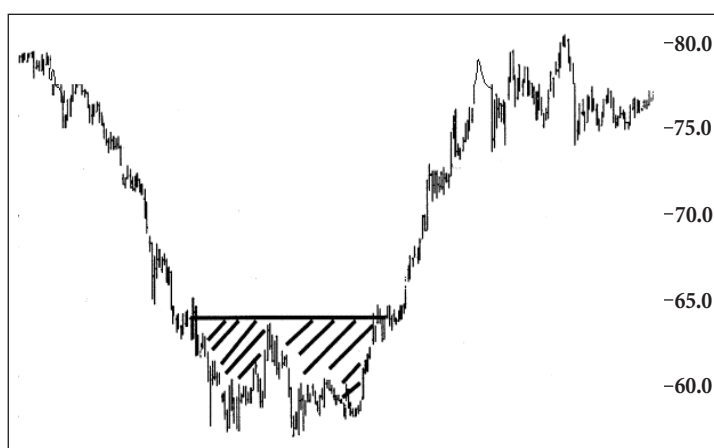
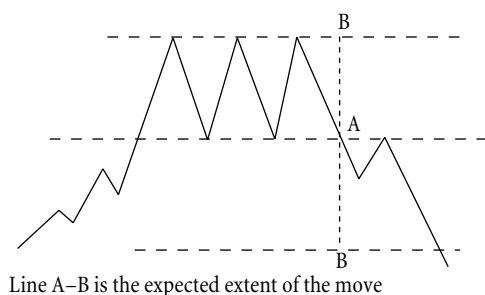


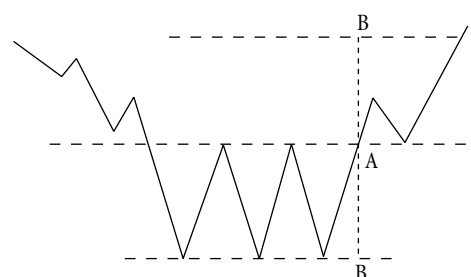
Figure 63.23: Double bottom. Price source: Bloomberg.

63.6.3 Triple top and triple bottom patterns

A triple top is essentially similar to a head-and-shoulders pattern, except that it has a flat centre peak that is approximately parallel with the other tops on both the left and right sides. The expected price range is again the length of a vertical line drawn from the middle top to the base of the pattern. Note that patterns rarely – if ever – are observed in the market as perfectly as the diagrammatic representations we use in this chapter.



Line A–B is the expected extent of the move



Line A–B is the expected extent of the move

Figure 63.24: Triple top.

Figure 63.25: Triple bottom.

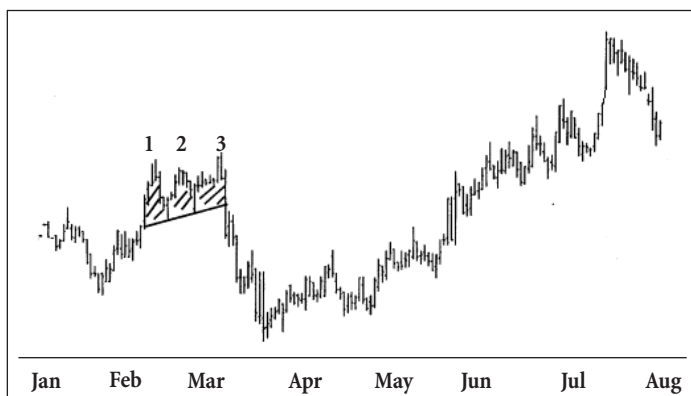


Figure 63.26: Triple top pattern. Price source: Bloomberg.

A triple top in reality will not be perfect, so it is important to remember not to look for a perfect shape in a chart. This is the case for all the patterns we discuss here, however triple tops, like diamond reversals, are relatively rare so the point about perfection is even more important. In practice triple tops are usually observed as extensions to a double top. Figures 63.24 and 63.25 show the pattern of a triple top and a triple bottom.

Figure 63.26 above illustrates a triple top from the German mark and Italian lira exchange rate in 1996.

63.6.4 Market spikes

A *spike* is a sudden and very fast market move, well outside the current range. It becomes a spike because the price action then snaps back, almost equally quickly, without prior indication, and reversing the previous trend. The reverse is known as a *spike bottom*. Spikes are commonly observed in commodity futures markets, as well as less frequently in other markets. Less spectacular spikes sometimes form part of the head in a head-and-shoulders pattern. Suffice it to say that it is often difficult to identify a spike as it is actually happening. Figure 63.27 is a diagrammatic representation of a spike, with Figure 63.28 illustrating a spike top from the crude oil contract on IPE contract during 1993–1994.

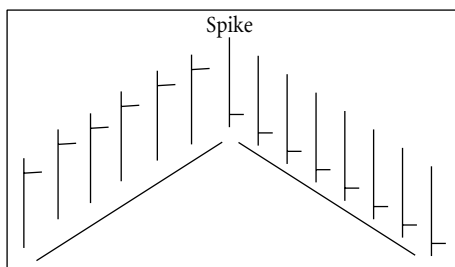


Figure 63.27: Market spike.

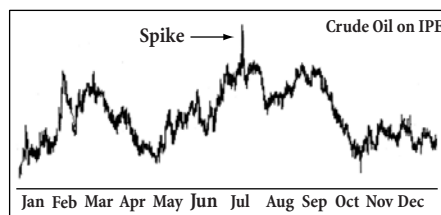


Figure 63.28: Market spike. Price source: IPE.

63.6.5 The broadening formation

A *broadening formation* is a longer term pattern; typically it requires five or six points in order to draw the formation line. Price points after that, the sixth or seventh point, are usually not reached in the price action and indicate a failure to return to the top of the pattern. A broadening pattern is usually indicative of confusion in the market and extreme price volatility, and is commonly observed at the end of a long bull market when market volumes are high. It is rarely seen in low volumes environments or at the end of a bear market.

Figure 63.29 is the diagrammatic representation while Figure 63.30 is an illustration of a broadening formation in the cable rate.

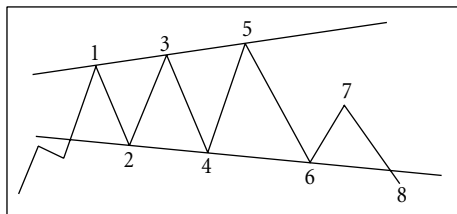


Figure 63.29: Symmetrical broadening formation.



Figure 63.30: Broadening formation.

63.6.6 Rounding tops and bottoms

A *rounding* pattern is commonly observed in markets experiencing low volatility and low trading volumes. It indicates a shift in supply and demand and an eventual reversal, however there is little if no predictive power in the pattern. Figure 63.31 is a diagrammatic representation of a rounding top.

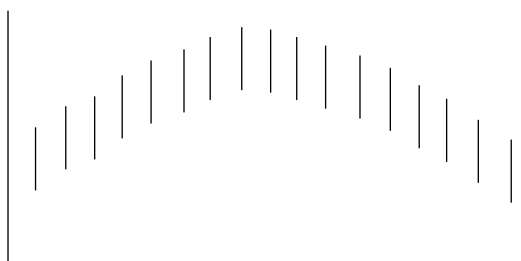


Figure 63.31: Rounding top.

63.6.7 Triangle tops

A *triangle* pattern is generally regarded as a *continuation pattern* (see next section), however if the observation time becomes over long the formation may terminate as a reversal pattern. For example a market that has been trading a narrow range for some time inside a triangle pattern is generally regarded as lacking both direction and interest, so that the eventual outcome is predicted to be a reversal. The widest point of the triangle is used as a target distance on any break out of the range. Figure 63.32 is the diagrammatic representation of a triangle top while Figure 63.33 illustrates a triangle top from the CBOT Eurodollar contract. Note the long time period over which the pattern formed, which would have made it difficult to spot.

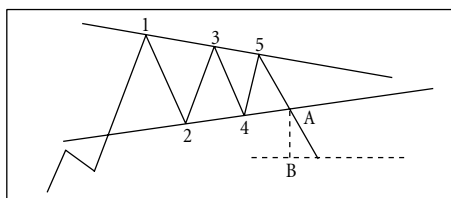


Figure 63.32: Triangle top.

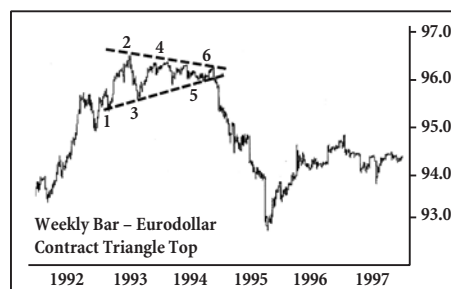


Figure 63.33: Triangle top.

63.6.8 Right-foot reversal

The *right-foot reversal* is a relatively new pattern, which is viewed as lying in between reversal and continuation patterns. It is similar in some respects to a broadening formation. The pattern is always bearish and represents a period of increased volatility, before the market moves downwards. It is the increased direction volatility characteristic that sets it apart from a continuation pattern. The formation begins with a strong advance followed by a period of distribution where the market makes progressively lower peaks, often more than four.⁷ The pattern is completed on a decline below the lower parallel line of the formation. Although it may be used as a short-term trading tool, the emergence of this pattern indicates a long-term bear phase. The pattern is illustrated with Figures 63.34 and 63.35.

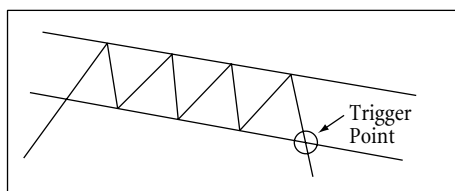


Figure 63.34: Right-foot reversal pattern.



Figure 63.35: Right-foot reversal.

63.6.9 Using reversal patterns

Although we have introduced reversal patterns in this brief section, the reader could carry out considerably more research and study in order to familiarise themselves with the detail. The key to using reversal patterns in trading is experience and learning from one's mistakes. It is important to maintain trading discipline when using technical analysis. For example, in the discussion on head-and-shoulders patterns, we noted that the neckline is the trigger level for market entry. The key issue here is that if one is prepared to take on a position on a break of the neckline level, equally one must be prepared to liquidate an existing position on a move back beyond the neckline. This is the key to disciplined trading. It is far more common to see traders breaking with discipline at such times in the hope that the market moves back in their favour.

Whatever pattern is observed, reversal patterns indicate the reversal of a prior trend. They do not become reversals until they are actually completed. Therefore if one is formulating a trade idea using a reversal it is important to wait until the formation is completed. A lack of trading discipline often sees traders entering the market in the hope that the pattern will be completed as they think. This is often a recipe for losses. At some point however the decision to enter the market must be made. We simply state here that if the basis for the decision is a technical one, the process must be followed through. It is always safer to wait for the pattern to emerge rather than deal before the emergence of one, but once the chart analysis indicates market entry, one should deal immediately.

63.7 Continuation patterns

A *continuation pattern* is just that, an indication of the continuation of a trend and not its reversal. It represents a pause or a consolidation of the main trend, and occur mainly when prices move too rapidly in the direction of the trend. They are always of shorter duration than reversal patterns.

The main types are:

- pennants;
- flags;
- head-and-shoulder continuation patterns;

⁷ These points are then considered by some to be similar to toes of the feet, hence the name.

- ascending, descending and symmetrical triangles;
- falling wedges and rising wedges;
- rectangle formations.

Continuation patterns are arguably more important to recognise than reversal patterns. This is because they indicate a trend, and technical analysis is a means of taking advantage of the trend and trading with it. Reversal patterns are more difficult to trade because they take a longer time to form and also represent higher volatility. Continuation patterns are usually smoother and form in a shorter time. For this reason it is easier to identify them. However the key point about continuation patterns is that they move against the primary trend. This reflects the fact that, in order for the trend to be sustained, there needs to be profit taking at the end of a secondary range, otherwise liquidity will dry up as everyone ends up with the same positions. Most continuation patterns only last for a few days to a month. They represent a consolidation of the market, a pause in a trend before moving on. As continuations are just that, a continuation of the trend, they present greater opportunity for profitable trading. Traders who follow technical analysis should ideally recognise these patterns and trade with them as often as possible, as this is where most of their profits are likely to be made.

In this section we briefly introduce each pattern.

63.7.1 Continuation pennant

A continuation *pennant* is observed in an upward trend environment. The pattern is normally formed in between one and three weeks. It is completed by a move out of the pennant. Trading volumes usually decrease in the body of the pattern and then increase on the break-out. The height of the *pole* (see Figure 63.36) measures the price range on a break out from the pennant's consolidation. The top of the pole is projected as being from the break-out point (the line A1–B1 in Figure 63.36), which gives an idea of the likely upward move.

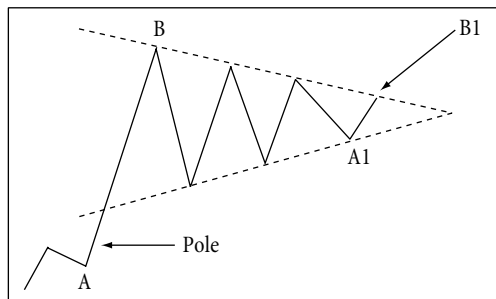


Figure 63.36: Continuation pennant.

A continuation *flag* is similar to a pennant and is also observed in an upward trend range.

63.7.2 Sideways congestion rectangle

The *rectangle* pattern is an example of a market congestion pattern. The market consolidates, resulting in a row of reaction high points (which are resistance levels) at the same level and a row of reaction low points (which are support) also at the same level. The pattern is triggered by a break to the upside over the previous reaction high points; the projected move upwards is based on the height of a line drawn at the top and bottom of the rectangle. The pattern is sometimes confused with a triple top reversal pattern. Figure 63.37 illustrates a bullish congestion pattern in an upward trend, while Figure 63.38 is a representation of a bearish congestion pattern in a downward trend. In the second case the market breaks the floor of the rectangle, and the target is the line drawn from the break-out point at the bottom of the rectangle.

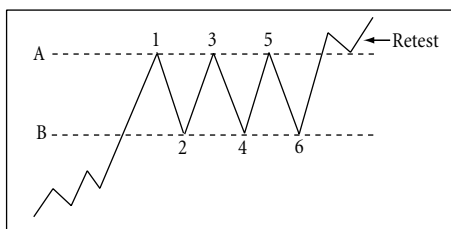


Figure 63.37: Sideways Congestion Rectangle Continuation Pattern in an Uptrend.

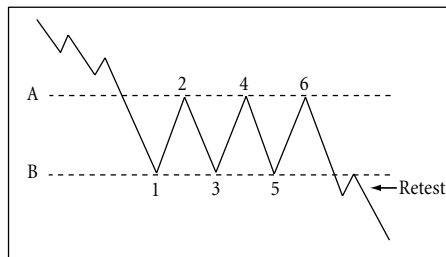


Figure 63.38: Sideways Congestion Rectangle Continuation Pattern in a Downtrend.

63.7.3 Head-and-shoulders continuation pattern

A head-and-shoulders continuation pattern has the same characteristics as the corresponding reversal pattern, with the exception that it is formed within the body of the trend. The key point to note is that the pattern is always in the direction of the trend. Figure 63.39 illustrates a head-and-shoulders continuation formation for the French notionnel contract during 1996.

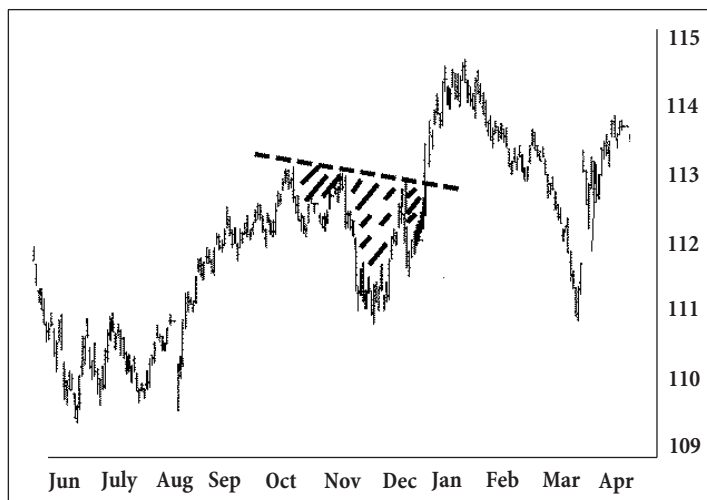


Figure 63.39: Head-and-shoulder continuation pattern. Price source: Bloomberg.

63.7.4 Symmetrical triangle

A symmetrical triangle is observed in an upward trend and usually is completed in one to three months. In our diagram the hypothetical market trades between two sloped lines, constricting the market towards the apex of the triangle. The market must break out before reaching the apex; if it does not, the pattern will drift or not emerge and the triangle loses its significance. A break-out is signalled by a closing price above the upper resistance line. The implied target on such a break-out is the highest point in the triangle, measured from the point of the break-out.

A *descending* triangle occurs in a downward trend. It also takes around one to three months to complete. It is similar to the triangle pattern observed in an upward trend.

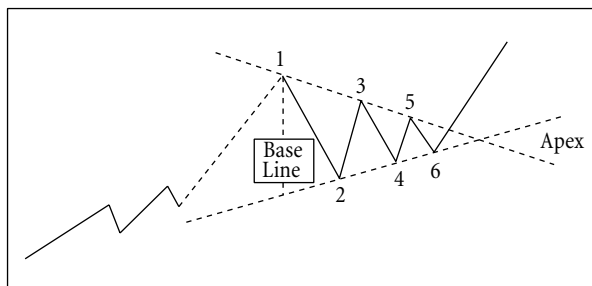


Figure 63.40: A Symmetrical Triangle in an Uptrend.

63.7.5 Falling and rising wedges

In an upward trend a bullish falling wedge results from a market moving against the trend in a constricted descending triangle formation. The trigger is a break of the upper resistance line. This is shown in Figure 63.41. A bearish rising wedge in a downward trend is similar to the falling wedge, except that it is a rising wedge.

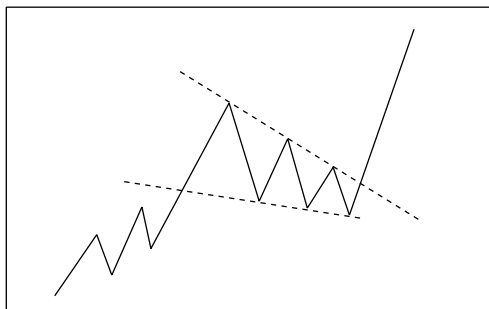


Figure 63.41: A Bullish Falling Wedge in an Uptrend.

63.8 Point and figure charting⁸

The *point-and-figure* method of charting dates from before bar charts; its first recorded use was in 1886. It was originally known as the “book method”. The essential purpose of using point-and-figure charts is to determine where price activity is consolidating. Once a consolidation area is observed, then (depending on where the price had been before the development of the consolidation area) a valid forecast may be made of future price activity. The consolidation area is defined by a range of price movements. That is, the greater the frequency of price movements, the less the price congestion; the more compact the price movements, the greater the congestion area. Once a price moves away from the congestion area, again depending on where it came from, prices will either reverse or continue. This approach is similar to the classification of bar charts into reversal or continuation patterns.

The charts are constructed using a series of noughts and crosses. The crosses depict rising prices and the noughts depict falling prices. No time axis is used, and the resulting chart is based purely on price. Volume cannot be shown on a point-and-figure chart and it is measured therefore by the activity shown on the chart, that is, by the width and length of the day’s price action.

The basic principle behind point-and-figure charting is as follows: an observation of a series of crosses rising above a previous column of crosses signals an upside break. In the case of a column of noughts the reverse would be true and the chart would be indicating a downside break or sell signal. This is shown in Figure 63.42 below.

Often time reference points are indicated by a different colour for each column (usually alternating between red and blue). A different colour is used for alternate days.

⁸ The author never quite understood it at the time, but hopes this exposition is a sufficiently sound one, especially if Messrs Baguley and Harrison are reading.

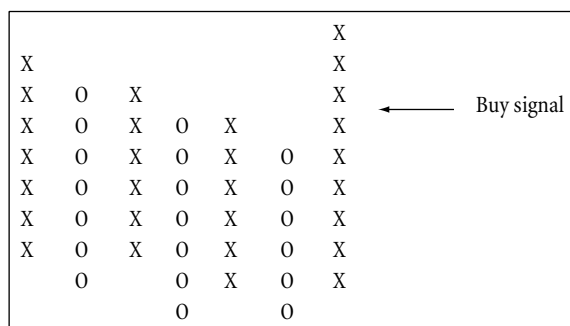


Figure 63.42: Point-and-figure chart.

63.8.1 Constructing a point-and-figure chart

Essentially there are two parameters to set when constructing a point-and-figure chart. These are the *box size* and the number of *reversal boxes*. The box size is simply the value of each box on the chart. For example if a trader is using a 1x3 point-and-figure chart to analyse foreign exchange rates, a box size of 1 would not be sufficient as it would not capture enough of the price action; the trader would need at least 20 points. To analyse say, a bond futures contract two points would be sufficient.⁹ In the currency markets therefore a box size of 20 points in a 1x3 would require a reversal 60 points while the bond futures contract would be a reversal of 6 points. The reversal box is defined as the number of boxes (and therefore points) required to complete a reversal in the trend. This acts as a filter so that the greater the number of boxes, the less sensitive the chart is to price changes.

The most common figures are a 1 point box against a 3 box reversal. Other frequently used examples are 1x5 and 2x7. For short term trades it is common to use one box reversal, while three box reversal is used to analyse intermediate trends and the five box reversal for longer-term trading horizons. Chartists often analyse point-and-figure patterns in the same way as bar and line graphs, and try to observe the same patterns.

63.8.2 Chart patterns

Congestion area analysis is one of the most important features of a point-and-figure chart. It helps to define areas of horizontal sideways price action. For instance if all the “X”s and “O”s are in the top end of the band then this represents selling pressure. Break-outs therefore are identified more easily. Another important feature of point-and-figure charting is how clearly they define support and resistance zones, within the horizontal congestion area. Some traders believe that it is easier to identify support and resistance levels on a point-and-figure chart than on a bar chart.

The *horizontal count* implies congestion width as a measure of vertical objectives. For example for intra-day trading, select a line to count across, the best one being through the middle of the reversal pattern, or the line that contains the most Xs and Os. This is of course an approximation. The general rule then is that the longer the pattern, the greater the subsequent move out of the range.

In terms of chart patterns it is possible to detect head-and-shoulders, triple tops and bottoms. These are analysed in the same way as for bar charts. In point-and-figure charting the term *fulcrum* is also used. A fulcrum is a congestion zone which usually occurs after long advances or declines. It is essentially a reversal pattern.

Traditional gaps are not detectable on a point-and-figure chart. This is because by definition there is a requirement to see a continuous string of noughts and crosses in order to facilitate a signal for the chart. However gaps may be marked onto the chart to show extreme strength or weakness; they are marked on to the columns. Other techniques that cannot be used are certain pattern types, including pennants, flags and triangles.

63.8.3 Advantages of point-and-figure charting

There are several advantages available in point-and-figure charting, indeed some traders use it almost exclusively rather than in conjunction with other techniques. The charts can be adapted to suit any market, by dint of varying

⁹ This is because FX rates extend out to “pips” which are hundredths of a hundredth of a point; for example, the cable rate might be quoted as 1.6542-52. Bond futures are in hundredths of a point, so a price might be 114.60-63.

the box and reversal size. It is also easy to update the monthly, weekly and daily chart. Other advantages are more contentious and not claimed by everyone; however some traders believe the charts indicate trading signals more precisely than bar charts, and provide specific entry and exit points. These attributes combined should help traders to achieve good trading discipline (conversely, the technique is best used by very disciplined traders).

Figure 63.43 is an example of the point-and-figure chart for the long gilt futures on LIFFE, for the period May–November 1999.

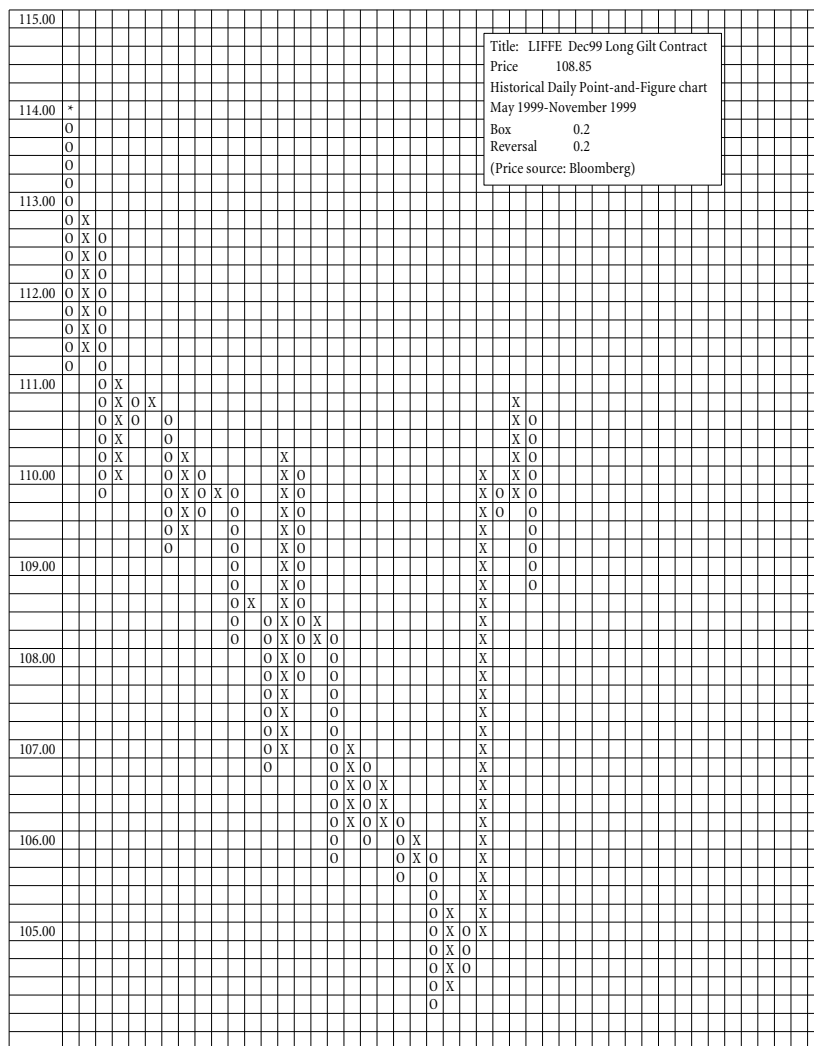


Figure 63.43: Point-and-figure chart, LIFFE long gilt contract, May–November 1999.

63.9 Mathematical approaches

Mathematical approaches to trading and technical analysis are relatively new. They were originally developed for longer-term analysis but have been adapted for use by intra-day trading as well. The main tools are:

- moving averages;
- parabolic charts;
- oscillators;

- relative strength indices;
- spreads;
- stochastics.

63.9.1 Moving averages

Moving averages are time series derived from a set of prices. For example the 20-day moving average on a particular date is the simple average of the previous 20 days' prices. As the date changes, so do the 20 prices from which the average is calculated. Moving averages have the effect of smoothing a set of data. They are designed to be a trend-following device and a signal for when a new trend has begun or an old one has ended or been reversed. When the closing price moves above or below the moving average it generates a buy or sell signal. By definition moving averages always lag behind the market. Therefore they can never anticipate but can only react. It is possible to construct short-term (five-day, 10-day) or long-term (30-day, 100-day) moving averages. The shorter term the moving average, the more sensitive it is in responding to price changes. A short-term moving average may give trend signals early in the move, but it may also generate false signals. The art is in striking the correct balance in this trade-off. Another technique is to await the crossing of a shorter-term moving average over a long-term moving average. This is often interpreted as trading signal.

In addition to the simple moving average, analysts also use exponentially smoothed moving averages, smoothed moving averages and weighted moving averages. The other types of moving average assign a weighting to earlier prices, to reduce their effect on the calculated figure. Within the context of the moving average itself, the actual set of data can be offset forward or backward, depending on what the trader wishes to do with the data.



Figure 63.44: Moving averages. Price source: Bloomberg.

Traders often use moving averages to back up trend analysis and the tools we described earlier. The main methods are:

- using a single moving average and a cross-over of a price over the moving average line;
- observing the cross-over of two moving average lines;
- observing the cross-over of three moving average lines (for example, using a five-day, 10-day and 20-day moving average);
- constructing a trading range using the high and low moving average lines;
- using long-term, such as 200-day moving averages to detect a trend;

- using a group of moving averages and observing if they all consolidate around a narrow range of the price action.

We reiterate however that moving averages are lagging indicators and should never be used in isolation to formulate trading ideas. Figure 63.44 illustrates the 21-day, 30-day, 100-day and 200-day moving averages for the short sterling contract on LIFFE during 1997–1998.

63.9.2 Pivot points¹⁰

Possibly a more useful application of moving averages is when they are combined with *pivot points*. These are used to indicate support and resistance levels. Figure 63.45 lists the significant pivots. In the author's experience brokers made more use of pivots than traders.

Level	Significance
Daily open	The previous day's opening level. Can act as a support and resistance in intra-day trading.
Daily high	The previous day's high. A potential resistance point.
Daily low	The previous day's low. A potential support point.
Daily close	The previous day's close. A potential closing price.
Pivot	Calculated as: $(\text{previous day's high} + \text{low} + \text{close})/3$. Psychological level the next day. A break of the pivot signifies a move to higher or lower price.
Regular high	Calculated as: $(2 \times \text{pivot level} - \text{daily low})$. This is the projected upside for the next day's trading. An early break should indicate a trend day.
Regular low	Calculated as: $(2 \times \text{pivot level} - \text{daily high})$. This is the projected downside objective for the next day's trading. An early break should indicate a trend day.
Extended high	Calculated as: $(\text{pivot level} + \text{high} - \text{low})$. Signifies a trend day. The market must move significantly above this point to move higher the next day.
Extended low	Calculated as: $(\text{pivot level} + \text{low} - \text{high})$. Signifies a trend day. The market must close below this level to imply a change in trend.
High break	Calculated as: $((\text{previous day's trading range} \times 0.75) + \text{close})$. Any break and close above this point from a new market low is an early warning signal of an imminent change in trend.
Low break	Calculated as: $((\text{previous day's trading range} \times 0.75) - \text{close})$. Any break and close below this point from a new market high is an early warning signal of an imminent change in trend.
Mid point	Calculated as: $(\text{daily high} + \text{daily low} - 2)$. Similar to the pivot point, the point at which to sell or buy crosses in moving average lines.
Pinnacle	The 2-day moving average of the close. Indicates short-term market strength or weakness. Similar to the pivot and the mid point.
Volatility break-out upside	The same as the high break.
Volatility break-out downside	The same as the low break.

Figure 63.45: Pivot points.

¹⁰ The author extends warm thanks to Martin Davies, formerly of GNI and Yamaichi Securities on LIFFE, for first introducing him to these concepts.

63.9.3 Oscillators

An *oscillator* may be constructed from two moving averages, a shorter one and a longer one. The shorter average oscillates about the longer average; the difference between the two is plotted as a histogram. The numbers of days used to construct the two averages are often taken from the *Fibonacci* series. This sequence of numbers is 1,1,2,3,5,8,13,21,34 and so on. It has the property that any number is the sum of the two preceding numbers. Fibonacci relationships occur frequently throughout nature. Oscillators are thought to be especially useful by some traders in non-trending markets, when prices are fluctuating within a trading range. They are used to signal short-term market extremes, or over-bought or over-sold conditions, and designed to give easily readable signals. The principal readings come from the crossing of the mid-point, *divergence analysis*, extreme readings, cyclical analysis, trend line oscillator break and chart patterns detected in oscillators.

There are three main types of oscillators:

- momentum oscillators;
- rate of change oscillators;
- moving average oscillators.

The author often used two types of oscillator on the same chart together with a bar chart, to compare signals and look for confirmation of trends or breaks.

- **Momentum oscillators.** This measures the acceleration or deceleration of the price rather than the actual price levels. Put another way, the momentum oscillator is constructed to measure the speed of rate of change. For example a five-day momentum oscillator is the difference between the current day's closing price and the closing price five days ago. Each day the positive or negative result is plotted around zero. There are no upper or lower boundaries. One of the benefits of momentum is that it leads price action at market turning points and then flattens out while the current trend is in effect. A point to note with this indicator however is that with no boundaries indicated, a trader has to try and observe support and resistance levels.
- **Rate of change oscillator.** This is simply a momentum oscillator in percentage rather than point form. For example a seven-day rate-of-change oscillator is calculated by dividing the present day's closing price by the closing price from seven days ago. This is then multiplied by 100 and shown in percentage times. In this case the market fluctuates as a fraction around 100%.
- **Moving average oscillator.** This is also known as a *daily departure*. It is the difference between two moving averages plotted between and around zero. The chart is used to identify divergences, note significant deviations caused by the short-term moving average expanding from the long-term moving average, to observe a cross-over more easily and to identify support and resistance levels.

The most frequently used oscillators are the *relative strength index* and *stochastics*.

63.9.4 The relative strength index

The relative strength index (RSI, invented by W. Wilder) is a type of oscillator that expresses the average of the daily increases in closing price as a percentage of the average magnitude of the daily changes. It is plotted on a vertical scale of zero to 100. Levels above 70 are considered overbought, and below 30 oversold. In extended bull markets the two indicators move to 80 and 20, so it is important to set the RSI parameters in the context of the current trend. RSI measure momentum in the market then, but is a smoothed oscillator. Trend lines can be drawn on the RSI and used in the same way as in a bar chart. In addition chart patterns emerge in an RSI and can be analysed in the conventional way. The RSI index calculation is given by (63.1), where x is a user-defined parameter.

$$RSI = 100 - \frac{100}{1 + RS} \quad (63.1)$$

where $RS = \frac{\text{Average of } x \text{ day's up closes}}{\text{Average of } x \text{ day's down closes}}$.

63.9.5 Stochastics

The stochastic process expands on the concept of RSIs. It was invented by George Lane and is based on the observation that in rising markets, prices tend to close near the upper end of their price range; in downtrends prices often close near the lower end. In a fast stochastics graph the more sensitive line records the latest close, expressed as a percentage of the total price range over a specified period of time. This line is called %K. A secondary line which is a moving average of this, known as %D, also plotted. Traders often prefer a more refined version which also uses two slower lines. In this case the faster line is %DS which is a moving average of %D; the other line is %DS-slow, a moving average of %DS. These increasingly smoothed lines are thought to generate more reliable signals than their respective counterparts. In both cases, as for RSIs, the vertical axis is scaled from zero to 100%; levels above 70 indicate the market is overbought, and levels below 30 indicate the market is oversold.

Stochastics is considered to be more sensitive than the RSI. The most frequently observed parameters are nine days and 14 days. The higher the number, the less sensitive is the oscillator.

63.9.6 Moving average convergence/divergence indicator

This technique (MACD) was developed by Gerald Appel. It is based on the theory that speculators “test” a trending price. In contrast to the simple arithmetic moving averages reviewed earlier, MACD uses two exponentially smoothed moving averages. The MACD1 line is the difference between these two averages, and the signal line is an exponentially-weighted average of MACD1. Changes in trends are thought to occur when MACD1 crosses the signal line, and this is used as an indicator. Some charts also provide a graph of the difference between the two lines. This oscillator is used to measure the trend acceleration or deceleration as well as the speed of the price moves.

63.9.7 Directional movement index

Trending markets can be determined using a directional movement index (DMI). It consists of two lines, +DIP and -DIP. The +DIP line measures the upward trend and -DIP line measures the downward trend. A third line, the Average Directional Movement or ADX is drawn to indicate how much the market is trending either up or down. The higher the ADX line, the more the market is trending and the more suitable it becomes for a trend-following system. A buy signal is indicated when +DIP crosses the -DIP line; a sell signal is given when the reverse happens. The ADX line can be used as a filter signal, that is the crossovers are considered to be false if it is below 25%. However if the ADX line moves back over 25%, this is a signal to enter the market.

63.10 Contrary opinion theory

This theory is often heard in the futures markets. The principle of contrary opinion holds that when a large majority of participants are long or short of a market, or are agreed on future expectations, they will be proved to be incorrect. Thus a contrarian will determine what the majority view of the market is, and then trade in the opposite direction. For example if on the eve of a meeting of the Monetary Policy Committee in the UK,¹¹ a newspaper carries an item stating that over 90% of economists polled believed that interest rates would remain unchanged, contrary opinion theory holds that rates will probably be increased and that short-dated interest rate futures are a sell. In another instance, if a market has been trading sideways for a few weeks and most participants are on small profits or small losses, and market comment is always bullish, the first piece of bearish news will push the market downwards as those who are running long positions get squeezed out, and the slide is accentuated.

In the author's experience some brokers fell into this line of thinking, suggesting that a market was worth a buy after a large move down in a short period of time. Without any supporting evidence though this strategy is risky, as it is always difficult (and therefore inadvisable) to try and pick the bottom, or indeed the top, of a market on gut feeling alone or after sudden price action. The primary principle must always be go with the trend if one is following technical analysis.

63.11 Volume and open interest

We alluded to trading volume and open interest at the start of the chapter, as being important measures in charting. We reiterate this here. It is always important to gauge the depth of a market from the volume of trading at particular

¹¹ The Monetary Policy Committee of the Bank of England is the body responsible for setting interest rates in the United Kingdom.

price levels. Chartists generally consider three measures as part of their analysis, the price, volume and open interest. This is most relevant in commodity and financial futures markets. In foreign exchange markets it is difficult to assess volume and open interest, if not impossible, but both measures are important in other markets.

Volume is the number of contracts traded during a given time period. *Open interest* is the total number of contracts outstanding at the close of the trading day, measured as the total long and short positions, and the net change from the previous day. The level of open interest is also used to measure the interest in the underlying cash market, because of the key uses of futures contracts is to hedge cash positions. A high level of open interest implies therefore a high level of interest in the cash market.

63.11.1 Characteristics of volume

The level of trading volume in a contract implies a measure of intensity or urgency in a move. Essentially if a market is rising on weak volume and weak open interest, traders may conclude that the market will fall back from the higher level sooner rather than later. The same would apply in a downward move with low volume. To detect or conclude a trend pattern requires heavy volume, and to a lesser extent high open interest. The market also expects volumes to increase in the direction of the trend, and decrease on (temporary) pullbacks against the trend. This is why charts are often set up to show the level of trading volume underneath the main picture; for example the line, bar, moving average and candle charts on a Bloomberg® terminal allow the user to select a volume measure to be included on the display. Volume levels are used to back up analysis when detecting patterns.

Volume levels are an important component of reversal patterns, particularly the head-and-shoulders pattern. For instance it is a characteristic of the head-and-shoulders to observe low volume in the “head” and high volume from the top of the pattern. The primary rule regarding volume however remains that volume must expand in the direction of the market trend.

63.11.2 Characteristics of open interest

The principle rules regarding open interest are:

- rising open interest in an upward trend is bullish;
- declining open interest in an upward trend is bearish;
- rising open interest in a downward trend is bearish;
- declining open interest in a downward trend is bullish.

Generally open interest is more important at the end of a trend. For example historically high open interest at the end of a long trend, together with a strong reversal, is an indicator of an impending change in trend.

63.12 Candlestick charting

The final technical analysis tool we introduce in this chapter is *candlestick charting*, which although a relatively new charting measure in western capital markets, originally dates from the rice markets of Osaka, in Japan in the early 1700s. They are usually credited to one Muneisha Homma, who was said to have dominated the Osaka rice markets at the start of the 18th century. Despite this pedigree no English translation of the technique was available until the late 1980s. Candle charts are now an accepted charting tool and are as common as bar charts. They perform a similar function to bar charts, but are associated with colourful terms describing specific market occurrences such as “hanging mans”, “Hirami” and “doji”. These and other terms are explained later.

63.12.1 Candle description

Figure 63.46 illustrates the standard bar, depicting the opening price at (A), the closing price at (B), the high at (C) and the low at (D). The representation of a Japanese candle at Figure 63.47 carries the same information, the opening, closing, high and low prices.

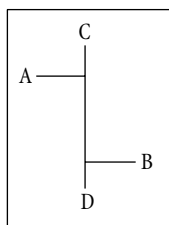


Figure 63.46:
Bar graph.

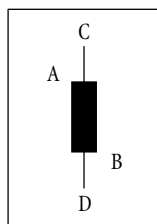


Figure 63.47:
Candle graph.

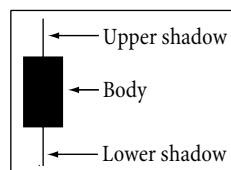


Figure 63.48

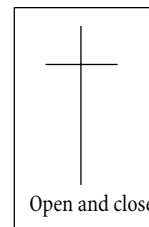


Figure 63.49:
Doji pattern.

The candle graph has a filled in “body” which distinguishes it from the bar graph. In the example at Figure 63.47 the closing price is below the open, which is indicated with a completely filled in or black body.¹² When the close is above the open, indicating a bullish period, the body is clear or white in colour. The lines outside the candle are known as “shadows”. This is shown at Figure 63.48.

Note how the body in Figure 63.48 is larger than the body in Figure 63.47. This is called a *spinning top* and depicts a smaller difference between the opening and closing prices of the trading period. Such a case indicates an equal strength of bulls and bears, so that the outcome of the day’s trading is not influenced in either direction.

A trading period that has a closing price at the same level as the opening price is known as a *doji*. This is shown at Figure 63.49. Combining the different types of candles forms patterns that can be interpreted much as bar charts are.

63.12.2 Candlestick reversal patterns

The observance of a reversal pattern in candlestick charting does not necessarily imply a complete reversal in trend (as is the case with bar graphs). It may indicate an end to the prior trend in advance of a sideways period of trading, or a consolidation before a resumption of the prior trend itself. Therefore reversal patterns are often interpreted as an indication to liquidate positions, whether long or short, and then waiting for confirmation of the reversal signal before re-entering the market again. The main reversal patterns are reviewed below.

- **Hanging man line.** This is a reversal signal at the top of a market, so called because it is meant to depict a head and dangling legs. The small body of the candle can be either black or white in colour although it is meant to signify a more bearish picture if black is used. The length of the shadows is important; there should be little or no upper shadow and a lower shadow that is at least twice as long as the body. A long lower shadow makes the shape itself more significant. To confirm the signal, in the next trading session the closing price needs to be lower than the body of the hanging man pattern. This then indicates lower prices over the longer term. The pattern is illustrated in Figure 63.50.

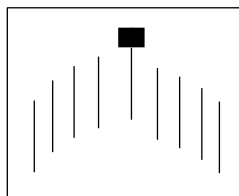


Figure 63.50: Hanging man line.

- **Hammer.** A hammer is identical to the hanging man except that it is observed at the bottom of a downward trend. Again the trader should wait for confirmation in the next session before assuming a reverse in the trend. The confirmation may be a close above the high of the day the “hammer” was observed. Hammers usually form after a prolonged upward trend.

¹² On Bloomberg blue and white colours are used for candle charts.

Figure 63.51 illustrates a hammer pattern and a failed hanging man from the long gilt contract during 1996.



Figure 63.51

- **Engulfing pattern.** This is either bullish or bearish in nature. The keys to the pattern are the bodies; in our illustration at Figure 63.53 the white body is larger than the black body. This indicates a strong bottom level reversal. A bearish engulfing line is shown in Figure 63.52.

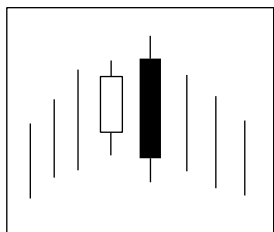


Figure 63.52: Bearish engulfing pattern.

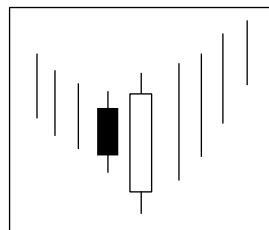


Figure 63.53: Bullish engulfing pattern.

The colours of the bodies in the pattern are always opposite, that is the black body engulfs the white body to indicate a bearish signal while a white body engulfing a black body indicates a bullish signal.

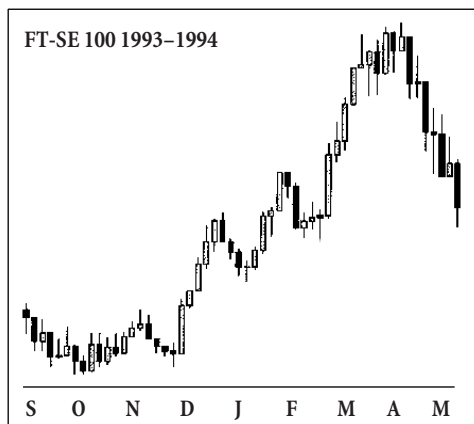


Figure 63.54: Bullish engulfing line in FT-SE 100, 1994.
Price source: Bloomberg.

- **Dark cloud cover.** This pattern is another reversal signal; that the pattern is interpreted as a warning is indicated in its name. A “pure” dark cloud cover is represented by an open on the black candle that is above the high of the prior (white body) day. If the closing price is below the previous day’s white candle open the pattern would actually be a bear engulfing line. The dark cloud pattern is shown at Figure 63.55 and illustrated using ordinary shares for an AIM-listed equity during 1998 at Figure 63.56.

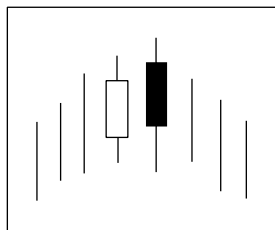


Figure 63.55: Dark cloud cover pattern.



Figure 63.56: Dark cloud cover for AIM-listed equity. Price source: Origin company.

- **Piercing line.** In the case of a piercing line the close of the white candle must be over 50% above the previous day’s black candle. The 50% rule is applied quite strictly otherwise the pattern is not considered to have emerged. The other characteristic of this pattern is as for the dark cloud cover, in that there is an opening gap, in our example (Figure 63.57) below the low price of the black candle.

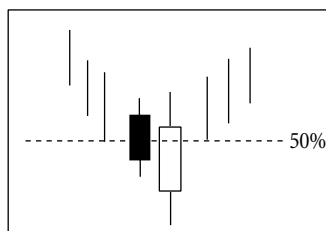


Figure 63.57: Piercing line.

- **Hirami patterns.** These patterns may appear at market tops or bottoms and are interpreted as a slowing down or temporary stopping of the previous trend, with a subsequent signal required to confirm the reversal in trend. The Hirami cross is considered to be more important than the Hirami line, as it includes a “doji”; it is known as a “petrifying pattern”.¹³ The pattern is regarded as a clear signal to liquidate one’s position. Hirami patterns are

¹³ When we said that candlestick chart terms are more colourful than bar or line chart patterns, we were not inaccurate!

equivalent to what is called an “inside day” by chartists. An inside day refers to a trading day where both the high and low prices are inside the previous day’s range, whereas a Hirami line or cross only requires the candle body to be inside the previous day’s candle body.

Figure 63.58 (i), (ii) and (iii) illustrate hypothetical Hirami patterns.

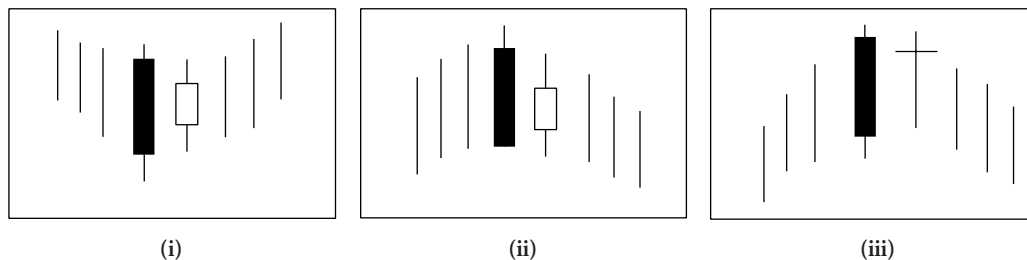


Figure 63.58: (i) Hirami line after downward trend, (ii) after upward trend, (iii) after upward trend and including doji (“petrifying pattern”).

- **Morning and evening stars.** Significant reversal indicators contain three candlesticks in their pattern. The reference to a morning star refers to something that appears before sunrise and warns of higher prices, so therefore morning stars are observed in a downward trend. They are comprised of a strong black candle, a “star” which is a “spinning top” that has resulted from a gap away from the prior black candle closing price, and a strong white candle whose range has moved into the first session black candle’s body. If this sounds like a mouthful, review the illustration at Figure 63.59.

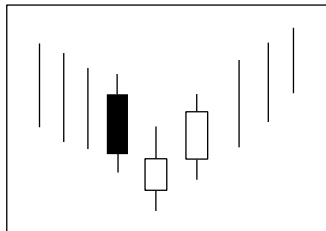


Figure 63.59: Morning star.

An *evening star* occurs after an upward trend; as suggested by the name it appears before sunset and is interpreted as a strong warning. If the “star” in the pattern (which is the second of the three candles) is a “doji” the pattern is considered an even more ominous warning. Figure 63.60 shows an evening star doji.

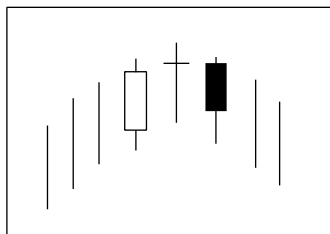


Figure 63.60: Evening star doji.

- **Shooting star and inverted hammer.** The shooting star pattern is observed after an upward trend and is regarded as a sign that the market has reached its top. It is not as strong a sign as an evening star however. An inverted hammer is similar to a shooting star except that it is observed at the bottom of a downward trend. It is

important to wait for confirmation of both of these patterns; for a shooting star this is provided by a lower close at the next trading session, while for an inverted hammer the confirmation is a higher close at the next session.

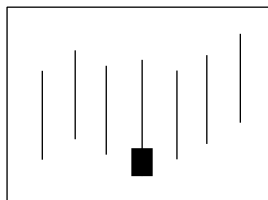


Figure 63.61: Shooting star.

Figure 63.62 illustrates a shooting star from the candlestick chart for the sterling/US dollar exchange rate during 1996.

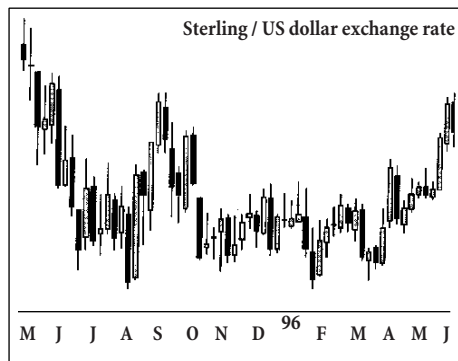


Figure 63.62: Shooting star cable rate.
Price source: Bloomberg.

63.12.3 Further candlestick patterns

The previous section reviewed the most important patterns in candlestick charting. There are a range of others as well, a few of which we introduce here.

- **Windows.** A “window” in a candlestick chart is the same as a gap in a bar chart. The equivalent expression for “filling the gap” in market-speak for candlestick charts is “closing the window”. The window is indicated by the space between the previous day’s candle upper shadow and the bottom of the next day’s candle. The depth of this window, which is the extent of the gap, is significant because the greater it is, the longer the time before the window is eventually closed. A pattern that exhibits more than one window will therefore see the price range indicated by the lower depth window to be closed first.
- **Tower top and bottom.** A tower top pattern occurs during an upward trend, which is followed by one or more white candles. This is interpreted as an acceleration of the market at the end of the trend, which is the same analysis for bar charts. After the white candles there appear small-body candles with lower highs. Finally the right-hand side of the tower is completed by one or more black candlesticks; this confirms the change in trend. Figure 63.63 shows the pattern for a tower top while Figure 63.64 illustrates the pattern for Woolwich plc ordinary shares during 1998/1999. A tower bottom is the reverse of a tower top.

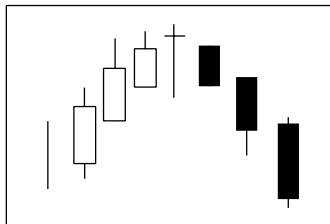


Figure 63.63: Tower top in an upward trend.

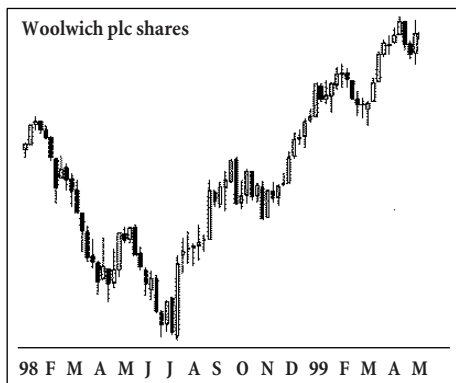


Figure 63.64: Illustration of tower top, Woolwich plc ordinary shares.
Price source: Dow Jones Telerate.

- **Dumpling top.** This pattern denotes a top reversal similar to the rounded top in a bar chart. It forms when an upward trend diminishes and a series of small-bodies candles appear in sideways trading, followed by a downside window to confirm the turn in market sentiment. The window is the “last gasp” of the upward trend. The next move is expected to a downward push to close the window. When the pattern is observed at market bottoms it is called a “frying pan bottom”. Figure 63.65 illustrates a dumpling top seen in LIFFE’s short sterling contract during 1995/1996.



Figure 63.65: Illustration of dumpling top, short sterling contract.

We have reviewed only a sample of candlestick patterns. Just as there are traders who combine bar charts with Gann lines and mathematical techniques such as moving averages and oscillators, it is common to see candlestick graphs used in combination with other charting techniques. Often traders will use one method to confirm signals given by another.

63.13 Elliott wave theory

The Elliott wave theory was developed by Ralph Elliott in 1932. Its basic principle is that price moves occur in “waves” of upward moves and downward moves. When Elliott first developed his ideas he used daily bar charts of stock market prices, since when the theory has been applied to movements in futures markets.

The basic principle behind Elliott wave theory is that market prices move as part of an “action-reaction” framework. That is, upward moves in prices are followed by downward corrections, which are in turn followed by upward moves. If one plots a graph of these moves one will discover repeated patterns, which are termed *impulse waves* if upward moves and *corrective ways* if downward moves. There is a pattern to impulse moves as well, which are said to be made up of five smaller-scale waves, three impulse waves and two corrective ways. An impulse wave is always followed by a corrective wave. Waves are individually designated by analysts, as primary waves and sub-waves of a main wave. At the end of a large-scale impulse wave one observes a large-scaled corrective wave.

A corrective wave also contains three constituent waves, with an impulse wave followed by a corrective wave followed by an impulse wave. The relationship between the various waves and sub-waves is described as a series of *fractals* in some text books, which is why this type of analysis is also called “fractal wave” theory. In a bull market then we have five waves which terminate at a high followed by three waves which retrace the upward trend. In a bear market we observe the reverse.

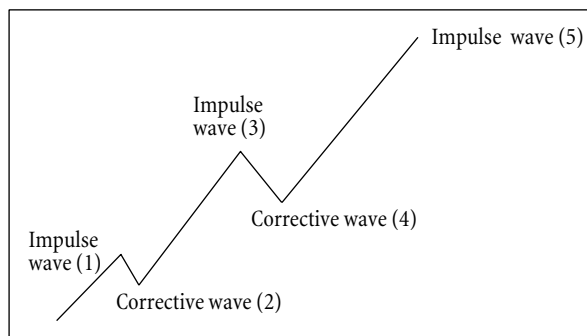


Figure 63.66: Waves of upside move.

The fractal relationship of the connecting waves is illustrated diagrammatically in Figures 63.66 and 63.67. Figure 63.66 shows the five basic waves that make up an upside move. In Figure 63.67 we show the component waves of the third wave itself, which are five smaller-scale waves on the upside.

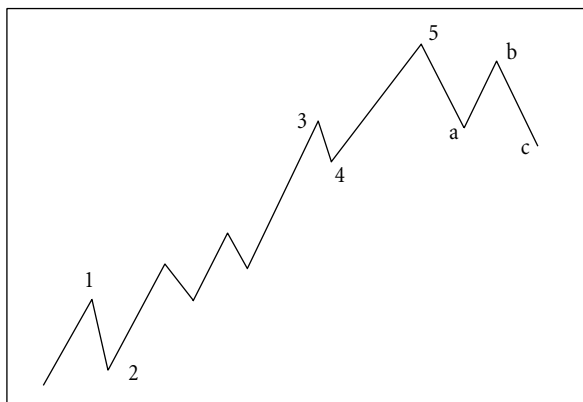


Figure 63.67: Fractal waves.

63.13.1 Data recording

The first issue in the application of Elliott wave theory is what level of data to record. Elliott himself analysed charts drawn using the price recorded at the end of each half-hour section of the trading session for the Dow Jones Industrial average. This is applied today although there is a problem when considering the daily trading sessions of stock exchanges where the total hours do not divide neatly into half-hour blocks. Another consideration is which closing price to use, although usually analysts use the last half-hour as originally proposed by Elliott. Some traders analyse waves using the session highs or lows; if we compared the patterns using closing prices to highs and lows, we would observe that the closing price chart would be bounded by the high price and low price charts. The analyst must determine the correct count of the corrective waves, so that extraneous data or “noise” must be filtered out. It is the presence of noise that renders the identification of waves and sub-waves difficult.

63.13.2 The predictive power of wave theory

As originally stated by Elliott and later also expounded by Prechter (1978), wave theory can be used to predict market moves, in the first instance according to a set of rules. These are summarised as:

- the first and fifth waves are usually shorter than the third wave;
- the “A” wave of a corrective wave is roughly the same in length, and over a similar time period, as the “B” wave;
- trading gaps usually occur in the third impulse wave;
- corrections during the A-B-C wave cycle rarely retrace to the range below the level of the fourth wave;
- any extensions to the secondary smaller-scale waves of the larger impulse usually take place during the third wave;
- the highest volume of trading is observed in the third wave;
- if wave 2 (a correction small-scale wave) of a larger-scale impulse wave is a simple structure, wave 4 (also a correction) will be complex in structure. The converse is also true.

The impulse wave has the highest predictive power. Observations of corrective waves are dependent on the impulse wave, although the impulse wave is not dependent on any of the components of the corrective waves.

The size and time duration of a corrective wave may have forecasting implications for the following trend. If the wave count is correct, the timing of the impending impulse wave can be calculated with accuracy. The fact that the market is trading in the opposite direction of the main trend should not deter a trader, as the theory suggests they should use this as an opportunity to buy (or sell short in a downward trend). The size of the corrective wave will determine the extent of the following impulse wave. If the correction is shorter than normal in time or price then the following impulse wave will be longer than usual in time or price. This is because the weight of market sentiment is usually sufficient to cut short the correction. When the impulse wave takes hold, this sentiment will carry the wave for longer than normal. The reverse would apply for a protracted corrective wave.

63.13.3 Refinements of the original theory

Over time there have been refinements to the original theory’s rules that took account of further market observations. These are listed below, but are sometimes ignored or overlooked by market participants who continue to apply the original rules only. The refined theory suggests that:

- the corrective wave 4 never falls (rises) lower than the peak (trough) of wave 1;
- if corrective wave 2 is a “zig-zag” pattern then the chances are that corrective wave 4 will be flat. This is known as Elliott’s “rule of alternation”;
- when waves continue for longer than expected, the wave is said to have extended. If one impulse wave extends, the other two waves are unlikely to be extended;
- when applying the principle to long-term price charts, a semi-logarithmic scale should be used. This is to counter the effects of inflation, which could interfere with the wave lengths.

Although originally designed for long-term trading, Elliott wave theory has been applied to day trading by some market participants. The theory states that profits are made mostly when prices move by the largest absolute

amount, and that this usually occurs in the third wave. The third wave is the longest and has the highest volume levels. The day-trader therefore attempts to determine the longest run in prices on a daily chart, which analyses shorter-span trading sessions compared to the conventional chart. Once the trader thinks they have identified the longest run, they track and label the future waves in the same way. In fact some traders believe that Elliott wave theory is better suited to short-term trading, because the long-term traders loses out on the profit potential of the third wave, since it takes a longer time to form. The day-trader on the other hand has a better opportunity to observe the formation of the third waves or sub-waves within it.

Considerable research has been conducted on the use and value of Elliott wave theory, and we note some relevant texts in the bibliography. It is probably one of the more difficult techniques to apply in the market. The theory may be viewed as useful as it allows the analyst to create forecasts; where a series of forecasts are shown to be inaccurate, the rules as we listed above are usually modified rather than thrown out. However it is not a technique that can be applied in half measures. The Elliott wave principle is a technique that requires considerable application and understanding. This section is an introduction to the topic, and interested readers will wish to consult further texts before employing the technique.

63.14 Stop losses

In this section we review stop-loss levels. Essentially a stop-loss is the point at which, a trade having gone off-side, market exit will be effected. A rough-and-ready rule of thumb one often encounters in the cash markets is that if one is expecting a gain of 10 basis points in a trade before profits are taken, the stop-loss should be placed at -5 basis points. This is probably too risky for some trades.

Stop-losses are an integral part of disciplined trading. As such they are also important in any technical analysis-based trading strategy. There is an art as to where to place a stop-loss however, and it is still common to encounter confusion on the subject. However traders following a technical analysis route to idea formulation probably can apply more logic to their decision than those following purely fundamental analysis. In many cases stop losses are placed at psychological levels, which prove to be precisely the point at which they should not be placed. The following are examples of stop-loss points used by some market participants, nearly all of which can be shown to be mistaken levels:

- a stop-loss placed on a significant support or resistance level;
- a stop-loss placed on a round number, for example 114.00;
- levels placed around trend lines;
- a stop-loss placed immediately below an important support or resistance level;
- stop-losses placed at the previous day's close;
- placing a stop-loss on the high or low of the prior week's trading range;
- a stop-loss placed on a Fibonacci or Elliott wave retracement level.

It is important to avoid these errors when placing stop levels. Generally the simplest approach is to analyse a chart and establish where the prior trading interest has been. A stop should not be placed on this level or close to it, but slightly beyond; this allows room for error. The general rule though is to avoid the pitfalls listed above, because they are well-known errors and market-makers and exchange locals are aware of them and will take them out, before the market moves back to where it was.

Stop levels are adopted ideally as part of the overall trading strategy. Disciplined traders adopt something approaching a 4:1 profit to loss trading ratio. Technical analysis essentially is about determining trigger points at which to initiate trades. If those trigger points are broken before the market moves back through them, that in effect acts as a stop-loss. However the key is to place your stops outside the common errors noted above while still observing the 4:1 profit/loss ratio. When a key support or resistance level is broken, a market generally moves into the new trading range very rapidly. Traders will bear this in mind when placing their stops.

63.15 Concluding remarks

This has been a lengthy chapter in which we have introduced the basics of technical analysis. None of the material is particularly new, indeed some tools have been in use for many years. However they are still actively employed by large numbers of traders and analysts working in virtually every bank and securities house in the markets. No one

technique has been proven to be more effective than the others, and indeed they all have their flaws. Readers may have noted that, while we have used examples from real life that illustrate the patterns that have been discussed, it is fairly clear after the event! How easy is it to identify a pattern that is still forming? Obviously the markets exist precisely because for each trader, having analysed their chart and decided it's time to buy, there is another who has exactly the opposite view – the counterparty to our first trader's buy order.

The price action of any asset reflects everything that is presently known or discounted about and relevant to the asset. Neither technical nor fundamental analysis can predict unforeseeable events or as yet unknown information. However charting does attempt to inject an element of science into trading. Some traders are purely technical traders while others will use charts to back up other ideas and techniques. The key to using charts is to be disciplined and follow the signals as they arise. Even if one is using charts in conjunction with fundamental analysis, discipline must be maintained so that rumours or market gossip, or simply waiting for your position to go back on-side, do not interfere with the trading decision. Successful traders are often those who trade from a known plan, and make entry decisions from a position of strength. A former colleague of the author's had another saying, "Never trade when you're angry." That would be trading from a position of weakness and that always ends in failure. Decisions taken in an atmosphere of frenzy or panic are usually bad decisions. It is important to take rational, calm decisions, which helps in making a profit but also helps in knowing when to cut a loss, and to understand why a trade did not work out. Technical analysis is not a panacea however. It tracks the market and simply reflects what the majority of participants are doing. As such it does not move the markets.

Questions and exercises

1. What is a Gann line?
2. Describe the major trend reversal patterns that may be observed in bar charting.
3. What is a "hanging man"?
4. An economist at a bank suggests that, as short-term interest rates have just been raised, after a long period of interest-rate stability, the country's currency should hold up and there should be increased inward foreign capital flows. However the technical analyst feels that over the short-term the currency will move lower. What factors could be used to explain this state of affairs?
5. Your junior trader suggests that the profit they are expecting on the two-year versus five-year bond spread position they have just put on is around nine basis points. They have put on a stop-loss at minus-five basis points. Are you happy with the stop-loss? Explain your answer.
6. What is the predictive power of a rounded bottom pattern?
7. Describe the main continuation patterns that are observed in the market.
8. A head-and-shoulders continuation pattern for the futures contract you are observing is followed by an ascending triangle. Did the market prediction indicated by the first pattern prove accurate? What if the following pattern had been a descending triangle?
9. You observe a "morning star" pattern that also includes a "doji," and are currently running a short position in the contract you are analysing. What should you do?
10. Using Gann lines on a chart a trader observes that the angle is fairly steep. What should the trader expect will happen, with regard to the previous trend?
11. From the previous question, imagine now that there is a short-term correction. Remembering again that the trend line was at a steep angle, how should the trader expect the correction will take place, through a directional move or through sideways price action?
12. A market breaks past a resistance point suddenly and very quickly, way beyond expectations. It then snaps back, again without warning, and reverses the previous trend. What is the name for this pattern?
13. What is the name for the pattern at Figure 63.68 below?

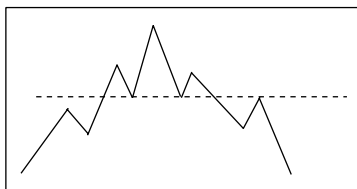


Figure 63.68

14. Consider Figure 63.69, below, which shows price movements in the S&P500 contract. The first chart is a daily bar with five-day, 13-day and 21-day moving averages. What pattern was developing at (a)? In the second chart, at what level is the value area forming? If we consider charts 3 and 4, which show 60-minute and 15-minute bars respectively, do the patterns at (d), (g) and (h) confirm your answer to (a)?

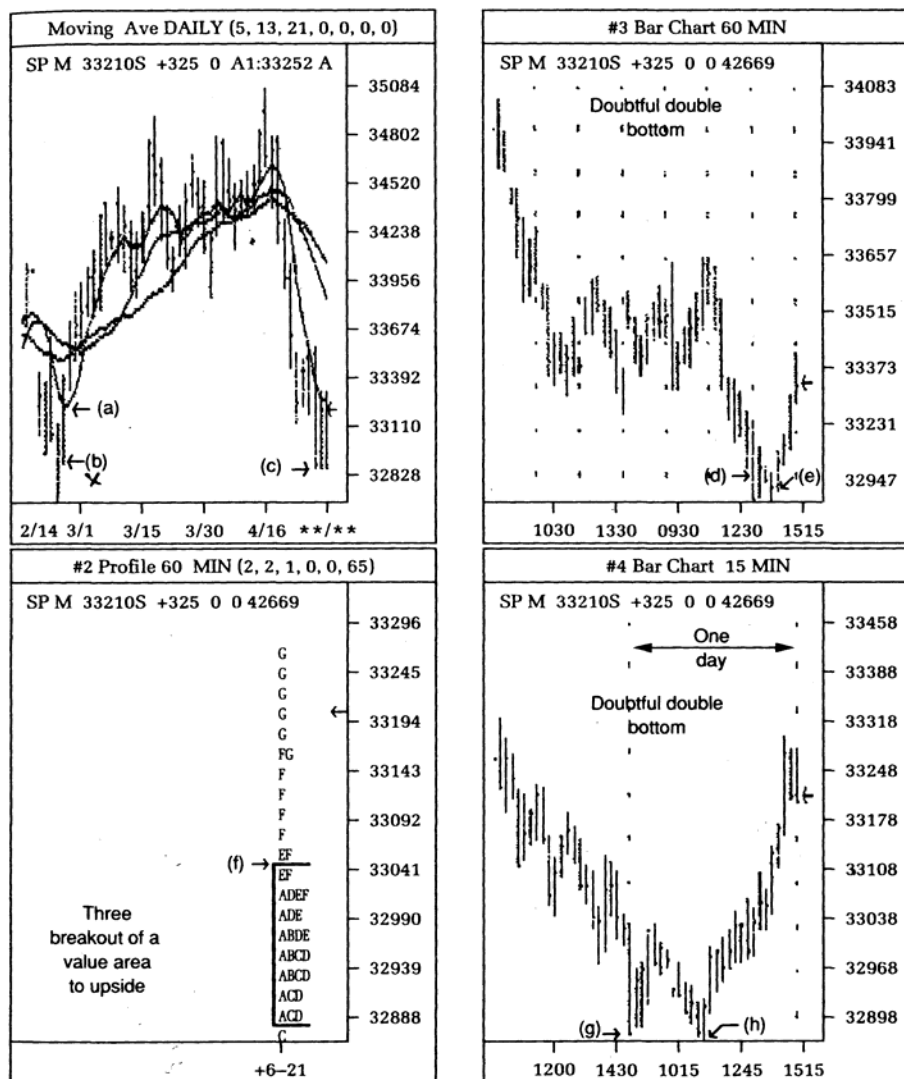


Figure 63.69: © William F. Eng. Reproduced from *The Day Trader's Manual*, John Wiley and Sons, 1993. Used with permission.

15. Analyse the chart shown in Figure 63.70, below, which depicts the gold contract on CBOT. Describe the patterns at (1) and (2). What happened to the contract at (3)? Did the pattern predict this?

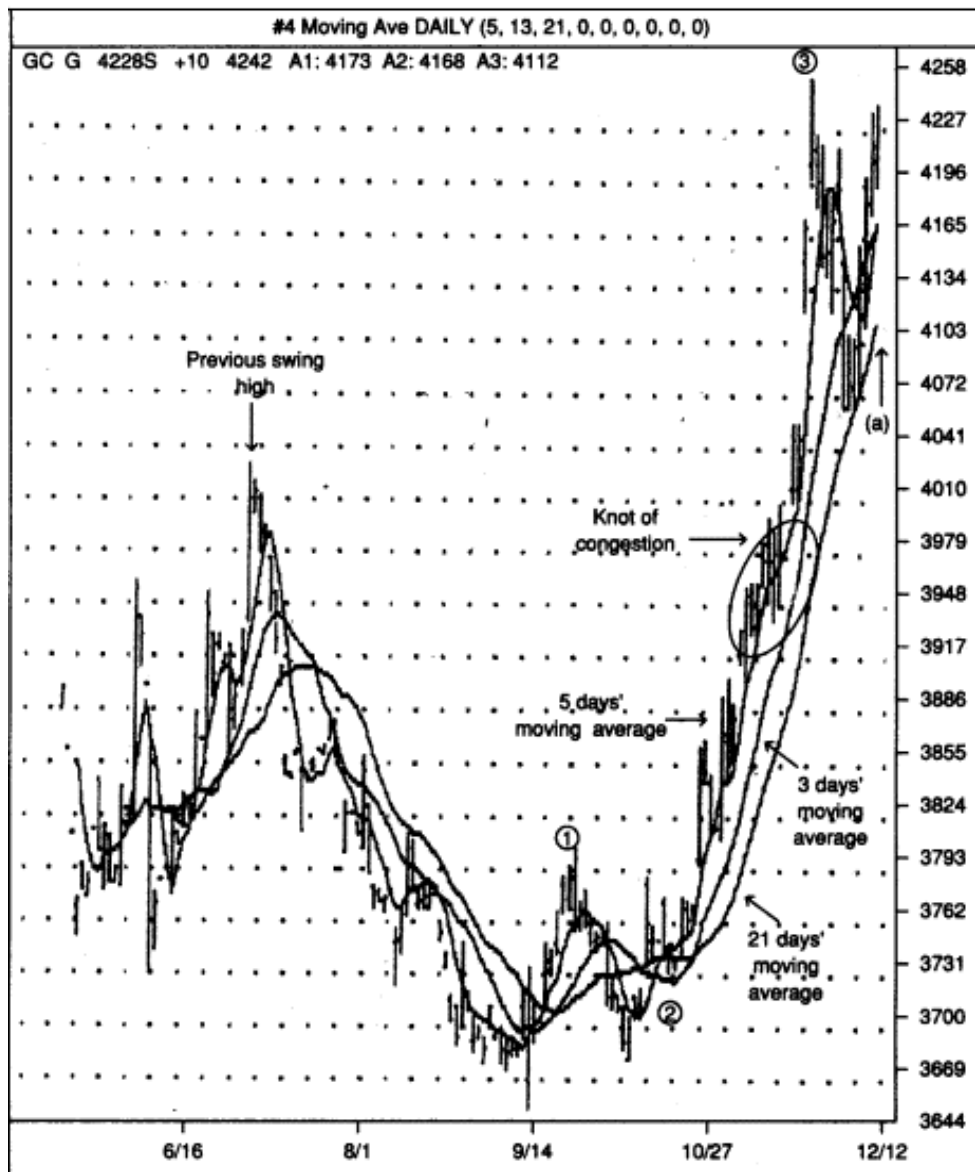


Figure 63.70: © William F. Eng. Reproduced from *The Day Trader's Manual*, John Wiley and Sons, 1993. Used with permission.

16. Figure 63.71, below, shows charts for the Japanese yen contract. All the charts are five-minute bars. Gann lines have been drawn onto the charts. Is the Gann line at L1 a valid trend line? What is the purpose of the upper trend line at L2? What are the resistance levels as indicated by the line? In the first chart, what is being formed at points A, B and C?

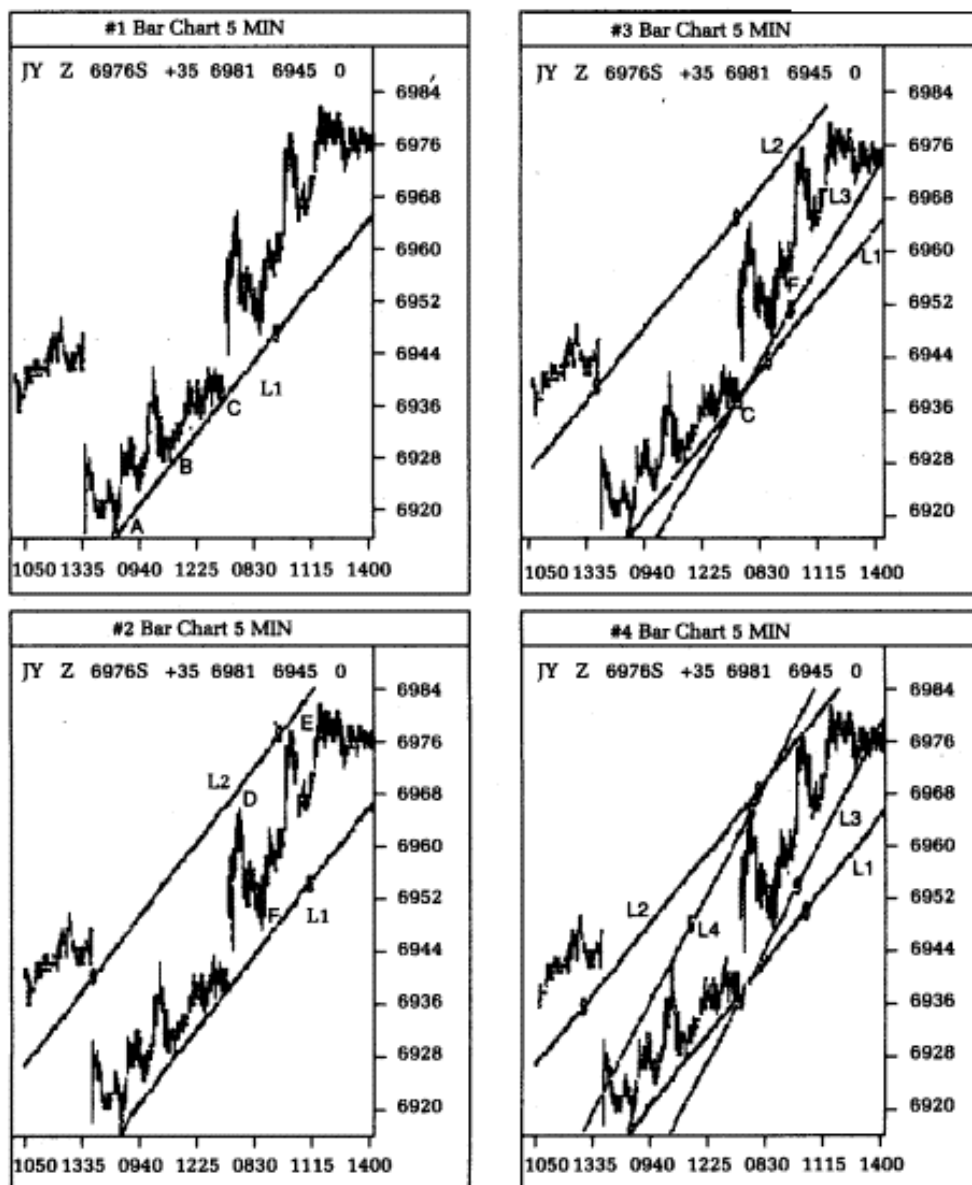


Figure 63.71: © William F. Eng. Reproduced from *The Day Trader's Manual*, John Wiley and Sons, 1993. Used with permission.

17. Consider Figure 63.72 below. Applying Elliott wave theory, analyse and discuss the patterns.

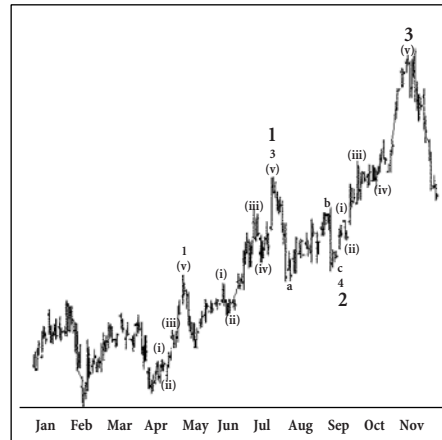


Figure 63.72

18. List the strengths and weakness of moving average indicators.
19. How does one construct a point-and-figure chart? Consider Figure 63.43; what signals (if any) are being given?
20. What is a “petrifying pattern”? When would one expect to see it?
21. Discuss the signals being indicated by the chart at Figure 63.73, which is a chart for the S&P500.

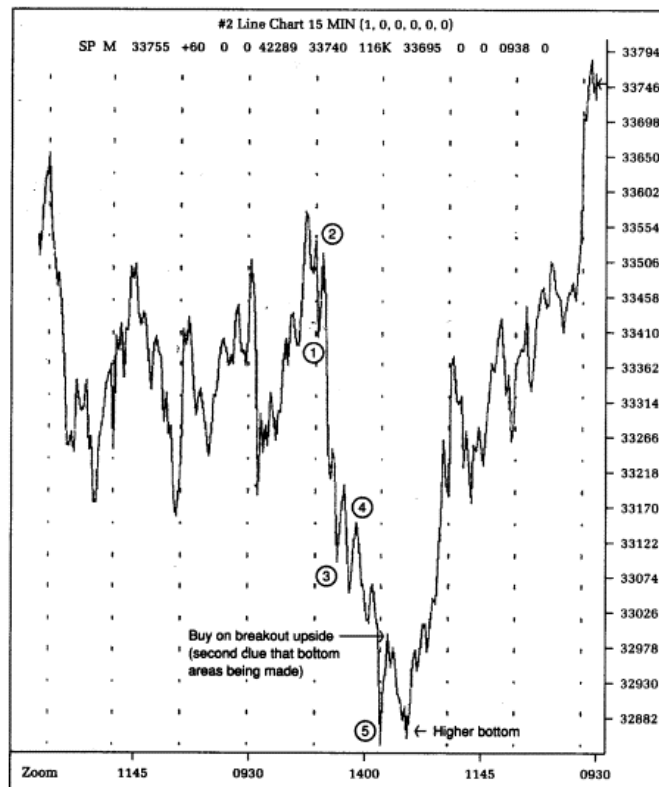


Figure 63.73: © William F. Eng. Reproduced from *The Day Trader's Manual*, John Wiley and Sons, 1993. Used with permission.

Selected bibliography

- Bayer, G., *Stock and Commodity Trader's Handbook of Trend Determination*, Author, 1940.
- Belveal, L.D., *Charting Commodity Market Price Behaviour*, Commodities Press, 1969.
- Bernstein, J., *Short-term trading in futures*, Probus Publishing, 1987.
- Cleeton, C.E., *Strategies for the Options Trader*, John Wiley and Sons, 1979.
- Davis, G.A., Jacobson, M.A., *Stock Option Strategies*, Badger Press, 1976.
- Edwards, R.D., Magee, J., *Technical Analysis of Stock Trends*, 5th edition, John Magee Publishing, 1973.
- Eng, W.F., *The Technical Analysis of Stocks, Options and Futures: Advanced Trading Systems and Techniques*, Probus Publishing, 1988.
- Gann, W.D., *The Tunnel Thru the Air or Looking Back from 1940*, Financial Guardian Publishing, 1927.
- Gastineau, G.D., *The Stock Options Manual*, McGraw-Hill, 1979.
- Hagin, R.L., *The Dow Jones-Irwin Guide to Modern Portfolio Theory*, Dow Jones-Irwin, 1979.
- Howard, B.J., *The Price Spiral Method*, Windsor Books, 1988.
- Jiler, W.L., *How Charts can Help you in the Stock Market*, Trendline, 1962.
- Kaufman, P.J., *Commodity Trading Systems and Methods*, John Wiley and Sons, 1978.
- Krow, H., *Stock Market Behavior: The Technical Approach to Understanding Wall Street*, Random House, 1969.
- Luskin, D.L., *Index Options and Futures: The Complete Guide*, John Wiley and Sons, 1987.
- McEvers, J. (ed.), *Financial Astrology for the 1990s*, Llewellyn Publications, 1989.
- Murphy, J.J., *Intermarket Technical Analysis: Trading Strategies for the Global Stock, Bond, Commodity and Currency Markets*, John Wiley and Sons, 1991.
- Murphy, J.J., *The Technical Analysis of the Futures Markets*, New York Institute of Finance, 1986.
- Plummer, T., *Forecasting Financial Markets: Technical Analysis and the Dynamics of Price*, John Wiley and Sons, 1991.
- Prechter, R.R., Jr., Frost, A.J., *Elliott Wave Principle: Key to Stock Market Profits*, New Classics Library, 1978.
- Prechter, R.R., Jr., *The Major Works of R.N. Elliott*, New Classics Library 1980.
- Pring, M., *How to Forecast Interest Rates*, McGraw-Hill, 1981.
- Tso, L., *Complete Investor's Guide to Listed Options*, Prentice Hall, 1981.
- Zwieg, M.E., *Understanding Technical Forecasting*, Dow Jones, 1980.

Part XI Introduction to Credit Derivatives

Credit derivatives are instruments that were originally developed to enable financial institutions to lay off and hedge credit risk exposure. During the late 1990s there was a marked rise in both volumes and liquidity of credit derivatives. The global credit derivatives market has expanded rapidly and what was once viewed as an exotic instrument is now used by a wide range of financial institutions and corporates. The majority of market participants remain banks however, and they are predominantly end-users of the instruments rather than market makers. The level of liquidity will vary considerably according to the market and type of instrument that is traded.

In Part XI of the book we introduce the subject of credit derivatives, and their application in the bond markets. A wide range of instruments fall into the category of “credit derivatives”. One of the more commonly used instruments is the *total return swap*, together with the *default swap* and the *credit linked note*. The key issues concerning credit derivatives are the need to ensure that one has a robust way to price them, and an effective way to manage a credit book. However as credit derivatives are still a relatively new instrument, banks are still developing both their systems, risk management processes and pricing models as they become more familiar with the product. The following three chapters review the main instruments as well as the pricing methodology used by some banks in the market. A more rigorous technical approach is contained in Choudhry *et al.* (2001).

64 Introduction to Credit Derivatives

In this chapter we introduce and review the market in credit derivatives, which are a relatively new instrument but have grown in importance fairly rapidly. They are increasingly common instruments, and are closely connected with the global market in corporate debt. Events associated with the Russian bond market in 1998 and subsequent bond market volatility led to a greater emphasis on the reduction and control of credit risk exposure, and credit derivatives are one way for banks to manage their credit risk.

64.1 Overview

64.1.1 Introduction

The emergence of a market in credit derivatives is one of the most important recent developments in financial risk management. Credit derivatives offer bankers a new way to establish themselves as intermediaries in the credit market. Using financial engineering techniques imported from other derivative markets, banks are busily transforming some readily definable blocks of credit risk into the kind of standardised credit-linked securities that investors demand. Credit Derivatives are bilateral financial contracts that transfer credit default risk from one counterparty to another. They represent a natural extension of fixed income (and equity) derivatives in that they isolate and separate the element of credit risk (arguably the largest part of a bank's risk profile) from other risks, such as market and operational risk. They exist in a variety of forms, perhaps the simplest is the *credit default swap*¹ which is conceptually similar to an insurance policy taken out against the default of a bond, for which the purchaser of the insurance pays a regular premium. However credit derivatives are different from other forms of credit protection such as guarantees and mortgage indemnity insurance because:

- the borrower is generally asked for a mortgage indemnity policy, or a guarantee;
- the credit derivative is requested by the lending bank, and the borrower doesn't have to know that the transaction has taken place;
- in theory credit derivatives are tradable, other forms of protection are generally not.

Unlike market risk where traders can move in or out of liquid markets in relatively homogeneous products, credit derivatives are long term illiquid investments. Each borrower is different and presents unique credit risk issues that cannot easily be compared to other parties. Unlike most other over-the-counter markets, there is no one single method used to price credit derivatives, and banks have adopted a number of different approaches to pricing. We consider some of these approaches later on in the chapter.

In this chapter we consider the main types of instrument traded in the market.

The currency and bond market volatility in Asia in 1997 and 1998 demonstrated the value of credit derivatives. For example in 1998 the International Finance Corporation of Thailand bought back \$500m of bonds several years before maturity because of a graduated put provision that was exercisable if the bank's credit rating fell below investment grade; the bond would have paid out an additional 50 basis points of yield if the bond fell two levels in credit worthiness and 25 basis points per additional level until the put threshold below investment grade was reached. This in fact occurred when Moody's re-rated Thailand to Ba1 grade. In volatile markets, investors are generally happy to give up yield in return for lower credit risk. Thus financial institutions have started focusing on credit as a separate asset class rather than treating counterparty credit risk as one of the risks associated with an asset.

In Europe the introduction of the euro has accelerated the development of a euroland corporate bond market, and inflation has remained relatively low, as has the euro interest rate; one consequence is that banks can no longer trade interest rates as profitably as they did in the past. At the same time countries are following the lead in the US and to an extent in the UK of maintaining control of budgets, paying off public sector debt so that the supply of

¹ Also known as *credit swaps* or *default swaps*.

government bonds has begun to decline. The euroland corporate bond market is in the process of changing from a predominantly high credit quality one to a more diverse one, with a range of credits similar to that found in the US. The higher credit risk associated with a more diverse corporate bond market can be hedged with credit derivatives.

A bank can use credit derivatives as part of its portfolio credit risk management. Corporates on the other hand can use them to hedge against default by any of their suppliers or creditors or joint venture partners. They also have regulatory applications; for instance under the current BIS capital rules, corporate risk is fully weighted at 100% whereas bank risk is weighted at 20%. Banks with capital constraints can get exposure via credit derivatives through other banks, so for instance in the recent past many Japanese banks have had to borrow funds at high rates because of the general problems of the Japanese banking industry. A credit derivative would give such banks the economic exposure to loans without having to fund them.

Another use of credit derivatives is as a hedge against a change in credit rating of an issuer, rather than outright default, as the price of the credit swap will move as a result of the rating change and the holder may be able to realise a gain, thus compensating for any loss suffered from a rating downgrade.

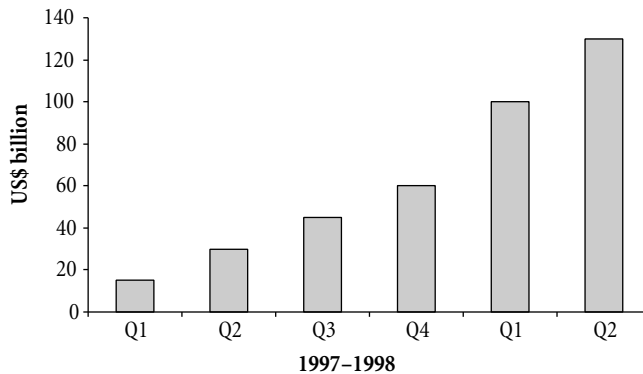


Figure 64.1: Credit derivative volumes 1997–1998.

Source: Kasapi 1999.

64.1.2 Credit derivative instruments

Credit derivatives exist in a number of different forms. In this section we introduce four of the main types. *Total return swaps* (TRS) are instruments that replicate the total performance of a loan asset, thereby allowing the synthetic purchase or investment in credit assets.

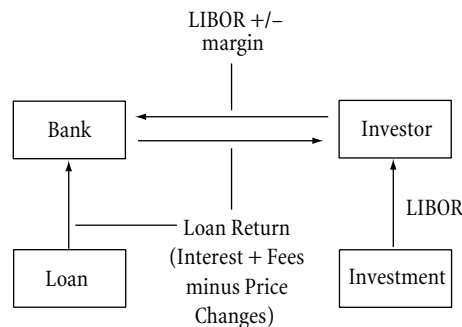


Figure 64.2: Total return swap.

The loan return is generally based on Libor which cancels out the Libor paid by the investor. The loan is marked to market at predetermined dates and the buyer receives the asset depreciation or the seller receives the asset appreciation, depending on what the price of the asset is at the time of the maturity of the swap. The TRS creates a synthetic long position for the protection seller, allowing the protection seller to receive the economic benefit of a loan without actually funding it. For example, consider a situation where ABC Bank plc, a clearing bank, has a loan

from a subsidiary of one of its core clients. The bank does not want to remain exposed to the subsidiary because it is approaching its internal limit on exposure to the parent company's industrial sector; however it does not want to sell the loan or ask for a guarantee from the parent because it does not want to endanger its relationship with the parent company. Now assume that another bank, XYZ Bank plc wants to increase its exposure to the same industry. It therefore executes a TRS with ABC Bank plc, giving XYZ plc the economic exposure to the loan to the subsidiary company, while the loan remains on ABC's balance sheet. This suits the requirements of both banks. The borrowing company is unaffected.

A *credit spread contract* isolates and captures value from any changes in credit quality, dependent of changes in interest rates.

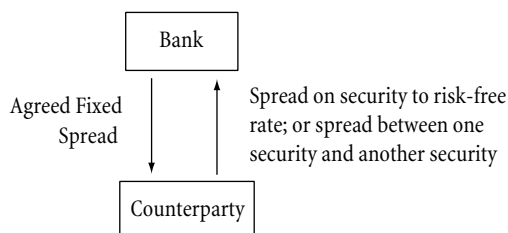


Figure 64.3: Credit spread contract.

Credit spread options are contracts that incorporate payoffs linked to changes in credit spreads on a specified asset. A *call spread option* is the right to buy the spread and benefits from a decreasing spread, while a *put option* is the right to sell the spread; it benefits from an increasing spread. These instruments allow debt issuers to lock in financing costs when rates are low, even though they may not plan to raise finance for a period into the future. For example in 1997 Hilton Hotels launched an offer of 10-year \$300m credit sensitive notes. As investors were worried about the impact of a possible Hilton take-over of ITT, the issue carried a provision that adjusts the yield upwards if Hilton made a big acquisition or its rating fell below investment grade within 3 years.

Credit-linked notes allow banks to transfer the default risk and funding of loans to investors via a special purpose vehicle. The bank retains the right to substitute loans. Also, the investors do not know the identity of individual loans, only that the portfolio of loans in a special purpose vehicle (SPV), formed for the purpose, that is structured to a minimum high quality credit rating.

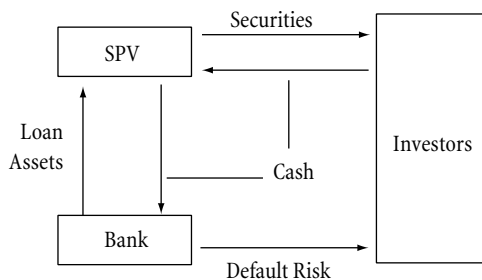


Figure 64.4: Credit-linked note.

They are set in the following way:

- the issuer sells loans to an SPV with a small capital cushion provided by the issuer;
- investors buy securities from the SPV with the money received paying for the loans from the bank;
- each tranche of securities represents loans of a particular credit rating with only the trustees knowing the name of the borrower behind the loans. The lower the credit rating, the higher the return on the tranche. The bank has the right to substitute loans as long as the rating of the tranche stays the same;
- any default on the loans is borne by the investors.

Credit default contracts are the most popular form of Credit Derivatives, with the British Bankers Association estimating that these represented over half of the Credit Derivatives booked in the London market during 1999.

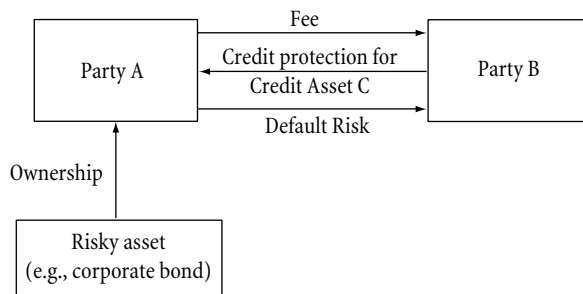


Figure 64.5: Credit default swap.

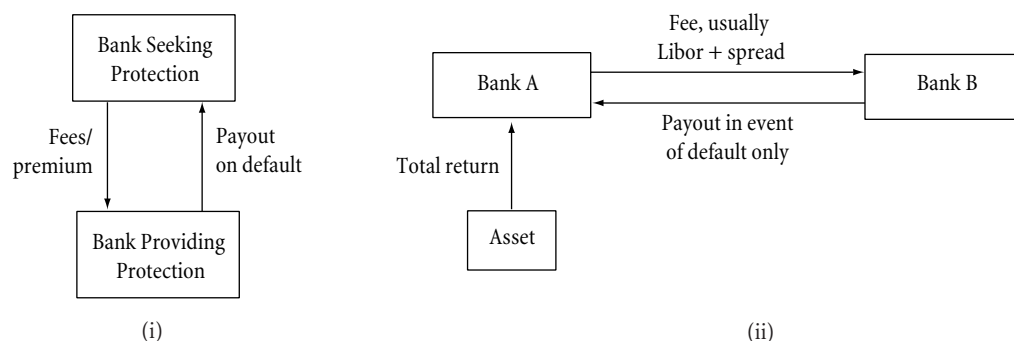


Figure 64.6: Credit default swap (i) and (ii).

Credit default swaps can be in vanilla form or in the form of default options. In a default swap, in the event of default the protection buyer delivers to the protection seller the asset that has been protected, and the protection seller pays the protection buyer the par value of the asset. If there is no default during the term of the swap, the protection seller will not pay out anything. During the life of the swap the protection buyer pays a regular spread, derived from the underlying asset. The level of this spread is the price of the credit swap and is a function of the perceived riskiness of the asset being protected. In a default option the protection seller receives a premium and agrees to pay in the event of default up to a predetermined limit. So for example if a bank has decided that it is overexposed to a particular market, but does not wish to or cannot sell any of the assets, it can buy a default option to protect its position.

64.2 Pricing

Let us recap on the four main types of credit derivatives.

- Credit default swaps allow separate trading of the default risk. Therefore, the pricing of the default swap should reflect the stripped-out value of the credit spread (which is effectively the compensation for assuming credit risk).
- Total return swaps are a mechanism for synthetically creating a loan or security. Given that the end economic outcome of a total return is to create the equivalent of the underlying loan or security (assuming that the total return swap is combined with a cash investment), it is equivalent to an investment in a risk-free cash asset combined with the sale of a credit default swap where the investor assumes the risk of default. The incremental return (above the risk-free rate) should reflect the value of the credit default swap or its equivalent stripped-out credit spread as noted above.

- A credit-linked note can be viewed as a collateralised credit default swap. The buyers of the note give the note seller cash and receive the economic performance of the assets behind the note. If one of the assets default the investors bear any loss.
- Credit spreads are inherent in both credit default and total return swap products as the essential return for credit risk. Credit spread products allow the trading of these spreads at forward dates in either forward or options formats. The pricing of the spot or current credit-spread should reflect the default risk. This is because if the credit spread is stripped out the remaining security should be equivalent to the risk-free return.

The above analysis highlights the fact that the key to pricing credit derivatives is the pricing of default risk and the derivation of credit spreads.

There is already a well-established industry in the analysis of credit. Banks, insurance companies, and credit rating institutions have departments that carry out evaluations of individual institutions. For example, a credit officer of a bank analyses fundamental data about a firm, industry, or country such as financial statements and economic data to determine the creditworthiness of a client. The findings of their efforts translate almost directly into the price of a loan or bond, or a decision on whether or not to buy a bond or make a loan. However this approach is not sufficient for pricing credit derivatives, because we require a model that describes the dynamics of credit risk. These dynamic models start with the creditworthiness of a borrower as above then describes the possible ways that the creditworthiness may evolve over time. This evolution depends on:

- the default risk of the borrower;
- the recovery if default should occur;
- the credit spread, which is the compensation for assuming credit risk.

64.2.1 Credit spreads

Credit spreads represent the margin relative to the risk-free rate designed to compensate the investor for the risk of default on the underlying security. Theoretically, one can construct a credit-risk adjusted yield curve as an input to a pricing module using the following calculation:

$$R_{spread} = r - r_f \quad (64.1)$$

where

R_{spread} is the credit spread
 r is the yield on the loan
 r_f is the risk-free yield.

This is analogous to the term structure of interest rates which is the main pricing tool for interest rate swaps. Practically, creating a credit-risk adjusted yield curve is hindered by:

- the absence in money markets of well-defined credit spread structures;
- the absence of a complete term structure of credit spreads with rather infrequent data points.

Where the term structure of credit spreads is not observable or the data is for any reason not available, credit spreads must be estimated. Market practitioners use the concept of a risk-neutral credit spread to overcome these problems. This approach involves estimating a credit spread which compensates the investor for the default risk assumed. The underlying logic being that the risk neutral spread would, in an efficient capital market, make an investor *indifferent* between a risky bond and a risk-free security. The approach is outlined in Appendix 64.1. In theory the credit spread should increase in line with increasing default risk and maturity; in practice, the credit spread appears to increase with maturity only for higher credit quality bonds. The credit spread decreases for lower credit quality bonds.

This practical observation reflects:

- the concept of “crisis at maturity”, predicated on the risk generated by the liquidity pressures created by the need to refinance near-term maturing debt which is often confronted by lower credit quality and highly leveraged firms;
- the pattern of marginal default risk for lower credit quality firms; however, while higher in absolute terms, the marginal default risk for lower rated firms decreases with maturity. In contrast the marginal default risk of higher rated firms increase with maturity.

The pattern of marginal default risk for lower credit quality firms is consistent with the following:

- the life cycle of ratings outlook, whereby lower rated firms face higher short term risk which is resolved by survival or default;
- mean reversion processes in rating outlook, where lower rated issuers improve, middle rated issues stay the same and higher rated firms tend to decline on average.

64.2.2 Pricing credit derivatives using historical data

The historical data method combines the loss exposure, default probability and recovery rate into a quantification of the credit risk of the transaction and the fair value which compensates for the risk. These three main inputs, which are described below, can be estimated from data available from external sources such as rating agencies, or internal information if available. Where portfolios are involved, the various correlations between the individual components, most significantly the default correlations, must be incorporated.

One input is the loss exposure; this is made up of expected, unexpected and extreme loss.

$$E[L] = l \times p(def) \times (1 - rec) \quad (64.2)$$

where

$E[L]$ is the expected loss
 l is the loss exposure
 $p(def)$ is the probability of default
 rec is the recovery rate.

Unexpected loss is often calculated as the expected loss plus the standard deviation of the expected loss based on a nominated level of confidence (say 95% or 99% confidence).

From market observation we conclude from the experience of credit losses that:

- the high frequency of losses resulting in small losses can be interpreted as mark to market losses caused by ratings downgrade;
- the low frequency of large losses can be interpreted as losses caused by default.

The graph is unlike the bell shaped curve of the normal distribution (the distribution used for modelling market risks).

Credit risk has both similarities and differences to other risks such as interest rate risk or equity risk. These latter risks are collectively known as market risks. The similarities are that credit risk can be traded, for example buying a bond is a means of obtaining credit exposure to a particular issuer. However, there are two main differences to market risk:

- the market for credit risk is illiquid;
- changes in credit risk often cause the price of the associated debt instrument to jump. And that jump can be very large, particularly when it is caused by default.

From Chapter 37 we know that the characteristics of market risk include:

- correlation between market risks particularly within, but even extending across, asset classes;

- the presence of these correlation relationships facilitates hedging as well as trading. By taking a position in one asset, which is correlated in price with the asset exposure one is seeking to hedge, risk can be reduced.

Default risk is the probability that a counterparty will fail to meet its obligations and is characterised by the following:

- the default risk for any borrower is dynamic, and subject to large fluctuations;
- the variability of default risk within a loan portfolio is substantial (that is, the largest default probability is significantly larger than the smallest default probability);
- the correlation between default risks of different borrowers is generally low (that is, low joint default frequency), although it can be significant for related companies, and smaller companies within the same domestic industry sector; this lack of correlation increases the difficulty of hedging portfolio default risk with tradable instruments but diversification can be used.

Recovery risk is the possibility that the recovery value of the defaulted contract may be less than its promised value. Experience suggests that recovery value is highly uncertain and may only be determined after a (possibly lengthy) negotiation process. Recovery risk is characterised by the following:

- in default the recovery value can be very large, particularly when it is caused by default;
- furthermore, the recovery value is highly uncertain and may only be determined after a (possibly lengthy) negotiation process.

The approach outlined here focuses on the standalone pricing for an individual transaction. It does not incorporate default correlations within a portfolio of credit exposures. Furthermore, it focuses purely on the risk of default and does not encompass the risk of changes in credit risk short of default. Such changes in credit risk (for example, as evidenced by rating migrations) have the potential to impact upon the value of the security materially. More sophisticated models would therefore need to encompass both the concept of rating migration for an individual counterparty and the impact of default correlations within a portfolio.

Appendix 64.4 sets out a simple calculation of the pricing of credit risk in a transaction.

64.2.3 Risk-neutral pricing

Risk-neutral pricing is applied to market risk instruments because the product can generally be hedged in such a way that the portfolio (instrument and hedge) is at least instantaneously risk free. If one can hedge credit risk, then risk-neutral pricing is equally appropriate for credit derivatives. The risk-neutral approach creates a structure to hedge the exposure, and the price is derived from this hedge. However, credit markets are not complete, for example, one may not efficiently “short” a corporate bond. So the assumptions in risk-neutral pricing theory are frequently not satisfied. On the other hand, risk-neutral pricing remains a useful tool. For credit derivatives it can be used to give a good approximation of the correct price. In practice, the estimated price may be adjusted to account for known shortcomings or vagaries in the model. In any case, so far the market seems to have decided to apply the risk-neutral model: most of the pricing models for credit derivatives are based on risk-neutral pricing theory.

To apply risk-neutral pricing theory, a model must start by modelling the risks that affect the price. This amounts to selecting a process that describes the evolution of future events which affect the cash flows of credit risky instruments. These future events may simply be future prices of credit risky instruments (the credit spread curves discussed above), or they may be future defaults and recoveries. Credit risk and market risk are the two risks that most obviously affect the price of a credit derivative. Usually the market risk is simply interest rate risk, for example the price of a corporate bond depends on the general level of interest rates. Thus all credit risk models should be integrated with interest rate risk models to various degrees. The second step in applying risk-neutral pricing is to replicate synthetically the credit derivative by means of a dynamic, self-financing portfolio of the underlying credit instruments and money market accounts. This step is implicit in the calibration process, whereby the model is adjusted so that it correctly prices known credit instruments. In the case of credit derivatives the underlying credit instruments are bonds or loans. As above, the bonds and/or loans are assumed to have an expected return equal to

the riskless rate. In the end the price of the credit derivative is the expected value of the present value of the future cash flows. Future cash flows are discounted back at the riskless rate.

Appendix 64.4 works through an hypothetical example. In general however pricing credit default swaps is not as simple as the example suggests. It was mentioned in the example that the bond tenor may well be longer than the swap tenor, but it is also often the case that the bond is not trading at par, or that coupon payments are fixed instead of floating. The price also depends on the particular terms of the swap agreement and the reference security.

64.2.4 Pricing using the asset-swap curve

An asset swap is a fixed-for-floating interest rate swap. For example, an investor with a \$10m five-year US Treasury bond yielding 6.5% payable semi-annually may enter into a swap (usually through a bank or broker counterparty) where it receives a floating rate from, say, a corporate rated A, through a floating-rate note. Increasingly, all credit assets are traded on the basis of a spread over Libor. This reflects the increased availability and liquidity of global swap and derivative markets which allow the ready transformation of assets irrespective of currency and interest-rate basis into a floating rate asset in US dollars in the first place and through a cross-currency basis swap into other currencies.

The availability of asset swaps and the depth and size of the market means that it provides a ready and reasonably priced transparent means for establishing the relative value of credit assets. This characteristic dictates that the asset-swap market currently serves as the principal market pricing source for credit pricing. The asset-swap market provides the benchmark returns which are used to derive the default swap values. In effect, it places a floor under these values as it provides a traded benchmark, and it allows a hedging mechanism for the traders.

64.2.5 Summary of main issues

Despite the limitation of risk-neutral pricing theory it is being used by a number of market participants pricing (with care) to price credit default swaps and other credit derivatives. It gives a good bound for the price of the credit default swap, but it does not give a single price because the assumptions of risk-neutral pricing (market completeness, liquidity and lack of transaction costs) do not apply. To reduce these shortcomings, pricing models are calibrated to the bond market, as opposed to calibrating to the historical price of credit. Sometimes looking at historical default rates and recovery rates serves as a good check of the final price. Secondly, the credit default swap is somewhere in-between a cash instrument and a derivative product. The more complicated credit default swaps may not be perfectly replicated in the cash markets, but the cash markets give some guidance to the correct price of default swaps.

64.3 Regulatory issues

Currently credit derivatives are traded as part of a global market, but under sometimes differing local regulations. For instance the European Union's Capital Adequacy Directive (CAD I) does not account for credit derivatives. The CAD I rule does not allow for default probability, expected loss or other developments in credit portfolio management. However, CAD II introduced during 1999 does provide more appropriate capital treatment. National regulators are also introducing new treatment rules. The UK's Financial Services Authority (FSA) released details of its revision of the regulatory treatment for credit derivatives for effect from September 1999. The FSA stated an intention to better reflect both the risks and the risk reducing properties that credit derivatives can have. The FSA have made significant steps towards the industry position, and have gone as far as they can within the limits set by the Basle Accord and EU directives. The new treatment included four key departures from prior practice:

- all credit default products are eligible for trading book treatment;
- capital weighting for a maturity mismatch on the banking book has been reduced from 100% to 70%. If there is no mismatch the weighting is 20%. The double charge for hedges on the trading book where there is a maturity mismatch has been removed. Previously there was not any offset recognised between the hedged positions and an additional specific risk charge was applied to the hedging instrument;
- simpler VAR models can now be used allowing partial hedge recognition to quantify risk offsets for partial hedges involving credit derivatives. This will allow participation from a broader number of Credit Derivative users;

- baskets products: if all reference assets are investment grade, then the capital treatment will be a specific risk charge rather than one based around the worst credit in the basket. When below investment grade the FSA will adopt the banking book approach of levying additional charges for each credit.

Financial regulators in other countries including the US, France, Canada, and Germany have also made favourable changes to their treatment of credit derivatives. The FSA revision has some common elements with these countries. Other issues include:

- **legal:** are credit derivatives guarantees or insurance contracts? Credit derivatives are structurally similar to guarantees. If they are treated as guarantees, they could be restricted by the existing case law applying to guarantees. This issue has not yet been addressed by the courts. However, in the UK both the Financial Law Panel and Counsel on behalf of ISDA have confirmed that Credit Derivatives would not constitute insurance business under UK law;
- **documentation:** until the introduction of a confirmation for default swaps to fit under the umbrella of the ISDA Master Agreement in 1999 there was not a uniform standard document for Credit Derivatives. All documents had to be negotiated on a bilateral basis which was expensive and time consuming. There are eight definitions of default listed under the ISDA confirmation: bankruptcy, credit event upon merger, cross acceleration, cross default, down grade, failure to pay, repudiation and restructuring. An ISDA confirmation for total return swaps was due by the end of 2000;
- **pricing:** this is not difficult if an issuer has issued paper across the maturity curve. If not, a price can be found by interpolation or investigating similar issues. However, the further away from the issuer and the relevant maturity, the greater the room for error. There are models available to determine ration migration and default probability (such as CreditMetrics, CreditRisk+) but these suffer from insufficient data. There is the risk of advanced analysis and extrapolation being done on a sample that is too small;
- **banks:** are concerned about selling key relationships. Some banks have inefficient credit monitoring systems, preventing them from determining their overall credit risk. As credit is a long term illiquid risk, credit derivatives cannot be treated like market risk instruments, hence banks might need to reassess their approach to credit risk management before they can use credit derivatives effectively. The secondary loan market is small (only 5% of US bank loans of \$900bn at end of 1998 have been traded) and growing only moderately. Credit derivatives are easier to execute. Banks will need to improve their credit pricing otherwise they will lose the better business to competitors and be left with the poorer credits that they cannot properly assess;
- **moral hazard:** if a bank has bought credit protection expiring tomorrow on a company that has developed serious financial problems it is easier to appoint receivers and pass the problem to the credit protection seller than help the company through its difficulties which is current policy in most banks.

These issues are by no means insurmountable and the market in credit derivatives is set to enlarge considerably in the near future. As previously mentioned the bank lending market is characterised by shrinking margins and consolidation in both the banking and corporate sectors, increasing concentration risk as borrowers restrict their number of counterparty banks. Credit derivatives will allow banks to diversify this risk without turning away prime corporate clients. Other drivers to market growth are:

- as also previously mentioned, the introduction of the euro leading to growth in the European corporate bond market. Credit Derivatives should allow smoother asset allocation and investor portfolio risk/return optimisation;
- pressure from shareholders to increase earnings. Banks will need to manage credit risk more dynamically;
- growing recognition by regulators that the BIS guidelines need updating;
- growth in credit as a separate asset class for the rapidly growing long term investment and savings market, particularly in Europe.

Appendices

APPENDIX 64.1 Risk-neutral spread calculation

The risk-neutral spread calculation typically assumes that we have:

- par bonds;
- a holding period equal to maturity or default (whichever occurs first), and
- risk neutrality and arbitrage free capital markets.

Within this framework, the risk-neutral spread can be as defined at (64.1). The risk-neutral credit spread is a function of the default risk of the risky bond. It is feasible to use marginal default risk and recovery rates estimates to approximate this value. Given the risk-free rate and the measure of the credit spread, we may solve for the yield of the risky asset. This requires weighting each cash flow of the risky bond by the probability of default and the recovery rate to calculate the default risk adjusted risky bond cash flows.

The generation of risk-neutral credit spreads allows the modelling of a complete term structure of risk from limited observations.

APPENDIX 64.2 Modelling default risk

The modelling of default risk is clearly central to pricing credit derivatives. Models for valuing default risk are not unique to credit derivatives, as they affect all financial transactions involving loss exposures in the event of the counterparty failing through insolvency to perform its obligations. There are two general classes of default risk models; one of which is a rating agency-based model that predicts the marginal and cumulative risk of default based on rating categories. The other type is the proprietary model developed for the purposes of default prediction.

Rating agency default models can be used to identify the risk of default for a counterparty with a known current rating. These models are based on historical default experience and incorporate macro-economic cycles of specific default risk as a function of two primary factors: current rating and time to maturity of the obligation. There are two distinguishable types of default risk:

- cumulative risk of default; this measures the total default probability of a counterparty over the term of the obligation;
- marginal risk of default, which measures the change in default probability of a counterparty over a sequence of time periods.

Previous research has shown that cumulative default probabilities increase with a decline in ratings levels, but that marginal default risks *decrease* in the lowest rating categories. The pattern of marginal default probabilities is consistent with the behaviour of credit spreads discussed previously. Where a firm or entity is not rated it is still possible to use rating agency default models and statistics. This will entail a three-step process:

- using the firm's financial data to calculate key accounting ratios. The accounting ratios usually used are those used by the rating agencies themselves;
- the accounting performance as captured by the ratios is compared with the comparable median for *rated* firms in both the industry and the universe of rated entities. The comparison is designed to allow a rating equivalent to be determined;
- based on the *theoretical rating*, the default probabilities appropriate for that particular rating categories are then used.

Proprietary default prediction models are, typically, based on the original thesis by Black and Scholes that the equity in a risky firm is equivalent to a call option on the net asset value of the firm. The net asset value is calculated as the market value of the firm's assets minus the claims on the assets which include traditional financial claims such as debt and other claims including erosion of asset values which may result upon default. Another way of restating this is to view the position of the bond holder as a combination of the long position in the underlying bond plus the sale of a put option on the company's assets where the option has a strike price equal to the value of the debt of the entity.

The dynamics of asset behaviour are as follows:

- asset values evolve over time as a function of volatility of the asset values;
- if asset values are less than the value of claims on the assets (that is, the liabilities) then the firm defaults, which is to say the call option held by the shareholders is abandoned or the shareholders exercise the put option written by the debt providers;
- the extent to which the asset values of the entity are below that of the value of the debt equates to the loss suffered on the bond as a result of default;
- the premium paid by the shareholders for the option which is equivalent to the share price paid by the shareholder is lost in the event of default.

This model allows derivation, calculated from the distribution of asset values, of the default probability as the probability that asset values will be lower than the value of the claims on the asset. The implementation of this type of model requires the estimation or calculation of parameter estimates for asset values, asset value volatility and claims on asset values.

The estimation of the required parameters is subject to some difficulties:

- the market value of real assets is difficult to determine because of the absence of liquid secondary markets, the difficulty in valuing intangible assets and the conventions and assumption underlying the measurement and presentation of accounting-based financial information. This difficulty is compounded by the fact that the asset value must be established to incorporate any potential diminution in the value of the asset in the event of default;
- the volatility of these asset values is similarly difficult to measure. This reflects the absence of traded markets and the requisite levels of price transparency in the real underlying assets;
- the measurement of liabilities is complicated by the fact that the claims may have different maturities and are governed by different credit conditions.

There are a number of estimation approaches which seek to overcome these difficulties, including those which underline this methodology. The asset volatility can be estimated using the option pricing approach developed by Black–Scholes, which captures the relationship between asset value and asset volatility and equity value and equity volatility. Using this approach the available equity value and equity volatility can be used to solve for the market value of asset and asset volatility. This approach requires an initial simplification that the claims are represented by a single liability due at a single date. If this simplification is not applicable then the approach remains the same, but a more complex process is required. The asset value, asset volatility and the cumulative liabilities allow the default risk of the firm to be calculated. The measure is calculated by determining the distance in volatility (standard deviations) measure between the asset value and the point at which the asset value will fail below the liabilities. The default probability is then determined based on this distance to default.

The major advantages of this type of approach include:

- the capacity to derive default within a volatility framework;
- the ability to estimate expected as well as *unexpected* default losses within a probability framework at specified confidence levels; and
- the determination of the cost of dynamically hedging credit risk and trading credit risk.

One difficulty with this type of approach is the requirement that the equity of the firm whose default risk is being modelled be publicly traded. This is necessary for the derivation of equity volatilities which is central to the calculation of default probabilities in allowing the asset price distribution to be generated. In practice, the absence of traded equity can be used by various proprietary models, which are designed to use measures, such as accounting ratios, to act as a proxy for traded equity enabling calculation of the asset price volatility.

APPENDIX 64.3 Modelling recovery rates

The concept of recovery rate focuses on the amount of any loss exposure likely to be recovered from a counterparty following default. In most defaults, often after a significant period, investors in the securities recover some portion of their investment. This recovery may take a number of forms including cash, securities (debt or equity) and occasionally assets of the business. The recovery rate may be defined as the percentage of par value of the security recovered by the investor. There are two separate elements to the recovery rate: first, the recovery rate itself, and secondly, the adjustment for time value reflecting the discounting of the recovery rate from the eventual date of recovery to the date of default. The potential for delay arises from the time taken to complete the legal processes required to facilitate the recovery of amounts owed as well as the time taken to realise the value of the counterparty's assets, if relevant. There are two general approaches to modelling recovery rates:

- using recovery experience on public and/or rated securities collated by the major rating agencies;
- information internal to the organisation based on its experience in the case of default.

Tables 64.1 and 64.2 sets out a recent set of recovery rates published by Moody's Investors Services. The recovery rates are, generally, based on the trading price of the defaulted instrument. This price is used as a proxy of the present value of the expected ultimate recovery. This approach is based on the fact that it provides an immediate measure of recovery and it corresponds to a traded market estimate of the anticipated recovery rate. It is valid insofar as an investor can effectively liquidate any position in the securities and cap its losses or achieve recovery at the recovery level equivalent to the spot default traded price of the bonds. This also has the advantage of avoiding the necessity of tracking payments on defaulted bonds for a potentially long period and then discounting back these payments. It also avoids potential valuation difficulties with securities issued in exchange for the original obligation which may prove problematic to value, given that they are issued by a defaulted issuer.

Class of debt	Recovery rate (%)
Secured	65
Senior unsecured	48
Senior subordinated	40
Subordinated	30
Junior subordinated	16

Table 64.1: Moody's recovery rates 1974–1993.

Class of debt	Recovery rate (%)	Standard deviation (%)
Senior secured bank debt	71.18	21.09
Senior secured public debt	63.45	26.21
Senior unsecured public debt	47.54	26.29
Senior subordinated public debt	38.28	24.74
Subordinated public debt	28.29	20.09
Junior subordinated public debt	14.66	8.67
All subordinated public debt	33.58	23.34
All public debt	41.25	26.55

Table 64.2: Moody's recovery rates 1989–1996.

Class of debt	Average	Median	Standard deviation	Percentile	
				10th	90th
Senior secured	53.11	56.00	24.27	18.50	85.32
Unsecured	49.86	46.56	26.32	11.46	87.88
Subordinated	32.83	30.17	19.67	9.18	60.40

Table 64.3: Moody's defaulted bond price distribution 1974–1995.

The recovery rates identified imply the following:

- the recovery rate, predictably, is related to the seniority of the obligation and its place in the counterparty's capital structure;
- the distribution of recovery rates does not appear to be normal;
- the recovery rates exhibit significant variability;
- the volatility statistics indicate that subordinated debt has a lower volatility than other causes of debt, indicating that the value of these obligations in the event of default is likely to result in recovery rates close to its indicated mean.

APPENDIX 64.4 Example of default pricing

The pricing is done for two different maturities, five and 10 years. In each case, the following assumptions are made:

- the credit exposure for loans is assumed to be the face value of the transaction, while for derivatives it is assumed to be percentages of face value (a lower percentage for average expected exposure and a higher for expected worst case exposure);
- the expected loss is calculated based on the Moody's expected default probabilities for the relevant maturity for the particular credit rating;
- the unexpected loss is subjectively set at a multiple of the expected loss;
- the recovery rate assumed is that for unsecured obligations;
- the cost of capital is assumed to be 15% p.a. pre-tax and the interest rate for amortisation purposes is set at the swap rate for the relevant maturity.

AA Rated Credit

Example (i)

Counterparty	AA
Transaction final maturity (years)	5.00
Type of transaction	Static loan or security exposure
Seniority of exposure	Senior, unsecured
Interest rate	6.500%
Cost of capital	15.00%
Face or notional value	£1,000,000
Average credit exposure (fully drawn)	£1,000,000
Worst case credit exposure (at default)	£1,000,000

Probability of counterparty default

Cumulative default probability	0.40%
Worst-case default probability	1.00%
Recovery rate	47.54%

Default Pricing

Expected Loss

Expected loss amount (pre-recovery)	£4000
Expected loss amount (post recovery)	$= 4\,000 \times (1 - 0.4754) = £2098.40$
Expected loss percentage (post recovery)	0.2098%

Unexpected loss

Unexpected loss	£10,000
Economic capital requirement	£10,000
Capital cost recovery (per annum)	£1,500
Capital cost recovery (£ present value)	£6,233.52

Unexpected loss charge	0.6234%
Total charge (%): (= Expected loss amount (post Recovery + Unexpected loss charge))	0.8332%
Total charge (£)	£8,332
Total charge (% per annum)	0.2005%

Note that 0.8332% is the present value of 0.2005% paid annually at 6.5% interest over 5 years.

Example (ii) Default pricing

Counterparty	AA
Transaction final maturity (years)	10.00
Type of transaction	Static loan or security exposure
Seniority of exposure	Senior, unsecured
Interest rate	6.500%
Cost of capital	15.00%
Face or notional value	£1,000,000.00
Average credit exposure (fully drawn)	£1,000,000.00
Worst case credit exposure (at default)	£1,000,000.00

Probability of counterparty default

Cumulative default probability	1.13%
Worst-case default probability	2.26%
Probability of positive exposures	100.00%
Recovery rate	47.54%

Default Pricing

Expected loss amount (pre-recovery)	£11,300
Expected loss amount (post recovery)	$= 4,000 \times (1 - 0.4754) = £5927.98$
Expected loss (post recovery)	0.5928%

Unexpected loss

Unexpected loss	£22,600
Economic capital requirement	£22,600
Capital cost recovery (per annum)	£3,390
Capital cost recovery (£ present value)	£24,370.13
Unexpected loss charge	2.4370%
Total charge (%)	0.8332%
(= Expected loss amount (post-recovery) + Unexpected loss charge)	£30,298

Total charge (% per annum)	0.4215%

3.0298% is the present value of 0.4215% paid annually at 6.5% interest over 10 years.

APPENDIX 64.5 The credit default swap

This example shows how risk-neutral pricing theory can be applied to price a credit default swap. The price is obtained by explicitly constructing a hedge from the underlying cash market instruments.

Suppose that two counterparties, a market maker and an investor, enter into a two-year credit default swap. They specify what is called the *reference* asset, which is a particular credit-risky bond issued by a third-party corporation or sovereign. For simplicity, let us suppose that the bond has exactly two years' remaining maturity and is currently trading at par value. The market maker agrees to make regular fixed payments to the investor for two

years, with the same frequency as the reference bond. In exchange the market maker has the following right. (For simplicity assume default can occur only at discrete times, namely, at the times just before the coupon payment is due.) If the third party defaults at any time within that two years, the market-maker makes their last regular fixed payment to the investor and puts the bond to the investor in exchange for the bond's par value plus interest. The credit default swap is thus a contingent put – the third party must default before the put is activated.

In this simple example there is little difference in terms of risk between the credit default swap and the reference bond. Because the swap and the bond have the same maturity, the market maker is effectively short the bond and the investor is long the bond. (In the real world, it is often the case that the bond tenor is longer than the swap tenor. In this case the swap counterparties have exposure to credit risk, but do not have exposure to the full market risk of the bond.) The simplicity of this example helps clarify how the instrument is priced. Pricing the credit default swap involves determining the fixed payments from the market-maker to the investor. In this case it is sufficient to extract the price from the bond market. One does not need to model default or any other credit risk process. To apply risk-neutral pricing theory one needs to construct a hedge for the credit default swap. In this simple example, it is sufficient to construct a static hedge. This means the cash instruments are purchased once, and once only, for the life of the credit default swap; they will not have to be sold until the termination of the credit default swap. The hedge is different for the market-maker and investor. If the market-maker were to hedge the credit default swap, then it would need to go long the bond. The market-maker borrows money in the funding markets at Libor and uses those funds to purchase the corporate bond, which pays Libor + x basis points. The hedge is paying the market-maker a net cash-flow of x basis points.

If the reference asset does not default, then at the termination of the swap the market-maker simply unwinds the hedge at no net cost. If the reference asset defaults, then the market-maker immediately unwinds the hedge. It delivers the bond to the investor in exchange for the par amount, and repays its borrowed funding with the principal. This perfectly hedges the market-maker's risk in the credit default swap.

Now apply the same reasoning to the investor. If the investor were to hedge the credit default swap, then they would need to short the bond. In order for the investor to borrow the bond, they must lend the face value of the bond to the repo market at what will certainly be a sub-Libor rate. Suppose the investor lends the par value of the bond at Libor – y basis points in exchange for borrowing the bond. The bond lender keeps the bond's coupon payments. The value y can be quite large, for two reasons. First the investor is making a collateralised loan. The bond is collateral against the loan, so the borrower expects a low borrowing rate. And secondly, the market for shortening the credit risky bond is inefficient. The value of y might be anywhere from 20 to 150 basis points. The investor then sells the bond in the debt markets and must pay Libor + x basis points to the bond buyer. The hedge is costing the investor a net cash flow of $x + y$ basis points which perfectly hedges the investor's risk in the credit default swap.

Notice that the hedges are not symmetric. The market maker is receiving x basis points from their hedge while the investor is paying $x + y$ basis points from their hedge. So the hedges determine the price of the credit default swap up to a range. The market is left with a spread of approximately y basis points which cannot be arbitrated away. Exactly where the price of the credit swaps falls in the range of X to $x + y$ depends on the counter-parties and their motivations. Counterparties to credit default swaps are entering into a customised, off-balance sheet transaction that has certain intangible advantages over the cash markets. Market-makers or commercial bank lenders looking for credit protection on a certain name might be willing to pay as much as $x + y$ or more. On the other hand an investor looking for some extra premium may be willing to accept as little as $x + y$ or less. Market makers with sub-Libor funding rates, and investors with above-Libor funding rates, would find the credit default swap even more favourable.

APPENDIX 64.6 Pricing default swaps in practice

In practice, the pricing of credit default derivatives is based on fundamental arbitrage relationships based on actual traded market instruments. This process is illustrated using the example below.

Assume a five-year asset swap for ABC Corporation trading at US dollars Libor plus 58 bps. The underlying asset is a bond with an interest rate swap with an AAA rated counterparty. Assume risk-free US dollar floating rate assets (taken in this instance to be AAA rated US dollars FRNs) are trading at Libor minus 10 bps.

This market scenario implies a credit spread for ABC of 68 basis points, calculated as follows:

Risk-free return (AAA rated FRN return)	LIBOR -10 bps p.a.
Risky return (ABC Corporation asset swap)	LIBOR +58 bps p.a.
Credit spread	68 basis points

The implied credit default cost can be calculated as shown below.

Asset-swap based pricing of default derivative:

Maturity (years)	5
Swap rate	6.5000%
Asset-swap pricing:	
Benchmark	Libor
Margin	0.5800%
Default-swap pricing:	
Risk-free (AAA) margin	-0.1000%
Required additional margin	0.0000%
Credit-default swap pricing	0.6800%
Credit-default swap pricing	2.8259%

The credit spread is the compensation received by the investor for the default risk assumed on ABC Corporation. Using a discount rate for five years of 6.50%, the net present value of this credit spread is 283 basis points. This present value amount can be equated to the price of a credit-default swap.

The relationship can be demonstrated by the process of replication. The investor can purchase the AAA rated risk-free FRN and enter into a credit default swap where it provides protection against the risk of default. The investor would require 283 bps up front or 68 bps p.a. to equate its position to that under the asset swap. The equivalence is necessitated by the fact that the asset swap and the FRN plus the default swap embody identical risk and therefore should attract identical returns. Failure of this condition would enable arbitrage to take place.

In practice, the asset swap market provides the benchmark return which are used to derive the default-swap values. In effect, it places a floor under these values as it allows, first, a traded benchmark, and, secondly, it allows a hedging mechanism for the traders. The practical approach (entailing stripping out the market price of credit risk from existing securities) and the more complex approach of derivation of the price of default from the default risk, recovery rate and loss expectations are consistent.

Selected bibliography and references

- Choudhry, M., Joannas, D., Pereira, R., Pienaar, R., *Capital Market Instruments: Analysis and Valuation*, FT Prentice Hall, 2001.
- Francis, J., Frost, J., Whittaker, J. G. (eds.), *Handbook of Credit Derivatives*, McGraw-Hill, 1999.
- Kaspari, A., *Mastering Credit Derivatives*, FT Prentice Hall, 1999.
- Kates, G., "Credit derivatives: risks and trends", *Risk Professional* 1(8), November 1999, pp. 28–31.
- Khuong-Huu, P., "The price of credit", *Risk*, December 1999, pp. 68–71.

65 Credit Derivatives II

In this chapter we consider some further approaches to credit derivatives pricing. Numerous techniques for credit default swaps and default options are considered. When pricing securities that are subject to credit risk, we can proceed in one of two ways. We can consider fundamental analysis, such as credit ratings, balance sheet (for firms), fiscal health, political situations, and market sentiment. From these factors we can determine, often heuristically, the proper premium on a credit risky security from a risk/reward standpoint. Alternatively, we can take information contained in the market prices for debt, equity and some liquid derivatives to derive a "no-arbitrage" price for the credit risk. From this framework we can employ existing results from finance theory to price credit derivatives, including default swaps and credit spread options, as well as impute market prices for credit risk in other transactions. In this chapter we consider this second approach. Although fundamental analysis continues to play a crucial role when making outright credit risk decisions, most derivatives can be synthetically created using existing market instruments, and it is important to use existing market information to compute fair-value of these derivatives.

65.1 Theoretical pricing models

65.1.1 Basic concept

The price of a credit derivative is essentially the price put by the bank on the likelihood of default of the issuer of the underlying asset. The price paid must reflect fair value in terms of compensation for taking on the credit risk of the issuer. A common approach used at the start of the market was the model presented by Fons (1994, from Kasapi (1999)). This is given at (65.1):

$$P = \sum_{t=1}^N \frac{S_t C + S_{t-1} p_t \mu (C + M)}{(1 + r)^t} + \frac{S_N M}{(1 + r)^N} \quad (65.1)$$

where

- S_t is the probability of reaching year t without any default (one minus the cumulative default rate for year t ; $1 - D_t$)
- p_t is the probability that the bond defaults in year t given that it has survived to year t without any default
- r is the yield on a government bond with N years to maturity
- C is the coupon on a bond with N years to maturity
- M is the face value of the bond
- μ is the recovery value of the bond in the event of default.

The Fons model follows the normal bond price/yield formula, but substitutes the bond cash flow with an expected value. The expected value of a risk-carrying cash flow is based on the likelihood (given by the default rate) of loss, and the extent of the loss, given by the recovery rate. The model can be used to measure the expected loss from holding a risk-carrying bond. This is calculated by setting the price of the bond to par and solving for the C coupon level. The expected loss is the coupon less the yield on a risk-free government bond of equivalent maturity. This expected loss is known as the risk-neutral spread, and gives us the price of the default swap, because at this price an investor would be indifferent between holding the risk-free bond or the risk-carrying bond, since they would be fully compensated for the expected loss. In practice a liquidity and bid-offer premium is added to the theoretical default spread on a risk-carrying bond.

65.1.2 Creating the credit term structure

We wish to illustrate basic concepts in creating a credit term structure. Let us set the following terms:

- $df(t)$ riskless discount factor for cash flows at time t (viewed from time 0)
- $F(t)$ survival factor for cash flows at time t (viewed from time 0)

R	assumed recovery rate
P_i	dirty price of coupon bond i
c_t	bond coupon at time t .

Given a coupon curve of credit risky bonds with dirty prices $\{P_1, P_2, \dots, P_N\}$ and a riskless discounting function $df(t)$, we can strip out an implied survival function $F(t)$ and use this survival function to price similar credit risky cash flows.¹ Each bond has a coupon time-vector $\{c_t\}$ and *a priori* given recovery rate R . If we assume that the default process is independent of the interest rate process,² we can solve for $F(t)$ by using the following relationship. For any P of maturity n :

$$P = \underbrace{\sum_i c(t_i) \cdot F(t_i) \cdot df(t_i)}_{\text{coupon from surviving balance}} + \underbrace{\sum_i (F(t_{i-1}) - F(t_i)) \cdot R \cdot df(t_i)}_{\text{recovery upon default before maturity}} + \underbrace{F(t_n) \cdot df(t_n)}_{\text{Non-defaulted principal paid at maturity}}. \quad (65.2)$$

Note that recovery (the second term) involves only recovery on principal. With bonds, recovery upon default is assumed to cover only the principal, not the coupon.

Using this equation, we can solve for the implied survival factors in a similar fashion to solving for discount factors when stripping a riskless curve. These survival factors become the basis for valuing all credit risky cash flows; given the survival function for an issuer, we can now price any general set of cash flows contingent on that issuer's default risk, including credit default swaps and credit spread options.

65.1.3 Default swap pricing

Here we use the previous terms, in addition to the following:

x	default swap spread (assumed constant)
α_i	length of accrual period (e.g. if there are 183 days in the coupon period and pay period is act/360, $\alpha_i = 183/360$).

In a default swap, one side, the protection receiver, agrees to pay a periodic margin x to the protection provider while the reference asset(s) are not in default. In the event that the reference asset defaults, the protection provider agrees to pay a certain amount to the receiver in exchange for the assets. In the most generic case, we can solve for default swap margin (or the default-adjusted present value of that margin) by equating the PV of the premium side (contingent upon survival), to the payoff in default as follows:

$$\sum_i x \alpha_i \cdot \text{prob}(\text{deftime} > t_i) \cdot df(t_i) = \sum_i (1 - R) \cdot \text{prob}(\text{deftime} = t_i) \cdot df(t_i). \quad (65.3)$$

Since x is constant we can factor it out of the left hand side, resulting in:

$$x = \frac{\sum_i (1 - R) \cdot \text{prob}(\text{deftime} \geq t_i) \cdot df(t_i)}{\sum_i \text{prob}(\text{deftime} > t_i) \cdot \alpha_i \cdot df(t_i)}. \quad (65.4)$$

Note that the probability of default time being greater than some time t_i is the survival factor at that time, which we calculated previously.

¹ The survival function for default is analogous to the discount function for interest rates. The survival function “discounts” cash flows with respect to credit risk.

² This independence assumption allows a more straightforward representation of the default risk in a bond and will allow comparisons between survival curves for the same issuer across different currencies, showing potential credit risk mispricings and arbitrage. We can relax this assumption by solving for risky discount factor instead of a survival function. Further, if we assume a normal distribution for both interest rates and hazard rate (for example, a Hull–White or Ho–Lee model) we can separate the risky discount function into a survival function, riskless discount function and a correlation term.

This analysis assumes that default can only occur at premium payment times. We can easily relax this assumption by creating a finer time grid partition and adjusting the premium payments accordingly.

65.2 Credit spread options

We define the following terms:

$df(t, t_i)$	risk-free discount factor from time t to t_i
L_{ti}	value of Libor at time t_i
s_T	asset swap margin at time T
s'	strike spread of option on the asset swap
c_t	bond coupon at time t
P_t	dirty bond price at time t .

Given the survival function, we can now solve for a option (either a put or call) on the asset swap spread. To avoid any ambiguities, we define a call on a spread as an option to purchase that asset at the price implied by the strike spread, and a put as an option to sell the asset. For a vanilla asset swap we have the following relationship:

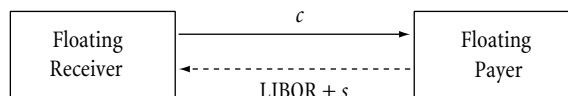


Figure 65.1

The floating receiver pays the coupon on the reference security and in exchange receives floating payments of LIBOR plus margin S . At the deal outset, the floating receiver pays par for the floating payments and receives the current price of the bond for providing the fixed payments. Finally, both sides make a “principal” payment of par at maturity.³ Therefore, if x is the equilibrium asset swap spread we have:

$$P_t + \sum_i (L_{ti} + s) \cdot df(t, t_i) + df(t, t_n) = 1 + \sum_i c_t \cdot df(t, t_i) + df(t, t_n). \quad (65.5)$$

The present value on the floating side of an asset swap at time t is given by:

$$PV(s) = P_t + \sum_i (L_{ti} + s) \cdot df(t, t_i). \quad (65.6)$$

The payoff on a *credit spread call* at exercise time T is:

$$f_T = \max(PV(s_T) - PV(s'), 0) \quad (65.7a)$$

where s' is the strike spread (determined at outset) and s_T is the time T asset swap spread.

$$\begin{aligned} f_t &= \max \left(\sum_i (L_i + s_T) \cdot df(t_i, T) + df(t_n, T) - \sum_i (L_i + s') \cdot df(t_i, T) - df(t_n, T), 0 \right) \\ &= \max \left(\sum_i (s_T - s') \cdot df(t_i, T), 0 \right) \\ &= \sum_i df(t, t_i) \cdot \max((s_T - s'), 0). \end{aligned}$$

For a European option, the value at time $t < T$ is the discounted expected value:

$$f_t = E_t \left[\sum_i df(t_i, T) \cdot \max((s_T - s'), 0) \right] \cdot df(t, T). \quad (65.7b)$$

By postulating stochastic processes for both the credit spread and interest rates we can value this option in a manner similar to the payoff on a European swaption.

³ As in an interest rate swap, these payments cancel and are usually ignored. They are included here for clarity of exposition.

65.3 Default options pricing

Default options give the purchaser protection against default by paying a lump sum in the event of a default on a reference bond. For example, a bank might purchase one-year default protection on a Sovereign emerging-market credit for say, 130 basis points per annum. In the event that the sovereign defaults on its bonds the purchaser would receive the difference between par and the price of the bonds after default. Otherwise, the bank must pay 65 basis points every 6 months. In addition, if the option is to be traded with accrued interest, in the case when the default occurs in-between two coupon dates, the purchaser of the option will pay the seller the portion of the next coupon that has “accrued” from the last coupon date until the default date.

Default options differ from Total Return Swaps in two very important ways. First, Total Return Swaps pay as interest the total coupon during the period plus the difference between the starting price and the ending price. In the case of default, the coupon would not be paid. A Default option will pay only in the case of default an amount equal to the difference between par and the price after default. Second, the purchaser will pay the carrying cost on a Total Return Swap. The purchaser of a Default option will pay an amount based on the pricing.

Although both Total Returns and Default options are ways to hedge default, Default options allow one to trade a select part of the curve. For this reason, the hedge of a total return is the asset, while a Default option would be more complex depending on the structure.

In this section, derivations and proofs are omitted, and the main formulae listed for reference purposes.

65.3.1 Pricing

The risk-neutral price of a default option is given by

$$P = P_{CF} + P_A + P_{payoff} \quad (65.8)$$

where P_{CF} is the risk-neutral expectation of the present value of the coupons paid by the purchaser until maturity, or default, whichever comes first. We have

$$P_{CF} = c \cdot \sum_i (1 - df(t_i)) M_i e^{-r_i t_i} \quad (65.9)$$

where

- c is the option's coupon
- t_i is the time of coupon i
- M_i is the option's notional at time t_i
- r_i is the corresponding risk-free rate
- P_A is the risk-neutral expectation of the present value of the accrued paid by the purchaser in the event of default, if the option trades with accrued.

P_{payoff} is the risk-neutral expectation of the present value of the payoff on the option in the event of default, which is equal to the difference between the strike and the *recovery price* of the bond immediately following the default, paid on the notional amount of the option at the time of default. This recovery price is equal to the recovery amount at the time of default, which was an input into building the risk-neutral probability of default curve. Thus, P_{payoff} , in turn, may be separated in two components:

$$P_{payoff} = P_{RR} - P_{SA}$$

where P_{RR} is the risk-neutral expectation of the present value of the underlying bond's recovery price, defined by the bond's recovery rate RR , and its payoff type; and P_{SA} is the risk-neutral expectation of the present value of paying the strike amount to the buyer, in the event of default.

Unlike P_{CF} , the quantities P_A and P_{payoff} depend on the type of the probability of default curve, and the underlying bond's payoff-at-recovery type, which was another curve input.

65.3.2 Discrete curve

For a *discrete* probability-of-default curve, there is a finite number of dates before the option's maturity on which the issuer's bonds prices may drop to their recovery prices, due to default. Let there be m such default dates, including the option's maturity, and $(t_{d_i} | 1 \leq i \leq m)$ be the corresponding default times, and set $t_{d_0} = 0$. Then,

assuming the option is trading with accrued, the seller receives the accrued on the option at time t_{d_i} , if and only if the default occurs during the time interval $(t_{d_{i-1}}, t_{d_i})$. Thus, we must have

$$P_A = \sum_{i=1}^m (df(t_{d_i}) - df(t_{d_{i-1}})) A(t_{d_i}) e^{-r_{d_i} t_{d_i}} \quad (65.10)$$

where $A(t_{d_i})$ is the accrued on the option at time t_{d_i} . Analogously, P_{SA} is given by:

$$P_{SA} = RP \sum_{i=1}^m (df(t_{d_i}) - df(t_{d_{i-1}})) M(t_{d_i}) e^{-r_{d_i} t_{d_i}} \quad (65.11)$$

where RP is the reference price, or “strike” of the option, and $M(t_{d_i})$ is the notional of the option at time t_{d_i} . Let RA_{d_i} , denote the bond’s recovery price at time t_{d_i} . Then, similar to above, P_{RR} is given by:

$$P_{RR} = \sum_{i=1}^m e^{-r_{d_i} t_{d_i}} (df(t_{d_i}) - df(t_{d_{i-1}})) (RA_{d_i} x M(t_{d_i})). \quad (65.12)$$

Now, the formula for RA_{d_i} depends on the recovery payoff type of the underlying bond.

Percentage of cash-flows method

When the recovery rate of the bond is RR , the recovery amount at time t_{d_i} , is all future guaranteed cash-flows, together with the fraction RR of all future risky cash-flows, present-valued to td . In turn, the corresponding recovery price is the ratio of the recovery amount to the bond’s notional at time t_{d_i} :

$$RA_{d_i} = \frac{\sum_j PV_{d_i}(gc_j(i)) + RR \sum_j PV_{d_i}(yc_j(i))}{M(t_{d_i})} \quad (65.13)$$

where $gc_j(i)$ are the cash-flows whose receipt is guaranteed at time t_{d_i} (as in say, Brady bonds), and $yc_j(i)$ are the non-guaranteed (risky) cash-flows after t . We assume here that the bond trades without accrued after default.

Percentage of face method

When the recovery rate of the bond is RR , it is also the recovery price at every time t_{d_i} , so that $RA_{d_i} = RR$.

65.3.3 Continuous curve

For a *continuous* probability-of-default curve, using the framework introduced in Part VIII, the default and payoff on the option may occur at any time up to the option’s maturity. Let T denote the time of default. The risk-neutral expectation of any function of the time of default, $f(T)$ is given by:

$$E[f(T)] = \int_0^{\infty} f(t) d'(t) dt \quad (65.14)$$

where:

$$\begin{aligned} d(t) &= P(T \leq t), \text{ for every } t, \text{ and} \\ d'(t) &= (1 - d(t_i)) p_i e^{-p_i(t-t_i)}. \end{aligned}$$

Let t be a positive time, and let t_i denote the largest default time in the curve that is smaller than t . Then, under exponential interpolation, we have

$$d(t) = P(T \leq t) = d(t_i) + (1 - d(t_i))(1 - e^{-p_i(t-t_i)}) = 1 - e^{-p_i(t-t_i)}(1 - d(t_i)). \quad (65.15)$$

These equations allow us relatively simple closed form solutions, when we assume that the accrual grows linearly in-between coupons, and either the notional grows exponentially (for capitalising bonds), or is a step function (for amortising bonds).

For example, let t_1 and t_2 be positive times such that $t_1 < t_2$, and the accrual grows linearly from t_1 to t_2 with a constant coefficient δ , and the probability grows exponentially with constant coefficient p . That is, for $\forall t \in [t_1, t_2]$

$$A(t) = A(t_1) + \delta \times (t - t_1) \quad (65.16)$$

$$d(t_1, t) = 1 - e^{-p(t-t_1)}. \quad (65.17)$$

Then, the risk-neutral expectation of the present value of the accrual paid by the purchaser in the case of default during the interval is $[t_1, t_2]$ given by

$$P_A(t_1, t_2) = \frac{pe^{-r_{t_1}}(1 - d(t_1))}{r_{t_1, t_2} + p} \left(A(t_1) - e^{-(r_{t_1, t_2} + p)(t_2 - t_1)} \left(A(t_1) - \delta \times \left(t_1 - t_2 - \frac{1}{r_{t_1, t_2} + p} \right) \right) \right) + \frac{\delta}{r_{t_1, t_2} + p}. \quad (65.18)$$

Now, to calculate the quantity P_A , we have to partition the time horizon $[0, M]$ (M is the maturity of the option) into segments satisfying the constant growth rates conditions, calculate the values $P_A(t_1, t_2)$ for every segment separately, and then add them together.

Analogously, let t_1 and t_2 be positive times such that $t_1 < t_2$, and the option's notional grows exponentially from t_1 to t_2 with a constant coefficient ϕ , and the probability grows exponentially with constant coefficient p . That is, for $\forall t \in [t_1, t_2]$

$$N(t) = N(t_1)e^{\phi \times (t - t_1)} \quad (65.19)$$

$$d(t_1, t) = 1 - e^{-p(t-t_1)}. \quad (65.20)$$

The exponential growth of the notional is a reasonable assumption for capitalising bonds, with $\phi = 0$ for non-capitalising bonds. Now, the risk-neutral expectation of the present value of the reference price paid by the seller in the case of default during the interval $[t_1, t_2]$ is given by

$$P_{SA}(t_1, t_2) = pe^{-r_{t_1}t_1}N(t_1)(1 - d(t_1)) \frac{RP(1 - e^{-(r_{t_1, t_2} + p - \phi)(t_2 - t_1)})}{r_{t_1, t_2} + p - \phi}. \quad (65.21)$$

One could also assume linear growth for the notional, and obtain an expression similar to $P_A(t_1, t_2)$.

As before, the value for P_{SA} can be calculated by partitioning the interval $[0, M]$ appropriately, integrating the segment separately, and adding up the individual $P_A(t_1, t_2)$ s.

Percentage of cash-flows method

At every time t , between the start and the end date of the option, in the case of default, the recovery price of the bond is given by

$$RA_t = \frac{\sum_j PV_t(gc_t(j)) + RR \sum_j PV_t(yt_t(j))}{M(t)} \quad (65.22)$$

where $gc_t(j)$ are the cash-flows whose receipt is guaranteed at time t , and $yt_t(j)$ are the non-guaranteed (risky) cash-flows after t . We assume here that the bond trades without accrued after default.

Let there be N cash-flows, and denote by $P_{RR}(i)$ the risk-neutral expectation of the present value of flow i 's contribution to the recovery amount, in the event of default; then,

$$P_{RR} = \sum_{i=1}^N P_{RR}(i). \quad (65.23)$$

Suppose the flow i is guaranteed, that is, the bond holder receives the flow if the issuer does not default before some time t_{gi} , less than the time of the flow t_i . Typically, in Brady bonds, the principal is fully guaranteed ($t_{gi} = 0$), whereas the coupons are guaranteed for one year prior to their payment ($t_{gi} = t_i - 1$).

Then, we have

$$P_{RR}(i) = c_i e^{-r_{t_i}t_i} (d(t_i) - d(t_{gi}))(1 - RR). \quad (65.24)$$

If the flow i is not guaranteed, we have

$$P_{RR}(i) = RRc_i e^{-r_i t_i} d(t_i). \quad (65.25)$$

Obviously, $P_{RR} = P_{SA}$ ($RP = RR$) – holds for the discrete model as well.

The break-even rate is by definition the rate X such that the value of the option is zero.

$$X \times 100\% \times \frac{P - P_{\text{payoff}}}{P_{CF}(100\%) + P_A(100\%)}. \quad (65.26)$$

Where $P_A(100\%)$ and $P_{CF}(100\%)$ are P_A and P_{CF} calculated for $c = 1$, respectively.

Selected bibliography and references

Das, S., *Credit Derivatives: Products, Applications and Pricing*, Wiley, 1998, Chapters 7–12.

Bingham, N., *Risk-Neutral Valuation: Pricing and Hedging of Financial Derivatives*, Kiesel, 1998.

66

Credit Derivatives III: Instruments and Applications

In this chapter we review in more detail some of the main credit derivative instruments and some of the ways they are used. Credit derivatives allow banks to strip out and isolate credit risk from other types of risk exposure. Once a bank can do this, it is able to hedge credit risk or, if it wishes, transfer it on to another entity. As we discussed briefly in Chapter 64, the main credit derivative instruments are credit default swaps, total return swaps, credit spread options and credit-linked notes. Later in the chapter we introduce some of the main applications of credit derivatives.

66.1 Credit default swaps

The earliest and perhaps the simplest manifestation of the credit derivative instrument is the *credit default swap*, also known as the *credit swap* or the *default swap*. They are akin to insurance contracts taken out on a specified asset, usually a corporate or risky sovereign bond, for which a premium is paid on a regular basis by the purchaser to the seller. In the event that the issuer of the assets defaults on the loan, the holder of the asset will receive a compensation payment from the seller of the default swap, which for bond assets is usually the par value of the bond. In order to receive the compensation payment, the buyer of the default swap hands over the asset to the seller before payment is effected. The value of the “premium” paid by the buyer of the swap to the seller is the price of the swap itself.

The above is a generic description of a credit default. It is important for the swap documentation to state clearly the terms and conditions under which credit protection is offered by the swap. A default swap does not have to be associated with an individual asset; they may be linked to a basket of assets. By entering into a credit default swap, the holder of a risky asset is able to transfer the credit risk exposure to the swap counterparty. In return for accepting this risk, the selling bank charges a premium expressed as a number of basis points of the nominal value of the swap contract. In practice this premium is expressed as spread over Libor. In the event of a *credit event*, the swap seller will make a payment to the buyer. If there is no credit event, then the seller does not make any payment. By entering into a credit default swap, the purchasing bank has obtained a hedge against default. The swap seller has conceptually lent funds, for which he receives some spread over Libor, and has taken on an off-balance sheet credit exposure.

There is no standard definition of “credit event”. The occurrence of a credit event will lead to the payout on the credit swap, so it is important that the swap documentation defines this as precisely as possible. A credit event may be one or more, of the following occurrences:

- bankruptcy;
- insolvency leading to administration;
- a technical default, as when an issuer does not meet a coupon payment on a bond as it falls due;
- a downgrade in credit rating;
- a change in the credit spread relative to an external benchmark, and above an agreed level.

The premium paid by the swap buyer to the swap seller is the price of the default swap. This price is a function of the credit quality of the asset being protected, the notional value of the asset, the probability of default, the expected recovery value of the asset and the term to maturity of the swap. The other important factor is the credit quality of the market maker that is selling the swap. A bank that is purchasing default protection on a risky asset will need to ensure that it does not simply trade credit risk for counterparty risk. That is why only the largest, highest quality financial institutions can involve themselves in credit derivatives market making to any meaningful extent. The impact of longer maturity, a lower credit quality asset and higher probability of default is to increase the price of a credit swap written on that asset. Note that the credit swap does not payout immediately after a credit event has occurred, but rather after a period of up to six months, to determine (in the event of bankruptcy) the extent of the recovery value of the asset. The payment made by the swap seller in the event of default can be either in the form of purchase of the asset at par, or the transfer of cash to the buyer, with value agreed beforehand. There may also be a pre-determined fixed percentage of the par value that is paid out.

The cash flow of a credit default swap, at any point prior to the occurrence of a credit event, is the regular flow of premiums from the swap buyer to the swap seller. The buyer of the credit swap is purchasing credit exposure and is therefore short the credit exposure, while the seller of the swap is long the credit exposure. As with an interest-rate swap, there is no exchange of principal. Credit default swaps are written for a fixed term; if in the period of the swap there is no credit event, the swap buyer will receive no payment and at the maturity of the swap it will expire.

The significance of credit swaps is that, because they can be written for any term to maturity, they can be used to construct a term structure of credit spreads. Consider for example a corporate that has issued bonds of say, ten and 20 years' maturity but nothing in between. Previously there would have only been a ten-year or 20-year yield spread available for this issuer. With a credit swap however, the holder of either of these bonds could purchase credit protection on the bond for any term to maturity. This would then enable the construction of a credit spread curve of up to 20 years' maturity, using the price of default swaps as the basis of the curve's construction.

EXAMPLE 66.1 Credit default swap

In October 1998 a credit default swap is written on the Sony Corporation 6.125% March 2003 bond, which is a US dollar issue. The swap has a maturity of three years and is priced by the market maker at 50–60 basis points. This means that the market maker bank will sell protection on the bond at 60 basis points and buy protection at 50 basis points. A holder of the bond wishes to protect himself against the effects of any possible default, but does not wish to sell the bond, and buys a credit default swap for a nominal holding of \$50 million. The swap will pay out the \$50 million multiplied by par in the event of default. Assuming annual cash flows, this would mean that the swap seller would receive $(0.0060 \times 50\,000\,000) = \$300\,000$ each year for three years. If at any time the issuer defaulted, the bond would be delivered to the swap seller, who would payout in line with the swap terms. If we assume that the bond has a recovery value of \$0.30 per \$1, this payout would be \$15 million.

66.2 Total return swap

A *total return swap* (TRS) is an agreement between two parties in which one counterparty pays the total return on an asset, including coupon payments and capital appreciation, in return for which it receives a regular payment, usually in the form of a spread over Libor. This means that the party that has purchased the TRS has transferred the market risk and the credit risk of the asset, but still owns the asset at the end of the trade. A TRS is a different beast from the credit default swap however; note that there is no credit event and no contingent payment. In the event of default by the issuer of the asset, the payment due at the next period is brought forward to the time of the default, and the contract is then terminated.

Buyers of TRS contracts, who pay the Libor rate, include bond dealers who wish to reduce or remove completely their exposure to an asset without selling it. A TRS may also be used to finance the synthetic purchase of an asset.

EXAMPLE 66.2 Total return swap

A bank wishes to gain exposure to a low-grade asset and enters into a six-month TRS dealt on a B-rated corporate bond. The terms of the swap are:

Asset	£10 million nominal B-rated bond, 8.00% March 2010
Trade date	7 March 2000
Maturity	7 September 2000
TRS payer	Counterparty
TRS receiver	Bank, pays Libor
Cost	Libor plus 50 basis points (assume Libor fixes at 6.00%)
Term	Six months
Price of bond at trade date	100.00 (dirty price)
Price of bond at maturity	104.00
Coupon value	400 000
Capital gain	200 000
Bank receives	600 000
Floating payment	324 109.59 (uses act/365)

The economic effects of a TRS are similar in some ways to a repo transaction. A TRS is essentially a financed purchase of a bond, similar to a repo, except that in the repo the total “return” generated by the asset is not paid by the repo buyer to the seller. However repo traders sometimes enter into a TRS trade if the rate payable by them is lower than it would be on the repo. They may also prefer the TRS as it allows them to remove the asset from the balance sheet for the term of the trade, which does not occur with a repo. Under what circumstances might a bank or other institution wish to enter into a short-term removal of assets from its balance sheet? One example is where the bank is expecting a visit or some other analysis from credit analysts, external auditors or ratings agencies. By entering into a TRS during the time when the balance sheet is being analysed, the bank may be able to receive a better credit rating or write-up of prospects.

Another example where either repo or a TRS may be used is given in Example 66.3.

EXAMPLE 66.3 Repo and Total Return Swap

A bond trader believes that the yield on a certain bond will rise in the next few months. He wishes to exploit this movement. He may enter into either of the following:

- the bond trader can short the bond, and cover the short in the repo market. In the repo trade, he will receive the repo rate on the bond. If the bond is “special”,¹ the rate the trader receives on the cash he has lent may be below the rate payable on the bond coupon, which would result in a net funding loss. Any capital gain received if the price of the bond falls will need to be above this funding rate if there is to be a net profit. If the bond is not special, it is likely that the trader will be able to fund the position cheaply or at a profit.
- the bond trader can enter into a TRS, in which he agrees to pay the “total return” on the bond. The price of the bond must fall by a sufficient amount to offset any negative funding costs on the bond itself. If the breakeven financing cost is lower with the TRS, then it will be used rather than the repo transaction. Therefore if the bond is special, it is more likely that a TRS will be used rather than a repo.

66.3 Credit options

A credit option is an instrument that has a payoff profile that is linked to the credit characteristics of an issuer, or one or more of the issuer’s assets. They were initially developed for the same reasons that default swaps were, to enable market participants to hedge and transfer credit risk exposure. There are essentially two types of credit options, those with a payoff profile linked to the value of the underlying asset, and those with a payoff linked to the change in the spread of the underlying asset over the risk-free interest rate. The main type of credit option is a put option where the write has agreed to compensate the buyer for any decline in value of the underlying asset below the strike price. As these are credit options, the specifications of the contract are usually set in terms of the acceptable default price of the underlying asset. This means that if the option is exercised the payoff is determined by subtracting the market price of the bond from the strike price. The strike price in this case is determined by taking the present value of the bond’s cash flows, discounted at the risk-free interest rate, and adding the strike price credit spread. The other type of credit option is a call option written on the level of the credit spread. These options are set so that the option is in-the-money when the credit spread is higher than the specified strike price spread level. The payoff is the difference in the credit spread multiplied by a specific notional value.

To illustrate, consider a case where ABC Bank sells an option to XYZ Bank, for which ABC Bank receives the option premium. The option gives XYZ Bank the right to sell the underlying bond to ABC Bank at a certain strike price, which is set in terms of the yield spread over the benchmark bond. On the expiry date, assuming a European option, if the spread of the underlying bond is below the strike price spread, the option will expire worthless and no action is taken. If the spread is higher, XYZ Bank delivers the bond to ABC Bank, and the purchase price paid by ABC is a yield spread over the benchmark that is equal to the strike price spread. There are a number of reasons why ABC Bank will wish to enter into the contract. For instance it may be enhancing the return on its bond book by generating premium income; or the bank may be taking a view on the yield spread that the bond is likely to trade at in the future.

¹ See Chapter 34.

66.4 Credit-linked notes

A *credit-linked note* is an instrument that enables an investor to purchase an asset that pays a return that is linked to the credit risk of that asset, as well as an additional credit risk that has been transferred between the issuer and the bank via a credit derivative. Therefore the structure is an instrument that incorporates the credit risk that has been transferred using a default swap into an otherwise vanilla bond. The holder of a credit-linked note will receive the coupon and redemption payments, unless there has been a credit event, whereupon the bond holder will receive the par value of the bonds.

The basic structure is illustrated at Figure 66.1.

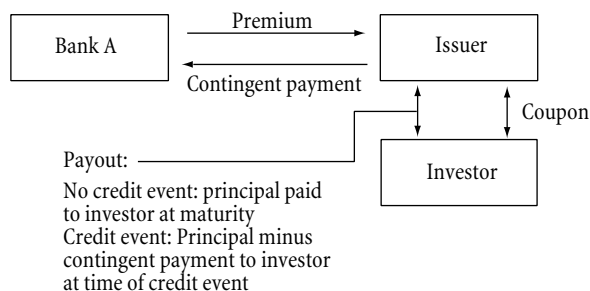


Figure 66.1: Credit-linked note.

Essentially there are two types of credit-linked notes. These are:

- where there is a link formed between a bond and underlying credit risk; this is the more traditional instrument and features a linkage of either the coupon or the principal or both to an underlying credit risk component that has been issued by a highly-rated issuer. The issuer also hedges out his exposure with a back-to-back credit derivative transaction;
- synthetic bonds, an issue of debt where the underlying credit risk exposure is created through a credit derivative transaction that has been set up to replicate the characteristics of a bond issued from the same issuer.

There are also *total rate of return credit-linked notes*. Such instruments are designed to help simulate an investment in an underlying loan asset. *Credit default notes* have the similar applications as default swaps. They are used mainly to assume or reduce counterparty default exposure. On the whole investors use credit default structures to assume default exposure to generate premium income, which enhances yield. Notes are constructed by combining a cash investment with a default swap. That is, they are created via the combination of a conventional security, with fixed or floating coupon, and the entry by the investor into a credit default swap, where the investor is the provider of the default protection. Credit default notes have the following advantages:

- they provide yield enhancement, in return for assuming default risk;
- they allow a bank to utilise a so-called credit risk assumption capacity, which may not have been available via other instruments;
- they enable the separation of default risk and principal risk;
- they present a capped and known loss in the event of default.

66.5 Applications

As a liquid, tradeable secondary market develops in credit derivatives, the range of uses to which they are put increases. In the US market the instruments are used as an alternative to cash market instruments, whereas both in the US and Europe they are mainly used for risk management purposes, and as an efficient tool by which cash risk can be reduced. As such credit derivatives provide participants with a mechanism by which they can enter (and leave) credit-sensitive asset markets. Credit derivatives have been used for a range of purposes; a summary of the main uses of each type of instrument is given below.

- **Credit default swap:** essentially these are purchased to provide default protection. They are also used for relative value purposes, for example to create a synthetic asset. A bank may use a default swap when the implied repo rate on the swap is trading at Libor, which means that the bank can realise the repo premium implied without having to finance the trade. Finally, default swaps enable investors to access a market that is potentially much larger than the cash bond market, because a desired credit exposure that is not available in the market can be created synthetically using a default swap. Note that the main use of default swaps remains that of hedging risk-carrying bond positions.
- **Total Return Swap:** the requirements of the investor can be adding to return on high-yield bonds, and entering into an off-balance sheet investment, while there are usually low levels of back-office activity and low capital requirements associated with the TRS transaction.

Variations of the TRS include the *secured loan trust note*. These are issued by a trust and are purchased by investors who wish to have exposure to a basket of assets that pay a high return. The selling bank receives Libor plus a spread for the term of the note.

What about the market maker in credit derivatives? A trader in the instruments will need to hedge the credit book. The following are the main factors to consider when hedging a credit derivatives book:

- Liquidity: credit swaps and other credit derivatives are bespoke products. It may be difficult to trade the instrument in the secondary market. This is very significant and will affect the realised profit & loss in a credit book. For instance, if someone is long a default swap that is subsequently downgraded, the value of the default swap is increased. However the holder can only realise the profit if the swap is sold to someone else. If the swap cannot be sold, there is no actual profit and if there is no default, the holder will receive no payment;
- The bid-offer spread may be very wide, reflecting the lack of liquidity in the secondary market;
- There is very little continuous trading in credit derivatives and this can contribute to an incomplete term structure of credit spreads.

These issues should diminish as the market in credit derivatives becomes more liquid.

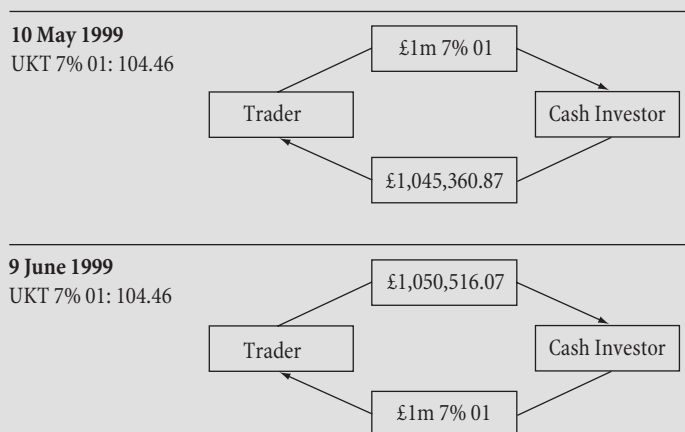
EXAMPLE 66.4 Repo and TRS

A trader wishes to finance a bond position for one month (30 days).

A cash investor agrees collateral of £1m nominal 7% 2001 gilt, which is trading at 104.46, plus 4 days' accrued interest which is 0.076087. The agreed repo rate is one month Libor flat, which is 6.00%. There is no haircut.

The following diagrams illustrate how the classic repo trade is structured compared to the total return swap.

■ Classic repo



In this transaction the interest at 6% for 30 days is £5,155.20, so termination money is £1,050,516.07.

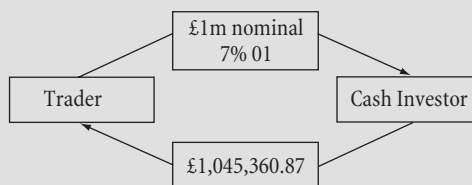
In the example there is no change in price on termination. In any case, if say on 9 June the 7% 2001 was trading at the higher price of 105.10, as this is a classic repo there would still be no change in the cash flows.

■ Total-Return-Swap (“short swap”)

10 May 1999

UKT 7% 01: 104.46

4 days accrued



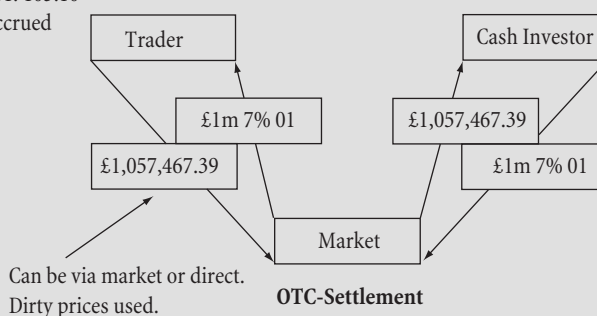
OTC Swap Agreement

Trader enters into a one-month swap in which he will **receive** the total return on a notional £1m nominal 7% 2001 and **pay** one-month Libor flat, that is 6%.

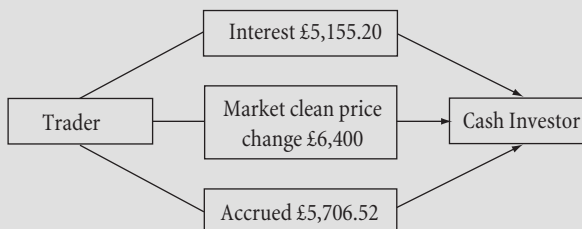
9 June 1999

UKT 7% 01: 105.10

34 days accrued



OTC-Settlement



$$1\text{m Libor } (6\%) \times 30/365 \times \text{Market Value stock} = \boxed{\text{£5,155.20}}$$

$$(\text{New dirty price} - \text{old dirty price}) \times 1\text{m} = \boxed{\text{£12,106.52}}$$

As there has been a rise in price the trader pays over the difference to the cash investor. If there had been a drop in price the difference would have been paid by the cash investor to the trader.

Selected bibliography and references

Chrofas, D., *Credit Derivatives and the Management of Risk*, New York Institute of Finance, 1999.

Tavakoli, J., *Credit Derivatives: A Guide to Instruments and Applications*, John Wiley & Sons, 1999.

Part XII

Emerging Bond Markets

The 1990s was undoubtedly the decade of emerging markets investment. This has been one of more fascinating developments in global debt capital; as economies around the world have engaged in market reform and restructuring, so they have attracted ever increasing amounts of capital. This has manifested itself most obviously in the bond markets of these countries. The shift in global capital flows has been striking, and during the 1980s and 1990s amounted to \$1 600 billion; the annual foreign investment flow from the developed economies to the developing ones is now running at over \$230 billion each year.¹ Both institutional and private investors, always engaged in a search for diversification and higher yield, have contributed to the growth of emerging debt markets. Despite spectacular market corrections in countries such as Thailand, Indonesia, Russia and Brazil in the second half of the 1990s, the emerging markets continue to be attractive to investors, due to the yield spreads on offer and the steadily improving credit quality, a testament to economic growth and investment in infrastructure. Returns have also been impressive; for example the average annual return in emerging fixed income markets from 1991 to 1997 was 19%, compared to 8% on US Treasury bonds and 7% on Standard & Poor's bond index.²

Of course, the term "emerging market" is almost a misnomer, as it implies that there is a homogeneous group of countries, or at least a regional bloc of countries with similar economic indicators and some form of convergence, as for example one might observe in continental European Union countries. Such a view would be very inaccurate. The countries grouped as "emerging" represent a large and diverse set, exhibiting considerable variation in economic and market performance. This is true not just across geographic regions but also within them. The stage of development ranges from the relatively well developed such as Portugal, Chile and Estonia to debt capital market beginners like Sri Lanka, Jordan and Zambia. From this small selection of just six countries we have a culturally and economically different set of countries; this effect is magnified when one adds in the remaining 150-odd other emerging markets! Within regions there are considerable variations in market development; for example in eastern Europe countries such as Poland have embarked on ambitious economic reform that has brought them to within perhaps only a few years entry to the EU, whereas close neighbours Romania and Bulgaria are a longer distance from such a prospect. The same can be observed in say, Latin America, if we compare Uruguay with Venezuela, while in 1999 Ecuador became the first country ever to default on a Brady bond. There are other factors for bond investors to consider; political instability is more evident, perhaps best illustrated by the market crisis observed in Indonesia in 1998.

Nevertheless the prospects for growth in emerging markets, and the high yields available, will contribute to the flow of capital towards them. The markets are now an acceptable asset class for international bond investors, and there is a wide range of instruments available, from sovereign Eurobonds and investment-grade corporate bonds to defaulted bank loans. The debt markets are also gradually seeing the introduction of more sophisticated derivative products, while the yield curve is also available, in certain countries, out to a maturity of 20 years and longer. The countries that are rated as investment-grade by agencies such as Moody's and Standard & Poor's include the Czech Republic, Poland, Chile, Malaysia and Thailand. In developed economies, the trend for lower inflation and lower interest rates, and the growth of bond (and equity) funds as more retail investors move out of bank deposits, has led to the growth in demand for emerging market instruments. The introduction of the euro, which has led to convergence in yields in the EU, will also lead to more institutional investment in emerging economies, as pension fund managers and insurance companies look to enhance the yields on their portfolios.

In this, the concluding part of the book, we look at the basic fundamentals of emerging country bond markets, and review some of the instruments that are traded, including Brady bonds. We also review the key issues in emerging market bond analysis. This is followed by further discussion of Brady bonds price sensitivity and the determinants of credit spread in emerging bond markets. This should help to convince the reader that the emerging bond markets are one of the most exciting and interesting sectors of the global debt market. It will indeed be

¹ Source: OECD.

² Source: UBS.

fascinating to observe which markets develop over time, in line with the development of their economies as a whole, and manage to shake off the “emerging” tag.

67

Emerging Bond Markets and Brady Bonds

The fixed income markets in developing countries, usually referred to as the *emerging markets*, are a well-established sector for international investors. Fund managers are able to invest in a wide range of bond instruments, with widely varying risks and return prospects, across a large number of markets. Many of the more popular markets for investors have only recently entered into economic and structural reforms that made their bond markets accessible for foreign investors, for example certain countries in eastern Europe. The demand for emerging market products, given economic developments in the developed countries, is without doubt set to continue to grow.

In this chapter we present a description of the main features of emerging bond markets, while stressing that the sector is by no means homogeneous and presents widely differing structures and risk exposures. We also present the basics of emerging market bond analysis, a review of Brady bonds and a look at selected markets, in this and the next chapter.

67.1 Overview

The market in developing country bonds presents higher yield potential, together with extra risk exposure, for international investors. Its growth in the 1980s and 1990s reflects the demand for such bonds, as well as the economic growth in the countries concerned. Although the market grew steadily in the 1970s, there was an element of complacency regarding the nature of the credit risk of developing countries, which was brought home during the Mexico debt default in 1982. Following the 1982 debt crisis the market renewed its growth as shown in Figure 67.1.

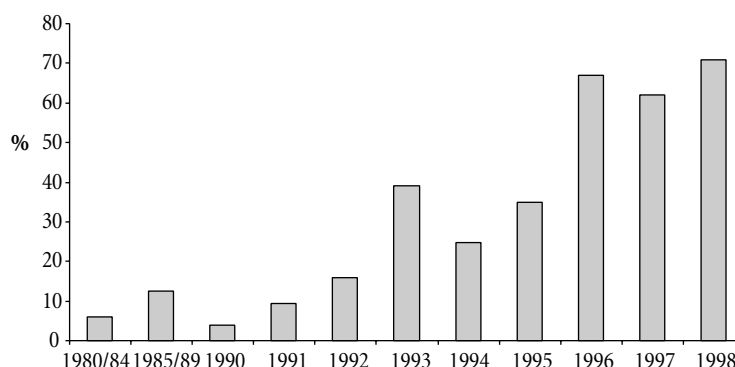


Figure 67.1: Total developing country bond issuance.
Issuers domiciled in countries rated at BBB or below. Source: EIU.

Until recently emerging market bonds available to overseas investors were essentially foreign currency securities. The domestic markets in many countries are not developed and therefore illiquid. However this situation is also changing and more countries are able to issue in their local currency as the market infrastructure develops. There have been only a few cases of default of government bonds in recent years, Costa Rica, Panama, Yugoslavia being some examples, as did Ecuador in 1999. Until recently there was a distinction between bank debt and debt in the form of bonds, and it was felt that it was relatively straightforward for a country to reschedule bank debt. However in November 1999 the government of Pakistan, which had changed recently in a military *coup d'etat*, proposed a rescheduling of the country's Eurobond debt. In the same year Ecuador became the first country to default on some of its Brady bond debt. However it remains easier for a country to negotiate rescheduling of debt with its bank creditors compared to its bond creditors, who may number in their thousand and also be anonymous.

A wide range of bond instruments are available to investors. During the 1980s many emerging markets raised funds in the international bond markets. Some of these such as Turkey, Greece and South Africa were not considered

surprises, however other countries such as Hungary, Bulgaria, Russia, India and Algeria have also subsequently issued bonds in the international markets.

There is now a distinct asset class known as emerging market bonds. The market can be divided into four sectors, which are:

- pre-restructured loans or “pre-Brady” debt;
- restructured debt, mainly in the form of Brady bonds;
- new debt which is mainly Eurobonds, usually issued in US dollars but also Yankee bonds;
- domestic bonds, issued in both US dollars and domestic currency.

The debt markets in emerging economies is essentially an over-the-counter one, with transactions taking place over the telephone and not (usually) on a stock exchange. The total volume of debt (in the form of bonds) in emerging markets exceeded \$380 billion at the end of 1998, with emerging defined as a rating of triple-B or lower. As well as restructured Brady debt, the Eurobond markets are being accessed on a steadily growing scale by more and more countries, and developing market Eurobond issuance reached \$75 billion in 1998. During the second half of the 1990s many countries raised financing in the Euro markets for the first time, including countries such as Kazakhstan, Slovenia, Morocco, Jordan, Mauritius and Estonia. Countries invariably seek a formal credit rating before issuing debt, from one of more of the established ratings agencies.

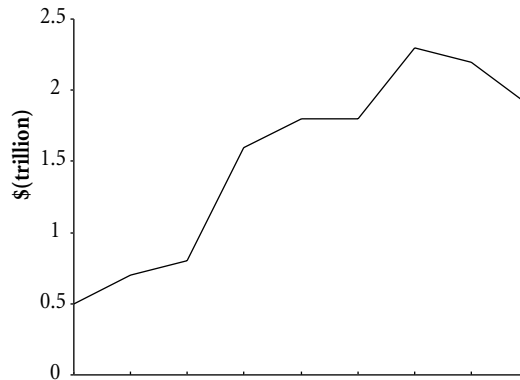


Figure 67.2: Value of stock markets in emerging economies. Source: EIU.

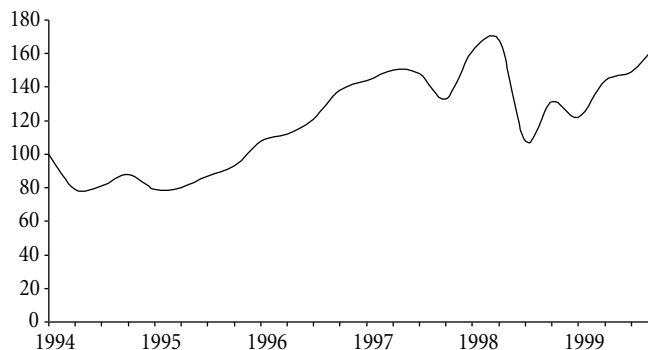


Figure 67.3: Emerging markets bond index. Source: JP Morgan; The Economist. (JP Morgan Emerging Market Composite Bond Index, December 1993 = 100).

67.2 Key features

Due to the variation in credit quality and level of infrastructure in emerging bond markets, investors must apply slightly different criteria when assessing the extent of their investments. We now review the key feature of the markets that make them stand out from developed country markets.

67.2.1 Seniority of debt

There are a number of different instruments trading in the emerging markets. The market does not treat the different instruments in the same way. The yields of bonds of the same issuer frequently differ, and this is because there would appear to be a hierarchy of debt, according to the form that it takes. Generally but not always, in descending order the seniority is viewed as being Eurobonds, Brady bonds, domestic US dollar debt and finally domestic currency debt. This is by no means a rigid rule; for instance previously Brady bonds would be rated by the agencies at one level below other sovereign debt of the same issuer. Now it is more common for them to be rated at the same level, and the practice of some countries buying back their Brady bonds and cancelling them (the debt is switched into foreign or domestic currency uncollateralised debt) has contributed to this. In fact investors that purchase Brady bonds are generally seeking an exposure to that country's market, rather than a form of collateral-backed bond that is linked to US Treasury bonds. It is expected that many countries will buy back and cancel their Brady bond debt in the future, so that many of the existing bonds today may not reach their stated maturity date.

67.2.2 Security and guarantee issues

Under the restructuring arrangements for Brady bonds, many bonds issued as part of the refinancing of debt obligations or defaulted loans carry collateral in the form of zero-coupon US Treasury bonds. Certain issues also incorporate an annual rolling interest payment guarantee. Such guarantees form an important part of the analysis of Brady bonds. Other bonds have been collateralised by other assets. In some cases the collateral has been stripped from the bonds and traded separately as pure sovereign risk at a higher yield. Some Brady bonds from oil-exporting countries have been issued with warrants or other options attached, for example "value recovery rights" that increase the level of the coupon payment if oil revenues rise above a pre-specified level. Certain Brady bonds issued by Uruguay have an option that is based on a trade-weighted basket of exports. Warrants and options are frequently detached from the host bond and traded separately.

Eurobonds held wholly or partly by foreign investors are frequently given sweeteners for investors in the form of ownership clauses that enable the bondholder to put the bond back to the issuer at par if the ownership structure changes.

An element of political stability is necessary if investors are to be convinced that a sovereign borrower has a willingness to honour its debt servicing obligations. It is the stability element that counts, for example the Soviet Union was considered a better credit by some in the markets than Russia is now. Another factor that is considered is a government's commitment to integration into the global financial system, and its relationship with overseas bondholders. An interesting illustration is provided by Nigeria; the country has foreign debt of over \$30 billion and has defaulted on several occasions in the past. However the negotiable debt of the country amounts to about \$3 billion of this total and has always been fully serviced. It is this debt that is important for maintaining relationships with foreign banks and is therefore a priority for the government.

67.2.3 Economic factors

One of the key evaluations made by overseas investors is their perception of a country's ability to pay. The factors that are assessed include capability for earning foreign currency, which is needed to service debt obligations. This is measured by ratios such as export earnings to debt service, and overall level of foreign currency reserves. Note that this does not mean that investors assess the ability to repay all its foreign debt if required to on demand at once; this is something that certain European Union countries could not do either. What investors seek to assess is an ability to service debt. Countries such as Sudan and Somalia have very restricted abilities to raise foreign exchange, required to service debt, and so the price of their bonds trade at a substantial discount to par.

Different factors are used to assess a country's ability to service its local currency debt. This is evaluated using more traditional analysis such as monetary policy, fiscal policy, the soundness of the domestic banking system, and the forecast for exchange rates. The key difference is that, unlike with foreign currency debt, as a last resort a government could simply print money to service a domestic currency debt. The significance of the exchange rate

forecast is that overseas investors have fewer opportunities to hedge local currency investments efficiently, and the exchange rate vis-à-vis the investor's currency will impact the value of repatriated profits. Equally important therefore are any restrictions on exchanging currency and taking funds out of the country. Although this is not as important as it once was, as more countries seek to integrate themselves into the global financial system, a change of government or new legislation can result in new restrictions.

67.3 Trading in the emerging bond markets

The different instruments that exist in developing markets trade under different conditions. The market in restructured debt is the preserve essentially of professional market participants, whereas Eurobonds frequently are purchased by private investors and may be illiquid. The market in domestic currency bonds is almost always dominated by local banks.

As the bond market is much less transparent and liquid in developing economies there is greater potential to gain advantage by engaging in relative value trading. This can include the following approaches:

- intra-country trades: for example positions in collateralised versus uncollateralised bonds. Certain Brady bonds for example are collateralised by strips (zero-coupon bonds) while others are not, for example the Mexico Par 6¼% 2019 versus Mexico 11½% 2026. Other trades have involved fixed- versus floating-rate notes, where there is a yield differential that may be earned, and curve spreads;
- inter-country trades: for example a relative value position in Brazil EI 2006 versus Argentina FRB 2005;
- cross-currency trades.

We consider some of the key features below.

67.3.1 The Brady bond market

Generally the market in restructured debt is highly volatile. The benchmark bonds exist in large issue size and are fairly liquid; it is possible to conduct large-size trades, say up to \$10 million, without affecting the market. The bonds themselves are usually long- or very long-dated and so carry high levels of interest-rate risk. The Brady bond has always attracted professional traders such as the proprietary and arbitrage desks of the large investment banks. This is changing somewhat though and more pure investors now hold Brady bonds. The level of transparency in the Brady bond market differs considerably. Where prices are not transparent and spreads wide, there is scope for profit from relative value and yield curve trading strategies. However because the bonds exist in many different forms, they often react differently to events. Fixed and floating-rate Brady bonds vary in price according to the spread over Libor.

The value of the collateral element of the bonds changes independently of developments in the local markets. Brady bonds are reviewed in greater detail later in this chapter.

67.3.2 Eurobonds

In general the liquidity of emerging market Eurobonds is low. This is mainly because issue sizes are comparatively low, between \$100–\$300 million is quite common. The bid-offer spread is sometimes as great as 1%, where the bond is available to trade. Where issues have been locked away an offer price is not available because there is no paper for the market maker to borrow.

Bonds of certain more well-established borrowers are more liquid. For example Argentinean and Mexican bonds have been issued in sizes of \$1 billion and more. However the most common feature of emerging market Eurobonds is that they are usually bought and held, rather than traded. The retail investor base places a strong emphasis on name recognition, so that eastern European borrowers raise finance in Europe more easily than say, Latin American borrowers. Due to the rule employed by the rating agencies that corporate borrowers cannot be rated higher than the country of their domicile, the credit ratings of certain corporates is lower than they might otherwise be. A good example of this is Gazprom in Russia.

67.3.3 Domestic markets

With one or two notable exceptions, the local bond market in emerging countries is illiquid, as most bonds are held to maturity. The yield curve may only go out to very short maturities; for example the yield curve in Zambia is only really completely liquid for Treasury bills, so that the curve does not extend out further than one year. In fact a large

number of domestic markets are composed mainly of T-bills, which are held by local banks for liquidity purposes (and, because there are few alternative instruments for them to hold). The lack of liquidity if repeated in the currency market means that foreign investors must trade with care, as there may be difficulty in repatriating any profits.

EMERGING MARKET BONDS

Jan 24	Red date	Coupon	S&P* Rating	Moody's Rating	Bid price	Bid yield	Day's chge yield	Mth's chge yield	Spread vs US
■ EUROPE €									
Croatia	03/06	7.375	BBB-	Baa3	98.5250	7.68	+0.01	-0.29	+1.02
Slovenia	03/09	4.875	A	A3	90.2430	6.31	-	+0.20	-0.42
Hungary	02/09	4.375	BBB	Baa1	85.1801	6.60	-0.02	+0.36	-0.13
■ LATIN AMERICA \$									
Argentina	09/27	9.750	BB	B1	87.6250	11.20	-0.03	+0.11	+4.51
Brazil	05/27	10.125	B+	B2	80.7500	12.64	-0.02	+0.64	+5.95
Mexico	05/26	11.500	BB	Ba1	113.8000	10.00	+0.03	+0.44	+3.31
■ ASIA \$									
China	12/08	7.300	BBB	A3	95.0494	8.09	-0.11	+0.47	+1.38
Philippines	01/19	9.875	BB+	Ba1	96.2381	10.33	-0.03	+0.35	+3.62
South Korea	04/08	8.875	BBB	Baa2	103.3750	8.30	-0.06	+0.36	+1.59
■ AFRICA/MIDDLE EAST \$									
Lebanon	07/00	9.125	BB-	B1	100.1633	8.41	-0.02	-0.45	+2.62
South Africa	10/06	8.375	BB+	Baa3	99.5000	8.47	+0.02	+0.06	+1.81
Turkey	09/07	10.000	B	B1	96.5000	10.68	+0.01	+0.09	+3.99
■ BRADY BONDS \$									
Argentina	03/23	6.000	BB	B1	64.3750	9.96	-0.08	+0.21	+3.26
Brazil	04/14	5.000	B+	B2	72.3750	12.00	-0.05	+0.39	+5.27
Mexico	12/19	6.250	BB	Ba1	77.3750	8.65	-0.03	+0.15	+1.94
Venezuela	03/20	6.750	B	B2	68.5000	10.55	+0.08	-0.18	+3.84

London closing. *Standard & Poor's.

Source: Interactive Data/FT Information.

Figure 67.4: Emerging Market Bonds, 25 January 2000.

© Financial Times, reproduced with permission.

67.3.4 Derivatives

As the cash markets have developed in emerging markets, so have associated derivative markets. In most cases derivatives have not developed to anything like the sophistication observed in developed markets, however this is only a matter of time. In this section we briefly describe some of the instruments that have been traded up to now.

The requirement of corporate customers, as well as the needs of market makers to hedge their cash market books, are the main driving forces behind the development of a viable derivatives market. It is the more advanced emerging market economies that offer liquid derivative instruments. For example the interest-rate markets in certain eastern European countries are relatively liquid. The market in Czech koruna interest-rate swaps has a maturity of up to seven years with an average daily volume traded of over Ck750 million.¹ Most trades are fixed versus the Prague interbank rate ("Pribor"). This growth can be attributed to a number of factors. Both overseas investors and domestic institutions have increased investment, in direct assets as well as the local currency, and during 1997 and 1998 over 100 corporate and supranational bond issues were made in Czech koruna. The nominal value of these issues amounted to over \$3 billion, and most issues were currency swapped into deutschmarks and US dollars. The rise in the number of new issues led to an extension of the domestic market yield curve, and in fact the swap market now has a greater maturity than the government bond cash market. On the other hand the interest-rate swap market in Poland developed along different lines, because of restrictions on the convertibility of the zloty. In Poland the market initially traded as non-deliverable forward swaps, with cash settlement in dollars. Generally however the infrastructure is not in place in many emerging markets for a swap market to develop in the near term. In the UK and US for instance, growth in the swaps markets was aided by the pre-existing infrastructure of market mechanisms and hedging instruments. This meant that futures markets, rate fixing mechanisms and known credit spreads were already available as the tools with which banks could price, trade and hedge swaps, as well as link them into existing government bond cash and repo markets. The absence of such market infrastructure will slow the

¹ Euromoney, February 1999.

development of emerging swap markets. In the near term growth will be limited by the size of the underlying market, as well as the lack of vanilla cash market facilities.

Non-deliverable synthetic assets

In emerging markets such as those in eastern Europe and certain Asian countries, synthetic assets dominate the market for derivatives. This is mainly because the domestic market is not developed enough to meet the requirements of overseas investors, so that an off-shore market in non-deliverable instruments develops. Until the onshore market develops sufficient volume and liquidity, synthetic assets will continue to be used. These instruments serve as a substitute for liquidity. The ability of an offshore bank to create synthetic long and short positions facilitates trading in assets that are otherwise difficult to source in illiquid onshore cash markets. An example of such an instrument was traded during 1997 and 1998 in the Polish debt market, the synthetic Polish T-bill basket note. This structure enabled offshore investors to gain an exposure to that country's T-bill risk who were otherwise not able to invest in securities in Poland, for regulatory reasons. Essentially it was a US dollar cash-settled fixed-rate instrument, with an interest rate based on Polish T-bill rates. By embedding the basket-weighted forward foreign exchange positions in the note, the instrument allowed the investor to match the Polish zloty basket peg and reduce the expected currency risk of the position. Other synthetic instruments have been traded in the Polish market, such as a *synthetic basis swap* but the requirement for these assets will diminish as the market is opened up to overseas investors, and as the currency is made fully convertible as part of Poland's requirements for joining the European Union.

Credit-spread products

A simple debt derivative instrument is the credit spread, expressed as the yield of an emerging market bond minus the yield of a risk-free bond. Certain instruments are available that allow investors to take a view on the specific credit spread or a basket of spreads. One way that this can be achieved is by using a credit-linked note, which is usually a private-placement bond created by a bank, usually via a special purpose vehicle. The bond contains provisions linking its value to the default risk on a specified emerging market credit. For example, consider where an investor wishes to purchase an exposure to a selected Latin American country such as Peru. The investor could only buy a long-dated Brady bond, as the country has not issued any short-term debt. However to meet the investor's needs, a bank will create a two- or five-year note, paying a relatively high coupon, in which the buyer takes on the Peruvian credit risk, but is guaranteed par on maturity should there be no default. How will this bond be priced? In fact the fair value of the bond is the two-year (or five-year) credit spread for Peruvian debt. Therefore the bank has created for its customer a specific exposure to credit for the term sought. Variations on this instrument include a step-up callable note and leveraged high-coupon note. The *step-up callable note* is a variation of the credit-linked note. The basic risk exposure inherent in the note is the credit risk of the note issuer and the reference credit; however the noteholder picks up additional yield by implicitly selling the issuer an option to call back the note on specified dates. In the volatile environment of emerging markets, such an option often carries a significant amount of value, and allows the investor to considerably enhance his yield. Step-up notes often appeal to investors who prefer a short-term exposure and cannot find investments that meet their yield targets. With a note that has a substantial step-up in coupon, there is a high probability that the note will be called on the step-up date. If the note is not called however the investor must be prepared to hold a longer-dated instrument, although one with a significantly higher yield. For instance, at one point during 1998 the two-year Argentina JPY note was trading at an equivalent of yen Libor plus 200 basis points, while five-year Argentina was trading at approximately yen Libor plus 350 basis points. An investor was able to purchase a five-year note in yen, callable at the end of the second year, that paid yen Libor plus 300 basis points for the first two years, and then stepped up to yen Libor plus 450 basis points if it was not called. As long as three-year Argentina yields two years from the trade date were below yen Libor plus 450 basis points, the note would be called, and the investor would have earned an excess spread of 100 basis points each year on the two-year investment. If the note had not been called (in fact it was) the investor would have locked in a yield for the remaining three years that was substantially higher than yields available at the time of the initial trade.

Option products are available in certain emerging markets, for instance those in Latin America. The market for plain vanilla call and put options on Brady bonds or large Eurobond issues is reasonably large and liquid. Quote spreads of $\frac{1}{4}$ to $\frac{1}{2}$ of one point are usual, with transaction sizes ranging from \$5 million to \$50 million nominal. The availability of more exotic instruments is more limited however. Exotic options have been traded on Brady bonds for

instance. One instrument that has been traded is a digital option written on the credit spread of a bond. One example of this was a digital option on a Bulgarian Brady bond. In early 1997 Bulgarian Brady bonds were trading at around 13–14% over US Treasuries, and the market consensus was that the chance of a sovereign default was high. At that time an investor could purchase a one-year digital option with a payoff on maturity of three times the premium, in the event that the Bulgarian credit spread remained below 15% over the Treasury yield. Certain market participants found this instrument attractive, as their risk was limited to the premium paid. It was a straight view on the Bulgarian credit spread. In fact the spread fell from a high of over 13% over Treasuries to just under 7% over Treasuries by the end of 1997.²

Another instrument is the *default swap with embedded collar* on US dollar Libor. A default swap is a simple off-balance sheet contract that allows an investor to create an identical exposure to the credit-linked note, but via a structure that requires no funding. One counterparty pays the other a fixed premium to take on the default risk of a specified credit. By using a default swap a market participant can effectively leverage the balance sheet and tailor their credit exposure to take advantage of available credit lines or limits. The swap with the embedded Libor collar allows an investor to earn a significantly higher spread, for combining two separate and distinct forms of risk in the same transaction. For example the market level for a bank to take on five-year default risk in say, in the Republic of Venezuela might be Libor plus 4%. This means that the bank can enter into a swap in which it receives Libor plus 4% for taking on the risk in the event of sovereign default, at which point the bank will pay out to the swap counterparty. However the same bank may choose to earn a higher yield of Libor plus 5.75% for the same five-year period, if they also have a view on another indicator, such as US interest rates. This is done by entering into a contract in which the bank bears the same sovereign default risk, with the Libor spread paid for every day that a US interest-rate reference such as six-month Libor stays in a specified range, say between 3% and 8%. For a bank to hedge this transaction, it will use a pricing model that incorporates both Venezuelan credit spreads and US interest rates, as well as the correlation between the two. By combining emerging market derivatives with more traditional derivative technologies, market participants can achieve risk exposure and reward profiles that are difficult to replicate in the cash markets.

67.4 Brady bonds

Brady bonds are bonds that have been issued as part of a restructuring of a country's commercial bank loans and other debt. Existing creditors tender their loans in exchange for the new bonds, which are sovereign bonds. The name refers to Nicholas Brady, chairman of the US Federal Reserve at the time the bonds were first introduced. The Brady market is characterised by high yields and liquidity levels ranging from very liquid to illiquid. Many Brady bonds are large size issues and trade in a liquid market. Due to the features of a Brady bond, they are traded by investors taking a view on the country risk, the yield spread to Treasuries or the volatility level.

There are a number of different types of bonds in existence, for example collateralised and uncollateralised, fixed-rate and floating-rate and so on. Although most of the issues to date have been long-dated bonds, short-term Brady bonds have also been issued. The bonds are denominated in US dollars, therefore they yield a spread over Treasury bonds. This makes the bonds very interesting to trade,³ as the yield on them reflects both the country credit risk as well as the shape and expectation of the Treasury yield curve. This is considered later in the chapter.

Countries that have completed Brady plans are listed in Table 67.1. At the end of 1998 there was over \$160 billion of total debt in existence.

Argentina	Ecuador	Panama	Venezuela
Brazil	Ivory Coast	Peru	Vietnam
Bulgaria	Jordan	Philippines	
Costa Rica	Mexico	Poland	
Dominican Republic	Nigeria	Uruguay	

Table 67.1: Countries that have issued Brady bonds.

² Source: Lehman Brothers.

³ Perhaps “interesting” is the wrong word! But they are certainly exciting products.

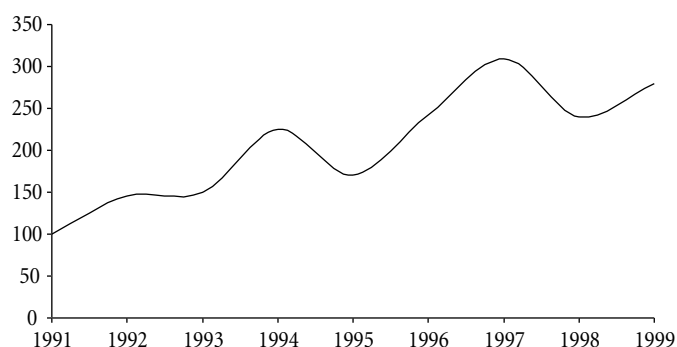


Figure 67.5: Brady bond index. Source: JP Morgan (December 1991 = 100).

Country	Bond	Bid	Offer
Bangladesh		73.00	78.00
Cambodia	Trade	18.00	22.00
Costa Rica	Principal A	88.25	89.25
Costa Rica	Principal B	85.25	86.25
Cuba	DEM denominated	7.00	9.00
Cuba	Yen	6.00	8.00
Guyana		20.00	30.00
Jamaica	Tranche A	90.75	92.75
Jamaica	Tranche B	86.50	87.75
Jordan	Par bonds	67.75	68.75
Jordan	Discount bonds	71.50	73.50
Jordan	IA bonds	97.00	98.00
Laos	Trade	12.00	15.00
Mongolia		10.00	15.00
Morocco	Restructured loans	92.00	93.00
Nicaragua		9.00	12.00
Sudan	Restructured loans	2.00	5.00
Surinam	Trade	23.00	27.00
Vietnam	PDI 3	53.80	54.30

Table 67.2: Exotic debt prices, US dollar denominated, February 2000. Source: IFR.

67.4.1 Brady bond structure

Brady bonds were introduced in the wake of the Latin American debt crisis of 1982. As countries threatened to default it was realised that the global banking system could be in danger if there was large-scale default. Therefore the debt was renegotiated and repacked as Brady bonds. The first step of the process is when the creditor banks negotiate with the debtor country to establish the level of borrowing that the country can realistically afford to service. This is sometimes undertaken in conjunction with the International Monetary Fund. The difference between the amount that the country can afford to service, and the actual level of its debt, is reconciled using one or more of the following:

- Discount bonds, where the principal amount is cut;
- Par bonds, where the debtor pays sub-market interest rates;
- a debt buy-back at an agreed discount rate.

There is an option available for creditors who do not wish to see the principal value of the loans reduced or receive below-market rates, in which case they must agree to lend additional funds in return for retaining the existing value of the obligation. Arrears in interest are usually rescheduled into a separate tradable bond.

The principal amount of Par and discount bonds is fully collateralised using zero-coupon US Treasury bonds. This provides comfort to creditors that the obligations will ultimately be repaid. In many cases interest payments, usually for the first few years of the issue, are also collateralised on a rolling basis; that is when the collateral backing a coupon payment is not used, it rolls forward to back the next coupon payment. This collateral arrangement therefore acts as a form of security for the investor in a Brady bond.

67.4.2 Types of Brady bond

The main type of Brady bond is the *collateralised fixed-rate par bond* or *Par bond*. These bonds are received in exchange for debt that is tendered at the face value. They offer the debtor permanent interest rate relief and protection from fluctuations in interest rates. The bonds are long-dated, usually of 25 or 30 years' maturity, and are conventional bullet bonds. The principal is collateralised with specially-issued zero-coupon US Treasury bonds which are held at the US Federal Reserve. There is also usually a rolling interest payment guarantee, collateralised by high-quality financial instruments rated at double-A or better, which cover 12–24 months of interest payments. In certain cases *value recovery rights* are attached to the bonds, which grant bondholders rights to payments under a formula linked to a commodity price index.

Collateralised floating-rate discount bonds or *Discount bonds* are received in exchange for eligible debt at a discount to face value, which results in permanent debt relief for the debtor in the form of partial relief on the debt obligation. In return for this the creditor receives higher interest payments, usually Libor plus 13–15%. When Mexico and Argentina issued discount bonds the yield spread was 35%, while for Bulgaria and Poland the spread was 50%. Again the bonds are long-dated, 25 or 30 years and have a single bullet payment at maturity. The principal is collateralised with zero-coupon Treasury bonds, and there is usually a rolling interest guarantee. Value recovery rights have also been attached to discount bonds.

Another type of Brady bond is the *front-loaded interest reduction bond* or *Flirb*. These bonds are received in exchange for debt at face value. They pay below-market interest rates for the first few years of their life, which is a temporary interest relief for the debtor. The principal is not collateralised, although there may be a rolling interest guarantee. As there is no collateralisation the bonds have a shorter average life, and amortise after an initial grace period of up to nine years.

Debt conversion bonds or *new money bonds* are received in exchange for debt at face value. They are conditional upon the creditor providing additional new money equivalent to a certain percentage of the amount of the eligible debt. Neither the principal nor the interest payments is collateralised, so the average lives of the bonds are shorter than par or discount bonds.

Interest arrears bonds are received in exchange for a creditor's claim on certain past due interest payments which have not been paid. There is no collateral backing for the bonds.⁴ The bonds are known by a number of names, including *Interest due and unpaid* or IDU bonds, *past due interest bonds* (PDI), or *interest arrears bonds* (IAB).⁵

The prices of selected Brady bonds on 5 April 2000 are given at Appendix 67.1.

67.4.3 Relative value

Since Brady bonds are (in most cases) collateralised instruments, we require additional techniques when assessing their value, beyond the simple gross redemption yield measure. Essentially a par bond has three elements: the principal, which is collateralised by US Treasuries, the collateralised rolling interest guarantee, and the (risk-carrying) remaining bond cash flows. The yield on a collateralised bond will be lower than that of a non-collateralised bond because of the credit protection. The country-specific risk factor, which is in the form of the spread to the Treasury yield, should in theory apply to the risk-carrying cash flows only. This means that an investor will calculate the present value of both the principal and the collateralised interest payments and subtract this from the price of the bond. This "stripped" price is then used when calculating the yield-to-maturity of the non-collateralised cash flows, and is known as the *stripped yield*. The stripped yield spread (as a spread to the Treasury bond) is viewed as

⁴ The exception to this was in the case of bonds issued by Costa Rica.

⁵ In the case of Russia the bonds are known as *interest arrears notes*.

the market's assessment of the sovereign risk. This approach enables the yields of collateralised and non-collateralised bonds to be compared. As we noted at the start of this section, the yield sensitivity of a Brady bond will reflect both the country credit risk and the Treasury yield curve. Separating a Brady bond's yield sensitivity to US interest rates and to credit risk is therefore of some importance for the market maker. We consider this here.

For a US Treasury bond we are familiar with the price/yield formula, given here as (67.1):

$$P = \frac{C/2}{(1 + \frac{1}{2}r)} + \frac{C/2}{(1 + \frac{1}{2}r)^2} + \frac{C/2}{(1 + \frac{1}{2}r)^3} + \dots + \frac{C/2 + 100}{(1 + \frac{1}{2}r)^{2n}}. \quad (67.1)$$

The most common Brady bond is the Par bond, for which the principal is collateralised. Given this security backing, the redemption payment can be taken as risk-free, which means that in theory it should be discounted at the Treasury yield for that maturity, rather than at a yield spread to the Treasury. Therefore the price/yield equation can be given as (67.2):

$$P = \frac{C/2}{(1 + \frac{1}{2}(r + s))} + \frac{C/2}{(1 + \frac{1}{2}(r + s))^2} + \dots + \frac{C/2}{(1 + \frac{1}{2}(r + s))^{2n}} + \frac{100}{(1 + \frac{1}{2}rs)^{2n}} \quad (67.2)$$

where

- r is the corresponding US Treasury yield
- s is the bond credit spread over the Treasury yield
- rs is the Treasury zero-coupon yield.

The s is the stripped spread.

We apply the same analysis when calculating interest-rate sensitivity for a Brady bond. The duration D of a Par bond is given by (67.3):

$$D = \frac{1}{P} \left[\frac{C/2}{(1 + \frac{1}{2}(r + s))} \cdot \frac{1}{2} + \frac{C/2}{(1 + \frac{1}{2}(r + s))^2} \cdot \frac{2}{2} + \dots + \frac{C/2}{(1 + \frac{1}{2}(r + s))^{2n}} \cdot n + \frac{100}{(1 + \frac{1}{2}rs)^{2n}} \times n \times \frac{1 + \frac{1}{2}(y + s)}{1 + \frac{1}{2}rs} \right]. \quad (67.3)$$

The final term in the expression at (67.3) receives a greater weight, because it is risk-free and therefore is discounted less heavily. This is the correct analysis; what it means is that a change in the Treasury yield curve will have a significant impact on the bond yield, as it affects all the bond's cash flows. However a change in the credit risk should have a smaller impact, because as it impact only the coupon payments. This has the effect of raising the duration of a Par bond, compared to the duration calculation carried out using the conventional approach. Using this analysis, the yield sensitivity of a Par bond, with respect to changes in the Treasury yield, is given by (67.4):

$$\frac{\Delta P/P}{\Delta y} = -\frac{D}{1 + \frac{1}{2}(r + s)}. \quad (67.4)$$

The credit risk sensitivity is a function of changes in the credit quality of the bond, and so it is not symmetrical with the bond's Treasury yield sensitivity. This is because the credit risk yield premium is only applicable to the bond's coupon payments, so the sensitivity measure will be lower. It is given by (67.5):

$$\frac{\Delta P/P}{\Delta s} = -\frac{D - (A/P)}{1 + \frac{1}{2}r + s} \quad (67.5)$$

where $A = n \cdot 100 \cdot \frac{1 + \frac{1}{2}(r + s)}{(1 + \frac{1}{2}rs)^{2n+1}}.$

The presence of the A/P term has the effect of reducing the impact of a change in the yield premium on the price of the bond compared to a change in the Treasury yield.

Appendices

APPENDIX 67.1 Brady bond prices

Argentina	Bond	Price
	Par	69.78
	Discount	84.13
	FRB	91.33
Brazil	IDU	99.59
	“C”	72.00
	Par	65.17
	Discount	76.54
	EI	88.69
	DCB	71.79
	NM94	83.98
	EXIT	68.92
Bulgaria	FLRB	76.17
	IAB	78.10
	Discount A	80.10
Croatia	FLB A	72.25
	FRN A	92.84
Ecuador	FRN B	84.88
	PDI	26.29
	Discount	41.07
Jordan	Par	36.67
	Discount	78.18
Mexico	Par	60.67
	Discount A	97.94
	Par A	83.41
Nigeria	Par B	84.88
	Par	71.70
Panama	PDI	83.73
	IRB	79.57
Peru	PDI	65.85
	FLRB	60.73
Philippines	FLB B	95.29
	Par B	85.08
Poland	Discount	99.75
	RSTA	66.67
	Par	62.17
Russia	IAN	27.88
	PRIN	25.51
Venezuela	FLB A	79.17
	FLB B	79.17
	DCB	77.88
	Discount A	76.42

Table 67.3: Brady bond prices, 5 April 2000. Source: Bloomberg.

Selected bibliography and references

Alumian. A., *The Emerging Bond Markets*, Mars publishing, 1998.

Brauer, J., Brady Bonds, in Fabozzi, F. (ed.), *The Handbook of Fixed Income Securities*, 5th edition, Dow-Jones Irwin, 1997.

The Economist, “Another coup”, *The Economist*, 27 November 1999.

Kamin, S., von Kleist, K., “The evolution and determinants of emerging market credit spreads in the 1990s”, *Bank for International Settlements Working Papers*, No. 68, May 1999.

68 Emerging Bond Markets II

Investors seeking to place funds in emerging market assets often employ techniques originally used in high-yield debt market analysis. The emerging markets are usually defined as those countries not in the OECD, which would include eastern Europe, Latin America, Asia, Africa and the Middle East.¹ There is a wide range of products available to investors across the emerging markets, including Brady bonds, Eurobonds and domestic bonds. Tradable loans are also available although the level of liquidity is lower than that of bonds. In the domestic market government T-bills and bonds are also available, at varying levels of liquidity, as well as corporate instruments such as commercial paper and bankers acceptances.

The emergence of significant trading volume in emerging bond markets means that they are an accepted segment of international capital markets. The total market size in all tradable instruments exceeded \$550 billion at the end of 1999,² and the potential future issuance of emerging market borrowers is viewed as approaching that of developed markets in the near- rather than long-term. It is important that institutional investors employ rigorous techniques when assessing the relative value that is available in these markets. In this chapter we introduce some of the key factors that need to be considered when analysing emerging fixed interest markets. In the second half of the chapter we present a summary of the debt markets in four selected emerging markets.

68.1 Analysing of relative value

An investor who is considering purchasing assets in a particular market needs to be aware of the methods by which relative value in that market is assessed. This is straightforward but also fundamental. For example, all the large independent credit rating agencies employ a rule that states that any corporate cannot be rated higher than the sovereign rating of the country that it is based in. This means that a corporate that is based in a country such as South Korea or India may well be rated higher were it to be domiciled in the United States. Understanding the factors that drive the analysis of relative value in a market is key to evaluating whether a particular analytical approach can be applied with success. In general the value of emerging market debt is a function of three key determinants:

- sovereign risk and cross-border risk;
- foreign capital flows, and technical factors that influence these;
- corporate issuers' economic health and prospects, also called *fundamental* factors.

In the case of sovereign debt, only the first two factors apply, whereas all three will be relevant in the case of corporate borrowers. We can consider each of these issues in greater detail.

68.1.1 Sovereign risk

The sovereign and cross-border risk in an emerging market investment is the exposure that arises from investing in a foreign country. The total foreign market risk includes political risk, which is the risk of political upheaval leading to a loss of investments, and social risk, which is the risk of loss arising from social upheaval (and which is closely related to political risk). Additional elements include risk of loss due to changes in the legal, tax or regulatory environment, which also are all related to political risk. Generally analysts define a specific segment of cross-border risk known as *sovereign risk*. This is the risk of loss due to default. Sometimes the risk exposure is extended to risk of loss due to excessive price volatility of sovereign debt. Sovereign risk therefore can be measured in terms of the probability of default, which is a function of the actions carried out by the sovereign power in the emerging market country. Sovereign risk is usually broken down into two elements, which are:

- **willingness to pay:** the risk of loss due to a sovereign simply refusing to honour existing debt. Exposure to this type of risk is difficult and problematic to predict, but its occurrence in practice has been rare. In the past it has taken place after changes in government brought on by revolution, such as in Russia in 1917 and China in 1949,

¹ For this analysis "Asia" is taken to exclude Japan, Singapore, South Korea and Taiwan.

² Source: EIU.

although Japan also refused to honour external debt in 1941. We can conclude that occurrences of this nature may take place in the event of significant and extreme changes in a country's internal social and ideological structure;

- **ability to pay:** the risk of loss due to a country being unable to honour, for economic reasons, its external debt. In the past a country has experienced an inability to honour external debt because of severe internal economic conditions, such as a balance of payments crisis, leading to the decision by the sovereign government to suspend debt service payments. This occurred for example in Mexico in 1982 (and spread to other Latin American countries), and Venezuela in 1994.

Hence there is a distinction between a country's willingness to pay and its ability to pay, although the first will and the second can lead to default. The second type can be modelled accurately enough to enable us to value default risk, thus allowing a price to be put on credit spreads. To assess a country's ability to pay, for instance because it is shortly going to access the international capital markets, we need to consider a range of factors, which can be used to determine if there will be sufficient cash flow available to service the debt issue. These include:

- **Foreign exchange reserves:** these represent funds required to service an international debt issue. The source of these reserves is also important, whether they are from exports or from inflows of foreign investment. A dependence on foreign investment capital flows can lead to difficulty in servicing debt if, due to events such as a market correction, investors decide to withdraw their funds. This was part of the problem behind the Asian market crisis in Thailand, which spread to other Asian countries, during 1997 and 1998. Although export earnings are regarded as the sign of a healthy economy, the nature of a country's exports are important. If exports are of a low value-added commodity product, for example raw timber or an agricultural commodity, the economy could be at risk of downturn due to an external shock that depressed commodity prices.
- **Balance of payments:** the overall balance of payments, or the *current account* of an economy reflects the impact of exports and foreign exchange reserves and is regarded as a key indicator. It may be that an increasing current account deficit is a sign of future upheaval, although it should not be the case that only a single indicator is used to assess the sovereign risk of a country. When breaking down the balance of payments analysts should also consider the extent of the country's imports, where these are more heavily weighted in terms of consumer goods or are capital products, as well as the methods used to finance the payments of deficit. To illustrate two extreme scenarios, a country that imported predominantly capital goods, financed via long-term funds would be viewed as a lower risk than a country that imported predominantly consumer goods using short-term funding.
- **Additional economic indicators:** these include the rate of inflation and inflation predictions, level of money supply, the independence or otherwise of monetary policy, and the levels and forecast of gross domestic product. It is important to build up a long-term trend of these indicators, as a few years' worth of data are not considered conclusive. It is also difficult to compare indicators across countries as they may have been calculated using different assumptions and methods.

68.1.2 Technical factors

The flow of capital into and out of an emerging market economy is sometimes influenced by technical factors, both in the target country and the country of origin. The creation of sudden interest in a particular market can lead to increased inflow of capital that drives down the price of that country's bonds, to a point where the bond yields do not reflect the sovereign risk of the issuer. It has been suggested³ that this occurred during 1992 and 1993, when technical pressures on investors led to ever increasing demand for emerging market debt. Spreads on such debt fell until the February 1994 rise in interest rates led to a bear market in bonds for the rest of the year.

Therefore it is important to be aware of investor-led demand leading to lower yields for bonds, to a point where yields do not reflect sovereign risk. Fundamental analysis of all the relevant factors remains essential, for all but the shortest-term investor, during a period of "hot money" that creates a demand for assets that depresses yields to unsustainable levels.

³ For example, see Vine (1997).

68.1.3 Fundamental risk

In many emerging market countries a market in corporate debt is developing alongside the main market in government bonds. In between the two categories of issuer are government agencies or nationalised industries or *parastatals*. Bonds issued by these issuers are analysed in terms of fundamental risk. This includes the bonds of parastatals, unless there is an explicit government guarantee for such issues. Fundamental analysis on corporate bonds can be carried out essentially in the same way that it is undertaken for developed country corporates. Where a difficulty may arise is in the availability of accurate financial and statistical data. This is magnified in the case of developing country banks, which often do not disclose lending data, making traditional financial analysis problematic. This means it is often impossible to gauge the financial strength of a bank, especially a state-owned one, until the institution is already in trouble. In many countries state-owned banks are not allowed to foreclose on bad loans, for political reasons, so that the extent of bad debt cannot be assessed. This is common where banks are required to finance nationalised industries, which are frequently run at a loss. This produces a paradoxical situation where a bank that is not nationalised may be forced to run a bad loan book, which represents government obligations, whereas its own debt in the form of bonds it has issued is not government debt and represents a potentially higher risk for investors.

Another common situation is where corporate debt represents a better credit risk for investors than the sovereign debt of the domicile country. Although the economic conditions in a country are very important, there may be times when the debt of a country can be regarded as better quality credit, despite the fact that it cannot have a better rating. An example of this is Gazprom, the natural gas exploration and utility company, and Russia. Investors may well feel that the former is a better credit, for a number of reasons, than the host country.

We have presented only the introductory points that are used in evaluating relative values in emerging bond markets. By combining all three elements in any analysis, namely cross-border and sovereign risk, technical factors and fundamental factors, an investor can arrive at an informed opinion of the risk exposures inherent in emerging bond markets.

68.2 Selected emerging bond markets

For the purposes of illustration we present here the key features of four emerging debt markets. The selection is purely arbitrary, and has been made to show the diversity of the different markets available for investors.

68.2.1 Poland

The government bond market in Poland was re-opened in 1991 and restrictions on foreign investment were removed in 1993. The central authority is the National Bank of Poland (NCB), which issues debt on behalf of the ministry of finance. It also carries out open market operations using government repo as the main tool, dealing direct with banks and government bond dealers.

Government bonds

The NCB issues eight-, 13-, 26-, 39- and 52-week bills each week via a competitive auction process. Bids must be above a set minimum price, and settlement is two days later. Treasury bills are settled in electronic book-entry form, and trade on the Warsaw stock exchange.

The total outstanding Polish government debt was Z264.5 billion at the end of 1999.⁴ Foreign investors held approximately 5% of this amount. The government raises finance using five different types of bonds. These are:

- One-year indexed bonds (IR and RP). These bonds pay interest on redemption at 3% over a specified base rate. The base rate is calculated using the consumer price index for the year, and there is a two-month indexation lag;
- Three-year floating-rate bonds (TZ). The interest rate on these bonds is fixed quarterly at 110% of the weighted average return of the first four 13-week T-bill auctions in the preceding month. Bonds are issued quarterly;
- Two-year fixed-rate bonds (OS). These were introduced in February 1994. They are conventional bonds issued via auction every month;
- Five-year fixed-rate bonds (OS). These are similar in form to the two-year bonds;

⁴ *Euromoney*, February 2000.

- Ten-year fixed-rate bonds (DZ). These bonds were first introduced in December 1995 and are issued quarterly via auction. The coupon is set at 100 basis points over the weighted-average return for 52-week T-bills during the two months prior to the first month of an interest period. The bonds are callable after the first five years.

Only the two-year and five-year bonds trade with any reasonable liquidity with average daily turnover of around \$200 million and \$100 million respectively.⁵ Bonds trade on the Warsaw stock exchange, with three-day settlement, however the dealing volume in the OTC market is larger. Both bills and bonds are bearer securities.

Other instruments

The Ministry of Finance issued the country's first Eurobond in July 1995, denominated in US dollars. This was a \$250 million issue, with a coupon at 7.75%, maturing in July 2000. A second DEM250 million nominal bond was issued subsequently, with 6.125% coupon and maturity in July 2001.

Poland is one of 17 countries to have renegotiated its external commercial bank debt using Brady bonds. There are six instruments, shown in Table 68.1.

Issue (\$)	Name	Coupon	Maturity
138m	New Money Bonds	6m Libor + 13/16	Oct-2009
2.7bn	Past Due Interest Bonds (PDIs)	3.25-7.00%	Oct-2014
393m	Debt Conversion Bonds (DCBs)	4.50-7.00%	Oct-2019
3bn	Collateralised Discount Bonds	6m Libor + 13/16	Oct-2024
930m	Collateralised Par Bonds	2.75-5.00%	Oct-2024
900m	Collateralised RSTA Bonds	2.75-5.00%	Oct-2024

Table 68.1: Polish Brady bond issues.

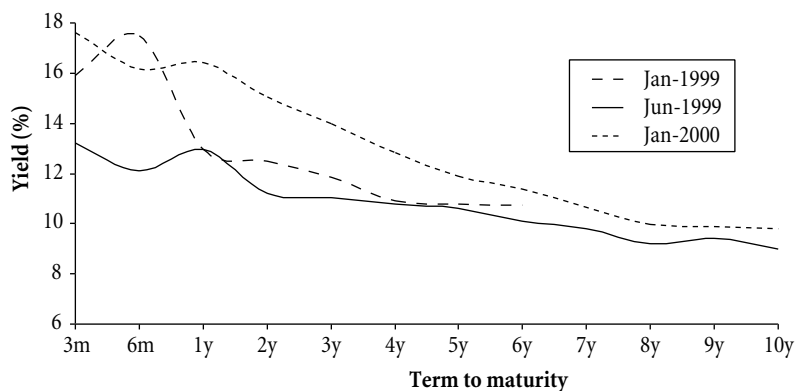


Figure 68.1: Polish government bond yield curve. Source: Bloomberg.

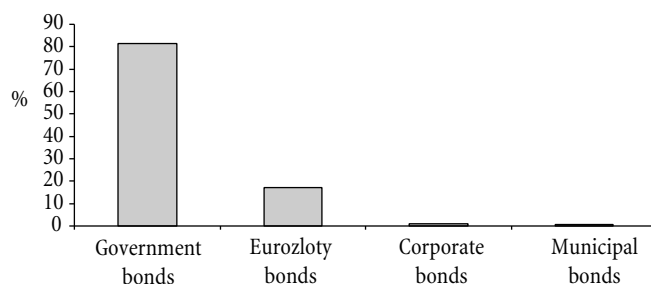


Figure 68.2: Zloty bond market volume outstanding, December 1999. Source: HypoVereinsbank.

⁵ *Euromoney*, February 2000.

Source: Bloomberg L. P. Used with permission.

With regard to market instruments, the central bank issues Treasury bills on behalf of the government, usually with 91-day maturities, but sometimes with one-year maturities. Currently foreign banks are not permitted to bid for bills at tender offers. Bonds are issued at maturities of between two and five years, although a maximum term of 15 years is allowed by statute. The bonds are issued in both registered and bearer form, to suit investor requirements. These bonds are known as Treasury bonds, and are issued on a regular monthly basis. The government also issues long-term development bonds, on an irregular basis, at maturities of 5–10 years. Development bonds trade on the secondary Amman Financial Market, and pay gross coupon on a semi-annual basis. A state utility, the Jordan

The money market is composed of commercial paper, Certificates of Deposit and bankers acceptances. Commercial paper is issued by private companies at maturities of between 30 days and 270 days. Unusually they are sold via auction. The secondary market in CP is illiquid, however there is a liquid market in bankers acceptances which trade both OTC and on the Bogota, Cali and Medellin stock exchange. All money market paper is in bearer form and is cash settled.

68.2.4 Malaysia

The central authorities in the Malaysian debt markets are the Ministry of Finance and the central bank, the Bank Negara Malaysia (BNM). The BNM and a second regulator, the Securities Commission report to the Ministry of Finance. The debt market is relatively liquid and well developed, and this reflects the government's policy of pushing the private debt market as an alternative to conventional bank borrowings to finance private sector investor. Both the government and corporate debt markets are run relatively efficiently and it is arguable that the market achieved its current level of sophistication mainly due to the active participation of the government. That said the markets are still influenced by external factors and experienced high volatility during 1997 and 1998, in common with other Asian, Latin American and eastern European emerging markets.

Government debt

The short-end of the yield curve is composed of Malaysian Treasury Bills (MTBs). The BNM issues 91-, 182- and 364-day bills on behalf of the federal Treasury at a regular tender auction. Issues are made to registered primary dealers who are obliged to bid for the bills at each tender. The 91-day bills are issued weekly, the 182-day bills fortnightly and the 364-day bills every month. The bills are conventional discount instruments. The central bank also issues Bank Negara Bills (BNBs) which are used in its open market operations, and also to absorb speculative cash flows. Banks can use BNBs as collateral for overnight repo transactions, as well as to meet liquidity requirements.

The longer-end of the yield curve is made up of Malaysian Government Securities (MGS). Bonds of between one- and 10-year maturity are offered at a "best price" tender basis to primary dealers, and yields are determined by the weighted-average rates of the successful bids. Longer-dated issues are placed with major institutional investors at par, at a pre-specified coupon. As at March 2000 the longest-dated MGS was the 6.844% issue, which expires in October 2009. Note that pension funds have a statutory requirement to hold MGS, and are the largest investors in them.

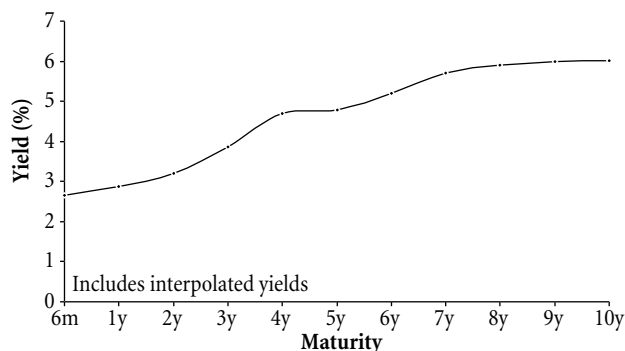


Figure 68.6: Malaysian government yield curve, March 2000. Source: Bloomberg.

Corporate debt

The Malaysian government has played an active part in developing a liquid corporate bond market, and it is possibly one of the most liquid in south-east Asia. As part of this development, the government set up the Rating Agency Malaysia Berhad (RAM) and the Malaysian Rating Corporation (MARC) in 1990 and 1996 respectively; in between in May 1992 the government made it mandatory for all domestic bonds to be rated. This enables issuers and investors to establish the credit quality of instruments in the market, which assists in the development of a liquid secondary market. To facilitate trading, clearing and settlement, in 1996 a central depository system called Sistem Pemindaham Elektronik Dana dan Sekuriti (SPEEDS) was introduced, which allowed for dematerialised electronic settlement. From this point it was also made compulsory for all new issues to be in paperless form and settled through the SPEEDS system. This was succeeded in July 1999 by the Real Time Transfer of Funds and Securities

Concluding Remarks

This book has highlighted the diverse nature of the world's debt capital markets. It has also shown how markets in bonds interact with other areas of the financial markets. Hence we have reviewed important topics that it is necessary for market participants to have an understanding of, such as risk management, value-at-risk, regulatory capital issues, bank asset and liability management, and the mathematics of stochastic processes. Every one of these subjects can be researched in their own right, and we hope that after having read this book, the reader is tempted to conduct further study in these areas, to greater depth. We have shown how the development of the market and the introduction of new instruments can be used to manage risk more effectively or make the market work more efficiently. The market in credit derivatives is a recent development, yet such instruments are already an established part of the operations of many banks and financial institutions, and they have allowed firms to hedge particular risks more effectively. The pace of development in only the last two decades of the twentieth century has been quite phenomenal. Consider that before 1980 many of the world's futures exchanges did not exist, and that no market existed in interest-rate swaps. Today instruments such as swaps, futures and forward-rate agreements are considered the plainest of plain vanilla instruments. We are probably aware that the largest companies now manage their risk exposures using tailored products, rather than standardised contracts; however even relatively small companies can now hedge their risk with tailored instruments. The chapter on exotic options illustrated the potential for variation and further development.

There is no doubt that further developments will continue to transform the structure of the debt markets, just as they have been transformed beyond almost all recognition from the 1980s onwards. For example, we can expect the use of the Internet to affect the way that bonds are issued and traded, just as they are already having an impact in the equity market, for both institutional and retail customers. In this final part of the book we suggest directions in which we expect the market to develop.

Developments in finance

The growth of the derivatives markets was partly a response to need for companies and financial institutions to lay off their risk exposure. Initially firms used standard instruments to hedge their risk, before moving on to instruments that were designed to match their specific requirements in a more tailored fashion. The key issue underpinning the development of new instruments is an ability by the market making bank to price, and hedge, this new instrument. In the past new developments in the field of mathematics, as well as in computer technology, have enabled banks to price risk and allocate capital more efficiently. Sharpe (1995) has proposed something known as "particle finance", named after the approach used in nuclear physics, which is the process of breaking down financial instruments into their smallest components, in an effort to price specific and often very esoteric forms of risk. Even without this approach, there is now a range of particular risks that can be hedged using financial instruments. For example, from September 1999 the Chicago Mercantile Exchange introduced weather derivatives, which are used by companies such as power generating utilities to hedge against changes in the weather. This followed the rapid growth of an over-the-counter market in weather contracts, used by power companies and also insurance companies to hedge against weather-related risks. In the United States the market in OTC weather contracts reached over \$4 billion during 1999¹. The CME's weather futures change in value with changes in the temperature, linked to an index that records the temperature in four cities in the United States. The market in energy derivatives, again used by power generating and power distribution utilities, is also a rapidly growing one.

More of the instruments offered as hedging products for companies are priced by insurance companies as well as banks. Such companies specialise in the pricing of very specific events, but connected with exposures that are often financial in nature. This process is known as *alternative risk transfer*, which involves the closer interaction of insurance and capital markets. It is made possible by developments in technology that enable rapid pricing of new instruments, as well as the entry of new firms into the markets. For example specialised arms of insurance and life

¹ *The Economist*, 1999.

assurance companies now offer loans and take deposits, often exclusively or partly over the Internet; therefore the facilitation of credit is no longer the exclusive preserve of banks. In related developments, insurance companies compete with investment banks to offer risk products to corporate customers, while fund managers are beginning to invest directly in asset holdings, rather than deal via an intermediary bank or securities house. In many cases diversification of financial institutions product base has been made possible by the Internet, which significantly reduces costs without, in theory, reducing efficiency.

Debt markets and the Internet

The global market in bonds, estimated at over \$34 trillion,² is largely an over-the-counter one. Transactions are conducted over the telephone, with investors buying paper largely via intermediaries in the shape of banks or stockbrokers. The market is transparent only for certain types of bonds, with only a few types as highly liquid as developed country government securities. Otherwise there is a lower level of transparency compared to say, equities, which are frequently dealt on an exchange and at a screen-advertised price (of course many equities are completely illiquid, despite being listed on an exchange, with perhaps only one or two market makers and with very wide spreads). Only recently have borrowers and underwriting banks begun to explore other means of issuing bonds. In January 2000 two bond issues were offered over the Internet, enabling investors to buy bonds direct from the issuer. Both issues, one by the US mortgage agency Freddie Mac and the other by the World Bank, were underwritten by a syndicate³, but allowed the issuers to broaden their traditional investor base, including individual retail investors. In the same month another US mortgage agency, Fannie Mae, also issued bonds over the Internet.

The impact of the Internet has not been limited to the United States. In February 2000 the European Investment Bank issued £500 million of a ten-year bond via Barclays Capital and Warburg Dillon Read; this was the first sterling “e-bond” deal and the EIB later stated that over half of the issue was ordered over the Internet. The immediate impact of the growing use of the Internet can be expected to be to a fall in the cost of issuing and buying bonds, possibly a reduction in bid-offer spreads. It may be that underwriters are no longer required for certain issues, although this is unlikely to be a possibility for all but the highest quality borrowers. Certain lenders may prefer to issue bonds via an underwriting bank, if certain investors do not wish to deal over the Internet but prefer their traditional relationship with their broker. Investment banks themselves can expect to see a reduction in their costs, and probably a reduction in their sales force, as trading bonds over the Internet rises to a larger share of total trading volume.

The Internet allows borrowers to target a larger group of investors, and it is possible that smaller investors, who previously did not take part in new issues, for example local authority pension funds and fund managers with less than £1 billion under management, will be able to take part in the primary market. It is only a matter of time before a complete trading mechanism, from marketing, offering, and placing to secondary market trading, is available over the Internet. The growth of corporate borrowing in the bond markets in Europe and elsewhere, which has been spurred on by the reduction of government deficits and the demand for yield from investors, should also result in a wide variety of bonds being available over the Internet. One can expect that investment banks that facilitate such a service will maintain a competitive edge over other banks and dominate bond offerings.

The importance of government debt

There is no doubt about the importance of developed country government debt markets in the functioning of the world’s capital markets. At the time of writing though, the United States Treasury had announced a programme of debt buy-backs, reflecting the country’s budget surplus that was estimated to rise to around 2% of gross domestic product by 2001, a turnaround from a budget deficit of 6% as recently as 1992.⁴ In the United Kingdom the forecast is of a budget surplus in 2001/2002, which should result in a reduction of supply in gilts. Similar developments are predicted for other developed economies, including European Union countries such as Denmark, Spain and the Netherlands as well as Australia and New Zealand.⁵ This has far-reaching implications. Government bond yields are

² Source: Bank for International Settlements, 2000.

³ For Freddie Mac, Warburg Dillon Read, Merrill Lynch and Salomon Smith Barney, and for the World Bank, Goldman Sachs and Lehman Brothers.

⁴ Source: OECD.

⁵ Source: JP Morgan, February 2000.

the benchmark against which all other debt market interest rates are set, and (as has already been observed in the long end of the gilt yield curve) a reduction in bond supply depresses yields to the point where they no longer remain accurate indicators of inflation expectations; it also results in calculated forward rates that may not reflect short-term market expectations. Banks are also required to hold government securities for liquidity purposes. A lack of supply therefore carries regulatory implications.

It is a moot point as to which instruments can be used as an adequate substitute for government bonds should the supply of the latter dry up. In the US market it has been suggested that agency securities be used, while in the UK the interest-rate swap curve is a possibility. Another alternative is the use of bonds issued by supranational bodies such as the World Bank or European Investment Bank. The use of any of these instruments is not without its problems however, including issues of liquidity and true default-free status. On the other hand there is a case for suggesting that where governments run budget surpluses, they should continue to issue government bonds, thus recognising the importance of national debt in capital markets. This currently occurs in Norway, where the government has run a budget surplus for a number of years but continues to issue bonds. The funds raised are invested in public sector investment and infrastructure projects. This approach may be more difficult for a large economy to adopt, such as the United States, where the size of such a public fund could influence asset prices both domestically and abroad. However it remains a possibility. On the other hand, government bond markets remain large and the total stock of world public debt exceeds \$15 trillion,⁶ a volume that will not disappear in quick time. It is also possible that budget surpluses quickly turn to deficits; this happened in the UK in the late 1980s. The subject matter is large and diverse and worthy of further study and research.

The one thing that we can expect to remain unchanged is the pre-eminence, and the importance, of debt markets in global economic development.

Bibliography

Sharpe, B., *Risk Management: Problems and Solutions*, McGraw-Hill, 1995.

The Economist, "The future of finance", *The Economist*, 11 December 1999, pp. 113–114.

⁶ Source: *The Economist*.

Glossary

A

Accreting: An accreting principal is one which increases during the life of the deal. See **amortising, bullet**.

Accreting swap: Swap whose notional amount increases during the life of the swap (opposite of **amortising swap**).

Accrued interest: The proportion of interest or coupon earned on a bond from the previous coupon payment date until the value date.

Accumulated value: The same as **future value**.

act/360: A day/year count convention taking the number of calendar days in a period and a “year” of 360 days.

act/365: A day/year convention taking the number of calendar days in a period and a “year” of 365 days. Under the ISDA definitions used for interest rate swap documentation, ACT/365 means the same as **act/act**.

act/act: A day/year count convention taking the number of calendar days in a period and a “year” equal to the number of days in the current coupon period multiplied by the coupon frequency. For an interest rate swap, that part of the interest period falling in a leap year is divided by 366 and the remainder is divided by 365.

Agency securities: In the United States, securities issued by government-sponsored entities, such as the Government National Mortgage Association (GNMA) and the Federal Home Loan Mortgage Corporation (FHLMC).

All-in price: See **dirty price**.

All or nothing: Digital option. This option’s **put** (call) pays out a pre-determined amount (“the all”) if the index is below (above) the strike price at the option’s expiration. The amount by which the **index** is below (above) the **strike** is irrelevant; the payout will be all or nothing.

American: An American option is one which may be exercised at any time during its life.

Amortising: An amortising principal is one which decreases during the life of a deal, or is repaid in stages during a loan. Amortising an amount over a period of time also means accruing for it pro rata over the period. See **accreting, bullet**.

Annuity: An investment providing a series of (generally equal) future cash flows.

Appreciation: An increase in the market value of a currency in terms of other currencies. See **depreciation, revaluation**.

Arbitrage: The process of buying securities in one country, currency or market, and selling identical securities in another to take advantage of price differences. When this is carried out simultaneously, it is in theory a risk-free transaction. There are many forms of arbitrage transactions. For instance in the cash market a bank might issue a money market instrument in one money centre and invest the same amount in another centre at a higher rate, such as an issue of three-month US dollar CDs in the United States at 5.5% and a purchase of three-month Eurodollar CDs at 5.6%. In the futures market arbitrage might involve buying three-month contracts and selling forward six-month contracts.

Arbitrageur: Someone who undertakes arbitrage trading.

Arch: (autoregressive conditional heteroscedasticity) A discrete-time model for a random variable. It assumes that variance is stochastic and is a function of the variance of previous time steps and the level of the underlying.

Asian: An Asian option depends on the average value of the underlying over the option’s life.

Ask: See **offer**.

Asset: Probable future economic benefit obtained or controlled as a result of past events or transactions. Generally classified as either current or long-term.

Asset & Liability Management (ALM): The practice of matching the term structure and cash flows of an organisation’s asset and liability portfolios to maximise returns and minimise risk. Also includes the deliberate mismatching of cash flows to take account of views on the short-term yield curve.

Asset allocation: Distribution of investment funds within an asset class or across a range of asset classes for the purpose of diversifying risk or adding value to a portfolio.

Asset securitisation: The process whereby loans, receivables and other illiquid assets in the balance sheet are packaged into interest-bearing securities that offer attractive investment opportunities.

Asset-backed security: A security which is collateralised by specific assets – such as mortgages – rather than by the intangible creditworthiness of the issuer.

Asset swap: An interest rate swap or currency swap used in conjunction with an underlying

asset such as a bond investment. See **liability swap**.

Asset-risk benchmark:

Benchmark against which the riskiness of a corporation's assets may be measured. In sophisticated corporate risk management strategies the dollar risk of the liability portfolio may be managed against an asset-risk benchmark.

At-the-money (ATM): An option is at-the-money if the current value of the underlying is the same as the strike price. See **in-the-money**, **out-of-the-money**.

Auction: A method of issuing bonds where institutions submit bids to the issuer on a price or yield basis. Auction rules vary considerably across markets.

Average cap: Also known as an average rate cap, a cap on an average interest rate over a given period rather than on the rate prevailing at the end of the period. See also **average price (rate) option**.

Average price (rate) option: Option on a currency's average exchange rate or commodity's average spot price in which four variables have to be agreed between buyer and seller: the premium, the **strike** price, the source of the exchange rate or commodity price data and the sampling interval (each day, for example). At the end of the life of the option the **average spot exchange rate** is calculated and compared with the strike price. A cash payment is then made to the buyer of the option that is equal to the face amount of the option times the difference between the two rates (assuming the option is **in-the-money**; otherwise it expires worthless).

B

Back-testing: The validation of a model by feeding it historical data and comparing the results with historical reality.

Backwardation: The situation when a forward of futures price for something is lower than the spot price (the same as forward discount in foreign exchange). See **contango**.

Balance sheet: Statement of the financial position of an enterprise at a specific point in time, giving assets, liabilities and stockholders' equity.

Band: The Exchange Rate Mechanism (ERM II) of the European Union links the currencies of Denmark and Greece in a system which limits the degree of fluctuation of each currency against the euro within a band (of 15 per cent for the drachma and 21/4 per cent for the krone) either side of an agreed par value.

Banker's acceptance: See **bill of exchange**.

Barbell: In its simplest form, a position consisting of long positions in short- and long-dated bonds, and no holding of a medium-dated bond. A barbell portfolio would consist of a portfolio of very short-dated and very long-dated bonds.

Bargain: A transaction undertaken on the London Stock Exchange.

Barrier option: A barrier option is one which ceases to exist, or starts to exist, if the underlying reaches a certain barrier level. See **knock out/in**.

Base currency: Exchange rates are quoted in terms of the number of units of one currency (the variable or counter currency) which

corresponds to one unit of the other currency (the base currency).

Basis: The underlying cash market price minus the futures price. In the case of a bond futures contract, the futures price must be multiplied by the conversion factor for the cash bond in question.

Basis point: In interest rate quotations, 0.01 per cent.

Basis rate: The rate applicable in a basis swap.

Basis risk: A form of market risk that arises whenever one kind of risk exposure is hedged with an instrument that behaves in a similar, but not necessarily identical way. For instance a bank trading desk may use three-month interest rate futures to hedge its commercial paper or euronote programme. Although eurocurrency rates, to which futures prices respond, are well correlated with commercial paper rates they do not always move in lock step. If, therefore, commercial paper rates move by 10 basis points but futures prices dropped by only 7 basis points, the 3 bp gap would be the basis risk.

Basis swap: An interest rate swap where both legs are based on floating rate payments.

Basis trade: Buying the basis means selling a futures contract and buying the commodity or instrument underlying the futures contract. Selling the basis is the opposite.

Basket option: Option based on an underlying basket of bonds, currencies, equities or commodities.

Bearer bond: A bond for which physical possession of the certificate is proof of ownership. The issuer does not know the identity of the bondholder.

Traditionally the bond carries detachable coupon, one for each interest payment date, which are posted to the issuer when payment is due. At maturity the bond is redeemed by sending in the certificate for repayment. These days bearer bonds are usually settled electronically, and while no register of ownership is kept by the issuer, coupon payments may be made electronically.

Bear spread: A spread position taken with the expectation of a fall in value in the underlying.

Benchmark: A bond whose terms set a standard for the market. The benchmark usually has the greatest liquidity, the highest turnover and is usually the most frequently quoted. It also usually trades expensive to the yield curve, due to higher demand for it amongst institutional investors.

Beta: The sensitivity of a stock relative to swings in the overall market. The market has a beta of one, so a stock or portfolio with a beta greater than one will rise or fall more than the overall market, whereas a beta of less than one means that the stock is less volatile.

Bid: The price at which a market maker will buy bonds. A tight bid-offer spread is indicative of a liquid and competitive market. The bid rate in a **repo** is the interest rate at which the dealer will borrow the **collateral** and lend the cash. See **offer**.

Big figure: In a bond price quotation, the price omitting the decimal portion. For example, if the 10-year benchmark is quoted as 109.15-21, the big figure is 109. See **points**.

Bid-offer: The two-way price at which a market maker will buy and sell stock.

Bilateral netting: The ability to offset amounts owed to a counterparty under one contract against amounts owed to the same counterparty under another contract – for example, where both transactions are governed by one master agreement. Also known as **cherry-picking**.

Bill: A *bill of exchange* is a payment order written by one person (the drawer) to another, directing the latter (drawee) to pay a certain amount of money at a future date to a third party. A bill of exchange is a bank draft when drawn on a bank. By accepting the draft, a bank agrees to pay the face value of the obligation if the drawer fails to pay, hence the term *banker's acceptance*. A *Treasury bill* is short-term government paper of up to one year's maturity, sold at a discount to principal value and redeemed at par.

Bill of exchange: A short-term, **zero-coupon** debt issued by a company to finance commercial trading. If it is guaranteed by a bank, it becomes a banker's acceptance.

Binomial tree: A mathematical model to value options, based on the assumption that the value of the underlying can move either up or down a given extent over a given short time. This process is repeated many times to give a large number of possible paths (the "tree") which the value could follow during the option's life.

BIS (Bank for International Settlements): Known as the central bank for central banks, an international organisation situated in Basel, Switzerland, which acts as the central bank for sovereign entities. It produced the "Basel accord" in 1988, implemented in 1992, which established capital requirements for banking institutions based on the risk-

weighting of their assets; it also produced rules on the capital treatment of derivative instruments.

Black-Scholes: A widely used option pricing formula devised by Fischer Black and Myron Scholes.

Blended interest rate swap: Result of adding forward swap to an existing swap and blending the rates over the total life of the transaction.

Bloomberg: The trading, analytics and news service produced by Bloomberg LP; also used to refer to the terminal itself. Introduced by ex-Salomon Brothers trader Mike Bloomberg.

Bond basis: An interest rate is quoted on a bond basis if it is on an **act/365**, **act/act** or **30/360** basis. In the short term (for accrued interest, for example), these three are different. Over a whole (non-leap) year, however, they all equate to 1. In general, the expression "bond basis" does not distinguish between them and is calculated as **act/365**. See **money-market basis**.

Bond-equivalent yield: The yield which would be quoted on a US treasury bond which is trading at par and which has the same economic return and maturity as a given treasury bill.

Bootstrapping: A method of deriving the term structure of interest rates from market bond prices and yields, using successive bonds to calculate the spot rate along the maturity term structure. In mathematics, a method of solving simultaneous equations in which the first equation contains one unknown, the next equation contains two unknowns and so on. The result obtained by solving the first equation is used to solve the second equation, and so on.

BPV: Basis point value. The price movement resulting from a one basis point change in yield.

Break forward: A product equivalent to a straightforward option, but structured as a forward deal at an off-market rate which can be reversed at a penalty rate.

Broken date: A maturity date other than the standard ones (such as 1 week, 1, 2, 3, 6 and 12 months) normally quoted.

Broker-dealers: Members of the London Stock Exchange who may intermediate between customers and market makers; may also act as principals, transacting business with customers from their own holdings of stock.

Bulldog: Sterling domestic bonds issued by non-UK domiciled borrowers. These bonds trade under a similar arrangement to Gilts and are settled via the Central Gilts Office (now CREST).

Bull spread: A spread position taken with the expectation of a rise in value in the **underlying**.

Bullet: A loan/deposit has a bullet maturity if the principal is all repaid at maturity. See **amortising**.

Buy/sell-back: Opposite of **sell/buy-back**.

C

Cable: The exchange rate for sterling against the US dollar.

CAD: The European Union's Capital Adequacy Directive.

Calendar spread: The simultaneous purchase/sale of a futures contract for one date and the sale/purchase of a similar futures contract for a different date. See **spread**.

Callable bond: A bond which provides the borrower with an

option to redeem the issue before the original maturity date. In most cases certain terms are set before the issue, such as the date after which the bond is callable and the price at which the issuer may redeem the bond.

Call option: An option to purchase the commodity or instrument underlying the option. See **put**.

Call price: The price at which the issuer can call in a bond.

Cancellable swap: Swap in which the payer of the fixed rate has the option, usually exercisable on a specified date, to cancel the deal (see also **swaption**).

Cap: A series of borrower's IRGs, designed to protect a borrower against rising interest rates on each of a series of dates.

Capital market: Long term market (generally longer than one year) for financial instruments. See **money market**.

Caption: Option on a **cap**.

Cash: See **cash market**.

Cash-and-carry: The US market term for basis trading, an arbitrage-type trade in which a simultaneous position in cash bond and associated bond futures contract is put on, with a view to exploiting price differentials between the spot price of the bond and the future price implied by the current price of the bond future. A cash-and-carry trade is a long bond/short futures position, while the opposite of short the bond and long the future is known as a *reverse cash-and-carry*.

Cash market: The market in instruments that represent actual cash, which is an on-balance sheet asset. For example, an interbank deposit of £10 million represents £10 million by value of funds

placed in the deposit institution. Bonds are cash instruments.

CBOT: The Chicago Board of Trade, one of the two futures exchanges in Chicago, USA and one of the largest in the world.

CD: See **certificate of deposit**

Cedel: Centrale de Livraison de Valeurs Mobilieres; a clearing system for Euro-currency and international bonds. Cedel is located in Luxembourg and is jointly owned by a number of European banks. Subsequently renamed **Clearstream** on merger with Deutsche Bourse.

Ceiling: The same as **cap**.

Central Gilts Office: The office of the Bank of England which runs the computer-based settlement system for gilt-edged securities and certain other securities (mostly bulldogs) for which the Bank acts as Registrar. Subsequently merged with **CREST**.

Central limit theorem: The assertion that as sample size, n , increases, the distribution of the mean of a random sample taken from almost any population approaches a normal distribution.

Certificate of Deposit (CD): A money market instrument of up to one year's maturity (although CDs of up to five years have been issued) that pays a bullet interest payment on maturity. After issue, CDs can trade freely in the secondary market, the ease of which is a function of the credit quality of the issuer.

CGBR: Central Government Borrowing Requirement.

CGBCR: Central Government Net Cash Requirement.

CGO reference prices: Daily prices of gilt-edged and other securities held in CGO which are used by CGO

in various processes, including revaluing stock loan transactions, calculating total consideration in a repo transaction, and DBV assembly. Also known as CREST prices or DMO prices.

Cheapest to deliver: (Or CTD). In a bond futures contract, the one underlying bond among all those that are deliverable, which is the most price-efficient for the seller to deliver.

Cherry-picking: See **bilateral netting**.

Classic repo: Repo is short for “sale and repurchase agreement” – a simultaneous spot sale and forward purchase of a security, equivalent to borrowing money against a loan of collateral. A reverse repo is the opposite. The terminology is usually applied from the perspective of the repo dealer. For example, when a central bank undertakes repos, it is lending cash (the repo dealer is borrowing cash from the central bank).

Clean deposit: The same as **time deposit**.

Clean price: The price of a bond excluding accrued coupon. The price quoted in the market for a bond is generally a clean price rather than a **dirty price**.

Clearing house: The body that settles trades on a futures exchange, and which acts as the counterparty to every transaction. The clearing house is able to guarantee settlement by charging **margin** to all exchange participants, which is used to establish a default fund.

Clearstream: The international bond market clearing system, resulting from a merger between CEDEL and Deutsche Bank.

Close-out netting: The ability to net a portfolio of contracts with a

given counterparty in the event of default. See also **bilateral netting**.

Closing leg: In a repo transaction, the termination of the trade when the bonds (or other assets) are returned against receipt of borrowed funds and repo interest.

CMO: Central Moneymarkets Office which settles transactions in Treasury bills and other money markets instruments, and provides a depository (in the UK). Now merged with CREST.

CMTM: Current **mark-to-market** value. See **current exposure** and **replacement cost**.

Collar: The simultaneous sale of a **put (or call) option** and purchase of a call (or put) at different strikes – typically both **out-of-the-money**.

Collateral: Something of value, often of good creditworthiness such as a government bond, given temporarily to a counterparty to enhance a party's creditworthiness. In a **repo**, the collateral is actually sold temporarily by one party to the other rather than merely lodged with it.

Commercial paper: A short-term security issued by a company or bank, generally with a zero coupon.

Commodity swap: Swap where one of the cash flows is based on a fixed value for the underlying commodity and the other is based on a floating index value. The commodity is often oil or natural gas, although copper, gold, other metals and agricultural commodities are also commonly used. The end-users are consumers, who pay a fixed-rate, and producers.

Competitive bid: A bid for the stock at a price stated by a bidder in an auction. **Non-competitive bid** is a bid where no price is

specified; such bids are allotted at the weighted average price of successful competitive bid prices.

Compound interest: When some interest on an investment is paid before maturity and the investor can reinvest it to earn interest on interest, the interest is said to be compounded. Compounding generally assumes that the reinvestment rate is the same as the original rate. See **simple interest**.

Compound option: Option on an option, the first giving the buyer the right, but not the obligation, to buy the second on a specific date at a pre-determined price. There are two kinds. One, on currencies, is useful for companies tendering for overseas contracts in a foreign currency. The interest rate version comprises **captions** and **floortions**.

Consideration: The total price paid in a transaction, including taxes, commissions and (for bonds) accrued interest.

Contango: The situation when a forward or futures price for something is higher than the spot price (the same as forward premium in foreign exchange). See **backwardation**.

Contingent option: Option where the premium is higher than usual but is only payable if the value of the underlying reaches a specified level. Also known as a contingent premium option.

Continuous compounding: A mathematical, rather than practical, concept of compound interest where the period of compounding is infinitesimally small.

Contract date: The date on which a transaction is negotiated. See **value date**.

Contract for differences: A deal such as an **FRA** and some futures contracts, where the instrument or commodity effectively bought or sold cannot be delivered; instead, a cash gain or loss is taken by comparing the price dealt with the market price, or an index, at maturity.

Conventional gilts (included double-dated): Gilts on which interest payments and principal repayments are fixed.

Conversion factor: In a bond futures contract, a factor to make each deliverable bond comparable with the contract's notional bond specification. Defined as the price of one unit of the deliverable bond required to make its yield equal the notional coupon. The price paid for a bond on delivery is the futures settlement price times the conversion factor.

Convertible currency: A currency that may be freely exchanged for other currencies.

Convexity: A measure of the curvature of a bond's price/yield curve (mathematically, $\frac{d^2P/dr^2}{\text{dirty price}}$).

Corporate bond: A cash debt instrument that has been issued by a corporate, and is therefore an IOU issued by the company and which it is obliged to redeem in accordance with its contractual terms.

Correlation matrices: Statistical constructs used in the value-at-risk methodology to measure the degree of relatedness of various market forces.

Corridor: The same as **collar**.

Cost of carry: The net running cost of holding a position (which may be negative) – for example, the cost of borrowing cash to buy a bond

less the coupon earned on the bond while holding it.

Counterparty risk weighting: See **risk weighting**.

Country risk: The risks, when business is conducted in a particular country, of adverse economic or political conditions arising in that country. More specifically, the credit risk of a financial transaction or instrument arising from such conditions.

Coupon: The interest payment(s) made by the issuer of security to the holders, based on the coupon rate and the face value.

Coupon swap: An interest rate swap in which one leg is fixed-rate and the other floating-rate. See **basis rate**.

Covariance: A statistical measure of how much two random variables are related to each other.

Cover: To cover an exposure is to deal in such a way as to remove the risk – either reversing the position, or hedging it by dealing in an instrument with a similar but opposite risk profile. Also the amount by how much a bond auction is subscribed.

Covered call/put: The sale of a covered call option is when the option writer also owns the underlying. If the underlying rises in value so that the option is exercised, the writer is protected by his position in the underlying. Covered puts are defined analogously. See **naked**.

Covered interest arbitrage: Creating a loan/deposit in one currency by combining a loan/deposit in another with a forward foreign exchange swap.

CP: See **commercial paper**.

Credit (or default) risk: The risk that a loss will be incurred if a

counterparty to a derivatives transaction does not fulfil its financial obligations in a timely manner.

Credit derivatives: Financial contracts that involve a potential exchange of payments in which at least one of the cash flows is linked to the performance of a specified underlying credit-sensitive asset or liability. Payment is made by the seller of credit protection, to the buyer, in the event of a specified credit event.

Credit-equivalent amount: As part of the calculation of the risk-weighted amount of capital the Bank for International Settlements (BIS) advises each bank to set aside against derivative credit risk; banks must compute a credit-equivalent amount for each derivative transaction. The amount is calculated by summing the **current replacement cost**, or market value, of the instrument and an **add-on factor**.

Credit risk (or default risk) exposure: The value of the contract exposed to default. If all transactions are marked to market each day, such positive market value is the amount of previously recorded profit that might have to be reversed and recorded as a loss in the event of counterparty default.

Credit spread: The interest rate spread between two debt issues of similar duration and maturity, reflecting the relative creditworthiness of the issuers.

Credit swaps: Agreement between two counterparties to exchange disparate cash flows, at least one of which must be tied to the performance of a credit-sensitive asset or to a portfolio or index of such assets. The other cash flow is usually tied to a **floating-rate index** (such as **Libor**) or a fixed

rate or is linked to another credit-sensitive asset.

Credit value-at-risk (CVAR): See **value-at-risk (VAR)**.

CREST: The paperless share settlement system through which trades conducted on the London Stock Exchange can be settled. The system is operated by CRESTCo and was introduced in 1996.

CRND: Commissioners for the Reduction of the National Debt, formally responsible for investment of funds held within the public sector e.g., National Insurance Fund.

Cross: See **cross-rate**.

Cross-rate: Generally an exchange rate between two currencies, neither of which is the US dollar. In the American market, spot cross is the exchange rate for US dollars against Canadian dollars in its direct form.

CTD: See **cheapest to deliver**.

Cum-dividend: literally “with dividend”, stock that is traded with interest or dividend accrued included in the price.

Cumulative default rate: See **probability of default**.

Current exposure: The risk exposure, whether this is market, credit or operational risk, to liabilities that are ongoing.

Currency option: The option to buy or sell a specified amount of a given currency at a specified rate at or during a specified time in the future.

Currency swap: An agreement to exchange a series of cash flows determined in one currency, possibly with reference to a particular fixed or floating interest payment schedule, for a series of cash flows based in a different currency. See **interest rate swap**.

Current assets: Assets which are expected to be used or converted to cash within one year or one operating cycle.

Current liabilities: Obligations which the firm is expected to settle within one year or one operating cycle.

Current yield: Bond coupon as a proportion of clean price per 100; does not take principal gain/loss or time value of money into account. See **yield to maturity, simple yield to maturity**.

Cylinder: The same as **collar**.

D

DAC-RAP: Delivery against collateral – receipt against payment. Same as **DVP**.

Daily range: The difference between the high and low points of a single trading day.

Day count: The convention used to calculate accrued interest on bonds and interest on cash. For UK gilts the convention changed to actual/actual from actual/365 on 1 November 1998. For cash the convention in sterling markets is actual/365.

DBV (delivery by value): A mechanism whereby a CREST/CGO member may borrow from or lend money to another CREST/CGO member against overnight gilt collateral. The CREST/CGO system automatically selects and delivers securities to a specified aggregate value on the basis of the previous night’s CREST/CGO reference prices; equivalent securities are returned the following day. The DBV functionality allows the giver and taker of collateral to specify the classes of security to included within the DBV. The options are: all classes of security held within CREST/CGO, including strips and bulldogs; coupon bearing gilts and

bulldogs; coupon bearing gilts and strips; only coupon bearing gilts.

DEaR: Daily earnings at risk.

Debenture: In the US market, an unsecured domestic bond, backed by the general credit quality of the issuer. Debentures are issued under a trust deed or indenture. In the UK market, a bond that is secured against the general assets of the issuer.

Debt Management Office (DMO): An executive arm of the UK Treasury, responsible for cash management of the government’s borrowing requirement. This includes responsibility for issuing government bonds (gilts), a function previously carried out by the Bank of England. The DMO began operations in April 1998.

Default correlation: The degree of covariance between the probabilities of default of a given set of counterparties. For example, in a set of counterparties with positive default correlation, a default by one counterparty suggests an increased probability of a default by another counterparty.

Default probability: See **probability of default**.

Default risk: See **credit risk**.

Default risk exposure: See **credit risk exposure**.

Default start options: Options purchased before their “lives” actually commence. A corporation might, for example, decide to pay for a deferred start option to lock into what it perceives as current advantageous pricing for an option that it knows it will need in the future.

Deferred strike option: Option where the strike price is established at a future date on the basis of the

spot foreign exchange price prevailing at that future date.

Deliverable bond: One of the bonds which is eligible to be delivered by the seller of a bond futures contract at the contract's maturity, according to the specifications of that particular contract.

Delivery: Transfer of bonds (in settlements) from seller to buyer.

Delivery versus payment (DVP): The simultaneous exchange of securities and cash. The assured payment mechanism of the CGO achieves the same protection.

Delta (Δ): The change in an option's value relative to a change in the underlying's value.

Depreciation: A decrease in the market value of a currency in terms of other currencies. See **appreciation, devaluation**.

Derivative: Strictly, any financial instrument whose value is derived from another, such as a forward foreign exchange rate, a futures contract, an option, an interest rate swap, and so on. Forward deals to be settled in full are not always called derivatives, however.

Devaluation: An official one-off decrease in the value of a currency in terms of other currencies. See **depreciation, revaluation**.

Digital option: Unlike simple European and American options, a digital option has fixed payouts and, rather like binary digital circuits, which are either on or off, pays out either this amount or nothing. Digital options can be added together to create assets that exactly mirror index price movements anticipated by investors. See **one touch all-or-nothing**.

Direct: An exchange rate quotation against the US dollar in which the

dollar is the **variable currency** and the other currency is the **base currency**.

Dirty price: The price of a bond including accrued interest. Also known as the "all-in" price.

Discount: The amount by which a bond is trading below par. In FX markets, amount by which a currency is cheaper, in terms of another currency, for future delivery than for spot.

Discount house: In the UK money market, originally securities houses that dealt directly with the Bank of England in T-bills and bank bills, or discount instruments, hence the name. Most discount houses were taken over by banking groups and the term is not now generally used, as the BoE now also deals directly with clearing banks and securities houses. Following closure of Gerard & King in 2000, the only remaining discount house is Cater Allen (part of Abbey National group).

Discount rate: The method of market quotation for certain securities (US and UK treasury bills, for example), expressing the return on the security as a proportion of the face value of the security received at maturity – as opposed to a **yield** which expresses the yield as a proportion of the original investment.

Discount swap: Swap in which the fixed-rate payments are less than the internal rate of return on the swap, the difference being made up at maturity by a balloon payment.

Dividend discount model: Theoretical estimate of market value that computes the economic or the net present value of future cash flows due to an equity investor.

DMO: The UK Debt Management Office.

DMR: The Debt Management Report, published annually by HM Treasury.

Down-and-in option: Barrier option where the holder's ability to exercise is activated if the value of the underlying drops below a specified level. See also **up-and-in option**.

Down-and-out option: Barrier option where the holder's ability to exercise expires if the value of the underlying drops below a specified level.

Dual currency option: Option allowing the holder to buy either of two currencies.

Dual currency swap: Currency swap where both the interest rates are fixed rates.

Dual strike option: Interest rate option, usually a **cap** or a **floor**, with one floor or ceiling rate for part of the option's life and another for the rest.

Duration: A measure of the weighted average life of a bond or other series of cash flows, using the present values of the cash flows as the weights. See **modified duration**.

Duration gap: Measurement of the interest rate exposure of an institution.

Duration weighting: The process of using the modified duration value for bonds to calculate the exact nominal holdings in a spread position. This is necessary because £1 million nominal of a two-year bond is not equivalent to £1 million of say, a five-year bond. The modified duration value of the five-year bond will be higher, indicating that its "basis point value" (bpv) will be greater, and that therefore £1 million worth of this bond represents greater sensitivity to a move in interest rates (risk). As

another example consider a fund manager holding £10 million of five-year bonds. The fund manager wishes to switch into a holding of two-year bonds with the same overall risk position. The basis point values of the bonds are 0.041583 and 0.022898 respectively. The ratio of the bpvs are $0.041583/0.022898 = 1.816$. The fund manager therefore needs to switch into $£10m \times 1.816 = £18.160$ million of the two-year bond.

DVP: Delivery versus payment, in which the settlement mechanics of a sale or loan of securities against cash is such that the securities and cash are exchanged against each other simultaneously through the same clearing mechanism and neither can be transferred unless the other is.

E

Early exercise: The exercise or assignment of an option prior to expiration.

ECU: The European Currency Unit, a basket composed of European Union currencies, now defunct following introduction of euro currency.

Effective rate: An effective interest rate is the rate which, earned as simple interest over one year, gives the same return as interest paid more frequently than once per year and then compounded. See **nominal rate**.

Efficient frontier method: Technique used by fund managers to allocate assets.

Efficient market: Attributed to Eugene Fama (1970), a market in which asset prices reflect all price-relevant information.

Embedded option: Interest-rate sensitive option within a debt instrument that affects its

redemption. Such instruments include **mortgage-backed securities** and **callable bonds**.

End-end: A money market deal commencing on the last working day of a month and lasting for a whole number of months, maturing on the last working day of the correlation.

Epsilon (€): The same as **vega**.

Equity: The residual interest in the net assets of an entity that remains after deducting the liabilities.

Equity options: Options on shares of an individual common stock.

Equity warrant: Warrant, usually attached to a bond, entitling the holder purchase share(s).

Equity-linked swap: Swap where one of the cash flows is based on an equity instrument or index, when it is known as an equity index swap.

Equivalent life: The weighted average life of the principal of a bond where there are partial **redemptions**, using the **present values** of the partial redemptions as the weights.

ERA: See **exchange rate agreement**.

Eta (η): The same as **vega**.

Euribor: The reference rate for the euro currency, fixed in Brussels.

Euro: The name for the domestic currency of the European Monetary Union. Not to be confused with **Eurocurrency**.

Euroclear: An international clearing system for Euro-currency and international securities. Euroclear is based in Brussels and managed by Morgan Guaranty Trust Company.

Eurocurrency: A Eurocurrency is a currency owned by a non-resident of the country in which the

currency is legal tender. Not to be confused with **euro**.

Euro-issuance: The issue of gilts (or other securities) denominated in euro.

Euromarket: The international market in which **Eurocurrencies** are traded.

European: A European **option** is one that may be exercised only at **expiry**. See **American**.

Exchange controls: Regulations restricting the free convertibility of a currency into other currencies.

Exchange rate agreement: A **contract for differences** based on the movement in a **forward-forward** foreign exchange swap price. Does not take account of the effect of **spot** rate changes as an **FXA** does. See **SAFE**.

Exchange-traded: Futures contracts are traded on a futures exchange, as opposed to **forward** deals which are **OTC**. **Option** contracts are similarly exchange traded rather than **OTC**.

Ex-dividend (xd) date: A bond's record date for the payment of coupons. The coupon payment will be made to the person who is the registered holder of the stock on the xd date. For UK gilts this is seven working days before the coupon date.

Exercise: To exercise an **option** (by the **holder**) is to require the other party (the **writer**) to fulfil the underlying transaction. Exercise price is the same as **strike** price.

Exotic option: Usually referred to simply as an "exotic", a non-standard or non-vanilla option that differs from conventional options, in one or more ways. Examples included path-dependent options such as Asian or lookback options.

Expected (credit) loss: Estimate of the amount a derivatives counterparty is likely to lose as a result of default from a derivatives contract, with a given level of probability. The expected loss of any derivative position can be derived by combining the distributions of credit exposures, rate of recovery and probabilities of default.

Expected default rate: Estimate of the most likely rate of default of a counterparty expressed as a level of probability.

Expected rate of recovery: See **rate of recovery**.

Expiry: An option's expiry is the time after which it can no longer be exercised.

Exposure: Risk to market movements.

Exposure profile: The path of worst-case or expected exposures over time. Different instruments reveal quite differently shaped exposures profiles due to the interaction of the diffusion and amortisation effects.

Extinguishable option: Option in which the holder's right to exercise disappears if the value of the underlying passes a specified level. See also **barrier option**.

Extrapolation: The process of estimating a price or rate for a particular value date, from other known prices, when the value date required lies outside the period covered by the known prices. See **interpolation**.

F

Face value: The principal amount of a security generally repaid ("redeemed") at maturity, but sometimes repaid in stages, on which the **coupon** amounts are

calculated. Also known as **nominal amount**.

Fixing: See **Libor fixing**.

Floating rate: An interest rate set with reference to an external index. Also an instrument paying a floating rate is one where the rate of interest is refixed in line with market conditions at regular intervals such as every three or six months. In the current market, an exchange rate determined by market forces with no government intervention.

Floating rate CD: CD on which the rate of interest payable is refixed in line with market conditions at regular intervals (usually six months).

Floating rate gilt: Gilt issued with an interest rate adjusted periodically in line with market interbank rates.

Floating rate note: Capital market instrument on which the rate of interest payable is refixed in line with market conditions at regular intervals (usually six months).

Floor: A series of lender's IRGs, designed to protect an investor against falling interest rates on each of a series of dates.

Floortion: Option on a **floor**.

Forward: In general, a deal for value later than the normal value date for that particular commodity or instrument. In the foreign exchange market, a forward price is the price quoted for the purchase or sale of one currency against another where the value date is at least one month after the **spot** date. See **short date**.

Forward band: Zero-cost collar, that is one in which the premium payable as a result of buying the **cap** is offset exactly by that obtained from selling the **floor**.

Forward break: See **break forward**.

Forward exchange agreement: A **contract for differences** designed to create exactly the same economic result as a foreign exchange cash **forward-forward** deal. See **ERA**, **SAFE**.

Forward price: The price agreed today for an asset that will be delivered to the buyer at a date in the future.

Forward rate agreement: Short-term interest rate **hedge**. Specifically, a contract between buyer and seller for an agreed interest rate on a notional deposit of a specified maturity on a predetermined future date. No principal is exchanged. At maturity the seller pays the buyer the difference if rates have risen above the agreed level, and vice versa.

Forward swap: **Swap** arranged at the current rate but entered into at some time in the future.

Forward-forward: Another term for an interest rate effective from a forward short date. Also a short-term exchange of currency deposits.

FRA: See **forward rate agreement**.

FRCD: See **floating rate CD**.

Fraption: Option on a forward rate agreement. Also known as an interest rate guarantee.

FSA: The Financial Services Authority, the body responsible for the regulation of investment business, and the supervision of banks and money market institutions in the UK. The FSA took over these duties from nine "self-regulatory organisations" that had previously carried out this function, including the Securities and Futures Authority (SFA), which had been responsible for

regulation of professional investment business in the City of London. The FSA commenced its duties in 1998.

FTSE-100: Index comprising 100 major UK shares listed on The International Stock Exchange in London. Futures and options on the index are traded at the London International Financial Futures and Options Exchange (**LIFFE**).

Funds: The **USD/CAD** exchange rate for value on the next business day (standard practice for **USD/CAD** in preference to **spot**).

Fungible: A financial instrument that is equivalent in value to another, and easily exchanged or substituted. The best example is cash money, as a £10 note has the same value and is directly exchangeable with another £10 note. A bearer bond also has this quality.

Future: A futures contract is a contract to buy or sell securities or other goods at a future date at a pre-determined price. Futures contracts are standardised and traded on an exchange.

Future exposure: See **potential exposure**.

Future value: The amount of money achieved in the future, including interest, by investing a given amount of money now. See **time value of money**, **present value**.

Futures contract: A deal to buy or sell some financial instrument or commodity for value on a future date. Unlike a **forward** deal, futures contracts are traded only on an exchange (rather than **OTC**), have standardised contract sizes and value dates, and are often only **contract for differences** rather than deliverable.

G

G7: The “Group of Seven” countries, the USA, Canada, UK, Germany, France, Italy and Japan.

Gamma (γ): The change in an option’s delta relative to a change in the underlying’s value.

Gap ratio: Ratio of interest-rate sensitive assets to interest-rate sensitive liabilities; used to determine changes in the risk profile of an institution with changes in interest rate levels.

GDP: Gross domestic product, the value of total output produced within a country’s borders.

GEMM: A gilt-edged market maker, a bank or securities house registered with the Bank of England as a market maker in gilts. A GEMM is required to meet certain obligations as part of its function as a registered market maker, including making two-way price quotes at all times in all gilts and taking part in gilt auctions. The Debt Management Office now make a distinction between conventional gilt GEMMs and index-linked GEMMs, known as IG GEMMs.

General collateral (GC): Securities, which are not “special”, used as collateral against cash borrowing. A repo buyer will accept GC at any time that a specific stock is not quoted as required in the transaction. In the gilts market GC includes DBVs.

GIC: Guaranteed investment contract.

Gilt: A UK Government sterling denominated, listed security issued by HM Treasury with initial maturity of over 365 days when issued. The term “gilt” (or gilt-edged) is a reference to the primary characteristic of gilts as an

investment: their security and risk-free status.

Gilt-edged market maker: See **GEMM**.

GNP: Gross national product, the total monetary value of a country’s output, as produced by citizens of that country.

Gross redemption yield: The same as **yield to maturity**; “gross” because it does not take tax effects into account.

GRY: See **gross redemption yield**.

H

Hedge ratio: The ratio of the size of the position it is necessary to take in a particular instrument as a hedge against another, to the size of the position being hedged.

Hedging: Protecting against the risks arising from potential market movements in exchange rates, interest rates or other variables. See **arbitrage**, **cover**, **speculation**.

Herstatt risk: See **settlement risk**.

High coupon swap: Off-market coupon swap where the coupon is higher than the market rate. The floating-rate payer pays a front-end fee as compensation. Opposite of low coupon swap.

Historical simulation methodology: Method of calculating **value-at-risk (VAR)** using historical data to assess the likely effect of market moves on a portfolio.

Historic rate rollover: A **forward rate swap** in FX where the settlement exchange rate for the near date is based on a historic **off-market** rate rather than the current market rate. This is prohibited by many central banks.

Historic volatility: The actual **volatility** recorded in market prices over a particular period.

Holder: The holder of an **option** is the party that has purchased it.

I

IDB: Inter-Dealer Broker, in this context a broker that provides facilities for dealing in bonds between market makers.

IG: Index-linked gilt whose coupons and final redemption payment are related to the movements in the Retail Price Index (RPI).

Immunisation: This is the process by which a bond portfolio is created that has an assured return for a specific time horizon irrespective of changes in interest rates. The mechanism underlying immunisation is a portfolio structure that balances the change in the value of a portfolio at the end of the investment horizon (time period) with the return gained from the reinvestment of cash flows from the portfolio. As such, immunisation requires the portfolio manager to offset interest-rate risk and reinvestment risk.

Implied repo rate: The break-even interest rate at which it is possible to sell a bond **futures contract**, buy a **deliverable bond**, and **repo** the bond out. See **cash and carry**.

Implied volatility: The **volatility** used by a dealer to calculate an **option** price; conversely, the volatility implied by the price actually quoted.

Index option: An **option** whose **underlying** security is an index. Index options enable a trader to bet on the direction of the index.

Indexed notes: Contract whereby the issuer usually assumes the risk

of unfavourable price movements in the instrument, commodity or index to which the contract is linked, in exchange for which the issuer can reduce the cost of borrowing (compared with traditional instruments without the risk exposure).

Index swap: Sometimes the same as a **basis swap**. Otherwise a swap like an **interest rate swap** where payments on one or both of the legs are based on the value of an index – such as an equity index, for example.

Indirect: An exchange rate quotation against the US dollar in which the dollar is the **base currency** and the other currency is the **variable currency**.

Initial margin: The excess either of cash over the value of securities, or of the value of securities over cash in a repo transaction at the time it is executed and subsequently, after margin calls.

Interbank: The market in unsecured lending and trading between banks of roughly similar credit quality.

Interest rate cap: See **cap**.

Interest rate floor: See **floor**.

Interest rate guarantee: An **option** on a specified interest rate, usually referenced to Libor.

Interest rate option: Option to pay or receive a specified rate of interest on or from a predetermined future date.

Interest rate swap: An agreement to exchange a series of cash flows determined in one currency, based on fixed or **floating** interest payments on an agreed **notional** principal, for a series of cash flows based in the same currency but on a different interest rate. May be combined with a **currency swap**.

Intermarket spread: A spread involving futures contracts in one market spread against futures contracts in another market.

Internal rate of return: The yield necessary to discount a series of cash flows to an **NPV** of zero.

Interpolation: The process of estimating a price or rate for value on a particular date by comparing the prices actually quoted for value dates either side. See **extrapolation**.

Intervention: Purchases or sales of currencies in the market by central banks in an attempt to reduce exchange rate fluctuations or to maintain the value of a currency within a particular band, or at a particular level. Similarly, central bank operations in the money markets to maintain interest rates at a certain level.

In-the-money: A **call (put) option** is in-the-money if the underlying is currently more (less) valuable than the **strike** price. See **at-the-money**, **out-of-the-money**.

Intrinsic value: The amount by which an option is in-the-money.

Inverse floater: A floating-rate note structured so that its coupon falls as interest rates rise, and vice-versa.

Invoice price: For exchange-traded bond futures contracts, the price received by the short future; calculated as (**conversion factor** × futures price) + accrued interest.

IRG: See **interest rate guarantee**.

IRR: See **internal rate of return**.

ISMA: The International Securities Market Association. This association compiled with the PSA (now renamed the Bond Market Association) the PSA/ISMA Global Master repurchase Agreement.

Issuer risk: Risk to an institution when it holds debt securities issued by another institution. (See also **credit risk**.)

Iteration: The repetitive mathematical process of estimating the answer to a problem, by trying how well this estimate fits the data, adjusting the estimate appropriately and trying against it, until the fit is acceptably close. Used, for example, in calculating a bond's **yield** from its price.

J

Junk bonds: The common term for high-yield bonds; higher risk, low rated debt.

K

Kappa (κ): An alternative term to refer to volatility; see **vega**.

Knock out/in: A knock out (in) **option** ceases to exist (starts to exist) if the underlying reaches a certain trigger level. See **barrier option**.

L

Lambda (λ): The same as **vega**.

Lender option: Floor on a single-period forward rate agreement.

Leptokurtosis: The non-normal distribution of asset-price returns. Refers to a probability distribution that has a fatter tail and a sharper hump than the normal distribution.

Level payment swap: Evens out those fixed-rate payments that would otherwise vary, for example, because of the amortisation of the principal.

Leverage: The ability to control large amounts of an underlying variable for a small initial investment.

Liability: Probable future sacrifice of economic benefit due to present obligations to transfer assets or provide services to other entities as a result of past events or transactions. Generally classed as either current or long-term.

Liability swap: An interest rate swap or currency swap used in conjunction with an underlying liability such as a borrowing. See **asset swap**.

LIBID: The London Interbank Bid Rate, the rate at which banks will pay for funds in the interbank market.

LIBOR: The London Interbank Offered Rate, the lending rate for all major currencies up to one-year set at 11am each day by the British Bankers Association.

Libor fixing: The Libor rate "fixed" by the British Bankers Association (BBA) at 11am each day, for maturities up to one year.

LIFFE: The London International Financial Futures and Options Exchange, the largest futures exchange in Europe.

Limean: The arithmetic average of Libor and Libid rates.

Limit up/down: **Futures** prices are generally not allowed to change by more than a specified total amount in a specified time, in order to control risk in very volatile conditions. The maximum movements permitted are referred to as limit up and limit down.

Liquidation: Any transaction that closes out or offsets a futures or options position.

Liquidity: A word describing the ease with which one can undertake transactions in a particular market or instrument. A market where there are always ready buyers and sellers willing to transact at competitive prices is regarded as

liquid. In banking, the term is also used to describe the requirement that a portion of a bank's assets be held in short-term risk-free instruments, such as government bonds, T-Bills and high quality Certificates of Deposit. This is the responsibility of the liquidity desk, part of Treasury.

Loan-equivalent amount: Description of derivative exposure which is used to compare the credit risk of derivatives with that of traditional bonds or bank loans.

Lognormal: A variable's **probability distribution** is lognormal if the logarithm of the variable has a normal distribution.

Lognormal distribution: The assumption that the *natural logarithm* of today's interest rate, for example, minus the *natural logarithm* of yesterday's rate is normally distributed.

Long: A long position is a surplus of purchases over sales of a given currency or asset, or a situation which naturally gives rise to an organisation benefiting from a strengthening of that currency or asset. To a money market dealer, however, a long position is a surplus of borrowings taken in over money lent out (which gives rise to a benefit if that currency weakens rather than strengthens). See **short**.

Long-dated forward: Forward foreign exchange contract with a maturity of greater than one year. Some long-dated forwards have maturities as great as 10 years.

Long-term assets: Assets which are expected to provide benefits and services over a period longer than one year.

Long-term liabilities: Obligations to be repaid by the firm more than one year later.

Lookback option: Option that allows the purchaser, at the end of a given period of time, to choose as the rate for exercise any rate that has existed during the option's life.

Low coupon swap: Tax-driven swap in which the fixed-rate payments are significantly lower than current market interest rates. The floating-rate payer is compensated by a front-end fee.

LSE: London Stock Exchange.

M

Macaulay duration: See **duration**.

Mapping: The process whereby a bank's trading positions are related to a set of risk "buckets", of fixed maturities.

Margin: Initial margin is **collateral**, placed by one party with a counterparty at the time of the deal, against the possibility that the market price will move against the first party, thereby leaving the counterparty with a credit risk. Variation margin is a payment or extra collateral transferred subsequently from one party to the other because the market price has moved. Variation margin payment is either in effect a settlement of profit/loss (for example, in the case of a **futures** contract) or the reduction of credit exposure (for example, in the case of a **repo**). In **repos**, variation margin refers to the fluctuation band or threshold within which the existing collateral's value may vary before further cash or **collateral** needs to be transferred. In a loan, margin is the extra interest above a **benchmark** (e.g., a margin of 0.5 per cent over **Libor**) required by a lender to compensate for the credit risk of that particular borrower.

Margin call: A request following marking-to-market of a repo transaction for the initial margin to

be reinstated or, where no initial margin has been taken, to restore the cash/securities ratio to parity. In the context of a futures exchange, call to make good trading losses and maintain initial margin levels.

Margin default rate: See **probability of default**.

Margin transfer: The payment of a **margin call**.

Market comparables: Technique for estimating the fair value of an instrument for which no price is quoted by comparing it with the quoted prices of similar instruments.

Market-maker: Market participant who is committed, explicitly or otherwise, to quoting two-way bid and offer prices at all times in a particular market.

Market risk: Risks related to changes in prices of tradeable macroeconomics variables, such as exchange rate risks.

Mark-to-market: The act of revaluing securities to current market values. Such revaluations should include both coupon accrued on the securities outstanding and interest accrued on the cash.

Matched book: Repo market making, or only trading to cover one's own requirements. It carries no implications that the trader's position is "matched" in terms of exposure, for example to short-term interest rates.

Maturity date: Date on which stock is redeemed. Also known as the expiry date.

Mean: Average.

Minmax option: One of the strategies for reducing the cost of options by forgoing some of the potential for gain. The buyer of a

currency option, for example, simultaneously sells an option on the same amount of currency but at a different strike price.

Modified duration: A measure of the proportional change in the price of a bond or other series of cash flows, relative to a change in yield. (Mathematically, $D/(1 + r)$.) See **duration**.

Modified following business day (MFBD): The convention that if a value date in the future falls on a non-business day, the value date will be moved to the next following business day, unless this moves the value date to the next month, in which case the value date is moved back to the last previous business day.

Momentum: The strength behind an upward or downward movement in price.

Money market: Short-term market (generally up to one year) for financial instruments. See **capital market**.

Money-market basis: An interest rate quoted on an act/360 basis is said to be on a money-market basis. See **bond basis**.

Monte Carlo simulation: Technique used to determine the likely value of a derivative or other contract by simulating the evolution of the underlying variables many times. The discounted **average** outcome of the simulation gives an approximation of the derivative's value. Monte Carlo simulation can be used to estimate the **value-at-risk (VAR)** of a portfolio. Here, it generates a simulation of many correlated market movements for the markets to which the portfolio is exposed, and the positions in the portfolio are revalued repeatedly in accordance with the simulated scenarios. This gives a probability

distribution of portfolio gains and losses from which the **VAR** can be determined.

Moosmüller: A method for calculating the yield of a bond.

Mortgage-backed security (MBS): Security guaranteed by a pool of mortgages.

Moving average convergence/divergence: The crossing of two exponentially smoothed moving averages that oscillate above and below an equilibrium line (MACD).

Multi-index option: **Option** which gives the holder the right to buy the asset that performs best out of a number of assets (usually two). The investor would typically buy a call allowing him or her to buy the **equity**.

N

Naked: A naked **option** position is one not protected by an offsetting position in the **underlying**. See **covered call/put**.

NAO: National Audit Office.

Negative divergence: When at least two indicators, indices or averages show conflicting or contradictory trends.

Negotiable: A security which can be bought and sold in a **secondary market** is negotiable.

Net present value: The net present value of a series of cash flows is the sum of the present values of each cash flow (some or all of which may be negative).

NLF: National Loans Fund, the account which brings together all UK Government lending and borrowing.

Noise: Fluctuations in the market which can confuse or impede interpretation of market direction.

Nominal amount: Another term for the **face value** of a security.

Nominal rate: A rate of interest as quoted, rather than the **effective rate** to which it is equivalent.

Normal: A normal **probability distribution** is a particular distribution assumed to prevail in a wide variety of circumstances, including the financial markets. Mathematically, it corresponds to the probability density function:

$$\frac{1}{\sqrt{2\pi}} e^{-\phi^2/2}$$

Notional: In a bond futures contract, the bond bought or sold is a standardised non-existent notional bond, as opposed to the actual bonds which are **deliverable** at maturity. **Contracts for differences** also require a notional principal amount on which settlement can be calculated.

Novation: Replacement of a contract or, more usually, a series of contracts with one new contract.

NPV: See **net present value**.

O

O/N: See **overnight**.

Odd date: See **broken date**.

Offer: The price at which a market maker will sell bonds. Also called “ask”.

Off-market: A rate which is not the current market rate.

Off-market coupon swap: Tax-driven swap strategy, in which the fixed-rate payments differ significantly from current market rates. There are high and low coupon swaps.

One touch all-or-nothing: Digital option. The option’s put pays out a predetermined amount (the “all”) if the index goes below (above) the strike price at any time during the

option’s life. How far below (above) the strike price the index moves is irrelevant; the payout will be the “all” or nothing.

Opening leg: The first half of a repo transaction (see **closing leg**).

Open interest: The quantity of **futures** contracts (of a particular specification) which have not yet been closed out by reversing. Either all **long** positions or all **short** positions are counted, but not both.

Operational Market Notice: Sets out the DMO’s (previously the Bank’s) operations and procedures in the gilt market.

Operational risk: Risk of loss occurring due to inadequate systems and control, human error, or management failure.

Opportunity cost: Value of an action that could have been taken if the current action had not been chosen.

Option: The right (but not the obligation) to buy or sell securities at a fixed price within a specified period.

Option-adjusted spread (OAS): For a bond with an embedded option or a mortgage-backed security, the additional spread earned over the term structure of returns that is implied from benchmark government yields in order for the value of the bond to be equal to its observed market price. The higher yield spread reflects the interest-rate option embedded in the callable bond or MBS security.

Option forward: See **time option**.

Ornstein–Uhlenbeck equation: A standard equation that describes mean reversion. It can be used to characterise and measure commodity price behaviour.

OTC: Over the counter. Strictly speaking, any transaction not conducted on a registered stock exchange. Trades conducted via the telephone between banks, and contracts such as FRAs and (non-exchange traded) options, are said to be “over-the-counter” instruments. OTC also refers to non-standard instruments or contracts traded between two parties; for example, a client with a requirement for a specific risk to be hedged with a tailor-made instrument may enter into an OTC structured option trade with a bank that makes markets in such products.

Out-of-the-money: A call (put) option is out-of-the-money if the underlying is currently less (more) valuable than the strike price. See **at-the-money**, **in-the-money**.

Outright: An outright (or **forward outright**) is the sale or purchase of one foreign currency against another value on any date other than spot. See **forward**, **short date**, **spot**, **swap**.

Overborrowed: A position in which a dealer's liabilities (borrowings taken in) are of longer maturity than the assets (loans out).

Overlent: A position in which a dealer's assets (loans out) are of longer maturity than the liabilities (borrowings taken in).

Overnight: A deal from today until the next working day (“tomorrow”).

P

$P(t, T)$: The price at time t of a risk-free zero-coupon bond that matures at time T (with $T > t$).

Paper: Another term for a bond or debt issue.

Par: When the price of a security is equal to the face value, usually expressed as 100, it is said to be trading at par. A par swap rate is the current market rate for a fixed **interest rate swap** against **Libor**. In foreign exchange, when the **outright** and **spot** exchange rates are equal, the **forward swap** is zero or par.

Par yield curve: A curve plotting maturity against **yield** for bonds priced at par.

Parity: The official rate of exchange for one currency in terms of another which a government is obliged to maintain by means of intervention.

Participation forward: A product equivalent to a straightforward **option** plus a **forward** deal, but structured as a forward deal at an **off-market** rate plus the opportunity to benefit partially if the market rate improves.

Path-dependent: A path-dependent **option** is one which depends on what happens to the **underlying** throughout the option's life (such as the **American** or **barrier** option) rather than only at expiry (a **European** option).

Peak exposure: If the worst case or the expected credit risk exposures of an instrument is calculated over time, the resulting graph reveals a credit risk exposure profile. The highest exposure marked out by the profile is the peak exposure generated by the instrument.

Perfect market: A theoretical market in which transactions may be undertaken without the need to pay a bid-offer spread or other transaction costs such as commission, taxes and so on. It also assumes that buy and sell trades can be undertaken in any size without affecting the market.

Periodic resetting swap: Swap where the floating-rate payment is an average of floating rates that have prevailed since the last payment, rather than the interest rate prevailing at the end of the period. For example, the average of six one-month **Libor** rates rather than one six-month Libor rate.

Pips: See **points**.

Plain vanilla: See **vanilla**.

Points: In bond markets, one whole unit of price. In FX markets, the last two decimal places in an exchange rate. For example, when EUR/USD is 1.0520/1.0530, the points are 20/30. See **bid figure**.

Portfolio variance: The square of the **standard deviation** of a portfolio's return from the mean.

Positive cash flow collar: Collar other than a zero-cost collar.

Potential exposure: Estimate of the future replacement cost, or positive market value, of a derivative transaction. Potential exposure should be calculated using probability analysis based on broad confidence intervals (e.g., two standard deviations) over the remaining term of the transaction.

Preference shares: These are a form of corporate financing. They are normally fixed interest shares whose holders have the right to receive dividends ahead of ordinary shareholders. If a company were to go into liquidation, preference shareholders would rank above ordinary shareholders for the repayment of their investment in the company. Preference shares (“prefs”) are normally traded within the fixed interest division of a bank or securities house.

Premium: The amount above par at which a bond is trading. In the FX market, the amount by which a

currency is more expensive, relative to another currency, for future delivery compared to spot delivery. This is the forward premium, and reflects the interest-rate differential between the two currencies.

Present value: The amount of money which needs to be invested now to achieve a given amount in the future when interest is added. See **future value**, **time value of money**.

Pre-settlement risk: As distinct from credit risk arising from intra-day settlement risk, this term describes the risk of loss that might be suffered during the life of the contract if a counterparty to a trade defaulted and if, at the time default, the instrument had a positive economic value.

Price–earnings ratio: A ratio giving the price of a stock relative to the earnings per share.

Price factor: See **conversion factor**.

Primary market: The market for new debt, into which new bonds are issued. The primary market is made up of borrowers, investors and the investment banks which place new debt into the market, usually with their clients. Bonds that trade after they have been issued are said to be part of the **secondary market**.

Probability distribution: The mathematical description of how probable it is that the value of something is less than or equal to a particular level.

Probability of default: The statistical measure of how likely it is that an institution will default on its debt obligations over the next 12 months. As calculated by the ratings agencies, this is based on historical measures of institutions in the same credit rating category.

Put: A put option is an option to sell the commodity or instrument **underlying** the option. See **call option**.

Q

Quanto: An option that has its final payoff linked to two or more underlying assets or reference rates.

Quanto swap: A **swap** where the payments of one or both legs are based on a measurement (such as the interest rate) in one currency but payable in another currency.

Quasi-coupon date: The regular date for which a **coupon** payment would be scheduled if there were one. Used for price/yield calculations for **zero-coupon** instruments.

R

Range forward: A **zero-cost collar** where the customer is obliged to deal with the same bank at spot if neither limit of the collar is breached at **expiry**.

Rate of recovery: Estimate of the percentage of the amount exposed to default – that is, the credit risk exposure – which is likely to be recovered if a counterparty defaults.

Record date: A **coupon** or other payment due on a security is paid by the issuer to whoever is registered on the record date as being the owner. See **cum-dividend**, **ex-dividend date**.

Redeem: A security is said to be redeemed when the principal is repaid.

Redemption yield: The rate of interest at which all future payments (coupons and redemption) on a bond are discounted so that their total

equals the current price of the bond (inversely related to price).

Redenomination: A change in the currency unit in which the nominal value of a security is expressed (in context, from sterling to euro). Also the **gross redemption yield**.

Reduced-cost option: Generic term for options for which there is a reduced premium, either because the buyer undertakes to forgo a percentage of any gain, or because the buyer offsets the cost by writing other See also **zero-premium option**.

Refer: The practice whereby a trader instructs a broker to put “under reference” any prices or rates he has quoted to him, meaning that they are no longer “firm” and the broker must refer to the trader before he can trade on the price initially quoted.

Register: Record of ownership of securities. For gilts, excluding bearer bonds, entry in an official register confers title.

Registered bond: A bond for which the issuer keeps a record (register) of its owners. Transfer of ownership must be notified and recorded in the register. Interest payments are posted (more usually electronically transferred) to the bondholder.

Registrar’s Department: Department of the Bank of England which maintains the register of holdings of gilts.

Reinvestment rate: The rate at which interest paid during the life of an investment is reinvested to earn interest-on-interest, which in practice will generally not be the same as the original yield quoted on the investment.

Relative performance option: Option whose value varies in line

with the relative value of two assets.

Replacement cost: The present value of the expected future net cash flows of a derivative instrument. Aside from various conventions dealing with the **bid/ask** spread, synonymous with the “market value” or “current exposure” of an instrument.

Repo: A collateralised loan. Usually refers in particular to **classic repo**. Also used as a term to include classic repos, **buy/sell-backs** and **securities lending**.

Repo rate: The return earned on a repo transaction expressed as an interest rate on the cash side of the transaction.

Repurchase agreement: See **repo**.

Return: The return on an investment, put simply, the ratio of the asset price at the start of the investment (P_0) and the asset price at the maturity of the investment (P_T), that is P_0/P_T . The market convention is to quote annualised returns.

Return on assets: The net earnings of a company divided by its assets.

Return on equity: The net earning of a company divided by its equity.

Return on value-at-risk: An analysis conducted to determine the relative rates of return on different risks, allowing corporations to compare different risk capital allocations and capital structure decisions effectively.

Revaluation: An official one-off increase in the value of a currency in terms of other currencies. See **devaluation**.

Reverse: See **reverse repo**.

Reverse repo: The other side of a **repo**.

Rho (ρ): The change in an option’s value relative to a change in interest rates.

Risk-free arbitrage: An arbitrage opportunity arising whenever it is possible to create two portfolios that have identical payoff profiles but different prices.

Risk-free return: Also known as riskless return, the return available to the holder of a risk-free asset, such as a gilt or US Treasury security. The interest rate on these instruments is the *risk-free interest rate*. The risk-free interest rate used in market analysis is the rate observed on a Treasury bill, this being the shortest-dated risk-free instrument available in any market.

Risk reversal: Changing a long (or short) position in a call option to the same position in a put option by selling (or buying) forward, and vice versa.

Risk weighting: Assigning risk exposure according to specified levels, according to the type of institution that is the counterparty. For example, bank risk capital, as applied under the Basle rules, assigned varying degrees of risk weighting depending on whether the counterparty is an OECD sovereign, banking institution or corporate.

Rollover: See **tom/next**. Also refers to the renewal of a loan.

Running yield: Same as **current yield**.

Rump: A gilt issue so designated because it is illiquid, generally because there is a very small nominal amount left in existence.

S

S/N: See **spot/next**.

SAFE: See **synthetic agreement for forward exchange**.

Secondary market: The market in instruments after they have been issued. Bonds are bought and sold after their initial issue by the borrower, and the marketplace for this buying and selling is referred to as the secondary market. The new issues market is the **primary market**.

Securities and Exchange

Commission (SEC): The central regulatory authority in the United States, responsible for policing the financial markets including the bond markets.

Securities lending: When a specific security is lent against some form of collateral. Also known as stock lending.

Securitisation: The process of raising finance, via a framework in which liquid or illiquid assets of an institution are transformed into a package of debt securities backed by these assets. Assets can include mortgages, corporate loans, credit card receivables, lease receivables and so on. To remove the assets from the originating company’s balance sheet, securities are often issued by a *special purpose vehicle* (SPV), an incorporated entity created specially for the process. The SPV is bankruptcy-remote, so that any financial difficulties of the originating institution will not affect the pool of financial assets held by the SPV.

Security: A financial asset sold initially for cash by a borrowing organisation (the “issuer”). The security is often negotiable and usually has a maturity date when it is redeemed.

Sell/buy-back: Simultaneous spot sale and forward purchase of a security, with the forward price calculated to achieve an effect equivalent to a classic repo.

Settlement: The process of transferring stock from seller to buyer and arranging the corresponding movement of funds between the two parties.

Settlement bank: Bank which agrees to receive and make assured payments for gilts bought and sold by a CGO member.

Settlement date: Date on which transfer of bonds and payment occur, usually one, two or three days after the trade is conducted.

Settlement risk: The risk that occurs when there is a non-simultaneous exchange of value. Also known as “delivery risk” and “Herstatt risk”.

Sharpe ratio: A measure of the attractiveness of the return on an asset by comparing how much risk premium the investor can expect it to receive in return for the incremental risk (volatility) the investment carries. It is the ratio of the risk premium to the volatility of the asset.

Short: A short position is a surplus of sales over purchases of a given currency or asset, or a situation which naturally gives rise to an organisation benefiting from a weakening of that currency or asset. To a money market dealer, however, a short position is a surplus of money lent out over borrowings taken in (which give rise to a benefit if that currency strengthens rather than weakens). See **long**.

Short date: The interest rate for short-term deposits, up to one month in maturity. Also a deal for value on a date other than spot but less than one month after spot.

Simple interest: When interest on an investment is paid all at maturity or not reinvested to earn interest on interest, the interest is

said to be simple. See **compound interest**.

Simple yield to maturity: Bond coupon plus principal gain/loss amortised over the time to maturity, as a proportion of the clean price per 100. Does not take time value of money into account. See **current yield**, **yield to maturity**.

Special: A security which for any reason is sought-after in the repo market, thereby enabling any holder of the security to earn incremental income (in excess of the *General Collateral* (GC) rate) through lending them via a repo transaction. The repo rate for a special will be below the GC rate, as this is the rate the borrower of the cash is paying in return for supplying the special bond as collateral. An individual security can be in high demand for a variety of reasons; for instance, if there is sudden heavy investor demand for it, or (if it is a benchmark issue) it is required as a hedge against a new issue of similar maturity paper.

Speculation: A deal undertaken because the dealer expects prices to move in his favour, as opposed to **hedging** or **arbitrage**.

Spot: In the money market, a deal to be settled on the day after the customary value date for that particular instrument. In the foreign exchange market, standard settlement for value in two working days' time.

Spot/next: A transaction from **spot** until the next working day.

Spot price: The price of an asset for delivery today, or at the earliest possible delivery date. Compare to the **forward price**, which is the price agreed today for delivery of the asset at a specified date in the future.

Spread: The difference between the bid and offer prices in a quotation. Also a strategy involving the purchase of an instrument and the simultaneous sale of a similar related instrument, such as the purchase of a **call option** at one **strike** and the sale of a call option at a different strike.

Square: A position in which sales exactly match purchases, or in which assets exactly match liabilities. See **long**, **short**.

Standard deviation (σ): A measure of how much the values of something fluctuate around its mean value. Defined as the square root of the **variance**.

Step-down swap: Swap in which the fixed-rate payment decreases over the life of the swap.

Step-up swap: Swap in which the fixed-rate payment increases over the life of the swap.

Stock lending: See **securities lending**.

Stock index future: Future on a stock index, allowing a hedge against, or bet on, a broad equity market movement.

Stock index option: Option on a stock index future.

Stock option: Option on an individual stock.

Straddle: A position combining the purchase of both a call and put at the same strike for the same date. See **strangle**.

Strangle: A position combining the purchase of both a call and a put at different strikes for the same date. See **straddle**.

Street: The “street” is a term for the market, originating as “Wall Street”. A US term for market convention, so in the US market it is the convention for quoting the

price or yield for a particular instrument.

Stress testing: Analysis that gives the value of a portfolio under a range of **worst-case** scenarios.

Strike: The strike price or strike rate of an option is the price or rate at which the holder can insist on the underlying transaction being fulfilled.

Strip: A zero-coupon bond which is produced by separating a standard coupon-bearing bond into its constituent principal and interest components. To strip a bond is to separate its principal amount and its coupons and trade each individual cash flow as a separate instrument ("separately traded and registered for interest and principal"). Also, a strip of **futures** is a series of short-term futures contracts with consecutive delivery dates, which together create the effect of a longer term instrument (for example, four consecutive 3-month futures contracts as a **hedge** against a one-year swap). A strip of **FRAs** is similar.

Swap: A foreign exchange swap is the purchase of one currency against another for delivery on one date, with a simultaneous sale to reverse the transaction on another value date. See also **currency swap**, **interest rate swap**.

Swaption: An **option** on an **interest rate swap** or **currency swap**.

Switch: In the gilt market, exchanges of one gilt holding for another, sometimes entered into between the DMO and a GEMM as part of the DMO's secondary market operations.

Synthetic: A package of transactions which is economically equivalent to a different transaction (for example, the

purchase of a **call option** and simultaneous sale of a **put option** at the same **strike** is a synthetic **forward purchase**).

Synthetic agreement for forward exchange: A generic term for **ERAs** and **FXAs**.

T

T/N: See **tom/next**.

Tail: The exposure to interest rates over a forward-forward period arising from a mismatched position (such as a two-month borrowing against a three-month loan). In the FX market, a forward foreign exchange dealer's exposure to **spot** movements.

Tap: The issue of a gilt for exceptional market management reasons and not on a pre-announced schedule.

Tenor: The term to maturity of a financial instrument.

Term: The time between the beginning and end of a deal or investment.

Term structure of interest rates: a plot of spot interest rates over time.

Theta (θ): The change in an option's value relative to a change in the time left to expiry.

Tick: The minimum change allowed in a futures price. Also, in some bond markets, 1/32nd of a point, or 0.03125.

Time deposit: A non-negotiable deposit for a specific term.

Time option: A forward currency deal in which the value date is set to be within a period rather than on a particular day. The customer sets the exact date two working days before settlement.

Time value of money: The concept that a future cash flow can

be valued as the amount of money which it is necessary to invest now in order to achieve that cash flow in the future. See **future value**, **present value**.

Today/tomorrow: See **overnight**.

Tom/next: "Tomorrow to the next". A transaction from the next working day ("tomorrow") until the day after ("next day" – i.e., **spot** in the foreign exchange market).

Total return swap: Swap agreement in which the total return of bank loans or credit-sensitive securities is exchanged for some other cash flow usually tied to **Libor**, or other loans, or credit-sensitive securities. It allows participants effectively to go **long** or **short** the credit risk of the **underlying asset**.

Traded option: Option that is listed on and cleared by an exchange, with standard terms and delivery months.

Tranche: One of a series of two or more issues with the same coupon rate and maturity date. The tranches become **fungible** at a future date, usually just after the first coupon date. In a structured product, different bonds in the same securitisation, usually with different credit ratings and coupon rates.

Transaction risk: Extent to which the value of transactions that have already been agreed is affected by market risk.

Transparent: A term used to refer to how clear asset prices are in a market. A transparent market is one in which a majority of market participants are aware of what level a particular bond or instrument is trading.

Trigger option: See **barrier option**

Treasury bill: A short-term security issued by a government, generally with a zero **coupon**.

Tunnel: The same as **collar**.

Tunnel options: Set of collars, typically zero-cost, covering a series of maturities from the current date. They might, for example, be for dates 3, 6, 9 or 12 months ahead. The special feature of a tunnel option is that the strike price on both sets of options is constant.

U

Uncovered option: When the writer of the option does not own the underlying security. Also known as a **naked option**.

Undated gilts: Gilts for which there is no final date by which the gilt must be redeemed.

Underlying: The underlying of a futures or option contract is the commodity or financial instrument on which the contract depends. The underlying for a bond option is the bond; the underlying for a short-term interest rate futures contract is typically a three-month deposit.

Underwriting: An arrangement by which a company is guaranteed that an issue of debt (bonds) will raise a given amount of cash. Underwriting is carried out by investment banks, who undertake to purchase any part of the debt issue not taken up by the public. A commission is charged for this service.

Unexpected default rate: The distribution of future default rates is often characterised in terms of an expected default rate (e.g., 0.05%) and a worst-case default rate (e.g., 1.05%). The difference between the worst-case default rate and the expected default rate is often termed the “unexpected

default” (that is:
 $1\% = (1.05 - 0.05\%)$).

Unexpected loss: The distribution of credit losses associated with a derivative instrument is often characterised in terms of an **expected loss** or a **worst-case loss**. The unexpected loss associated with an instrument is the difference between these two measures.

Up-and-away option: See **up-and-out option**.

Up-and-in option: Type of barrier option which is activated if the value of the underlying goes above a predetermined level. See also **down-and-in option**

Up-and-out option: Type of barrier option that is extinguished if the value of the underlying goes above a predetermined level. See also **down-and-out option**.

V

Value-at-risk (VAR): Formally, the probabilistic bound of market losses over a given period of time (known as the holding period) expressed in terms of a specified degree of certainty (known as the confidence interval). Put more simply, the VAR is the worst-case loss that would be expected over the holding period within the probability set out by the confidence interval. Larger losses are possible but with a low probability. For instance, a portfolio whose VAR is \$20 million over a one-day holding period, with a 95% confidence interval, would have only a 5% chance of suffering an overnight loss greater than \$20 million.

Value date: The date on which a deal is to be consummated. In some bond markets, the value date for coupon accruals can sometimes differ from the settlement date, as

the latter can only be a working day.

Vanilla: A vanilla transaction is a straightforward one.

VAR: See **value-at-risk**.

Variable currency: Exchange rates are quoted in terms of the number of units of one currency (the variable or counter currency) which corresponds to one unit of the other currency (the **base currency**).

Variance (σ^2): A measure of how much the values of something fluctuate around its mean value. Defined as the average of $(x - \bar{x})^2$. See **standard deviation**.

Variance-covariance methodology: Methodology for calculating the **value-at-risk** of a portfolio as a function of the **volatility** of each asset or liability position in the portfolio and the correlation between the positions.

Variation margin: The band agreed between the parties to a repo transaction at the outset within which the value of the collateral may fluctuate before triggering a right to call for cash or securities to reinstate the initial margin on the repo transaction.

Vega: The change in an option's value relative to a change in the underlying's **volatility**.

Volatility: The **standard deviation** of the continuously compounded return on the **underlying**. Volatility is generally annualised. See **historic volatility**, **implied volatility**.

W

Warrant: A security giving the holder a right to subscribe to a share or bond at a given price and from a certain date. If this right is

not exercised before the maturity date, the warrant will expire worthless.

Warrant-driven swap: Swap with a warrant attached allowing the issuer of the fixed-rate bond to go on paying a floating rate in the event that he or she exercises another warrant allowing him or her to prolong the life of the bond.

When-issued trading: Trading a bond before the issue date; no interest is accrued during this period. Also known as the “grey market”.

Worst-case (credit risk) exposure: Estimate of the highest positive market value a derivative contract or portfolio is likely to attain at a given moment or period in the future, with a given level of confidence.

Worst-case (credit risk) loss: Estimate of the largest amount a derivative counterparty is likely to lose, with a given level of probability, as a result of default from a derivatives contract or portfolio.

Worst-case default rate: The highest rates of default that are likely to occur at a given moment or period in the future, with a given level of confidence.

Write: To sell an option is to write it. The person selling an option is known as the **writer**.

Writer: The same as “seller” of an option.

X

X: Used to denote the strike price of an option; sometimes this is denoted using the term *K* or *S*.

Y

Yield: The interest rate which can be earned on an investment, currently quoted by the market or

implied by the current market price for the investment – as opposed to the **coupon** paid by an issuer on a security, which is based on the coupon rate and the face value. For a bond, generally the same as yield to maturity unless otherwise specified.

Yield curve: Graphical representation of the maturity structure of interest rates, plotting yields of bonds that are all of the same class or credit quality against the maturity of the bonds.

Yield-curve option: Option that allows purchasers to take a view on a yield curve without having to take a view about a market’s direction.

Yield-curve swap: Swap in which the index rates of the two interest streams are at different points on the yield curve. Both payments are refixed with the same frequency whatever the index rate.

Yield to equivalent life: The same as **yield to maturity** for a bond with partial redemptions.

Yield to maturity: The **internal rate of return** of a bond – the yield necessary to discount all the bond’s cash flows to an **NPV** equal to its current price. See **current yield**, **gross redemption yield**, **simple yield to maturity**.

YTM: See **yield to maturity**.

Z

Zero-premium option: Generic term for options for which there is no premium, either because the buyer undertakes to forgo a percentage of any gain or because the buyer offsets the cost by **writing** other options.

Zero-cost collar: A **collar** where the premiums paid and received are equal, giving a net zero cost.

Zero-coupon: A zero-coupon security is one that does not pay a

coupon. Its price is correspondingly less to compensate for this. A zero-coupon **yield** is the yield which a zero-coupon investment for that term would have if it were consistent with the **par yield curve**.

Zero-coupon bond: Bond on which no coupon is paid. It is either issued at a discount or redeemed at a premium to face value.

Zero-coupon swap: Swap converting the payment pattern of a zero-coupon bond, either to that of a normal, coupon-paying **fixed-rate** bond or to a **floating rate**.

Index

Note:

top level entries are in **bold type**,
sub-entries are in “normal type”,
sub-sub-entries are in *italic type*.

A

AAA-rated, meaning of 498
AIM 35
All-in price 45
Alpine bonds 11
American options, pricing of 801
AMEX Composite 35
Amortised bonds 12
Analysis 983
 of bonds, spot and forward rates
 858–60
 candlestick charts 1010–16
 charts, types of 987
 candlestick 989, 1010
 point-and-figure 988, 1003
 continuation patterns 1000–3
 directional movement index 1009
 Dow theory 986–7
 Elliott wave theory 1017–18
 Gann line 991
 market spikes 998
 mathematical approaches 1005–6
 moving averages 1006, 1009
 moving average oscillator 1008
 and pivot points 1007
 open interest 1010
 oscillators, types of 1008
 pricing gaps 993–4
 principles of 984–5
 relative strength index 1008
 reversal patterns 995–7, 1000
 candlestick 1011
 rounding patterns 999
 sideways congestion rectangle 1001
 stochastics 1009
 stop losses 1019
 trading volume 1009–10
 of trends 989–93
 triangle patterns 999
Anderson and Sleath model 905, 909
Annualised interest rates 17, 19
Annuity 26, 107
 and conventional bonds 41
 pension 27
 perpetuity 27
 present value, calculating 26

Anti-dilution clause 359
Approximate duration 167
Arbitrage strategies, and options 824–5
Arbitrage transactions 478
Arbitrage-free modelling 868
Arbitrage-free pricing, introduction to
 342–5
Ask price, defined 7
Asset convertibles 382
Asset and liability committee 536
Asset and liability management 119 –
 see also **Management**
Asset-backed bonds 90
 analysis of 93
 average life 93
 call option 93
 callable bonds 93–4
 credit enhancement 92
 credit rating 91–2
 Permanent Interest Bearing Shares
 93–4
 Hoare Govett Securities Limited 93
 redemption mechanism 92–3
 securitisation 90–1
Asset-backed securities 12, 459
 collateralised mortgage obligations
 (CMOs) 459, 460, 464–5
 commercial mortgage-backed
 securities 467–8
 credit card asset-backed securities
 470–3
 interest-only class 463–4
 motor-car-backed securities 468–9
 non-agency CMO bonds 466
 planned amortisation classes 459–61
 principal-only class 463–4
 sequential-pay classes 462
 static spread analysis 473–5
 synthetic coupon pass-throughs 463
 targeted amortisation classes 459,
 462
 Z-class bonds 462–4
At-the-money 737
Auction market preferred stock
 (AMPs) 423–4
Australia 294, 305
 government bonds 294
 derivatives 296
 market structure 294–5
 Reserve Bank of Australia 295
 yield curves 296
 Sydney Futures Exchange 296

Australian Options Market 35
Average redemption price 69

B

B-splines, yield curve fitting 904
Balance sheet 533–4
Bank of England 203
 Anderson and Sleath model 909
 forward interest rates and monetary
 policy 902
 gilt-edged market makers (GEMMs)
 216
 gilt warrants, introduction 397
 Monetary Policy Committee 203
 role of, in gilt market
 Central Gilts Office (CGO) 217, 221
 CRESTCo Limited 221
 Svensson yield curve model 905
Bank for International Settlements 526
 and repo transactions 564
Bank of Italy 285
Bank of Japan 293
Bankers acceptance 507, 515–16
Banking book 533
 capital adequacy ratio 534
 and repo 564
Banking capital, international
 definitions 527
Barbell portfolios 195
Barbell strategies 239
Barings Bank 526, 621
Barone-Adesi and Whaley model
 774–5
Barrier options 836
Basis point value 166, 175
 concepts 175
 dollar value of 175
 hedging 178–80
 portfolio 177
 price, value of 175
Basis trading 723
 repo market 582
Basket options 838
Basle rules 527–8, 530–1
 applying, issues in 529
 changes to, proposed 529–30
BBB+ credit rating, meaning of 498
Bear spread – see **Options**
Bermudan options 832
Bid price, defined 7
Bid-offer spread 7
Bill of exchange 507
Binary options 832

- Binomial distribution** 352–3, 793
- Binomial interest-rate tree** 341
 binomial distribution 352–3
 binomial process, example 351–2
- Binomial model**
 Cox, Ross and Rubinstein (1979) 365
 parameters 370–1
 and pricing mortgage-backed securities 453
 and valuation of convertible bonds 365
- Binomial (options) pricing model** 788
 binomial lattice, introduction to 791–2
 compared to Black–Scholes model 791
 Cox, Ross and Rubinstein 788
 dividend-paying assets 789
 interest-rate options 790
 introduction 788–9
 lattice approach 788
- Binomial tree** 452
 and pricing mortgage-backed securities 453
- Black model, and interest-rate options** 772–3
- Black Monday** 372
- Black–Derman–Toy model** 453
 of interest rates 884
- Black–Scholes model** 734, 762, 794
 and Barone–Adesi and Whaley model 774–5
 compared to binomial model 791
 calculation, example 769
 critique of 773–4
 and Greek sensitivity measures 810
 and interest-rate options 772
 and options 771, 755
 limitations of 794
 and pricing caps and floors 821–2
 pricing derivatives 764–6,
 Monte Carlo simulation 770
 as a spreadsheet 770, 784
 zero-coupon bond 771
- Bond, definition of** 3
- Bond futures** 720
 basis trading 723, 726, 725–6
 cash-and-carry trading 723, 725
 cheapest-to-deliver 723, 725
 Chicago Board of Trade 721
 Chicago Mercantile Exchange 730
 clearing house 720
 contracts 721–2
 conversion factor 722, 730–2
 counterparty risk 720
 defined 720
 exchange traded futures 720
 futures contract, defined 720
 hedging with 726–9
 LIFFE 720
 margin process 730
 payoff profile 735
 price quotes, example 721
 pricing 723–5
 settlement 720
 bond futures contract 197
 Bond Market Association 562
- Bond market yields** 62
 vs. money market yields 62
- Bond options**
 interest-rate options (pricing models) 794
 Cox–Ingersoll–Ross (1985) 794, 796
 Heath–Jarrow–Morton (1992) 794, 797
 Vasicek 794, 796
 pricing 795–6
- Bond ownership privileges** 3
- Bond portfolio, yield of** 76–7
- Bond valuation, and forward rates** 112
- Bond warrants** 358, 397
 and convertibles, compared 398
 example 398
- Bonds, features of, 4–5**
- Bootstrapping** 106, 132, 236
 deriving a discount function (case study) 140–1
 from par yield curve 131
- Brady bonds** 1061–2, 1067
 bond index 1068
 countries that have issued 1067
 relative value 1069
 structure of 1068
 types of 1069
- Breakeven inflation** 208
- Breakeven principle** 106
 and forward rates 137–8
- Breakeven rate** 31
- Brennan and Schwartz (1982)** 892
- Brokers, role of** 7
- Brokers rates screen** 31
- Brownian motion** 746–8, 752, 868
 and Heath–Jarrow–Morton model 889
- Bull spread** 813 – see also **Options**
- Bulldog bond** 11, 379
- Bullet bond** 3, 12
- Bullet portfolios** 195
- Bundesbank** 282
- Bunds** 11
- Butterfly spreads** 816–17, 850
- Butterfly strategies** 239
- ## C
- Call feature** 12, 94
- Call option** 66, 96
 of asset-backed bonds 93
- Call protection** 95
- Call provision, of embedded options** 341
- Call risk, of corporate bonds** 330
- Call schedule** 66, 95
- Call structure, issue prospectus** 66
- Callable bonds** 66, 94
 analysis 95, 98
 call feature 94
 call price 95–6
 call protection 95
 call schedule 66, 95
 callable repo 591
 constituents of 96
 convexity, deriving 354–5
 and modified duration 167
 operative life 67
 operative yield 67
 price/yield relationship 95
 pricing 345–7
 putable bond 96
 refunding 96
 sinking fund 97
 yield analysis 66
 yield to first call 67
 yield to next call 67
- Canada**
 credit rating 305
 government bonds 298
 Canadian Depository for Securities 299
 derivatives 300
 primary market 298–9
 secondary market 299
 T-bill market 299–300
 yield curves 300
- Candlestick charts** – see **Analysis**
- Capital Asset Pricing Model** 374–5, 944–7
- Capital markets** 3, 6
- Capital-indexed bonds** 99
- Cash flows** 4, 41
 annuities, present value, calculating 26
 diagram 4
 internal rate of return 30

- multiple 23, 26, 28
- net present value 30
- perpetual 27
- Cedel 391**
- Central Gilts Office (CGO) 221, 571**
 - CRESTCo Limited 221
- Central Government Net Cash Requirement (CGNCR) 203**
- Central Moneymarkets Office (CMO) 522**
- Certificate of Deposit 507, 509–11**
 - defined 509
 - more than one coupon 511–2
 - yields 510–1
- Channel Tunnel rail link, financing of 3**
- Chartists 983**
- Cheapest-to-deliver bond 198**
- Chicago Board of Trade 269, 721**
- Chicago Mercantile Exchange 269, 730**
- Chooser option 833**
- Clean bond prices 45**
- Clearing systems 391**
- Cliquet options 834**
- Collared FRNs 88**
- Collateralised debt obligations (CDOs) 478**
 - analysis 484–5
 - arbitrage CDOs 478–9, 482
 - balance sheet CDOs 479
 - cash flow CDOs 482–3
 - collateralised bond obligations 478
 - collateralised loan obligations 478–9
 - compared to other asset-backed securities 481–2
 - credit derivatives 485
 - application 486, 488*
 - use in CDO market 485–6*
 - Master Trust structure 479
 - overview 478–81
 - product evolution 483
 - synthetic CDOs 481
- Collateralised mortgage obligations 91**
- Colombia, bond market 1078**
- Commercial mortgage-backed securities 467**
- Commercial paper 414, 507, 516**
 - asset-backed commercial paper 415
 - credit-supported commercial paper 415
 - Eurocommercial 414–5
 - issue methods 415
 - programmes 415
 - US market 414–5
 - yields 416
- Compound interest calculations 18**
 - compounding frequency 18
 - exponential function 19, 22
- Compounding, and forward rates 140**
- Condors, long condor 819**
- Confidence interval 653**
 - and value-at-risk 632, 665
- Consols 43**
- Consortium yield 57**
- Contingent options 833**
- Conventional bond 3, 11, 74**
 - pricing 41
 - yield of vs. index-linked bond 74
- Conversion factor 722**
 - for long gilt future 730–2
 - repo market 583
- Conversion ratio 359**
- Convertible bonds 12, 357, 364**
 - advantages of 362–3
 - anti-dilution clause 359
 - basic properties of 357–9
 - bond with warrant 358
 - Capital Asset Pricing Model 374–5
 - conversion price 359
 - conversion ratio 359, 364
 - credit ratings 373
 - credit spread 373
 - default premium 373
 - default risk 372
 - discount convertible 358
 - and Eurobonds 381
 - exercise or strike price 359
 - fair price, defined 365
 - intrinsic value 359
 - investor analysis 359–62
 - liquid yield option note (LYON) 358
 - parity 359
 - premium put convertible 358
 - rolling put convertible 358
 - step-up convertible 358
 - valuation 357, 359, 371–2
 - binomial model 365–6, 368*
 - binomial tree 365, 367–9*
 - call and put features 369–70*
 - parameters, of model 370–1*
 - traditional methods 364*
 - zero-coupon convertible 357
- Convertible preference shares 421**
- Convertible preferred stock 357**
- Convexity 155, 182–3, 194, 347–8**
 - approximating 186–7
 - bias 711–12
 - calculating 184–5
 - cash value 187
 - and derivatives value-at-risk 640
 - and duration 182–3
 - effective, of embedded options 350
 - of gilt strips 230
 - measuring 190
 - and mortgage-backed securities 446–7
 - negative 167
 - properties summary 188–9
 - as second-order measure of interest rate risk 182, 186
 - using 186
- Corporate bonds 10, 319**
 - basic features of 321
 - classification of 323
 - credit analysis 496, 498
 - art of 503*
 - and financial analysis 500–1*
 - industry-specific 502–3*
 - and issuer industry 499*
 - credit rating agencies 329, 496, 498
 - credit risk 496
 - debentures 327
 - default risk 934–5
 - spread duration 935*
 - spread risk 934*
 - default spread 935–7
 - distribution by currency 9
 - interest payments 325
 - issuance 319–20
 - junk bonds 332
 - liquidity of 9
 - and medium-term notes (MTNs) 403
 - mortgage bonds 326–7
 - offering circular 332–7
 - pricing with option models 797–8
 - primary market 322–3
 - redemption provisions 328–9
 - risks associated with 329–32
 - secondary market 323–4
 - secured debt 320
 - security 326–7
 - term to maturity 325
 - underwriting services 322
 - unsecured debt 320
 - yield 321–2
- Corporate finance, methods of 9**
- Correlation, and value-at-risk analysis 627**
- Corridor FRNs 89**
- Cost of capital, estimating 32**
- Coupon, defined 4–5**
- Coupon frequency, effect on discounting and redemption yield 62**

Coupon interest, and bond pricing 45
Coupon yield curve 104
Covariance, and value-at-risk analysis 627
Covenants, and Eurobonds 388
Cox–Ingersoll–Ross model 794, 796, 868
 of interest rates 880
Credit default contracts 1032
Credit default swap 1029, 1032, 1052
Credit derivatives 1052
 applications of 1055–6
 credit default swap 1029, 1032, 1042–3, 1052, 1053, 1056
 credit options 1054
 credit spread options, pricing model 1047
 credit-linked notes 1055
 default options 1048
 continuous curve, probability of default 1049–50
 discrete curve, probability of default 1048–9
 pricing 1048
 risk-neutral price 1048
 default risk, modelling of 1038–9
 four main types 1032–3
 instruments 1030
 credit default contracts 1032
 credit spread contract 1031
 credit spread options 1031
 total return swaps 1030
 introduction to 1029–30
 portfolio credit risk management 1030
 pricing 1032, 1045
 credit spread options 1047
 credit spreads 1033–4
 default, example of 1041–4
 default options 1048
 risk-neutral pricing 1035–6
 using the asset-swap curve 1036
 using historical data 1034–5
 pricing models, theoretical 1045
 credit term structure 1045–6
 default swap pricing 1046
 Fons (1994) 1045
 recovery rates, modelling of 1040
 regulatory issues 1036
 European Union's Capital Adequacy Directive 1036
 FSA 1036–7
 risk-neutral spread, calculation of 1038

total return swap 1053
 applications 1056
 example 1053
 and repo 1054, 1056–7
 uses of 1030
Credit rating agencies 496
 Duff & Phelps Credit Rating Co 496
 Dun & Bradstreet 496
 Fitch Investors Service, Inc 496
 Moody's Investors Service 496
 Standard & Poor's Corporation 496
Credit ratings 496
 bond ratings, meaning of 498
 by country 305
 and issuer industry 499
 and municipal bonds 430
 purpose of 496
Credit risk
 of corporate bonds 329
 CreditMetrics 647
 RiskMetrics 647
 and swaps 696
 and value-at-risk 645
 applications of 649
 default, probability of 646
Credit spread contract 1031
Credit spread options 1031
CreditMetrics 647
CREST 571
CRESTCo 221, 571
Cubic spline, and fitting yield curve data 907
Cubic spline, yield curve modelling 128, 149–50
Cum dividend 45
 Eurobonds 47
 US Treasuries 47
Cumulative preference shares 420
Currency convertibles 382
Current yield 54

D

David Bowie, bond issue 3
Daycount basis
 actual/365 convention 47
 government bond market conventions 47
 harmonisation in Europe 48
Daycount convention, of gilts market 214
Dealers, defined 7
Debt capital, bonds as 3
Debt Management Office (DMO) 203
 electronic trading, developments in 247

gilt auction calendar 218
 and gilt-edged market makers (GEMMs) 216
 reference prices, list of 251
 and secondary market trading 220
Delta, of an option 802–3
 delta-neutral 804
 of an option instrument, defined 370
Delta-gamma VaR 640, 642
Derivatives 599
 call options, modelling 767–9
 Eurodollar futures 611
 forward contracts 607
 forward rate agreement 599
 definition 599
 example 600
 hedging, FRA position 605–6
 long-dated 606
 mechanics 600–1
 pricing 601–3
 valuation, of existing FRA 604
 interest rate future 599
 hedge ratio 614
 hedging with 611, 613–4, 616
 pricing 608–9
 short-term 607
 pricing 762–3– see also **Black–Scholes model**
 complete market 764
 factors influencing 762
 future volatility 763
 martingale method 763
 and value-at-risk analysis 640
 volatility 763
 world exchanges, list of 742–4
Developing countries – see **Emerging bond markets**
Digital options 832
Dilution, and convertible bond valuation 371–2
Dirty bond prices 45
 components of 47
Discount bonds 12
Discount convertible 358
Discount factors
 discount function 131
 table of 38
 zero-coupon 107–8, 131
Discount function 131, 28
 deriving (case study) 140–2
 for swaps, deriving 701
 and zero-coupon swaps 682
Discount rate, and bond pricing 41
Discounted margin 75

Discounting – see **Time value of money**
Dispersion 183, 189
Dividends, of bonds, tax deductibility 9
Doji pattern 1011
Dollar convexity 187
Dollar duration 167–8
Dollar value of a basis point 175
Dow Jones 35
Dow theory, and technical analysis 986
Drop-lock bond 382
Duff & Phelps Credit Rating Co 496
Dun & Bradstreet 496
Duration 155, 171, 191
 approximate 167
 basic concepts 158
 and convexity 182
 dollar duration 167–8
 effective 167
 and embedded options 166–7, 172, 349–50
 example calculation 158
 expression for 159
 deriving 159, 161
 factors affecting 164
 as first-order approximation to
 interest rate risk 182–3
 and floating rate notes 172–3
 fulcrum 159
 futures contracts 172
 and hedging strategy 197
 and interest rate swaps 173
 limiting 162–3
 Macaulay duration 158–9, 161, 165, 171
 modified 165
 calculation example 162
 deriving, for bonds with embedded options 353–4
 portfolio 169–71
 yield curve changes 195
 price duration 167
 reshaping durations 195
 short-end duration 196
 uneven cash flow 164–5
 unexpectedly long-end duration 196
Duration fulcrum 159
Duration hedge ratio 197–8
E
Effective convexity 186
Effective duration 167, 196
 and embedded options 349–50
 and mortgage-backed securities 446–7
Effective interest rate 20

 and interest calculations 19
Egypt 303, 305
 government bonds 303–4
Electronic trading, of swaps 716
 SwapsWire 716
 trading platforms, list of 717
Elliott wave theory 1017
Embedded options 12
 analysis of 339
 arbitrage-free pricing 342–4
 Black–Scholes model, use of with certain options 340
 basic features of 340
 binomial interest-rate tree 341
 call feature 12
 call provision 341
 callable bonds 66
 price/yield relationship 348
 pricing of 345–7
 convexity 347
 effective 350
 duration of 166, 172
 effective duration 349–50
 option-adjusted spread 348
 price/yield relationship 348
 purchase fund 67
 put feature 12
 putable bonds 66
 sinking fund 66–7, 71, 350–1
 understanding, option elements 339
 volatility 349
 yield calculations 65–6
 yield curves 348
 negative convexity 347–8
 yield spread 348
 measuring 348–9
 yield-to-worst 339
Emerging bond markets 1061, 1073
 bond issuance, totals 1061
 Brady bonds 1061–4, 1067–71
 classification of 1062
 Colombia 1078
 Costa Rica 1061
 credit risk of 1061
 definition of, and OECD 1073
 Ecuador 1061
 Estonia 1062
 government default, examples 1061
 Greece 1061
 Jordan 1062, 1077
 Kazakhstan 1062
 key features of 1063
 Malaysia 1079
 Mauritius 1062

 Morocco 1062
 overview 1061
 Pakistan 1061
 Panama 1061
 Poland 1075–6
 relative value, analysis of 1073–5
 Slovenia 1062
 South Africa 1061
 sovereign risk 1073–4
 trading in 1064–6
 Turkey 1061
 Yugoslavia 1061
Equilibrium modelling 868
Equity capital, defined 3
Equivalent life 70
Escrow fund, and exotic municipal bonds 431
Eta, of an option 806
EURIBOR 509
Euro, bonds issued in 319
Euro Exchange Rates 306
Eurobonds 10, 378
 associations 392
 clearing systems 391–2
 conventional 380
 convertible 381–2
 covenants 388
 cum dividend 47
 defined 378
 and emerging economies 1064
 Eurowarrants 382
 ex-dividend period 47
 expenses, issue launch 385–6
 fees, placing and underwriting 385
 first issue of 379
 fiscal agent 391
 floating rate notes 380
 forms of 390–1
 grey market 387
 interest payments 380
 International Primary Market Association 392
 issuing process 382–7
 legal issues 394
 and Poland 1076
 price quotation of 45
 pricing 386
 registered 391
 secondary market 393–5
 settlement 393
 settlement date 42
 swap transactions 395
 taxation issues 394
 trust services 388–90

- vs. foreign bonds 379
- zero-coupon bonds 381
- Eurodollar futures** 611
- European Central Bank (ECB)** 282
- European Union's Capital Adequacy Directive (CAD I)** 526, 1036
- Event risk, of corporate bonds** 331
- Ex-dividend date** 45
- Ex-dividend period** 47
- Ex-dividend, and gilts** 214
- Exchange rates, fixed to floating, move to** 8
- Exchangeable security** 358
- Exchequer gilts** 5
- Expectations hypothesis** 115–16
- Expected values, introduction to** 869
- Exponentially-weighted moving average** 633
- Extrapolation, of interest rates** 31

F

- Fabozzi, Frank J.** 264, 340, 434–5, 469, 485
- Fair price, defined** 42
- Fair value** 53
- Fangmeyer** 283
- Federal Agency Bonds** 267
- Federal Home Loan Corporation** 434
- Federal Home Loan Cost of Funds Index** 89
- Federal Home Loans Mortgage Corporation** 91
- Federal National Mortgage Association** 434
- Financial Accounting Standards Board** 533
- Financial engineering** 12
- Financial futures** 244
- Financial intermediaries, in debt markets** 6
- Financial Services Act (1986)** 216
- Financial Services Authority (FSA)** 216
- Fiscal agent** 391
- Fischer Black** 734
- Fisher identity, and average expected inflation** 224
- Fitch Investors Service, Inc** 496
- Fixed interest instruments, value-at-risk analysis** 636
- Flat yield** 54
- Flat yield curve** 121
- Floating rate notes (FRNs)** 11, 75, 86, 173
 - analysing 86–8
 - callability 86
 - caps 86

- collared FRNs 88
- corridor FRNs 89
- coupon reset date 86
- discount margin equation 100
- duration 172
 - and Eurobonds 380–1
- floors 86
- inverse floating-rate notes 89
- modified duration 90
- and municipal bonds 432
- redemption yield of 86
- reference interest rate 86
- step-up recovery FRNs 88
- Floating rate bonds** 75–6
- Foreign bonds** 11
 - Alpine 11
 - Bulldog 11
 - vs. Eurobonds 379
 - Matador 11
 - Samurai 11
 - Yankee 11
- Foreign exchange market** 517
 - Central Moneymarkets Office (CMO) 522
 - currency quotation system 517
 - forward exchange rates 519–21
 - spot exchange rates 518–9
 - SWIFT/ISO currency codes, list of 524
- Forward contracts** 607
- Forward rate** 110
 - and bond valuation 112
 - bootstrapping from par yield curve 131
 - breakeven principle 137–8
 - calculating spot rate from 112
 - and compounding 140
 - and embedded options 341
 - implied 132, 135, 137, 146–7
 - understanding 140
 - using 130
 - zero-coupon rate 112
- Forward rate agreements** 599
 - definition 599
 - example 600
 - hedging, FRA position 605–6
 - and interest-rate swaps 680
 - long-dated 606
 - mechanics of 600–1
 - pricing 601–3
 - valuation, of existing FRA 604
 - value-at-risk analysis 638
- Forward start options** 833
- Forward transactions** 110

- Forward yield curve** 110
- Forward yields** 110
- France**
 - credit rating 305
 - government bonds 287–9
 - repo market 593
- FSA, and credit derivatives** 1036–7
- FTSE-100** 34–5
- Futures contract** 244, 582
 - defined 720
 - duration 172
 - LIFFE long gilt contract 244
- Futures trading**
 - approaches to 848–50
 - designatory letters (H, M, U and Z) 849
 - LIFFE short-sterling contract 848

G

- Gamma, of an option** 804–5
- Gann line** 991
- Gap analysis** 535 – 6, 541
- GARCH models** 634
- General Motors** 415
- General Motors Acceptance Corporation (GMAC)** 401
- Germany**
 - credit rating 305
 - government bonds 282–3
 - repo market 593
- Ghost feature, and value-at-risk analysis** 633
- Gilt Repo Code of Best Practice** 563, 565, 572
 - Custody 572
 - Default and Close-Out 572
 - Legal Agreement 572
 - Margin 572
 - Market Professionals 572
 - Preliminary Issues 572
- Gilt Repo Legal Agreement** 563
- Gilt-edged market makers (GEMMs)**
 - brokers 216
 - and repo market 565
- Gilt-edged stock lending agreement, and repo market** 565
- Gilts**
 - 2.5% I-L Treasury 2009
 - breakeven inflation 227
 - duration matching 225
 - real yields 211, 224
 - AAA-rating 203
 - breakeven inflation 208, 226–7
 - calculating 224
 - Deacon and Derry 225, 227

- effect of taxation* 226
 - implied forward inflation rate* 227
 - brokers 216
 - broker functions* 217
 - cash flow analysis 240
 - Central Government Net Cash Requirement (CGNCR) 203
 - classification of 213
 - conventional 206
 - auction issue procedure* 218
 - yield spread* 208
 - Debt Management Office
 - bidding for conventional stock* 221
 - bidding for I-L stocks* 221
 - bidding for rump stock* 221
 - closing reference prices* 221
 - electronic trading, developments in* 247
 - reference prices* 251
 - sale of stock from official portfolios* 221
 - and secondary market trading* 220
 - supplying stock for use in repo* 221
 - switches of stock* 221
 - double-dated 212
 - ex-dividend 214
 - ex-dividend date 206
 - Financial Services Authority (FSA) 216
 - floating rate 212
 - futures 244
 - 3% Gas 1995/98 206
 - Gerrard & National 205
 - I-L Treasury 2006, DMO calculations 211
 - I-L Treasury 2009 207–8, 225
 - and inflation expectations* 224
 - index-linked 207–8
 - analytics* 223
 - auction issue procedure* 219, 223
 - breakeven inflation* 224, 226
 - cash flow calculation* 209
 - duration matching* 225
 - and inflation expectations* 223
 - real yields* 212
 - yield spread* 208
 - inflation expectations
 - Fisher identity* 224
 - and index-linked gilts* 223–4
 - issuing 218–19
 - issuing authority 203
 - Debt Management Office (DMO)* 203, 218
 - King & Shaxson Bond Brokers 204
 - LIFFE gilt 245
 - long gilt* 244
 - London Stock Exchange, role of, 217, 221
 - market makers 216
 - Bank of England, role of* 217
 - brokers* 217
 - gilt-edged market makers (GEMMs)* 216, 250
 - market screens 214
 - Butlers Gilts screen* 215
 - market structure 216
 - market trading
 - conventions* 214
 - European bond markets* 214
 - price quote* 214
 - strategy* 235
 - market turnover 206, 214
 - daily average* 206
 - measure of return 210
 - minimum funding requirement 246
 - Public Sector Borrowing Requirement 203
 - Public Sector Debt Requirement 203
 - repo market 246
 - and retail prices index 207, 223
 - indexation lag* 207
 - settlement 221
 - Central Gilts Office (CGO)* 221
 - CRESTCo Limited* 221
 - delivery by value* 222
 - delivery versus payment* 221
 - Merger of CGO, CMO and CREST* 222
 - strips 213, 228–9
 - bond replication* 235
 - characteristics* 233
 - developments in* 242
 - interest rate risk* 230
 - Macaulay duration* 233
 - market anomalies* 235
 - market mechanics* 228
 - pricing convention* 230
 - slow build-up to trading, reasons for* 232
 - uses of* 231
 - value, determining* 234
 - vs. coupon bonds* 233
 - taxation 215
 - trading strategy 235–40
 - 7% Treasury 2002 206
 - 5% Treasury 2004 206
 - 5.75% Treasury 2009 207, 224
 - yield and cash flow analysis* 240–2
 - 9% Treasury 2012
 - duration matching* 225
 - 6% Treasury 2028 204
 - Treasury bills 213
 - undated 213
 - 3.5% War Loan 206, 213
 - yield 210
 - analysis* 240
 - DMO calculations* 210
 - Fisher identity* 210
 - and inflation* 208, 211
 - nominal* 210
 - real yields vs. gross redemption* 211–2
 - yield curves 205
 - of zero-coupon bonds* 235
 - yield spread 208
 - Global capital markets** 10
 - Global Master Repurchase Agreement** 562, 565
 - Government National Mortgage Association** 434
 - Greeks, and option pricing** 802
 - delta 802–3
 - epsilon 806
 - eta 806
 - gamma 804–5
 - kappa 806
 - lambda 808
 - rho 808
 - theta 805–6
 - vega 806
 - Grey market, and Eurobonds** 387
 - Gross Domestic Product, effect on yield** 51
 - Gross National Product, effect on yield** 51
 - Gross price** 45
 - Gross redemption yield** 56, 58, 102
 - calculating, example 58
 - defined 5
- ## H
- Harrison and Kreps (1979)** 763
 - Harrison and Pliska (1981)** 763, 890
 - Heath–Jarrow–Morton model** 794, 797
 - of interest rates 888
 - Hedge ratio, with futures** 614
 - Hedging**
 - an FRA position 605–6
 - approaches to 847, 856–7
 - and basis point value 178
 - bond portfolio 728
 - exotic options 838–9, 841
 - hedge ratio 178

hedge size, with basis point value
 178–80
 interest-rate swaps 707
 nominal hedge position 178
 optimum hedge equation, derivation
 861
 with options 823–4
 and portfolio duration 170–1
 strategy and duration 197–8
 swap transaction using futures
 contracts 709–11
 using bonds and swaps 707–8
 using futures 726
 using interest-rate futures 611
 via repo markets 568, 578
 volatility weighting 179
Her Majesty's Treasury 203, 243
 auction calendar 218
 Debt Management Office (DMO) 203,
 243
 derivatives and repo markets 244
High-yield bonds 489–90
 market, growth in 489
 performance 492
 level of default 493
 yields 493
 pricing 494–5
 types of 490–2
Ho and Lee model, of interest rates 881
Hoare Govett Securities Limited 93
Holding-period yield 65
Hull–White model 779–80, 788, 790–1
 of interest rates 882–4
 Markov process 788
 and volatility 774
Humped yield curves 120
Hungary
 credit rating 305
 government bonds 301
 yield curves 301

I

Implied volatility 166, 179
In-the-money 737
Indenture 359
Index-linked bonds 12, 71–2, 98
 analysis, advanced 918
 capital-indexed bonds 99
 coupon payment 71
 duration 918–20
 indexed-annuity bonds 99
 interest rates, real term structure
 921–4
 interest-indexed bonds
 structure of 98–9
 yield of 71–4
 vs. conventional bond 74
 zero-coupon indexed bonds 99
Indexed-annuity bonds 99
Indices 34
 AIM 35
 AMEX Composite 35
 Australian Options Market 35
 CAC 40 35
 Dow Jones 35
 equally-weighted 35
 FTSE-100 34
 as measure of fund managers'
 performance 35
 Nikkei 225 35
 NYSE Composite 35
 price-weighted 35
 S&P 500 35
 SMI 35
 TOPIX 35
 Toronto 35 35
 Value Line 35
 value-weighted 35, 36
Indices (bond) 36, 972
 bond market indices, selection of 977
 composition 974
 events, response to 974
 index value, calculation of 975–6
 maturity of 973–4
 objectives of 972
 types 972–3
Inflation
 and calculating gilt yields 208
 expectations, and index-linked gilts
 223
 headline rate 33
 and index-linked bond yields 73
 measuring 33
 and rate of return 33
 retail prices index 33
 RPIX 33
 RPIY 33
Integrals, defined 759
Interest calculations 17
 annualised interest rates 17
 compound interest 18
 compounding, principle of 18
 effective interest rate 19
 of money market instruments
 507–8
 mortgage interest 436
 example 437
 rates, market quoting conventions 20

 simple interest 17
Interest rate risk 155
 convexity 155
 as second-order measure 182, 186
 duration 155, 158–9
 Macaulay duration 158–9, 161,
 163
 duration as first-order
 approximation 182
 of gilt strips 230
 modified duration 155, 162
 of mortgage-backed bonds 455
 portfolio duration 169, 170
Interest rate swaps, duration 173
Interest rates
 brokers rates screen 31
 extrapolation of 31–2
 interpolation of 31
 liquidity premium 15
 market determination and present
 value calculations 13
 market quoting conventions 20
 real interest rate 14
 stochastic behaviour and yield curves
 121
Interest-indexed bonds 99
Interest-rate gap 541
Interest-rate models 873–4, 876
 arbitrage-free 880–1, 888
 Black–Derman–Toy model 884
 Ho and Lee model 881–2, 888
 Hull–White model 882–4
 assessing 895–8
 Brennan and Schwartz (1982) 892
 classification of 874
 Cox–Ingersoll–Ross model 880
 fitting to data 884–5
 Heath–Jarrow–Morton model 888
 Brownian motion 889
 initial probability measure 889–90
 Itô's lemma 889
 multi-factor model 892, 894
 no-arbitrage, principle of 891
 single-factor model 889–90
 Weiner process 889
 jump models 894
 Bjork (1996) 894
 Das (1997) 894
 Jarrow and Madan (1991) 894
 Merton model 879
 process dynamics 874
 Gaussian or normal 874–5
 lognormal 875
 Ornstein–Uhlenbeck process 876

- square root or squared Gaussian* 874
- Vasicek model 876–7
 - mean reversion* 876
 - Ornstein–Uhlenbeck process* 876
- Interest-rate options**
 - binomial pricing model 790
 - Ho–Lee model* 790
 - Hull–White model (1993)* 790
 - and the Black model 772–3
 - interest-rate processes 795
 - conditions of certainty* 795
 - conditions of uncertainty* 795–6
 - pricing, models for 794
 - Cox–Ingersoll–Ross 794
 - Heath–Jarrow–Morton 794
 - Merton model 799
 - Vasicek 794
- Interest-rate parity theorem** 783
- Interest-rate risk** 536, 661, 663
- Internal rate of return (IRR)** 29, 37
 - and bond yield 53
 - breakeven rate 31
 - calculating 30, 37
 - relationship to NPV 30
 - and yield to maturity 31, 56–7
- International government bond markets** 277
 - Australia 294
 - benchmark bonds 280
 - Canada 298
 - credit ratings, by country 305
 - domestic debt 277
 - Egypt 303
 - European Central Bank (ECB) 282
 - European Union 279
 - France 287
 - Germany 279, 282
 - Hungary 301
 - International Debt Securities 278
 - Italy 284
 - Japan 279, 292
 - list of 277–9
 - New Zealand 297
 - Overview 279
 - primary dealers 281
 - primary market 279, 280
 - repo market 281
 - secondary market 281
 - South Africa 301
 - United States 279
- International investment** 978, 979
- International Securities Identification Number (ISIN)** 392
- International Securities Market Association** 392, 562
- International Swap Dealers Association** 669
- Interpolation**
 - cubic spline 149
 - of interest rates 31
 - of yield curve 127
- Inverse floating-rate notes** 89
- Investors, types of** 7
- Irredeemable bonds, pricing of** 43
- Issuers** 3–5, 8
- Italy**
 - Bank of Italy 284–5
 - credit rating 305
 - government bonds 284–7
 - repo market 594
- Itô calculus** 752
 - Itô’s lemma 752, 757–8, 890
- J**
- Japan**
 - credit rating 305
 - government bonds 292
 - Bank of Japan* 293
 - Bond Futures Contract Calculation* 306
 - derivatives* 294
 - primary market* 292
 - secondary market* 293
 - Tokyo Stock Exchange* 294
- Jarrow and Madan (1991)** 894
- Jensen’s inequality** 149
- Jordan, bond market** 1077
- Jump diffusion model, and stochastic volatility** 778
- Jump risk** 661
- Junk bonds** 332
- K**
- Kappa, of an option** 806
- Kidder Peabody** 526, 621
- King & Shaxson Bond Brokers Limited** 579
- L**
- Ladder options** 834
- Lambda, of an option** 808
- LIBID** 509
- LIBOR (London Inter-bank Offer Rate)** 75, 509
 - defined 509
 - EURIBOR 509
 - and floating rate notes 86
 - LISBOR 509
 - STIBOR 509
- LIFFE (London International Financial Futures and Options Exchange)** 7, 720
 - long gilt contract 244, 582
 - long gilt future 582–3
- Limiting duration** 162–3
- Liquid yield option note (LYON)** 358
- Liquidation, and preference shares** 419
- Liquidity** 7
 - and bid–offer spread 7
 - of corporate bond markets 9
 - defined 3
 - funding, management of 532, 536
 - and medium-term notes 404
 - premium 15
- Liquidity book** 533
- Liquidity gap** 536–7
- Liquidity management** 540–1
- Liquidity preference theory** 118
- Liquidity premium** 118
- Liquidity risk** 536
 - of corporate bonds 330
 - and value-at-risk 664
- LISBOR** 509
- Local expectations hypothesis** 115
- Locals, meaning of** 7
- Lognormal distribution** 652
 - of returns 782
- London Clearing House** 720
- London Stock Exchange, role of, in gilt market** 217, 221
- Long call strategy** 822
- Long put strategy** 823
- Long-dated bond**
 - pricing 929
 - yield, analysis of 926–8, 931
 - convexity bias 932
 - forward rates, impact of 929–30
- Long-end spread** 196
- Long-term institutional investors** 7
- Lookback options** 834
- M**
- Macaulay duration** 158, 171, 190
 - asset-backed bonds 93
 - of gilt strips 230, 233
 - and mortgage-backed securities 444–5
 - properties of 163–4
- McCulloch** 922
 - term structure of interest rates, model of 922–3
- McCulloch’s cubic spline model** 914
- Malaysia, bond market** 1079

Management

banking assets and liabilities 532
 ALM (asset and liability management) 532
 ALM Desk 534, 536
 asset and liability committee 536
 banking book 533
 capital adequacy ratio 534
 and liquidity gap 537
 cash level requirement, banking minimum 533
 developments in 536
 gap analysis 535, 541
 gap ratio 535
 portfolio modified duration 543
 interest-rate gap 541
 interest-rate risk 536
 liquidity 532–3, 540–1
 liquidity gap 536–40
 liquidity risk 536
 major areas of 534
 portfolio modified duration gap 543
 position management 547–8
 securitisation 544–7
 traditional approach 534
 of risk 621– see also **Value-at-Risk**
 management function, of bank 622–3
 market failures 624
 non-value-at-risk 624
 types of risk 621–2
Margin method 86–7
Market makers, defined 6, 7
Markov process 788
Martingale, and Heath–Jarrow–Morton model 889
Martingale method 763
Master Trust structure 479
Matador bonds 11, 379
Mathematics primer 868
 expected values 869
 probability distributions 869
 random variables 869
 regression analysis 870
 stochastic calculus 871
 stochastic processes 870
Matrix multiplication 657–8
Matrix, and variance-covariance value-at-risk 629
Maturity, defined 4
Medium-term notes (MTNs) 400
 and corporate bonds 403
 defined 400

Euro-MTN 408–9, 413
 first MTN issue 401
 global market, size of 400
 issue options 404–5
 issue process 405–6
 issue size 404
 liquidity 404
 primary market 401–3
 secondary market 406–7
 Securities and Exchange Commission (SEC) 401
 structured MTNs 408
 Walt Disney issue 401
Merton model 799
 of interest rates 879
Minimum Funding Requirement 246
Mixed horizon institutional investors 7
Modified duration 90, 155, 165, 183, 191, 543
 approximate nature of 182
 approximation 168
 using Microsoft Excel 168–9
 and convexity 182
 deriving, for bonds with embedded options 353–4
 of gilt strips 230
 measure derivation 173
 portfolio modified duration gap 543
 using 166–7
 and volatility 169
 weakness of and price volatility 183
Monetary Policy Committee 203, 1009
Money market debt, defined 3
Money markets 507
 Bankers Acceptance 507, 514–16
 Bill of Exchange 507
 Central Moneymarkets Office (CMO) 522
 CMO settlement 522
 Certificate of Deposit 507, 509–10
 discount, as quotation basis 512
 commercial paper 516
 foreign exchange 517
 cross-rates 518
 currency quotation system 517
 forward exchange rates 519–20
 spot exchange rates 518
 SWIFT/ISO currency codes, list of 524
 instruments, two classes of 507
 interest calculation 507–8
 LIBOR 509
 sterling money market rates 508
 time deposit 507

Treasury bill 507, 513
 bond equivalent yield 514
 discount and pricing 513
 Federal Funds 514
 Prime Rate 514
 US Treasury Bills 514
 yield 508–10, 515
Money markets yields, vs. bond market yields 62
Money substitute hypothesis 119
Monte Carlo simulation 627
 of mortgage cash flow 448–9
 and pricing mortgage-backed securities 453
 simulation for Black–Scholes model (option pricing) 770
 and value-at-risk calculations 626, 635–6
 stress testing, of VaR models 644
Monthly Statement of the Public Debt 257
Moody's Investors Service 496
Moosmüller yield 62, 283
Morgan Guaranty Trust Company 391
Mortgage bonds 326
Mortgage pass-through certificates 91
Mortgage-backed bonds 91, 452
 defined 452
 holding period return 452
 interest rate risk 455–6
 portfolio, performance 457
 prepayment 452
 pricing 452–3
 binomial model 453
 Monte Carlo simulation 453
 valuation process 452
 option-adjusted spread 454–5
Mortgage-backed securities 434 – see also Asset-backed bonds
 advantages of 435
 agency mortgage-backed securities 439
 cash flows
 Monte Carlo simulation 448–9
 prepayment analysis 440–3
 prepayment models 443–4
 static cash flow model 445
 collateralised mortgage obligation 439
 collateralised mortgage securities 439
 defined 434, 439
 evaluation and analysis 444–6
 market, growth of 435

- modelling, Monte Carlo simulation 448
- mortgage pass-through securities 439, 441
- mortgage risk 438
 - default risk* 438
 - prepayment risk* 435, 438
- mortgage terms, US market 435, 437
- mortgages, defined 434
- securitisation 434
- stripped mortgage-backed securities 439
- Mortgages 434**
 - balloon mortgage 437
 - calculations 436–7
 - endowment 434
 - graduated payment 438
 - growing equity 438
 - interest only 434
 - mortgagee 434
 - mortgagor 434
 - prepayment risk 435, 438
 - servicing fee 437
 - variable rate 437
- Motor-car-backed securities 468**
- Municipal bonds 427**
 - compared to Treasury bonds 427
 - credit ratings 430
 - defined 427
 - exotic 431
 - general obligation bonds 428
 - insurance against default 431
 - issue prices, Standard & Poor's Blue List 429
 - liquidity 429
 - market 429
 - maturity date 428
 - price of 432
 - revenue bonds 428
 - and taxation 427, 429–31
 - yield 429
 - zero-coupon 427
- Municipal money market instruments 432**
- Myron Scholes 734**
- N**
 - Negative carry 55**
 - Negative convexity 167**
 - Negative funding 55**
 - Nelson–Siegel model 905**
 - Net present value (NPV) 29**
 - calculating 29–30
 - relationship to IRR 30
 - and value-at-risk 547
 - Net redemption yield 58**
 - Netherlands, Rembrandt bonds 379**
 - New Zealand**
 - credit rating 305
 - government bonds 297–8
 - Newton–Raphson method 783**
 - Nikkei 225 35**
 - Non-conventional bonds 11–12**
 - Non-parallel shift 180**
 - Normal distribution 650–1**
 - cumulative, approximation of 782
 - and technical analysis 985
 - and value-at-risk 665
 - NYSE Composite 35**
- O**
 - Off-the-run 262**
 - Offer price, defined 7**
 - Offering circular, of corporate bond 332**
 - Oil prices 8**
 - On-the-run 262**
 - Option book 801**
 - managing 801, 809, 832
 - option prices, behaviour of 801
 - option risk, the Greeks 802–8
 - Option smile 809**
 - Option-adjusted spread 454–5**
 - Options 734, 812**
 - American or European style 737–8
 - arbitrage strategies 824
 - Bermudan style 737
 - binomial pricing model 788
 - binomial lattice approach* 791
 - compared to Black–Scholes model* 791
 - bull spread 813
 - call option 734
 - caps 820–1
 - collars 820–1
 - defined 734
 - exercising 735
 - exotic, pricing 838–9
 - exotic, types of 832–7
 - floors 820–1
 - hedging with 823
 - covered call hedge* 823
 - protective put hedge* 824
 - instruments 738
 - interest-rate options, pricing 790
 - Ho–Lee model* 790
 - Hull–White model* 790
 - option spread 812–15
 - payoff profiles 739, 814
 - bear put spread* 814
 - bull call spread* 813–4
 - synthetic structures* 739
 - premium 734
 - pricing 740–1
 - put option 734
 - terminology 736–7
 - trading strategies 739–40
 - types of 812
 - using 822–3
 - valuation of 771
 - American* 775–7
 - Black–Scholes model* 771
 - models, summary of* 780–1
 - volatility trades 816–19
 - writer 734
- Origination, defined 7**
- Ornstein–Uhlenbeck process 876**
- Out-of-the-money 737**
- Over-the-counter markets 8**
- P**
 - Par 3, 104**
 - as approximation to yield 60
 - defined 5
 - premium to par 45
 - and price quoting 45
 - yield to par 60
 - Par yield curve 104**
 - bootstrapping from 131
 - Parallel change 194**
 - Parallel shift 180, 194, 661**
 - Parametric techniques, fitting yield curves 904**
 - Participating preference shares 421**
 - Pension contributions 25, 27**
 - Perfect market model, and the yield curve 122**
 - Permanent Interest Bearing Shares 93–4**
 - Hoare Govett Securities Limited 93
 - Perpetual bonds, pricing of 43**
 - Perpetuity 27**
 - Plain vanilla bond 3, 11**
 - Planned amortisation class 460**
 - Poisson distribution 784**
 - Poland, bond market 1075**
 - Polynomial models, of yield curves 127**
 - Portfolio**
 - barbell and bullet 195
 - case study 825
 - using traded options* 825–9
 - generic, interest-rate swaps 677
 - hedging 728
 - of mortgage-backed securities 457

- and value-at-risk analysis 627, 641
 - fixed interest instruments* 636
 - volatility, modelling of and value-at-risk analysis 635
 - Portfolio basis point value** 177
 - Portfolio duration** 169
 - changes to yield curve 195
 - and hedging 170–1
 - Portfolio management** 941, 948
 - active 942, 943, 948
 - adjustment 944
 - Capital Asset Pricing Model 944–7
 - international 979–80
 - passive 941
 - performance 966–7
 - Portfolio measurement, of yields** 969
 - approximating 970
 - value-weighted 970
 - Preference shares** 418, 420
 - auction market preferred stock 423–4
 - capital, cost of 422–3
 - defined 418–9
 - liquidation, rights in a market size 418, 419
 - and sinking fund 422
 - types of 420–1
 - voting rights 420
 - Preferred habitat theory** 120
 - Preferred stock** 418
 - Premium put convertible** 358
 - Prepayment risk, of mortgages** 435, 438
 - Present value** 13, 17, 58 – see also **Time value of money**
 - of coupon payments 58
 - of redemption payment 58
 - Price duration** 167
 - and volatility 169
 - Price quotations** 45
 - all-in price 45
 - conventions 45, 47
 - dirty price 45
 - offer price 45
 - premium to par 45
 - tick, meaning of 45
 - trading at a discount 45
 - Price sensitivity hedge ratio** 197
 - Price value of a basis point** 175
 - Price/yield function, Taylor expansion** 192
 - Price/yield relationship** 155–7
 - callable bonds 95
 - convex 157
 - and convexity 184
 - drawing tangents to 182
 - and embedded options 347–8
 - vanilla bonds 155
 - volatility 157
 - Pricing** 45
 - of bonds 41–3, 45
 - coupon interest 45
 - present value analysis, weakness of 45
 - yield, relation to bond price 48, 50
 - yield, relation to coupon 49
 - Primary market**
 - international government bonds 280
 - and medium-term notes 401
 - par yield curve 104
 - Principal, defined** 5
 - Probability distributions** 650
 - introduction to 869
 - Lognormal 652
 - Normal 651, 655
 - Poisson 784
 - Public Sector Borrowing Requirement** 203
 - Public Sector Debt Requirement** 203
 - Public Securities Association** 562
 - Put feature** 12
 - Put option** 96
 - Puttable bonds** 66
 - analysis of 96
 - call and put 67
 - constituents of 96
 - and modified duration 167
 - put option 96
 - yield analysis 67
- ## Q
- Quanto options** 837
- ## R
- Rainbow options** 837
 - Random variables, introduction to** 869
 - Rate of return** 32
 - average, arithmetic and geometric 33
 - cost of capital, estimating 32
 - geometric mean 32
 - historical performance 32
 - inflation-adjusted 33
 - simple, calculating 32
 - time-weighted 32
 - Redeemable preference shares** 421
 - Redemption date** 43
 - Redemption yield** 53
 - calculation of 58, 59, 62, 64
 - net vs. gross 58
 - Refunding, of callable bonds** 96
 - Registered bonds** 391
 - Regression analysis** 870
 - Regression models, of yield curve** 129
 - Regulation, banking capital requirements** 526
 - Bank for International Settlements 526
 - Barings Bank 526
 - capital adequacy 527
 - Basle rules* 527, 529–31
 - Daiwa Securities 526
 - European Union's Capital Adequacy Directive 526
 - Kidder Peabody 526
 - types of capital, international definitions 527
 - Yamaichi Securities 526
 - Reinvestment risk** 103
 - Reinvestment yield** 65
 - Repo, and total return swap** 1054, 1056–7
 - Repo market** 550
 - accounting 563
 - basis trading 582, 595
 - cash and carry trading* 582, 585, 588
 - cheapest to deliver bond* 584–5, 595–6
 - delivery mechanism* 589
 - gross basis* 586, 589
 - practical implications* 590
 - reverse cash-and-carry* 590
 - capital adequacy 564
 - central bank repo 593
 - classic repo 551, 554
 - example* 554
 - conversion factor 583–4
 - credit intermediation 577
 - cross-currency, example 592
 - daily turnover, estimated (1998) 550
 - development of 550
 - economic functions of 552
 - electronic trading 579
 - King & Shaxson Bond Brokers Limited* 579–80
 - example 551–2
 - futures contract 582–3
 - hedging via repo 568
 - tools* 578–9
 - implied repo rate 582, 587
 - calculating* 587
 - definition* 582
 - initial margin 556
 - international government bonds 281
 - legal issues 562

- Bond Market Association* 562
- Gilt Repo Code of Best Practice* 563, 572
- Gilt Repo Legal Agreement* 563
- Global Master Repurchase Agreement* 562
- International Securities Market Association* 562
- Public Securities Association* 562
- LIFFE Long Gilt future 583
- margin 558
 - variation margin, example* 559–60
- market structures 590–1
- matched book trading 577–8
 - example* 578
- mechanics of 554
- netting 580–1
- open market 570–1
- overseas markets 593
- participants, in market 565
- pricing 558
 - example* 559
- RepoClear system 581
- risks 560
 - dealing with* 561–2
 - financial market* 560–1
- securities lending 552
- sell/buy back 552, 554
 - example* 555
- settlement 571
 - CREST reference prices* 571
 - CREST service* 571
 - Real Time Gross Settlement System* 571
- specials trading 569, 576–7
- stock lending, *example* 555
- strategy, trading 572–4, 576
- structures 556
 - bonds borrowed/collateral pledged* 558
 - borrow/loan vs. cash* 558
 - cross-currency* 558
 - hold-in-custody* 557
 - tri-party* 556–7
- taxation 563–4
- UK gilt repo 564
 - gilt-edged market makers* 564
 - market growth* 566
 - market participants* 567
 - market structure* 567–8
 - market volumes* 566
- uses of 552
 - as a financing transaction* 553
 - funding bond positions* 552
- yield curve
 - arbitrage* 573–4, 576
 - environment* 572–3
 - impact of* 568
- Reserve Bank of Australia** 295
- Retail prices index** 33
 - and gilts 207, 223
 - list of 250
- Return-to-maturity expectations hypothesis** 115
- Rho, of an option** 808
- Risk management** – see **Management, of risk**
- Risk measurement** 625
- RiskMetrics** 664
 - J P Morgan 623, 625, 630, 637
 - grid points* 631
 - and modelling credit risk* 647
- Rolling put convertible** 358
- S**
- S&P 500** 35
- Samurai bonds** 11, 379
- Schuldscheine** 282
- Secondary market** 3
 - and Eurobonds 393–5
 - international government bonds 281
 - and medium-term notes 406
 - settlement date 42
- Securities and Exchange Commission (SEC)** 401
 - Rule 415, and MTNs 401
- Securitisation** 90
 - and asset and liability management 544–7
 - benefits* 544–5
 - process of* 544
 - investor advantage 91
 - and mortgage-backed securities 434
- Segmentation Hypothesis** 119
 - preferred habitat theory 120
- Sequential-pay classes** 462
- Settlement** 523
 - Central Moneymarkets Office 522
 - CREST 571
 - CRESTCo 571
 - and Eurobonds 393
 - Real Time Gross Settlement System 571
 - repo markets 571
- Settlement date** 43, 45
 - and accrued interest 45
 - conventions 42
 - definition of 42
 - Eurobonds 42
- secondary market 42
 - and value date 43
- Share price, and binomial valuation model** 371
- Shares, as equity capital** 3
- Short call strategy** 822
- Short-end duration** 196
- Short-end spread** 196
- Short-term institutional investors** 7
- Short-term instruments** 8
- Short-term interest rate futures** 607
 - description 607–8
 - hedging with 611
 - pricing 608–9
- Simple interest, calculations** 17
- Simulation**
 - of call options 766
 - methods, for Black–Scholes 770
 - methods for Value-at-Risk 635
 - stress testing, of VaR models* 643–4
- Sinking fund** 97
 - corporate bond redemption 328
 - of embedded options 350
 - and preference shares 422
- SMI** 35
- SONIA swaps** 691–2
- South Africa**
 - credit rating 305
 - government bonds 301–3
- Spain, repo market** 594
- Spens clause, of corporate bonds** 331
- Spot rates**
 - calculating from forward rates 112
 - bootstrapping from par yield curve* 131
 - unbiased expectations hypothesis* 112
 - implied 132, 137
 - use in bond analysis 109, 130
 - and zero-coupon rate 105
- Spread, defined** 7
- Standard & Poor's "Blue List"** 429
- Standard & Poor's Corporation** 496
- Standard deviation** 650
 - and value-at-risk analysis 627
- Static spread** 109
- Statistical concepts** 650
 - confidence intervals 653–4
 - mean 650
 - normal distribution 650
 - tables* 655
 - probability distributions 650
 - standard deviation 650
 - summary of 781

Step-up convertible 358
Step-up recovery FRNs 88
Sterling, origin of 203
Sterling money markets 566
STIBOR 509
Stochastic calculus 752, 871
 Itô calculus 752
Stochastic processes, interest rates and yield curves 121
Stochastic processes, introduction to 870
Stochastic processes (asset price dynamics) 745
 bond price equation 753
 Brownian motion 746–8, 752
 geometric Brownian motion 750
 Lévy theorem 752
 continuous processes 745
 definition and concepts 745, 757
 deterministic element 746
 drift 746
 mean reversion 746
 Newtonian term 748
 noise 746
 probability density function 745
 random function 745
 random variable 745
 stochastic differential 752
 diffusion coefficient 752
 dynamic asset prices, a model for 750–1
 Itô formula, generalised 753
 Itô process 750
 Itô calculus 752
 Itô's lemma 752, 757
 lognormal distribution 753
 Lévy theorem 752
 Martingale property 748–9
 perfect capital markets 755–6
 Poisson process 746–7
 risk-free interest rate 751
 stochastic calculus 752
 stochastic integrals 753
 volatility 748
 Weiner process 746, 748, 752
 generalised 749–50
 standard 750
Stochastic volatility, describing 778
 Hull–White model 779
 Jump diffusion model 778
Stock exchanges
 geographical distribution 8
 list of, world 13
Straddles 817

Straight line interpolation 31
Strangle 818–9
Strips – see **Gilts**
Swap transaction 13
 and Eurobonds 395
Swaps 669, 705
 accreting and amortising swap 690
 asset-linked swaps 705–6
 basis swap 691
 cancelling 692
 constant maturity swap 690
 convexity bias 711
 calculating 714, 717
 in futures contracts 712
 impact of 715
 credit risk 696
 cross-currency swaps 669, 694
 defined 694
 market, growth in 671
 valuation 694
 defined 669
 differential swap 691
 discount function, deriving 701
 documentation, example 696–700
 electronic trading 716
 trading platforms, list of 717
 Eurocurrency swaps 709
 SwapsWire 716
 forward-start swap 691
 hedging 709
 using futures contracts 709–11
 interest, calculation of 673
 interest-rate swaps 669–71, 707
 defined 672
 forward rate from spot rate
 discount factors, calculating 684–5
 and forward-rate agreements 680
 and futures 712
 generic, pricing of 686–7
 generic portfolio 677
 hedging 707
 market, growth in 671
 non-vanilla 688
 pricing, principles summary 688
 swaptions 688
 valuation 688
 International Swap Dealers Association 669
 liability-linked swaps 705
 Libor-in-arrears swap 690
 margin swap 691
 marked-to-market 672
 netting 716

 off-market swap 691
 SONIA swaps 691–2
 swaptions 688, 692
 call swaption 692
 defined 692
 put swaption 692
 terminology, of market 673
 using 705
 floating-floating currency swap 706–7
 floating-rate asset to fixed-rate 706
 floating-rate liability to fixed-rate 705–6
 valuation, of generic swap 680–1
 vanilla swap, example 674–7
 warehousing 669
 yield curve 678–9
 zero-coupon swap 681–2, 684, 690
 discount factors 684
 discount function 682–3
 pricing 681–2
 valuation 693
SwapsWire 716
Swaptions 688
 call swaption 692
 defined 692
 put swaption 692
 valuation 693
Sydney Futures Exchange 296
Synthetic coupon pass-throughs 463

T

T-bills – see **Treasury bill**
Targeted amortisation class 462
Tate & Lyle, warrants example 396
Tax deductibility, of bond dividends 9
Taxation, and Eurobonds 394
Taxation, and gilts 215
Taxation, and the repo market 563
Taylor's expansion, of the price/yield function 192
Taylor's expansion, and value-at-risk 701
Technical analysis – see **Analysis**
Technical default 320
Term structure 902
Term structure of interest rates 102
Term to maturity
 altering 4
 corporate bonds 325
 defined 4
 and mortgage-backed bonds 444
 and volatility 5
The Federal Reserve 258
The Wall Street Journal 258

Theta, of an option 805
Tick, meaning of 45
Time deposit 507
Time horizon, and credit risk analysis (VaR) 648
Time value of money 20
 calculating yield 21
 discount factors 21
 calculating from prices of government bonds 22–3
 with compound interest 22
 discount rate 22
 with simple interest 22
 future value 20
 of multiple cash flows 23, 26
 internal rate of return 29
 breakeven rate 31
 net present value 29
 pension contributions, calculating 25
 present value 20–1
 of multiple cash flows 26
 multiple discounting 23
 single payments 20
Time-weighted rate of return 107
Tokyo Stock Exchange 294
TOPIX 35
Total return swap 1030, 1053
Tracker index 972
Traders, defined 7
Trading, approaches to 847–9
Trading clean 45
Trading flat 45
Treasuries 11
Treasury bill 213, 263–4, 507, 513
 bond equivalent yield 514
 Federal Funds 514
 prime rate 514
 risk free yield 513
 US Treasury Bills 514
Treasury bonds, compared to municipal bonds 427
Treasury gilts 5
Treasury notes 257
Treasury strips 264
Trust services, and Eurobonds 388–90

U

UK gilts, yield and sensitivity analysis 198
UK national debt 203
 growth in 204
 history of 203
Unbiased expectations hypothesis 112, 115
 testing 148

Undated bonds 27
 pricing of 43
 3.5% War Loan 27
Unexpectedly long-end duration 196
United States repo market 593
United States government bonds 11
US Treasuries, cum dividend 47
US Treasury 257
 Monthly Statement of the Public Debt 257, 259
 Separate Trading of Registered Interest and Principal Securities programme 265
 The Federal Reserve 258
 yield curve 267–9
US Treasury Bills 514
US Treasury bonds 257
 auction calendar 262
 derivatives 269
 Chicago Board of Trade (CBOT) 269
 Chicago Mercantile Exchange (CME) 269
 Treasury futures 269
 Federal Agency Bonds 267
 federally-related institutions 267
 government sponsored enterprises 267
 inflation-protected 265
 market developments 266
 market structure 266
 list of 274
 market conventions 261
 price quotations 260
 primary market 261
 repo market 266
 Federal Funds interest rate 266
 general collateral 266
 secondary market 262
 Treasury 6% 2009 263
 yield analysis 263
 Treasury bills 263–4
 discount rate 264
 Treasury bonds 257
 Treasury futures 269
 conversion factor 269–70
 Treasury notes 257
 Treasury strips 264–5

V

Value date 43, 45
Value line 35
Value-at-Risk 625, 661
 calculation, methods of 626

example 628
 historical simulation 626
 Monte Carlo simulation 626–7
 correlation 627
 credit risk 645
 applications of 649
 CreditMetrics 647
 default, probability of 646
 modelling of 647
 portfolio approach 647
 time horizon 648
 critique of 661, 663
 confidence intervals 665
 liquidity risk 664
 normal distribution 665
 RiskMetrics 664
 defined 625
 derivatives 640
 convexity 640–1
 delta-gamma VaR 640, 642
 and duration-based risk
 measurement, compared 663
 fixed interest instruments 636
 bond portfolio 637
 forward-rate agreements 638
 GARCH model 634
 historical methodology 635
 interest-rate risk 661
 risk exposure, interpreting 663
 sensitivity, of interest rates 663
 interest-rate swaps 639–40
 and net present value 547
 normal distribution 626, 654
 approximation of returns 636
 assumption of 654
 tables 655
 portfolio volatility 635
 RiskMetrics 625, 637
 exponentially-weighted moving average model 633
 grid points 631
 simulation methods 635
 Monte Carlo 636
 and portfolio volatility 635
 stress testing 643
 statistical concepts 650
 stress testing, of models 643–4
 variance-covariance 628
 autocorrelation 632
 calculation of 628–9
 confidence intervals 632
 equally-weighted moving average 633
 mapping 630, 632

- portfolio volatility* 635
- variance matrix* 629
- volatility 628
 - autocorrelation* 632
 - capturing* 632
 - equally-weighted moving average* 633
 - exponentially-weighted moving average* 633
 - generalised autoregressive conditional heteroscedasticity* 634
 - ghost feature* 633
 - implied volatility* 632
 - portfolio* 635
- Vanilla bonds** 53, 105, 155
- Variable-rate note** 12, 75, 86
- Variable-rate demand obligation (VRDO)** 432
- Vasicek model** 794, 796, 868, 876
- Vega, of an option** 806
- Volatility** 7
 - Barone-Adesi and Whaley model 774–5
 - and binomial valuation model 371
 - and the Black–Scholes model 774
 - critique of 773–4
 - and derivatives pricing 763
 - of embedded options 349
 - example calculation 769
 - Hull–White model 774
 - implied, and value-at-risk analysis 628
 - implied volatility 166, 179
 - and interest-rate options 772
 - level, and convertible bond valuation 372
 - and modified duration 169
 - of options, 771
 - pricing derivatives 764–6
 - assumptions* 764
 - Monte Carlo simulation* 770
 - and price/yield relationship 157
 - smile curve 809
 - as a spreadsheet 770
 - in Microsoft Excel* 784
 - and stochastic process modelling 748
 - and term to maturity 5
 - weakness of modified duration 183
 - weighting and hedging 179
 - zero-coupon bond 771
- Volatility trades** 816
 - butterfly spreads 816
 - ratio spreads 819–20
- straddles 817–8
- strangle 818
- W**
- Waggoner model** 907
- 3.5% War Loan** 27, 43, 206, 213
 - present value of 27
- Warrants** 12, 396
 - analysis 396–7
 - bond warrants 397–8
 - and convertibles, compared 398
 - defined 396
 - equity warrant 396
 - host bond 396
 - Tate & Lyle 396
- Weiner process** 746–8, 752, 868
 - generalised 749–50
 - and Heath–Jarrow–Morton model 889
 - and interest rate models 879
- World bond markets** 8
 - France 9
 - Germany 9
 - growth in, since 1970s 9
 - Japanese 9
 - relative sizes of 9
 - UK 9
 - US Treasury securities 9, 11
- X–Z**
- Yankee bonds** 11, 379
- Yield**
 - annualised, calculating 63–4
 - of asset-backed bonds 93
 - bond equivalent yield 64
 - bond portfolio 76
 - bond price, relationship to 48
 - calculating zero-coupon 108
 - calculations, German 283
 - of certificates of deposit 510
 - of commercial paper 416
 - corporate bonds 321
 - coupon, relationship to 49
 - current yield, formula for 54
 - defined 54
 - determinants of 51
 - effect base rate or minimum lending rate* 51
 - Gross Domestic Product* 51
 - Gross National Product* 51
 - embedded options, calculation of yield 65–6
 - first call 67
 - flat yield 54
- floating-rate bonds 75
 - and LIBBOR* 75
- forward yield 110
- gilts 208
- high-yield bonds 493
- holding-period 65
- index-linked bonds 71
- interest yield 54
- of long-dated bond 926
- main quoted, and government bonds 51
- measures, comparison of 59
- modifying 61
 - by discounting the cash flow* 62
- money market vs. bond market 62
 - making comparisons* 62
- Moosmüller 62
- of municipal bonds 429
- negative carry 55
- negative funding 55
- next call 67
- operative 67
- par yield 60
- portfolio 970
 - approximating* 970–1
 - managing* 969
- putable bonds 67
- running cost 54
- running yield 54–5
- of short-date final coupon bond 62
- to average life 67
- to first call 67
- and volatility 157
- of zero-coupon bond 60
- Yield curve** 102, 194, 304, 661, 851–2, 866
 - analysing and interpreting 114, 124
 - Expectations Hypothesis* 115
 - liquidity preference theory* 118
 - money substitute hypothesis* 119
 - preferred habitat theory* 120
 - Segmentation Hypothesis* 119–20
 - annuity 114
 - basic shape of 114
 - combined theory* 120
 - downward sloping or inverted or negative* 114
 - explanations for* 115, 122–3
 - flat yield curve* 121
 - humped* 114, 118, 120
 - normal or conventional* 114
 - upward sloping or positive or rising* 114
 - changes to 194

- flattening* 195
- main types* 195
- parallel change* 194
- parallel shift* 194
- and portfolio duration* 195
- steepening* 195
- twists* 195
- convexity, and value-at-risk analysis 640
- coupon yield curve 104
- curve modelling 126
- curve-fitting techniques
 - James and Webber* 905
 - Nelson–Siegel model* 905–6
 - parametric* 904–5, 915
 - spline-based* 905, 907, 915
 - Svensson parametric model* 911
- definition of 15
- dynamics of, introduction to 865
- expectations hypothesis 115
- factors influencing 866
- fitting data 901
 - Anderson–Sleath model* 909–10
 - calibration* 901, 909
 - cubic spline method* 907–9
 - curve functions, estimating* 904
 - kernel approximations* 901
 - linear programming* 901
 - spot yield curve* 902
 - zero-coupon yield curve* 901
- forecasting and portfolio management 948
- forward yield 110, 137
- of government bonds 851–2
 - Australian* 296
 - Canadian* 300
 - French* 289
 - German* 283
 - Hungarian* 301
 - Italian* 286
 - New Zealand* 298
 - South African* 303
- humped curve 195
- Jensen’s inequality 149
- modelling 127
 - approaches to* 867
 - cubic splines* 128
 - interpolation* 127
 - logarithmic interpolation* 127
 - polynomial models* 127–8
- par yield curve 104–5, 137
- and the perfect market model 122
- regression models 129
- and repo markets 568, 572–3
- spot 137
- and stochastic interest rate
 - behaviour 121
- and swaps 678–9
- and trading basis 610
- UK gilt curve 123–6
- US treasury 267
- uses of 102–3
- and value-at-risk 663
- yield to maturity curve 103–4
- zero-coupon yield curves 105–7
 - fitting data to* 901–2
- Yield spread** 51, 855
 - bond spread weighting 855
 - bond spreads, types of 856
- Yield to call** 95
- Yield to equivalent life** 70
- Yield to first call** 67
- Yield to maturity** 53, 55, 102
 - calculating 56
 - consortium yield 57
 - and internal rate of return 31, 56–7
 - for semi-annual coupon bond 56–7
 - simple yield 55
- Yield to next call** 67
- Yield to par** 60
- Yield-to-average life** 68, 83
 - average redemption price 69
 - sample spreadsheet calculation 83
- Yield-to-maturity expectations hypothesis** 115
- Yield-to-worst** 339
- Z-class bonds** 462
- Zero-coupon bond** 5, 12
 - and Eurobonds 381
 - and municipal bonds 427
 - discount bonds 12
- Zero-coupon convertible** 357
- Zero-coupon discount factors** 107
 - example calculation 108
 - spot yields 107, 109
- Zero-coupon indexed bonds** 99
- Zero-coupon rate** 131
 - forward 112
 - and spot rate 105
- Zero-coupon (spot) rate curve**
 - deriving theoretical 144–6
- Zero-coupon swap** 681, 690
- zero-coupon yield curves** 105
- Zero-coupon yields** 108
- Zero-volatility spread** 109